



Name: Reza Shisheie

Login ID: reshishe

Due date: 9/23/2019

1. Use a drawing tool to draw figure 6.5 (c) and (d) on page 165:

Figure.1 and 2 show the two figures from book:

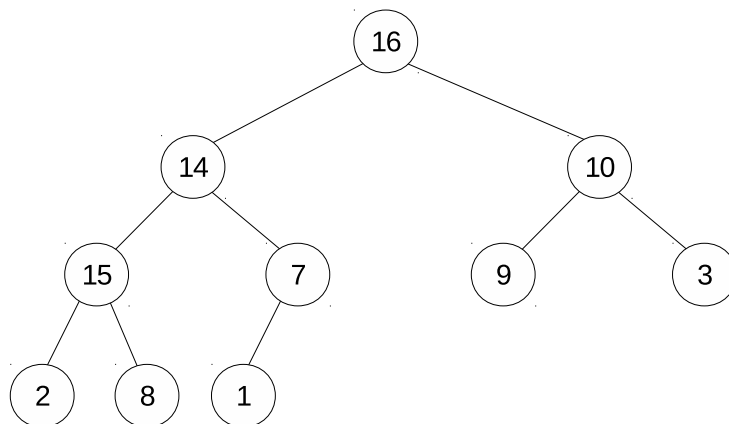


Figure 1: From book, Figure 6.5 (c) on page 165

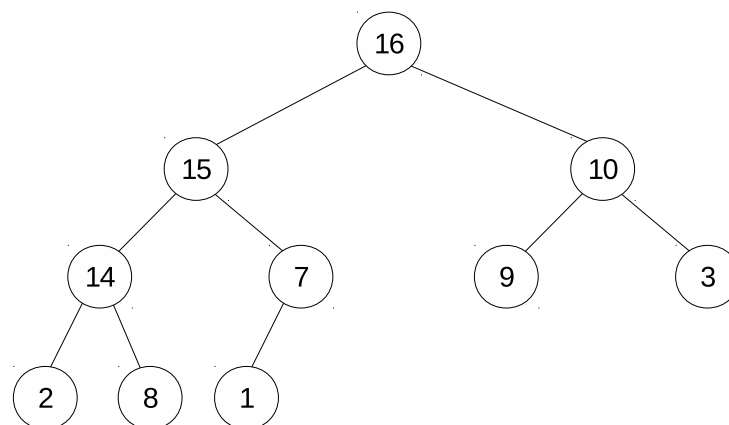


Figure 2: From book, Figure 6.5 (d) on page 165

2. Problem 3-4f (Textbook page 62):

If $f(n) = O(g(n)) \implies \exists c, n_0 > 0$ such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$

If $g(n) = \Omega(f(n)) \implies \exists c', n_0 > 0$ such that $0 \leq c'f(n) \leq g(n)$ for all $n \geq n_0$

Considering the two inequalities, if the first inequality is divided by c and $c' = 1/c$, then second inequality is achieved. Thus, two inequalities are the same and $c' = 1/c$.

3. Problem 4.3-9 (Textbook page 88):

$$T(n) = 3T(\sqrt{n}) + \log(n) \xrightarrow{n=2^m} T(2^m) = 3T(\sqrt{2^m}) + \log(2^m) \implies T(2^m) = 3T(2^{m/2}) + m$$

$$S(m)=T(2^m) \implies S(m/2)=T(2^{m/2}) \implies S(m) = 3S(m/2) + m$$

- METHOD 1: According to Figure 4.7 of textbook on page 99, the tree expansion of $S(m) = 3S(m/2) + m$ will be like Figure.3 in the following:

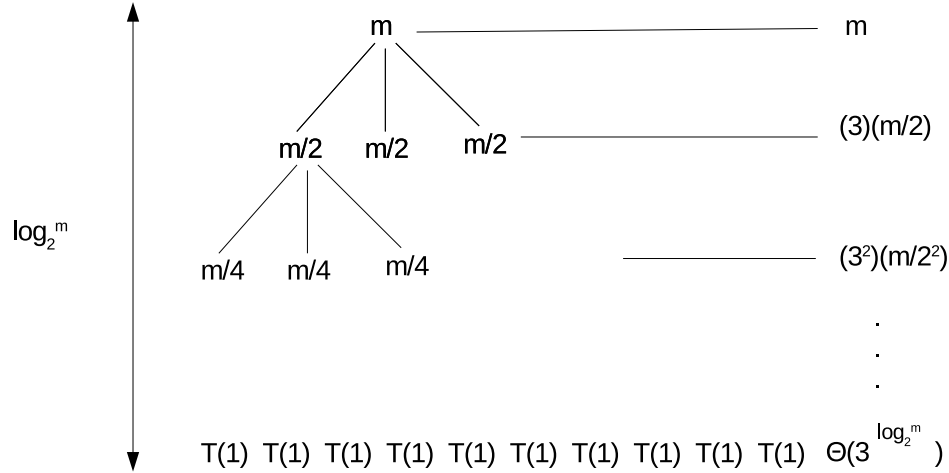


Figure 3: Expanded tree

$$S(m) = m + \frac{3}{2}m + \left(\frac{3}{2}\right)^2 m + \dots + \left(\frac{3}{2}\right)^{\log_2 m - 1} m + \Theta(3^{\log_2 m})$$

$$S(m) = \sum_{k=0}^{\log_2(m)-1} \left(\frac{3}{2}\right)^k m + \Theta(3^{\log_2 m})$$

Since $\Theta(3^{\log_2 m})$ is the dominant term in the bracket (leave dominant), $S(m)$ can be simplified to :

$$S(m) = \Theta(3^{\log_2 m})$$

By flipping m and 3, $S(m)$ becomes:

$$S(m) = \Theta(m^{\log_2 3}) \xrightarrow{m=\log_2 n} T(n) = \log_2 n^{\log_2 3}$$

- METHOD 2:

Using Master theorem for $S(m) = 3S(m/2) + m$:

$$a = 3, b = 2, f(m) = m$$

$$m^{\log_b a - \epsilon} = m^{\log_2 3 - \epsilon} = m^{1.58 - \epsilon} \stackrel{\epsilon=0.58}{=} m \implies f(m) = O(m^{\log_b a - \epsilon}) : \epsilon = 0.58$$

$$\xrightarrow{\text{Case 1}} S(m) = \Theta(m^{\log_b a}) = T(2^m) = T(n) \xrightarrow{m=\log_2 n} T(n) = \log_2 n^{\log_b a} \implies$$

$$T(n) = \log_2 n^{\log_2 3}$$

As you see, Method 1 and Method 2 yield the same result.

4. Problem 4.5-1a (Textbook page 96):

$$T(n) = 2T(n/4) + 1$$

$$a = 2, b = 4, f(n) = 1$$

$$n^{\log_b a - \epsilon} \stackrel{\epsilon=0.5}{=} n^{(\log_4 2) - 0.5} = n^0 = 1 \implies f(n) = O(n^{\log_b a - \epsilon}) : \epsilon = 0.5$$

$$\stackrel{\text{Case 1}}{\implies} T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_4 2}) = \Theta(\sqrt{n}) \implies T(n) = \Theta(\sqrt{n})$$

5. Problem 4-1b (Textbook page 107):

$$T(n) = T(7n/10) + n$$

$$a = 1, b = 10/7, f(n) = n,$$

$$n^{\log_b a + \epsilon} = n^{\log_{10/7} 1 + \epsilon} = n^{0 + \epsilon} \stackrel{\epsilon=1}{=} n^1 = n \implies f(n) = \Omega(n) \implies f(n) = \Omega(n^{\log_b a + \epsilon}) : \epsilon = 1$$

$$\text{If } c = 8/10 \text{ and } c < 1 \implies \left\{ \begin{array}{l} af(n/b) = f(7n/10) = 7n/10 \\ cf(n) = 8n/10 \end{array} \right\} \implies af(n/b) \leq cf(n) \text{ is valid!}$$

$$\stackrel{\text{Case 3}}{\implies} T(n) = \Theta(f(n)) \implies T(n) = \Theta(n)$$

6. Problem 4.1c (Textbook page 107):

$$T(n) = 16T(n/4) + n^2$$

$$a = 16, b = 4, f(n) = n^2,$$

$$n^{\log_b a} = n^{\log_4 16} = n^2 \implies f(n) = \Theta(n^{\log_b a} \cdot \log n)$$

$$\stackrel{\text{Case 2}}{\implies} f(n) = \Theta(n^2 \cdot \log n)$$