CS512

AS1

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4. (a) Light in the scene is called surface radiance and light in the image is called image irradiance.

Surface radiance is how much light is radiated ~~reflected~~ from the surface because of reflection. Image irradiance is how much light is collected at the image sensor.

(b) $E(p) = L(p) \frac{\pi}{4} \left(\frac{d}{f}\right)^2 (\cos \alpha)^4$

where,

$E(p)$ = Image irradiance

$L(p)$ = Surface radiance

d = diameter of lens

f = focal length

α = angle between principal axis and surface normal.

(c) $I_{ref} = I \cdot \rho \cdot \cos\theta$

I_{ref}: Intensity of reflection

I: Intensity of source

ρ: Surface albedo $\in [0,1]$ reflection coefficient

θ: angle of reflection

Surface albedo is a measure of how well a surface reflects light.

(d) The RGB color model was used to represent colors in the visible spectrum in order to match the colors perceived by human vision.

(e) (0,0,0) refers to black and (1,1,1) refers to white. Connecting them would give the gray axis.

(f) ~~RGB colors are first transformed into CIE. XYZ system and then~~
RGB colors are mapped to real world colors by linear transformations (i.e. by multiplying with matrices).
The 3 curves for RGB correspond to the values of red, green and blue at any given wavelength of light. The linear Transformation matrices were derived through trial and error in order to match a known color.

(g) Luminance (Y) sets the intensity. ~~in an analog TV~~ The advantage is that if we have a TV set that can show only Black and white, it will only display the luminance.

(h) Lab color space is a non linear color conversion method where euclidean distance in Lab space correspond to human perception.

1. ~~(a)~~ (c) The bigger the focal length is, the bigger the image will be. When the distance to the object gets bigger, the image gets smaller.

(d) ~~(1,1) 2D →~~ $(1,1)_{2D} \rightarrow (1,1,1)_{2DH}$
Another 2DH corresponding to the same 2D point is $(2,2,2)_{2DH}$

(e) $(0.5, 0.5)_{2D}$

(f) The point (1, 1, 0) represents direction.

(g) ~~Converting cartesian coordinates to homogenous coordinate system can~~ Multiplying by a scaler, α, to the homogenous coordinates temporarily postpones the non-linearity until we convert it back to cartesian coordinate system.

(h) M is a 3X4 projection matrix and K and I have 3x3 dimensions and O has 3x1 dimensions.

(i) $\cancel{p = MP}$

$$p = MP$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}_{3 \times 4} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}_{4 \times 1}$$

$$= \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

The 2D point is $(1.8, 4.6)$.

(a) $u = f \frac{x}{z} = 10 \cdot \frac{3}{1} = 30$

$v = f \frac{y}{z} = 10 \frac{2}{1} = 20$

So, the image point is $(30, 20)$.

(b)

2. (a)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \underset{3\times3}{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

So, the coordinates are $(3,4)$.

(b)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

So, the coordinates are $(2,2)$.

(c)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1.41 \\ 1 \end{bmatrix}$$

So, the coordinates are $(0, 1.41)$

(d) $R_{p,u}(\theta) = T(p)\, R_u(\theta)\, T(-p)$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1.41 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0.59 \\ 1 \end{bmatrix}$$

(e) TR

(f) It will scale the ~~axi~~ x-axis 3 times and the y-axis 2 times

(g) It will translate the point p by $(1,2)$.

(h) Need to multiply the coordinates by the inverse of M.

$$M^{-1} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) $M = R(45)\, T(1,2)$

$$M^{-1} = \left(R(45)\, T(1,2)\right)^{-1}$$

$$= T^{-1}(1,2)\, R^{-1}(45)$$

$$= T(-1,-2)\, R(-45)$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) & 0 \\ \sin(-45^\circ) & \cos(-45^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.71 & 0.71 & -1 \\ -0.71 & 0.71 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

(j) $\vec{u} = (1,3)$
$\vec{v} = (a, b)$

$\vec{u}.\vec{v} = a + 3b = 0$

$a = 3$
$b = -1$

So, vector perpendicular is $(3,-1)$

(k) $\vec{u} = (1, 3)$
$\vec{v} = (2,5)$

Projection of $\vec{u}$ in the direction of $\vec{v} = \dfrac{1 \times 2 + 3 \times 5}{\sqrt{2^2 + 5^2}}$

$= \dfrac{17}{\sqrt{29}}$

3. (a) To account for the ~~angle~~ field of view of the camera. We know that by changing the focal length of a zoom lens on a real camera, we can change how much we see of a scene (the extent of the scene). We want our camera to work in the same way.

(b) TR

(c) $p = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix}$

$\cancel{P = R\ 0\ 0}$

$\cancel{0\ R}$

(d) $R^* = R^T \rightarrow$ R transpose
$T^* = -R^T t \rightarrow$ coordinate of translation

(e) $M_{i \leftarrow c} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & 512 \\ 0 & k_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$

(f) K^* is intrinsic and $[R^*|T^*]$ is extrinsic parameters of the camera.

(g) Skew parameters are given out during camera calibration as the camera has some small amount of inherent skewness. ~~This~~ Setting this parameter helps to account for that.

(h) To account for radial lens distortion, we need to multiply the transformation matrix by another matrix having a scale factor. As the distance from centre increases, the radial lens distortion also increases. The complication is that we need to know the ~~Lin~~ distortion coefficients in order to calculate. We need to warp the image to correct for distortion.

~~(i)~~

(i) Weak perspective camera refers to the ~~mod~~ situation when the observed object is far from the camera relative to its size. An affine camera is a linear mathematical model to approximate the perspective projection followed by an ideal pinhole camera.