

NAME: _____

Instructor: JP Wheeler/Grader:

HOMEWORK 1 - INTRO TO LINEAR PROGRAMMING AND THE SIMPLEX METHOD
MATH 1101 - AN INTRODUCTION TO OPTIMIZATION
THE UNIVERSITY OF PITTSBURGH - FALL 2018

Submit the following problems at the beginning of class Friday, September 14. Your work is expected to be clear and legible. Additionally, attach this sheet to the front of your work.

INSTRUCTIONS **For questions 1-3**, do each of the following:

I. Solve the following linear programming problems by graphing the feasible region then evaluating the objective function at each corner point. “Solve” means state the optimal value of the objective function and ***all points*** in the feasible region at which this optimal value occurs. II. Solve each problem a second time using the Simplex Method clearly stating the model after the introduction of slack, surplus, and artificial variables. You may use a calculator or computer to do the row operations, but write down the obtained simplex tableau after each iteration of the method. At each iteration identify the pivot element. III. Check your work using a software package of your choice (Solver, Matlab, etc.). Print and submit your answer screen and please make clear what software you have used.

1.

$$\begin{aligned} &\text{Maximize and minimize } P(x, y) = 5x + 2y \\ &\text{Subject to } x + y \geq 2 \\ &\quad 2x + y \geq 4 \\ &\quad x, y \geq 0 \end{aligned}$$

For this question only (that is, Question 1), when finding the minimum and using the Simplex method (part II above), at each iteration state which variables are basic and which are nonbasic. Also, at each iteration state the value of the objective function.

2.

$$\begin{aligned} &\text{Maximize } P(x, y) = 20x + 10y \\ &\text{Subject to } x + y \geq 2 \\ &\quad x + y \leq 8 \\ &\quad 2x + y \leq 10 \\ &\quad x, y \geq 0 \end{aligned}$$

3.

$$\begin{aligned} &\text{Maximize and minimize } P(x, y) = 20x + 10y \\ &\text{Subject to } 2x + 3y \geq 30 \\ &\quad 2x + y \leq 26 \\ &\quad -2x + 5y \leq 34 \\ &\quad x, y \geq 0 \end{aligned}$$

4. In this problem, there is a tie for the choice of the first pivot column. When you do your work using the simplex method use the method twice to solve the problem two different ways; first by choosing column 1 as the first pivot column and then for your second solution effort, solve by choosing column 2 as the first pivot column. You may use a computer or calculator to perform the Simplex Method, but do write down the results of each iteration.

$$\begin{aligned} &\text{Maximize } P(x, y) = x + y \\ &\text{Subject to } 2x + y \leq 16 \\ &\quad x \leq 6 \\ &\quad y \leq 10 \\ &\quad x, y \geq 0 \end{aligned}$$

5. In Example 2 in class, we used the dual to solve

$$\begin{aligned} &\text{Minimize } C(x_1, x_2, x_3) = 40x_1 + 12x_2 + 40x_3 \\ &\text{Subject to } 2x_1 + x_2 + 5x_3 \geq 20 \\ &\quad 4x_1 + x_2 + x_3 \geq 30 \\ &\quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

The dual problem has as its first constraint

$$2y_1 + 4y_2 \leq 40. \tag{1}$$

Replace this constraint by its simplified version

$$y_1 + 2y_2 \leq 20 \tag{2}$$

then proceed with the Simplex Method. Compare your answer with the one obtained in class and explain what causes the different answer. Follow the instructions in II from questions 1-3. [Note: the purpose of this question is to illustrate Warning 4.3.2 on page 47 of the notes.]