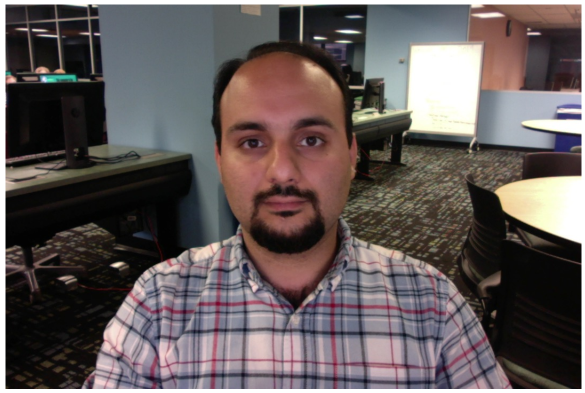
CIS 606 **Homework 3** **Fall 2019**



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1. Exercise 4.2-6 (Textbook page 83):

2. Exercise 4.2-7 (Textbook page 83):

3. Exercise 4.5-2 (Textbook page 97):

4. Problem 3-4f (Textbook page 62):

   If $f(n) = O(g(n)) \implies \exists\, c,\, n_0 > 0$ such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$

   If $g(n) = \Omega(f(n)) \implies \exists\, c',\, n_0 > 0$ such that $0 \leq c'f(n) \leq g(n)$ for all $n \geq n_0$

   Considering the two inequalities, if the first inequality is divided by c and $c' = 1/c$, then second inequality is achieved. Thus, two inequalities are the same and $c' = 1/c$.

5. Problem 4.3-9 (Textbook page 88):

$$T(n) = 3T(\sqrt{n}) + \log(n) \overset{n=2^m}{\Longrightarrow} T(2^m) = 3T(\sqrt{2^m}) + \log(2^m) \implies T(2^m) = 3T(2^{m/2}) + m$$
$$\overset{S(m)=T(2^m) \implies S(m/2)=T(2^{m/2})}{\Longrightarrow} S(m) = 3S(m/2) + m$$

- METHOD 1: According to Figure 4.7 of textbook on page 99, the tree expansion of $S(m) = 3S(m/2) + m$ will be like in the following:
  $S(m) = m + \frac{3}{2}m + (\frac{3}{2})^2 m + ... + (\frac{3}{2})^{\log_2 m - 1} m + \Theta(3^{\log_2 m})$
  $S(m) = \sum_{k=0}^{\log_2 (m-1)} (\frac{3}{2})^k m + \Theta(3^{\log_2 m})$
  Since $\Theta(3^{\log_2 m})$ is the dominant term in the bracket (leave dominant), $S(m)$ can be simpliefied to :
  $S(m) = \Theta(3^{\log_2 m})$
  By flipping m and 3, $S(m)$ becomes:
  $S(m) = \Theta(m^{\log_2 3}) \overset{m=\log_2 n}{\Longrightarrow} T(n) = \log_2 n^{\log_3 2}$

- METHOD 2:
  Using Master theorem for $S(m) = 3S(m/2) + m$:
  $a = 3,\ b = 2,\ f(m) = m$
  $m^{\log_b a - \epsilon} = m^{\log_2 3 - \epsilon} = m^{1.58 - \epsilon} \overset{\epsilon=0.58}{=} m \implies f(m) = O(m^{\log_b a - \epsilon}) : \epsilon = 0.58$
  $\overset{\text{Case1}}{\Longrightarrow} S(m) = \Theta(m^{\log_b a}) = T(2^m) = T(n) \overset{m=\log_2 n}{\Longrightarrow} T(n) = \log_2 n^{\log_b a} \implies$
  $T(n) = \log_2 n^{\log_2 3}$

As you case Method 1 and Method 2 yield the same result.

6. Problem 4.5-1a (Textbook page 96):

   $T(n) = 2T(n/4) + 1$

   $a = 2,\ b = 4,\ f(n) = 1$

   $n^{\log_b a - \epsilon} \overset{\epsilon=0.5}{=} n^{(\log_4 2)-0.5} = n^0 = 1 \implies f(n) = O(n^{\log_b a-\epsilon}) : \epsilon = 0.5$

   $\overset{\text{Case1}}{\implies} T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_4 2}) = \Theta(\sqrt{n}) \implies T(n) = \Theta(\sqrt{n})$

7. Problem 4-1b (Textbook page 107):

   $T(n) = T(7n/10) + n$

   $a = 1,\ b = 10/7,\ f(n) = n,$

   $n^{\log_b a+\epsilon} = n^{\log_{10/7} 1+\epsilon} = n^{0+\epsilon} \overset{\epsilon=1}{=} n^1 = n \implies f(n) = \Omega(n) \implies f(n) = \Omega(n^{\log_b a+\epsilon}) : \epsilon = 0.5$

   If $c = 8/10$ and $c < 1 \implies \left\{ \begin{array}{l} af(n/b) = f(7n/10) = 7n/10 \\ cf(n) = 8n/10 \end{array} \right\} \implies af(n/b) \leq cf(n)$ is valid!

   $\overset{\text{Case3}}{\implies} T(n) = \Theta(f(n)) \implies T(n) = \Theta(n)$

8. Problem 4.1c (Textbook page 107):

   $T(n) = 16T(n/4) + n^2$

   $a = 16,\ b = 4,\ f(n) = n^2,$

   $n^{\log_b a} = n^{\log_4 16} = n^2 \implies f(n) = \Theta(n^{\log_b a}.\log n)$

   $\overset{\text{Case2}}{\implies} f(n) = \Theta(n^2.\log n)$