MCE/EEC 647/747
Homework 3 - Spring 2019
Due 03/07/19 at the beginning of class. Do not email m-files, but include code listings with comments.

1. (20 pt) Consider the quadratic equation:

$$116x_0^2 + 56y_0^2 + 44z_0^2 + 64x_0y_0 + 16x_0z_0 - 8y_0z_0 = 9$$

It defines an ellipsoid in coordinate frame 0. Use the singular value decomposition to do the following:

- Find the principal axis lengths

- Find a new coordinate frame 1 where the equation of the ellipsoid appears in the canonical form

$$x_1^2/a^2 + y_2^2/b^2 + z_2^2/c^2 = 1$$

  Give the rotation matrix between frames 0 and 1 and the Euler angles defining the orientation of the ellipsoid in frame 0. Use this information to make a hand sketch of the ellipsoid.

- Download the Matlab ellipsoidal toolbox by Alex Kurzhanskiy, available through the Matlab File Exchange. Use it to obtain a 3D plot of the ellipsoid.

- Verify your calculations by plotting the three principal axis as obtained from the U and Σ matrix of the singular value decomposition, overlaying the plot of the ellipsoid. Show one 3D view and three plan views where the axes can be easily compared to the ellipsoid's boundary.

2. (80 pt) Consider again the PP robot with spherical wrist used for HW2. Refer to the solution posted on the website and use the DH frame assigments from there. Do the following:

- Obtain all transformations necessary to compute the Jacobian. As a verification reference, set $q_1 = 1, q_2 = -1, q_3 = \pi/2, q_4 = \pi/2, q_5 = 0$. Your H_5^0 should evaluate to:

$$\begin{bmatrix} 0 & 0 & -1 & -1 \\ -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

- Find the velocity Jacobian J_v using symbolic processing.

- As a cautionary note, try the `rank` command on the symbolic J_v, with unspecified values for q. Then evaluate J_v using the test values of q above, entering `sym(pi)` for π. Comment on the results.

- Based on the above, can J_v be singular at certain q values? For example which ones?

- Select a q that produces singularity and provide a physical/geometric explanation of how this occurs.

- Compute Yoshikawa's manipulability measure $\mu(q)$ using symbolic processing. Provide a mathematical and/or physical explanation for the result.

- For the above test values of q, describe the set of all joint velocities that result in an instantaneous zero velocity for P. Give an example of such a joint velocity vector (other than the zero vector) and verify it with a forward calculation.

- Calculate the angular velocity Jacobian J_w and exclude the first two columns. This is equivalent to considering the wrist only, and only orientation kinematics.

- For the above reduced angular velocity Jacobian, compute Yoshikawa's manipulability measure in symbolic form. It predicts singularity for certain values of q_4. Interpret geometrically.