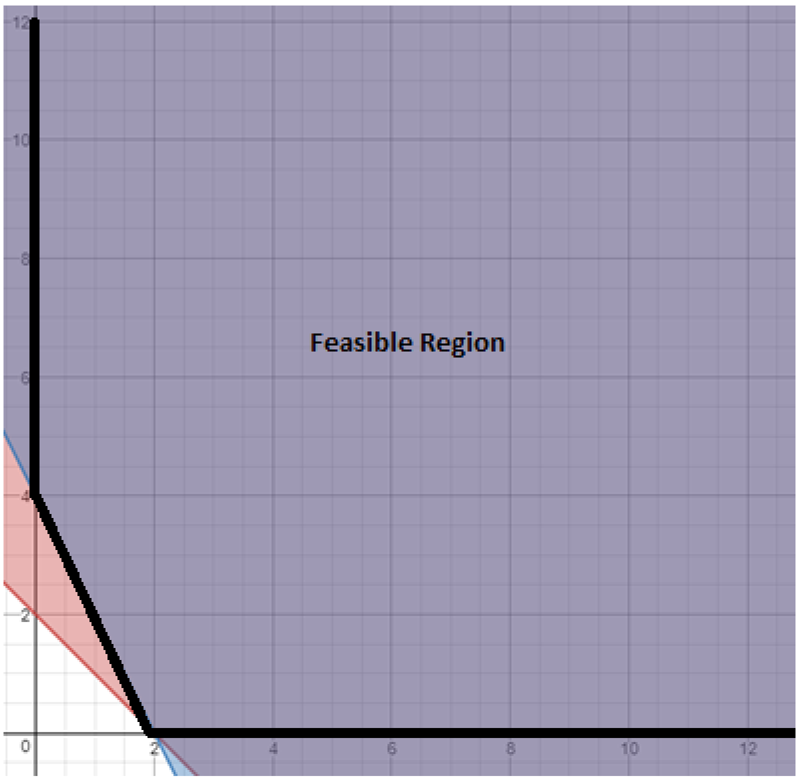
James Hahn
MATH1101
Homework #1

1. Feasible region is colored in purple below:



I. Corner points = (2, 0) and (0, 4)
P(2, 0) = 5*2 + 2*0 = 10
P(0, 4) = 5*0 + 2*4 = 8
Minimum exists at (0, 4)
Maximum does not exist due to unbounded feasible region

II. Minimize and Maximize P(x, y) = 5x + 2y
Subject to:

$x + y - s_1 = 2$
$2x + y - s_2 = 4$
$x, y, s_1, s_2 \geq 0$

This is a dual problem, so the initial tableau's transpose acts as the initial simplex tableau. Also, by Remark 4.1.3, since the feasible region is unbounded and coefficients of the objective function are positive, there exists a minimum, but no maximum, so below are the tableaus to solve *only* the minimization problem. There exists no maximal solution.

x	y	s1	s2	P	
1	2	1	0	0	5
1	1	0	1	0	2
-2	-4	0	0	1	0

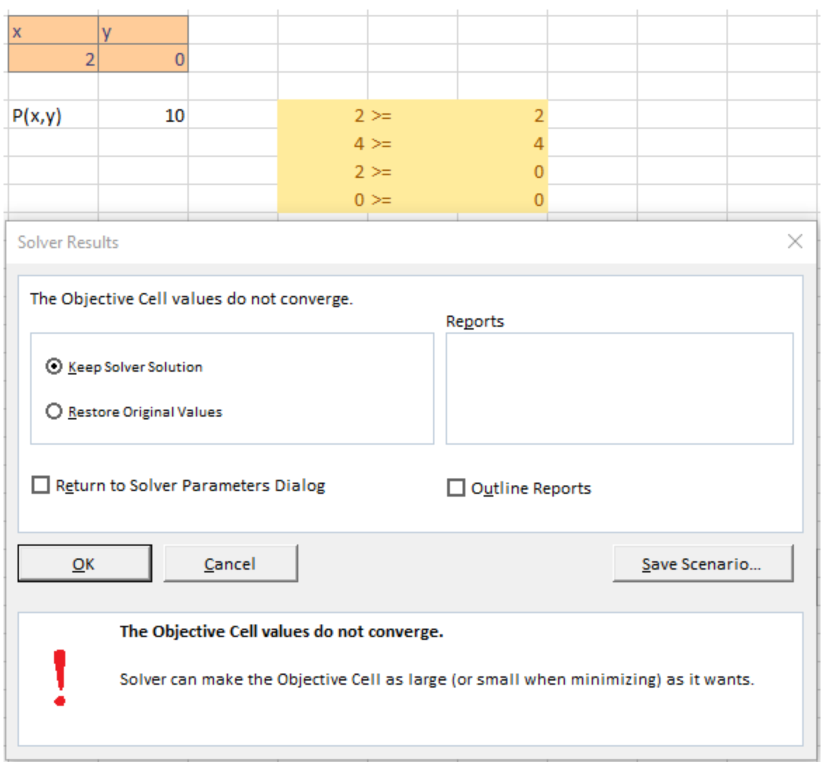
Pivot: **pivot column: 2nd, pivot row: 2nd, pivot element: 1**
Basic variables: **s1, s2, P**
Objective function = **0**

x	y	s1	s2	P	
-1	0	1	-2	0	1
1	1	0	1	0	2
2	0	0	4	1	8

No more negative entries in bottom row. The tableau has converged to a minimum at (0, 4).
Basic variables: **x, y, P**
Objective function = **8**

III. I used Excel's Solver add-in to solve this problem. Below is the answer report:

Results when solving for the **maximum** (not found):



Results when solving for the **minimum**:

Microsoft Excel 16.0 Answer Report
Worksheet: [Book2]Sheet1
Report Created: 9/11/2018 2:43:33 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 6 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$J$5	P(x,y) y	0	8

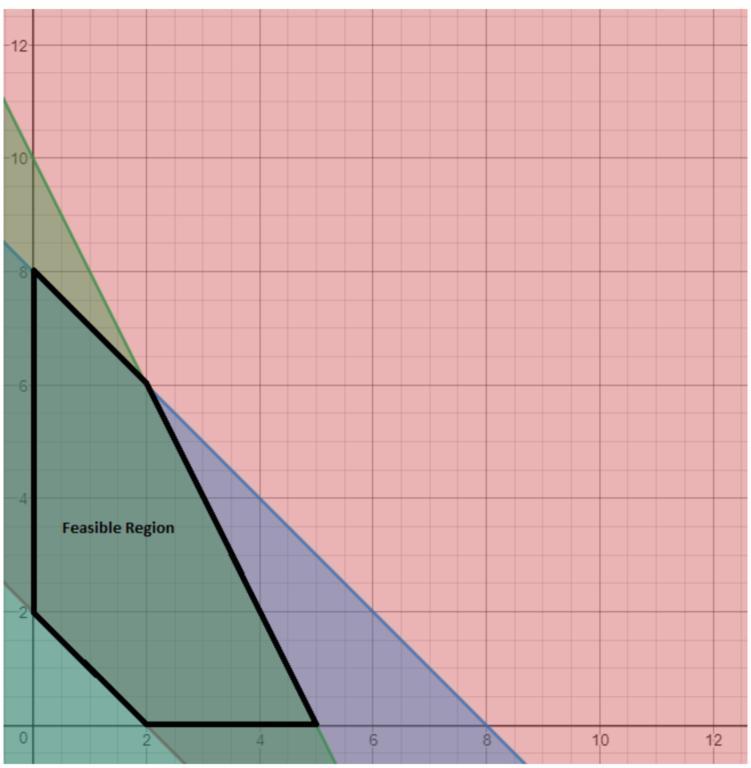
Variable Cells

Cell	Name	Original Value	Final Value	Integer
$I$3	x	0	0	Contin
$J$3	y	0	4	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$L$5	P(x,y)	4	$L$5>=$N$5	Not Binding	2
$L$6	>=	4	$L$6>=$N$6	Binding	0
$L$7	>=	0	$L$7>=$N$7	Binding	0
$L$8	>=	4	$L$8>=$N$8	Not Binding	4

2. Feasible region is marked on the graph below:



I. Corner points = (0, 2) and (2, 0) and (5, 0) and (2, 6) and (0, 8)
P(0, 2) = 20
P(2, 0) = 40
P(5, 0) = 100
P(2, 6) = 100
P(0, 8) = 80

There are two corner points with maximum values. Therefore, the solution is the line connecting both points. So, the solution **S = { (x, y) | y = 2x + 10 for 2 ≤ x ≤ 5 }.**

II. Maximize P(x, y) = 20x + 10y
Subject to:

$x + y - s_1 + a_1 = 2$
$x + y + s_2 = 8$
$2x + y + s_3 = 10$
$x, y, s_1, s_2, s_3, a_1 \geq 0$

Let M = 50 in the initial tableau calculations:

x	y	s1	s2	s3	a1	P	
1	1	-1	0	0	1	0	2
1	1	0	1	0	0	0	8
2	1	0	0	1	0	0	10
-70	-60	50	0	0	0	1	-100

Pivot column: 1st column, Pivot row: 1st row, Pivot element: 1

x	y	s1	s2	s3	a1	P	
1	1	-1	0	0	1	0	2
0	0	1	1	0	-1	0	6
0	-1	2	0	1	-2	0	6
0	10	-20	0	0	70	1	40

Pivot column: 3rd column, Pivot row: 3rd row, Pivot element: 2

x	y	s1	s2	s3	a1	P	
1	0.5	0	0	0.5	0	0	5
0	0.5	0	1	-0.5	0	0	3
0	-0.5	1	0	0.5	-1	0	3
0	0	0	0	10	50	1	100

There are no more negative elements on the last row. We have converged to the **maximum objective function value of 100 at (5, 0)**.

III. I used Excel's Solver add-in to solve this problem. Below is the answer report:

Results when solving for the **maximum**:

Microsoft Excel 16.0 Answer Report
Worksheet: [Book2]Sheet1
Report Created: 9/11/2018 2:21:53 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 2 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$C$15	P(x,y) y	0	100

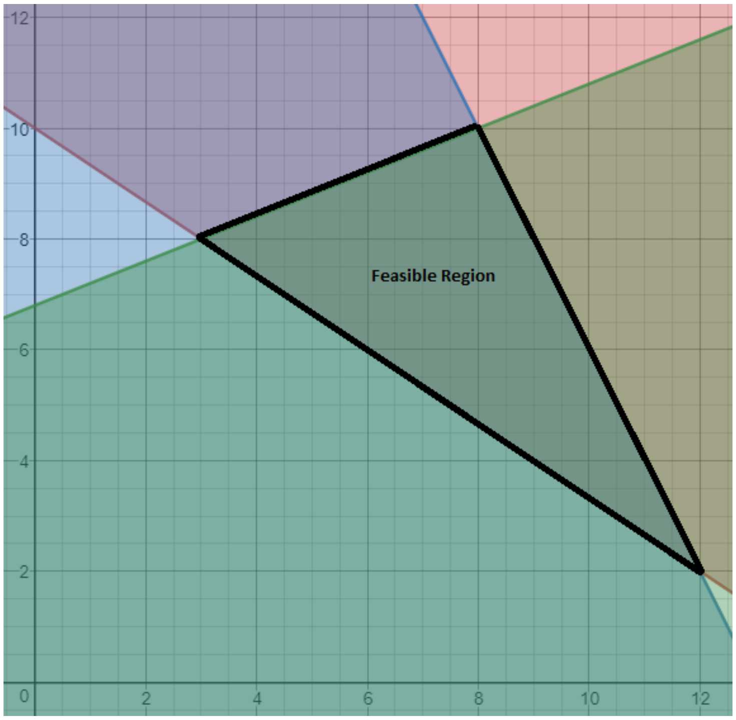
Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$13	x	0	5	Contin
$C$13	y	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$15	P(x,y)	5	$E$15>=$G$15	Not Binding	3
$E$16		5	$E$16<=$G$16	Not Binding	3
$E$17		10	$E$17<=$G$17	Binding	0

3. Feasible region is marked on the graph below:



I. Corner points = (3, 8) and (8, 10) and (12, 2)
P(3, 8) = 140
P(8, 10) = 260
P(12, 2) = 260

There are two coordinates with maximum values. Therefore, the **maximal solution** is the line connecting them. So, the solution **S = { (x, y) | y = -2x + 26 }**. The **minimum solution is located at (3, 8)**.

II. Maximize and Minimize P(x, y) = 20x + 10y
Subject to:

$2x + 3y - s_1 + a_1 = 30$

$2x + y + s_2 = 26$

$-2x + 5y + s_3 = 34$

$x, y, s_1, s_2, s_3, a_1 \geq 0$

Iterations of the tableaus for the maximization problem (let M = 50):

x	y	s1	s2	s3	a1	P	
2	3	-1	0	0	1	0	30
2	1	0	1	0	0	0	26
-2	5	0	0	1	0	0	34
-120	-160	50	0	0	0	1	-1500

Pivot column: 2nd column, Pivot row: 3rd row, Pivot element: 5

x	y	s1	s2	s3	a1	P	
3.2	0	-1	0	-0.6	1	0	9.6
2.4	0	0	1	-0.2	0	0	19.2
-0.4	1	0	0	0.2	0	0	6.8
-184	0	50	0	32	0	1	-412

Pivot column: 1st column, Pivot row: 1st row, Pivot element: 3.2

x	y	s1	s2	s3	a1	P	
1	0	-0.3125	0	-0.1875	0.3125	0	3
0	0	0.75	1	0.25	-0.75	0	12
0	1	-0.125	0	0.125	0.125	0	8
0	0	-7.5	0	-2.5	57.5	1	140

Pivot column: 3rd column, Pivot row: 2nd row, Pivot element; 0.75

x	y	s1	s2	s3	a1	P	
1	0	0	0.416667	-0.0833333	0	0	8
0	0	1	1.33333	0.333333	-1	0	16
0	1	0	0.166667	0.166667	0	0	10
0	0	0	10	0	50	1	260

The tableau has converged to a **maximum objective function value of 260 at (8, 10).**

Iterations of the tableaus for the minimization problem can be found below. We need to maximize the negative objective function, so maximize -P(x, y) = -20x – 10y :

x	y	s1	s2	s3	a1	P	
2	3	-1	0	0	1	0	30
2	1	0	1	0	0	0	26
-2	5	0	0	1	0	0	34
-80	-140	50	0	0	0	1	-1500

Pivot column: 2nd column, Pivot row: 3rd row, Pivot element: 5

x	y	s1	s2	s3	a1	P	
3.2	0	-1	0	-0.6	1	0	9.6
2.4	0	0	1	-0.2	0	0	19.2
-0.4	1	0	0	0.2	0	0	6.8
-136	0	50	0	28	0	1	-548

Pivot column: 1st column, Pivot row: 1st row, Pivot element: 3.2

x	y	s1	s2	s3	a1	P	
1	0	-0.3125	0	-0.1875	0.3125	0	3
0	0	0.75	1	0.25	-0.75	0	12
0	1	-0.125	0	0.125	0.125	0	8
0	0	7.5	0	2.5	42.5	1	-140

There are no more negative entries on the last row. We have converged to an optimal solution for the original minimization problem with objective function value = -140 = -P(x, y), so **P(x, y) = 140 at (3, 8).**

III. I used Excel's Solver add-in to solve this problem. Below are the answer reports:

For the maximization problem:

Microsoft Excel 16.0 Answer Report
Worksheet: [Book2]Sheet1
Report Created: 9/11/2018 2:24:29 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 3 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$C$24	P(x,y) y	0	260

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$22	x	0	8	Contin
$C$22	y	0	10	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$24	P(x,y)	46	$E$24>=$G$24	Not Binding	16
$E$25		26	$E$25<=$G$25	Binding	0
$E$26		34	$E$26<=$G$26	Binding	0

For the minimization problem:

Microsoft Excel 16.0 Answer Report
Worksheet: [Book2]Sheet1
Report Created: 9/11/2018 2:26:14 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 2 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$J$24	P(x,y) y	0	140

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$I$22	x	0	3	Contin
$J$22	y	0	8	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$L$24	P(x,y)	30	$L$24>=$N$24	Binding	0
$L$25	<=	14	$L$25<=$N$25	Not Binding	12
$L$26	<=	34	$L$26<=$N$26	Binding	0

4. Solving by choosing column 1 as the first pivot column:

x	y	s1	s2	s3	P	
2	1	1	0	0	0	16
1	0	0	1	0	0	6
0	1	0	0	1	0	10
-1	-1	0	0	0	1	0

Pivot column: 1st column, Pivot row: 2nd row, Pivot element: 1

x	y	s1	s2	s3	P	
0	1	1	-2	0	0	4
1	0	0	1	0	0	6
0	1	0	0	1	0	10
0	-1	0	1	0	1	6

Pivot column: 2nd column, Pivot row, 1st row, Pivot element: 1

x	y	s1	s2	s3	P	
0	1	1	-2	0	0	4
1	0	0	1	0	0	6
0	0	-1	2	1	0	6
0	0	1	-1	0	1	10

Pivot column: 4th column, Pivot row: 3rd row, Pivot element: 2

x	y	s1	s2	s3	P	
0	1	0	0	1	0	10
1	0	0.5	0	-0.5	0	3
0	0	-0.5	1	0.5	0	3
0	0	0.5	0	0.5	1	13

We have arrived at a solution of **P = 13 at (3, 10)**.

Solving by choosing column 2 as the first pivot column:

x	y	s1	s2	s3	P	
2	1	1	0	0	0	16
1	0	0	1	0	0	6
0	1	0	0	1	0	10
-1	-1	0	0	0	1	0

Pivot column: 2nd column, Pivot row: 3rd row, Pivot element: 1

x	y	s1	s2	s3	P	
2	0	1	0	-1	0	6
1	0	0	1	0	0	6
0	1	0	0	1	0	10
-1	0	0	0	1	1	10

Pivot column: 1st column, Pivot row: 1st row, Pivot element: 2

x	y	s1	s2	s3	P	
1	0	0.5	0	-0.5	0	3
0	0	-0.5	1	0.5	0	3
0	1	0	0	1	0	10
0	0	0.5	0	0.5	1	13

We have arrived at a solution of **P = 13 at (3, 10)**.

5. The dual problem becomes:

Maximize $C(y_1, y_2) = 20y_1 + 30y_2$

Subject to:

$y_1 + 2y_2 + x_1 \leq 20$

$y_1 + y_2 + x_2 \leq 12$

$5y_1 + y_2 + x_3 \leq 40$

$y_1, y_2, x_1, x_2, x_3 \geq 0$

y1	y2	x1	x2	x3	P	
1	2	1	0	0	0	20
1	1	0	1	0	0	12
5	1	0	0	1	0	40
-20	-30	0	0	0	1	0

Pivot column: 2nd column, Pivot row: 1st row, Pivot element: 2

y1	y2	x1	x2	x3	P	
0.5	1	0.5	0	0	0	10
0.5	0	-0.5	1	0	0	2
4.5	0	-0.5	0	1	0	30
-5	0	15	0	0	1	300

Pivot column: 1st column, Pivot row: 2nd row, Pivot element: 0.5

y1	y2	x1	x2	x3	P	
0	1	1	-1	0	0	8
1	0	-1	2	0	0	4
0	0	4	-9	1	0	12
0	0	10	10	0	1	320

We have converged to a maximum. The values for the final solution are of **$x_1 = 10$, $x_2 = 10$, and $x_3 = 0$ with $C = 320$**. The objective function value is the same as we obtained in class, $C = 320$. However, the final solution values are different. In class, we obtained $x_1 = 5$, $x_2 = 10$, and $x_3 = 0$. As such, by changing the first constraint in the dual problem, we reached different solution parameter values. Lastly, note if we plug $x_1 = 10$, $x_2 = 10$, and x_3 into the original objective function, $C = 520$, which provides us with an incorrect minimum as well. This perfectly illustrates the warning described in the notes.