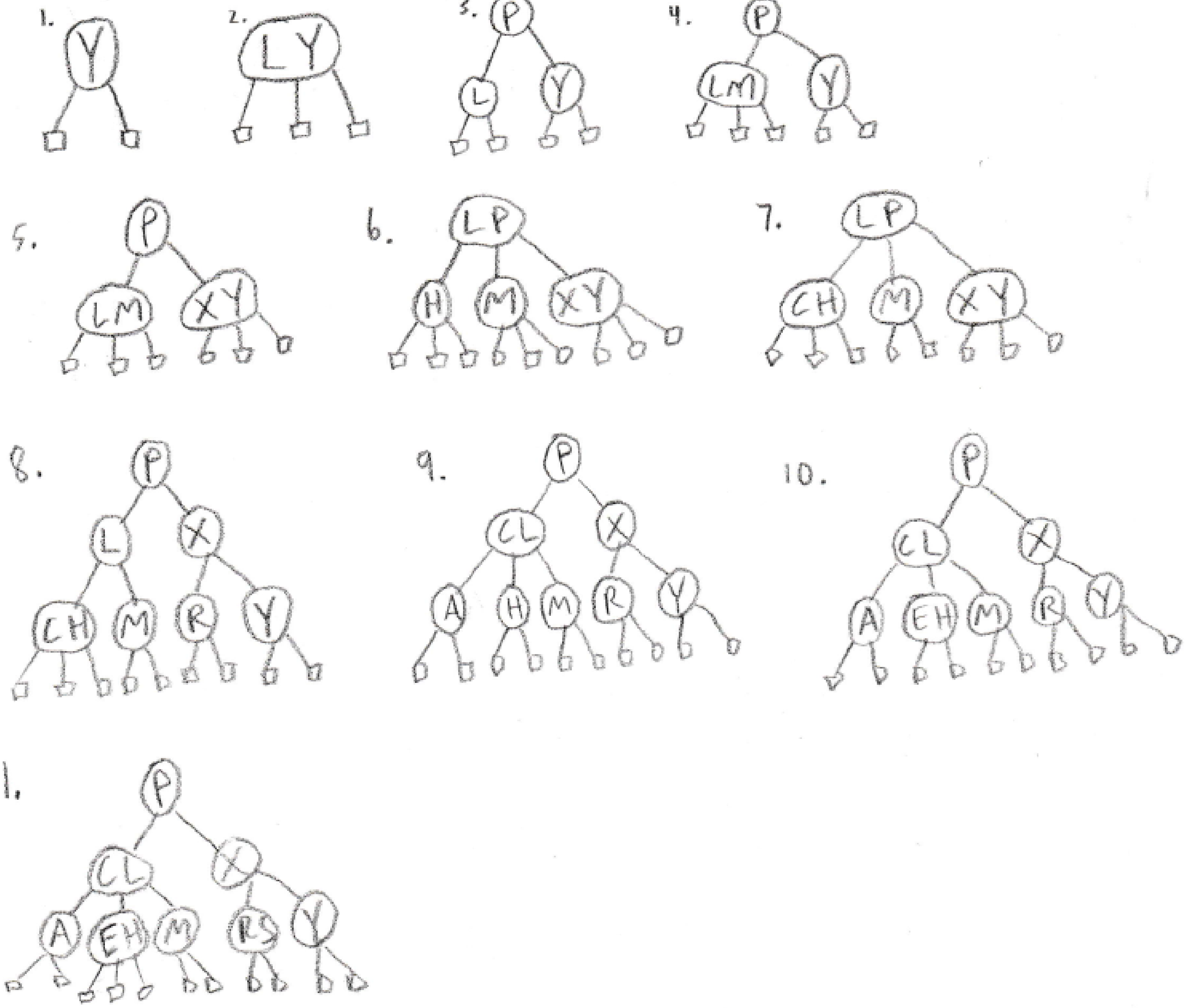
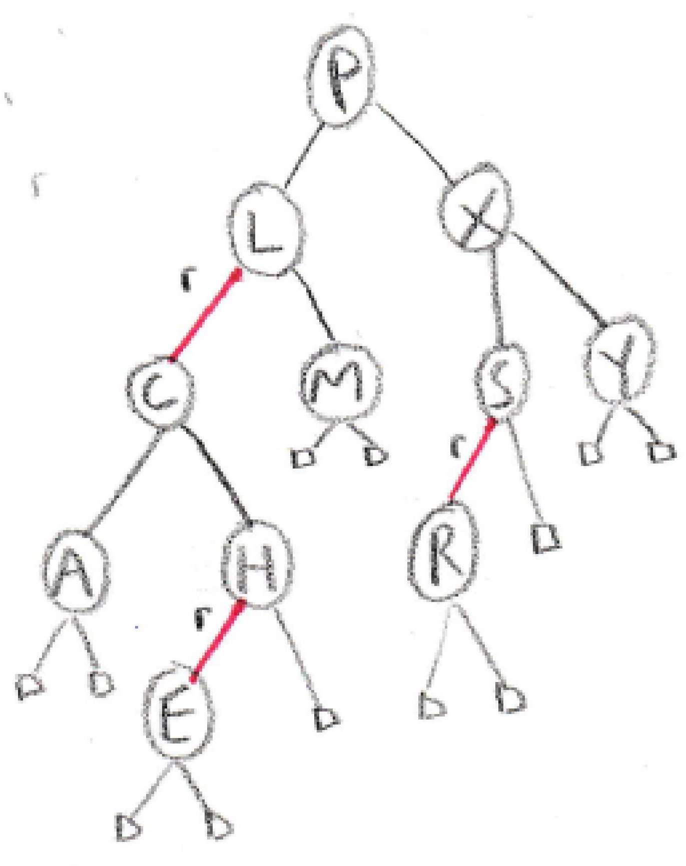
CSC 226 Written Assignment 2

connor Schultz
V00872923

1.

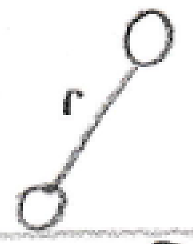


CORRESPONDING RED BLACK BST (Assumes Left Leaning):



2.

2-Keys:



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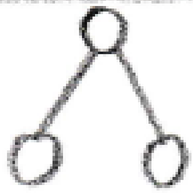
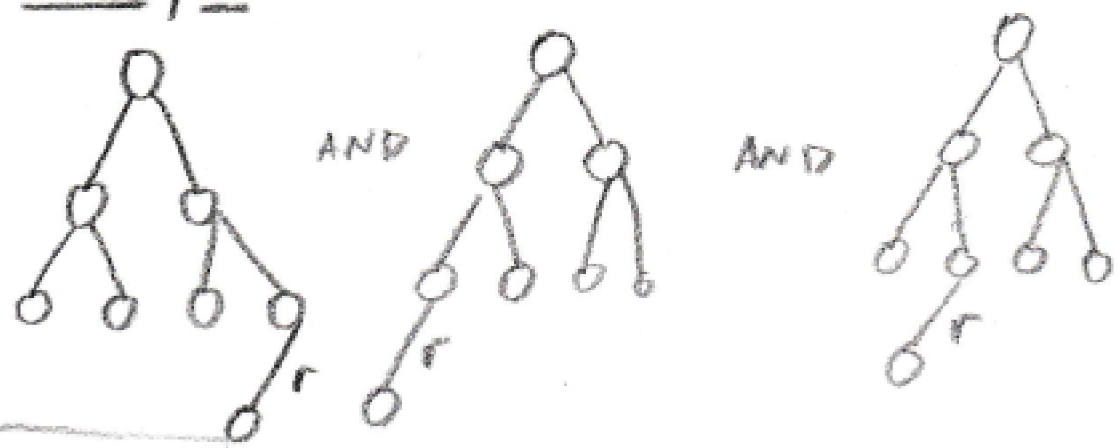
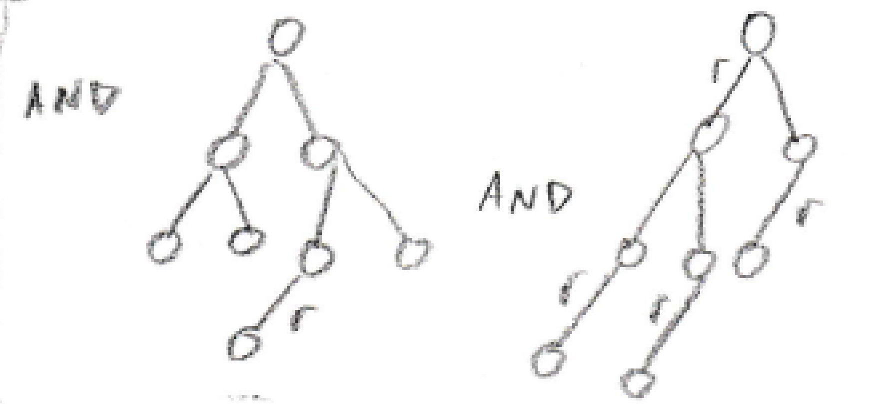
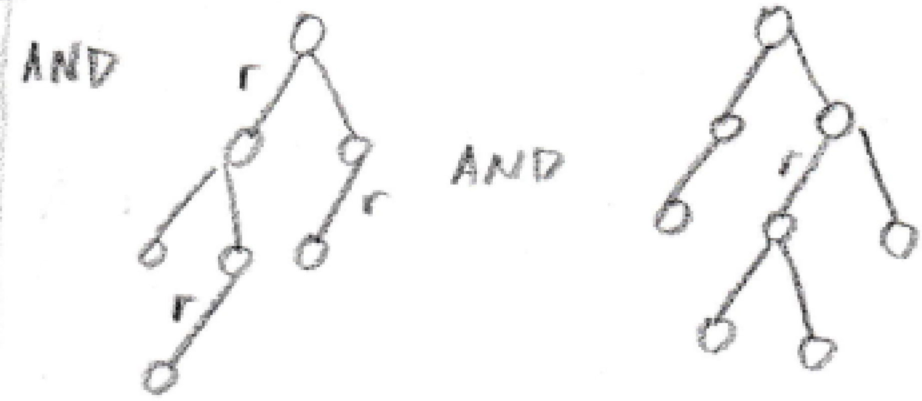


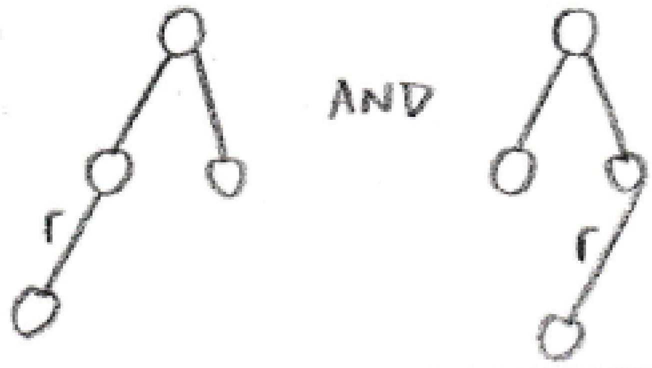
8-Keys:







4-keys:



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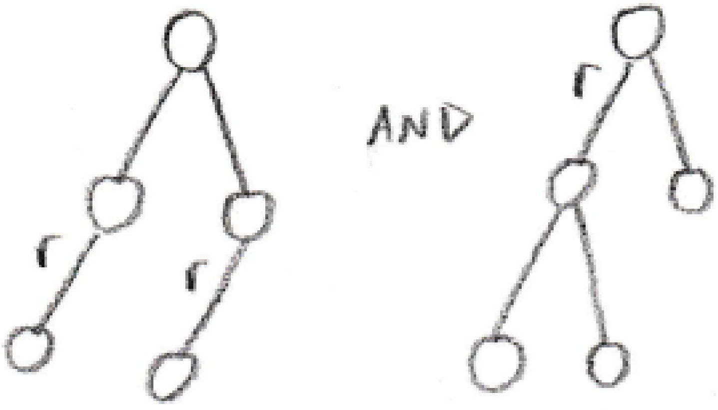
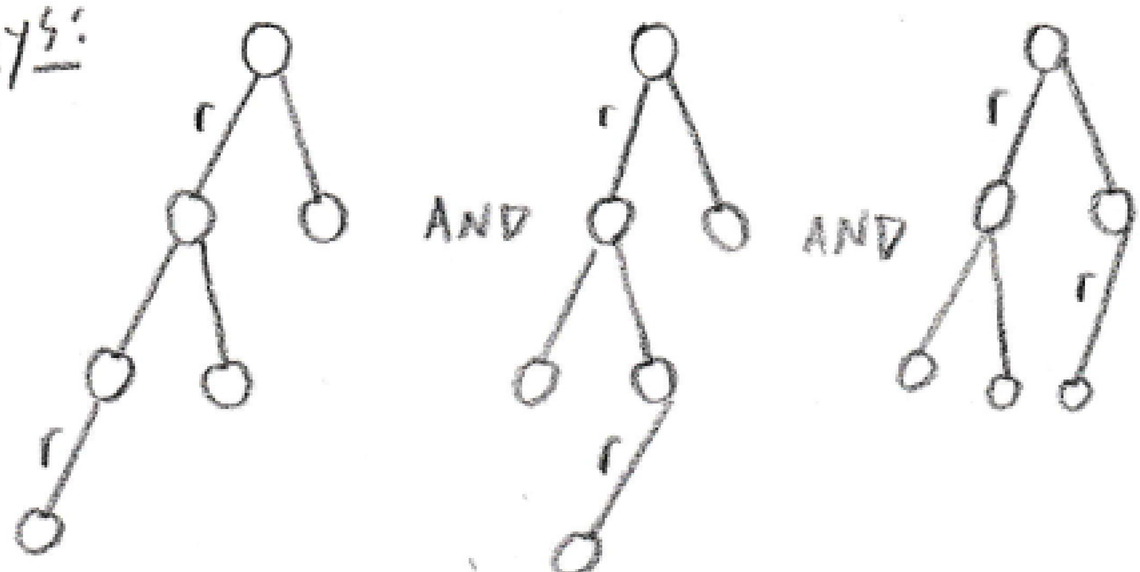
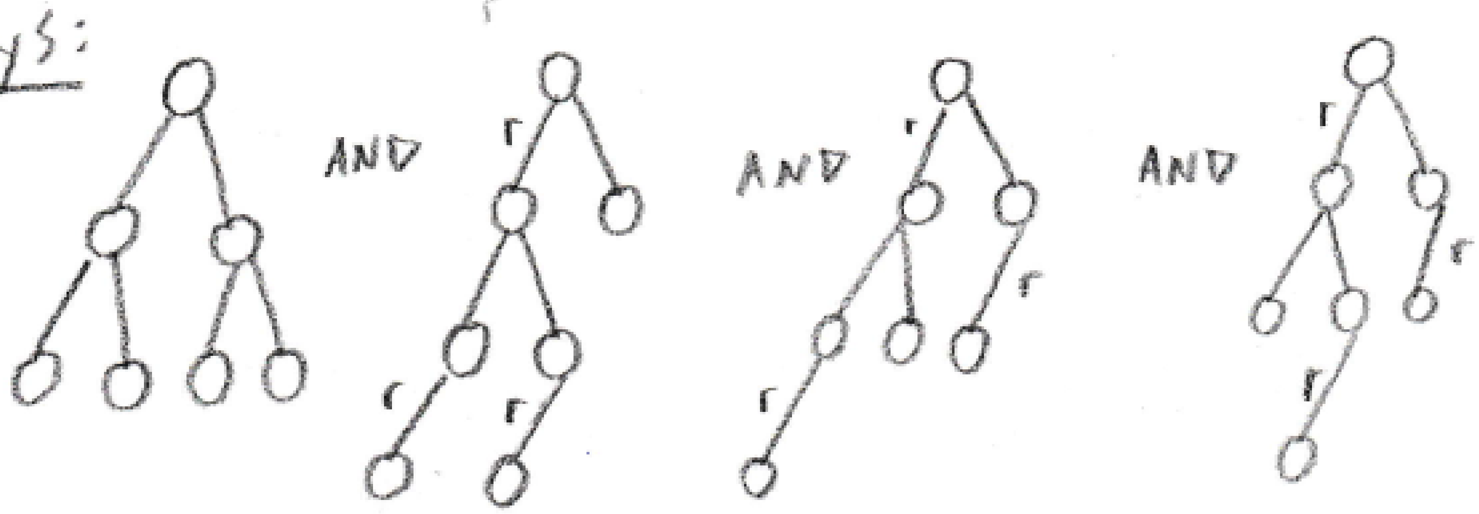


6-keys:



7-keys:



3.

- The maximum number of inversions in a list occurs when the input is in reverse order. This means that all inputs are inserted in the right subtree of their parent node. When the opposite occurs (list is inserted in sorted order) there are no inversions and all inputs are inserted to the left.

- In order to count the inversions using a RB BST, when a node is input to the right of a parent node the number of inversions is equal to:

  current inversions + size(T_R) + 1

  This is because all nodes in T_R are larger then the node and will hence cause inversions.
  If a node is inserted into T_L, no new inversions are added.

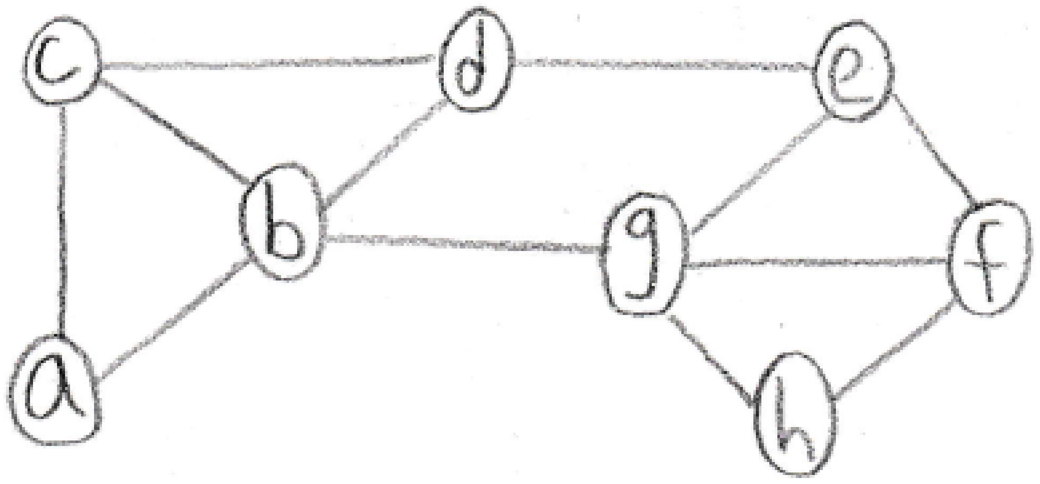
put() method:

- need a global variable result to keep track of inversions init. 0
- After insert left:

  result = result + size(node.right) + 1

- At the end of all inserts the correct number of inversions will be stored in result in $O(n \log n)$

4. a)



b)

1	1	ac cd de ef fh
	2	ac cd de ef fg gh
2	3	ac cd de eg gh
	4	ac cd de eg gf fh
	5	ab bc cd de ef fh
	6	ab bc cd de ef fg gh
	7	ab bc cd de eg gh
	8	ab bc cd de eg gf fh
3	9	ab bg ge ef fh
4	10	ab bg gf fh
5	11	ab bg gh
	12	ac cb bg ge ef fh
6	13	ac cb bg gf fh
7	14	ac cb bg gh
8	15	ab bd de ef fh
	16	ab bd de ef fg gh
9	17	ab bd de eg gh
	18	ab bd de eg gf fh

22 possible paths from a to h

19	ac, cd, db, bg, gh
20	ac, cd, db, bg, gh
21	ac, cd, db, bg, gh
22	ac, cd, db, bg, gh

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c) 9 possible paths of length ≤ 5

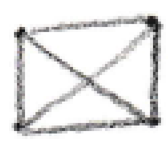
1: ac, cd, de, ef, fh
2: ac, cd, de, eg, gh
3: ab, bg, ge, ef, fh
4: ab, bg, gf, fh
5: ab, bg, gh
6: ac, cb, bg, gf, fh
7: ac, cb, bg, gh
8: ab, bd, de, ef, fh
9: ab, bd, de, eg, gh

5.

Base Case: $n=1$  $|V|=1, |E|=0, 2\cdot 0 \leq 0^2-0$

$n=2$  $|V|=2, |E|=1, 2\cdot 1 \leq 2^2-2$

$n=3$  $|V|=3, |E|=3, 2\cdot 3 \leq 3^2-3$

$n=4$  $|V|=4, |E|=6, 2\cdot 4 \leq 4^2-4$

I.H: Assume that the statement $2m \leq K^2-K$ is true for some $K \geq n$.

I.S: Need to show that $2m+K \leq (K+1)^2-(K+1)$ as adding a new nodes adds at most K edges.

$$2m+K \leq (K+1)^2-(K+1)$$
$$2m+K \leq K^2+2K+1-K+1$$
$$2m+K \leq (K^2-K)+2K+1+1$$
$$2m+K \leq 2m+2K+1+1 \quad [\text{I.H.}]$$
$$K \leq 2K+1+1$$
$$0 \leq K+2$$