

Dynamic Pricing under Competition on Online Marketplaces: A Data-Driven Approach

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ABSTRACT

Most online markets are characterized by competitive settings and limited demand information. Due to the complexity of such markets, efficient pricing strategies are hard to derive. We analyze stochastic dynamic pricing models in competitive markets with multiple offer dimensions, such as price, quality, and rating. In a first step, we use a simulated test market to study how sales probabilities are affected by specific customer behaviors and the strategic interaction of price reaction strategies. Further, we show how different state-of-the-art learning techniques can be used to estimate sales probabilities from partially observable market data. In a second step, we use a dynamic programming model to compute an effective pricing strategy which circumvents the curse of dimensionality. We demonstrate that the strategy is applicable even if the number of competitors is large and their strategies are unknown. We show that our heuristic can be tuned to smoothly balance profitability and speed of sales. Further, our approach is currently applied by a large seller on Amazon for the sale of used books. Sales results show that our data-driven strategy outperforms the rule-based strategy of an experienced seller by a profit increase of more than 20%.

CCS CONCEPTS

• **Applied computing** → **Multi-criterion optimization and decision-making**; *Online shopping*;

KEYWORDS

dynamic pricing, demand learning, decision making, e-commerce

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1 DYNAMIC PRICING UNDER COMPETITION

The problem of dynamic pricing can be loosely defined as follows: Given a number of items to sell and a given sales horizon, adaptively adjust prices over time to maximize expected profits. Uncertain

customer demand, steady competition, changing markets, as well as remaining inventory levels have to be taken into account.

Dynamic pricing is crucial in revenue management, especially in e-commerce, since prices are a key marketing factor and have a huge impact on sales and profits. Naturally, in the Big Data era revenue management systems provide automated demand estimation and price adjustment methods that assist managers in this challenging and important task [19].

Stochastic dynamic pricing under competition and incomplete demand information is a major open problem in revenue management. Practical relevance is enormous, but the problem appears intrinsically hard. The challenge is (i) to predict sales probabilities based on observable *private market data*, and (ii) to derive approaches that allow for optimized automated price reactions with minimal computation times [21].

In this paper, we solve dynamic pricing problems for large input specifications that are common in real-world systems, e.g., competition with dozens of firms, thousands of distinct products, multiple offer dimensions (e.g., product qualities, seller ratings, cf. [12]), and several financial constraints (e.g., holding costs, discounting). Existing re-pricing techniques cannot handle such scenarios efficiently and hence, force managers to limit the scope of pricing strategies, e.g., by using deterministic or highly stylized demand functions [22], monopoly settings, or by pricing less frequently. This simplification negatively affects the quality of strategies.

The inefficiency of existing techniques stems from several factors that reflect the challenges behind dynamic pricing. First, the space of possible solutions grows exponentially with the number of time periods, products, and competitors. This is an inherent property of the problem (curse of dimensionality), since pricing decisions interact in terms of their impact and hence, simple heuristics do not lead to good solutions. Moreover, automated techniques depend on accurate data-driven estimations of sales probabilities in order to predict the impact of price decisions in a specific market situation. Limited demand information, combined with the large solution space, implies a very high problem complexity, thus increasing computation time dramatically.

Recent machine learning methods for demand estimation have been a major development in the area, as they can accurately quantify demand probabilities. However, these methods only speed up a single component of the process. The fundamental issue remains that the solution space is too large, i.e., it is necessary to circumvent the dimensionality curse.

In this paper, we derive price reaction strategies to be applied on Amazon Marketplace. Our aim is to deal with the following assumptions: (i) limited demand information, (ii) unknown competitors' strategies, and (iii) partially observable market data. To compute

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viable pricing strategies in competitive settings, we combine data-driven demand estimations and a dynamic price optimization model that circumvents the curse of dimensionality.

1.1 Related Work

Selling products is a classical application of revenue management theory. The problem is closely related to the field of dynamic pricing, which is summarized in the books [26] and [32]. The surveys [4] and [7] provide an excellent overview of recent pricing models under competition. Oligopoly pricing models are studied by, e.g., [16], [29], and [30]. Dynamic pricing models under competition that also include strategic customers are analyzed by [14] and [15].

In most existing models, the demand intensity is assumed to be known. Dynamic pricing competition models with limited demand information are analyzed by [2], [6], [27], and [31] using robust optimization and learning approaches. As many models are not flexible enough, their practical applicability is often limited.

Data-driven pricing applied in real-life is studied in the recent publication by [11]. Their multi-product model allows to include substitution effects in demand. Their framework is characterized by a deterministic, finite horizon monopoly with unrestricted supply and static pricing decisions. While they use least squares and orthogonal matching pursuit method to estimate demand, their price optimization is based on binary quadratic programming.

Another important aspect – especially in the context for competitive settings – are pricing simulations. Simulating the performance of automated pricing strategies is important as testing is potentially hazardous when done in production. [13] and [9] present two approaches, which are, however, limited in their capabilities: Simulations run on a single machine, offer a limited set of consumer behaviors, simulate solely short sales horizons, and pricing updates or orders are restricted to predefined discrete points in time. [25] presents a continuous time framework – called Price Wars [20] – that allows to simulate various oligopoly settings and to study strategic interaction of self-adapting data-driven strategies [3].

1.2 Contributions

The main contributions of our work are as follows:

Comparison of Demand Learning Techniques: We show how to predict (conditional) sales probabilities from partially observable market data (Section 3). Using a reproducible test model¹, we are able to analyze strategic interaction and to measure sales probabilities in competitive markets characterized by a given customer behavior and a specific set of competing pricing strategies. This allows comparing the quality of different estimation methods.

Efficient Algorithm for Dynamic Price Optimization under Competition: We derive a novel effective dynamic pricing approach that builds on estimated sales probabilities and a decomposed dynamic programming formulation (Section 4). Using reproducible examples, we show that our approach is even applicable when the number of competitors is large and the competitors' strategies are unknown. Moreover, our strategy can be used to smoothly balance profitability and speed of sales. Our concept is easy to implement and enables pricing computations in milliseconds.

¹The framework used for data generation and demand learning can be found on Github: <https://git.io/pricing>. The framework is self-contained and ensures reproducible results.

Experiments on Amazon Marketplace: We demonstrate the effectiveness of our pricing strategy on Amazon (Section 5). Our strategy outperforms the established rule-based strategy of an experienced seller both in profitability and speed of sales.

2 MODEL DESCRIPTION

We consider the situation in which a firm wants to sell a finite number of durable goods on a digital market platform (e.g., Amazon or eBay). The time horizon is not restricted. We allow that customers compare the offers of different competitors. Arriving customers take prices as well as product conditions (quality) and ratings of different competitors into account.

We assume that items cannot be reproduced or reordered. A firm k 's initial number of items to sell is denoted by $N^{(k)}$, $N^{(k)} < \infty$. If a sale takes place shipping costs c have to be paid, $c \geq 0$. A sale of one item at price a leads to a profit of $a - c$.

Moreover, we consider inventory holding costs. We assume that each unsold item leads to inventory costs of l per period (e.g., one hour or one day), $l \geq 0$. We also include discounting in the model using the discount factor δ , $0 < \delta < 1$, for one unit of time. This corresponds to a discount rate α , $\alpha > 0$, given by $\alpha = \ln(\delta^{-1})$.

Due to customer choice, the sales intensity of a firm particularly will depend on a firm's offer (characterized by price a , quality q , and rating r) and the competitors' offers. Further, we allow the sales intensity also to depend on time, e.g., the time of the day or the weekday. We assume that the time-dependence is periodic and has a finite cycle length of J periods.

In our model, the sales intensity denoted by λ is a function of the current market situation denoted by \vec{s} . We assume that \vec{s} is a vector which includes time t and the current offers of all market participants, i.e., all prices $\vec{p} = (p^{(1)}, \dots, p^{(K)})$, product conditions $\vec{q} = (q^{(1)}, \dots, q^{(K)})$, and customer ratings $\vec{r} = (r^{(1)}, \dots, r^{(K)})$, where K is the number of market participants at time t .

To highlight that the sales intensity of a firm k 's offer $(a, q, r) = (p^{(k)}, q^{(k)}, r^{(k)})$, $k = 1, \dots, K$, given a market situation $\vec{s} = (t, \vec{p}, \vec{q}, \vec{r})$ particularly depends on its offer price a and time t , we consider the jump intensity, $0 \leq t < \infty$, $a \geq 0$,

$$\lambda_t^{(k)}(a, \vec{s}) \quad (1)$$

A firm k 's random inventory level at time t is denoted by $X_t^{(k)}$, $t \geq 0$, $k = 1, \dots, K$. The end of sale is the random time $\tau^{(k)}$, when all $N^{(k)}$ products are sold, that is $\tau^{(k)} := \min_{t \geq 0} \{t : X_t^{(k)} = 0\}$; for all remaining $t \geq \tau^{(k)}$, we let $\lambda_t^{(k)}(\cdot, \cdot) := 0$, cf. (1).

At some points in time t , a firm k 's offer price a can be updated. We call strategies $(a_t)_t$ admissible if they belong to the class of Markovian feedback policies, i.e., pricing decisions $a_t \geq 0$ may depend on time t , the inventory level $X_t^{(k)}$, and the current random market situation denoted by \vec{s}_t .

A firm k 's profits are characterized by its sales and its holding costs which are connected to the inventory process $X_t^{(k)}$. Given a pricing strategy $(a_t)_t$, a firm k 's random accumulated future profits from time t on (discounted on time t) amount to, $t \geq 0$, $k = 1, \dots, K$,

$$G_t^{(k)} := \int_t^{\tau^{(k)}} e^{-\alpha \cdot (u-t)} \cdot (a_{u-} - c) |dX_u^{(k)}| - \int_t^{\tau^{(k)}} e^{-\alpha \cdot (u-t)} \cdot l \cdot X_u^{(k)} du \quad (2)$$

A firm k 's objective is to determine a non-anticipating (Markovian) feedback pricing policy that optimizes a given objective, e.g., to maximize the expected total discounted profit given $N^{(k)}$ items at time 0 and an initial market situation \vec{s}_0 , $k = 1, \dots, K$,

$$E(G_0^{(k)} | X_0^{(k)} = N^{(k)}, \vec{S}_0 = \vec{s}_0) \quad (3)$$

In the next section, we introduce a simulation model and show how sales probabilities can be estimated in competitive markets with incomplete information. In Section 4, we compute heuristic pricing strategies for scenarios with many competitors and unknown strategies. The applicability of our strategy is demonstrated for the simulation model. In Section 5, we measure the performance of our pricing strategy when being applied on the Amazon Marketplace.

3 DATA-DRIVEN DEMAND ESTIMATION

The goal of this section is to estimate sales probabilities from observable market data. We will consider different demand learning approaches. To measure the quality of those approaches, we introduce a reproducible simulation model that is able to mimic the dynamics of specific markets. This way we are able to assess which approaches are suitable to estimate sales probabilities under specific competitive settings and a representative customer behavior.

3.1 Reproducible Test Model

Our continuous time model supports multiple competitors and offers with multiple features, such as price, quality, rating, etc. Customers are assumed to arrive at random times and to decide for a competitor's offer according to their individual willingness to pay and a weighted scoring function that allows ranking the competitors' offers.

We assume K competitors. The initial market situation is given by $\vec{s}_0 = (\vec{p}_0, \vec{q}_0, \vec{r}_0)$, i.e., the price, quality, and rating of all current market participants. We assume quality levels 1-5 where 1 is the best. Seller ratings range from 0-100 (%) where 100 (%) is the best score. In e-commerce merchants typically adjust their prices on a regular basis. However, reaction times are not exactly equidistant which is due to delays caused by the marketplace or in order not to act predictably, cf. [23].

Hence, in our model each competitor k , $k = 1, \dots, K$, adjusts its price $p^{(k)}$ at (uniformly distributed) points in time $t_j^{(k)}$, $j = 1, 2, \dots$, where $t_0^{(k)} \sim U(0, 1)$ and $t_j^{(k)} := t_{j-1}^{(k)} + U(0.8, 1.2)$. Considering a time interval $[0, T]$, we obtain that market situations denoted by $\vec{s}_t = (\vec{p}_t, \vec{q}_t, \vec{r}_t)$, $t \in [0, T]$, change at all points in time

$$t \in \bar{T} := \bigcup_{k=1, \dots, K; j=1, \dots, J^{(k)}} \{t_j^{(k)} \wedge T\}$$

where for all firms k the total number of their price adjustments $J^{(k)}$ is defined by $J^{(k)} := \min_{j=1, 2, \dots} \{j | t_j^{(k)} > T\}$ and the final point in time is $t_{J^{(k)}}^{(k)} = T$, $k = 1, \dots, K$.

Further, we assume a random stream of interested customers that arrive at (exponentially distributed) times $t_i^{(c)}$, $i = 1, 2, \dots$, where $t_1^{(c)} \sim 2 \cdot \text{Exp}(1)$ and $t_i^{(c)} := t_{i-1}^{(c)} + 2 \cdot \text{Exp}(1)$. We assume heterogeneous customers. An interested customer arriving at time t assigns scores $v_t^{(k)}$ to each offer $k = 1, \dots, K$, e.g., determined by

$$v_t^{(k)} := p_t^{(k)} + U(0, 1) \cdot q_t^{(k)} + U(0, 0.5) \cdot (100 - r_t^{(k)}) \quad (4)$$

where the random coefficients are the same for each offer k . A customer buys a firm k 's offer if $\arg \min_{k=1, \dots, K} \{v_t^{(k)}\} = k$ and the utility score is good enough compared to the customer's individual (uniformly distributed) reference score (cp. willingness to pay), i.e.,

$$\min_{k=1, \dots, K} v_t^{(k)} < U(5, 15) \quad (5)$$

Our framework allows to model various market dynamics which can be determined using different parameter settings. The model can be easily extended by further offer dimensions as well as the exit or entry of firms.

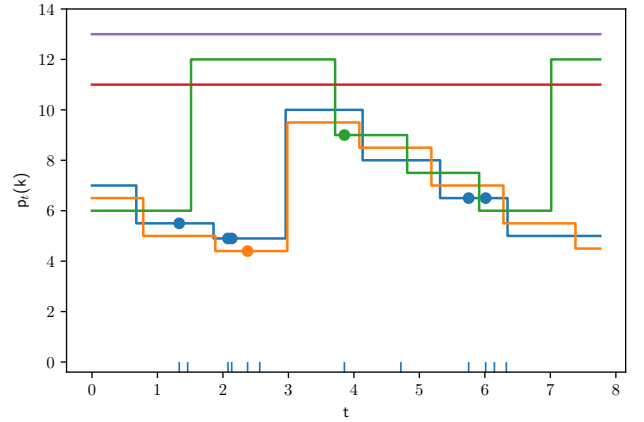


Figure 1: Illustration of customer arrivals (ticks), buying decisions (dots), and price reactions of 5 firms over time.

Figure 1 illustrates examples of price trajectories over time as well as the arrival of interested (indicated by ticks on the x-axis) and buying customers (indicated by dots) in case of $K = 5$ and $\vec{q}_t = (2, 1, 4, 3, 2)$, $\vec{r}_t = (98, 99, 95, 96, 97)$. Note, due to different qualities and ratings of competitors customers do not always choose the cheapest competitor.

In markets, it can be observed, that competitors undercut each other (e.g., by ε) and raise the price (e.g., to D) if the price level is too low (e.g., below d), i.e., a firm k adjusts its prices via the strategy

$$F(\vec{p}; d, D, \varepsilon) := \begin{cases} D & , \min_{j \in \{1, \dots, K\} \setminus \{k\}} p^{(j)} < d \\ \min_{j \in \{1, \dots, K\} \setminus \{k\}} p^{(j)} - \varepsilon & , \text{else} \end{cases} \quad (6)$$

The application of such response strategies leads to cyclic price patterns over time, cf. Edgeworth cycles, see, e.g., [17], [18]. We call such kind of strategies “two bound” strategies, see also [13], [25].

3.2 Simulation of Oligopoly Settings

In this subsection, we illustrate the impact of strategic interaction on a firm's sales probabilities for the time between two own price adjustments. In the following example, we define different settings of competing strategies, which include (i) randomized prices and (ii) two bound strategies that strategically undercut competitors' prices. In a third setting (iii), we consider a mixture of active and passive competitors, which can be often found in practice.

Example 3.1. We use the simulation setting described in Section 3.1 with $K = 5$. Qualities and ratings of firms are chosen randomly but fixed over time $q_0^{(k)} := \text{round}(U(0.5, 5.5))$, $r_0^{(k)} := U(90, 100)$. We define three settings of interacting price reaction strategies:

- (i) All firms use randomized prices, $0 \leq t \leq T$,
 $p_t^{(k)} := U(0, 15)$, $k = 1, \dots, K$
- (ii) All firms use two bound strategies, $0 \leq t \leq T$, cf. (6),
 $p_t^{(k)} := F(\vec{p}_t; 5, 10, 0.5)$, $k = 1, \dots, K$
- (iii) Firms use either two bound strategies or fix price strategies
 $p_t^{(1)} := F(\vec{p}_t; 5, 10, 0.5)$, $p_t^{(2)} := F(\vec{p}_t; 4, 9, 0.5)$,
 $p_t^{(3)} := F(\vec{p}_t; 6, 12, 0.5)$, $p_t^{(4)} := 11$, $p_t^{(5)} := 13$

Our simulation model, cf. Section 3.1, can be used to evaluate competing strategies over time and to simulate buying events. In the following, we illustrate how strategic interaction affects a firm's sales probabilities. Due to price reactions of the other firms, we can expect that the market situation might change over time according to the different settings (i)-(iii) described in Example 3.1.

Example 3.2. Assume firm 1 does not adjust its price until $t = 1$. We simulate multiple market evolutions each starting in the *fixed* market situation \vec{s}_0 at time $t = 0$ with $\vec{p}_0 = (6.5, 7, 9, 11, 13)$, $\vec{q}_0 = (2, 2, 2, 2, 2)$, $\vec{r}_0 = (98, 98, 98, 98, 98)$. To determine the average sales intensities of firm 1 within the time interval $(0, 1)$ for the different strategy settings (i)-(iii), we used 10 000 simulation runs starting in \vec{s}_0 and counted the observed sales of firm 1 over time.

Figure 2 illustrates how the chance to sell an item at price $p^{(1)} = 6.5$ decreases over time due to price reactions of competitors, cf. Example 3.1. The random points in time in which potential customers arrive and competitors adjust their prices were simulated as described previously. As expected, we observe that setting (ii) is the most competitive one.

Finally, such simulations allow quantifying the conditional probabilities, $k = 1, \dots, K$, $t \geq 0$, $h > 0$, $i = 0, 1, \dots$,

$$P_{t, t+h}^{(k)}(i, p^{(k)} | \vec{s}_t) \quad (7)$$

to sell i items at price $p^{(k)}$ during the interval $(t, t+h)$ in a certain strategy setting given that the market situation at time t is \vec{s}_t . Conditional probabilities are integrated sales intensities over time and correspond to the areas under the curves depicted in Figure 2.

Remark 3.1. The conditional sales probabilities, cf. (7), are characterized by the following effects:

- (i) the arrival intensity of potential customers,
- (ii) the buying behavior of customers,
- (iii) the pricing strategies of competing merchants, and
- (iv) the adjustment frequencies of competing merchants

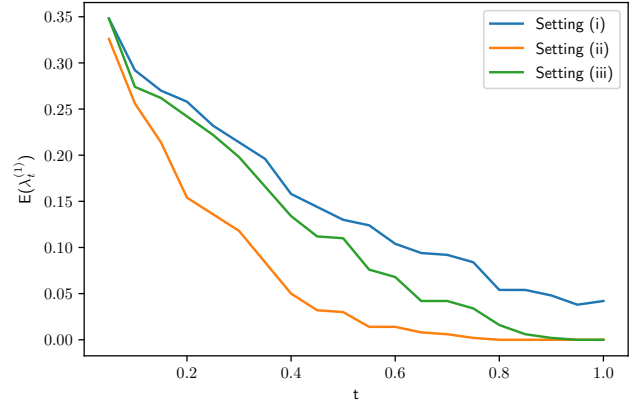


Figure 2: Decay of average sales intensity of firm 1 over one period of time due to competitors' price reactions in different competitive settings (i)-(iii), starting in market situation \vec{s}_0 ; Example 3.1 - 3.2.

The goal of the next two subsections is to estimate such conditional probabilities from observable data.

3.3 Observable Market Data

In real-life applications, merchants cannot continuously track markets over time. Typically, merchants have to request the marketplace to observe the current market situation. Then, based on the current market situation a price adjustment is sent back according to a certain repricing rule or strategy. Each merchant can also observe realized sales as *private* knowledge.

The idea of data-driven demand learning is to quantify how the number of observed sales within different time intervals is affected by the relation of a firm's offer (i.e., a firm's price, quality, rating) and the competitors' offers.

We assume that firms observe current market situations every time they adjust its prices. W.l.o.g. in the following, we take the perspective of firm $k = 1$. According to our model, cf. Section 3.1, within a time frame $[0, T]$ firm 1 observes market situations \vec{s} at $J^{(1)}$ points in time as well as its realized number of sales denoted by $y_j^{(1)}$ within the $J^{(1)}$ time intervals $(t_j^{(1)}, t_{j+1}^{(1)})$, see Table 1.

The data of observed market situations \vec{s} consists of firm 1's prices $p^{(1)}$, quality $q^{(1)}$, and rating $r^{(1)}$ as well as each competitor k 's price $p^{(k)}$, quality $q^{(k)}$, and rating $r^{(k)}$, $k = 2, \dots, K$, at times $t_j^{(1)}$, $j = 0, 1, \dots, J^{(1)} - 1$.

Note, the other merchants k , $k = 2, \dots, K$, can observe similar (asymmetric) data for their corresponding points in time $t_j^{(k)}$, $j = 0, 1, \dots, J^{(k)} - 1$.

3.4 Estimating Conditional Sales Probabilities

A firm k that plans to set a price at time t for the length of, e.g., h units of time, seeks to estimate sales probabilities for the period $(t, t+h)$ given the current market situation \vec{s} observed at time t . While its offer features $q^{(k)}, r^{(k)}$ are fixed the offer price $p^{(k)} = a$ can be chosen from a set A of admissible prices. In this context, firm

j	$t_j^{(1)}$	$y_j^{(1)}$	$p_{t_j^{(1)}}^{(1)}$	$q_{t_j^{(1)}}^{(1)}$	$r_{t_j^{(1)}}^{(1)}$	$p_{t_j^{(1)}}^{(2)}$	$q_{t_j^{(1)}}^{(2)}$	$r_{t_j^{(1)}}^{(2)}$	\dots	$p_{t_j^{(1)}}^{(K)}$	$q_{t_j^{(1)}}^{(K)}$	$r_{t_j^{(1)}}^{(K)}$
0	0.0	1	15	2	98	14	3	99	...	/	/	/
1	1.7	0	17	2	98	14	3	99	...	7	1	96
2	2.5	0	13	2	98	9	3	99	...	7	1	96
3	2.8	1	11	2	98	12	1	95	...	/	/	/
...
$J^{(1)} - 1$	$T - 0.6$

Table 1: Illustration of observable and private data of firm 1: Market situations \vec{s} at discrete points in time $t_j^{(1)}$ and number of sales between $t_j^{(1)}$ and $t_{j+1}^{(1)}$, $j = 0, 1, \dots, J^{(1)} - 1$

k seeks to derive an estimation of the conditional sales probabilities, cf. (7), $k = 1, \dots, K$, $a \in A$, $h > 0$, $i = 0, 1, \dots$,

$$\tilde{P}_{t, t+h}^{(k)}(i, a | \vec{s}) \quad (8)$$

where $\vec{s} = (t, \vec{p}, \vec{q}, \vec{r})$ is characterized by the competing firms' offers.

There are several approaches to estimate sales probabilities, cf. (8), from data sets as described in Table 1. Common approaches are, e.g., least squares, gradient boosted trees (e.g., XGBoost [5]), neural networks, etc. If, as in our use-case the dependent variable $y^{(1)}$ (i.e., the number of sales) is predominantly binary (0 or 1), logistic regression approaches are also admissible.

In the following, we want to compare different demand learning approaches in order to investigate which approach fits best to our use-case, i.e., the simulation model described in Section 3.1. The different approaches are typically based on explanatory variables. The raw data, cf. Table 1, usually does not serve as suitable explanatory variables. Instead, it is advisable to use the raw data to express characteristic quantities that might be responsible for customers' decisions. In Definition 3.1, we give simple examples of such explanatory variables.

Definition 3.1. Consider the following data of firm 1: market situations $\vec{s} = (\vec{p}, \vec{q}, \vec{r})$ including our own offer $(p^{(1)}, q^{(1)}, r^{(1)})$ at the begin of the interval $(t_j^{(1)}, t_{j+1}^{(1)})$ of observation j , $j = 0, 1, \dots, J^{(1)} - 1$. We define the following explanatory variables $\vec{x} = \vec{x}^{(j)}(\vec{s}_{t_j^{(1)}}, t_j^{(1)}, t_{j+1}^{(1)})$:

- $x_1^{(j)} := 1$, constant / intercept
- $x_2^{(j)} := \text{rank}(p^{(1)}; \vec{p})$, rank of price $p^{(1)}$ within prices \vec{p} at $t_j^{(1)}$
- $x_3^{(j)} := 1_{\{\text{rank}(p^{(1)}; \vec{p}) = 1\}}$, is price $p^{(1)}$ at rank 1 at $t_j^{(1)}$ (yes/no)
- $x_4^{(j)} := \text{rank}(q^{(1)}; \vec{q})$, rank of quality $q^{(1)}$ within qualities \vec{q}
- $x_5^{(j)} := \text{rank}(r^{(1)}; \vec{r})$, rank of rating $r^{(1)}$ within ratings \vec{r} at $t_j^{(1)}$
- $x_6^{(j)} := p^{(1)}$, price $p^{(1)}$ at $t_j^{(1)}$
- $x_7^{(j)} := p^{(1)} - \min_{k=2, \dots, K} \{p^{(k)}\}$, price gap to best competitor
- $x_8^{(j)} := q^{(1)}$, quality $q^{(1)}$ at $t_j^{(1)}$
- $x_9^{(j)} := r^{(1)}$, rating $r^{(1)}$ at $t_j^{(1)}$
- $x_{10}^{(j)} := 1_{\{\arg \min_{k=1, \dots, K} \{p^{(k)} + 0.5 \cdot q^{(k)} + 0.25 \cdot r^{(k)}\} = 1\}} \in \{0, 1\}$, is firm 1's score the best for a fixed weighting (yes/no)

In this general framework, further explanatory variables can be easily defined to capture the impact of various effects, such as time, duration of intervals, etc.

Next, we use the explanatory variables defined above to estimate demand probabilities for different competitive oligopoly settings.

Example 3.3. We use the simulation setting described in Section 3.1 with $K = 5$. We distinguish the three settings of interacting pricing strategies, cf. Example 3.1. For each setting (i)-(iii), we simulate 1 000 scenarios of market data each for a time horizon of $T = 100$, resulting in ca. 100 000 observed market situations for each firm. Based on the explanatory variables defined in Definition 3.1, for firm 1 we apply the following regression approaches:

- (LR) logistic regression
- (LS) least squares
- (XGB) gradient boosted trees
- (MLP) multi-layer perceptron

Table 2 shows the results of the different regression approaches for the three competitive settings of Example 3.1. As information criteria (goodness-of-fit) we used the McFadden Pseudo R^2 . We used 80% of the data for training and 20% for validation.

Oligopoly setting	LR	XGB	MLP	LS
(i)	0.249	0.249	0.252	0.233
(ii)	0.394	0.393	0.395	0.311
(iii)	0.214	0.217	0.207	0.179
fitting time in ms	58	443	530	5
prediction time in μs	0.116	1.938	0.791	0.066

Table 2: Comparison of McFadden Pseudo R^2 s and fitting times of different demand learning approaches for data derived from simulations of the settings (i)-(iii); Example 3.3.

We observe that almost all estimation approaches yield satisfying results for our use-case. Logistic regression (LR), multi-layer perceptron (MLP), and boosted trees (XGB) achieved the best results. We also tested random forest (RF) and support vector machine (SVM) approaches; as their results were worse, we concentrated on the other approaches. Our examples predominantly serve to illustrate that different approaches can be applied and compared. While not focus of the paper all methods can be further tuned and improved or tested for different settings of the model.

Regression results also depend on the size of the training data. Our model can be used to study the impact of the size of the data on regression results, and in turn, to determine how many observations are needed to obtain good sufficient results.

Further, the quality of estimations of sales probabilities does not have to be the same for the entire range of prices. While for some merchants, it is desirable to consistently estimate probabilities for all prices, for others it might be more favorable to accurately estimate probabilities for prices that often occur during the competition.

In general, regression results are better if prices are more randomized, cf. [8]. In this context, our model can also be used to study the impact of a selection bias caused by firm 1's strategy as well as the competitors' strategies. Moreover, the impact of various effects of the model can be studied, such as distribution and length of reaction times, customer arrival intensity, customers' buying behavior, cf. (4)-(5), or number of competitors, etc.

3.5 Validation of Demand Estimations

In this subsection, we validate the quality of estimations of (8) for different learning approaches, cf. Section 3.4, by comparing them to their true counterparts (7), which can be derived using repeated Monte Carlo simulations, cf. Section 3.2.

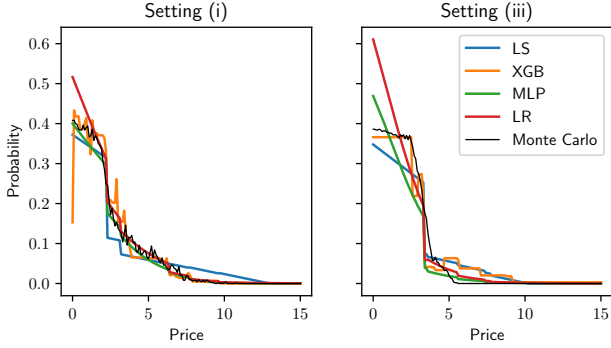


Figure 3: Illustration of comparisons of evaluated Monte Carlo probabilities $P(\bar{p}) := 1 - P_{0,1}^{(1)}(0, \bar{p}|\vec{s}_0)$ (10 000 simulation runs for each price) and different predicted probabilities $\tilde{P}(\bar{p}) := 1 - \tilde{P}_{0,1}^{(1)}(0, \bar{p}|\vec{s}_0)$, $\bar{p} = 0, 0.05, \dots, 15$, for two random market situations for setting (i) and (iii); Example 3.1 - 3.3.

Figure 3 illustrates an example of estimated sales probabilities and true (simulated) probabilities. For a given setting and an initial market situation \vec{s}_0 in $t = 0$, we let firm 1 adjust its price to $p_0^{(1)} := \bar{p}$ for the time interval $(0, 1)$. For each situation and price, we used 10 000 simulation runs to compute accurate approximations of the correct probabilities $P_{0,1}^{(1)}(i, \bar{p}|\vec{s}_0)$, cf. (7), for $\bar{p} = 0, \dots, 15$, see Figure 3. This way, we are able to evaluate the quality of the estimations of different approaches in specific price ranges.

Table 3 summarizes average prediction errors (bias, SMRE, and deviation of expected short-term profits) for single prices and different estimation approaches. We observe that estimations are almost unbiased and prediction errors are modest.

We obtain that different estimation approaches can be used to accurately approximate the true probabilities for all prices. For small prices the estimations are worse if prices are rarely applied, cf. setting (ii)-(iii). Prices that do not exceed production costs or shipping costs, however, are often irrelevant in practice.

Remark 3.2. Our simulation model is simple yet reasonable and allows studying the quality of different demand learning approaches, cf. Example 3.3. We find that estimation approaches (LR), (LS), (XGB), (MLP) yield good performance results. To choose a suitable approach, practitioners might also take additional aspects into account, such as (i) accessibility, (ii) interpretability, (iii) computation time, and (iv) scalability.

Finally, the conditional probabilities (7) are affected by both, the customer behavior as well as the strategic interplay of competitors' price adjustments, cf. Remark 3.1. Note, a firm's demand learning does neither anticipate competitors' strategies nor their reaction

\bar{p}	approach	$E(\tilde{P}(\bar{p}) - P(\bar{p}))$	$E(\tilde{P}(\bar{p}) - P(\bar{p}) ^{0.5})^2$	$E(\tilde{P}(\bar{p}) - P(\bar{p})) \cdot \bar{p}$
4	LR	0.0353	0.0550	0.273
4	LS	0.0346	0.0562	0.263
4	XGB	0.0252	0.0679	0.322
4	MLP	0.0054	0.0568	0.272
8	LR	0.0003	0.0092	0.086
8	LS	-0.0127	0.0114	0.123
8	XGB	-0.0191	0.0179	0.166
8	MLP	-0.0030	0.0099	0.093
12	LR	-0.0011	0.0010	0.013
12	LS	-0.0017	0.0002	0.021
12	XGB	-0.0075	0.0060	0.089
12	MLP	-0.0015	0.0014	0.018

Table 3: Comparison of average prediction quality of different estimation approaches for prices $\bar{p} = 4, 8, 12$; 100 random initial market situations, setting (iii); Example 3.1 - 3.3.

times. The estimated probabilities (8), however, allow to indirectly measure the average impact of competitors' price adjustments and, thus, account for the fact that market situations may change between two price adjustments of a firm.

Remark 3.3. The estimation of conditional sales probabilities, cf. (8), allows taking the complex interplay of different effects, cf. Remark 3.1, implicitly into account. We have the following effects:

- (i) The adjustment frequencies of competing merchants, and their strategic interaction determine the expected evolution of sales intensities, cf. Figure 2, and hence the size of conditional sales probabilities, cf. Figure 3.
- (ii) The arrival intensity of potential customers mirrors the maximum sales probability, cf. y -intercept of Figure 3.
- (iii) The buying behavior of customers determines the slope of sales probabilities, see Figure 3.

Moreover, our model can be easily extended by additional features, e.g. such as, (i) limited inventory levels, (ii) potential changes in quality and ratings over time, or (iii) exit and entry of firms.

Our basic model allows to test and to validate different demand learning approaches in different competitive markets. In addition, components of the demand estimation can be further improved (exploration phases, sampling, feature selection, etc.). While not focus of the paper, further issues, such as missing variables, IIA assumption, unobservable demand shocks, etc., can be addressed, cf. to recent literature, e.g., [1], [10], or [28]. To this end, our model can be used to study to which extent such effects influence the quality of different demand learning approaches.

The goal of the next section is to derive an efficient pricing mechanism that is based on estimated conditional probabilities for current market situations, cf. (8). Further, we seek to account for different inventory levels, holding costs, and discounting.

4 PRICING STRATEGIES TO BALANCE PROFITABILITY AND SPEED OF SALES

In this section, we look for viable data-driven pricing strategies for markets with restricted supply capacities, many competitors, and

multiple offer dimensions, cf. Section 2 and 3. Further, the goal is to be able to balance profitability and speed of sales.

There are two major problems to derive applicable pricing strategies in competitive markets: (i) as demand is affected by many parameters (e.g., dozens of competitors' prices) a model's state space explodes and the problem becomes intractable, and (ii) in general, as competitors' strategies are not known, their price adjustments cannot be effectively anticipated.

4.1 Dynamic Programming Approach

Our approach deals with both problems. Most importantly, instead of computing complete feedback strategies we compute prices for one period only based on the current market situation that occurs during a sales process. To compute prices for single time periods, in general, the current state as well as potential future states have to be taken into account. As price reactions of competitors occur with a certain delay the short-term evolution of the market can be well approximated by the current market situation. The long-term evolution of the market, however, can hardly be predicted. Our approach is motivated by the fact that the optimal price for one period mostly depends on the current state and is much less affected by specific potential states in the future.

For a current state, we manage problem (i) as follows: We roughly approximate future market situations by using sticky prices. While the degree of inaccuracy is acceptable, we gain a structure that makes it possible to circumvent the curse of dimensionality, cf. problem (i), as the states of our dynamic system (i.e., the market situation) are not coupled and can be decomposed. Thus, for single states decisions can be computed independently, which makes it possible to consider current market situations only.

The second key idea is to compensate the model's inaccuracy as well as the lack of price anticipations, cf. problem (i), by frequent price adjustments, which in turn are possible as the model's simplicity allows for fast re-computations.

Due to price adjustments, exits, or entries of firms, in general, market situations are not stable. In our model, we consider conditional sales probabilities, cf. (8). W.l.o.g., we consider firm $k = 1$ with an average adjustment delay of $h = 1$. We consider the (estimated) probabilities, $a \geq 0$, $t = 0, 1, \dots$, $i = 0, 1, \dots$,

$$\tilde{P}_t(i, a|\vec{s}) := \tilde{P}_{t,t+1}^{(1)}(i, a|\vec{s}) \quad (9)$$

for selling i items within the time span $(t, t+1)$ at price a under the condition that at time t the market situation is \vec{s} (and may change within the period due to competitors' price reactions).

As described in the beginning of this section, we use a simplified dynamic programming approach in which the best expected discounted future profits $E(G_t|X_t = n, \vec{S}_t = \vec{s})$, cf. (2) - (3), are described by the value function $V_t^*(n, \vec{s})$, $t \geq 0$, $n = 0, 1, \dots, N$. If all items are sold, no future profits can be made, i.e., for any \vec{s} and t the natural boundary condition for the value function is given by

$$V_t^*(0, \vec{s}) = 0 \quad (10)$$

The time dependence in our infinite horizon model is assumed to be seasonal or cyclic (daily/weekly effects) with a given a cycle length of J periods. Hence, for all t , where $t \bmod J = j$, we have $\tilde{P}_t(i, a|\vec{s}) = \tilde{P}_j(i, a|\vec{s})$, for all \vec{s} , $i \geq 0$, $j = 0, 1, \dots, J-1$. Since, we

can assume that $V_t^*(n, \vec{s}) = V_{t \bmod J}^*(n, \vec{s})$ for all t , we just have to determine the values $V_t^*(n, \vec{s})$ for $t = 0, 1, \dots, J-1$.

Finally, the value function is characterized by the associated Hamilton-Jacobi-Bellman equation, $t = 0, 1, \dots, J-1$, $n = 1, \dots, N$,

$$V_t^*(n, \vec{s}) = \max_{a \in A} \left\{ \sum_{i \geq 0} \tilde{P}_t(i, a|\vec{s}) \cdot \left((a - c) \cdot \min(n, i) - n \cdot l + z \cdot \delta \cdot V_{(t+1) \bmod J}^*((n-i)^+, \vec{s}) \right) \right\} \quad (11)$$

where z , $z \geq 0$, is an additional penalty/discount parameter which allows (i) to control the *aggressiveness* of the feedback pricing policy, and (ii) to account for expected general *long-term* market trends (decay of average prices, product attractiveness, etc.). The set of admissible prices A can be chosen arbitrarily. The solution of the system of equations (10) - (11) can be derived using standard methods like value iteration or policy iteration. Alternatively, the system can also be solved using a nonlinear solver.

Value iteration does not need a solver to approximate the value function. For a given "large" number \tilde{T} , $\tilde{T} \gg J$, we let $V_{\tilde{T}}^*(n, \vec{s}) := 0$ for all numbers n and market situations \vec{s} . Using the recursion, $t = 0, 1, \dots, \tilde{T}-1$, $n = 1, \dots, N$,

$$V_t(n, \vec{s}) = \max_{a \in A} \left\{ \sum_{i \geq 0} \tilde{P}_t(i, a|\vec{s}) \cdot \left((a - c) \cdot \min(n, i) - n \cdot l + z \cdot \delta \cdot V_{t+1}((n-i)^+, \vec{s}) \right) \right\} \quad (12)$$

we can compute the values $V_t(n, \vec{s})$, $t = 0, 1, \dots, J-1$. The number of iteration steps \tilde{T} can be chosen such that the approximation error between V and V^* is sufficiently small. Finally, the associated optimal pricing strategy $a_t(n, \vec{s})$, $t = 0, 1, \dots, J-1$, $n = 1, \dots, N$, is given by the *arg max* of (12) and (11), respectively.

Note, due to the size of the state space it is *not* possible to compute prices $a_t(n, \vec{s})$ for all states \vec{s} in advance. The following algorithm, however, circumvents the curse of dimensionality and allows to derive viable heuristic pricing strategies in competitive markets with a large number of competitors.

Algorithm 4.1. We propose the following pricing heuristic:

Step 1: For every period t observe the new state, i.e., the current inventory level X_t and the current market situation \vec{S}_t . Compute the probabilities $\tilde{P}_t(i, a|\vec{S}_t)$, $i = 0, 1, \dots, X_t$, $a \in A$.

Step 2: Solve $V_{t \bmod J}^*(X_t, \vec{S}_t)$, cf. (11), or use $\tilde{T} - t$ recursion steps to compute the specific value $V_{t \bmod J}(X_t, \vec{S}_t)$, cf. (12), and obtain the associated offer price $a_{t \bmod J}(X_t, \vec{S}_t)$.

The key idea is to just compute prices for single market situations and to regularly refresh prices in response to changing market situations. Due to the small dimensionality of the state space, a single re-computation is very fast. Further, our solution is *scalable* as the algorithm's complexity does neither increase with the number of competitors nor the dimensionality of market situations.

In case demand can be assumed to be independent of time the computational effort of Algorithm 4.1 via (11) or (12), can be even further reduced.

Theorem 4.1. If demand is *time homogeneous* then $V^*(n, \vec{s})$ can be expressed explicitly, $n > 1$,

$$V^*(n, \vec{s}) = \max_{a \in A} \left\{ \frac{\sum_{i>0} \tilde{P}(i, a|\vec{s}) \cdot \left(\frac{(a-c) \cdot \min(n, i) - n \cdot l}{-z \cdot \delta \cdot V^*((n-i)^+, \vec{s})} \right)}{1 - \tilde{P}(0, a|\vec{s}) \cdot z \cdot \delta} \right\} \quad (13)$$

Proof. Consider (10) - (11) for $n = 1$ and *fixed* prices \tilde{a} , i.e., using $\tilde{A} := \tilde{a}$. Then solve for $V^*(1, \vec{s}; \tilde{a})$ (where $V^*(0, \vec{s}) = 0$) and maximize $V^*(1, \vec{s}; \tilde{a})$ over $\tilde{a} \in A$ to obtain $V^*(1, \vec{s})$. Do the same for $n = 2, \dots, N$ in increasing order (where $V^*(n-1, \vec{s})$ was derived previously).

Using the explicit formula (13) of Theorem 4.1 in Algorithm 4.1 makes it possible to adjust prices in milliseconds.

4.2 Application of the Heuristic Strategy

Using a numerical example that can be reproduced by the reader, we demonstrate the applicability of our approach in markets with many competitors and unknown strategies.

Example 4.1. We consider the setting of Example 3.1. We let $c = 0$, $l = 0.001$, $\delta = 0.9999$, $n = 1, 2, 3, 5, 10$, $K = 5$, and $z = 0.5, 0.9, 0.95, 0.98, 0.99, 1$. We assume the market situation $\vec{p}_0 = (\bar{p}, 5, 7, 11, 13)$, $\vec{q}_0 = (3, 2, 2, 1, 1)$, $\vec{r}_0 = (99, 98, 97, 99, 98)$. We consider estimated logit probabilities of setting (iii) determined by explanatory variables defined in Definition 3.1 and the corresponding beta coefficients: $\vec{\beta} = (-5.39, -0.44, -0.54, -0.03, 0.13, -0.30, -0.17, 0, -0.14, 0.07, 0.31)$. We compute prices using (13), cf. Theorem 4.1.

$a(n, \vec{s}_0), (\tilde{P}(n \vec{s}_0))$	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 10$
$z = 1$	14.48 (0.00)	12.99 (0.00)	12.99 (0.00)	10.99 (0.01)	10.99 (0.01)
$z = 0.99$	9.11 (0.03)	6.99 (0.13)	6.99 (0.13)	6.99 (0.13)	5.00 (0.33)
$z = 0.98$	6.99 (0.13)	6.99 (0.13)	6.90 (0.13)	5.00 (0.33)	5.00 (0.33)
$z = 0.95$	6.99 (0.13)	6.19 (0.18)	5.00 (0.33)	5.00 (0.33)	4.24 (0.41)
$z = 0.9$	6.33 (0.17)	5.00 (0.33)	5.00 (0.33)	4.24 (0.41)	4.02 (0.43)
$z = 0.5$	5.00 (0.33)	4.03 (0.43)	3.89 (0.45)	3.83 (0.45)	3.83 (0.45)

Table 4: Feedback prices $a(n, \vec{s}_0)$ (and associated expected sales probabilities $\tilde{P}(n|\vec{s}_0) := 1 - \tilde{P}(0, a(n, \vec{s}_0)|\vec{s}_0)$) for different inventory levels n , degrees of aggressiveness z , and a specific market situation (predictions via LR); Example 4.1.

Table 4 illustrates how prices of Algorithm 4.1 are affected by (i) the inventory level, (ii) the current market situation, and (iii) the aggressiveness factor z . Next, we demonstrate the applicability of Algorithm 4.1's strategy *dynamically* over time.

Example 4.2. We consider the setting of Example 4.1. The competitors adjust their prices according to the strategies of setting (iii), Example 3.1. Firm 1 adjusts its prices according to the infinite horizon model of Algorithm 4.1 and (13). We use different aggressiveness factors z and simulate sales (10 000 runs) over a finite test horizon of $[0, T]$, $T = 10$. At time 0, we let $N^{(k)} = 10$, $k = 1, \dots, 5$.

Table 5 summarizes sales results of firm 1, 2, and 3 for different degrees of aggressiveness z for firm 1's strategy, cf. Example 4.2. The accumulated profits $R_T^{(k)}$ of firm $k = 4$ and $k = 5$ are omitted as they are close to 0. We observe that the average number of

sales of firm 1 decreases in z while profit per sales increases. The results demonstrate that certain versions of our pricing strategy can dominate competitors' results in both sales *and* profit per sale.

Firm	Average sales $N - X_T^{(k)}$			Total profit $R_T^{(k)}$			Profit per sale		
	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
$z = 1$	0.17	2.86	1.91	1.10	18.80	13.30	7.62	6.56	6.94
$z = 0.99$	2.31	3.66	0.92	12.71	17.97	5.55	5.24	4.96	6.02
$z = 0.98$	3.25	2.81	0.95	15.73	12.99	5.47	4.83	4.63	5.80
$z = 0.95$	4.36	2.40	0.64	20.57	10.50	3.78	4.58	4.39	5.88
$z = 0.9$	4.50	2.32	0.63	18.71	9.97	3.55	4.05	4.29	5.73
$z = 0.5$	5.06	1.94	0.49	20.50	8.23	2.75	3.98	4.25	5.60

Table 5: Average sales results of Algorithm 4.1's strategy applied over time (from 0 to T) for different z values compared to active competitors of setting (iii), $T = 10$; Example 4.2.

Note, to have an effective strategy that also allows to smoothly *balance* sales and profitability (via the factor z) is useful in practice as it allows to control cash-flows, overall inventory levels, etc. according to a manager's needs.

5 APPLICATION IN PRACTICE

To evaluate the performance of our approach, we applied our heuristic strategy on Amazon Marketplace. Online market platforms such as Amazon or eBay are highly dynamic as sellers can regularly observe the current market situation and adjust their prices instantly. This dynamic is hard to manage as pricing decision requires handling a multitude of dimensions for each competitor (e.g., price, quality, shipping, rating). Further, a firms' supply is typically limited and sales events are private knowledge.

In this experiment, we partner with a German bookseller. The seller is among the top 10 sellers for used books on Amazon in Germany and has an inventory of over 100 000 distinct books (ISBN), each with multiple items (1-20). Our seller can decide – to some extent – on the replenishment of used books (via purchase prices for second-hand books). However, supply is limited and it is not possible to directly reorder specific items ([24]). Hence, the challenge is to extract as much profit as possible from a given number of books (inventory level) in a reasonable amount of time.

The pricing strategy of our project partner is characterized by a rule-based system that has been developed over years by carefully adjusting rules to lessons learned from selling books on Amazon. As our project partner has more than ten years of experience in the market, we consider his strategy to be effective and accurate. However, market dynamics are increasingly sophisticated making rule-based strategies increasingly hard to handle and maintain.

We applied the pricing strategy derived in the previous sections. As our approach is designed to be applied in practice, we need to calibrate the model, particularly the (conditional) sales probabilities. The data set that we use for calibration contains both the requested market situations from Amazon as well as the seller's own data (offers, sales, and inventory). The seller requests market situations for each offered book every two hours (i.e., > 20 M market situations per month which result in > 140 M single competitor observations per month). In an exploration phase, also randomized offer prices were used by the seller.

	Offer Dimension	Range/Unit
Product-Specific Parameters	time	seconds
	Amazon sales rank of ISBN	1 – 5 000 000 by 1
	weight	gram
	original price	0.01 – 500 Euro by 0.01
	number of used offers	0 – 20 by 1
Competitor-Specific Parameters	price	0.01 – 500 Euro by 0.01
	condition/quality	new – acceptable (6 levels)
	rating	0% – 100%
	feedback count	0 – 5 000 000 by 1
	shipping time	0 – 30 days by 1
	shipping costs	0 – 10 Euro by 0.01
	domestic shipping	yes / no

Table 6: Product and merchant-specific offer dimensions.

Our estimations of sales probabilities for realized sales of a specific book in a certain time interval are based on market situations at the time of our firm’s price adjustment. Market situations are characterized by product-specific features as well as offer dimensions (e.g., price, quality, ratings, feedback count, shipping time) for each present competitor, see Table 6.

Based on the offer dimensions given in Table 6, we defined 30 customized features to describe the relative competitiveness of our offer in a particular market situation. Among others, we used features similar to Definition 3.1, i.e., the price rank of our offer price within the competitors’ prices, etc.

We tested different demand learning techniques to quantify how offer prices and specific market situations affect sales. We decided for a logistic regression approach to estimate sales probabilities for offer price in specific market situations (mainly driven by the requirement to yield interpretable models).

We calibrated our model based on the estimated (conditional) sales probabilities (\tilde{P}). Using our heuristic approach, we computed optimized prices adjustments for current market situations. The application of our dynamic pricing strategy works as described in the previous sections, cf. Algorithm 4.1.

Finally, we used the calibrated model to determine heuristic pricing strategies to be applied on the Amazon Marketplace. In our experiment, we used four different z factors ($z^{(1)} > z^{(2)} > z^{(3)} > z^{(4)}$) to vary the strategy’s aggressiveness, cf. strategy $S^{(1)} - S^{(4)}$ in Table 7. $S^{(4)}$ is the most aggressive strategy. Over three months, we compared our data-driven strategies with the seller’s rule-based benchmark strategy. To each strategy, we randomly assigned a test group of over 5 000 books. The price adjustment frequencies were the same for all five strategies.

Table 7 summarizes a comparison of sales, revenues per sale, and profits per sale of the different strategies. Profits are defined as revenue minus costs, i.e., shipping, Amazon provision, tax (7%), packing, additional costs (warehouse rent, electricity costs, staff costs), and the average purchase price per item.

As expected, the speed of sales increases and profitability decreases the more aggressive strategies are defined. Hence, the aggressiveness of our strategy can be used to actively control the

Strategy	Test group size	% Sold	Revenue per sale (EUR)	Profit per sale (EUR)
Benchmark	5 534	41.71 (100.0%)	7.54 (100.0%)	2.56 (100.0%)
$S^{(1)}$	5 206	29.37 (−29.6%)	8.84 (+17.2%)	3.58 (+40.0%)
$S^{(2)}$	5 407	36.62 (−12.2%)	8.15 (+ 8.1%)	3.03 (+18.7%)
$S^{(3)}$	5 241	44.61 (+ 7.0%)	8.03 (+ 6.5%)	2.94 (+15.0%)
$S^{(4)}$	5 200	44.92 (+ 7.7%)	7.50 (− 0.5%)	2.52 (− 1.2%)

Table 7: Comparison of sales and profits for our data-driven strategies and the seller’s rule-based benchmark strategy.

trade-off between profitability and speed of sales. Moreover, strategy $S^{(3)}$ reveals that our approach can sell faster (+7%) and at the same time more profitable (+15%) as the seller’s benchmark strategy.

In Table 8, we compare the accumulated profits of all strategies. The relative accumulated profit denotes the quantity “profit per sale (EUR) \times % of items sold” compared to the corresponding value of the benchmark strategy. Results show that with our strategy applied, cf. $S^{(3)}$, profits can be increased by more than 20%.

	Benchmark	$S^{(1)}$	$S^{(2)}$	$S^{(3)}$	$S^{(4)}$
Relative accum. profit	100.0%	−1.5%	+4.3%	+23.1%	+6.4%

Table 8: Comparison of strategies’ accumulated profits.

6 CONCLUSIONS

Using a reproducible test market, we studied the impact of the customer behavior and the complex interplay of price reaction strategies. Moreover, we verified and compared the suitability of different demand learning techniques.

Further, we have proposed a novel data-driven approach to compute dynamic pricing strategies under competition. We have demonstrated that our heuristic approach is applicable even if the number of competitors is large and the competitors’ strategies are unknown. Our approach allows for frequent price adjustments as the computation of prices is efficient and fast. Furthermore, the aggressiveness of the strategy can be used as a management instrument to smoothly balance profitability and speed of sales.

Our approach combines key features that are important for real-life applications. First, the approach is applicable if many competitors are involved and offers have multiple dimensions. Second, market dynamics do not have to be explicitly known, but they can be indirectly taken into account using data-driven demand estimations. Third, computation of prices is efficient, easy to implement, and allows for frequent price adjustments.

Finally, the performance of our data-driven pricing strategy was measured in a real-life experiment on Amazon. We outperformed the rule-based strategy of an experienced seller by more than 20%.

In future research, we will extend our model to also address problems (i) with a finite horizon framework (perishable products) as well as (ii) competitive multi-product models, in which demand is characterized by substitution effects, see [11].

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