CSC 226 SPRING 2019
ALGORITHMS AND DATA STRUCTURES II
ASSIGNMENT 2 - WRITTEN
UNIVERSITY OF VICTORIA

1. Draw the 2-3 trees that result when you insert the keys Y L P M X H C R A E S in that order into an initially empty tree. There should be 11 trees in all. Use the final tree to construct the corresponding red-black tree.

2. Draw all the structurally different red-black trees (i.e. no specific keys) with n keys for n from 2 to 8.

3. An inversion in a sequence is an out-of-order pair; i.e., $i < j$ but $a_i > a_j$. Inversions are discussed briefly in the book on page 252. For example, the sequence $(5,3,2,1,4)$ has 7 inversions. Explain carefully how to use red-black trees to compute the number of inversions in a permutation in time $O(n \log n)$. Effectively, you may need to modify the code for Algorithm 3.4 on page 439. Explain in detail any changes that you would make to method put().

4. Let $G = (V, E)$, where $V = \{a, b, c, d, e, f, g, h\}$ and $E = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{b, g\}, \{c, d\}, \{d, e\}, \{e, f\}, \{e, g\}, \{f, g\}, \{f, h\}, \{g, h\}\}$.
   (a) Draw the corresponding graph with no edges crossing.
   (b) How many paths are there in G from a to h?
   (c) How many of these paths have length less than or equal to 5? List them.

5. Let $G = (V, E)$ be an undirected graph, with no parallel edges or self-loops. Let $|V| = n$ and $|E| = m$. Prove by induction that $2m \leq n^2 - n$ for all $n \geq 1$.