



Momentum resolution of a tracking detector

Data analysis 2025 – Group project I

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1 Motivation

In this project, you are going to simulate a simple particle tracking detector for the reconstruction of the trajectories of charged particles. One goal in the reconstruction of such trajectories is to estimate the position at which the particle was created, the second goal is to measure its momentum. To measure the momentum, a magnetic field is applied. The magnetic field exerts a Lorentz force on moving charged particle. If the field is homogeneous, the particle is forced onto a circular trajectory with radius of curvature

$$\rho = \frac{p_T}{q \cdot B}$$

where B is the strength of the magnetic field, q is the charge of the particle, and p_T is the component of its momentum transverse to the magnetic field lines. Expressing ρ in metres, B in T and q in units of the elementary charge, the transverse momentum in GeV is

$$p_T \approx 0.3 \cdot e \cdot B \cdot \rho.$$

For illustration, Fig. 1 shows an “event display” from the LHCb experiment at CERN. You are going to estimate the spatial resolution and the momentum resolution for a similar but simplified detector setup.

An important cross check in simulation studies is the so-called “pull”, defined as

$$\text{pull} = \frac{\text{reconstructed quantity} - \text{generated quantity}}{\text{uncertainty on reconstructed quantity}},$$

where the “uncertainty on reconstructed quantity” is for example that returned from your fit to the data. If the reconstruction is unbiased and the uncertainties on the reconstructed quantities is estimated correctly, the distribution of the pulls over many simulated experiments should follow a Gaussian distribution with $\mu = 0$ and $\sigma = 1$. If μ deviates significantly from zero, the measurement is biased. If σ deviates significantly from one, the uncertainties are not estimated correctly.

2 Setup

The detector setup is illustrated in Fig. 2. It consists of n detection planes that measure the x coordinate of the trajectory and are spaced at equal intervals Δz along the z axis, followed by a

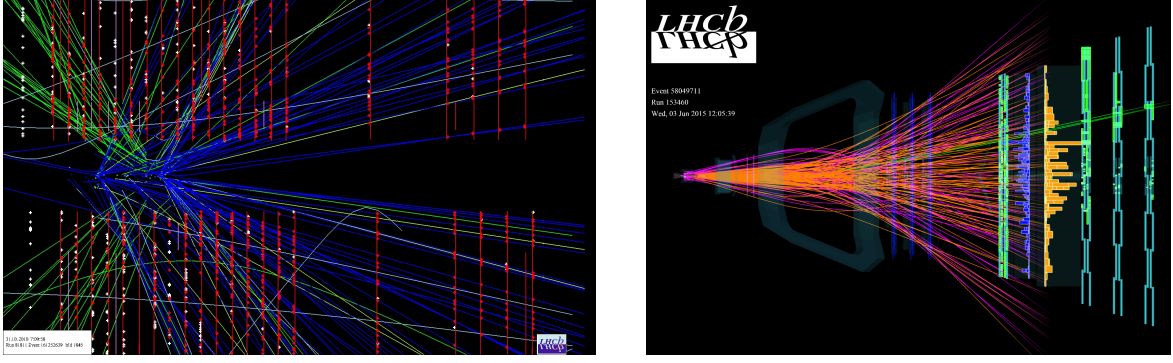


Figure 1: Two events displays from the LHCb experiment at CERN. The display on the left shows a close-up of the “interaction region” where protons from the two beams circulating in the LHC are brought into collisions. The detection planes of the tracking detector are illustrated by thin, red vertical lines, hits in these detection layers are illustrated by little red rectangles. Blue, green and yellow lines indicate reconstructed trajectories. There are two regions in which lots of trajectories seem to intersect, indicating that in this event two proton-proton collisions seem to have occurred. The display on the right shows reconstructed trajectories of charged particles as they traverse the full detector from left to right. The trajectories are bent by the field of the LHCb dipole magnet. Positively and negatively charged particles are bent in opposite directions and low-momentum particles are bent more than high-momentum particles.

magnet with length L and field lines orthogonal to the z and x axes, and another three detection planes spaced at equal intervals Δz .

Each detection plane consists of $500 \mu\text{m}$ wide cells that give a signal when a particle passes through the cell. The detector does not provide position resolution within the cell, i.e. for the reconstruction of the trajectory you have to use the position of the cell that was hit.

The field is constant (homogeneous) inside the magnet and zero outside the magnet. This means particles follow a straight trajectory before and after the magnet and a circular trajectory inside the magnet.

3 Tracking resolution

In this first part of your study you make use only of the first n detection planes before the magnet. To start with, set $n = 5$ and $\Delta z = 2 \text{ cm}$.

As a warm-up exercise, generate a single particle trajectory starting at $z_0 = 0$ and $x_0 = x_0^{\text{true}}$ and propagating towards positive z with a small slope s_0 . Draw x_0^{true} from a Gaussian distribution with mean 0 and standard deviation 1 mm and draw s_0^{true} from a Gaussian distribution with mean 0 and standard deviation 0.1 rad.

- Generate a detector response (a hit) at each intersection with a detection layer, i.e. calculate the x position of the trajectory at the z position of the detection layer, determine which cell was hit and store the position of this cell as the position of the hit in this layer.
- Discuss and decide how you want to model the uncertainty on the measured hit positions. Use this uncertainty model, fit a straight line to the reconstructed hit positions from all n detection layers, and determine the reconstructed slope s_0^{reco} and impact parameter x_0^{reco} with their uncertainties.

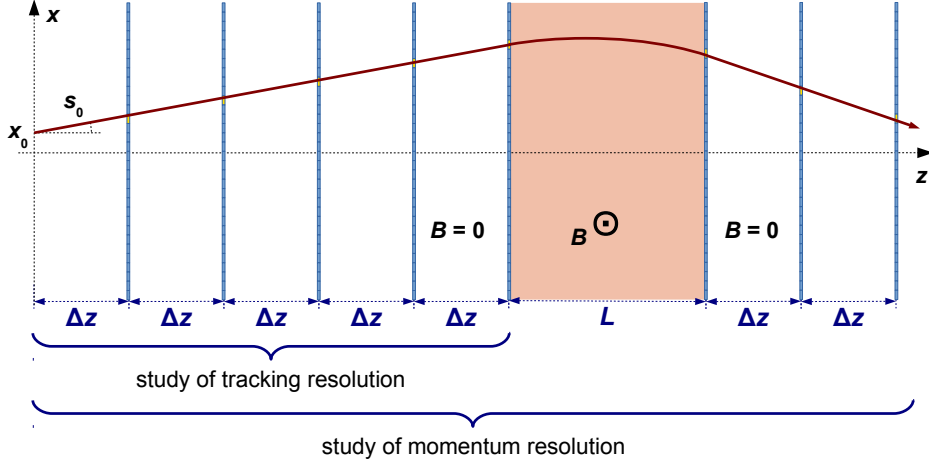


Figure 2: Illustration of the setup.

- (c) Draw the generated trajectory as a dashed line, the calculated hit positions with their assigned uncertainty as points with errorbars, and the reconstructed trajectory as a full line.

To estimate the resolution of your detector in x_0 and s_0 , generate 1'000 trajectories with $z_0 = 0$, x_0^{true} Gaussian distributed with mean 0 and standard deviation 1 mm and s_0^{true} Gaussian distributed with mean 0 and standard deviation 0.1 rad.

- (d) For each of the 1'000 trajectories, generate hit positions in the n detection layers and determine s_0^{reco} and x_0^{reco} with their uncertainties as you did in the warm-up exercise.
- (e) Generate histograms of the residual distributions, $s_0^{\text{reco}} - s_0^{\text{true}}$ and $x_0^{\text{reco}} - x_0^{\text{true}}$. Determine the mean and standard deviation for each of the two distributions.
- (f) Calculate and histogram the pulls for s_0^{reco} and x_0^{reco} . Determine the means and standard deviations of the two pull distributions.

4 Momentum resolution

For this second part of your study you make use of the full setup, including the magnet and the three detection layers behind the magnet. Set $n = 5$, $\Delta z = 2$ cm, $L = 10$ cm and $B = 0.5$ T.

As a warm-up exercise, generate a single particle trajectory for a particle with $p_T = 0.3$ GeV, x_0^{true} drawn from a Gaussian distribution with mean 0 and standard deviation 1 mm and with s_0^{true} drawn from a Gaussian distribution with mean 0 and standard deviation 0.1 rad. Randomly assign charge $q = +1$ or $q = -1$ to the particle.

- (a) Extrapolate the trajectory to the z positions of the eight detection layers (five before and three after the magnet). Generate hit positions as you did in Sec. 3. Draw the trajectory and the hit positions with their uncertainties.
- (b) Fit straight lines to the hits in the stations before and after the magnet and determine p_T^{reco} and its uncertainty from the angle between these two lines (see Appendix).

To estimate the momentum resolution of your detector, generate 1'000 trajectories with x_0^{true} Gaussian distributed with mean 0 and standard deviation 1 mm and with s_0^{true} Gaussian distributed with mean 0 and standard deviation 0.1 rad. Randomly assign charge $q = +1$ or $q = -1$ to each

particle.

- (c) For each of the 1'000 trajectories, generate hit positions in the detection layers and determine p_T^{reco} with its uncertainty as you did in the warm-up exercise.
- (d) Fill a histogram with $p_T^{\text{reco}} - p_T^{\text{true}}$. Determine the mean and the standard deviation of the distribution.
- (e) Calculate and histogram the pulls for p_T^{reco} . Determine the mean and the standard deviation of the pull distribution.

Repeat the exercise for $p_T^{\text{true}} = 0.1, 1, 2, 5, 10, 20 \text{ GeV}/c$. Observe how the resolution on p_T changes as a function of p_T^{true} .

Repeat the exercise for $p_T^{\text{true}} = 0.3 \text{ GeV}$ and $B = 1.0, 1.5, 2.0 \text{ T}$. Observe how the resolution on p_T changes as a function of B .

Appendix

Assuming the bending radius of the particle trajectory is much larger than the length of the magnetic field, the p_T of the particle can be estimated from the bending angle

$$\theta \approx \frac{L}{\rho} = \frac{L \cdot q \cdot B}{p_T}$$

as illustrated in Fig. 3.

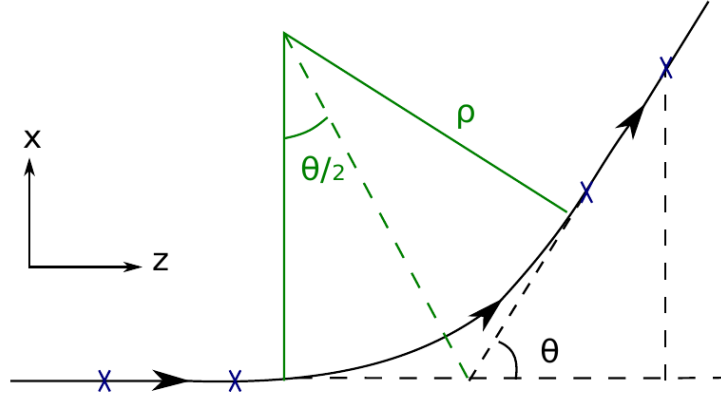


Figure 3: Reconstructing the p_T of the particle from the bending angle θ in its trajectory.