Math 1080: Spring 2019

Homework #9 (due April 12)

Problem 1:

Determine one eigenvalue of the following matrix using Rayleigh Quotient iteration, starting with initial guess $v^{(0)} = [0 \quad 1]^{\mathrm{T}}$ and initial eigenvalue estimate $\lambda^{(0)} = \left(v^{(0)}\right)^T A v^{(0)}$. Terminate iteration after 3 steps, i.e., after you obtain $\lambda^{(3)}$. What is the approximate eigenvector $v^{(3)}$? What is the error of each $\lambda^{(k)}$?

$$A = \begin{bmatrix} -6 & 2 \\ 2 & -3 \end{bmatrix}$$

Problem 2:

Perform the first two iterations of the QR algorithm (i.e., compute $A^{(2)}$ and $\tilde{Q}^{(2)}$) for the following matrix. How close are the diagonal elements of $A^{(2)}$ to the eigenvalues of A?

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Problem 3:

Reduce the following matrix to Hessenberg form using Householder reflector.

$$A = \begin{bmatrix} 3 & -2 & 4 & 4 \\ -2 & 1 & 9 & -4 \\ 4 & 9 & 2 & -4 \\ 4 & -4 & -4 & 2 \end{bmatrix}$$

Problem 4:

Let Q and R be the QR factors of a symmetric tridiagonal matrix H. Show that the product $K = RQ$ is again a symmetric tridiagonal matrix.

(Hint: Prove the symmetry of K. Show that Q has Hessenberg form and that the product of an upper triangular matrix and a Hessenberg matrix is again a Hessenberg matrix. Then use the symmetry of K.)