CS512

AS3

Khalid Saifullah
A20423546

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1.a) Problem with using the slope and y-intercept as line parameters is that the range of possible values for these parameters is infinite. This can lead to a large parameter space, which can be computationally expensive to search.

b) When using the polar representation of lines in the Hough transform, each point in the image space votes for a sinusoidal curve in the parameter space.

d) Lines are detected in the Hough transform by identifying the intersections of the sinusoidal curves generated by the votes of edge points in the parameter space. These intersections represent the parameters of the lines that are most likely to be present in the image.

e) In general, if the desired accuracy of the Hough transform is high, a smaller bin size should be used, even if it results in longer processing times and higher memory usage. Conversely, if the desired accuracy is lower and efficiency is a concern, a larger bin size can be used to reduce processing times and memory requirements.

f) Instead of searching for lines in a full range of possible theta values which is typically from 0 to 180 degrees, θ can be $\theta_n \pm \Delta\theta$.

g) 3 dimensions: circle center and radius

h) Implicit line equation: $x\cos\theta + y\sin\theta - d = 0$

$$y\sin\theta = -x\cos\theta + d$$

$$y = -\frac{\cos\theta}{\sin\theta}x + \frac{d}{\sin\theta}$$

[Because $\theta \in [45°, 135°]$
$\therefore \sin\theta \neq 0$]

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The reason why we scan x and compute y for nearly horizontal line:

i) It can ~~do~~ avoid zero denominator because $\cos(90) = 0$ in denominator otherwise

ii) The slope of the line is very small, which means that changes in y have ~~a~~ a smaller effect on the value of the left-hand side of the equation than changes in x. This leads to gaps in the representation of the line in raster graphics.

i)

$$x\cos\theta + y\sin\theta = d$$

$$x = -\frac{\sin\theta}{\cos\theta}\, y + \frac{d}{\cos\theta}$$

$\therefore \cos 0^\circ = 1$ & $\sin 0^\circ = 0$ since $\theta \in [-45^\circ, 45^\circ]$

If we scan x, the y-value of the line will change by more than 1. This leads to gaps in the representation of the line in raster graphics, ~~incremen~~ since the increments, or the jumps, in the y-values are larger than 1.

c)

$$x\cos\theta + y\sin\theta = d$$

$\cancel{1\cos\theta}$; $1\cos(0^\circ) + 1\sin(0^\circ) = d$

$$\therefore \underbrace{d = 1}_{\text{vote}}$$

2 a) $y = ax + b$ cannot model vertical or near vertical lines because the slope would have to be infinite. When choosing a model for line fitting, it's important to consider a model that can accurately describe all possible observations, not just a subset of them.

c) $m = \frac{20-10}{20-10} = \frac{10}{10} = 1$

So, $y = x$

$\therefore x - y = 0 \longrightarrow$ implicit eq^n

And,

normalized normal vector $[1/\sqrt{2}, -1/\sqrt{2}]^T$

e) ~~d)~~ $ax + by + c = 0$

$$y = \frac{-ax - c}{b}$$

$$= \frac{-1 \times 2 - 3}{2}$$

$$= -2.5$$

f) Implicit Equation: $ax + by + c = l^T x = 0$

we need to solve:

$$\begin{cases} E(l) = l^T S l \\ l^* = \arg\min_{\{l\}} E(l) \end{cases}$$

where

$$S = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & n \end{bmatrix}$$

$$E(l) = l^T S l$$

Now, $\nabla E(l) = 0$

$$S l = 0$$

The solution l^* is the eigen vector of S belonging to zero/smallest eigenvalue.

g) $S = \sum_{i=1}^{n} P_i P_i^T = D^T D$ where $D = \begin{bmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 6 & 1 \end{bmatrix}$

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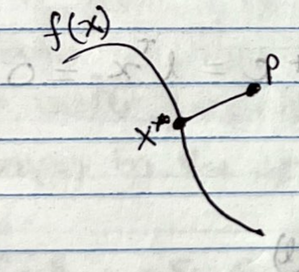
$$\therefore S = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$$

(h) The geometric distance between two points in space is the length of the shortest path connecting them. On the other hand, the ~~geometric distance between two~~ algebraic distance between two points is simply the absolute value of the ~~different~~ difference between their coordinates in each dimension.

i) 

closest point on the curve

arbitrary point

$$d(P, f) = |P - X^*| = \frac{|f(P)|}{|\nabla f(X^*)|}$$

geometric distance

$$\cong \frac{|f(P)|}{|\nabla f(P)|}$$ algebraic distance [approximation]

Reduce the algebraic distance for points with large ~~gra~~ gradients $|\nabla f(P)|$. The reason for approximation is that we don't know X^* on the curve.

j) $$d(P, f) = \frac{|f(P)|}{|\nabla f(P)|}$$

$$= \frac{1}{2}$$

Algebraic distance 1.

(k) $d(p, f) = \dfrac{|f(p)|}{|\nabla f(p)|}$

$= \dfrac{1}{2}$

(l) $\cancel{E(\varphi)}$ $E(\varphi(s)) = \int (\alpha E_{continuity} + \beta E_{curvature} + \gamma E_{image})\, ds$

where $E_{continuity} = \left|\dfrac{\partial \varphi}{\partial s}\right|^2$, $E_{curvature} = \left|\dfrac{\partial^2 \varphi}{\partial s^2}\right|^2$;

$E_{image} = \Sigma - |\nabla I(P_i)|^2$

(m) $P_1 = (1, 2)$, $P_2 = (2, 3)$ and $P_3 = (3, 4)$

At point P_2,

$E_{continuity} = |P_2 - P_1|^2 = |(1,1)|^2 = \left(\sqrt{1^2 + 1^2}\right)^2 = 2$

$E_{curvature} = |(P_3 - P_2) - (P_2 - P_1)|^2 = |(1,1) - (1,1)|^2 = 0$

(n) $\beta = 0$

3.(a) Outliers are datapoints that lie far away from the ~~m~~ majority of other data points in a dataset. The fundamental ~~poin~~ problem associated with outliers when fitting a models is that they can disproportionately affect the results of the analysis, leading to inaccurate or unreliable conclusions.

(b) $E(\theta) = \sum_{i=1}^{n} \xi_\sigma(d(x_i;\theta))$

In robust estimation, the function gives low weight for high value outlier, however the least squares objective function gives ~~k~~ higher weight for high value outlier.

(c) The Geman-McClure function is defined as,

$$\xi_\sigma(x) = \frac{x^2}{x^2+\sigma^2}$$

The advantage is that we can control the loss function and the weight of the outliers has upper bound which makes it more robust. Intially σ is set to a large value and it is gradually decreased with convergence

$$\sigma^{(n)} = 1.5 \times \text{median}\{d(x_i;\theta^{(n-1)})\}$$

(d) $\xi_\sigma(x) = \frac{1}{1+1} = \frac{1}{2}$

(e) The RANSAC (Random Sample Consensus) algorithm is a robust method for fitting models to data in the presence of outliers. The basic principle of the algorithm is to randomly select a subset of the data points, fit a model to this subset, and then use the model to identify inliers and outliers. ~~(data points that do not fit the model well)~~ The process is then repeated a number of times to obtain ~~a~~ the best model that fits the inliers.

The number of points drawn at each attempt should be small. Small number of points drawn can avoid including more outliers.

(f) Parameter of RANSAC algorithm :

n = no. of points at each evaluation
d = minimum no. of points needed
t = threshold to identify inliers
k = no. of trials

Equation : $k = \dfrac{\log(1-p)}{\log(1-w^n)}$

(g) The number of experiments, $k = \dfrac{\log(1-p)}{\log(1-w^n)}$

$$= \frac{\log(1-0.99)}{\log(1-0.9^n)}$$

n	k
1	1.99
2	2.77
3	3.53
4	4.31
5	5.16