Why different results?

(if $r=0$)

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① BS-formula:

$$C_0 = \mathbb{E}(C_T)$$
$$= \mathbb{E}((S_T-K)_+)$$
$$= S_0 N(d_1) - K N(d_2)$$

where $d_1 = \frac{\ln(S_0/K) + (\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \xrightarrow{T\to+\infty} +\infty$

$d_2 = \frac{\ln(S_0/K) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \xrightarrow{T\to+\infty} -\infty$

$$\Rightarrow C_0 \xrightarrow{T\to+\infty} S_0 N(+\infty) - Ke^{-rT}N(-\infty) = S_0$$

② Monte Carlo: $S_T = S_0 e^{(r-\frac{\sigma^2}{2})T + \sigma\sqrt{T}Z}$ $\quad Z \sim N(0,1)$

$\xrightarrow{T\to\infty} S_0 e^{-\infty} = 0 \quad$ (a.s.)

$\therefore \quad C_T = (S_T-K)_+$

$\xrightarrow{T\to\infty} 0 \quad$ (a.s.)

$C_0 = \mathbb{E}(C_\infty) = 0$

$$\lim_{t\to\infty} \mathbb{E}(C_t) \neq \mathbb{E}(\lim_{t\to\infty} C_t)$$

(The same with $\sigma \to +\infty$)