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1 Determine whether or not there are any positive integral solutions of the simultaneous equations

$$x_1^2 + x_2^2 + \cdots + x_{1985}^2 = y^3,$$
$$x_1^3 + x_2^3 + \cdots + x_{1985}^3 = z^2$$

with distinct integers $x_1, x_2, \ldots, x_{1985}$.

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2 Determine each real root of

$$x^4 - (2 \cdot 10^{10} + 1)x^2 - x + 10^{20} + 10^{10} - 1 = 0$$

correct to four decimal places.

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3 Let A, B, C, D denote four points in space such that at most one of the distances AB, AC, AD, BC, BD, CD is greater than 1. Determine the maximum value of the sum of the six distances.

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4 There are n people at a party. Prove that there are two people such that, of the remaining $n-2$ people, there are at least $\lfloor \frac{n}{2} \rfloor - 1$ of them, each of whom either knows both or else knows neither of the two. Assume that knowing is a symmetric relation, and that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

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5 Let $a_1, a_2, a_3, \cdots$ be a non-decreasing sequence of positive integers. For $m \geq 1$, define $b_m = \min\{n : a_n \geq m\}$, that is, b_m is the minimum value of n such that $a_n \geq m$. If $a_{19} = 85$, determine the maximum value of

$$a_1 + a_2 + \cdots + a_{19} + b_1 + b_2 + \cdots + b_{85}.$$

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