Math 1080: Spring 2019

Homework #1

Due Jan 18

Problem 1:

Let B be a 4x4 matrix to which we apply the following operations:

1. Double column 3
2. Add row 2 to row 1
3. Interchange columns 2 and 3
4. Halve row 4
5. Replace column 4 by sum of columns 1 and 3

Each of these operations can be performed by multiplying B on the left or on the right by a specific matrix E_k (where k stands for the operation above) Find the matrices E_k. Then find matrices A and C such that the result is obtained as a product ABC

Problem 2:

Consider the matrix

$$Q = \frac{1}{3}\begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{bmatrix}$$

Show that Q is an orthogonal matrix. What transformation of $\mathbb{R}^3$ does it correspond to?

(Hint: Find the vector a that is invariant under Q. Pick a vector b orthogonal to a. Find the angle α between b and Qb. If this angle is independent of the choice of b, then Q corresponds to a rotation about a by the angle α. Think about other possibilities.)

Problem 3:

Find the 2x2 orthogonal matrix Q that corresponds to the reflection over the line $2x - 3y = 0$.

Problem 4:

Let u, v be two vectors and $A = I + uv^T$ a matrix. Show that if A is invertible, its inverse is the matrix $A^{-1} = I + \alpha uv^T$ and find the scalar α.When is A singular?

Problem 5:

(a) Compute the norms $\|w\|_1, \|w\|_2, \|w\|_\infty$ for the vector $w = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$

(b) Compute the norms $\|A\|_1, \|A\|_2, \|A\|_\infty$ for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \end{bmatrix}$

(c) Verify the inequalities $\|Aw\|_p \leq \|A\|_p \|w\|_p$.