HW II   Reza Shisheie   Due 2/19/19

PROBLEM 1

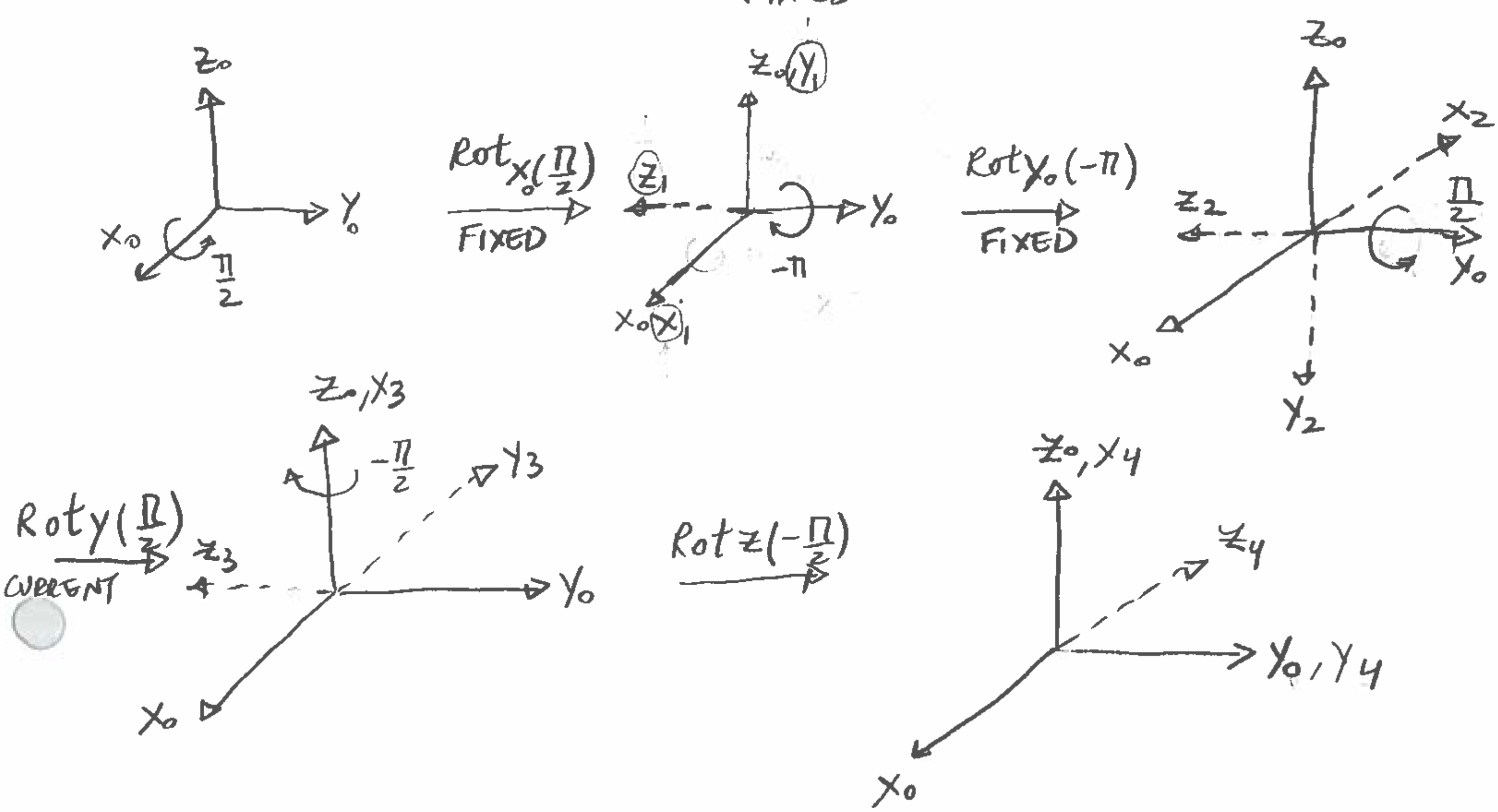
$$\underset{\text{FIXED}}{Rot_{x_0}\left(\tfrac{\pi}{2}\right)} \rightarrow \underset{\text{FIXED}}{Rot_{y_0}(-\pi)} \rightarrow \underset{\text{CURRENT}}{Rot_y\left(\tfrac{\pi}{2}\right)} \rightarrow \underset{\text{FIXED}}{Rot_{z_0}\left(-\tfrac{\pi}{2}\right)}$$

$$\underset{④}{Rot_z\left(-\tfrac{\pi}{2}\right)^{F}} \cdot \underset{②}{\underset{F}{Rot_y(-\pi)}} \cdot \underset{①}{\underset{F}{Rot_x\left(\tfrac{\pi}{2}\right)}} \cdot \underset{③}{\underset{C}{Rot_y\left(\tfrac{\pi}{2}\right)}}$$

FIXED

CURRENT

FIXED



The solution is as computed by MATLAB CODE

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\theta = \operatorname{atan2}\left(\sqrt{1 - r_{33}^2},\ r_{33}\right)$$
$$\phi = \operatorname{atan2}(r_{23},\ r_{13})$$
$$\psi = \operatorname{atan2}(r_{32},\ -r_{31})$$

$$\downarrow$$

$$[\theta = 180,\ \phi = 180,\ \psi = 0]$$

$$\theta = \operatorname{atan2}\left(-\sqrt{1 - r_{33}^2},\ r_{33}\right)$$
$$\phi = \operatorname{atan2}(-r_{23},\ -r_{13})$$
$$\psi = \operatorname{atan2}(-r_{32},\ r_{31})$$

$$[\theta = 0,\ \phi = 0,\ \psi = 180]$$

$$\downarrow$$

PLEASE CHECK MATLAB CODE FOR VALIDATION

Both results yield the same rotation matrix so ther are 2 solutions

Code

```
clc;clear all; close all;
format short
format compact

%% computing rotations
syms q1 q2 q3 q4
R1 = Rx(q1)
R2 = Ry(q2)
R3 = Ry(q3)
R4 = Rx(q4)

%% computing final rotation
Rot = R4*R2*R1*R3

%% evaluating rotation matrix
q1=pi/2
q2=-pi
q3=pi/2
q4=-pi/2
Rot = eval(Rot)
%Res_deg = radtodeg(tr2eul(Rot))

%% computing euler angles
theta1 = atan2( sqrt(1-Rot(3,3)^2), Rot(3,3));
phi1 = atan2(Rot(2,3),Rot(1,3));
psi1 = atan2(Rot(3,2),-Rot(3,1));
res1 = [phi1, theta1, psi1 ]
%res1_d = radtodeg([phi1, phi1, psi1 ])

theta2 = atan2( -sqrt(1-Rot(3,3)^2), Rot(3,3));
phi2 = atan2(-Rot(2,3),-Rot(1,3));
psi2 = atan2(-Rot(3,2),Rot(3,1));
res2 = [phi2, theta2, psi2 ]
%res2_d = radtodeg([phi2, phi2, psi2 ])

%% checking to see if the same rotation is resulted
Rot_check_1 = Rz(phi1)*Ry(theta1)*Rz(psi1) - Rot
Rot_check_2 = Rz(phi2)*Ry(theta2)*Rz(psi2) - Rot
```

Result

```
R1 =
[ 1,       0,        0]
[ 0, cos(q1), -sin(q1)]
[ 0, sin(q1),  cos(q1)]
R2 =
[  cos(q2), 0, sin(q2)]
[        0, 1,       0]
[ -sin(q2), 0, cos(q2)]
R3 =
[  cos(q3), 0, sin(q3)]
[        0, 1,       0]
[ -sin(q3), 0, cos(q3)]
R4 =
[ 1,       0,        0]
```

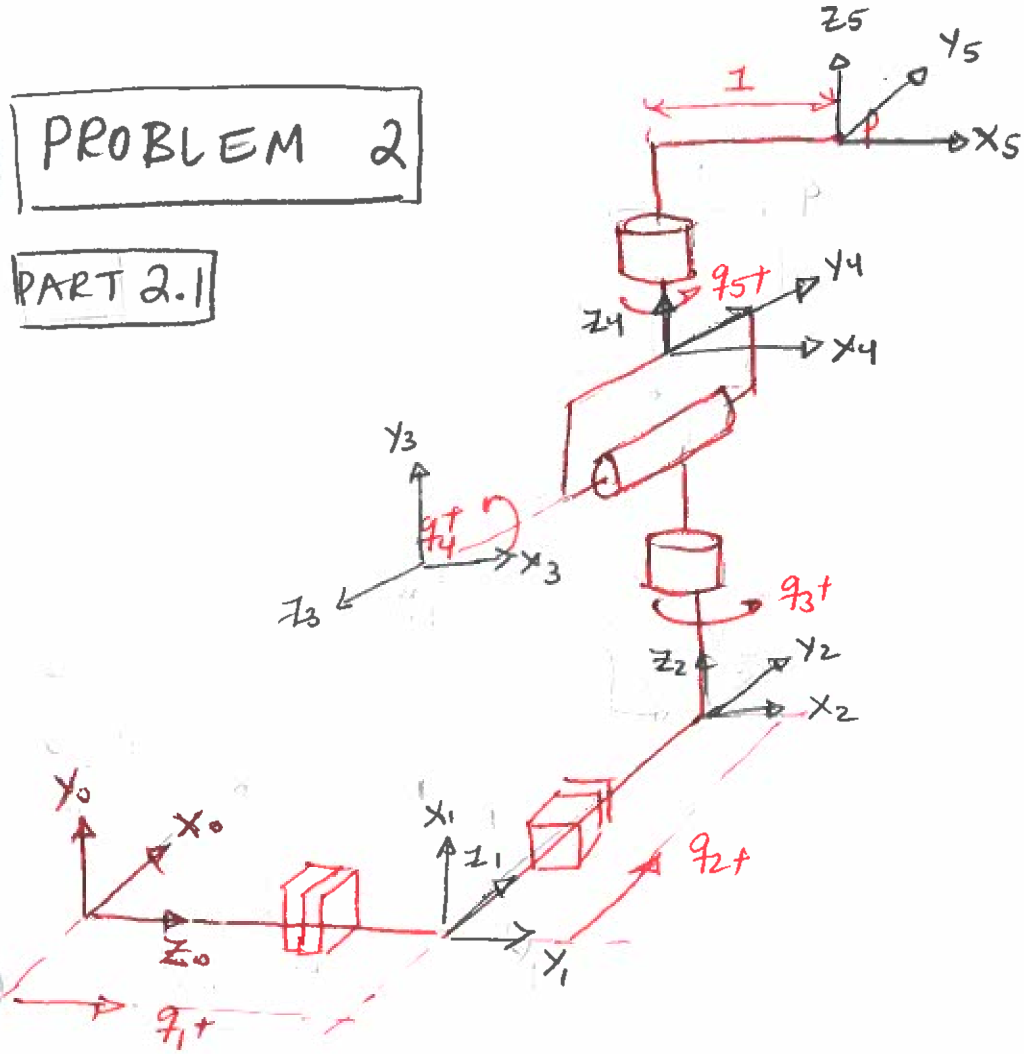
```
[ 0, cos(q4), -sin(q4)]
[ 0, sin(q4),  cos(q4)]
Rot =
[                                     cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3),
sin(q1)*sin(q2),                                         cos(q2)*sin(q3) +
cos(q1)*cos(q3)*sin(q2)]
[ sin(q3)*(cos(q4)*sin(q1) + cos(q1)*cos(q2)*sin(q4)) + cos(q3)*sin(q2)*sin(q4),
cos(q1)*cos(q4) - cos(q2)*sin(q1)*sin(q4),   sin(q2)*sin(q3)*sin(q4) -
cos(q3)*(cos(q4)*sin(q1) + cos(q1)*cos(q2)*sin(q4))]
[ sin(q3)*(sin(q1)*sin(q4) - cos(q1)*cos(q2)*cos(q4)) - cos(q3)*cos(q4)*sin(q2),
cos(q1)*sin(q4) + cos(q2)*cos(q4)*sin(q1), - cos(q3)*(sin(q1)*sin(q4) -
cos(q1)*cos(q2)*cos(q4)) - cos(q4)*sin(q2)*sin(q3)]
q1 =
    1.5708
q2 =
   -3.1416
q3 =
    1.5708
q4 =
   -1.5708
Rot =
   -0.0000   -0.0000   -1.0000
    0.0000   -1.0000    0.0000
   -1.0000   -0.0000    0.0000
res1 =
    3.1416    1.5708   -0.0000
res2 =
   -0.0000   -1.5708    3.1416
Rot_check_1 =
   1.0e-31 *
         0         0         0
         0         0         0
         0         0   -0.1233
Rot_check_2 =
   1.0e-31 *
         0         0         0
         0         0         0
         0         0   -0.1233
```

THEY GENERATE THE SAME MATRIX

As you see both of the checked results yield 0 matrix which means that both solutions are valid

PROBLEM 2

PART 2.1



$$A = Rot_{z,\theta} * Tran_{z,d} * Trans_{x,a} * Rot_{x,\alpha}$$

	$Rot_{z,\theta}$	$Tran_{z,d}$	$Trans_{x,a}$	$Rot_{x,\alpha}$
A_1^0	$\pi/2$	q_1	–	$\pi/2$
A_2^1	$\pi/2$	q_2	–	$\pi/2$
A_3^2	q_3	–	–	$\pi/2$
A_4^3	q_4	–	–	$-\pi/2$
A_5^4	q_5	–	1	0

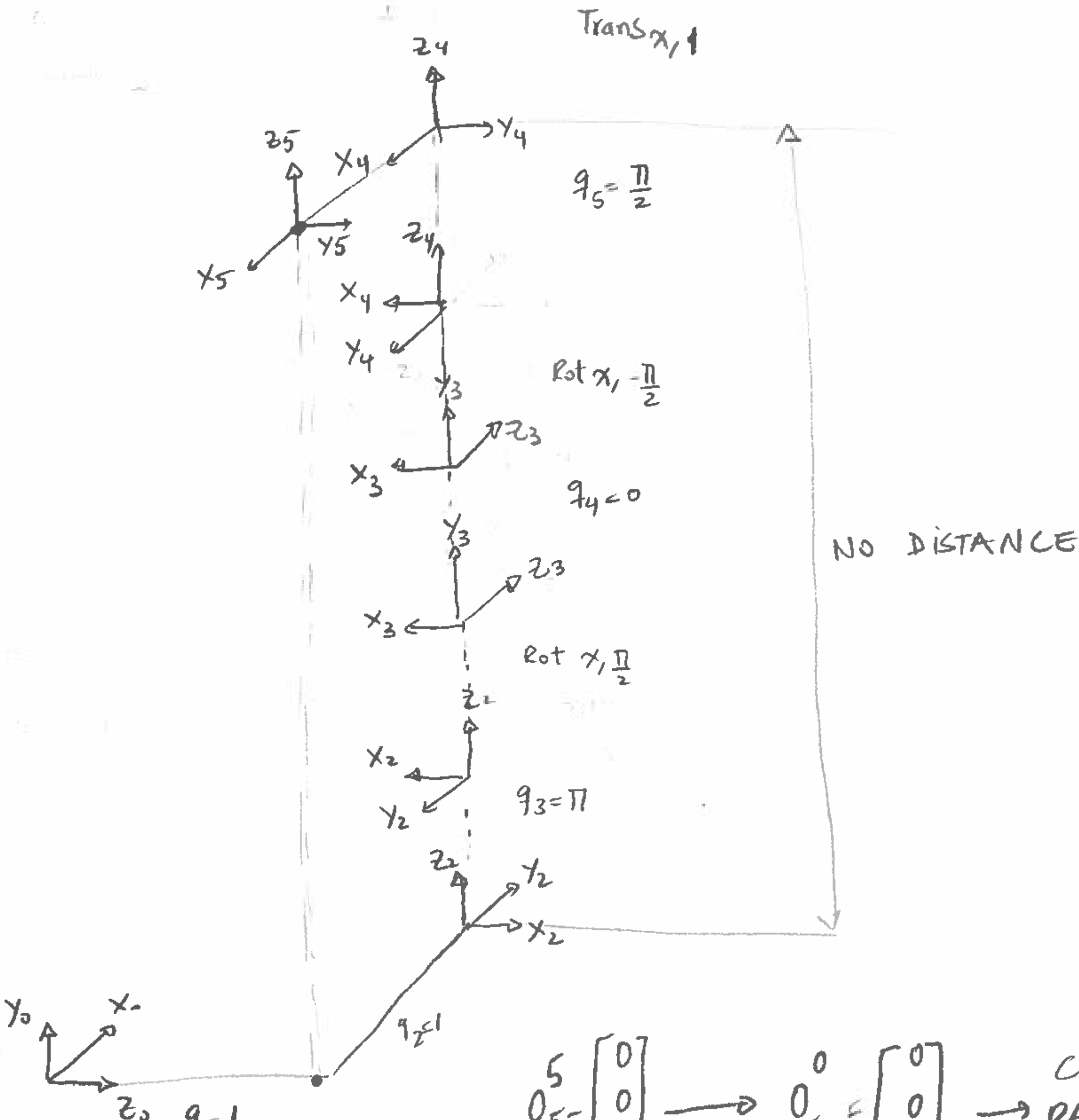
PART 2.2:

```
clc;clear all; close all;
format short
format compact

%% computing rotations
syms q1 q2 q3 q4 q5
H10 = Rotz(pi/2)*Transz(q1)*Transx(0)*Rotx(pi/2);
H21 = Rotz(pi/2)*Transz(q2)*Transx(0)*Rotx(pi/2);
H32 = Rotz(q3)*Transz(0)*Transx(0)*Rotx(pi/2);
H43 = Rotz(q4)*Transz(0)*Transx(0)*Rotx(-pi/2);
H54 = Rotz(q5)*Transz(0)*Transx(1)*Rotx(0);

vpa(H10,2)
vpa(H21,2)
vpa(H32,2)
vpa(H43,2)
vpa(H54,2)

H50 = H10*H21*H32*H43*H54;
vpa(H50,2);

%% PART A
q1 = 1;
q2 = 1;
q3 = pi;
q4 = 0;
q5 = pi/2;

%% PART B
% q1 = 1;
% q2 = -1;
% q3 = pi/2;
% q4 = pi/2;
% q5 = 0;

%% RESULTS
H50 = eval(H50)
p55 = [0,0,0,1]
p05 = (H50*p55')'
```

As you see all 4 transformations Rotz,Transz, Transx, Rotx are included in each line. It must be noted that if the value corresponding to each 4x4 matrix is zero, it yields a 4x4 identity matrix which has not effect on the final outcome.

PART 2.3 - A

$q_1 = 1$, $q_2 = 1$, $q_3 = \pi$, $q_4 = 0$, $q_5 = \pi/2$



$$O_5^5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow O_5^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ → CHECK NEXT PAGE FOR CODE

PART 2.3.A:

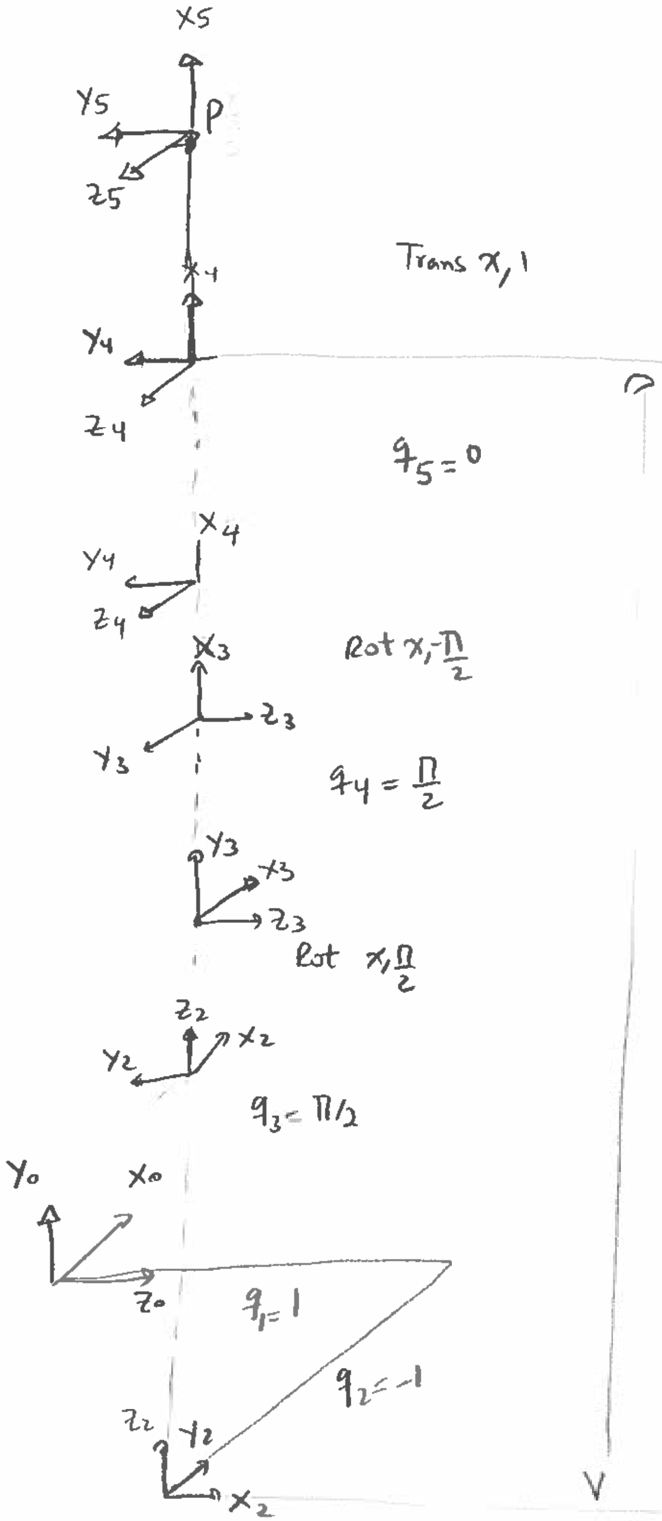
Solution for q1 = 1; q2 = 1; q3 = pi; q4 = 0; q5 = pi/2;

```
ans =
[ 6.1e-17, -6.1e-17,      1.0,   0]
[     1.0,  3.7e-33, -6.1e-17,   0]
[       0,      1.0,  6.1e-17,  q1]
[       0,        0,        0, 1.0]
ans =
[ 6.1e-17, -6.1e-17,      1.0,   0]
[     1.0,  3.7e-33, -6.1e-17,   0]
[       0,      1.0,  6.1e-17,  q2]
[       0,        0,        0, 1.0]
ans =
[ cos(q3), -6.1e-17*sin(q3),      sin(q3),   0]
[ sin(q3),  6.1e-17*cos(q3), -1.0*cos(q3),   0]
[       0,              1.0,      6.1e-17,   0]
[       0,                0,            0, 1.0]
ans =
[ cos(q4), -6.1e-17*sin(q4), -1.0*sin(q4),   0]
[ sin(q4),  6.1e-17*cos(q4),      cos(q4),   0]
[       0,             -1.0,      6.1e-17,   0]
[       0,                0,            0, 1.0]
ans =
[ cos(q5), -1.0*sin(q5),   0, cos(q5)]
[ sin(q5),      cos(q5),   0, sin(q5)]
[       0,            0, 1.0,       0]
[       0,            0,   0,     1.0]
H50 =
   -1.0000   -0.0000    0.0000    0.0000
    0.0000    0.0000    1.0000    0.0000
   -0.0000    1.0000   -0.0000    1.0000
         0         0         0    1.0000
p55 =
     0     0     0     1
p05 =
    0.0000    0.0000    1.0000    1.0000
```

The location of origin in the 5th coordinate which is [0,0,0] is mapped to [0,0,1] in the world frame. This proves the sketch in previous page

PART 2.3 – B

$q_1 = 1 \qquad q_2 = -1 \qquad q_3 = \pi/2 \qquad q_4 = \frac{\pi}{2} \qquad q_5 = 0$



$$p_5^5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow p_5^0 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

NUMERICAL PROOF IS IN NEXT PAGE

PART 2.3.B:

Solution for q1 = 1; q2 = -1; q3 = pi/2; q4 = pi/2; q5 = 0;

```
ans =
[ 6.1e-17, -6.1e-17,      1.0,   0]
[     1.0,  3.7e-33, -6.1e-17,   0]
[       0,      1.0,  6.1e-17,  q1]
[       0,        0,        0, 1.0]
ans =
[ 6.1e-17, -6.1e-17,      1.0,   0]
[     1.0,  3.7e-33, -6.1e-17,   0]
[       0,      1.0,  6.1e-17,  q2]
[       0,        0,        0, 1.0]
ans =
[ cos(q3), -6.1e-17*sin(q3),       sin(q3),   0]
[ sin(q3),  6.1e-17*cos(q3), -1.0*cos(q3),   0]
[       0,              1.0,      6.1e-17,   0]
[       0,                0,            0, 1.0]
ans =
[ cos(q4), -6.1e-17*sin(q4), -1.0*sin(q4),   0]
[ sin(q4),  6.1e-17*cos(q4),      cos(q4),   0]
[       0,             -1.0,      6.1e-17,   0]
[       0,                0,            0, 1.0]
ans =
[ cos(q5), -1.0*sin(q5),   0, cos(q5)]
[ sin(q5),      cos(q5),   0, sin(q5)]
[       0,            0, 1.0,       0]
[       0,            0,   0,     1.0]
H50 =
    0.0000    0.0000   -1.0000   -1.0000
    1.0000   -0.0000    0.0000    1.0000
   -0.0000   -1.0000   -0.0000    1.0000
         0         0         0    1.0000
p55 =
     0     0     0     1
p05 =
   -1.0000    1.0000    1.0000    1.0000
```

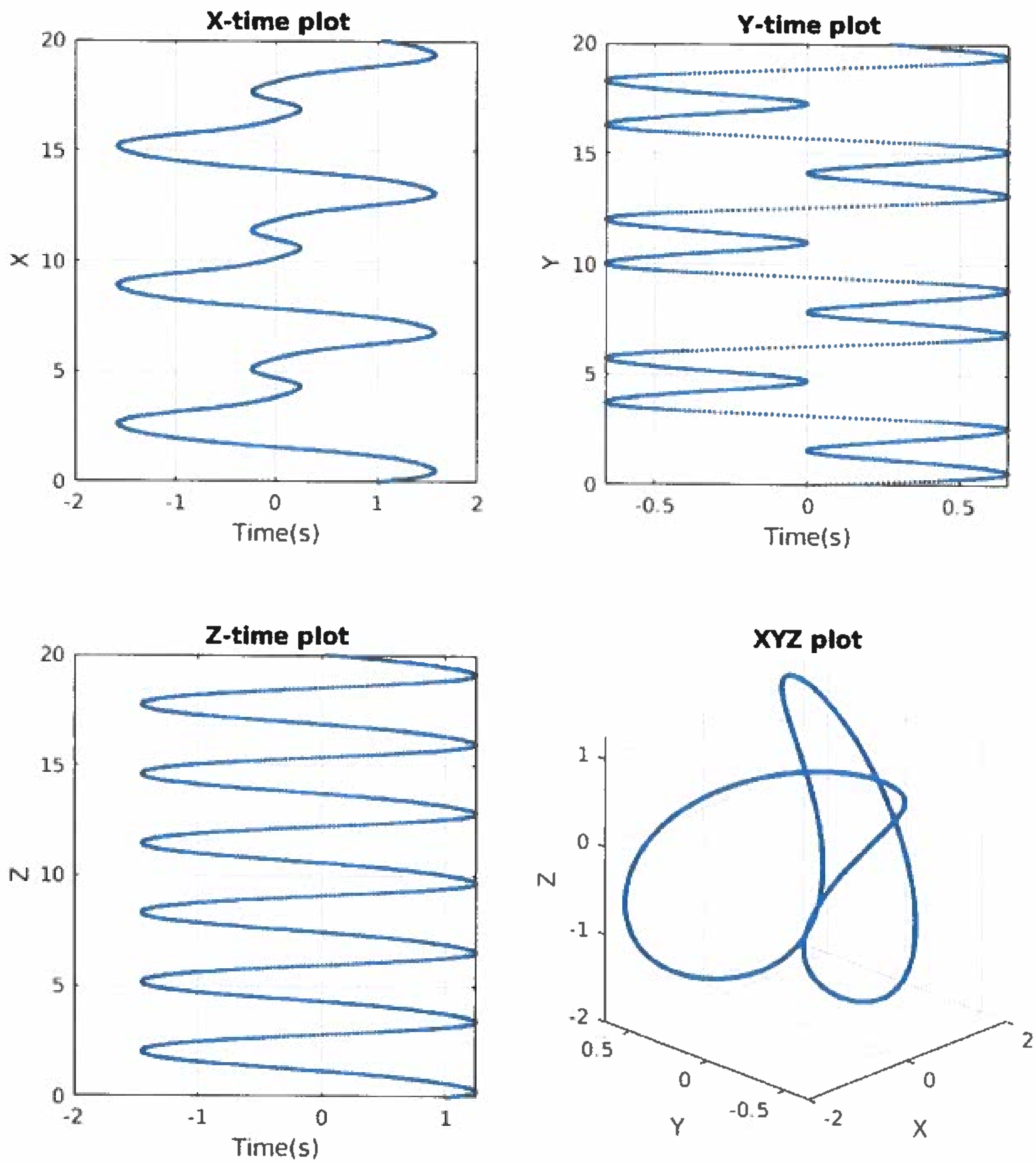
The location of origin in the 5th coordinate which is [0,0,0] is mapped to [-1,1,1] in the world frame. This proves the sketch in previous page

PART 2.4: COMPUTE q as a function

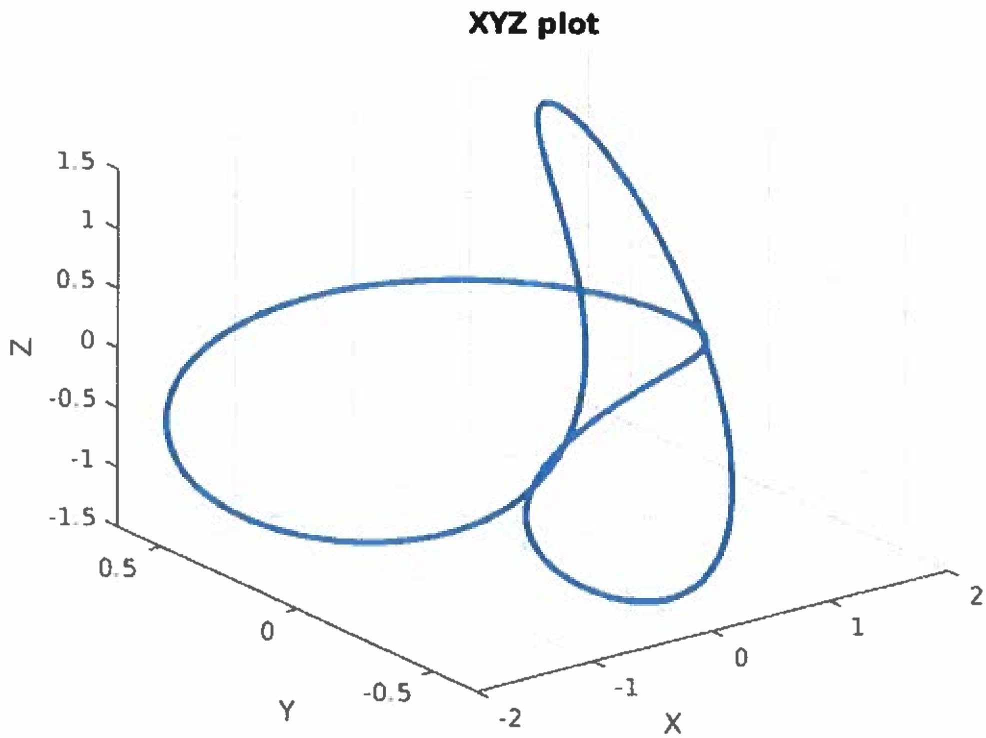
```
clc;clear all; close all;
format short
format compact

%% computing rotations
syms q1 q2 q3 q4 q5
H10 = Rotz(pi/2)*Transz(q1)*Transx(0)*Rotx(pi/2);
H21 = Rotz(pi/2)*Transz(q2)*Transx(0)*Rotx(pi/2);
H32 = Rotz(q3)*Transz(0)*Transx(0)*Rotx(pi/2);
H43 = Rotz(q4)*Transz(0)*Transx(0)*Rotx(-pi/2);
H54 = Rotz(q5)*Transz(0)*Transx(1)*Rotx(0);

H50 = H10*H21*H32*H43*H54;
x(1) = 0;
y(1) = 0;
z(1) = 0;

i = 1;
for t=0:.01:20
    q1 = sin(2*t);
    q2 = cos(t);
    q3 = t;              --> q(t)
    q4 = sin(2*t);
    q5 = t;
    H = eval(H50);
    p55 = [0,0,0,1];
    p05 = (H*p55')';
    x(i) = p05(1,1);
    y(i) = p05(1,2);
    z(i) = p05(1,3);
    time(i)=t;
    i = i+1;
    t
end


%% PART 2.5
subplot(2,2,1);
plot(x,time,'.')
xlabel('Time(s)')
ylabel('X')
title('X-time plot')
grid on

subplot(2,2,2);
plot(y,time,'.')
xlabel('Time(s)')
ylabel('Y')
title('Y-time plot')
grid on

subplot(2,2,3);
plot(z,time,'.')
xlabel('Time(s)')
ylabel('Z')
```

```
title('Z-time plot')
grid on

subplot(2,2,4);
plot3(x,y,z,'.')
xlabel('X')
ylabel('Y')
zlabel('Z')
title('XYZ plot')
grid on


%% PART 2.6
figure
plot3(x,y,z,'.')
xlabel('X')
ylabel('Y')
zlabel('Z')
title('XYZ plot')
grid on


%% PART 2.7
figure
subplot(2,2,1);
plot(x,y,'.')
xlabel('X')
ylabel('Y')
title('XY projection')
grid on

subplot(2,2,2);
plot(x,z,'.')
xlabel('X')
ylabel('Z')
title('XZ projection')
grid on

subplot(2,2,3);
plot(y,z,'.')
xlabel('Y')
ylabel('Z')
title('YZ projection')
grid on

subplot(2,2,4);
plot3(x,y,z,'.')
xlabel('X')
ylabel('Y')
zlabel('Z')
title('XYZ plot')
grid on
```

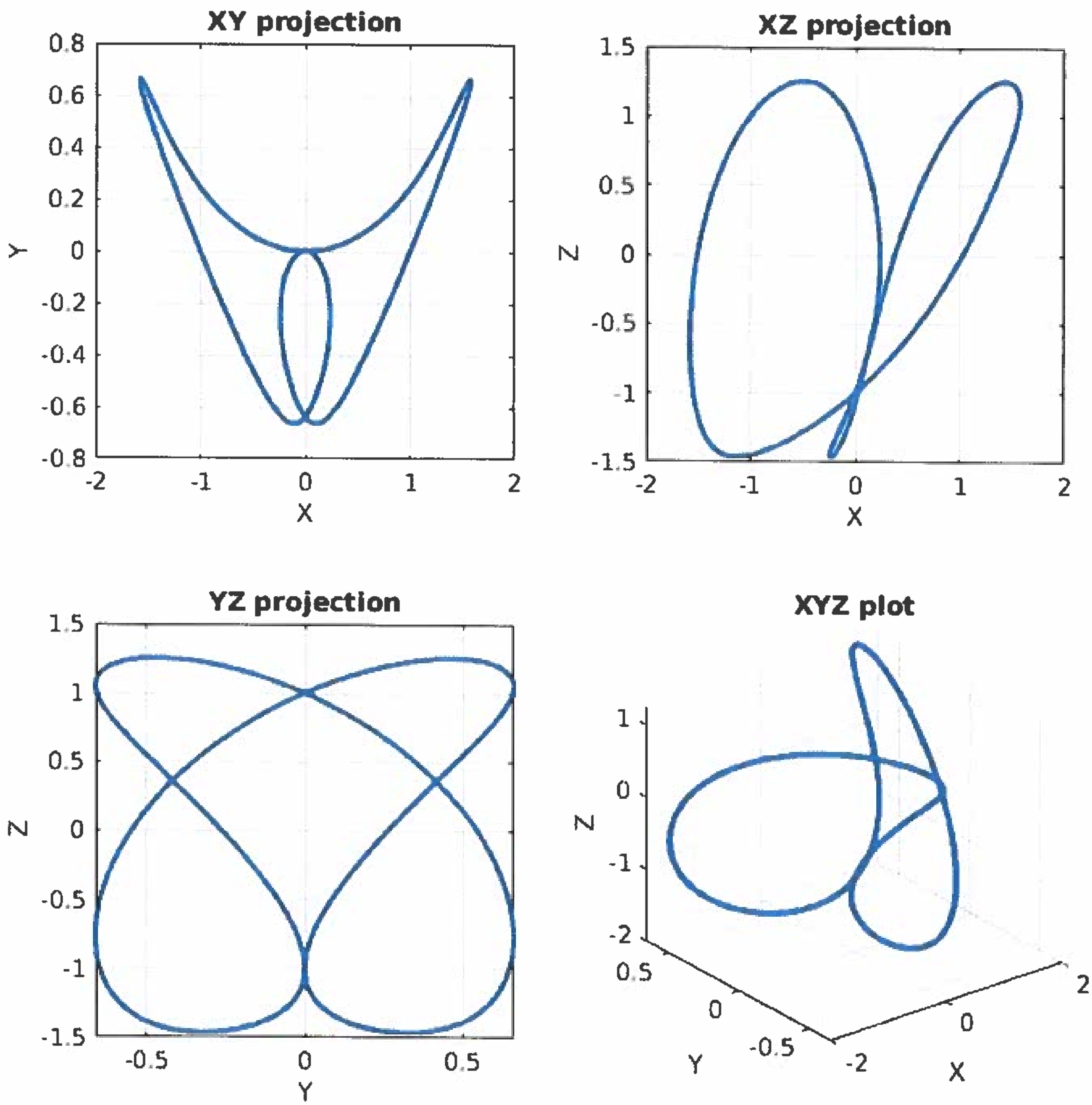
PART 2.5:


X-time plot
X
Time(s)
Y-time plot
Y
Time(s)
Z-time plot
Z
Time(s)
XYZ plot
Z
Y
X

PART 2.6:



PART 2.7:


XY projection
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Y
-2
-1
0
1
2
X
XZ projection
1.5
1
0.5
0
-0.5
-1
-1.5
Z
-2
-1
0
1
2
X
YZ projection
1.5
1
0.5
0
-0.5
-1
-1.5
Z
-0.5
0
0.5
Y
XYZ plot
1
0
-1
-2
Z
0.5
0
-0.5
Y
-2
0
2
X

2.8:

Here is a list of all the functions. All of these functions are available in the file directory:

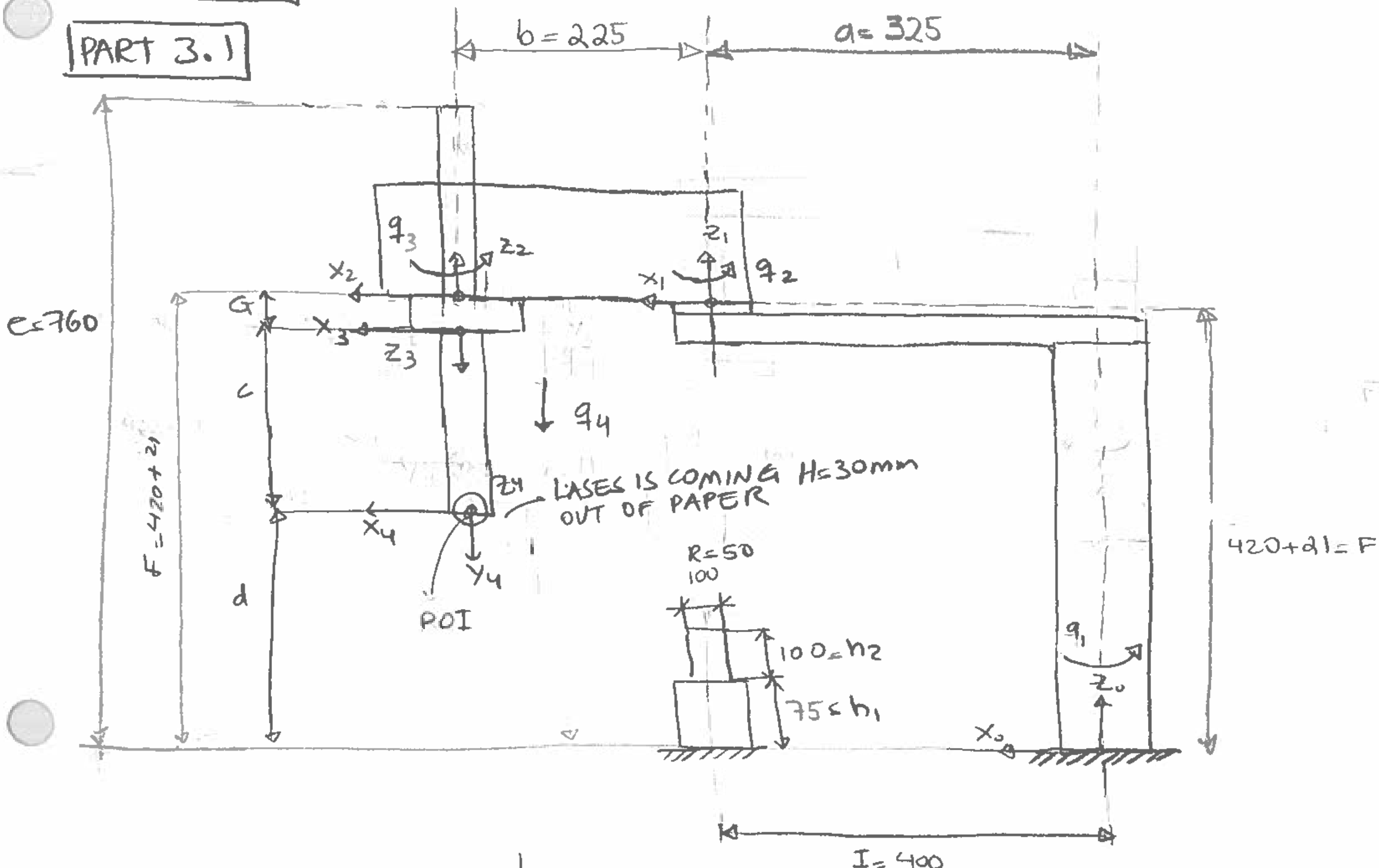
Rotz(q)
Transz(d)
Transx(a)
Rotx(q)

Rx(q)
Ry(q)
Rz(q)

PROBLEM 3

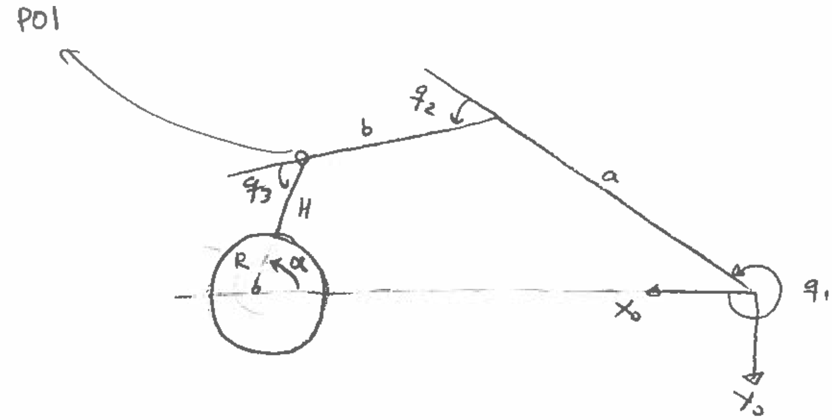
$G = 420 + 21 - d - c$

PART 3.1



a = 325
b = 225
c = 150
d = 225
e = 760
F = 420 + 21
G = F − d − c
H = 30
I = 400
R = 50
$h_2 = 100$
$h_1 = 75$

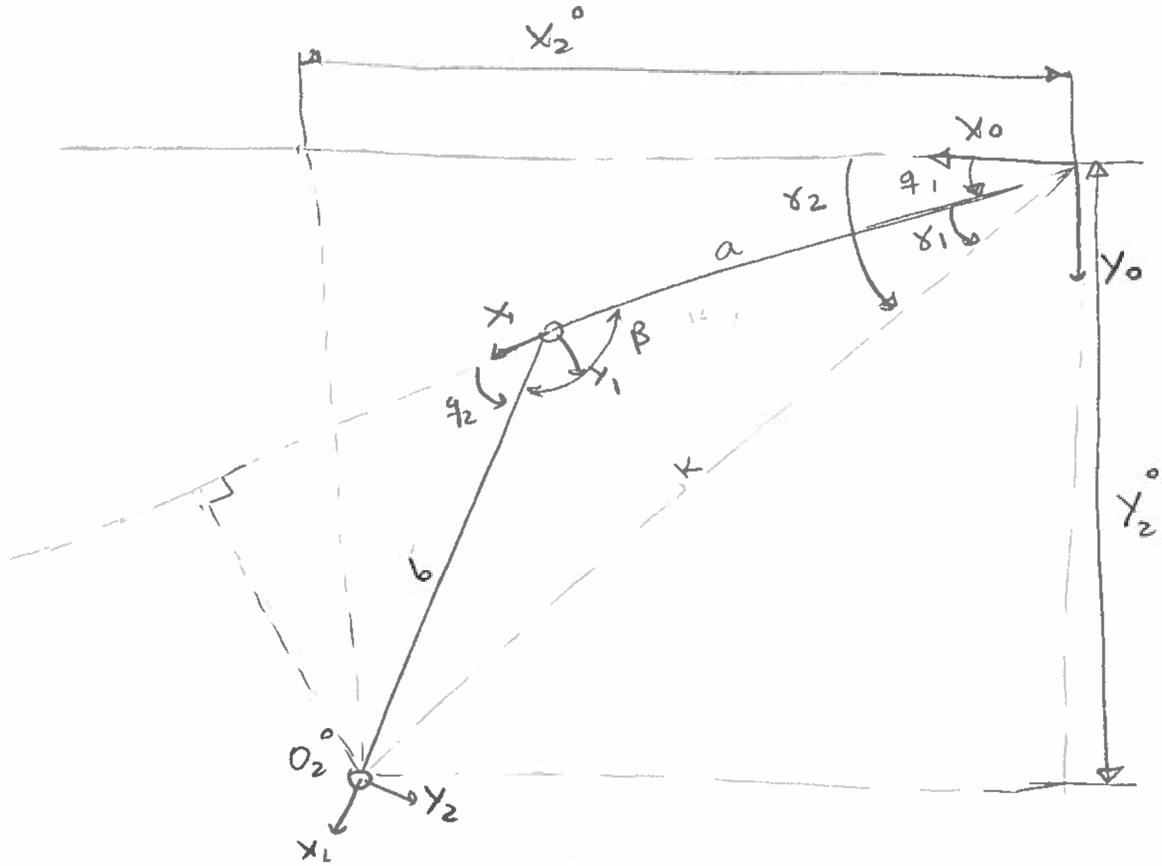
	Rotz, θ	Trans z, d	Trans x, a	Rot x, α
${}^{0}A_1$	q_1	F	a	—
${}^{1}A_2$	q_2	—	b	—
${}^{2}A_3$	q_3	− G	—	π
${}^{3}A_4$	—	q_4	—	$+\frac{\pi}{2}$



LETS DEVIDE PROBLEM TO TWO SUB PROBLEMS :

$A^0_2 \longrightarrow$ Solve for q_1 and q_2

$A^2_4 \longrightarrow$ Solve for q_3 and q_4

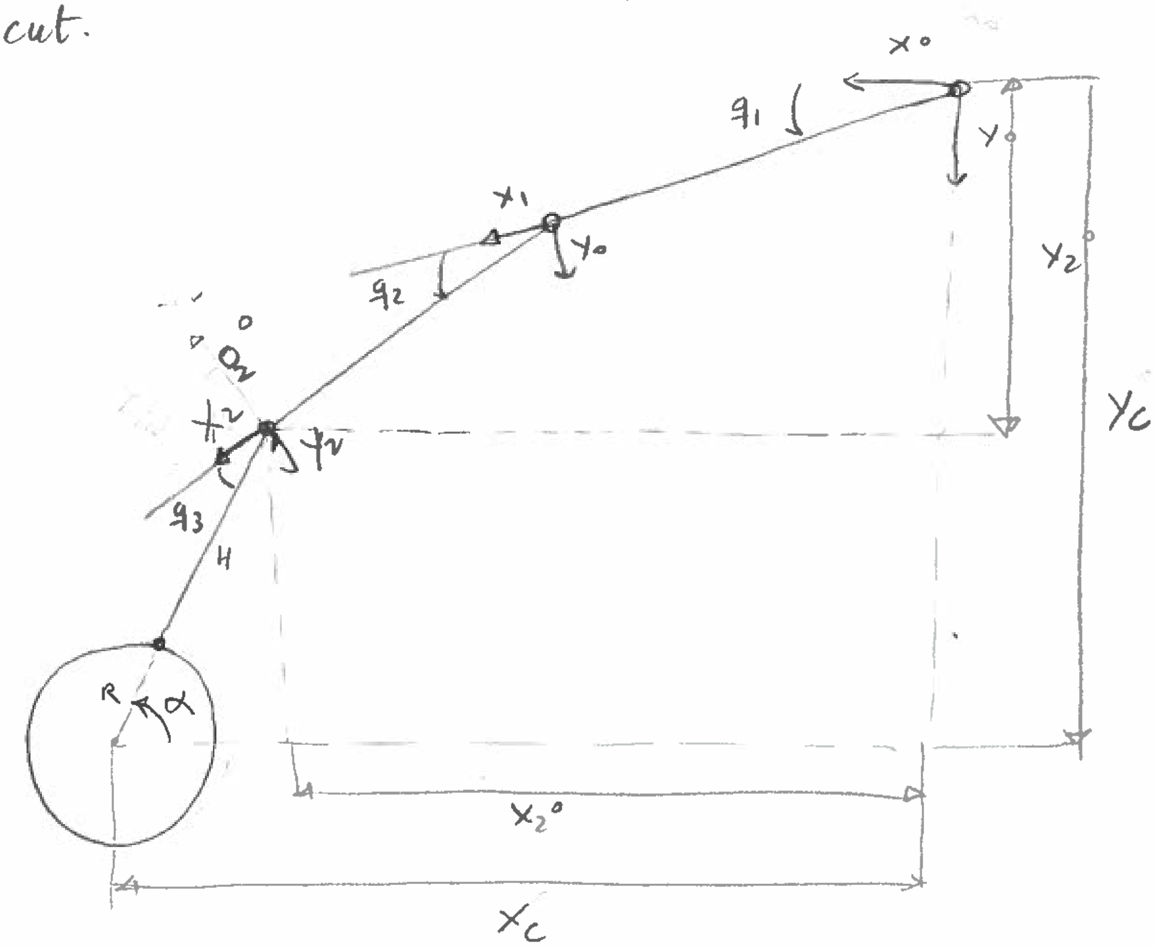


$$K = \sqrt{(X_2^0)^2 + (Y_2^0)^2}$$

$$\beta = \cos^{-1}\left(\frac{a^2+b^2-K^2}{2ab}\right) \longrightarrow \boxed{q_2 = \pi - \beta}\ ✓$$

$$\left.\begin{aligned} \gamma_1 &= tg^{-1}\left(\frac{b\sin(q_2)}{a+b\cos(q_2)}\right) \\ \gamma_2 &= tg^{-1}\left(\frac{Y_2^0}{X_2^0}\right) \\ \gamma_2 - \gamma_1 &= q_1 \end{aligned}\right\} \quad \boxed{q_1 = tg^{-1}\left(\frac{Y_2^0}{X_2^0}\right) - tg^{-1}\left(\frac{b\sin(q_2)}{a+b\cos(q_2)}\right)}\ ✓$$

α is a variable which defines the angle to which window is cut.

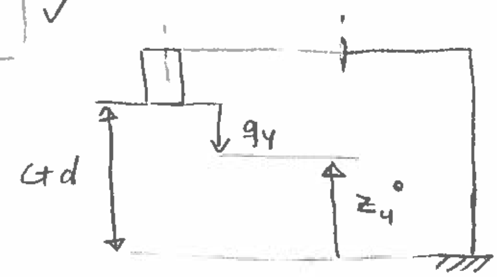


$$x_2^{\circ} = x_c - (R+H)\cos\alpha$$

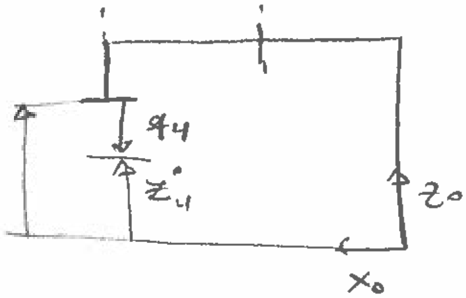
$$y_2^{\circ} = y_c - (R+H)\sin\alpha$$

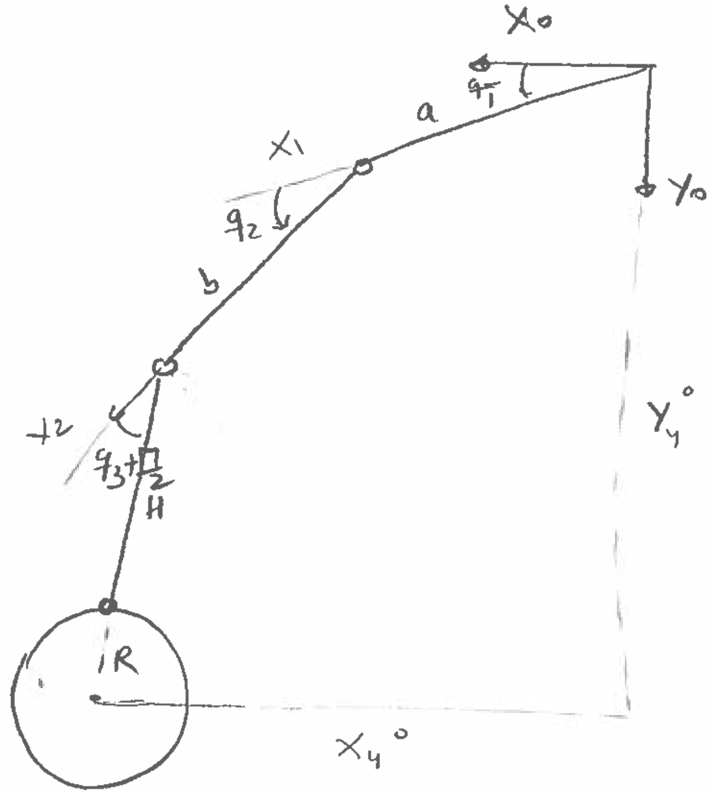
$$q_1 + q_2 + q_3 = \alpha \longrightarrow \boxed{q_3 = \alpha - q_1 - q_2}$$ ✓

$$z_c = c+d - q_4 \longrightarrow \boxed{q_4 = c+d - z_c}$$ ✓



FORWARD KINEMATICS :





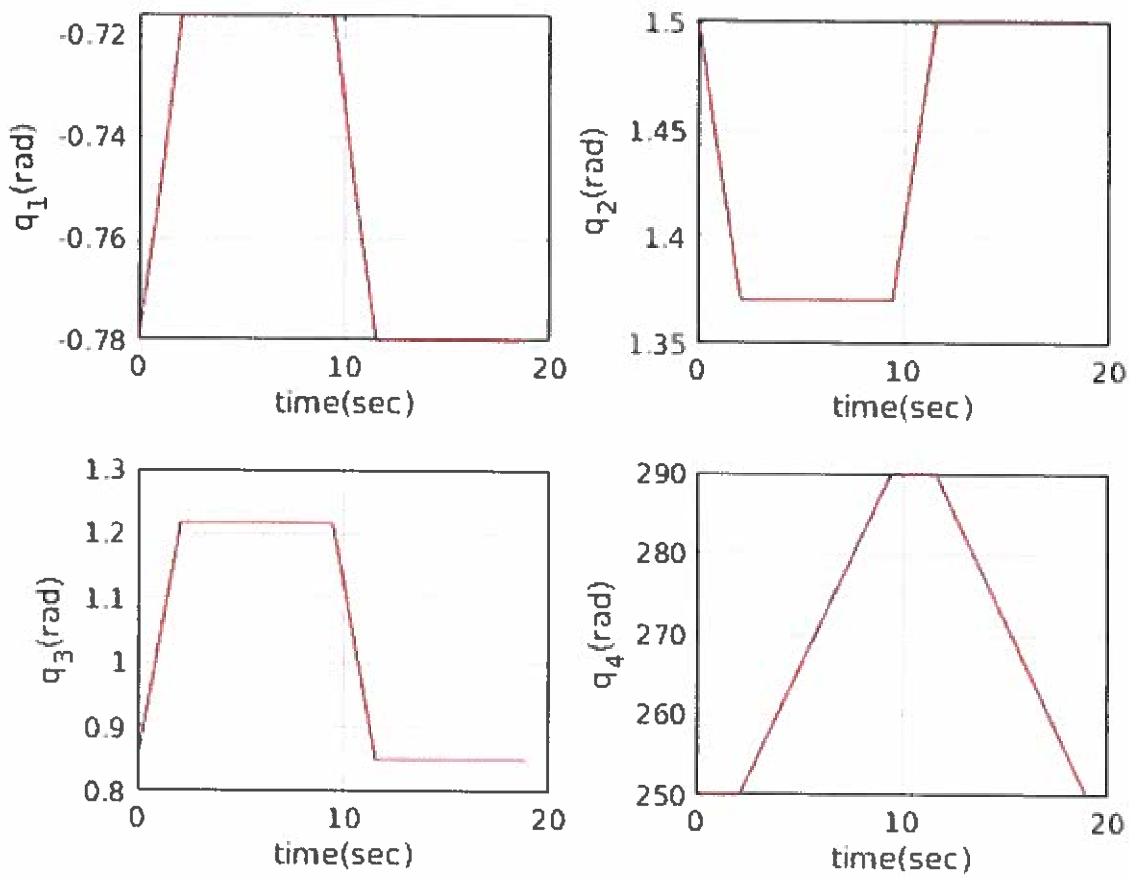
$$x_4^0 = a\cos(q_1) + b\cos(q_1+q_2) + (H+R)\cos\left(q_1+q_2+q_3+\frac{\pi}{2}\right)$$

$$y_4^0 = a\sin(q_1) + b\sin(q_1+q_2) + (H+R)\sin\left(q_1+q_2+q_3+\frac{\pi}{2}\right)$$

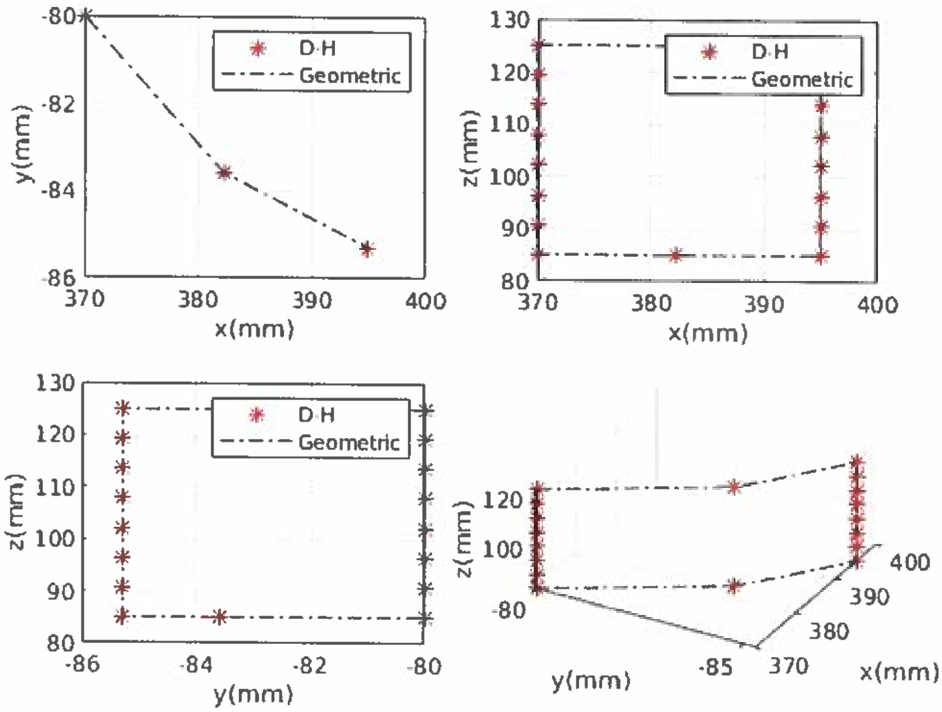
$$z_4^0 = c+d-q_4$$

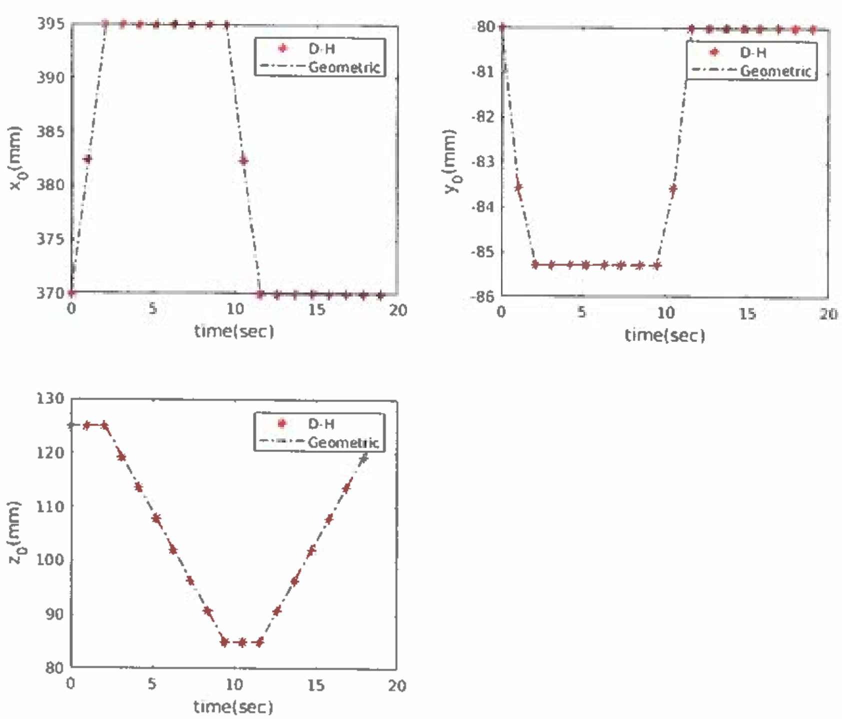
FORWARD KINEMATICS WILL BE USED TO CHECK CORRECTNESS OF INVERS KINEMATICS RESULTS

Here is q1 q2 q3 q4 as outcome of the inverse kinematic.

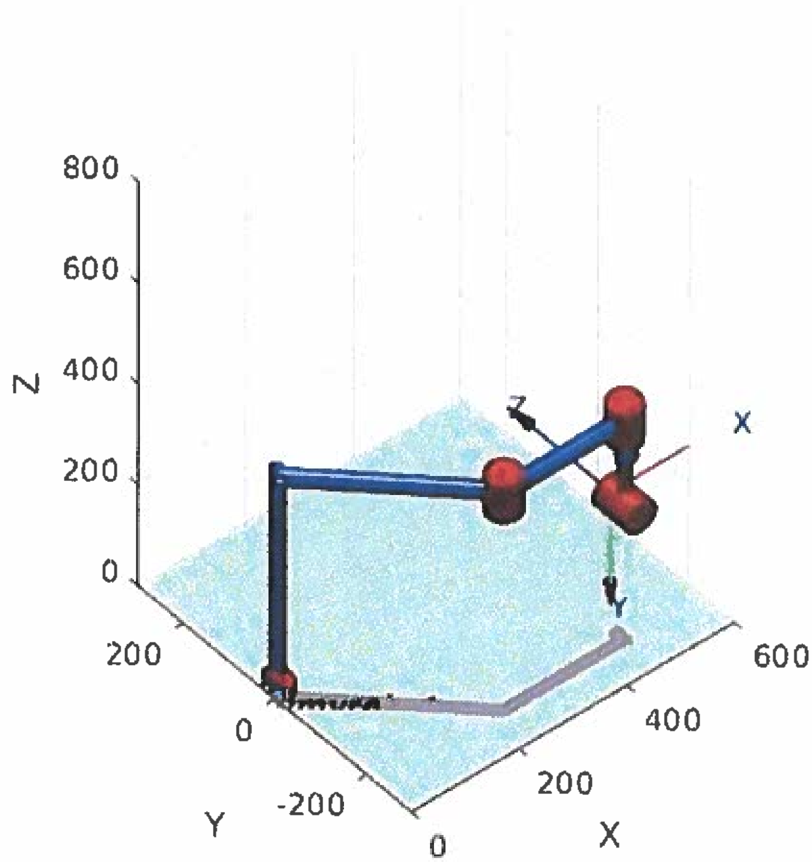


To check the accuracy and correctness of the above results x y x of end-effector coordinates are plotted using both the DH convention and geometric approach. As you see both outcomes match each other.





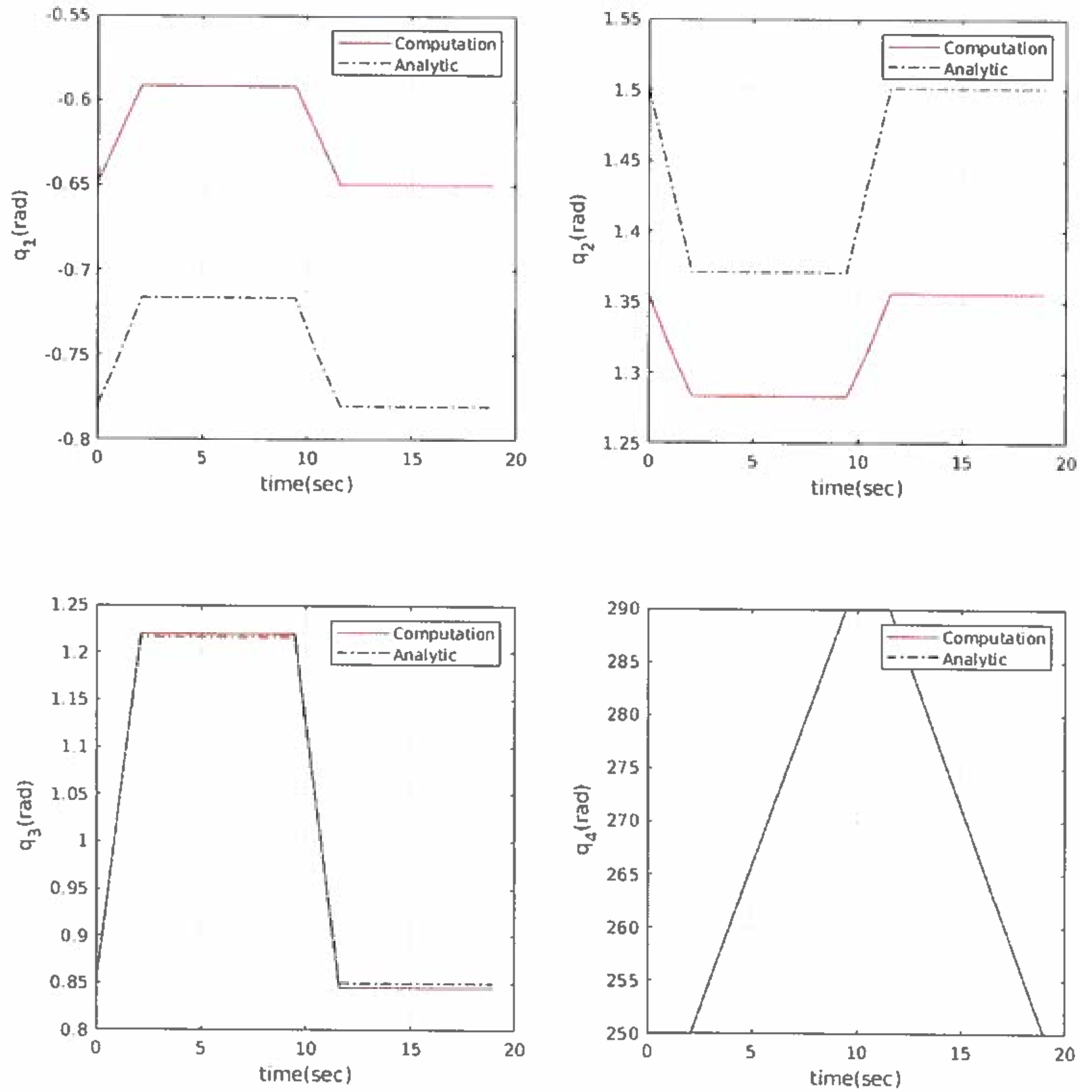
Here is the location and orientation of the robot in the following configuration:
q1 = -pi/4
q2 = pi/4
q3 = 0
q4 = 100



This orientation exactly matches with analytical and DH convention solution.

3-9

The following is the solution of numerical computation. There is a perfect match between the result of analytic inverse kinematic and computational of q3 and q4 but results for q1 and q2 were diverging.



I experienced the following error which I could not resolve:

```
Warning: solution diverging at step 998, try reducing alpha
> In SerialLink/ikine (line 260)
  In hw_3_4 (line 286)
```