Assignment 6

Deadline:-30/November/2020 11:55 PM IST

1 Theory

1. Compute the depth of a B Tree built on n elements which has at most m children per node. 2

2. Eggs break when dropped from great enough height. Specifically, there must be a floor f in any sufficiently tall building such that an egg dropped from the fth floor breaks, but one dropped from the $(f-1)$st floor will not. If the egg always breaks, then $f = 1$. If the egg never breaks, then $f = n + 1$. You seek to find the critical floor f using an n-story building.

   The only operation you can perform is to drop an egg off some floor and see what happens. You start out with 2 eggs, and seek to drop eggs as few times as possible. Broken eggs cannot be reused.

   Let $E(n)$ be the minimum number of egg droppings that will always suffice. Show that $E(n) = \Theta(\sqrt{n})$ 4

3. Given an array of n real numbers, consider the problem of finding the maximum sum in any contiguous subvector of the input. For example, in the array
   $\{31, -41, 59, 26, -53, 58, 97, -93, -23, 84\}$
   the maximum is achieved by summing the third through seventh elements, where 59 + 26 + (-53) + 58 + 97 = 187. When all numbers are positive, the entire array is the answer, while when all numbers are negative, the empty array maximizes the total at 0. Give a $\Theta(n)$-time dynamic programming algorithm for this problem. 4

4. Suppose that we are given a weighted, directed graph $G(V, E)$ in which edges that leave the source vertex s may have negative weights, all other edge weights are nonnegative, and there are no negative-weight cycles. Argue that Dijkstra's algorithm correctly finds shortest paths from s in this graph. 4

5. Suppose you are given an undirected graph $G(V, E)$, a source vertex s and destination vertex d as part of the input . Prove that finding the shortest path from s to d is as hard as finding the shortest path from s to all the other vertices in V. 2