AIME Problems 1983

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1 Let x, y, and z all exceed 1 and let w be a positive number such that

$$\log_x w = 24, \quad \log_y w = 40 \quad \text{and} \quad \log_{xyz} w = 12.$$

Find $\log_z w$.

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2 Let $f(x) = |x - p| + |x - 15| + |x - p - 15|$, where $0 < p < 15$. Determine the minimum value taken by $f(x)$ for x in the interval $p \leq x \leq 15$.

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3 What is the product of the real roots of the equation

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45} \ ?$$

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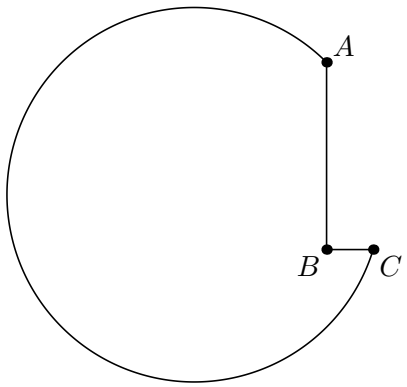
4 A machine-shop cutting tool has the shape of a notched circle, as shown. The radius of the circle is $\sqrt{50}$ cm, the length of AB is 6 cm, and that of BC is 2 cm. The angle ABC is a right angle. Find the square of the distance (in centimeters) from B to the center of the circle.



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5 Suppose that the sum of the squares of two complex numbers x and y is 7 and the sum of the cubes is 10. What is the largest real value that $x + y$ can have?

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6 Let $a_n = 6^n + 8^n$. Determine the remainder on dividing a_{83} by 49.

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7 Twenty five of King Arthur's knights are seated at their customary round table. Three of them are chosen - all choices of three being equally likely - and are sent off to slay a troublesome dragon. Let P be the probability that at least two of the three had been sitting next to each other. If P is written as a fraction in lowest terms, what is the sum of the numerator and denominator?

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8 What is the largest 2-digit prime factor of the integer $n = \binom{200}{100}$?

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9 Find the minimum value of

$$\frac{9x^2 \sin^2 x + 4}{x \sin x}$$

for $0 < x < \pi$.

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10 The numbers 1447, 1005, and 1231 have something in common: each is a four-digit number beginning with 1 that has exactly two identical digits. How many such numbers are there?

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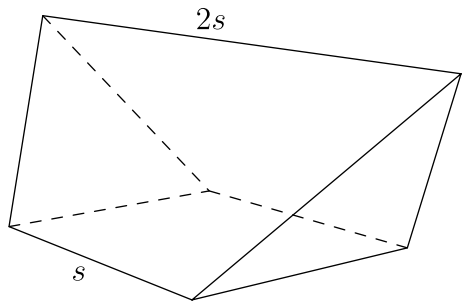
11 The solid shown has a square base of side length s. The upper edge is parallel to the base and has length $2s$. All other edges have length s. Given that $s = 6\sqrt{2}$, what is the volume of the solid?



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12 Diameter AB of a circle has length a 2-digit integer (base ten). Reversing the digits gives the length of the perpendicular chord CD. The distance from their intersection point H to the center O is a positive rational number. Determine the length of AB.

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13 For $\{1, 2, 3, \ldots, n\}$ and each of its nonempty subsets a unique **alternating sum** is defined as follows: Arrange the numbers in the subset in decreasing order and then, beginning with the largest, alternately add and subtract successive numbers. (For example, the alternating sum for $\{1, 2, 4, 6, 9\}$ is $9 - 6 + 4 - 2 + 1 = 6$ and for $\{5\}$ it is simply 5.) Find the sum of all such alternating sums for $n = 7$.

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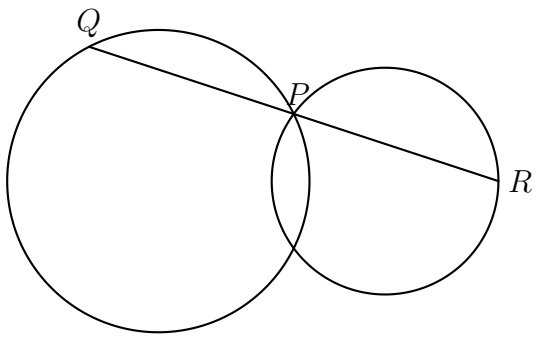
14 In the adjoining figure, two circles of radii 6 and 8 are drawn with their centers 12 units apart. At P, one of the points of intersection, a line is drawn in such a way that the chords QP and

PR have equal length. Find the square of the length of QP.



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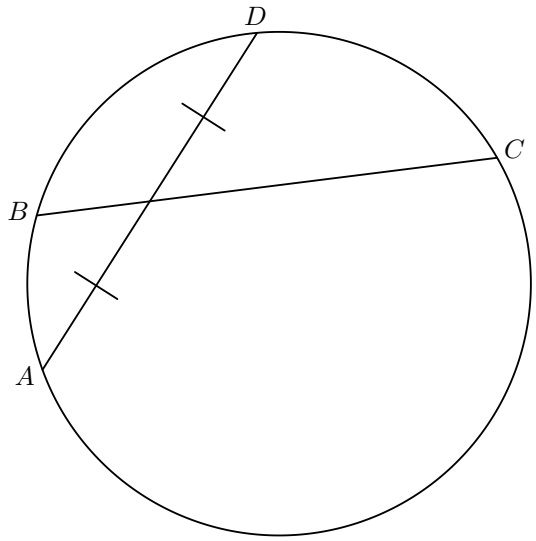
15 The adjoining figure shows two intersecting chords in a circle, with B on minor arc AD. Suppose that the radius of the circle is 5, that $BC = 6$, and that AD is bisected by BC. Suppose further that AD is the only chord starting at A which is bisected by BC. It follows that the sine of the minor arc AB is a rational number. If this fraction is expressed as a fraction m/n in lowest terms, what is the product mn?



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