AMATH 422 HW 3

Question 1

In [1]:
```
#imports
import matplotlib.pylab as plt
%matplotlib inline
import numpy as np
import scipy.optimize as opt
import scipy.linalg as la
import numpy as np
from scipy.linalg import eig
import sys
np.set_printoptions(threshold=sys.maxsize)
```

In [2]:
```
#A matrix from email.
age_max = 50
A_mat = np.zeros([age_max+1,age_max+1])
A_mat[0,3:age_max+1] = 0.24

for i in np.arange(0,3):
    A_mat[i+1,i] = 0.0722**(1/3)

for i in np.arange(3,age_max):
    A_mat[i+1,i] = 0.942

#print(A_mat)
print(A_mat.shape)
```

```
(51, 51)
```

In [3]:
```
eigvals, eigvecs = eig(A_mat)
dominant_eigval = np.real(eigvals[0])
w = np.real(eigvecs[:, 0])
w = w / np.sum(w)  # Normalize the right eigenvector

# Calculate the left eigenvector (v)
v = np.real(eig(np.transpose(A_mat))[1][:, 0])
v = v / (v @ w)  # Normalize left eigenvector so that v^T w = 1

# Calculate the elasticity matrix
elasticity_matrix = np.zeros_like(A_mat)
for i in range(age_max):
    for j in range(age_max):
        if A_mat[i, j] != 0:  # Only calculate for non-zero entries
            elasticity_matrix[i, j] = (A_mat[i, j] / dominant_eigval) * (v[i] * w[j])


print(f"left eigenvectors: {v}")
print(f"right eigenvectors: {w}")
print(f"elasticity matrix shape: {elasticity_matrix.shape}")
#print(f"elasticity_matrix: {elasticity_matrix}")
```

```
left eigenvectors: [0.20537054 0.46553003 1.05525459 2.39203095 2.34450964 2.29689293
 2.24918061 2.2013725  2.15346841 2.10546815 2.05737152 2.00917832
 1.96088837 1.91250147 1.86401743 1.81543604 1.76675712 1.71798047
 1.66910589 1.62013319 1.57106217 1.52189263 1.47262437 1.4232572
 1.37379092 1.32422533 1.27456022 1.2247954  1.17493067 1.12496584
 1.07490068 1.02473502 0.97446864 0.92410134 0.87363292 0.82306317
 0.7723919  0.7216189  0.67074396 0.61976688 0.56868746 0.51750549
 0.46622076 0.41483307 0.36334221 0.31174797 0.26005015 0.20824854
 0.15634293 0.10433311 0.05221887]
right eigenvectors: [0.17957669 0.07922102 0.03494869 0.01541776 0.01538687 0.01535604
 0.01532527 0.01529457 0.01526392 0.01523334 0.01520282 0.01517235
 0.01514195 0.01511162 0.01508134 0.01505112 0.01502096 0.01499087
 0.01496083 0.01493085 0.01490094 0.01487108 0.01484129 0.01481155
 0.01478187 0.01475225 0.0147227  0.0146932  0.01466376 0.01463438
 0.01460505 0.01457579 0.01454659 0.01451744 0.01448835 0.01445932
 0.01443035 0.01440144 0.01437258 0.01434379 0.01431505 0.01428636
 0.01425774 0.01422917 0.01420066 0.01417221 0.01414381 0.01411547
 0.01408719 0.01405896 0.0140308 ]
elasticity matrix shape: (51, 51)
```

In [4]:
```
fecundity_elasticities = elasticity_matrix[0, 3:age_max + 1]  # Elasticities for fecundity
survival_elasticities = elasticity_matrix[np.arange(1, 51), np.arange(50)]

print("\nFecundity Elasticities (ages 3 to age_max):")
print(fecundity_elasticities)

print("\nSurvival Elasticities (ages 0 to age_max-1):")
print(survival_elasticities)
```

```
Fecundity Elasticities (ages 3 to age_max):
[0.0008051  0.00080348 0.00080188 0.00080027 0.00079866 0.00079706
 0.00079547 0.00079387 0.00079228 0.0007907  0.00078911 0.00078753
 0.00078595 0.00078438 0.00078281 0.00078124 0.00077967 0.00077811
 0.00077655 0.000775   0.00077344 0.00077189 0.00077035 0.0007688
 0.00076726 0.00076572 0.00076419 0.00076266 0.00076113 0.00075961
 0.00075808 0.00075657 0.00075505 0.00075354 0.00075203 0.00075052
 0.00074902 0.00074752 0.00074602 0.00074452 0.00074303 0.00074154
 0.00074006 0.00073857 0.00073709 0.00073562 0.00073414 0.        ]

Survival Elasticities (ages 0 to age_max-1):
[0.03687976 0.03687976 0.03687976 0.03607467 0.03527118 0.03446931
 0.03366904 0.03287037 0.03207331 0.03127784 0.03048397 0.02969168
 0.02890099 0.02811188 0.02732435 0.02653839 0.02575402 0.02497121
 0.02418997 0.0234103  0.02263219 0.02185564 0.02108064 0.0203072
 0.01953531 0.01876496 0.01799616 0.0172289  0.01646317 0.01569898
 0.01493632 0.01417519 0.01341559 0.0126575  0.01190094 0.01114589
 0.01039235 0.00964032 0.0088898  0.00814079 0.00739327 0.00664725
 0.00590273 0.0051597  0.00441816 0.0036781  0.00293953 0.00220243
 0.00146682 0.        ]
```

Question:

(a) Is the elasticity for fecundity values f_a the same for all ages a? (b) Is the elasticity for annual survival probabilities values p_a the same for all ages a? (c) Give an intuitive explanation for your findings in two to three sentences, and state one possible implication for management plan

Answer:

(a) No, the elasticity for the fecundity values of f_a is not the same for all age of a. According to my calculations the elasticity values of the fecundity decreases as owls get older. The intuitive explanation for this phenomena is that the younger owls are more vulnerable to changes (elastic) in the ability to produce new offspring than the older owls. Which makes sense since older owls probably produce substantially less owls each than the younger owls, meaning older owls are not changed as much by external factors.

(b) No the annual survival probability values p_a are not the same for all ages a. According to my calculations the annual survival probabilties of the fecundity values decreases as owls get older. This may be because of the varying contributions of the younger age class to the population growth rate (reflected in the eigenvectors). Meaning the survival rate may be the same for each age, but the amount of owls that are in each age contributes to the population growth rate differently. The younger owls contribute to the population more than the older owls because it is more likely that they are alive.

(c) One possible solutions for management plans would be to focus efforts on the age classes that contribute the most to population growth. This can be done by enhancing the survival rates of the younger age classes through some engineered device and ensuring reproductive success.

In [5]:
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print(A_mat)
print(f"elasticity_matrix: {elasticity_matrix}")
```

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[0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
```

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 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.942 0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.   ]
[0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.942 0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.   ]
[0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.942
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.   ]
[0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.942 0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.   ]
[0.    0.    0.    0.    0.    0.
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 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.942 0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.   ]
[0.    0.    0.    0.    0.    0.
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 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.942 0.    0.
 0.    0.    0.    0.    0.    0.
 0.    0.    0.   ]
[0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.        ]
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.942
  0.          0.          0.          0.          0.          0.
  0.          0.          0.        ]
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.942       0.          0.          0.          0.          0.
  0.          0.          0.        ]
 [0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.942       0.          0.          0.          0.
  0.          0.          0.        ]
 [0.          0.          0.          0.          0.          0.
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.942       0.          0.          0.
  0.          0.          0.        ]
 [0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.942       0.          0.
  0.          0.          0.        ]
 [0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
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  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.942       0.
  0.          0.          0.        ]
 [0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.
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  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.942
  0.         0.         0.        ]
 [0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.942      0.         0.        ]
 [0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.942      0.        ]]
elasticity_matrix: [[0.         0.         0.         0.0008051  0.00080348 0.00080188
  0.00080027 0.00079866 0.00079706 0.00079547 0.00079387 0.00079228
  0.0007907  0.00078911 0.00078753 0.00078595 0.00078438 0.00078281
  0.00078124 0.00077967 0.00077811 0.00077655 0.000775   0.00077344
  0.00077189 0.00077035 0.0007688  0.00076726 0.00076572 0.00076419
  0.00076266 0.00076113 0.00075961 0.00075808 0.00075657 0.00075505
  0.00075354 0.00075203 0.00075052 0.00074902 0.00074752 0.00074602
  0.00074452 0.00074303 0.00074154 0.00074006 0.00073857 0.00073709
  0.00073562 0.00073414 0.        ]
 [0.03687976 0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.        ]
 [0.         0.03687976 0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.        ]
 [0.         0.         0.03687976 0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.        ]
 [0.         0.         0.         0.03607467 0.         0.
```

```
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.03527118 0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.03446931
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
 0.03366904 0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
 0.         0.03287037 0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
 0.         0.         0.03207331 0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
```

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[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.        ]
[0.         0.         0.         0.         0.         0.
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 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.01493632 0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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[0.         0.         0.         0.         0.         0.
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