Math 1080: Spring 2019

Homework #7 (due March 20)

Problem 1:

Show that if $A = \begin{bmatrix} a_{11} & w^{\mathrm{T}} \\ w & K \end{bmatrix}$ is symmetric and positive definite, then $a_{11} > 0$ and both K and $K - ww^{\mathrm{T}} / a_{11}$ are symmetric and positive definite.

(Hint: Use the definition of positive definite matrix. Assume $x = \begin{bmatrix} \beta \\ y \end{bmatrix}$ where β is a scalar and y is an n-1 dimensional vector.)

Problem 2:

Use necessary conditions to test positive definiteness of the symmetric matrix A. If conditions are satisfied, compute the Cholesky factorization:

a) $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$

Problem 3:

Solve the following system of equations by Cholesky factorization (if the coefficient matrix is positive definite) or by Gaussian elimination (otherwise)

$$\begin{aligned} 4x_1 + 2x_2 \phantom{{}-6x_3} - 2x_4 &= 6 \\ 2x_1 + 10x_2 - 6x_3 + 2x_4 &= 36 \\ -6x_2 + 8x_3 \phantom{{}+2x_4} &= -30 \\ -2x_1 + 2x_2 \phantom{{}-6x_3} + 4x_4 &= 6 \end{aligned}$$