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Homework 5 - Theory of Linear Programming
Math 1101 - An Introduction to Optimization
The University of Pittsburgh - Fall 2018

Please submit the following problems at the beginning of class Wednesday, November 15. Additionally, please staple this sheet to the front of your solutions.

1. Let $P(\overrightarrow{x}) = P(x_1, x_2, \ldots, x_n)$ be the objective function of a Linear Programming problem that has a feasible area F. Show

$$P(\overrightarrow{x}^*) = \max\{P(\overrightarrow{x}) \mid \overrightarrow{x} \in F\} \Leftrightarrow -P(\overrightarrow{x}^*) = \min\{-P(\overrightarrow{x}) \mid \overrightarrow{x} \in F\},$$

   i.e. the max of $P(\overrightarrow{x})$ occurs at the same location as the min of $-P(\overrightarrow{x})$.

2. Let $F = \{[0,0]^T, [1,2]^T, [4,1]^T\}$. Present (without proof) a graph of
   (a) $aff(F)$.
   (b) $conv(F)$.
   (c) $coni(F)$.

3. Determine for each of the following sets if the set is affine, convex, or a convex cone with vertex at the origin ("determine" means "prove or disprove"). As well, (without proof) state whether each set is a convex polyhedron and/or a convex polytope. If a set is not convex, state (without proof) its convex hull.
   (a) $S_1 = \{[x,y]^T \mid x + y \leq 5, y \geq 0, x \geq 1\}$.
   (b) $S_2 = \{[x,y]^T \mid y \leq |x|, y \geq 0, x \geq 0\}$.
   (c) $S_3 = \{[x,y]^T \mid y - x^2 + 6x \leq 9, y \geq 0, x \geq 0\}$.

4. Determine the extreme (corner) point(s) $P = \{\overrightarrow{P}_1, ..., \overrightarrow{P}_s\}$ of S_2 in the previous question. Then find direction vectors $D = \{\overrightarrow{d}_1, ..., \overrightarrow{d}_t\}$ of S_2 such that

$$S_2 = conv(P) + coni(D)$$

   as in the Finite Basis Theorem and Fundamental Theorem of Linear Programming.