

[Mathematics Homework 1]

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1. Use the Bisection method to find P_3 for $f(x) = \sqrt{x} - \cos(x) = 0$ on $[0, 1]$.

MATHEMATICS Homework

① Use the Bisection method to find P_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$

Soln:

$f(x) = \sqrt{x} - \cos x$ on $[0, 1]$ (i) $f(0) = -1 < 0$ (ii) $f(1) = 0.459 \approx 0.46$
 $0.46 > 0$

Iteration	a_n	b_n	$f(a_n)$	$f(b_n)$	$P_n = \frac{a+b}{2}$	$f(P_n)$
1	0	1	-1	0.46	0.5	-0.1704
2	0.5	1	-0.1704	0.46	0.75	0.1343
3	0.5	0.75	-0.1704	0.1343	0.625	-0.020

∴ Therefore at the third iteration, we can set $[a_3, b_3]$ as $[0.5, 0.75]$

and $P_3 = \frac{a_3 + b_3}{2} = \frac{0.5 + 0.75}{2}$

$P_3 = 0.625$

Matlab script:

```

y = @(x) sqrt(x) - cos(x);
x1 = input('Enter the value of x1: ');
x2 = input('Enter the value of x2: ');
if y(x1)*y(x2) > 0
    fprintf('No root exist within the given interval \n');
    return
end

if y(x1) == 0
    fprintf('x1 is one of the roots \n')
    return
elseif y(x2) == 0
    fprintf('x2 is one of the root \n')
    return
end

for i = 1: 3
    xh = (x1+x2)/2; % bisection
    if y(x1)*y(xh) < 0
        x2 = xh;
    else
        x1 = xh;
    end
    if abs(y(x1)) < 1.0E-6

```

```

        break
    end
end
fprintf('The root: %f\n The number of bisections: %d\n', x1, i)

```

2. Let $f(x) = 3(x+1)(x-1/2)(x-1)$. Use the Bisection method on the following intervals to find P_3 . a. $[-2, 1.5]$.

② Let $f(x) = 3(x+1)(x-1/2)(x-1)$. Use the Bisection method on the following interval ① $[-2, 1.5]$ ② $[1.25, 2.5]$ find P_3 :

① at interval of $E \in [-2, 1.5]$

Iteration	a_n	b_n	$f(a_n)$	$f(b_n)$	$P_n = \frac{a_n + b_n}{2}$	$f(P_n)$
1	-2	1.5	-22.5	3.75	-0.25	2.109
2	-2	-0.25	-22.5	2.109	-1.125	-1.295 2.491
3	-1.125	-0.25	-1.295	-1.495 2.491 2.109	2.109 -1.5625 -0.6875	-1.495 2.491 1.878

$\therefore P_3 = -1.5625$ at $[-2, -1.125]$

$\therefore P_3 = -0.6875$ at $[-1.125, -0.25]$

- Let $f(x) = 3(x+1)(x-1/2)(x-1)$. Use the Bisection method on the following intervals to find P_3 . a. $[-1.25, 2.5]$.

② $f(x) = 3(x+1)(x-1/2)(x-1)$ at $[-1.25, 2.5]$ find P_3 :

Iteration	a_n	b_n	$f(a_n)$	$f(b_n)$	$P_n = \frac{a_n + b_n}{2}$	$f(P_n)$
1	-1.25	2.5	-2.953	31.5	0.625	-0.2285
2	0.625	2.5	-0.2285	31.5	1.563	4.6016 0.3508
3	0.625	1.563	-0.2285	4.6016	1.094	0.3508

$\therefore P_3 = 1.094$ at $[0.625, 1.563]$

Matlab script for question 2 a and b:

```
%y = @(x) 3*((x+1) * (x - (1/2)) * (x-1));
y = @(x) 3*x^3 - 1.5*x^2 - 3*x + 1.5;
x1 = input('Enter the value of x1: ');
x2 = input('Enter the value of x2: ');
if y(x1)*y(x2) > 0
    fprintf('No root exist within the given interval \n');
    return
end

if y(x1) == 0
    fprintf('x1 is one of the roots \n')
    return
elseif y(x2) == 0
    fprintf('x2 is one of the root \n')
    return
end

for i = 1:3
    xh = (x1+x2)/2; % bisection
    if y(x1)*y(xh) < 0
        x2 = xh;
    else
        x1 = xh;
    end
    if abs(y(x1)) < 1.0E-6
        break
    end
end
fprintf('The root: %f\n The number of bisections: %d\n',x1,i)
```