AMC 12/AHSME 1950

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1 If 64 is divided into three parts proportional to 2, 4, and 6, the smallest part is:

(A) $5\frac{1}{3}$ **(B)** 11 **(C)** $10\frac{2}{3}$ **(D)** 5 **(E)** None of these answers

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2 Let $R = gS - 4$. When $S = 8$, $R = 16$. When $S = 10$, R is equal to:

(A) 11 **(B)** 14 **(C)** 20 **(D)** 21 **(E)** None of these

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3 The sum of the roots of the equation $4x^2 + 5 - 8x = 0$ is equal to:

(A) 8 **(B)** -5 **(C)** $-\frac{5}{4}$ **(D)** -2 **(E)** None of these

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4 Reduced to lowest terms, $\frac{a^2-b^2}{ab} - \frac{ab-b^2}{ab-a^2}$ is equal to:

(A) $\frac{a}{b}$ **(B)** $\frac{a^2-2b^2}{ab}$ **(C)** a^2 **(D)** $a-2b$ **(E)** None of these

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5 If five geometric means are inserted between 8 and 5832, the fifth term in the geometric series:

(A) 648 **(B)** 832 **(C)** 1168 **(D)** 1944 **(E)** None of these

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6 The values of y which will satisfy the equations $2x^2 + 6x + 5y + 1 = 0, 2x + y + 3 = 0$ may be found by solving:

(A) $y^2 + 14y - 7 = 0$ **(B)** $y^2 + 8y + 1 = 0$ **(C)** $y^2 + 10y - 7 = 0$ **(D)** $y^2 + y - 12 = 0$ **(E)** None of these equations

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7 If the digit 1 is placed after a two digit number whose tens' digit is t, and units' digit is u, the new number is:

(A) $10t+u+1$ **(B)** $100t+10u+1$ **(C)** $100t+10u+1$ **(D)** $t+u+1$ **(E)** None of these answers

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8 If the radius of a circle is increased 100%, the area is increased:

(A) 100% **(B)** 200% **(C)** 300% **(D)** 400% **(E)** By none of these

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9 The area of the largest triangle that can be inscribed in a semi-circle whose radius is r is:

(A) r^2 **(B)** r^3 **(C)** $2r^2$ **(D)** $2r^3$ **(E)** $\frac{1}{2}r^2$

10 After rationalizing the numerator of $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$, the denominator in simplest form is:

(A) $\sqrt{3}(\sqrt{3}+\sqrt{2})$ **(B)** $\sqrt{3}(\sqrt{3}-\sqrt{2})$ **(C)** $3-\sqrt{3}\sqrt{2}$
(D) $3+\sqrt{6}$ **(E)** None of these answers

11 If in the formula $C=\frac{en}{R+nr}$, n is increased while e, R and r are kept constant, then C:

(A) Increases **(B)** Decreases **(C)** Remains constant **(D)** Increases and then decreases
(E) Decreases and then increases

12 As the number of sides of a polygon increases from 3 to n, the sum of the exterior formed by extending each side in succession:

(A) Increases **(B)** Decreases **(C)** Remains constant **(D)** Cannot be predicted
(E) Becomes $(n-3)$ straight angles

13 The roots of $(x^2-3x+2)(x)(x-4)=0$ are:

(A) 4 **(B)** 0 and 4 **(C)** 1 and 2 **(D)** $0,1,2$ and 4 **(E)** $1,2$ and 4

14 For the simultaneous equations

$$2x-3y=8$$
$$6y-4x=9$$

(A) $x=4, y=0$ **(B)** $x=0, y=\frac{3}{2}$ **(C)** $x=0, y=0$
(D) There is no solution **(E)** There are an infinite number of solutions

15 The real factors of x^2+4 are:

(A) $(x^2+2)(x^2+2)$ **(B)** $(x^2+2)(x^2-2)$ **(C)** $x^2(x^2+4)$
(D) $(x^2-2x+2)(x^2+2x+2)$ **(E)** Non-existent

16 The number of terms in the expansion of $[(a+3b)^2(a-3b)^2]^2$ when simplified is:

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

17 The formula which expresses the relationship between x and y as shown in the accompanying table is:

x	0	1	2	3	4
y	100	90	70	40	0

(A) $y=100-10x$ **(B)** $y=100-5x^2$ **(C)** $y=100-5x-5x^2$
(D) $y=20-x-x^2$ **(E)** None of these

18 Of the following
(1) $a(x-y)=ax-ay$
(2) $a^{x-y}=a^x-a^y$
(3) $\log(x-y)=\log x-\log y$
(4) $\frac{\log x}{\log y}=\log x-\log y$
(5) $a(xy)=ax\times ay$

(A) Only 1 and 4 are true
(B) Only 1 and 5 are true
(C) Only 1 and 3 are true
(D) Only 1 and 2 are true
(E) Only 1 is true

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19 If m men can do a job in d days, then $m+r$ men can do the job in:

(A) $d+r$ days **(B)** $d-r$ days **(C)** $\frac{md}{m+r}$ days **(D)** $\frac{d}{m+r}$ days **(E)** None of these

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20 When $x^{13}+1$ is divided by $x-1$, the remainder is:

(A) 1 **(B)** -1 **(C)** 0 **(D)** 2 **(E)** None of these answers

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21 The volume of a rectangular solid each of whose side, front, and bottom faces are 12 in^2, 8 in^2, and 6 in^2 respectively is:

(A) 576 in^3 **(B)** 24 in^3 **(C)** 9 in^3 **(D)** 104 in^3 **(E)** None of these

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22 Successive discounts of 10% and 20% are equivalent to a single discount of:

(A) 30% **(B)** 15% **(C)** 72% **(D)** 28% **(E)** None of these

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23 A man buys a house for 10000 dollars and rents it. He puts $12\frac{1}{2}\%$ of each month's rent aside for repairs and upkeep; pays 325 dollars a year taxes and realizes $5\frac{1}{2}\%$ on his investment. The monthly rent is:

(A) 64.82 dollars **(B)** 83.33 dollars **(C)** 72.08 dollars **(D)** 45.83 dollars **(E)** 177.08 dollars

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24 The equation $x+\sqrt{x-2}=4$ has:

(A) 2 real roots **(B)** 1 real and 1 imaginary root **(C)** 2 imaginary roots **(D)** No roots **(E)** 1 rea

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25 The value of $\log_5 \frac{(125)(625)}{25}$ is equal to:

(A) 725 **(B)** 6 **(C)** 3125 **(D)** 5 **(E)** None of these

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26 If $\log_{10} m = b - \log_{10} n$, then m=

(A) $\frac{b}{n}$ **(B)** bn **(C)** $10^b n$ **(D)** $b - 10^n$ **(E)** $\frac{10^b}{n}$

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27 A car travels 120 miles from A to B at 30 miles per hour but returns the same distance at 40 miles per hour. The average speed for the round trip is closest to:

(A) 33 mph **(B)** 34 mph **(C)** 35 mph **(D)** 36 mph **(E)** 37 mph

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28 Two boys A and B start at the same time to ride from Port Jervis to Poughkeepsie, 60 miles away. A travels 4 miles an hour slower than B. B reaches Poughkeepsie and at once turns back meeting A 12 miles from Poughkeepsie. The rate of A was:

(A) 4 mph **(B)** 8 mph **(C)** 12 mph **(D)** 16 mph **(E)** 20 mph

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29 A manufacturer built a machine which will address 500 envelopes in 8 minutes. He wishes to build another machine so that when both are operating together they will address 500 envelopes in 2 minutes. The equation used to find how many minutes x it would require the second machine to address 500 envelopes alone is:

(A) $8 - x = 2$ **(B)** $\frac{1}{8} + \frac{1}{x} = \frac{1}{2}$ **(C)** $\frac{500}{8} + \frac{500}{x} = 500$ **(D)** $\frac{x}{2} + \frac{x}{8} = 1$
(E) None of these answers

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30 From a group of boys and girls, 15 girls leave. There are then left two boys for each girl. After this 45 boys leave. There are then 5 girls for each boy. The number of girls in the beginning was:

(A) 40 **(B)** 43 **(C)** 29 **(D)** 50 **(E)** None of these

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31 John ordered 4 pairs of black socks and some additional pairs of blue socks. The price of the black socks per pair was twice that of the blue. When the order was filled, it was found that the number of pairs of the two colors had been interchanged. This increased the bill by 50%. The ratio of the number of pairs of black socks to the number of pairs of blue socks in the original order was:

(A) $4:1$ **(B)** $2:1$ **(C)** $1:4$ **(D)** $1:2$ **(E)** $1:8$

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32 A 25 foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet, then the foot of the ladder will slide:

(A) 9 ft **(B)** 15 ft **(C)** 5 ft **(D)** 8 ft **(E)** 4 ft

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33 The number of circular pipes with an inside diameter of 1 inch which will carry the same amount of water as a pipe with an inside diameter of 6 inches is:

(A) 6π **(B)** 6 **(C)** 12 **(D)** 36 **(E)** 36π

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34 When the circumference of a toy balloon is increased from 20 inches to 25 inches, the radius is increased by:

(A) 5 in **(B)** $2\frac{1}{2}$ in **(C)** $\frac{5}{\pi}$ in **(D)** $\frac{5}{2\pi}$ in **(E)** $\frac{\pi}{5}$ in

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35 In triangle ABC, $AC = 24$ inches, $BC = 10$ inches, $AB = 26$ inches. The radius of the inscribed circle is:

(A) 26 in **(B)** 4 in **(C)** 13 in **(D)** 8 in **(E)** None of these

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36 A merchant buys goods at 25% of the list price. He desires to mark the goods so that he can give a discount of 20% on the marked price and still clear a profit of 25% on the selling price. What percent of the list price must he mark the goods?

(A) 125% **(B)** 100% **(C)** 120% **(D)** 80% **(E)** 75%

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37 If $y = \log_a x$, $a > 1$, which of the following statements is incorrect?

(A) If $x = 1, y = 0$
(B) If $x = a, y = 1$
(C) If $x = -1, y$ is imaginary (complex)
(D) If $0 < x < z, y$ is always less than 0 and decreases without limit as x approaches zero
(E) Only some of the above statements are correct

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38 If the expression $\begin{pmatrix} a & c \\ d & b \end{pmatrix}$ has the value $ab - cd$ for all values of a, b, c and d, then the equation $\begin{pmatrix} 2x & 1 \\ x & x \end{pmatrix} = 3$:

(A) Is satisfied for only 1 value of x
(B) Is satisified for only 2 values of x
(C) Is satisified for no values of x
(D) Is satisfied for an infinite number of values of x
(E) None of these.

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39 Given the series $2 + 1 + \frac{1}{2} + \frac{1}{4} + ...$ and the following five statements:
(1) the sum increases without limit
(2) the sum decreases without limit
(3) the difference between any term of the sequence and zero can be made less than any positive quantity no matter how small
(4) the difference between the sum and 4 can be made less than any positive quantity no matter how small

(5) the sum approaches a limit
Of these statments, the correct ones are:

(A) Only 3 and 4 **(B)** Only 5 **(C)** Only 2 and 4 **(D)** Only $2, 3$ and 4 **(E)** Only 4 and 5

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40 The limit of $\frac{x^2-1}{x-1}$ as x approaches 1 as a limit is:

(A) 0 **(B)** Indeterminate **(C)** $x-1$ **(D)** 2 **(E)** 1

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41 The least value of the function ax^2+bx+c with $a>0$ is:

(A) $-\dfrac{b}{a}$ **(B)** $-\dfrac{b}{2a}$ **(C)** b^2-4ac **(D)** $\dfrac{4ac-b^2}{4a}$ **(E)** None of these

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42 The equation $x^{x^{x}}\ldots = 2$ is satisfied when x is equal to:

(A) ∞ **(B)** 2 **(C)** $\sqrt[4]{2}$ **(D)** $\sqrt{2}$ **(E)** None of these

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43 The sum to infinity of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+\ldots$ is:

(A) $\frac{1}{5}$ **(B)** $\dfrac{1}{24}$ **(C)** $\dfrac{5}{48}$ **(D)** $\dfrac{1}{16}$ **(E)** None of these

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44 The graph of $y=\log x$

(A) Cuts the y-axis
(B) Cuts all lines perpendicular to the x-axis
(C) Cuts the x-axis
(D) Cuts neither axis
(E) Cuts all circles whose center is at the origin

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45 The number of diagonals that can be drawn in a polygon of 100 sides is:

(A) 4850 **(B)** 4950 **(C)** 9900 **(D)** 98 **(E)** 8800

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46 In triangle ABC, $AB=12$, $AC=7$, and $BC=10$. If sides AB and AC are doubled while BC remains the same, then:

(A) The area is doubled
(B) The altitude is doubled
(C) The area is four times the original area
(D) The median is unchanged
(E) The area of the triangle is 0

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47 A rectangle inscribed in a triangle has its base coinciding with the base b of the triangle. If the altitude of the triangle is h, and the altitude x of the rectangle is half the base of the rectangle, then:

(A) $x = \frac{1}{2}h$ **(B)** $x = \frac{bh}{b+h}$ **(C)** $x = \frac{bh}{2h+b}$ **(D)** $x = \sqrt{\frac{hb}{2}}$ **(E)** $x = \frac{1}{2}b$

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48 A point is selected at random inside an equilateral triangle. From this point perpendiculars are dropped to each side. The sum of these perpendiculars is:

(A) Least when the point is the center of gravity of the triangle
(B) Greater than the altitude of the triangle
(C) Equal to the altitude of the triangle
(D) One-half the sum of the sides of the triangle
(E) Greatest when the point is the center of gravity

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49 A triangle has a fixed base AB that is 2 inches long. The median from A to side BC is $1\frac{1}{2}$ inches long and can have any position emanating from A. The locus of the vertex C of the triangle is:

(A) A straight line $AB, 1\dfrac{1}{2}$ inches from A
(B) A circle with A as center and radius 2 inches
(C) A circle with A as center and radius 3 inches
(D) A circle with radius 3 inches and center 4 inches from B along BA
(E) An ellipse with A as focus

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50 A privateer discovers a merchantman 10 miles to leeward at 11:45 a.m. and with a good breeze bears down upon her at 11 mph, while the merchantman can only make 8 mph in her attempt to escape. After a two hour chase, the top sail of the privateer is carried away; she can now make only 17 miles while the merchantman makes 15. The privateer will overtake the merchantman at:

(A) $3{:}45$ p.m. **(B)** $3{:}30$ p.m. **(C)** $5{:}00$ p.m. **(D)** $2{:}45$ p.m. **(E)** $5{:}30$ p.m.

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