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1. True value $(V_T) = \pi$

Approximated value $(V_a) = 3.14$

$\therefore$ Absolute Error $(E_a) = |V_T - V_a| = |\pi - 3.14| = 0.00159$

Relative Error $(E_p) = \frac{E_a}{V_T} = \frac{0.00159}{\pi} = 0.0005069$

Percentage Error $(E_p) = E_n \times 100\% = 0.05\%$

(Ans)

2. True value $(V_T) = \cancel{2.77}\ 2.71828$

Approximated value $(V_a) = 2.71937$

$\therefore$ Absolute error $(E_a) = |V_T - V_a| = |2.71828 - 2.71937|$

$= 0.00109$

Relative error $(E_n) = \frac{0.00109}{2.71828} = 0.000400$

$\therefore$ percentage error $(E_p) = E_n \times 100\% = 0.0004 \times 100\% = 0.04\%$

(Ans)

3. True value $= \frac{4}{3}$

Approximated value $(V_a) = 1.3333$

$\therefore$ Absolute Error $(E_a) = |4/3 - 1.3333| = 0.000033$

Relative Error $(E_n) = \frac{E_a}{V_T} = \frac{0.000033}{4/3} = 0.00002475$

$\therefore$ Percentage Error $(E_p) = 0.0024\%$

4. Prove that $\Delta f(n) = 0$ where $f(n)$ is a constant function.

Let $f(n) = k \ \forall \ x \Rightarrow f(x+h) = k$

$\therefore \Delta f(n) = f(x+h) - f(x)$

$= k - k$

$= 0$ (proved)

5. prove that $\Delta \cdot \nabla \equiv \Delta \cdot \nabla$

$\Delta \cdot \nabla f(n) = \Delta(\nabla f(n))$

$= \Delta(f(n) - f(x-h))$

$= \Delta f(n) - \Delta f(x-h)$

$= \{(f(n+h) - f(n)) - (f(n-\cancel{h}+\cancel{h}) - f(n-h))\}$

$= f(n+h) - f(n) - f(n) + f(n-h)$

$= f(n+h) - f(n) - (f(n) - f(x-h)) = \nabla - \Delta$

(Proved)

6. Prove that $E \equiv 1 + \Delta$

$E f(n) = f(x+h)$ where h is ~~ste~~ the step length

$\{\Delta f(n) = f(n+h) - f(n)\}$ — ①

$= f(n+h) - f(n) + f(n)$ [adding and substracting $f(n)$]

$= \Delta f(n) + f(\Delta n)$ (using ①)

$Ef(n) = f(n)(1+\Delta)$

$E \equiv 1 + \Delta$ (Proved)

7. prove that $E \equiv e^{hD}$, $D \equiv \frac{d}{dx}$

$$Ef(x) = f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \dots \infty$$

or, $$Ef(x) = f(x) + hDf(x) + \frac{h^2D^2}{2!}f(x) + \frac{h^3D^3}{3!}f'''(x) + \dots \infty$$

or, $$Ef(x) = \left[1 + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \dots \infty\right] f(x)$$

or, $$Ef(x) = e^{hD} f(x) \quad \left[\because 1 + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \dots \infty = e^{hD}\right]$$

$\therefore E = e^{hD}$ (proved)

8. prove that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

$$\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

$$= \log\{f(x+h) - f(x)\}$$

$$= \log f(x+h) - \log f(x)$$

$$= \log\left(\frac{f(x+h)}{f(x)}\right)$$

$$= \log\left(\frac{f(x+h) - f(x) + f(x)}{f(x)}\right)$$

$$= \log\left(\frac{\Delta f(x)}{f(x)} + \frac{f(x)}{f(x)}\right)$$

$$= \log\left(\frac{\Delta f(x)}{f(x)} + 1\right) \quad \text{(proved)}$$

9. ~~prove~~ Evaluate $\Delta^2(ax^2 + bx + c)$

$$f(x) = \Delta^2(ax^2 + bx + c)$$

$$= \Delta(\Delta f(x))$$

$$\Delta f(x) = \Delta(ax^2 + bx + c)$$

$$= a\Delta x^2 + b.\Delta x + \Delta c$$

$$= a(2xh + h^2) + b(h) + 0$$

$$= 2axh + h^2 + bh + 0 = h^2 + (2ax + b)h$$

$$\Delta^2 f(x) = \Delta(h^2 + 2axh + bh)$$

$$= \Delta h^2 + 2ah\,\Delta x + \Delta bh$$

$$= 0 + h \times 2a + 0 = 2ah^2 \text{ (Ans)}$$

10. prove that $\Delta^2(\cos 2x) = -\Delta \sin^2 h \cos 2(x+h)$

$$\Delta^2(\cos 2x) = \Delta(\Delta \cos 2x)$$

$\therefore \Delta \cos 2x = \cos 2(x+h) - \cos 2x$ $\left[\cos c - \cos D = 2\sin\left(\frac{c+D}{2}\right) - \sin\left(\frac{D-c}{2}\right)\right]$

$$= 2\sin\left(\frac{2(x+h)+2x}{2}\right)\sin\left(\frac{2x-2(x+h)}{2}\right)$$

$$\Delta \cos 2x = 2\sin(x+h)\sin(-h)$$

$$\Delta^2 \cos 2x = -2\sin h \, \Delta \sin(2x+h)$$

$$= -2\sin h\left[\sin\{2(x+h)+h\} - \sin(2x+h)\right]$$

$$= -2\sin h \cdot 2\cos\frac{\{2(x+h)+h\}+(2x+h)}{2}\sin\frac{\{2(x+h)+h\}-(2x+h)}{2}$$

$$= -4\sin h\cos\left[\frac{(2x+2h+h+2x+h)}{2}\right]\sin\left[\frac{2x+3h-2x-h}{2}\right]$$

$$= -4\sin h\cos\left[2\frac{(2x+2h)}{2}\right]\sin\frac{2h}{2}$$

$$= -4\sin h\cos(2x+2h)\sin h$$

$$= -4\sin^2 h\cos(2x+2h). \quad \text{(proved)}$$

11. Estimate the missing terms from the following set of data.

x	2	4	6	8	10
$f(x)$	5.6	8.6	13.9	*	35.6

~~Sei~~ Since y is known for 4 data of x i.e the functional value is given for four information, from this given information, we can construct a polynomial of degree 3.

As $f(x)$ will be a polynomial of degree 3, the third order difference of $f(x)$ will be constant and 4th order difference of $f(x)$ will be zero.

$$\Delta^4 f(x) = 0 \quad [E^h f(x) = f(x+nh)]$$

$$(E-1)^4 f(x) = 0$$

$$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) f(x) = 0$$

$$\Rightarrow E^4 f(x) - 4E^3 f(x) + 6E^2 f(x) - 4Ef(x) + f(x) = 0$$

$$\Rightarrow f(x+4h) - 4f(x+3h) + 6f(x+2h) - 4f(x+h) + f(x) = 0$$

We have,

$\rightarrow f(10) - 4f(8) + 6f(6) - 4f(4) + f(2) = 0$

$\rightarrow 8.56 - 4f(8) + 6 \times 13.9 - 4 \times 8.6 + 5.6 = 0$

$\rightarrow 4f(8) = 90.2$

$\therefore f(8) = \frac{90.2}{4} = 22.55$ (ans)

12. Estimate the missing value from the following set of data.

x	0	1	2	3	4	5
$f(x)$	0	*	8	15	*	35

since y is known for 4 datas i.e we have informations for 4 datas therefore we can create a ~~polygram~~ polynomial of 3 degree as the $f(x)$ is of 3 degree. Therefore, The third order will be a constant and 4th order will be 0.

$\therefore$ we have two missing terms, here we need 2 eqs

$\therefore$ let us consider $\Delta^4 f(x) = 0$ for 1st term

$\Delta^4 f(x) = 0$ for 2nd term

Saving for 1st variable

$E = 1 + \Delta \qquad \therefore \Delta\ E - 1$

$(E-1)^4 f_0(x) = 0$

$\Rightarrow (4c_0 E^4 - 4c_1 E^3 + 4c_2 E + 4c_4 E^0) f_0(x) = 0$

$\Rightarrow E^4 f_0(x) - 4E^3 f_0(x) + 6E^2 f_0(x) + 4E f_0(x) + f_0(x) = 0$

$\Rightarrow f_0^4(x) - 4f_0^3(x) + 6f_0^2(x) - 4f_0'(x) + f_0(x) = 0$

$\Rightarrow f_4(x) - 4 \times 15 + 6 \times 8 - 4f_0'(x) + f_0(x) = 0$

$\Rightarrow f_4(x) - 60 + 48 - 4f_1(x) = 0$

$f_4(x) - 4f_1(x) = 12 \quad \text{-----} \quad (1)$

$(E-1)^4 f_1(x) = 0$

$\Rightarrow 4c_0 E^4 (f_1(x)) - 4c_1 E^3 f_1(x) + 4c_2 E^3 f_1(x) + 4c_3 E f_1(x) + 4c_4 E^0 f_1(x) = 0$

$\Rightarrow E^4 f_1(x) - 4E^3 f_1(x) + 6E^2 f_1(x) - 4Ef_1(x) + f_1(x) = 0$

$\Rightarrow f_5(x) - 4f_4(x) + 6f_3(x) - 4f_2(x) + f_1(x) = 0 \quad [\because E^n f_m(x) = f_{m+n}(x)]$

$\Rightarrow 35 - 4f_4(x) + 6\times 15 - 4\times 8 + f_1(x) = 0$

$\Rightarrow 4f_4(x) - f_1(x) = 93 \quad \ldots\ldots (2)$

$\therefore$ from eq (1) ~~and~~ and eq (2)

$$f_4(x) - 4f_1(x) = 12$$
$$4f_4(x) - f_1(x) = 93$$
$$4f_4(x) - 16f_1(x) = 48$$
$$-4f_4(x) + f_1(x) = -93$$
$$-15f_1(x) = -45$$

$\therefore f_1(x) = 3$

$$\therefore f_1(x) = \frac{45}{15} = 3$$

Replacing the value of $f_1(x)$ in eq (2)

$$4f_4(x) - 3 = 93$$
$$f_4(x) = \frac{96}{4} = 24$$

The two missing terms are $f_1(x) = 3$ and $f_4(x) = 24$

13. Derive Newton's interpolation formula.

Suppose a function known for $(n+1)$ distinct equi-spaced arguments $x_0, x_1, x_2, \ldots x_n$ and the corresponding functional value are $y_0 = f(x_0)$, $y_1 = f(x_1)$, $\ldots$ $y_n = f(x_n)$

Here we have to construct a polynomial $p(x)$ which is of degree n and $p(x)$ replaces the function $f(x)$ at the set of interpolation point so we have $x_i = x_0 + ih$; h is the step length, $i = 0, 1, 2 \ldots n$ — (1)

Also we have $y_i = f(x_i)$ $i = 0, 1, 2, \ldots n$ — (2)

Also at the set of interpolation point, the functional value and the value of the polynomial are same i.e $P(x_i) = f(x_i) = y_i$ where $i = 0, 1, 2 \ldots n$ —— (3)

We can construct the polynomial as follows:

$$P(x) = A_0 + A_1(x_1 - x_0) + A_2(x - x_0)(x - x_1) + A_3(x - x_0)(x - x_1)(x - x_2)$$
$$+ \cdots A_n(x - x_0)(x - x_1) \cdots (x - x_n) \quad \cdots (4)$$

In equation (4) $A_0, A_1, \ldots A_n$ are the unknown which we have to determine using (1) - (3).

Substitute $x = x_0$ in equation (4)

$$P(x_0) = A_0$$

$$\Rightarrow f(x_0) = y_0 = A_0 \Rightarrow A_0 = y_0$$

Substitute $x = x_1$ in equation (4)

$$P(x_1) = A_0 + A_1(x_1 - x_0)$$

$$\Rightarrow f(x_1) = y_1 = A_0 + A_1 h$$

$$\Rightarrow y_1 = y_0 + A_1 h \Rightarrow A_1 = \frac{y_1 - y_0}{h} = \frac{\Delta y_0}{h}$$

Substitute $x = x_2$ in equation (4)

$$P(x_2) = A_0 + A_1(x_2 - x_0) + A_2(x_2 - x_0)(x_2 - x_1)$$

$$\Rightarrow y_2 = A_0 + A_1(2h) + A_2(2h)(h)$$

$$\Rightarrow y_2 = y_0 + \frac{\Delta y_0}{h} 2h + A_2 \lfloor 2 h^2$$

$$\Rightarrow \lfloor 2 h^2 A_2 = y_2 - y_0 - 2\Delta y_0$$
$$= y_2 - y_0 - 2(y_1 - y_0)$$
$$= y_2 - 2y_1 + y_0$$
$$= (y_2 - y_1) - (y_1 - y_0)$$
$$= \Delta y_1 - \Delta y_0 = \Delta^2 y_0$$

$$\therefore A_2 = \frac{\Delta^2 y_0}{\lfloor 2 h^2}$$

In general we have,

$$A_n = \frac{\Delta^n y_0}{\lfloor n h^n} \quad \text{for } n = 0, 1, 2, \ldots n \quad \cdots (5)$$

Substituting all these equations in equation (4) we have

$$P(x) = y_0 + \frac{\Delta y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{\lfloor 2 h^2}(x - x_0)(x - x_1) + \frac{\Delta^3 y_0}{\lfloor 3 h^3}(x - x_0)(x - x_1)(x - x_2)$$
$$+ \cdots + \frac{\Delta^n y_0}{\lfloor n h^n}(x - x_0)(x - x_1) \cdots (x - x_{n-1}) \cdots (6)$$

Let $u = \frac{x - x_0}{h}$

$x = uh + x_0$

Also from equation ① $x_i = x + ih$

$x - x_i = (u-i)h, \; i = 0, 1, 2, \ldots, n$ —— ⑦

substituting equation ⑦ in ⑥

$$P(x) = y_0 + u\Delta y_0 + \frac{u\Delta^2 y_0}{L_2} + \frac{u(u-1)(u-2)\Delta^3 y_0}{L_3} + \cdots + \frac{u(u-1)(u-2)\cdots(u-u_{n-1})\Delta^n y_0}{L_n}$$

(Proved)

13. Derive newton's back word interpolation formula.

Suppose a function known for $(n+1)$ distinct equi-spaced argument $x_0, x_1, \ldots x_n$ and the corresponding functional values are $y_0 = f(x_0)$, $y_1 = f(x_1)$, $\ldots y_n = f(x_n)$. here also we have to predict a polynomial function al $P(x)$ which replaces $(f(x))$ at the set of interpolation points.

$x_i = x_0 + ih \quad i = 0, 1, 2, \ldots n$ —— ①

$y_i = f(x_i)$ for $i = 0, 1, 2, \ldots n$ — ②

Also at the set of interpolation points, the value of the function $f(x)$ and the value and the value of the polynomial $P(x)$ are the same i.e

$y_i = f(x_i) = P(x_i)$ for $i = 0, 1, 2, 3 \ldots n$ — ③

while we have to extend the polynomial in terms of u always $|u| \ll 1$ (if $u = (x - x_0)/h$)

so keeping this in mind, we express the series as -

$$P(x) = B_n + B_{n-1}(x - x_n) + B_{n-2}(x - x_n)(x - x_{n-2}) + \cdots + B_0(x - x_n)(x - x_{n-1})(x - x_{n-2}) \cdots (x - x_1)$$

~~Substitute $x = x_n$ in equation ④~~

~~$P(x) = B_n$~~

Here our task is to determine $B_0, B_1, \ldots B_n$ subjected to the use of equation ①-③ —— ④

$$P(x) = B_n + B_{n-1}(x - x_n) + B_{n-2}(x - x_n)(x - x_{n-1}) + \dots + B_0(x - x_n)(x - x_{n-1}) \dots (x - x_1)$$

Substitute $x = x_n$ in equ^n (4)

$$P(x) = B_n \Rightarrow f(x_n) = y_n = B_n \Rightarrow B_n = y_n$$

Substitute $x = x_{n-1}$ in equ^n (4)

$$P(x_{n-1}) = B_n + B_{n-1}(x_{n-1} - x_n)$$

$$\Rightarrow y_{n-1} = f(x_{n-1}) = y_n + B_{n-1}(-h)$$

$$\Rightarrow B_{n-1} = (y_n - y_{n-1})/h \Rightarrow B_{n-1} = \frac{\nabla y_n}{h}$$

Substitute $x = x_{n-2}$ in eq (4)

$$P(x_{n-2}) = B_n - B_{n-1}(x_{n-2} - x_n) + B_{n-2}(x_{n-2} - x_n)(x_{n-2} - x_{n-1})$$

$$y_{n-2} = y_n + \frac{\nabla y_n}{h}(-2h) + B_{n-2}(-2h)(-h)$$

$$\Rightarrow \lfloor 2 h^2 B_{n-2} = y_{n-2} - y_n + 2\nabla y_n$$

$$= y_{n-2} - y_n + 2(y_n - y_{n-1})$$

$$= y_n - 2y_{n-1} + y_{n-2}$$

$$= (y_n - y_{n-1}) - (y_{n-1} - y_{n-2}) = \nabla y_n - \nabla y_{n-1}$$

$$= \nabla^2 y_n$$

$$B_{n-2} = \frac{\nabla^2 y_n}{2 \lfloor h^2} \qquad B_{n-n} = \frac{\nabla^2 y_n}{\lfloor n h^2} \qquad n = 0, 1, 2, \dots n$$

$$P(x) = y_n + \frac{\nabla y_n}{h}(x - x_n) + \frac{\nabla^2 y_n}{h}(x - x_n) + \frac{\nabla^2 y_n}{\lfloor 2 h^2}(x - x_n)(x - x_{n-1})$$

$$+ \dots\dots + \frac{\nabla^n y_n}{\lfloor n h^n}(x - x_n)(x - x_{n-1}) \dots (x - x_1)$$

let $u = (x - x_n)/h$

$$\Rightarrow x = x_n + uh$$

since $x_i = x_0 + ih$ so, $x_{n-n} = x_0 + (n-n)h \Rightarrow x - x_n = uh$

$$\Rightarrow x_n = x_0 + nh$$

$$x - x_n = (x_n + uh) - (x_0 + nh - nh)$$

$$= x_n - x_0 + uh - nh + nh$$

$$= nh - nh + (u+n)h$$

$$x - x_{n-1} = (u+n)h, \quad n = 0, 1, 2 \dots n$$

$$P(x) = y_n + u\nabla y_n + \frac{u(u-1)}{\lfloor 2}\nabla^2 y_n + \cdots + \frac{u(u+1)(u+2)\cdots(u+u_{n-1})}{\lfloor n}\nabla^n y_n$$

5. Derive Langrange's interpolation formula.

Suppose a function $f(x)$ is Know for $n+1$ distinct arguments $x_0, x_1 \cdots x_n$ which may not be equally spaced and corresponding functional values are $y_0 = f(x_0)$, $y_1 = f(x_1), \cdots ; y_n = f(x_n)$.

We have to construct a polynomial $L(x)$ of degree n such that at the interpolation points x_i, the value of the functional and the value of the polynomial will be same.

given, $y_i = f(x_i)$, $i = 0, 1, 2, \cdots n$ ——— (1)

And at the interpolation points x_i, $L(x_i) = f(x_i) = f_i = 0, 1, 2, \cdots n$ ——— (2)

Considering the polynomial as

$$L(x) = A_0(x-x_1)(x-x_2)\cdots(x-x_n) + A_1(x-x_0)(x-x_2)\cdots(x-x_n) + A_2(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)\cdots + A_n(x-x_0)(x-x_1)\cdots(x-x_{n-1})$$

Substitute $x = x_0$ in equation (3) $y_0 = L(x_0) = A_0(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)$ ——— (3)

$\Rightarrow A_0 = y_0 / \{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)\}$

Substitute $x = x_1$ in equation (3) $y_1 = L(x_1) = A_1(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)$

$\Rightarrow A_1 = y_1 / \{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)\}$

In general, $A_i = y_i / \{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)\}$

$$L = A_0(x-x_1)(x-x_2)\cdots(x-x_n) + A_1(x-x_0)(x-x_2)\cdots(x-x_n) + \cdots + A_n(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})$$

$$L(x) = \frac{(x-x_1)(x-2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_0)\cdots(x_1-x_n)}y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n+1})}y_n \quad \text{—— (iv)}$$

Give the following set of data

x	40	45	50	55
$f(x)$	15.22	13.99	12.62	11.13

Construct the langrange polynomial and hence evaluate $f(50.22)$ upto 5 decimal places

$$L_n(x) = \frac{(x-45)(x-50)(x-55)}{(40-45)(40-50)(40-55)} \times 15.22 + \frac{(x-40)(x-50)(x-55)}{(45-40)(45-50)(45-55)} \times 13.99$$

$$+ \frac{(x-40)(x-45)(x-55)}{(50-40)(50-45)(50-55)} \times 12.62 + \frac{(x-40)(x-45)(x-50)}{(55-40)(55-45)(55-50)} \times 11.13$$

$$= 2.7 \times 10^{-5} x^3 - 6.4 \times 10^{-3} \times^2 + 15.3 \times 10^{-2} x + 17.62$$

$$f(50.22) = 12.5828 \text{ (ans)}$$

Given the following set of Data from which compute $f(1.02)$ and $f(1.29)$ up to 4 decimal places.

x	1.00	1.10	1.20	1.30
$f(x)$	0.8415	0.8912	0.9320	0.9636

To compute $f(1.02)$ we will use Newton's forward interpolation formula

$$f(a+hu) = f(a) + \frac{u}{1!} \Delta f(x) + \frac{u(u-1)}{2!} \Delta^2 f(x) + \frac{u(u-1)(u-2)}{3!} \Delta^3 f(x) + \cdots$$

$a + hu = 1.02$ $\quad a = 1.00 \quad h = 0.10$

$1.00 + (0.10)u = 1.02$

$u = 0.2$

Constructing the difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1.00	0.8415			
		0.0497		
1.10	0.8912		-0.0089	
		0.0408		-0.0003
1.20	0.9320		-0.0092	
		0.0316		
1.30	0.9636			

$$0.8415 + 0.2 \times 0.0497 + \frac{0.2(0.2-1) - 0.0089}{2} + \frac{0.2(0.2-1)(0.2-2) - 0.0003}{6}$$

$= 0.8521$ (ans)

to compute $f(1.29)$ we will use Newton's backward interpolation formula

$f(a+hu) = 1.29$

$a = 1.30$, $h = 0.10$ $\qquad a + hu = 1.29$

$1.30 + 0.10u = 1.29$

$\therefore u = \frac{1.29 - 1.30}{0.10} = -0.1.$

$$f(a) + u \nabla f(x) + \frac{u(u+1)}{2} \nabla^2 f(x) + \frac{u(u+1)(u+2)}{6} \nabla^3 f(x)$$

$$= 0.9636 + (-0.1) \times 0.0316 + \frac{(-0.1)(-0.1+1) - 0.0092}{2} + \frac{-0.1(-0.1+1)(-0.1+2) - 0.0003}{6}$$

$= 0.9608$ (ans)

18. Using suitable interpolation formula compare $f(2.1)$ and $f(7.5)$ from the following data upto a decimal points.

x	0	2	4	6
$f(x)$	2	6	10	15

To compute $f(2.1)$ we will use Newton's forward interpolation formula.

$$f(a+hu) = \Delta f(x) = f(a) + \frac{u}{1!} \Delta f(x) + \frac{u(u-1)}{2!} \Delta^2 f(x) + \frac{u(u-1)(u-2)}{3} \Delta^3 f(x) + \cdots$$

$a + hu = 2.1$ $\qquad a = 2$ $\quad h = 2$

$2 + 2u = 2.1$ $\qquad u = 0.05$

Constructing the difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2			
		4		
2	6		0	
				1
4	10	9		
			1	
6	15	5		

$$f(a+hu) = f(a) + u\Delta f(x) + \frac{u(u-1)}{2}\Delta^2 f(x)$$

$$= 6 + 0.05 \times 4 + \frac{0.05(0.05-1)}{2} \times 1$$

$$= 6 + 0.2 + (-0.02375)$$

$$= 6.17625 = 6.1763 \text{ (ans)}$$

To compute $f(7.5)$ → $f(a+hu) = 7.5$ $\quad a+hu = 7.5$

$6+2u = 7.5 \Rightarrow u = 0.75$

$$f(7.5) = 15 + 0.75 \times 5 + \frac{0.75(0.75-1)}{2} \times 1 = 15 + 3.6562 = 13.6562 \text{ (Ans)}$$

19. Evaluate $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule taking $n=8$ up to 4 decimal places.

$f(x) = \frac{1}{1+x}$, here $a=0$, $b=1$, $n=8$ $\quad \therefore h = \frac{b-a}{n} = \frac{1}{8} = 0.125$

x_i	$y_i = \frac{1}{1+x}$;
$x_0 = 0$	$y_0 = 1$
$x_1 = 0.125$	$y_1 = 0.888889$
$x_2 = 0.25$	$y_2 = 0.8$
$x_3 = 0.375$	$y_3 = 0.727273$
$x_4 = 0.5$	$y_4 = 0.666667$
$x_5 = 0.625$	$y_5 = 0.615384$
$x_6 = 0.75$	$y_6 = 0.571428$
$x_7 = 0.875$	$y_7 = 0.533333$
$x_8 = 1.000$	$y_8 = 0.5$

by trapezodial Formula,

$$\int_0^1 \frac{1}{1+x}dx = h\left[\frac{y_0+y_1}{2} + y_1+y_2+y_3+y_4+y_5+y_6+y_7\right]$$

$$= 0.125\left[\frac{1+0.5}{2} + 0.888889 + 0.8 + 0.727273 + 0.66667 + 0.615384 + 0.571428 + 0.53333\right] = 0.6941 \text{ (Ans)}$$

20. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using simpson one-third rule taking $n=10$ upto 4 decimal places and hence approximate the value of π.

Here, $n=10$, $a=0$, $b=1$

$$y = f(x) = \frac{1}{(1+x^2)}$$

So, $h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$

x_i	y_i
$x_0=0$	$y_0=1$
$x_1=0.1$	$y_1=0.99$
$x_2=0.2$	$y_2=0.96$
$x_3=0.3$	$y_3=0.92$
$x_4=0.4$	$y_4=0.86$
$x_5=0.5$	$y_5=0.8$
$x_6=0.6$	$y_6=0.73$
$x_7=0.7$	$y_7=0.67$
$x_8=0.8$	$y_8=0.61$
$x_9=0.9$	$y_9=0.55$
$x_{10}=1.0$	$y_{10}=0.5$

By Simpson's 1/3 rule,

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{h}{3}\left[(1+0.5) + 4(0.99+0.92+0.8+0.67+0.55) + 2(0.96+0.86+0.73+0.61)\right]$$

$$= \frac{0.1 \times 23.54}{3} = 0.7846 \text{ (ans)}$$

$$\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1 = \left[\tan^{-1}1 - \tan^{-1}0\right] = \frac{\pi}{4}$$

$$= \frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2} = 0.7846$$

$$\Rightarrow \pi/4 = 0.7846 \Rightarrow \pi = 0.7846 \times 4 = 3.1384 \approx 3.14 \text{ (ans)}$$

21. Evaluate $\int_0^1 \frac{x^2\,dx}{1+x^3}$ using simpson's one third rule taking $n=10$ upto 4 decimal places and hence approximate the value of $\log_e 2$, Here $a=0$, $b=1$ $h = \frac{b-a}{n} = \frac{1}{10} = 0.1$

By simpson 1/3 rule,

$$\int_0^1 \frac{x^2\,dx}{1+x^3} = \frac{h}{3}\left[(y_0+y_{10}) + 4(y_1+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{h}{3}\left[(1+0.49) + 4(0.5+0.32+0.19+0.14+0.11) + 2(0.44+0.25+0.16+0.12)\right]$$

$$= \frac{0.1}{3} \times 7.07 = 0.235666$$

$$= 0.2357 \text{ (ans)}$$

$$\int_0^1 \frac{x^2\,dx}{1+x^3} = \frac{1}{3}\left[\frac{dz}{z}\right]_0^1 = \frac{1}{3}\left[\log z\right]_0^1 = \frac{1}{3}\left[\log(1+x^3)\right]_0^1$$

$$= \frac{1}{3}\left[\log 2 - \log 1\right] = \frac{1}{3}\log 2$$

let $1+x^3 = z$

$3x^2\,dx = dz$

$x^2\,dx = \frac{1}{3}dz$

$$\Rightarrow \frac{1}{3}\log_e 2 = \int_0^1 \frac{x^2\,dx}{1+x^3} = 0.2357$$

$$\therefore \frac{1}{3}\log_e 2 = 0.2357$$

$$\Rightarrow \log_e 2 = 3 \times 0.2357 = 0.7071 \text{ (ans)}$$

22. The following table gives the value of accelaration (f) of a partical in cm/sec^2 at equal interval of time (t) in sec. Find the velocity of the position partical at t = 2 sec using suitable formula upto 4 decimal places.

t	0.0	0.5	1.0	1.5	2.0
f	0.3989	0.3521	0.2420	0.1295	0.0540

velocity $v = \int_0^1 f(t)\,dt$

$h = \frac{2.0}{4} = \frac{1}{2} = 0.5$

where $f(0) = 0.3989$

$f(0.5) = 0.3521$

$f(1.0) = 0.2420$

$f(1.5) = 0.1295$

$f(2.0) = 0.0540$

Now by simpsson 1/3 rule

$$\int_0^2 f(t)\,dt = \frac{h}{2}\left[f(0) + f(2) + 4\{f(0.5) + f(1.5)\} + 2f(1)\right]$$

$$= \frac{h}{3}\left[0.3989 + 0.0540 + 4\{0.3521 + 0.1295\} + 2 \times 0.2420\right]$$

$$= \frac{0.5}{3} \times 2.8633$$

$$= 0.4772 \text{ (ans)}$$