Computational Astrophysics (AST245 / AST246)

Lectures every week at Tuesday 12:15-14:00 in room **Y23 G04** of UniZH Irchel campus

Course website on a *Trello Board*. It will contain any relevant info as well as lecture slides, material and instructions for projects.

See https://trello.com/b/SPmzepm6/computational-astrophysics-2024

Teacher: Prof. Lucio Mayer (lmayer@physik.uzh.ch)
Department of Astrophysics (DA), Office Y11 F86

Teaching Assistants (DA offices at F floor of Y11):
Floor van Donkelaar(floor.vandonkelaar@uzh.ch)

Textbooks:

(1)Hockney & Eastwood (Computer Simulation using Particles - partially available at https://books.google.ch/books/.
(2)Saas-Fee Lecture notes by V. Springel (http://arxiv.org/abs/1412.5187),

(3)Galactic Dynamics (Binney & Tremaine 1998, 2008)

Class Assessment

Class will be assessed via computational project to be presented during the exam. Students will write a computer program in language of choice that implements an important numerical algorithm (or more algorithms) used in the modelling of astrophysical systems. It will be based on techniques learned in class.

2 Projects will be released on topics covered during the class (after mid November), each structured and several tasks. One project centered on N-Body methods, one on methods for hydrodynamics.

Tutoring by assistants and by teacher.

Goal; to master the fundamental modelling notions and numerical techniques needed to create your first astrophysical simulation.

- **For the 6 ECTS version**: aim to complete one of the two projects.

- **For the 10 ECTS version**: aim to complete the whole project package

Meeting time with teacher/assistants: by appointment

Outline of Lectures

- Introduction on mathematical models for (astro)physical systems of interacting and non-interacting bodies: from the Vlasov equation to the Euler equation

- Discretization methods and particles: mesh and particles

- The gravitational problem: direct N-body, discretization methods on meshes (FFTs, multigrid) and without meshes (treecode)

- Time integration schemes for ODEs

- Astrophysical hydrodynamics I: particle-based (SPH) methods

- Astrophysical hydrodynamics II: grid-based and unstructured mesh eulerian and lagrangian methods

Computer Simulation in Astrophysics

Astrophysics and Cosmology have always been (since the 70s) ideal research areas where to employ computer simulations because experiments cannot be done in the first place!

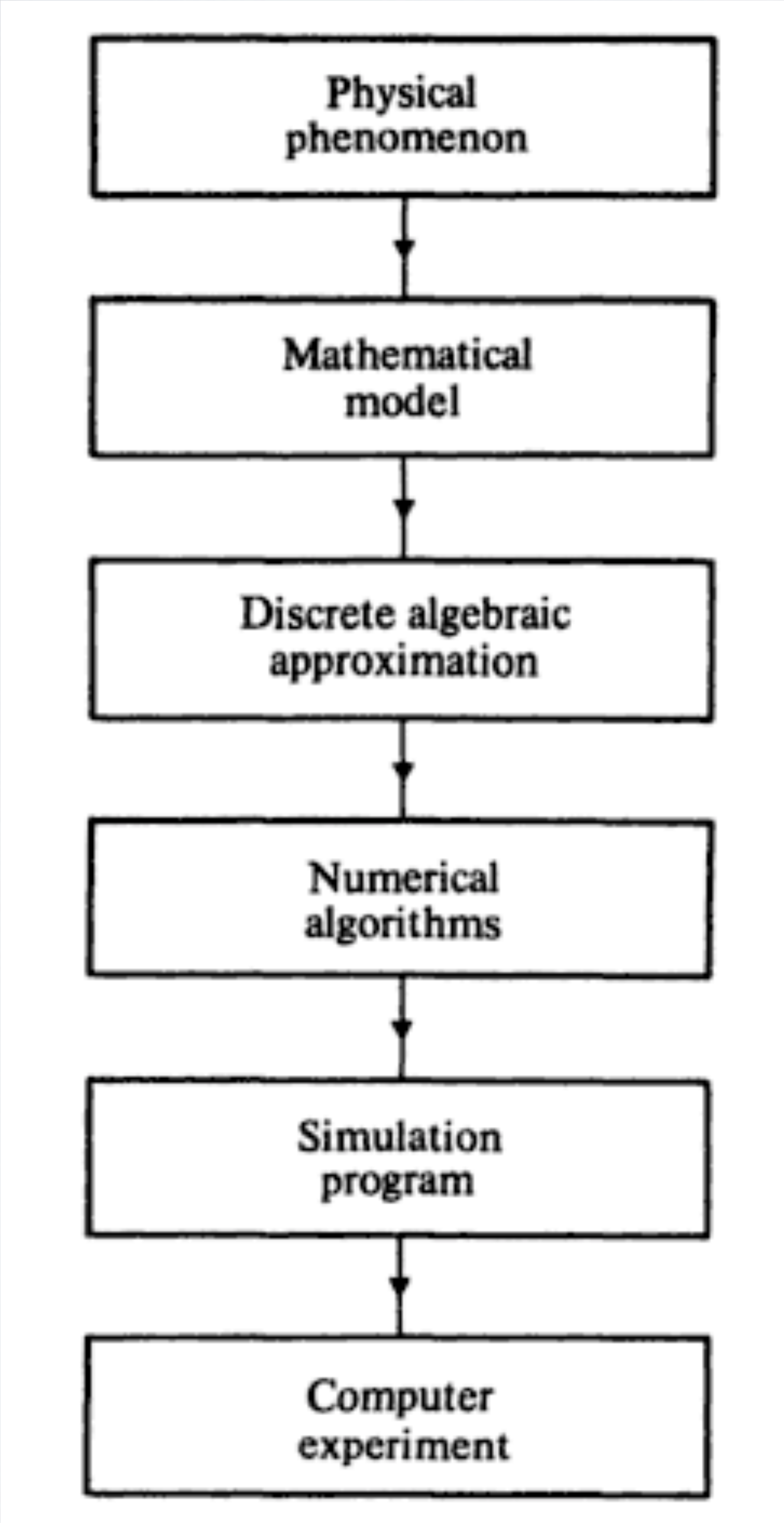
Systems under study, like galaxies or stars or extrasolar planets, are indeed too far away and the timescales of processes of interest (eg galaxy rotation or stellar evolution) are too long in human standards (million to billion of years) for any experiment to be possible

Telescopes (from ground and space) do provide data (normally at fixed time, rarely with time-dependent information) to compare theory with but no direct interaction with such data is possible, hence observations cannot replace experiments

In modern science, computer simulation has become the 3rd pillar, along with conventional theory and experiment

The advent of high performance parallel computers (hardware) combined with continued development of increasingly accurate and fast algorithms (software) is the reason behind the increasingly central role of computer simulation in science

Initially aimed at increasing accuracy and applicability of theory in the regime of complex systems (many variables and degrees of freedom) it has begun to show ability to predict new phenomena



Conventional scheme of a computational experiment.

Note: mathematical model is usually some set of ordinary or partial differential equations that describe a physical system

It is a “model” because such equations are often based on some approximation (eg the fluid approximation to describe interstellar gas in galaxies or the behaviour of dark matter)

The mathematical model, which can contain
a complex set of nonlinear ODEs and/or PDEs,
is *discretized to yield a simpler form of the equations*
that can be efficiently treated numerically
(for example, reduce to a new set of linear equations)

The *simulation program is a collection of many*
numerical algorithms, each of them designed to
solve the discretized equations describing the
the mathematical model

The *computer experiment* is what we usually call *simulation.*
It is a product of the simulation program, which normally
can be capable of generating multiple simulations of many
different (astrophysical) systems

Theory , Computation and Astronomical (Telescope) Observation

In the last two decades computational has grown tremendously to the point of becoming a major driver of new theoretical developments as analytical theory has reached the limits of the questions that it can answer in most situations.

For example, the origin of cosmic structure, from small to large scales (from planets to galaxies) is by nature dealing with complex systems described by systems of time-dependent coupled ODEs and PDEs describing gravity,hydrodynamics, radiation and sometimes also magnetic fields (newtonian or relativistic) simultaneously ---> ideal use of large parallel supercomputers

Systems of interacting and non-interacting particles: I - direct simulations vs. simulation of a model

Examples: molecular dynamics or star cluster (A) dynamics vs. galactic dynamics (B)

In (A) we can simulate directly a bunch of molecules or stars by using "particles" obeying the relevant physical laws, normally expressed by a field equation.
No discretization of the system necessary.

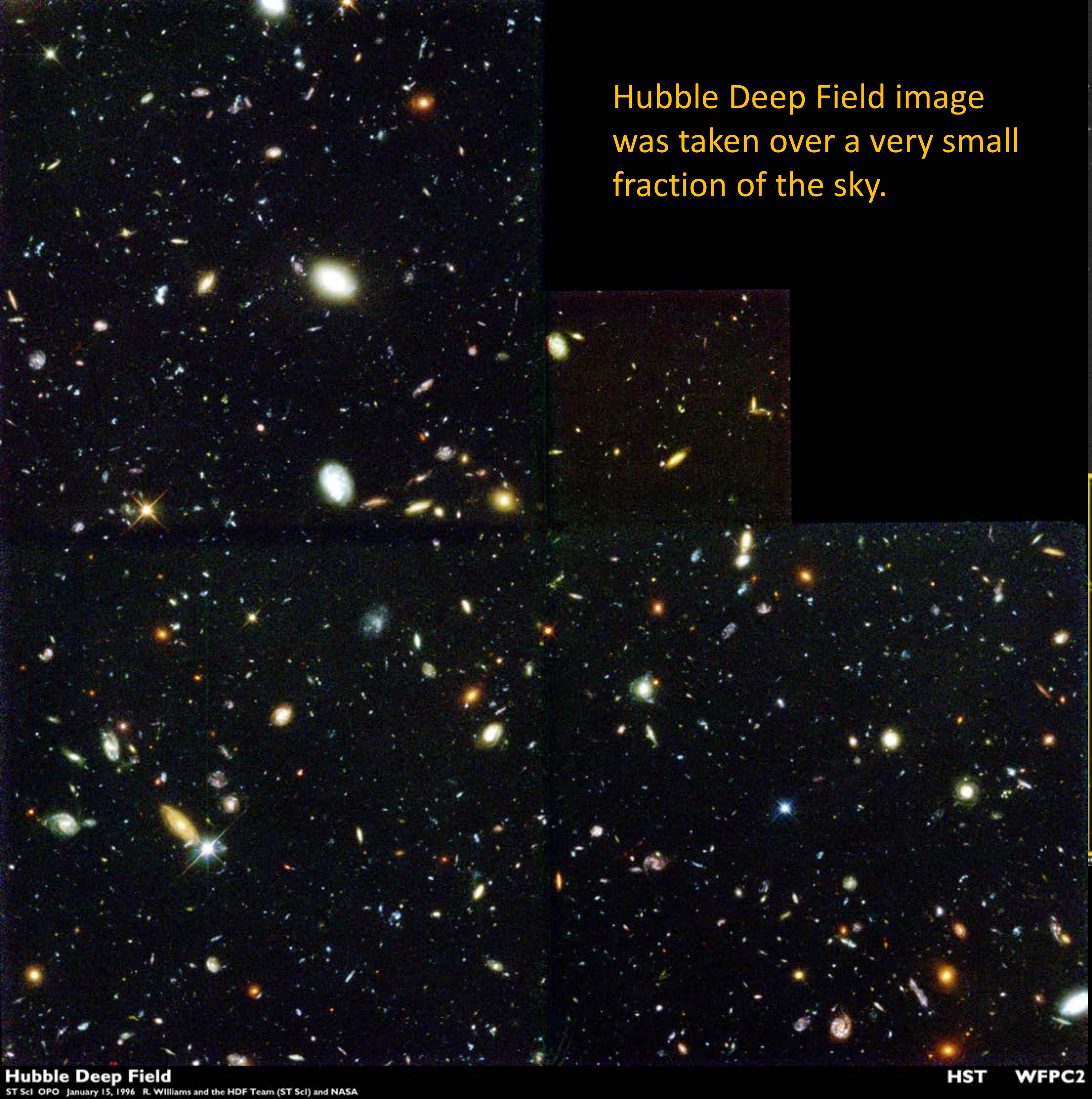
Note: Star clusters contain $N_p \sim 10^4$-10^6 particles interacting via gravity - computing mutual forces is possible with such low N_p on modern computers.
Note2: in this case the governing field equation is the Poisson equation

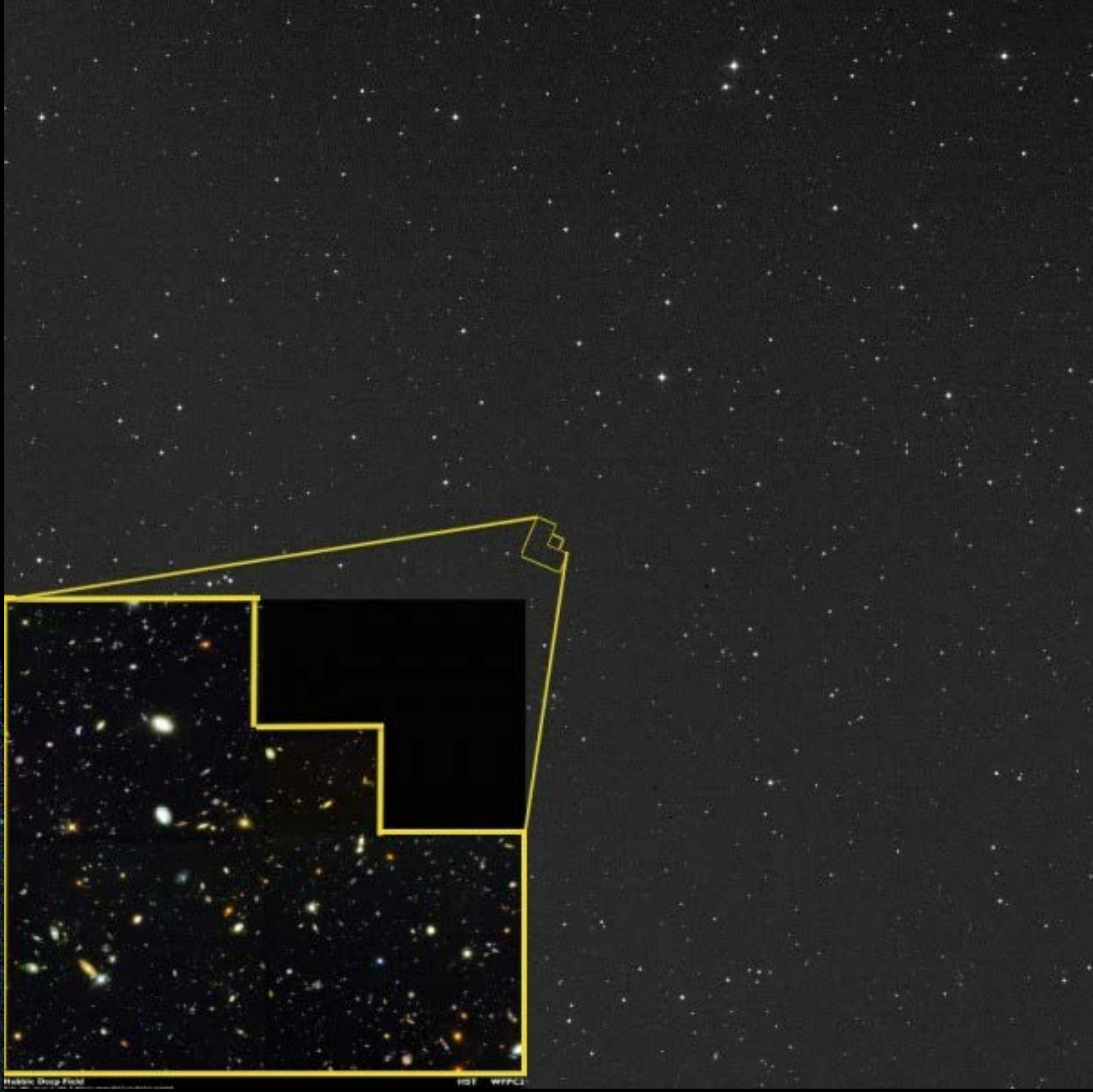
In (B) we would have to compute gravitational interactions between Np ~ 10^{11} stars, which is too much even for modern parallel computers!

So we have to discretize, creating “superparticles” or “mesh points” each of which represents a cluster of stars of, say, 10^4-10^5 Mo.
Now we can do the computation (a galaxy is more than just a collection of stars as it contains interstellar gas and dark matter, but for the moment we will forget about it).

The discretized model, to be physically meaningful, has to be based on a robust mathematical model of the system under study. As we will see, the physical nature of a galaxy is such that it can be correctly described using a discretized model due to the low interaction rate of stars (-->> collisionless systems)

Hubble Deep Field image was taken over a very small fraction of the sky.





Euclid will cover all of the sky at something approaching this level of resolution!

The Euclid Flagship v2.0 Simulation

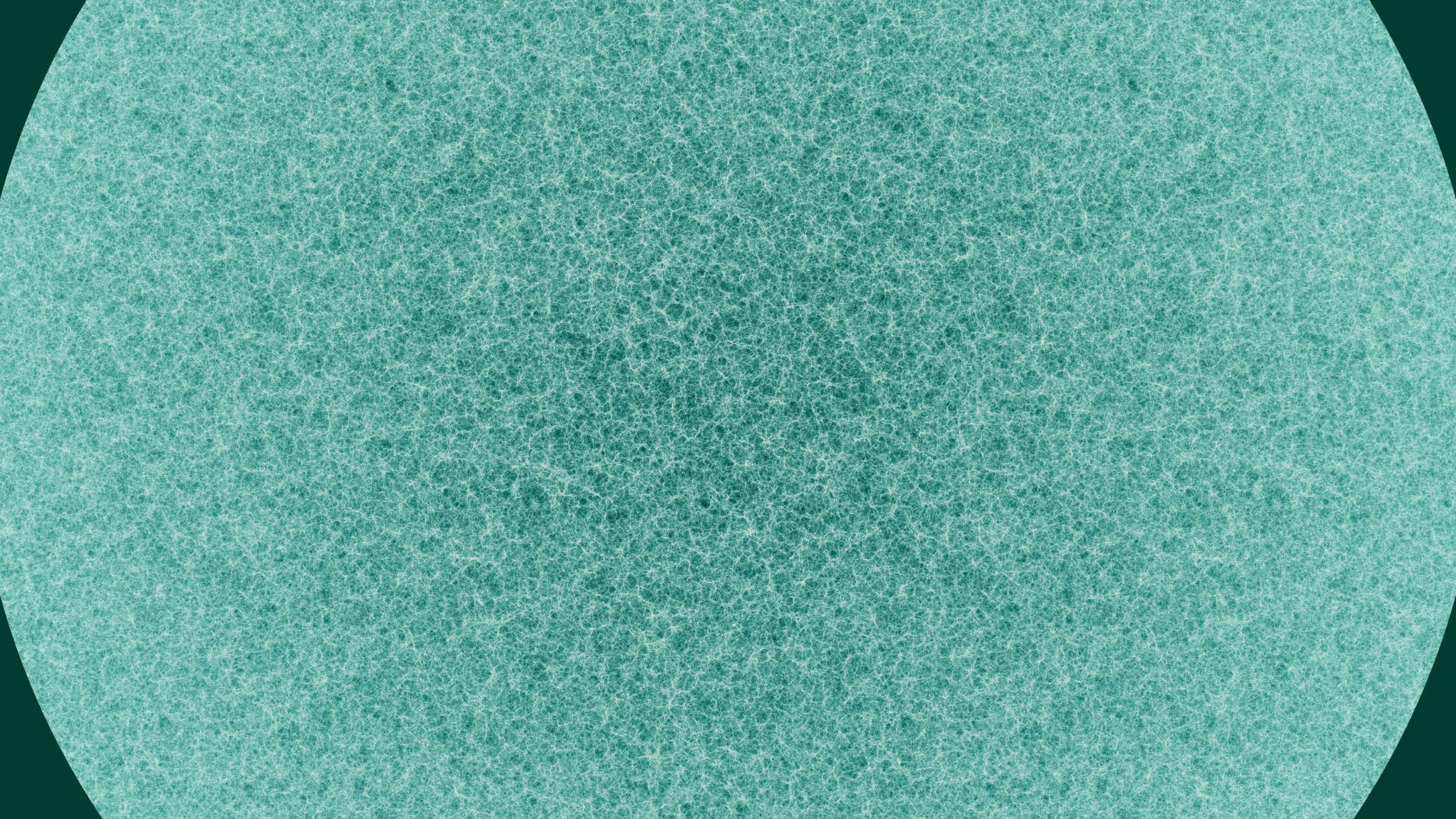
4×10^{12} Particles

$(16000)^3$

$L=3600\ h^{-1}\mathrm{Mpc}$

$m_p=10^9\ h^{-1}M_\odot$

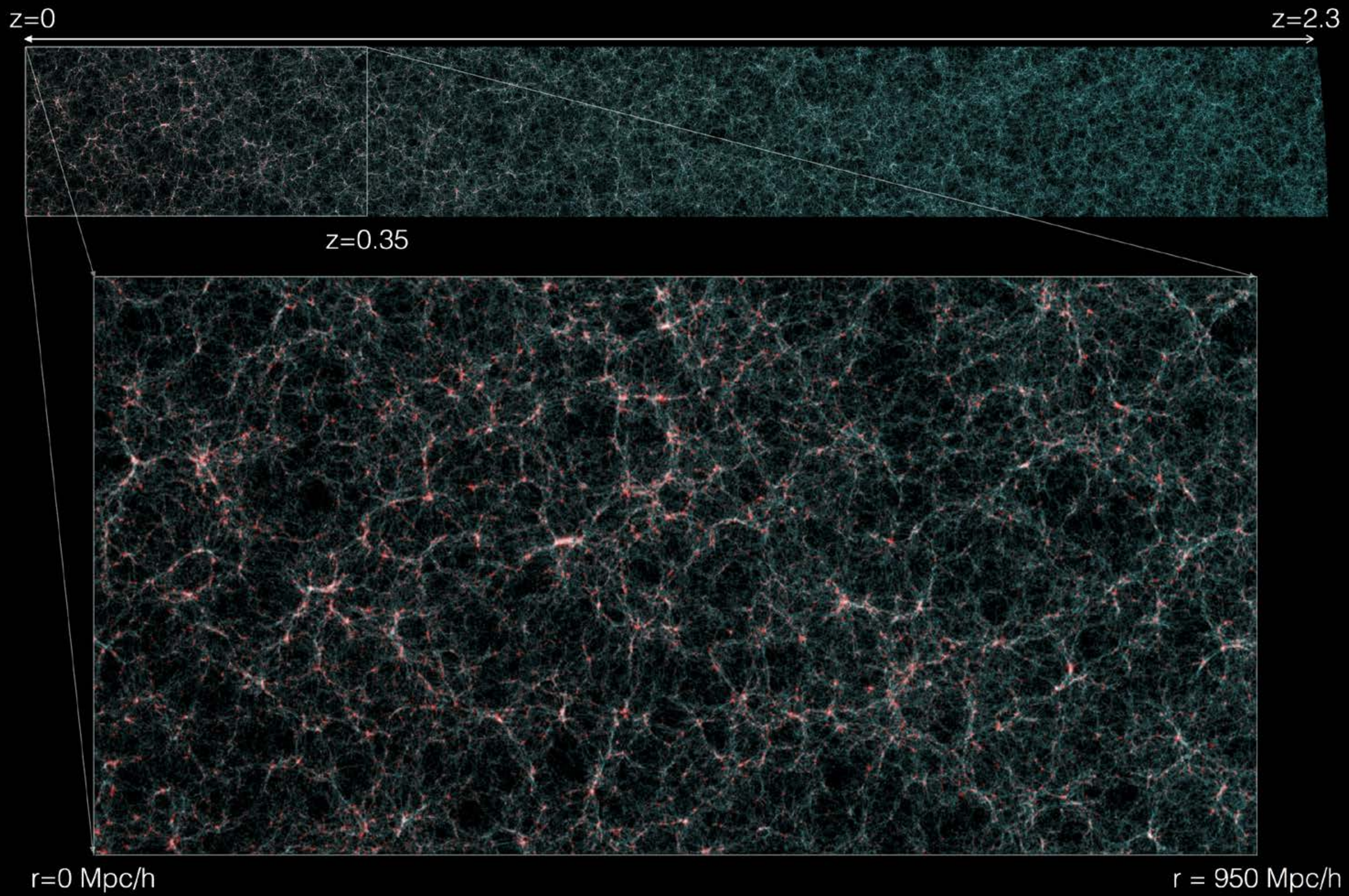
Doug Potter Joachim Stadel Romain Teyssier



Piz Daint – over 5000 GPU Nodes

Swiss National Computing Center (CSCS) in Lugano, Switzerland

Flagship mock galaxy catalog



II - Towards a general mathematical model for astrophysical systems: the distribution function *f* (**x,v**,t)

Any system of particles or superparticles can be described by a distribution function *f* which expresses the *probability density* to have a particle (or superparticle) with position between **x** and **x + dx** and velocity between **v** and **v + dv** at time t.
In other words it describes how phase space (**w= (x,v)**) is filled at any given time, by "real" particles or superparticles.

Next question is: how does *f* evolve in time? Or equivalently, how particles/superparticles evolve in phase-space?

First we must specify a field by which these particles interact (eg gravity or electromagnetic field, so we give them a mass or a charge),then find the dynamical equation that *f* obeys under the action of such a field.

Uncorrelated and correlated systems

Consider a system of *N* particles subject to some mutual force
If the particles are *uncorrelated*, namely the evolution of a particle in phase space is independent from that of another particle, at any given time *t* the state of the system the two-particle distribution function can be written as:

$$f_2\,(\mathbf{r},\mathbf{v},\mathbf{r'},\mathbf{v'},t) = f_1(\mathbf{r},\mathbf{v},t)\; f_1{}'(\mathbf{r'},\mathbf{v'},t)$$

which can be extended to N-particle distribution function as:

$$f_N = f_1\;(\mathbf{r_i},\,\mathbf{v_i},\,t)\ldots f_n(\mathbf{r_n},\,\mathbf{v_n},\,t)$$

and holds when the direct interaction between particles (through whatever force field) is negligible. We will see that for gravitational systems this means the acceleration induced by *individual* particles is negligible, yet particles will still respond to the *mean gravity field*

A note on the statistical meaning of *f*.....

First define the *exact* distribution describing state of a system of N particles at time t (occupation of states in phase space) as:

$$F(\mathbf{r},\mathbf{v},t) = \sum_{i=1}^{N} \delta(\mathbf{r}-\mathbf{r}_i(t)) \cdot \delta(\mathbf{v}-\mathbf{v}_i(t)).$$

Then *f* the ensambled-average distribution function, namely the exact distribution function weighted by the probability density of states in phase space:

$$f_1(\mathbf{r},\mathbf{v},t) = \langle F(\mathbf{r},\mathbf{v},t)\rangle = \int F \cdot p \cdot \mathbf{dr}_1\, \mathbf{dr}_2 \cdots \mathbf{dr}_N\, \mathbf{dv}_1\, \mathbf{dv}_2 \cdots \mathbf{dv}_N$$

This is important to remember as it means, in practice, that *f* can be correctly evaluated only after considering many realizations of a system of *N* particles and then taking their weighted average in phase space. We will return to this when dealing with numerical models

The dynamical equations obeyed by f follow from the notion of conservation of probability density in phase space (formally from Liouville's equation in statistical mechanics).

Conservation of probability in phase space can be understood less formally with the analogy with mass conservation in fluid flow. For an arbitrary volume *V* in phase space we can define the probability P of finding a particle in V as:

$$P = \int_V \mathrm{d}^6\mathbf{w}\, f(\mathbf{w}). \qquad \mathbf{w=(r,v)}$$

While f can evolve P must be conserved, which can then be expressed with a continuity equation *in phase space*:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{w}} \cdot (f\dot{\mathbf{w}}) = 0.$$