Math 1080: Spring 2019

Homework #5

Due Feb 22

Problem 1:

Find the absolute and relative condition number for the following problems. Comment on the values of *x* for which the problem would be considered well-conditioned or ill-conditioned.

a) $f(x) = (\ln x)^2$

b) $f(x) = \|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$

c) $f(A) = [trace(A) \quad \det(A)]$ for 2x2 matrix *A*

Use $\|.\|_\infty$ norm in the formula for the condition number and treat the input as a vector of dimension 4, so your Jacobian becomes a 2 x 4 matrix.

d) $f(x,y) = \begin{bmatrix} xy & x^2 \\ y^2 & xy \end{bmatrix}$

Use $\|.\|_1$ norm in the formula for the condition number and treat the output as a vector of dimension 4, so your Jacobian becomes a 4 x 2 matrix.

Problem 2:

Determine whether the following algorithms are backward stable, stable, or unstable:

a) Computation of $f(x,y) = x^2 - y^2$ as $\tilde{f}(x,y) = [fl(x) \otimes fl(x)] \ominus [fl(y) \otimes fl(y)]$

b) Computation of $f(x,y) = x^2 - y^2$ as $\tilde{f}(x,y) = [fl(x) \oplus fl(y)] \otimes [fl(x) \ominus fl(y)]$

c) Computation of $f(x) = 1/(1+x)$ as $\tilde{f}(x,y) = 1 \oslash [1 \oplus fl(x)]$

Problem 3:

Determine the accuracy of the algorithms in Problem 2. Which of the algorithms a) and b) is more accurate?