



University of Calgary

# Codetoads

Rosa Shah, Charlie Zheng, Kingsley Zhong

ICPC North American Championship 2023

May 29, 2023

## 1 Contest

## 2 Data structures

### Contest (1)

#### template.cpp

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
    cin.tie(0)->sync_with_stdio(0);
    cin.exceptions(cin.failbit);
}
```

#### .bashrc

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++14 \
-fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #
caps = <
```

#### hash.sh

```
# Hashes a file, ignoring all whitespace and comments.
Use for
# verifying that code was correctly typed.
# Usage: ./hash.sh < FILE (make executable first: chmod +
x hash.sh)
# cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |
cut -c-6
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |
awk '{print substr($1, 1, 6)}'
```

#### troubleshoot.txt

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
```

1 Confusing N and M, i and j, etc.?

1 Are you sure your algorithm works?

What special cases have you not thought of?

Are you sure the STL functions you use work as you think?

Add some assertions, maybe resubmit.

Create some testcases to run your algorithm on.

Go through the algorithm for a simple case.

Go through this list again.

Explain your algorithm to a teammate.

Ask the teammate to look at your code.

Go for a small walk, e.g. to the toilet.

Is your output format correct? (including whitespace)

Rewrite your solution from the start or let a teammate do it.

Runtime error:

Have you tested all corner cases locally?

Any uninitialized variables?

Are you reading or writing outside the range of any vector?

Any assertions that might fail?

Any possible division by 0? (mod 0 for example)

Any possible infinite recursion?

Invalidated pointers or iterators?

Are you using too much memory?

Debug with resubmits (e.g. remapped signals, see Various)

Time limit exceeded:

Do you have any possible infinite loops?

What is the complexity of your algorithm?

Are you copying a lot of unnecessary data? (References)

How big is the input and output? (consider scanf)

Avoid vector, map. (use arrays/unordered\_map)

What do your teammates think about your algorithm?

Memory limit exceeded:

What is the max amount of memory your algorithm should need?

Are you clearing all data structures between test cases?

### Data structures (2)

#### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null\_type.

**Time:**  $\mathcal{O}(\log N)$

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;

void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9));
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
```

```
assert(*t.find_by_order(0) == 8);
t.join(t2); // assuming T < T2 or T > T2, merge t2 into
t
}
```

#### HashMap.h

**Description:** Hash map with mostly the same API as unordered\_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(*x
C); }
};
__gnu_pbds::gp_hash_table<ll, int, chash> h({}, {}, {}, {}, {
1<<16});
```

#### SegmentTree.h

**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.

**Time:**  $\mathcal{O}(\log N)$

```
struct Tree {
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative
fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    }
    T query(int b, int e) { // query [b, e)
        T ra = unit, rb = unit;
        for (b += n, e += n; b < e; b /= 2, e /= 2) {
            if (b % 2) ra = f(ra, s[b++]);
            if (e % 2) rb = f(s[--e], rb);
        }
        return f(ra, rb);
    }
};
```

#### LazySegmentTree.h

**Description:** Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.

**Usage:** Node\* tr = new Node(v, 0, sz(v));

**Time:**  $\mathcal{O}(\log N)$ .

```
"../various/BumpAllocator.h"
const int inf = 1e9;
struct Node {
    Node *l = 0, *r = 0;
    int lo, hi, mset = inf, madd = 0, val = -inf;
    Node(int lo, int hi) : lo(lo), hi(hi) {} // Large interval
of -inf
    Node(vi& v, int lo, int hi) : lo(lo), hi(hi) {
        if (lo + 1 < hi) {
```

```

    int mid = lo + (hi - lo)/2;
    l = new Node(v, lo, mid); r = new Node(v, mid, hi);
    val = max(l->val, r->val);
}
else val = v[lo];
}
int query(int L, int R) {
    if (R <= lo || hi <= L) return -inf;
    if (L <= lo && hi <= R) return val;
    push();
    return max(l->query(L, R), r->query(L, R));
}
void set(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) mset = val = x, madd = 0;
    else {
        push(), l->set(L, R, x), r->set(L, R, x);
        val = max(l->val, r->val);
    }
}
void add(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) {
        if (mset != inf) mset += x;
        else madd += x;
        val += x;
    }
    else {
        push(), l->add(L, R, x), r->add(L, R, x);
        val = max(l->val, r->val);
    }
}
void push() {
    if (!l) {
        int mid = lo + (hi - lo)/2;
        l = new Node(lo, mid); r = new Node(mid, hi);
    }
    if (mset != inf)
        l->set(lo,hi,mset), r->set(lo,hi,mset), mset = inf;
    else if (madd)
        l->add(lo,hi,madd), r->add(lo,hi,madd), madd = 0;
}
};

```

### UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().

**Usage:** int t = uf.time(); ...; uf.rollback(t);

**Time:**  $\mathcal{O}(\log(N))$

de4ad0, 21 lines

```

struct RollbackUF {
    vi e; vector<pii> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return sz(st); }
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool join(int a, int b) {
        a = find(a), b = find(b);

```

```

        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
};

```

### SubMatrix.h

**Description:** Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).

**Usage:** SubMatrix<int> m(matrix);

m.sum(0, 0, 2, 2); // top left 4 elements

**Time:**  $\mathcal{O}(N^2 + Q)$

c59ada, 13 lines

```

template<class T>
struct SubMatrix {
    vector<vector<T>> p;
    SubMatrix(vector<vector<T>>& v) {
        int R = sz(v), C = sz(v[0]);
        p.assign(R+1, vector<T>(C+1));
        rep(r,0,R) rep(c,0,C)
            p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r]
                [c];
    }
    T sum(int u, int l, int d, int r) {
        return p[d][r] - p[d][l] - p[u][r] + p[u][l];
    }
};

```

### Matrix.h

**Description:** Basic operations on square matrices.

**Usage:** Matrix<int, 3> A;

A.d = {{{{1,2,3}}}, {{4,5,6}}, {{7,8,9}}};

vector<int> vec = {1,2,3};

vec = (A^N) \* vec;

c43c7d, 26 lines

```

template<class T, int N> struct Matrix {
    typedef Matrix M;
    array<array<T, N>, N> d{};
    M operator*(const M& m) const {
        M a;
        rep(i,0,N) rep(j,0,N)
            rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
        return a;
    }
    vector<T> operator*(const vector<T>& vec) const {
        vector<T> ret(N);
        rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
        return ret;
    }
    M operator^(ll p) const {
        assert(p >= 0);
        M a, b(*this);
        rep(i,0,N) a.d[i][i] = 1;
        while (p) {
            if (p&1) a = a*b;
            b = b*b;
            p >>= 1;
        }
        return a;
    }
};

```

};

### LineContainer.h

**Description:** Container where you can add lines of the form  $kx+m$ , and query maximum values at points  $x$ . Useful for dynamic programming ("convex hull trick").

**Time:**  $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```

struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x->p >= y->p)
            isect(x, erase(y)));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

```

### Treap.h

**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.

**Time:**  $\mathcal{O}(\log N)$

9556fc, 55 lines

```

struct Node {
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {};

```

```

if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(
    k)
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};
} else {
    auto pa = split(n->r, k - cnt(n->l) - 1); // and just
    "k"
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};
}
}

Node* merge(Node* l, Node* r) {
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        l->recalc();
        return l;
    } else {
        r->l = merge(l, r->l);
        r->recalc();
        return r;
    }
}

Node* ins(Node* t, Node* n, int pos) {
    auto pa = split(t, pos);
    return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r] to index k
void move(Node*& t, int l, int r, int k) {
    Node *a, *b, *c;
    tie(a,b) = split(t, l); tie(b,c) = split(b, r - 1);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
}

```

### FenwickTree.h

**Description:** Computes partial sums  $a[0] + a[1] + \dots + a[\text{pos} - 1]$ , and updates single elements  $a[i]$ , taking the difference between the old and new value.

**Time:** Both operations are  $\mathcal{O}(\log N)$ .

e62fac, 22 lines

```

struct FT {
    vector<ll> s;
    FT(int n) : s(n) {}
    void update(int pos, ll dif) { // a[pos] += dif
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
    }
    ll query(int pos) { // sum of values in [0, pos]
        ll res = 0;
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];
        return res;
    }
    int lower_bound(ll sum) { // min pos st sum of [0, pos]
        >= sum
        // Returns n if no sum is >= sum, or -1 if empty sum
        is.

```

```

if (sum <= 0) return -1;
int pos = 0;
for (int pw = 1 << 25; pw; pw >= 1) {
    if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
}
return pos;
}
};

```

### FenwickTree2d.h

**Description:** Computes sums  $a[i,j]$  for all  $i < I$ ,  $j < J$ , and increases single elements  $a[i,j]$ . Requires that the elements to be updated are known in advance (call `fakeUpdate()` before `init()`).

**Time:**  $\mathcal{O}(\log^2 N)$ . (Use persistent segment trees for  $\mathcal{O}(\log N)$ .)

"FenwickTree.h"

157f07, 22 lines

```

struct FT2 {
    vector<vi> ys; vector<FT> ft;
    FT2(int limx) : ys(limx) {}
    void fakeUpdate(int x, int y) {
        for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
    }
    void init() {
        for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v))
        ;
    }
    int ind(int x, int y) {
        return (int)(lower_bound(all(ys[x]), y) - ys[x].begin
        ()); }
    void update(int x, int y, ll dif) {
        for (; x < sz(ys); x |= x + 1)
            ft[x].update(ind(x, y), dif);
    }
    ll query(int x, int y) {
        ll sum = 0;
        for (; x &= x - 1)
            sum += ft[x-1].query(ind(x-1, y));
        return sum;
    }
};

```

### RMQ.h

**Description:** Range Minimum Queries on an array. Returns  $\min(V[a], V[a+1], \dots, V[b-1])$  in constant time.

**Usage:** `RMQ rmq(values);`

`rmq.query(inclusive, exclusive);`

**Time:**  $\mathcal{O}(|V| \log |V| + Q)$

510c32, 16 lines

```

template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k)
            {
                jmp.emplace_back(sz(V) - pw * 2 + 1);
                rep(j, 0, sz(jmp[k]))
                    jmp[k][j] = min(jmp[k-1][j], jmp[k-1][j + pw
                    ]);
            }
    }
    T query(int a, int b) {
        assert(a < b); // or return inf if a == b

```

```

    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
}
};

```

### MoQueries.h

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge  $(a, c)$  and remove the initial add call (but keep in).

**Time:**  $\mathcal{O}(N\sqrt{Q})$

a12ef4, 49 lines

```

void add(int ind, int end) { ... } // add a[ind] (end = 0
    or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
    int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk &
    1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t
    ]); });
    for (int qi : s) {
        pii q = Q[qi];
        while (L > q.first) add(--L, 0);
        while (R < q.second) add(R++, 1);
        while (L < q.first) del(L++, 0);
        while (R > q.second) del(--R, 1);
        res[qi] = calc();
    }
    return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int
    root=0){
    int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
    add(0, 0), in[0] = 1;
    auto dfs = [&](int x, int p, int dep, auto& f) -> void
        {
            par[x] = p;
            L[x] = N;
            if (dep) I[x] = N++;
            for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
            if (!dep) I[x] = N++;
            R[x] = N;
        };
    dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk
    & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t
    ]); });
    for (int qi : s) rep(end, 0, 2) {
        int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; }
        \
            else { add(c, end); in[c] = 1; } a = c;
        }

```

```
    while (!(L[b] <= L[a] && R[a] <= R[b]))  
        I[i++] = b, b = par[b];  
    while (a != b) step(par[a]);  
    while (i--) step(I[i]);  
    if (end) res[qi] = calc();  
}  
return res;  
}
```

# Techniques (A)

## techniques.txt

159 lines

```

Recursion
Divide and conquer
    Finding interesting points in N log N
Algorithm analysis
    Master theorem
    Amortized time complexity
Greedy algorithm
    Scheduling
    Max contiguous subvector sum
    Invariants
    Huffman encoding
Graph theory
    Dynamic graphs (extra book-keeping)
    Breadth first search
    Depth first search
    * Normal trees / DFS trees
    Dijkstra's algorithm
    MST: Prim's algorithm
    Bellman-Ford
    Konig's theorem and vertex cover
    Min-cost max flow
    Lovasz toggle
    Matrix tree theorem
    Maximal matching, general graphs
    Hopcroft-Karp
    Hall's marriage theorem
    Graphical sequences
    Floyd-Warshall
    Euler cycles
    Flow networks
    * Augmenting paths
    * Edmonds-Karp
    Bipartite matching
    Min. path cover
    Topological sorting
    Strongly connected components
    2-SAT
    Cut vertices, cut-edges and biconnected components
    Edge coloring
    * Trees
    Vertex coloring
    * Bipartite graphs (=> trees)
    * 3^n (special case of set cover)
    Diameter and centroid
    K'th shortest path
    Shortest cycle
Dynamic programming
    Knapsack
    Coin change
    Longest common subsequence
    Longest increasing subsequence
    Number of paths in a dag
    Shortest path in a dag
    Dynprog over intervals
    Dynprog over subsets
    Dynprog over probabilities
    Dynprog over trees
    3^n set cover
    Divide and conquer

```

```

Knuth optimization
Convex hull optimizations
RMQ (sparse table a.k.a 2^k-jumps)
Bitonic cycle
Log partitioning (loop over most restricted)
Combinatorics
    Computation of binomial coefficients
    Pigeon-hole principle
    Inclusion/exclusion
    Catalan number
    Pick's theorem
Number theory
    Integer parts
    Divisibility
    Euclidean algorithm
    Modular arithmetic
    * Modular multiplication
    * Modular inverses
    * Modular exponentiation by squaring
    Chinese remainder theorem
    Fermat's little theorem
    Euler's theorem
    Phi function
    Frobenius number
    Quadratic reciprocity
    Pollard-Rho
    Miller-Rabin
    Hensel lifting
    Vieta root jumping
Game theory
    Combinatorial games
    Game trees
    Mini-max
    Nim
    Games on graphs
    Games on graphs with loops
    Grundy numbers
    Bipartite games without repetition
    General games without repetition
    Alpha-beta pruning
Probability theory
Optimization
    Binary search
    Ternary search
    Unimodality and convex functions
    Binary search on derivative
Numerical methods
    Numeric integration
    Newton's method
    Root-finding with binary/ternary search
    Golden section search
Matrices
    Gaussian elimination
    Exponentiation by squaring
Sorting
    Radix sort
Geometry
    Coordinates and vectors
    * Cross product
    * Scalar product
    Convex hull
    Polygon cut
    Closest pair

```

```

Coordinate-compression
Quadrees
KD-trees
    All segment-segment intersection
Sweeping
    Discretization (convert to events and sweep)
    Angle sweeping
    Line sweeping
    Discrete second derivatives
Strings
    Longest common substring
    Palindrome subsequences
    Knuth-Morris-Pratt
    Tries
    Rolling polynomial hashes
    Suffix array
    Suffix tree
    Aho-Corasick
    Manacher's algorithm
    Letter position lists
Combinatorial search
    Meet in the middle
    Brute-force with pruning
    Best-first (A*)
    Bidirectional search
    Iterative deepening DFS / A*
Data structures
    LCA (2^k-jumps in trees in general)
    Pull/push-technique on trees
    Heavy-light decomposition
    Centroid decomposition
    Lazy propagation
    Self-balancing trees
    Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
    Monotone queues / monotone stacks / sliding queues
    Sliding queue using 2 stacks
    Persistent segment tree

```