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Yukang Zang

Homework 2 - Integer Programming via Branch and Bound Method and Cut-Planes
Math 3060 - An Introduction to Optimization
The University of Pittsburgh - Fall 2018

Please submit the following problem at the beginning of class Monday, September 24. Additionally, please staple these sheets to the front of your solutions.

1. Solve the following problem by incrementally using Dakin's Branch and Bound Method:

$$\text{Maximize:} \quad 4x_1 + 3x_2 + 3x_3$$

Subject to:

$$4x_1 + 2x_2 + x_3 \leq 10 \tag{1}$$
$$3x_1 + 4x_2 + 2x_3 \leq 14 \tag{2}$$
$$2x_1 + x_2 + 3x_3 \leq 7 \tag{3}$$

where x_1, x_2, x_3 are nonnegative integers.

Please draw a decision tree similar to what we did in class for your answers. As well, for each iteration of the particular path you are following, please clearly state the LP problem you are answering. You may use Solver (or a program of your choice) to answer each of these individual LP problems (you do not have to show row operations...).

2. User Solver (or a program of your choice) to answer the above question as a LP problem. Observe the computation time (should be incredibly fast). Return to the original problem, solve it a second time but choose"integer" as a constraint for each of the decision variables. Do this a third time but change the tolerance under "Options" to 1% or less. Observe the computation time for the IP problem. This should be a little longer, but for this small of a problem the difference in computation time is probably not very noticeable. Submit the Answer Report for the third run of the problem (if you are using software or a program other than Solver, submit a screenshot of the program's output).

3. Solve the following problem by hand incrementally using Cut-Planes:

$$\text{Minimize:} \quad x - y$$

Subject to:

$$3x + 4y \leq 6 \tag{4}$$
$$x - y \leq 1 \tag{5}$$

where x and y are nonnegative integers.

1. State the modified Linear Programming problem (what we have after introducing slack, surplus, and artificial variables, etc.);
2. graph the feasible region;
3. provide the initial simplex tableau;
4. begin iteratively introducing Gomory cuts until an integer solution is attained where for each iteration:
   (a) clearly show work supporting why you have introduced a particular cut-plane (i.e. the new constraint),
   (b) write down the new LP problem (the one from the previous iteration plus the new constraint) and the new initial Simplex Tableau,
   (c) provide a diagram showing the feasible region for the decision variables and
   (d) use any computer resource to find the final simplex tableau for this iteration's LP and provide the final Simplex tableau (a screen shot is acceptable).

4. Solve the following problem by hand incrementally using Cut-Planes:

$$\text{Maximize:} \quad 4x_1 + 3x_2 + 3x_3$$

Subject to:

$$4x_1 + 2x_2 + x_3 \leq 10 \tag{6}$$
$$3x_1 + 4x_2 + 2x_3 \leq 14 \tag{7}$$
$$2x_1 + x_2 + 3x_3 \leq 7 \tag{8}$$

where x_1, x_2, x_3 are nonnegative integers.

1. State the modified Linear Programming problem (what we have after introducing slack, surplus, and artificial variables, etc.);
2. provide the initial simplex tableau;
3. begin iteratively introducing Gomory cuts until an integer solution is attained where for each iteration:
   (a) clearly show work supporting why you have introduced a particular cut-plane (i.e. the new constraint),
   (b) write down the new LP problem (the one from the previous iteration plus the new constraint) and the new initial Simplex Tableau, and
   (c) use any computer resource to find the final simplex tableau for this iteration's LP and provide the final Simplex tableau (a screen shot is acceptable).

5. User Solver (or a program of your choice) to answer the following IP question (you are not to answer this one by hand!). Do note that solving the problem requires making certain decision variables binary (i.e. 0-1) variables. If you are running Solver, there is an option in the drop-down list box for the binary constraint (this is the box with "<=", etc.).

   (From Ragsdale) In his position as vice president of research and development (R&D) for CRT Technologies, Mark Schwartz is responsible for evaluating and choosing which R&D projects to support. The company received 18 R&D proposals from its scientists and engineers and identified six projects as being consistent with the company's mission. However, the company does not have the funds available to undertake all six projects, so Mark must determine which projects to select. The funding requirements for each project are summarized in the following table along with the NPV (Net Present Value; let's not worry about what that means) the company expects each project to generate.

Project	Expected NPV	Year 1 CR	Year 2 CR	Year 3 CR	Year 4 CR	Year 5 CR
1	\$141	\$75	\$25	\$20	\$15	\$10
2	\$187	\$90	\$35	\$0	\$0	\$30
3	\$121	\$60	\$15	\$15	\$15	\$15
4	\$83	\$30	\$20	\$10	\$5	\$5
5	\$262	\$100	\$25	\$20	\$20	\$20
6	\$127	\$50	\$20	\$10	\$30	\$40

KEY: NPV = Net Present Value; CR = Capital Required.
NOTE: all dollar values are in \$1,000's
(So think of NPV as revenue and CR as costs.)

   The company currently has \$250,000 available to invest in new projects. It has budgeted \$75,000 for continued support for these projects in year 2 and \$50,000/year for years 3,4,and 5. Surplus funds in any year are reappropriated for other uses within the company and may not be carried over to future years (note: this actually makes the problem easier).

   So, what projects should the company select in order to maximize NPV? (Note to all future analysts; please begin your solution by clearly stating what your decision variables are and what they represent and clearly state the model.) Submit your model and the answer report.

James Hahn
MATH1101
Homework #2

2. 1) 0.016 seconds
   2) 0.017 seconds
   3) 0.017 seconds (tolerance = 0.5%)
   All simulations were ran on my desktop computer, so these times are almost indistinguishable.

Microsoft Excel 16.0 Answer Report
Worksheet: [Book1]Sheet1
Report Created: 9/22/2018 6:09:44 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.017 Seconds.
Iterations: 2 Subproblems: 4
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance .5%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$A$5	Objective	0	13

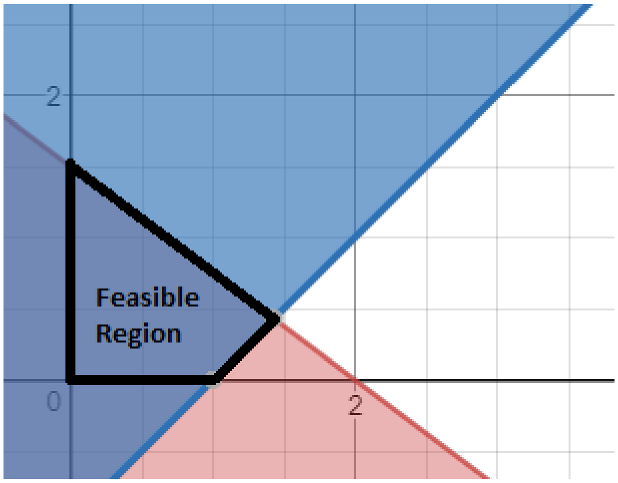
Variable Cells

Cell	Name	Original Value	Final Value	Integer
$A$2	x1	0	1	Integer
$B$2	x2	0	2	Integer
$C$2	x3	0	1	Integer

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$1	x3	9	$E$1<=$G$1	Not Binding	1
$E$2		13	$E$2<=$G$2	Not Binding	1
$E$3		7	$E$3<=$G$3	Binding	0
$A$2:$C$2=Integer					

3. i) Minimize P = x – y => Maximize P = y - x
   Subject to:
   $3x + 4y + s_1 = 6$
   $x - y + s_2 = 1$
   $x, y, s_1, s_2 >= 0$

ii) 

iii) Initial simplex tableau:

x	y	s1	s2	P	
3	4	1	0	0	6
1	-1	0	1	0	1
-1	1	0	0	1	0

iv) Solution to initial LP problem; $x = 0$, $y = 3/2$, $P = 3/2$:

x	y	s1	s2	P	
3/4	1	1/4	0	0	3/2
7/4	0	1/4	1	0	5/2
7/4	0	1/4	0	1	3/2

Choose 2nd row as the next cut plane since it has the largest RHS:

$3/4x + y - \frac{1}{4} s_1 = 3/2$

$(0 + \frac{3}{4})x + (1+0)y + (0 + \frac{1}{4})s_1 = (1 + \frac{1}{2})$

$1 - 3/4x - 1/4s_1 = -1/2$

$-3/4x - 1/4s_1 + \frac{1}{2} <= 0$ is our new constraint, which converts to:

$-3/4x - 1/4s_1 + t_1 = -1/2$

<u>New problem</u>

Maximize $y - x$

Subject to:

$3x + 4y + s_1 = 6$

$x - y + s_2 = 1$

$-3/4x - 1/4s_1 + t_1 = -1/2$

$x, y, s_1, s_2, t_1 >= 0$

Initial simplex tableau:

x	y	s1	s2	t1	P	
3/4	1	1/4	0	0	0	3/2
7/4	0	1/4	1	0	0	5/2
-3/4	0	-1/4	0	1	0	-1/2
7/4	0	1/4	0	0	1	3/2

Feasible Region:

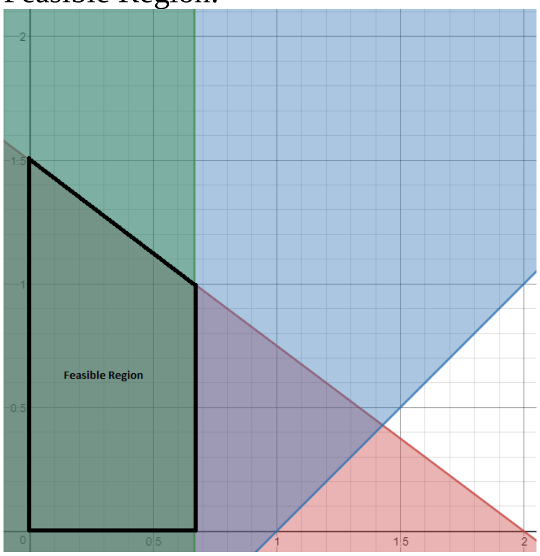


Tableau after solving the LP problem:

x	y	s1	s2	t1	P	
0	1	0	0	1	0	1
1	0	0	1	1	0	2
3	0	1	0	-4	0	2
1	0	0	0	1	1	1

x is not a basic variable and we cannot reduce the tableau, so x = 0 (reasoning for this is on page 48 of the notes) and y = 1 with P = 1. Thus, we have an integer solution and we do not need anymore cuts. To convert this into the solution for the minimization problem, negate the P value. So, the final solution is: **x = 0, y = 1, P = -1**.

4.

Maximize $P = 4x_1 + 3x_2 + 3x_3$
Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$
$3x_1 + 4x_2 + 2x_3 + s_2 = 14$
$2x_1 + 1x_2 + 3x_3 + s_3 = 7$
$x_1, x_2, x_3, s_1, s_2, s_3 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	p	
4	2	1	1	0	0	0	10
3	4	2	0	1	0	0	14
2	1	3	0	0	1	0	7
-4	-3	-3	0	0	0	1	0

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	p	
1	0	0	2/5	-1/5	0	0	6/5
0	1	0	-1/5	2/5	-1/5	0	11/5
0	0	1	-1/5	0	2/5	0	4/5
0	0	0	2/5	2/5	3/5	1	69/5

We get a solution at $x_1 = 6/5$, $x_2 = 11/5$, $x_3 = 4/5$ with P = 69/5 .
Choose the 1st row as the next cut plane since we just need to choose one at random:
$x_1 + 2/5s_1 – 1/5s_2 = 6/5$
$(1 + 0)x_1 + (0 + 2/5)s_1 + (-1 + 4/5)s_2 = (1 + 1/5)$
$-2/5s_1 – 4/5s_2 + 1/5 <= 0$
$-2/5s_1 – 4/5s_2 + t_1 = -1/5$ is the new constraint

<u>New problem</u>

Maximize $P = 4x_1 + 3x_2 + 3x_3$
Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$
$3x_1 + 4x_2 + 2x_3 + s_2 = 14$
$2x_1 + 1x_2 + 3x_3 + s_3 = 7$
$-2/5s_1 – 4/5s_2 + t_1 = -1/5$
$x_1, x_2, x_3, s_1, s_2, s_3, t_1 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	P	
1	0	0	2/5	-1/5	0	0	0	6/5
0	1	0	-1/5	2/5	-1/5	0	0	11/5
0	0	1	-1/5	0	2/5	0	0	4/5
0	0	0	-2/5	-4/5	0	1	0	-1/5
0	0	0	2/5	2/5	3/5	0	1	69/5

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	P	
1	0	0	1/2	0	0	-1/4	0	5/4
0	1	0	-2/5	0	-1/5	1/2	0	21/10
0	0	1	-1/5	0	2/5	0	0	4/5
0	0	0	1/2	1	0	-5/4	0	1/4
0	0	0	1/5	0	3/5	1/2	1	137/10

We get a solution at $x_1 = 5/4$, $x_2 = 21/10$, $x_3 = 4/5$ with P = 137/10
Choose the 2nd row as the next cut plane at random:
$x_2 – 2/5s_1 – 1/5s_3 + 1/2t_1 = 21/10$
$(1 + 0)x_2 + (-1 + 3/5)s_1 + (-1 + 4/5)s_3 + (0 + ½)t_1 = (2 + 1/10)$
$-3/5s_1 – 4/5s_3 – 1/2t_1 + 1/10 <= 0$
$-3/5s_1 – 4/5s_3 – 1/2t_1 + t_2 = -1/10$ is the new constraint

<u>New problem</u>

Maximize $P = 4x_1 + 3x_2 + 3x_3$
Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$
$3x_1 + 4x_2 + 2x_3 + s_2 = 14$
$2x_1 + 1x_2 + 3x_3 + s_3 = 7$
$-2/5s_1 – 4/5s_2 + t_1 = -1/5$
$-3/5s_1 – 4/5s_3 – 1/2t_1 + t_2 = -1/10$
$x_1, x_2, x_3, s_1, s_2, s_3, t_1, t_2 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	t2	P	
1	0	0	1/2	0	0	-1/4	0	0	5/4
0	1	0	-2/5	0	-1/5	1/2	0	0	21/10
0	0	1	-1/5	0	2/5	0	0	0	4/5
0	0	0	1/2	1	0	-5/4	0	0	1/4
0	0	0	-3/5	0	-4/5	-1/2	1	0	-1/10
0	0	0	1/5	0	3/5	1/2	0	1	137/10

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	t2	P	
1	0	0	0	0	-2/3	-2/3	5/6	0	7/6
0	1	0	0	0	1/3	5/6	-2/3	0	13/6
0	0	1	0	0	2/3	1/6	-1/3	0	5/6
0	0	0	0	1	-2/3	-5/3	5/6	0	1/6
0	0	0	1	0	4/3	5/6	-5/3	0	1/6
0	0	0	0	0	1/3	1/3	1/3	1	41/3

We get a solution at $x_1 = 7/6$, $x_2 = 13/6$, $x_3 = 5/6$ with P = 41/3
Choose the 1st row as the next cut plane at random:
$x_1 – 2/3s_3 – 2/3t_1 + 5/6t_2 = 7/6$
$(1 + 0)x_1 + (-1 + 1/3)s_3 + (-1 + 1/3)t_1 + (0 + 5/6)t_2 = (1 + 1/6)$
$-1/3s_3 – 1/3t_1 – 5/6t_2 + 1/6 <= 0$

$-1/3s_3 - 1/3t_1 - 5/6t_2 + t_3 = -1/6$ is the new constraint

<u>New problem</u>

Maximize $P = 4x_1 + 3x_2 + 3x_3$
Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$
$3x_1 + 4x_2 + 2x_3 + s_2 = 14$
$2x_1 + 1x_2 + 3x_3 + s_3 = 7$
$-2/5s_1 - 4/5s_2 + t_1 = -1/5$
$-3/5s_1 - 4/5s_3 - 1/2t_1 + t_2 = -1/10$
$-1/3s_3 - 1/3t_1 - 5/6t_2 + t_3 = -1/6$
$x_1, x_2, x_3, s_1, s_2, s_3, t_1, t_2, t_3 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	t2	t3	P	
1	0	0	0	0	-2/3	-2/3	5/6	0	0	7/6
0	1	0	0	0	1/3	5/6	-2/3	0	0	13/6
0	0	1	0	0	2/3	1/6	-1/3	0	0	5/6
0	0	0	0	1	-2/3	-5/3	5/6	0	0	1/6
0	0	0	1	0	4/3	5/6	-5/3	0	0	1/6
0	0	0	0	0	-1/3	-1/3	-5/6	1	0	-1/6
0	0	0	0	0	1/3	1/3	1/3	0	1	41/3

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	t2	t3	P	
1	0	0	0	0	-1	-1	0	1	0	1
0	1	0	0	0	3/5	11/10	0	-4/5	0	23/10
0	0	1	0	0	4/5	3/10	0	-2/5	0	9/10
0	0	0	0	1	-1	-2	0	1	0	0
0	0	0	1	0	2	3/2	0	-2	0	1/2
0	0	0	0	0	2/5	2/5	1	-6/5	0	1/5
0	0	0	0	0	1/5	1/5	0	2/5	1	68/5

We get a solution at $x_1 = 1$, $x_2 = 23/10$, $x_3 = 9/10$ with $P = 68/5$
Choose the 2nd row as the next cut plane at random:
$x_2 + 3/5s_3 + 11/10t_1 - 4/5t_3 = 23/10$
$(1 + 0)x_2 + (0 + 3/5)s_3 + (1 + 1/10)t_1 + (-1 + 1/5)t_3 = (2 + 3/10)$
$-3/5s_3 - 1/10t_1 - 1/5t_3 + 3/10 <= 0$
$-3/5s_3 - 1/10t_1 - 1/5t_3 + t_4 = -3/10$ is the new constraint

<u>New problem</u>

Maximize $P = 4x_1 + 3x_2 + 3x_3$
Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$
$3x_1 + 4x_2 + 2x_3 + s_2 = 14$
$2x_1 + 1x_2 + 3x_3 + s_3 = 7$
$-2/5s_1 - 4/5s_2 + t_1 = -1/5$
$-3/5s_1 - 4/5s_3 - 1/2t_1 + t_2 = -1/10$

$-1/3s_3 - 1/3t_1 - 5/6t_2 + t_3 = -1/6$
$-3/5s_3 - 1/10t_1 - 1/5t_3 + t_4 = -3/10$
$x_1, x_2, x_3, s_1, s_2, s_3, t_1, t_2, t_3, t_4 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	P	
1	0	0	0	0	-1	-1	0	1	0	0	1
0	1	0	0	0	3/5	11/10	0	-4/5	0	0	23/10
0	0	1	0	0	4/5	3/10	0	-2/5	0	0	9/10
0	0	0	0	1	-1	-2	0	1	0	0	0
0	0	0	1	0	2	3/2	0	-2	0	0	1/2
0	0	0	0	0	2/5	2/5	1	-6/5	0	0	1/5
0	0	0	0	0	-3/5	-1/10	0	-1/5	1	0	-3/10
0	0	0	0	0	1/5	1/5	0	2/5	0	1	68/5

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	P	
1	0	0	1/2	0	0	-1/4	0	0	0	0	5/4
0	1	0	-3/8	0	0	9/16	0	0	-1/4	0	35/16
0	0	1	-1/4	0	0	-1/8	0	0	1/2	0	5/8
0	0	0	1/2	1	0	-5/4	0	0	0	0	1/4
0	0	0	-3/8	0	0	-7/16	0	1	-5/4	0	3/16
0	0	0	-1/2	0	0	-1/4	1	0	-1	0	1/4
0	0	0	1/8	0	1	5/16	0	0	-5/4	0	7/16
0	0	0	1/8	0	0	5/16	0	0	3/4	1	215/16

We get a solution at $x_1 = 5/4$, $x_2 = 35/16$, $x_3 = 5/8$ with P = 215/16
Choose the 2nd row as the next cut plane at random:
$x_2 - 3/8s_1 + 9/16t_1 - 1/4t_4 = 35/16$
$(1 + 0)x_2 + (-1 + 5/8)s_1 + (0 + 9/16)t_1 + (-1 + 3/4)t_4 = (2 + 3/16)$
$-5/8s_1 - 9/16t_1 - 3/4t_4 + 3/16 <= 0$
$-5/8s_1 - 9/16t_1 - 3/4t_4 + t_5 = -3/16$ is the new constraint

<u>New problem</u>

Maximize $P = 4x_1 + 3x_2 + 3x_3$
Subject to:
$4x_1 + 2x_2 + 1x_3 + s_1 = 10$
$3x_1 + 4x_2 + 2x_3 + s_2 = 14$
$2x_1 + 1x_2 + 3x_3 + s_3 = 7$
$-2/5s_1 - 4/5s_2 + t_1 = -1/5$
$-3/5s_1 - 4/5s_3 - 1/2t_1 + t_2 = -1/10$
$-1/3s_3 - 1/3t_1 - 5/6t_2 + t_3 = -1/6$
$-3/5s_3 - 1/10t_1 - 1/5t_3 + t_4 = -3/10$
$-5/8s_1 - 9/16t_1 - 3/4t_4 + t_5 = -3/16$
$x_1, x_2, x_3, s_1, s_2, s_3, t_1, t_2, t_3, t_4, t_5 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	t5	P	
1	0	0	1/2	0	0	-1/4	0	0	0	0	0	5/4
0	1	0	-3/8	0	0	9/16	0	0	-1/4	0	0	35/16
0	0	1	-1/4	0	0	-1/8	0	0	1/2	0	0	5/8
0	0	0	1/2	1	0	-5/4	0	0	0	0	0	1/4
0	0	0	-3/8	0	0	-7/16	0	1	-5/4	0	0	3/16
0	0	0	-1/2	0	0	-1/4	1	0	-1	0	0	1/4
0	0	0	1/8	0	1	5/16	0	0	-5/4	0	0	7/16
0	0	0	-5/8	0	0	-9/16	0	0	-3/4	1	0	-3/16
0	0	0	1/8	0	0	5/16	0	0	3/4	0	1	215/16

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	t5	P	
1	0	0	0	0	0	-7/10	0	0	-3/5	4/5	0	11/10
0	1	0	0	0	0	9/10	0	0	1/5	-3/5	0	23/10
0	0	1	0	0	0	1/10	0	0	4/5	-2/5	0	7/10
0	0	0	0	1	0	-17/10	0	0	-3/5	4/5	0	1/10
0	0	0	0	0	0	-1/10	0	1	-4/5	-3/5	0	3/10
0	0	0	0	0	0	1/5	1	0	-2/5	-4/5	0	2/5
0	0	0	0	0	1	1/5	0	0	-7/5	1/5	0	2/5
0	0	0	1	0	0	9/10	0	0	6/5	-8/5	0	3/10
0	0	0	0	0	0	1/5	0	0	3/5	1/5	1	67/5

We get a solution at $x_1 = 11/10$, $x_2 = 23/10$, $x_3 = 7/10$ with $P = 67/5$

Choose the 1st row as the next cut plane at random:

$x_1 – 7/10t_1 – 3/5t_4 + 4/5t_5 = 11/10$

$(1 + 0)x_1 + (-1 + 3/10)t_1 + (-1 + 2/5)t_4 + (0 + 4/5)t_5 = (1 + 1/10)$

$-3/10t_1 – 2/5t_4 – 4/5t_5 + 1/10 <= 0$

$-3/10t_1 – 2/5t_4 – 4/5t_5 + t_6 <= -1/10$

<u>New problem</u>

Maximize $P = 4x_1 + 3x_2 + 3x_3$

Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$

$3x_1 + 4x_2 + 2x_3 + s_2 = 14$

$2x_1 + 1x_2 + 3x_3 + s_3 = 7$

$-2/5s_1 – 4/5s_2 + t_1 = -1/5$

$-3/5s_1 – 4/5s_3 – 1/2t_1 + t_2 = -1/10$

$-1/3s_3 – 1/3t_1 – 5/6t_2 + t_3 = -1/6$

$-3/5s_3 - 1/10t_1 - 1/5t_3 + t_4 = -3/10$

$-5/8s_1 – 9/16t_1 – 3/4t_4 + t_5 = -3/16$

$-3/10t_1 – 2/5t_4 – 4/5t_5 + t_6 <= -1/10$

$x_1, x_2, x_3, s_1, s_2, s_3, t_1, t_2, t_3, t_4, t_5, t_6 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	t5	t6	P	
1	0	0	0	0	0	-7/10	0	0	-3/5	4/5	0	0	11/10
0	1	0	0	0	0	9/10	0	0	1/5	-3/5	0	0	23/10
0	0	1	0	0	0	1/10	0	0	4/5	-2/5	0	0	7/10
0	0	0	0	1	0	-17/10	0	0	-3/5	4/5	0	0	1/10
0	0	0	0	0	0	-1/10	0	1	-4/5	-3/5	0	0	3/10
0	0	0	0	0	0	1/5	1	0	-2/5	-4/5	0	0	2/5
0	0	0	0	0	1	1/5	0	0	-7/5	1/5	0	0	2/5
0	0	0	1	0	0	9/10	0	0	6/5	-8/5	0	0	3/10
0	0	0	0	0	0	-3/10	0	0	-2/5	-4/5	1	0	-1/10
0	0	0	0	0	0	1/5	0	0	3/5	1/5	0	1	67/5

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	t5	t6	P	
1	0	0	0	0	0	-1	0	0	-1	0	1	0	1
0	1	0	0	0	0	9/8	0	0	1/2	0	-3/4	0	19/8
0	0	1	0	0	0	1/4	0	0	1	0	-1/2	0	3/4
0	0	0	0	1	0	-2	0	0	-1	0	1	0	0
0	0	0	0	0	0	1/8	0	1	-1/2	0	-3/4	0	3/8
0	0	0	0	0	0	1/2	1	0	0	0	-1	0	1/2
0	0	0	0	0	1	1/8	0	0	-3/2	0	1/4	0	3/8
0	0	0	1	0	0	3/2	0	0	2	0	-2	0	1/2
0	0	0	0	0	0	3/8	0	0	1/2	1	-5/4	0	1/8
0	0	0	0	0	0	1/8	0	0	1/2	0	1/4	1	107/8

We get a solution at $x_1 = 1$, $x_2 = 19/8$, $x_3 = 3/4$ with $P = 107/8$

Choose the 2nd row as the next cut plane at random:

$x_2 + 9/8t_1 + 1/2t_4 – 3/4t_6 = 19/8$

$(1 + 0)x_2 + (1 + 1/8)t_1 + (0 + ½)t_4 + (-1 + ¼)t_6 = (2 + 3/8)$

$-1/8t_1 – 1/2t_4 – 1/4t_6 + 3/8 <= 0$

$-1/8t_1 – 1/2t_4 – 1/4t_6 + t_7 = -3/8$ is the new constraint

New problem

Maximize $P = 4x_1 + 3x_2 + 3x_3$

Subject to:

$4x_1 + 2x_2 + 1x_3 + s_1 = 10$

$3x_1 + 4x_2 + 2x_3 + s_2 = 14$

$2x_1 + 1x_2 + 3x_3 + s_3 = 7$

$-2/5s_1 – 4/5s_2 + t_1 = -1/5$

$-3/5s_1 – 4/5s_3 – 1/2t_1 + t_2 = -1/10$

$-1/3s_3 – 1/3t_1 – 5/6t_2 + t_3 = -1/6$

$-3/5s_3 - 1/10t_1 - 1/5t_3 + t_4 = -3/10$

$-5/8s_1 – 9/16t_1 – 3/4t_4 + t_5 = -3/16$

$-3/10t_1 – 2/5t_4 – 4/5t_5 + t_6 <= -1/10$

$-1/8t_1 – 1/2t_4 – 1/4t_6 + t_7 = -3/8$

$x_1, x_2, x_3, s_1, s_2, s_3, t_1, t_2, t_3, t_4, t_5, t_6, t_7 >= 0$

Initial simplex tableau:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	t5	t6	t7	P	
1	0	0	0	0	0	-1	0	0	-1	0	1	0	0	1
0	1	0	0	0	0	9/8	0	0	1/2	0	-3/4	0	0	19/8
0	0	1	0	0	0	1/4	0	0	1	0	-1/2	0	0	3/4
0	0	0	0	1	0	-2	0	0	-1	0	1	0	0	0
0	0	0	0	0	0	1/8	0	1	-1/2	0	-3/4	0	0	3/8
0	0	0	0	0	0	1/2	1	0	0	0	-1	0	0	1/2
0	0	0	0	0	1	1/8	0	0	-3/2	0	1/4	0	0	3/8
0	0	0	1	0	0	3/2	0	0	2	0	-2	0	0	1/2
0	0	0	0	0	0	3/8	0	0	1/2	1	-5/4	0	0	1/8
0	0	0	0	0	0	-1/8	0	0	-1/2	0	-1/4	1	0	-3/8
0	0	0	0	0	0	1/8	0	0	1/2	0	1/4	0	1	107/8

Tableau at end of LP iteration:

x1	x2	x3	s1	s2	s3	t1	t2	t3	t4	t5	t6	t7	P	
1	0	0	0	-1	0	1	0	0	0	0	0	0	0	1
0	1	0	0	2/3	0	-1/6	0	0	0	0	0	-1/3	0	5/2
0	0	1	0	2/3	0	-7/6	0	0	0	0	0	2/3	0	1/2
0	0	0	0	2/3	0	-7/6	0	0	0	0	1	-4/3	0	1/2
0	0	0	0	1/3	0	-1/3	0	1	0	0	0	-5/3	0	1
0	0	0	0	2/3	0	-2/3	1	0	0	0	0	-4/3	0	1
0	0	0	0	-2/3	1	5/3	0	0	0	0	0	-5/3	0	1
0	0	0	0	-1/3	0	5/6	0	0	1	0	0	-4/3	0	1/2
0	0	0	0	1	0	-3/2	0	0	0	1	0	-1	0	1/2
0	0	0	1	2	0	-5/2	0	0	0	0	0	0	0	1/2
0	0	0	0	0	0	0	0	0	0	0	0	1	1	13

We get a solution at $x_1 = 1$, $x_2 = 5/2$, $x_3 = 1/2$ with P = 13
It was at this point the online matrix calculator ran out of columns. It began with 15 columns, and after 7 cuts, there are not enough columns to continue further cuts unless I decided to do all the row operations by hand. However, by best judgement, that is not completely necessary. Through these 7 cuts, we have noticed several unique patterns:

1) x_1 has been hovering around the value of 1 for six cuts now and has even reached the value of 1 on several iterations. We can estimate the true integer value of x_1 is 1.
2) x_2 has been hovering around the value of 2 for five cuts now. We can estimate the true integer value of x_1 is 2.
3) Our objective function, P, began at a value of 13.6, hovered around that number, and has slowly decreased since, slowly plateauing around the value of 13. We can estimate this may be one of the possibilities for the ground-truth objective function value for this integer linear programming problem.
4) With the above estimates, we notice x_3 has been hovering around the value of 1 on and off for iterations. With moderate to low confidence, we can conclude at this point that x_3 converges to integer 1. However, undergoing many more cuts should unveil its true value of 1, as we have learned in question #1 of this homework.
5) With some confidence, we can safely predict the solution to this integer linear programming problem is $\mathbf{x_1 = 1, x_2 = 2, x_3 = 1}$ **with P = 13**. Undergoing a countless number of further cuts will require much more computational time, handwritten work to discover new constraints, and a large matrix calculator. However, due to the limits of all three variables showing themselves across 7 cuts/iterations, we can assume to an extent this is a safe maximum solution to our integer linear programming problem.

5.

	NPV	Year 1 Cost	Year 2 Cost	Year 3 Cost	Year 4 Cost	Year 5 Cost	Selected
Project 1	141	75	25	20	15	10	1
Project 2	187	90	35	0	0	30	0
Project 3	121	60	15	15	15	15	0
Project 4	83	30	20	10	5	5	1
Project 5	262	100	25	20	20	20	1
Project 6	127	50	20	10	30	40	0
Objective							
486	Maximum total NPV of all selected projects (in thousands of dollars)						

Constraints			
205	<=	250	Cost of first year <= 250k
70	<=	75	Cost of second year <= 75k
50	<=	50	Cost of third year <= 50k
40	<=	50	Cost of fourth year <= 50k
35	<=	50	Cost of fifth year <= 50k
All project 'selected' values are binary			
The 'selected' column indicates that project was chosen in the optimal solution			

Microsoft Excel 16.0 Answer Report
Worksheet: [Book1]Sheet1
Report Created: 9/23/2018 2:53:54 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.157 Seconds.
Iterations: 4 Subproblems: 10
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 0.5%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$A$17	Objective	486	486

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$H$9	Project 1 Selected	1	1	Binary
$H$10	Project 2 Selected	0	0	Binary
$H$11	Project 3 Selected	0	0	Binary
$H$12	Project 4 Selected	1	1	Binary
$H$13	Project 5 Selected	1	1	Binary
$H$14	Project 6 Selected	0	0	Binary

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$J$10	Project 2 Constraints	70	$J$10<=$L$10	Not Binding	5
$J$11	Project 3 Constraints	50	$J$11<=$L$11	Binding	0
$J$12	Project 4 Constraints	40	$J$12<=$L$12	Not Binding	10
$J$13	Project 5 Constraints	35	$J$13<=$L$13	Not Binding	15
$J$9	Project 1 Constraints	205	$J$9<=$L$9	Not Binding	45
$H$9:$H$14=Binary					

In this problem, we have several variables to consider. For the objective function, it is simply the sum of the all NPV revenue from the selected R&D projects. In addition, there are six main constraints: the total cost per year of the projects for years 1 through 5, as well as each project being selected or not (i.e. project 1 is selected = 1, project 1 is **not** selected = 0), so it's a binary possibility of whether a project is selected in the end. Finally, a table was created to monitor all costs and revenues for all five years, as well as the final maximum NPV brought into the company. The results can be found below:

> Selected projects are **Project 1**, **Project 4**, and **Project 5**.
> Total cost for the first year investment is $205k, which is $45k under the $250k limit.
> Total cost for the second year investment is $70k, which is $5k under the $75k limit.
> Total cost for the third year investment is $50k, which matches the $50k limit.
> Total cost for the fourth year is $40k, which is $10k under the $50k limit.
> Total cost for the fifth year is $35k, which is $15k under the $50k limit.
> The total NPV generated from these three selected projects is $486k.

As such, Project 1, 4, and 5 will require a total investment of $400k across all five years, while bringing in an NPV of $486k. These three projects also fit within the money constraints of all five year investments, so the company is provided with $75k to reallocate across the rest of the company.