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MATH1080
Coding Assignment #6

In this project, we implement both the QR algorithm with shifting and the Hessenberg transformation algorithm, which are both used to efficiently compute the eigenvalues and eigenvectors of a matrix A. With the provided matrix in the coding assignment, we first use just the QR algorithm with shifting to compute the eigenvalues and eigenvectors. Then, as an alternative simulation, we reduce A to its Hessenberg form H, and find the eigenvalues and eigenvectors for H. As seen below, these produce nearly the same eigenvalues and similar eigenvectors:

QR with shifting	**Hessenberg + QR with shifting**
QR Shift without Hessenberg form: Q: 0.2935 -0.3357 0.4371 -0.1449 0.4957 -0.5860 0.6132 -0.0045 -0.2708 0.3914 -0.4867 -0.4008 0.2373 -0.4268 0.4842 -0.3391 -0.5333 0.3573 0.4133 -0.2947 -0.6267 -0.4275 0.3047 0.2718 0.1031 -0.4119 0.1022 0.7262 0.3000 0.4381 0.5477 0.6698 0.3128 -0.0277 0.2239 0.3203 L: 29.0434 -0.0000 -0.0000 -0.0000 0.0000 0.0000 0.0000 -22.4325 -0.0000 -0.0000 -0.0000 -0.0000 0.0000 -0.0000 14.4364 0.0000 -0.0000 -0.0000 -0.0000 0.0000 -0.0000 10.3213 -0.0000 -0.0000 0.0000 -0.0000 0.0000 -0.0000 -9.1617 -0.0000 0.0000 -0.0000 -0.0000 -0.0000 -0.0000 3.7931	QR Shift with Hessenberg form: Q: -0.2935 -0.3357 -0.4371 -0.1449 -0.4957 -0.5860 0.5713 -0.5904 0.3913 0.0868 -0.3983 0.0756 0.6422 0.5836 -0.0817 -0.0774 -0.1296 -0.4663 0.3858 -0.4172 -0.5286 -0.1335 0.6145 -0.0468 -0.1613 -0.1550 0.5788 0.0119 0.4475 -0.6438 -0.0114 0.0062 0.1860 -0.9734 -0.0302 0.1296 L: 29.0434 -0.0000 -0.0000 0.0000 0.0000 -0.0000 0.0000 -22.4325 -0.0000 -0.0000 0.0000 -0.0000 -0.0000 -0.0000 14.4364 0.0000 0.0000 0.0000 -0.0000 0.0000 -0.0000 10.3213 0.0000 0.0000 -0.0000 0.0000 -0.0000 0.0000 -9.1617 -0.0000 0.0000 -0.0000 0.0000 0.0000 -0.0000 3.7931
182 iterations	145 iterations

As we can see, both these approaches finish after a different number of iterations (discounting the time taken to convert A to its Hessenberg form). Additionally, we observe the approximate eigenvalues on the diagonals of the L matrix. With an online calculator, these eigenvalues were verified to be the correct values to the ten-thousandths decimal place. So, both approaches produce very good estimates of the eigenvalues of large 6x6 matrices in a trivial amount of time on everyday computing machines.