CSC 225 FALL 2018
ALGORITHMS AND DATA STRUCTURES I
ASSIGNMENT 2 - WRITTEN
UNIVERSITY OF VICTORIA

- Submission guidelines:
  - Assignments should be uploaded to Connex->Assignments. You can write your solutions using a text editor and upload the PDF file or you can write it by hand and take a CLEAR photo or scan it, and then upload it.
  - Include your V number and your name as it appears on Connex Roster, otherwise, the TAs may not be able to enter your grades.
  - Due date is Monday October 15th 3:30 pm. Late assignments are not accepted.

1. [10 marks] For the following recurrence, prove using the substitution method (i.e. induction) that $T(n) = O(2^n)$. You can assume that $n_0 = 1$, and the exact form of induction is $T(n) \leq c2^n$.

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + 2^n & \text{if } n > 1 \end{cases}$$

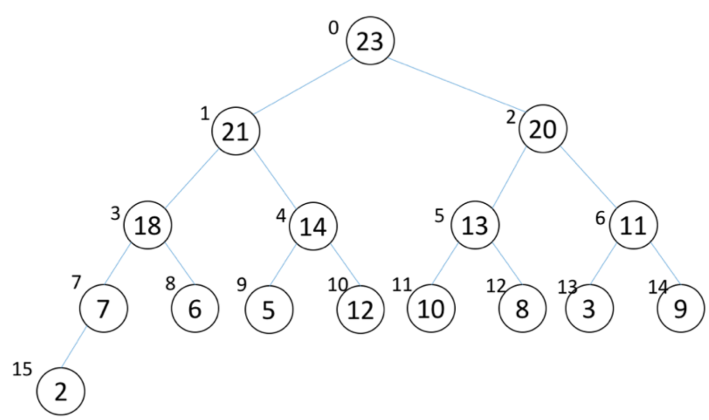
   Note that you should also show for what values of constant c this inequality holds just like any other Big-O proof.

2. [20 marks] Assume that we have a binary heap that we are representing as a 0-based index array. (Similar to the binary heap in class with the difference that indices start from 0). Use these three methods below to answer to parts (a) and (b):

PARENT(i)	LEFT(i)	RIGHT(i)
return $\lfloor (i-1)/2 \rfloor$	**return** $2i+1$	**return** $2i+2$

   a) Write a **recursive** pseudocode for a method called PRINT-GRANDPARENTS(i). For a given index i, this method prints i, then parent-of-parent of i, and so on.
   For example, on the following heap PRINT-GRANDPARENTS(15) outputs “2 18 23”. Also, PRINT-GRANDPARENTS(8) outputs “6 21”.

   b) Write a pseudocode for a method called PRINT-AUNT(i). For a given index i, this method prints the sibling of the node’s parent.
   For example, on the following heap PRINT-AUNT(5) outputs 21, and PRINT-AUNT(7) outputs 14.



3. [5 marks] Say we have m balls numbered from 1 to m, and we have n bins numbered from 1 to n. We are throwing these balls one by one into the bins. When we throw a ball, it goes with equal probability of $1/n$ into any of the n bins.
   What is the **expected number of balls in bin number 1** after throwing all m balls into the bins?

   Hint: This problem is very easy if we define the random variables as follows. Assume that random variable Y represents answer which is the number of balls in bin 1. Also, assume that for each ball i we have a random variable X_i. X_i is 1 if the ball i goes to bin 1 and is 0 otherwise. So, we can say that $Y = X_1 + X_2 + \ldots + X_m$, because basically for each ball that goes into bin 1 we are adding 1 to the value of Y.
   Now, compute $E[X_i]$ for each i, and then compute $E[Y]$ using linearity of expectation.

4. [15 marks] For each of the following recurrences say which case of the Master method it falls into, and then write the solution to the recurrence.
   a) $T(n) = 2T(n/16) + n^{0.25}$
   b) $T(n) = 8T(n/2) + \Theta(n^2)$
   c) $T(n) = 27\,T(n/3) + n^3 \log n$