CIS 606 **Homework 2** **Fall 2019**

Name: **Reza Shisheie**

Login ID: **reshishe**

1. Use a drawing tool to draw figure 6.5 (c) and (d) on page 165:

   Figure.1 and 2 show the two figures from book:

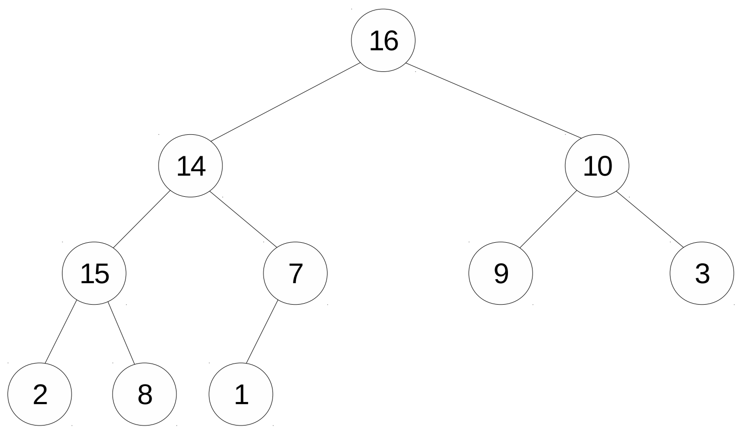

Figure 1: From book, Figure 6.5 (c) on page 165

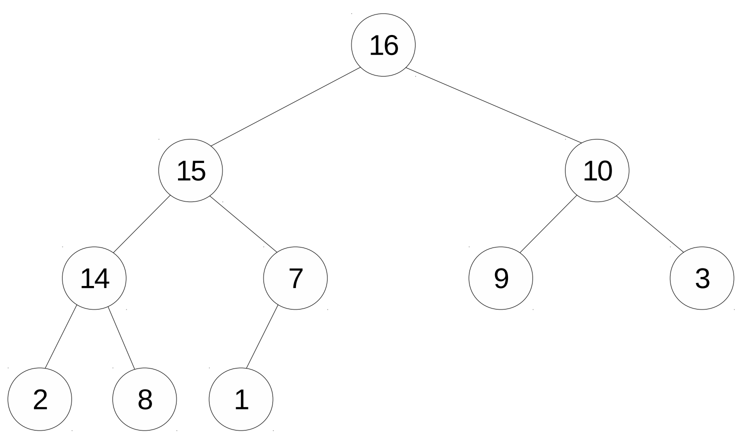

Figure 2: From book, Figure 6.5 (d) on page 165

2. Problem 3-4f (Textbook page 62):

   If $f(n) = O(g(n)) \implies \exists\ c,\ n_0 > 0$ such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$

   If $g(n) = \Omega(f(n)) \implies \exists\ c',\ n_0 > 0$ such that $0 \leq c' f(n) \leq g(n)$ for all $n \geq n_0$

   Considering the two inequalities, if the first inequality is divided by c and $c' = 1/c$, then second inequality is achieved. Thus, two inequalities are the same and $c' = 1/c$.

3. Problem 4.3-9 (Textbook page 88):

$$T(n) = 3T(\sqrt{n}) + \log(n) \overset{n=2^m}{\Longrightarrow} T(2^m) = 3T(\sqrt{2^m}) + \log(2^m) \Longrightarrow T(2^m) = 3T(2^{m/2}) + m$$
$$\overset{S(m)=T(2^m) \Longrightarrow S(m/2)=T(2^{m/2})}{\Longrightarrow} S(m) = 3S(m/2) + m$$

- METHOD 1: According to Figure 4.7 of textbook on page 99, the tree expansion of $S(m) = 3S(m/2) + m$ will be like Figure.3 in the following:

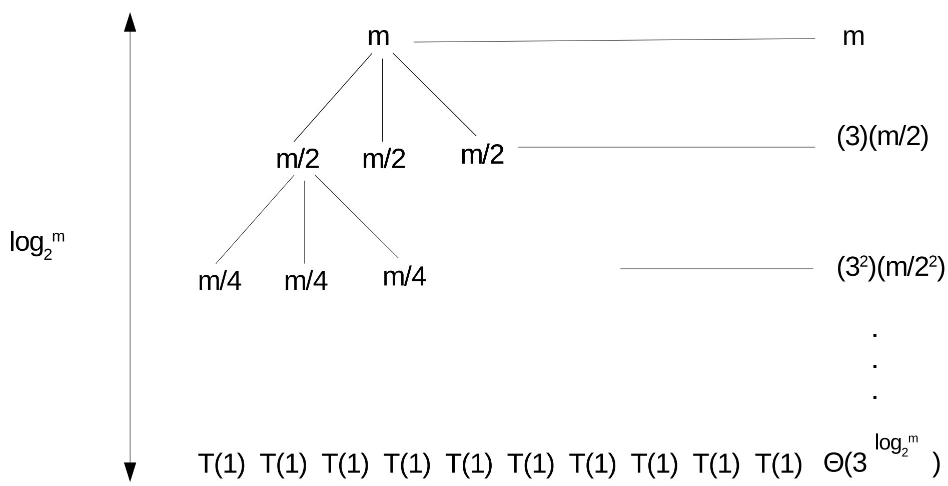

Figure 3: Expanded tree

$S(m) = m + \frac{3}{2}m + (\frac{3}{2})^2 m + ... + (\frac{3}{2})^{\log_2 m - 1} m + \Theta(3^{\log_2 m})$
$S(m) = \sum_{k=0}^{\log_2 (m-1)} (\frac{3}{2})^k m + \Theta(3^{\log_2 m})$
Since $\Theta(3^{\log_2 m})$ is the dominant term in the bracket (leave dominant), $S(m)$ can be simplified to :
$S(m) = \Theta(3^{\log_2 m})$
By flipping m and 3, $S(m)$ becomes:
$S(m) = \Theta(m^{\log_2 3}) \overset{m=\log_2 n}{\Longrightarrow} T(n) = \log_2 n^{\log_3 2}$

- METHOD 2:
Using Master theorem for $S(m) = 3S(m/2) + m$:
$a = 3,\ b = 2,\ f(m) = m$
$m^{\log_b a - \epsilon} = m^{\log_2 3 - \epsilon} = m^{1.58 - \epsilon} \overset{\epsilon=0.58}{=} m \Longrightarrow f(m) = O(m^{\log_b a - \epsilon}) : \epsilon = 0.58$
$\overset{\text{Case1}}{\Longrightarrow} S(m) = \Theta(m^{\log_b a}) = T(2^m) = T(n) \overset{m=\log_2 n}{\Longrightarrow} T(n) = \log_2 n^{\log_b a} \Longrightarrow$
$T(n) = \log_2 n^{\log_2 3}$

As you case Method 1 and Method 2 yield the same result.

4. Problem 4.5-1a (Textbook page 96):

   $T(n) = 2T(n/4) + 1$

   $a = 2,\ b = 4,\ f(n) = 1$

   $n^{\log_b a - \epsilon} \overset{\epsilon=0.5}{=} n^{(\log_4 2) - 0.5} = n^0 = 1 \implies f(n) = O(n^{\log_b a - \epsilon}) : \epsilon = 0.5$

   $\overset{\text{Case1}}{\implies} T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_4 2}) = \Theta(\sqrt{n}) \implies T(n) = \Theta(\sqrt{n})$

5. Problem 4-1b (Textbook page 107):

   $T(n) = T(7n/10) + n$

   $a = 1,\ b = 10/7,\ f(n) = n,$

   $n^{\log_b a + \epsilon} = n^{\log_{10/7} 1 + \epsilon} = n^{0+\epsilon} \overset{\epsilon=1}{=} n^1 = n \implies f(n) = \Omega(n) \implies f(n) = \Omega(n^{\log_b a + \epsilon}) : \epsilon = 1$

   If $c = 8/10$ and $c < 1 \implies \left\{ \begin{array}{l} af(n/b) = f(7n/10) = 7n/10 \\ cf(n) = 8n/10 \end{array} \right\} \implies af(n/b) \leq cf(n)$ is valid!

   $\overset{\text{Case3}}{\implies} T(n) = \Theta(f(n)) \implies T(n) = \Theta(n)$

6. Problem 4.1c (Textbook page 107):

   $T(n) = 16T(n/4) + n^2$

   $a = 16,\ b = 4,\ f(n) = n^2,$

   $n^{\log_b a} = n^{\log_4 16} = n^2 \implies f(n) = \Theta(n^{\log_b a}. \log n)$

   $\overset{\text{Case2}}{\implies} f(n) = \Theta(n^2. \log n)$