Math 1080: Spring 2019

Homework #3

Due Feb 1

Problem 1:

Show that the Householder reflector $F = I - 2ww^{\mathrm{T}}$, with $\|w\| = 1$, is symmetric and orthogonal. Find the eigenvalues and eigenvectors of *F*.

Problem 2:

Use Householder triangularization to find the QR factorization of

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Problem 3:

Use QR factorization to solve the least squares minimization problem $r = \min_x \|Ax - b\|_2$ for the following data. Provide both the vector x and the minimum residue r.

$$A = \begin{bmatrix} 2 & 2 \\ -2 & 3 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Problem 4:

(a) Show that the product of two upper triangular matrices is an upper triangular matrix.

(b) Show that the inverse of an upper triangular matrix (if it exists) is an upper triangular matrix.