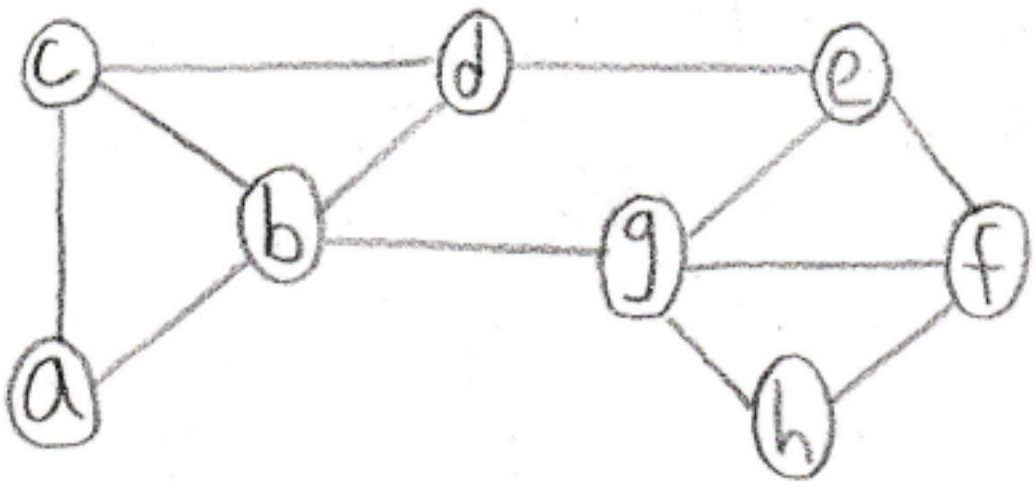
4. a)



b)

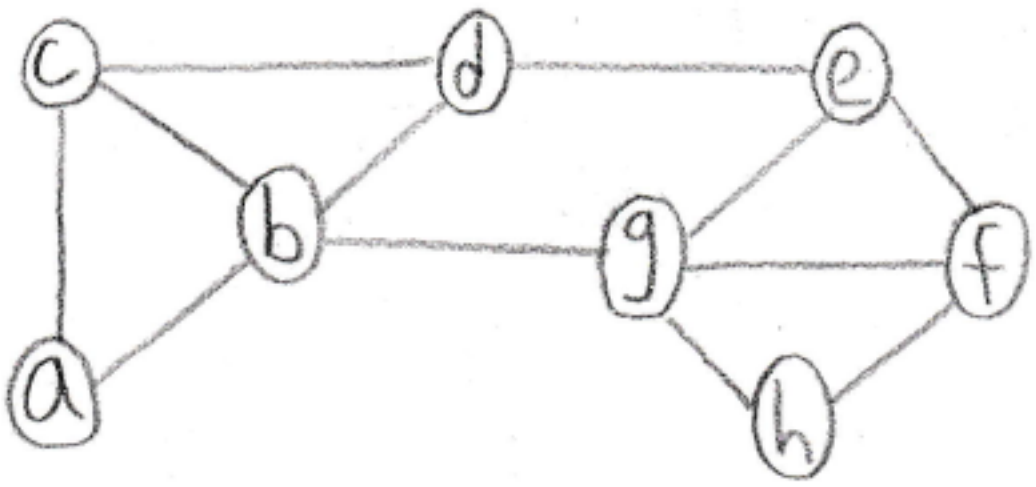
1	1	ac cd de ef fh
	2	ac cd de ef fg gh
2	3	ac cd de eg gh
	4	ac cd de eg gf fh
	5	ab bc cd de ef fh
	6	ab bc cd de ef fg gh
	7	ab bc cd de eg gh
	8	ab bc cd de eg gf fh
3	9	ab bg ge ef fh
4	10	ab bg gf fh
5	11	ab bg gh
	12	ac cb bg ge ef fh
6	13	ac cb bg gf fh
7	14	ac cb bg gh
8	15	ab bd de ef fh
	16	ab bd de ef fg gh
9	17	ab bd de eg gh
	18	ab bd de eg gf fh
	19	ac, cd, db, bg, gh
	20	ac, cd, db, bg
	21	ac, cd, db, bg
	22	ac, cd, db, bg

22 possible paths from a to h

c) 9 possible paths of length ≤ 5

1: ac, cd, de, ef, fh
2: ac, cd, de, eg, gh
3: ab, bg, ge, ef, fh
4: ab, bg, gf, fh
5: ab, bg, gh
6: ac, cb, bg, gf, fh
7: ac, cb, bg, gh
8: ab, bd, de, ef, fh
9: ab, bd, de, eg, gh

4. a)



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	6	ab bc cd de ef fg gh
	7	ab bc cd de eg gh
	8	ab bc cd de eg gf fh
3	9	ab bg ge ef fh
4	10	ab bg gf fh
5	11	ab bg gh
	12	ac cb bg ge ef fh
6	13	ac cb bg gf fh
7	14	ac cb bg gh
8	15	ab bd de ef fh
	16	ab bd de ef fg gh
9	17	ab bd de eg gh
	18	ab bd de eg gf fh
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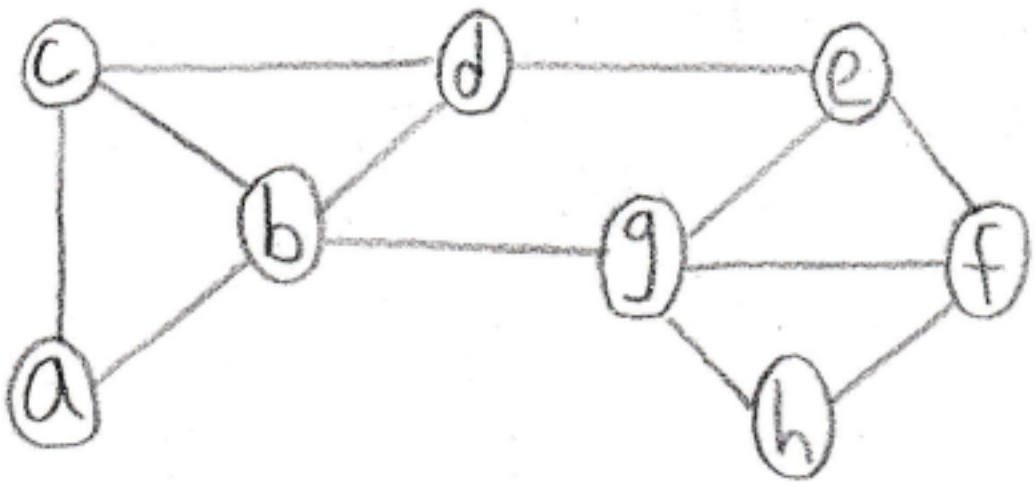
4. a)



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4	10	ab bg gf fh
5	11	ab bg gh
	12	ac cb bg ge ef fh
6	13	ac cb bg gf fh
7	14	ac cb bg gh
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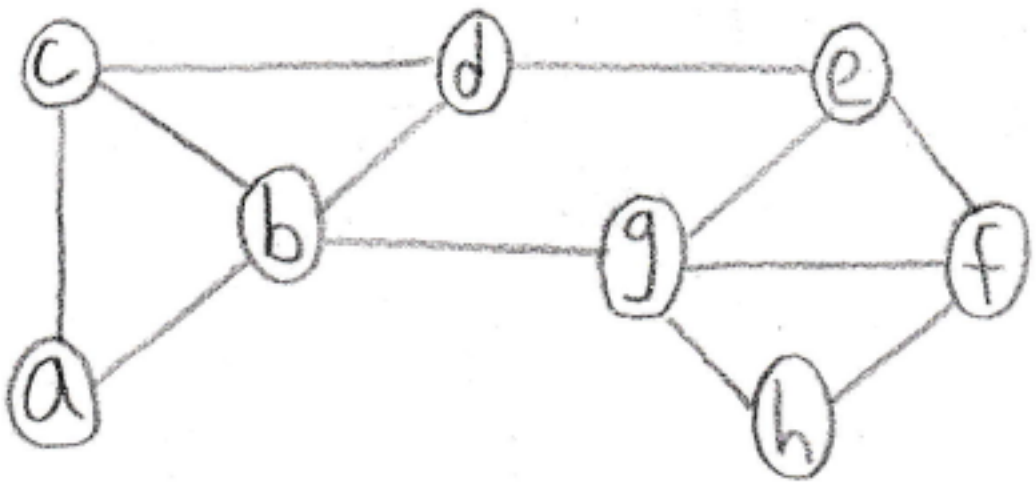

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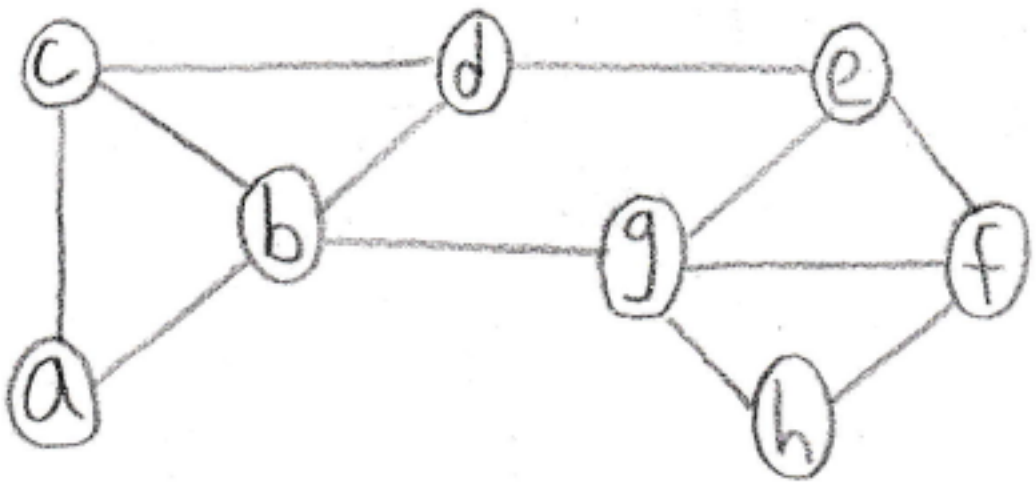

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5	11	ab bg gh
	12	ac cb bg ge ef fh
6	13	ac cb bg gf fh
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7: ac, cb, bg, gh
8: ab, bd, de, ef, fh
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5.

Base Case: $n=1$ • $|V|=1, |E|=0, 2\cdot 0 \leq 0^2-0$

$n=2$  $|V|=2, |E|=1, 2\cdot 1 \leq 2^2-2$

$n=3$ $|V|=3, |E|=3, 2\cdot 3 \leq 3^2-3$

$n=4$  $|V|=4, |E|=6, 2\cdot 4 \leq 4^2-4$

I.H.: Assume that the statement $2m \leq k^2-k$ is true for some $k \geq n$.

I.S.: Need to show that $2m+k \leq (k+1)^2-(k+1)$ as adding a new nodes adds at most k edges.

$$2m+k \leq (k+1)^2-(k+1)$$
$$2m+k \leq k^2+2k+1-k+1$$
$$2m+k \leq (k^2-k)+2k+1+1$$
$$2m+k \leq 2m+2k+1+1 \quad [\text{I.H.}]$$
$$k \leq 2k+1+1$$
$$0 \leq k+2$$