Math 1080: Spring 2019

Homework #2

Due Jan 25

Problem 1:

Find the orthogonal projector P onto range(A) where

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$$

What is the nullspace of P? What is the image under P of the vector $[3 \quad 3 \quad 0]^T$?

Problem 2:

Let A be $m \times n$ matrix with $m > n$, and let $A = \hat{Q}\hat{R}$ be a reduced QR factorization. Show that A has full rank if and only if all the diagonal entries of $\hat{R}$ are nonzero.

Problem 3:

Using Gram-Schmidt orthogonalization compute the QR factorization of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & -4 \\ -2 & 6 & 5 \end{bmatrix}$$

Problem 4: Show that if P is a projector, then $\|P\|_2 \geq 1$.

(Hint: For (a), take an arbitrary vector and decompose as $x = Px + (I - P)x$. Use the triangle inequality to conclude that $\|x\|_2 \leq \|Px\|_2$. Now use the definition of $\|P\|_2$ to conclude the result.)