NAME: ______________________________ (Instructor: J Wheeler)

Homework 3 - Geometric (Nonlinear) Programming
Math 1101 - an Introduction to Optimization
The University of Pittsburgh - Summer 2018

Please submit the following problems at the beginning of class Monday, October 8. Additionally, please staple these questions to the front of your solutions.

1. (Cobb-Douglas Production Function) By investing x units of labor and y units of capital, a low-end watch manufacturer can Produce $P(x, y) = 50x^{0.4}y^{0.6}$ watches. Find the maximum number of watches that can be produced on a budget of $\$20,000$ if labor costs $\$100$ per unit and capital costs $\$200$ per unit (hence we have the constraint function $g(x, y) = 100x + 200y - 20000 = 0$).

   Please do this problem using

   (a) partial derivatives (include using the Second Derivative Test in order to justify that we do have a minimum), and

   (b) Lagrange multipliers (you may use a calculator, software, etc. to solve the system of equations).

   (c) NOTE: this can also be solved using the **AGM**.

2. (a) Use the AGM to solve the Nonlinear Programming problem (if you are having difficulty determining the δ_i's, you may use Method 6.3.1 in the notes):

$$\begin{aligned} \text{Maximize: } & Q(x, y, z) = xyz \\ \text{Subject to: } & 3x + 4y + 12z = 96 \\ & x, y, z > 0 \end{aligned}$$

   (b) Use Excel's Solver (or Mathematica, etc) to answer part (a). Note on using Solver:
   -since we do not have "> 0" as an option in the constraints but rather ≥ 0, use ≥ 0.00000001 or something similar
   -you must have initial values for x, y, z, so initially try $(4, 15, 2)$ as the starting point. Then try $x = 31.9995, y = z = 0.0001$ and watch what happens. Include a print out of your answer reports with your HW.

3. (a) Use the Method 6.3.1 (MART) from the notes to solve

$$\begin{aligned} \text{Minimize: } & g(x, y) = \frac{1000}{xy} + 2x + 2y + xy \\ \text{Subject to: } & x, y > 0. \end{aligned}$$

   The feasible set in this situation consists of more than 1 vector. You may use a calculator or computer software to solve any system of equations you encounter.

(b) Use Solver (or Mathematica, etc.) to answer part (a). Include a print out of your answer report with your HW.

4. Do the first two iterations of Steepest Descent for the following problems:

   (a) Minimize $f(x, y) = 2x^2 + y^2 - 2xy$ with a starting point of $(2, 3)$.

   (b) Minimize $g(x, y) = x^2 + y^2 - 2x - 2y - xy$ with a starting point of $(0, 0)$. Comment on what happens here.

   (c) Use Solver, etc. to find solutions to these two problems with the stated starting point. Repeat this with at least two different starting points of your choice for both problems. Include a print out of your answer reports with your HW.

5. Repeat part a) of the previous problem doing two iterations of Newton's Method.

6. (Extra Credit) In class, we found $\min P = 5$ where $P(x, y) = \frac{2}{xy} + xy + x + y$ with $x, y \geq 0$ using the **AGM**. Show that the method of Steepest Decent converges to $\min P = 5$ when using the starting point $(2, 3)$.

7. Application: a location problem: AllTalk Communications (from *Ragsdale*)

   - AllTalk Communications provides cellular phone service in several midwestern states.
   - They seek to expand their operations by providing inter-city service between four cities in northern Ohio.
   - To do this, a new tower must be built to accommodate the existing towers in the cities.
   - The tower will have a 40 mile transmission radius.
   - AllTalk would like to minimize the distance between the new tower and the existing towers.

   We can express the location of each tower as an ordered pair (x, y) where x represents the distance east (in miles) the tower is from an arbitrary, but fixed, reference point, and y represents the distance north (in miles) the tower is from the same reference point.

   The Cleveland tower is located at $(5, 45)$, the Akron tower at $(12, 21)$, the Canton tower at $(17, 5)$, and the Youngstown tower at $(52, 21)$. Let (x, y) represent the location of the new tower.

   Clearly state the mathematical model for this problem then use Solver, etc. to locate the optimal position (the one that minimizes the total distance) of the new tower. Recall that the distance between points (a, b) and (x, y) is given by $d = \sqrt{(x - a)^2 + (y - b)^2}$.

   Once you have obtained an answer, prepare a short written presentation of your recommendation as a consultant to AllTalk Communications (i.e.

pretend you are a consultant that wants to continue being a consultant and write up a very short presentation of your solution and proposal to AllTalk).