Homework 2

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Problem 1

As $n \to \infty$, the mass function

$$f_{T_n}(t) = (8n - 8n^2 t)\, I_{(1/2n,1/n)}(t)$$

approaches different values in its upper and lower bounds in t. At $t = 1/n$,

$$\lim_{n\to\infty}(8n - 8n^2 t) = \lim_{n\to\infty} 8n - \frac{8n^2}{n} = 0.$$

while at $t = 1/2n$,

$$\lim_{n\to\infty}(8n - 8n^2 t) = \lim_{n\to\infty} 8n - \frac{8n^2}{2n} = \lim_{n\to\infty} 4n = \infty.$$

Meanwhile the domain of t becomes $(1/2n, 1/n) = (0, 0)$. Thus the limiting distribution for f is a vertical line at $t = 0$, where the image of f is all non-negative real numbers.

Problem 2

For

$$f_{X_n}(x) = \begin{cases} \frac{1}{n}, & x = n^2, \\ \frac{n-1}{n}, & x = 0, \\ 0, & \text{otherwise.} \end{cases}$$

the moment-generating function is calculated by

$$M_X(t) = \sum_{x\in A} e^{tx} p(x) = e^{t\cdot 0}\left(\frac{n-1}{n}\right) + e^{tn^2}\left(\frac{1}{n}\right) = \frac{1}{n}\left(e^{tn^2} + n - 1\right).$$

To calculate the $E(X_n)$ we need the first t derivative of M_X,

$$M_X'(t) = \frac{d}{dt}\frac{1}{n}\left(e^{tn^2} + n - 1\right) = ne^{tn^2},$$

so that

$$E(X_n) = M_X'(0) = ne^{0\cdot n^2} = n.$$

For $T_n = X_n - E(X_n)$,

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} X_n - \lim_{n\to\infty} E(X_n).$$

$E(X) = n$ therefore $E(X)$ approaches infinity. For X_n, the probability that $x = n^2$ is $1/n$ and approaches zero. The probability that $x = 0$ approaches 1. Then

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} X_n - \lim_{n\to\infty} E(X_n) = 0 - \infty = -\infty.$$

Therefore, there is no limiting distribution for f.