Math 1080: Spring 2019

Homework #8 (due April 5)

Problem 1:

Find the diagonalization $A = X\Lambda X^{-1}$ of the following matrix:

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 3 & 1 \\ -2 & 0 & 4 & 0 \\ 2 & -2 & 1 & 2 \end{bmatrix}$$

Problem 2:

Find the Schur factorization $A = QTQ^T$ for the following matrix.

(Hint: Follow the proof of existence of the factorization.)

$$A = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 4 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

Problem 3:

Calculate the Rayleigh quotients $r_k = r(x_k)$ for the following matrix A and given vectors x_k. How far is each r_k from the closest eigenvalue of A?

$$A = \begin{bmatrix} 4 & 6 & 1 \\ 6 & 4 & 6 \\ 1 & 6 & 4 \end{bmatrix},$$

$$x_1 = \begin{bmatrix} 1.5 \\ 2 \\ 1 \end{bmatrix}, \qquad x_2 = \begin{bmatrix} 1 \\ 2.1 \\ 1 \end{bmatrix}, \qquad x_3 = \begin{bmatrix} 1 \\ 0 \\ -1.1 \end{bmatrix}, \qquad x_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Problem 4:

Let A be a symmetric matrix and let $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$ be its eigenvalues. Show that for any $x \ne 0$ the Rayleigh quotient $r(x) = \frac{x^T A x}{x^T x}$ obeys $\lambda_1 = \min_{x \ne 0} r(x)$ and $\lambda_n = \max_{x \ne 0} r(x)$.

(Hint: Use orthogonal diagonalization of the matrix A.)