CS577 – Homework 0

Solution

A.

1. $2A - B = [-2 \quad -1 \quad 0]'$

2. The unit vector in the direction of A is: $\widehat{A} = \frac{1}{\sqrt{14}}\begin{bmatrix}1\\2\\3\end{bmatrix}$

3. $|A| = \sqrt{14}$. The Angle of A relative to positive X axis is: $\arccos(\frac{1}{\sqrt{14}})$

4. The direction cosines of A are: $(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}})$

5. The angle between A and B is: $\arccos(\frac{A \cdot B}{|A||B|}) = \arccos(\frac{32}{\sqrt{14 \cdot 77}})$

6. $A \cdot B = B \cdot A = 4 + 10 + 18 = 32$

7. $A \cdot B = |A| \cdot |B| \cdot \cos\theta = \sqrt{14} \cdot \sqrt{77} \cdot \frac{32}{\sqrt{14 \cdot 77}}) = 32$

8. $s = \frac{B \cdot \hat{A}}{|\hat{A}|} = B \cdot \hat{A} = \frac{32}{\sqrt{14}}$

9. $[x \;\; y \;\; z]\begin{bmatrix}1\\2\\3\end{bmatrix} = 0 \quad \Rightarrow \quad \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$

10. $A \times B = \begin{bmatrix}-3\\6\\-3\end{bmatrix} \qquad B \times A = -A \times B = \begin{bmatrix}3\\-6\\3\end{bmatrix}$

11. A vector perpendicular to A and B is: $A \times B = \begin{bmatrix}-3\\6\\-3\end{bmatrix}$.

12. $aA + bB = cC = 0 \quad \Rightarrow \quad 3A - B - C = 0$

13. $A^T B = 32, AB^T = \begin{bmatrix}4 & 5 & 6\\8 & 10 & 12\\12 & 15 & 18\end{bmatrix}$

B.

1. $2A - B = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$

2. $AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$ $BA = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$

3. $(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$ $\quad B^T A^T = (AB)^T$

4. $|A| = 55$. Because of A–10 we get: $|C| = 0$.

5. The matrix in which the row vectors form an orthogonal set is the matrix B.

6. $A^{-1} = \frac{1}{55}\begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$. Because of B–5 we get: $B^{-1} = \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 2/6 & 1/21 & -2/14 \\ 1/6 & -4/21 & 1/14 \end{bmatrix}$

7. $C^{-1} = \frac{adj(C)}{|C|}$, whereas |C| = 0. Thus C^{-1} does not exist.

8. $Ad = \begin{bmatrix} 14 \\ 9 \\ 7 \end{bmatrix}$

9. $a1 = \frac{A_1 \cdot d}{||d||^2} d = \frac{14}{14} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, a2 = $\frac{A_2 \cdot d}{||d||^2} d = \frac{9}{14} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9/14 \\ 18/14 \\ 27/14 \end{bmatrix}$,

   a3 = $\frac{A_3 \cdot d}{||d||^2} d = \frac{7}{14} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7/14 \\ 14/14 \\ 21/14 \end{bmatrix}$

10. $\begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} \cdot 1 + \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \cdot 2 + \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix} \cdot 3 = \begin{bmatrix} 14 \\ 9 \\ 7 \end{bmatrix}$

11. x = $B^{-1}d = \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 2/6 & 1/21 & -2/14 \\ 1/6 & -4/21 & 1/14 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

12. Since C can not be inverted, no solution for Cx = d.

C.

1. $\lambda_1 = -1, \lambda_2 = 4, e_1 = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}, e_2 = \begin{bmatrix} -0.555 \\ -0.832 \end{bmatrix}$
2. $e_1 \cdot e_2 = -0.196$
3. $e_1 \cdot e_2 = 0$
4. Since B is a symmetric real matrix, its eigenvectors are orthogonal.
5. x = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
6. x = $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$, x = $\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ (satisfy x_1 = -2x_2)
7. x = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, because the row of D is linearly independent.

D.

1. $f'(x) = 2x, \;\; f''(x) = 2$
2. $\frac{\partial g}{\partial x} = 2x, \;\; \frac{\partial g}{\partial y} = 2y$
3. $\nabla g(x, y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$
4. with chain rule: $\frac{d}{dx}\left(f\left(g(x)\right)\right) = \frac{d}{dg(x)} f\left(g(x)\right) \cdot \frac{d}{dx} g(x) = 2x^2 \cdot 2x = 4x^3$

   without chain rule: $\frac{d}{dx}\left(f\left(g(x)\right)\right) = \frac{d}{dx}(x^4 + 3) = 4x^3$