Math 1080: Spring 2019

Homework #6

due Monday, March 4

Problem 1: Compute LU factorization of the matrix

$$A = \begin{bmatrix} 3 & 0 & -2 & -2 \\ 0 & -1 & 3 & -1 \\ -2 & 0 & 3 & 0 \\ 2 & -2 & 1 & 2 \end{bmatrix}$$

Problem 2:

Solve the following system of equations by both LU factorization and QR factorization:

$$\begin{aligned} 2x_1 - x_3 &= -7 \\ 2x_1 + 2x_2 + 3x_3 &= 1 \\ x_1 + x_2 + 3x_3 &= 2 \end{aligned}$$

Problem 3:

Let A be nonsingular $n \times n$ matrix. Show that A has LU factorization $A = LU$ (no pivoting) with the diagonal terms of the matrix U all nonzero if and only if for each $1 \leq k \leq n$ the upper left $k \times k$ submatrix $A_{1:k,1:k}$ is nonsingular.

(Hint: Use induction argument).

Problem 4:

Compute the LU factorization with partial pivoting, (i.e., find P, L, U such that $PA = LU$) for the following matrix

$$A = \begin{bmatrix} -1 & -2 & -1 & -2 \\ 3 & -1 & 1 & -1 \\ 3 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$