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Chance Discovery and Analysis of Data via Multi-Agent Logics

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Abstract

We study applications of mathematical logic to Information Sciences in theirs particular part - Chance Discovery (which is a popular area in Knowledge Representation and CS). Main used tool is multi-agent logic based at modal-like temporal logic. In particular, we consider more thin case when the time is not supposed to be transitive. The semantics of our logical approach is based at relational models for modelling computational processes and analysis of databases (with incomplete information, for instance, with information forgotten in the past, etc). We assume that the agent's accessibility relations may have lacunas; agents may have no access to some potentially known and stored information. Satisfiability and decidability issues are in focus of research. We find algorithms solving satisfiability problem. Illustrating examples are given and application areas are suggested.

Keywords: chance discovery; multi-agent logics; many–valued logics; deciding algorithms;

1. Introduction

Chance discovery (CD) is a new popular area in Information Sciences and Knowledge Engineering Systems. The focus point there is the elicitation of the notion CD, modelling it by various mathematical and computational tools and usage CD in applications. An essential point is determining the significance of some piece of information about an event and then using this new knowledge in decision making. The area became popularity in last two decades (cf. Ohsawa and McBurney [5], Abe and Ohsawa [1])), that now form a solid direction in Artificial Intelligence (AI) it analyzes important events with uncertain information, incomplete past data, so to say, *chance* events, where *a chance* is defined as some event which is significant for decision-making in a specified domain.

The technique of CD is often focused to practical applications (cf. e.g. [1, 2, 6, 7, 8, 3, 4]). Though, some attention was already paid to theoretical research the notion of CD in mathematical terms via symbolic logic. Properties of

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CD in logical systems originating in AI (as well as some interpretations of CD-like operations via existence, without direct reference to CD) were already considered (cf. Rybakov [18] – [22]) In this approach we often consider CD as a various kind of logical operations.

In this our paper we will also use symbolic mathematical logic for analysis CD. Nowadays applications of non-standard logics for verification of computational processes in multi-thread environment is a popular area. A particulary important case is verification and modelling processes of reasoning and derivation information from collected reliable facts. That processes are non-monotonic by nature and collected and proven facts may be refuted and updated. In particular, for that reason, non-standard logics are often used for that kind of analyses. These logics often are modal-like or temporal ones or others close to epistemic logics.

They, in general, are devised to capture and represent defeasible reasoning when reasoners draw tentative conclusions, enabling reasoners to retract their conclusion based on further evidence (for origin the research in non-monotonic logic cf. e.g. W. Lukaszewicz and other authors in [12, 13, 14, 16]). Reasoning about knowledge is another good issue for applications non-monotonic logic. That logics include formulae that may mean that something is known or not known, therefore these logics should not be monotonic. These non-monotonic logics may be various multi-modal, temporal or agent's logics (when agent's knowledge logical operations are meant as special modal-like operations). These researches are well represented in publications of many authors, cf. [9, 11, 15, 17, 19, 20, 21, 23, 24, 25].

We absorb in a sense technique developed in these publications, though most technique we use here may be found in Rybakov [25, 26]. We develop technique to represent and analyze CD by logical tools. In particular, we consider more thin case with relational temporal models when the time is not supposed to be transitive. The relational models for modelling computational processes and analysis of databases represent the cases with incomplete information, for instance, with information forgotten in the past (we assume that the agent's accessibility relations may have lacunas; agents may have no access to some potentially known and stored information). Mathematical part of our research deals with computational algorithms; we provide illustrative examples and discuss the application areas.

2. Logical Language, Definitions, Examples

To make the paper easy readable we first recall all definitions and instruments from symbolic mathematical logic which are necessary for our research in CD. We use the language of temporal logic extended by agent's operations. So it consists of propositional letters P, operation $\mathbf{N}$ (next), operations $\mathbf{U}_i$ (until) for any agent i and the agent's knowledge operations $A_i, i \in Ag$ (where Ag is a fixed finite set of agents) which may be applied to only letters - for all $p \in P, A_i(p)$ is a formula. The formation rules for compound formulas are as always.

Any letter from P is a formula; the set of all formulas is closed w.r.t. applications of Boolean operations, the unary operation $\mathbf{N}$ (next) and the binary operations $\mathbf{U}_i$ (until by opinion of the agent i); as it is pointed above the operation A_i (agent i knows that it is true) may be applied only to propositional letters. For representation and modelling CD in multi-agent environment we will use the following Kripke-Hinttikka like models.

Definition 1. *An intransitive multi-agent frame is a tuple*

$$\mathcal{F} := \langle N, \leq, \text{Next}, \bigcup_{j\in N}[R_j]\rangle,$$

components of which are as follows. N denotes the set of all natural numbers and
(i) $N = \bigcup_{i\in In\subset N}[i, m_i]$ ($[i, m_i]$ is the interval of all natural numbers situated between i and m_i). The set In is a set of indexes - it is a subset of N;
(ii) $\forall i_1, i_2 \in In,\ i_1 \neq i_2 \Rightarrow (i_1, m_{i_1}) \cap (i_2, m_{i_2}) = \emptyset$;
(iii) $\forall i \in In\ \ (m_i > i)$; for any $j \in Ag$ (where Ag is the set of agents) any R_j is a subset of the standard linear order on the interval $[i, m_i]$; so – R_j the accessibility relation on the interval $[i, m_i]$ for the agent j;
(iv) Next *is the standard NEXT relation on N:* **n** Next **m** *if $m = n + 1$.*

It is clear that by the definition $\bigcup_{j\in N}[R_j]$ is not a transitive relation as well as all R_j are transitive only inside of the interval under consideration, and overall R_j nay be not transitive on whole the frame. Besides note that any R_j may have lacunas - places of states in the interval which are not accessible from the proceeding sates. For the sequel we fix notation: $t(i) := m_i$ - boundary of transitivity for i.

The multi-agent's models $\mathcal{M}$ on such frames $\mathcal{F}$ are defined by fixing valuations $V_i, i \in Ag, ||Ag|| < \infty$ for a set of letters P - agents valuations for truth of letters $p \in P$, i.e. $\forall i,\ \forall p \in P,\ V_i(p) \subseteq N$.

Ag is a set of indexes for agents, for each model it may be different (any model may have its own fixed agents, their quantity may be different). For all n, $n \in V_i(p)$ is interpreted as p is true at the state n by opinion of the agent i. Also we consider the agreed (global) valuation V for letters from P: –

$$V(p) = \{n \mid n \in N, ||\{i \mid i \in Ag, n \in V_i(p)\}|| > k\},$$

where k is a fixed rational number (for this given model), which is e.g. bigger than $||Ag||/2$.

That is k is the *threshold*, which shows that the number of the agents which are sure that p is true in the given state (world) is big enough, e.g. bidder than half. The particular value of k may vary from model to model - each one has its *own threshold*. That is k is the *threshold*, which shows that the number of the agents which are sure that p is true in the given state (world) is big enough, e.g. bidder than half. The particular value of k may vary from model to model - each one has its *own threshold*.

The question how to define the global opinion of the agents is actually a deep one. Ways to construct V out of all V_j may be different. E.g. we may consider

(I) $(\mathcal{M}, a) \Vdash_{V_0} p \ \Leftrightarrow$

$$||\{j \mid (\mathcal{M}, a) \Vdash_{V_j} p, j \neq 0\}|| > ||(\mathcal{M}, a) \nVdash_{V_j} p, j \neq 0\}||,$$

This means the majority of agents believe that p is true.

(II) $(\mathcal{M}, a) \Vdash_{V_0} p \ \Leftrightarrow$

$$||\{j \mid (\mathcal{M}, a) \Vdash_{V_j} p, j \neq 0\}|| \geq ||(\mathcal{M}, a) \nVdash_{V_j} p, j \neq 0\}||,$$

This would mean that p is plausible.

(III) $(\mathcal{M}, a) \Vdash_{V_0} p \Leftrightarrow$

$$(||\{j \mid (\mathcal{M}, a) \Vdash_{V_j} p, j \neq 0\}||)/(||(\mathcal{M}, a) \nVdash_{V_j} p, j \neq 0\}||) > 3,$$

(for $||(\mathcal{M}, a) \nVdash_{V_j} p, j \neq 0\}|| \neq 0$). This would mean p is true from viewpoint of dominating majority of agents.

We may mean only definition of V out of V_j using the threshold as defined earlier, etc. There are very many ways to express what means global valuation and what indeed means the dominant part of agents. Maybe the agent's opinion may be considered with an appropriate prescribed weights; maybe depending on different states, the rules to compute global valuation may be different, etc. In the very limit point we may assume V to be arbitrary, which does not depend on all V_j - it is the opinion of a total dominant - the only true what V thinks to be true.

But the rules of computation V out of V_js to be uniform and the same for all models. We refer to V in the sequel as for given single valuation V. So, for all p, $V(p) \subseteq N$. Now any such model $\mathcal{M}$ is a multi-valued model - with a finite number of different valuations. The rules for calculation truth values of compound formulas are as follows (they are standard for Boolean logical valuations, but new, non-standard – for agents operations).

$$\forall p \in P, \forall a \in N\ (\mathcal{M}, a) \Vdash_V p \Leftrightarrow a \in N \land\ a \in V(p);$$

$$(\mathcal{M}, a) \Vdash_V (\varphi \land \psi) \Leftrightarrow (\mathcal{M}, a) \Vdash_V \varphi \land (\mathcal{M}, a) \Vdash_V \psi;\quad (\mathcal{M}, a) \Vdash_V (\varphi \lor \psi) \Leftrightarrow$$

$(\mathcal{M}, a) \Vdash_V \varphi \vee (\mathcal{M}, a) \Vdash_V \psi$;

$(\mathcal{M}, a) \Vdash_V \neg\varphi \Leftrightarrow not[(\mathcal{M}, a) \Vdash_V \varphi]$;

$\forall a \in N,\ (\mathcal{M}, a) \Vdash_V \mathbf{N}\varphi \ \Leftrightarrow\ [(a\ Next\ b) \Rightarrow (\mathcal{M}, b) \Vdash_V \varphi]$.

$\forall a \in N,\ (\mathcal{M}, a) \Vdash_V (\varphi\, \mathbf{U}_j\, \psi) \ \ \Leftrightarrow$

$\exists b[(aR_jb) \wedge ((\mathcal{M}, b) \Vdash_V \psi) \wedge \forall c[(aR_jc < b) \Rightarrow (\mathcal{M}, c) \Vdash_V \varphi]]$;

$\forall a \in N, \forall i \in Ag,\ (\mathcal{M}, a) \Vdash_V A_i(p) \Leftrightarrow a \in V_i(p)$.

The latter one – $(\mathcal{M}, a) \Vdash_V A_i(p)$ says that the agent i knows that p is true at a. That allows to clarify knowledge (information) which the agent for sure, definitely, has at the current state, which is useful in applications to make distinction from aggregated knowledge.

3. Discussion, Results

The logic overall we wish to introduce is the collection of all general statements, formulas, which are valid in all models.

Definition 2.*The multi-agent non-transitive logic L^{MA} is the set of all formulas which are valid in all models $\mathcal{M}$.*

Here we will not specify exactly which models and agent's valuations are fixed, so the results will hold for chosen collections of models.

Recall that $(\mathcal{M}, a) \Vdash_V A_i(p)$ says that the agent i knows that p is true at a (and recall that the operations A_i may not be extended from only letters). That allows to clarify knowledge (information) which the agent for sure, definitely, has at the current state, which is useful in applications to make distinction from aggregated knowledge.

Clearly we may vary such logic taking particular models as the semantics, - all ones, or some with particular rules for definition of the agreed valuation, with particular restrictions on the agent's accessibility relations R_j, etc. As always we may definite modal operations via temporal ones. For example *possibility* for the agent j may be defined as follows:

$\Diamond_j p := \top U_j p.$

The operation *necessary* then to be expressed as: $\Box_j p := \neg\ U_j \neg p$, and it is easy to se that

$(\mathcal{M}, a) \Vdash_V \Diamond_j\varphi \ \Leftrightarrow\ \left[\exists b(aR_jb)\ \&\ (\mathcal{M}, b) \Vdash_V \varphi\right]$;

$(\mathcal{M}, a) \Vdash_V \Box_j\varphi \ \Leftrightarrow\ \left[\forall b(aR_jb) \Rightarrow\ (\mathcal{M}, b) \Vdash_V \varphi\right]$,

so the defined agent's modal operations work as expected – in accordance with meaning of modal operations. Now we will give some examples illustrating how the chosen framework may model agent's relations including non-transitivity, lack of monotonicity and possible lacunas in agent's accessibility relations.

Examples.

(1) The formula $\mathbf{N}p \wedge \neg\Diamond_1 p \wedge \mathbf{NN}\Diamond_1 p$ says that the relation R_1 has lacunas – the next sate is not accessible by R_1 but some next after next one is.

(2) Consider the formula $\Diamond_1 p \wedge \neg\Diamond_2 p$, it being true w. r. t. a valuation V says that the accessibility relation for the agent 2 has a hole (lacuna) which nonetheless has inside states accessible for the agent 1.

(3) The formula $\Diamond_1\Diamond_1 p \wedge \neg\Diamond_1 p$ says that the relation R_1 is not transitive for a given state a – in the transitivity interval (b-path) where a is situated truth of p is impossible by relation R_1, but in the next b-path p is possible.

(4) To illustrate multi-agency consider the formula $\varphi_{op} := [\Box_1 p \to \Box_2\neg p] \wedge [\Box_2 p \to \Box_1\neg p]$. It says that these both agents are totally opposite in their opinion for stable facts at all states accessible for them.

(5) The formula $[(\Box_1 p \to \Box_2 p) \wedge (\Box_2 p \to \Box_1 p)] \wedge \Diamond_1 \mathbf{N}\varphi_{op}$ says that the agents may be agree at all visible time but after this they may be in a complete opposition.

(6) Total recall: $\Diamond_1 p \wedge \Box_1(p \to \Diamond_1[\neg p \wedge \Diamond_1 p]) \wedge \Diamond_1\Diamond_1\Box_1 p$. This formula says that the agent 1 always swapping its opinion about the truth of p from true to false and vise versa or lose p during the whole initial interval of time, but after some time it decides p to be always true.

Now we motivate briefly why time might be non-transitive in revision of computation, reasoning.

Computations view. Inspections of protocols for computations are limited by time resources and have non-uniform length. Therefore, if we interpret our models as the ones reflecting inspection of protocols for computation, the amount of check points is finite. In any point of inspection we may refer to stored protocols, and any one has limited length. Thus the inspections look as non-transitive accessibility relations.

Agent's-admin's view. We may consider states (worlds of our model) as checkpoints of admins (agents) for any inspection of state of a network in past. Any admin has allowed amount of inspections for states, but only within the areas of its(his/her) responsibility (by security or another reasons). So, the accessibility is not transitive again, the admin (a1) can reach a state, and there in the admin (a2) responsible for this state (it may be a new one or the same yet), has again some allowed amount of inspections to past. But, in total, (a1) cannot inspect all states accessible for (a2).

Now we explain how CD may be modelled in such framework. First of all - interpretation via possibility:

$$(\mathcal{M}, b) \Vdash_V \bigwedge_j \Diamond_j p,$$

all agents think that p may be true - possible. Though - definitely there is a chance. The next one is a slimmer chance:

$$(\mathcal{M}, b) \Vdash_V \bigvee_j \Diamond_j p$$

at least one agent knows that p is possible - there is a slim chance.

Clear that this way we may grade the value of the CD - moving from believe of only one agent to some majority and to all agents.

Next possible way is to use operations until $\mathbf{U}_j$ in a similar manner - chance to find information until some pre-condition still holds.

Also we may consider the operations A_j together with modal and temporal ones - to model chances with some particular restrictions on distinct V_j – individual knowledge of agents. Thus, this framework very well works for modelling and study CD.

Our Mathematical Results. We study problems satisfiability and decidability in logics based on distinct classes of suggested models (e.g. with various rules of computation agreed valuations, restrictions for accessibility relations of the agents). We suggest solving algorithms in the suggested framework applying technique which is borrowed from [25, 26].

We briefly recall here technique which we use here for computation of satisfiability and decidability Recall that a (sequential) (inference) rule is an expression (statement)

$$\mathbf{r} := \frac{\varphi_1(x_1, \ldots, x_n), \ldots, \varphi_l(x_1, \ldots, x_n)}{\psi(x_1, \ldots, x_n)},$$

where $\varphi_1(x_1,\ldots,x_n),\ldots,\varphi_l(x_1,\ldots,x_n)$ and $\psi(x_1,\ldots,x_n)$ are formulas constructed out of letters $x_1,\ldots,x_n$. The letters $x_1,\ldots,x_n$ are the variables of $\mathbf{r}$, we use the notation $x_i \in Var(\mathbf{r})$. A meaning of a rule $\mathbf{r}$ is that the statement (formula) $\psi(x_1,\ldots,x_n)$ (which is called conclusion) follows (logically follows) from statements (formulas) $\varphi_1(x_1,\ldots,x_n),\ldots,$ $\varphi_l(x_1,\ldots,x_n)$ which are called premises.

Definition 3. *A rule* $\mathbf{r}$ *is said to be* valid *in a model* $\langle\mathcal{M},V\rangle$ *(we will use the notation* $\mathcal{M}\Vdash_V r$*) if*

$$[\forall a\,((\mathcal{M},a)\Vdash_V \bigwedge_{1\leq i\leq l}\varphi_i)] \Rightarrow \forall a\,((\mathcal{M},a)\Vdash_V \psi).$$

Otherwise we say $\mathbf{r}$ *is* refuted in $\mathcal{M}$, *or* refuted in $\mathcal{M}$ *by* V, *and write* $\mathcal{M}\nVdash_V \mathbf{r}$. *A rule* $\mathbf{r}$ *is* valid *in a frame* $\mathcal{M}$ *(notation* $\mathcal{M}\Vdash\mathbf{r}$*) if, for any valuation* V*, the following holds* $\mathcal{M}\Vdash_V\mathbf{r}$.

For any formula φ, we can transform φ into the rule $x\to x/\varphi$ and employ a technique of reduced normal forms for inference rules as follows.

Lemma 4. *For any formula* φ*,* φ *is a theorem of a logic* L *iff the rule* $(x\to x/\varphi)$ *is valid in any frame* $\mathcal{M}$*.*

Definition 5. *A rule* $\mathbf{r}$ *is said to be in* reduced normal form *if* $\mathbf{r}=\varepsilon/x_1$ *where*

$$\varepsilon := \bigvee_{1\leq j\leq l}[\bigwedge_{1\leq i\leq n} x_i^{t(j,i,0)} \wedge \bigwedge_{1\leq i\leq n}(\mathbf{N}x_i)^{t(j,i,1)} \wedge \bigwedge_{1\leq i,k\leq n, i\neq k, j\in Ag}(x_i\mathbf{U}_j x_k)^{t(j,i,k,1)}]$$

and, for any formula α *above,* $\alpha^0 := \alpha$*,* $\alpha^1 := \neg\alpha$*.*

Definition 6. *Given a rule* $\mathbf{r_{nf}}$ *in reduced normal form,* $\mathbf{r_{nf}}$ *is said to be a normal reduced form for a rule* $\mathbf{r}$ *iff, for any frame* F*,*

$$F \Vdash \mathbf{r} \Leftrightarrow F \Vdash \mathbf{r_{nf}}.$$

Theorem 7. *There exists an algorithm running in (single) exponential time, which, for any given rule* $\mathbf{r}$*, constructs its normal reduced form* $\mathbf{r_{nf}}$.

So, we may use rules in reduced form instead of usual rules and hence instead of formulas to identify truth of formulas, this allows to avoid consideration of formulas with nested logical operations (which crucially simplifies proofs and constructions of computational algorithms).

So we obtain theorems (distinct for distinct models classes)

Theorem 8.. *The logics have decidable satisfiability problem, we propose an algorithm computing satisfiability.*

Applications. Our suggested techniques may be used for

(i) Identifying possibility for chance to find a common satisfactory decision for different agents in future time in changing environment (e.g. increasing recourses, etc).

(ii) Investigations of changing criteria for opinion (valuations) of agents bearing in mind possible revisions of priority and rank of competence for agents.

(iii) Analysis existing networks for failure which may be resulted by interaction of different computational threads and possible wrong transmissions.

(iv) Computation of consistency and reliability collected information via interpretation of agents as distinct techniques of verification with a final ranking obtained results.

(v) Recommendations for clearing data when analysis shows inefficiency and low use of information occupying significant part of databases.

4. Conclusion

This paper investigates multi-agent logics for study CD. Our models reflects well typical tasks form CS - computational runs and inspection of protocols of completed computations or the state at any check point of a computation. Besides these models nicely represent interpretation of knowledge and information collected in past, as via approach with individual experts – as agents, as well as with voted values for facts to be knowledge.

We investigate these models from logical viewpoint and suggest a multi-agent non-transitive temporal logic, the later one allows to flexibly formalize the concept of *knowledge* and CD expressed by logical means. We consider main basic problems for any logical system - decidability and satisfiability problems, and solve these problems, computational algorithms are found. The suggested approach looks very flexible and allows to develop logical technique for many other non-trivial aspects of CD and its applications.

There are many remaining open interesting problems in our suggested framework. For example the case with totally unbounded limits of the agents accessibility relations remain open. The basic logical problems such as axiomatizability and unifiability also remain open. Areas of applications in Information Sciences overall look also attractive and promising. Towards useful possible applications (based at the developed in this paper technique) it is interesting to look for possibilities to embed in the existing (suggested) multi-agents systems some elements for verification consistency and correctness information for multi-agent environment, a logical approach, – using elements of first order logic with usage quantifiers. The exiting completed research towards this works already, but until now it has been not a big part from combination of temporal and multi-agent logic, which we offer. Even now we give only some parts of possible algorithms which do not cover most general case. That may be implemented for extracting hidden information which follow from collected observations and facts.

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