AMC 12/AHSME 1957

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1 The number of distinct lines representing the altitudes, medians, and interior angle bisectors of a triangle that is isosceles, but not equilateral, is:

(A) 9 **(B)** 7 **(C)** 6 **(D)** 5 **(E)** 3

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2 In the equation $2x^2 - hx + 2k = 0$, the sum of the roots is 4 and the product of the roots is -3. Then h and k have the values, respectively:

(A) 8 and -6 **(B)** 4 and -3 **(C)** -3 and 4 **(D)** -3 and 8 **(E)** 8 and -3

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3 The simplest form of $1 - \frac{1}{1+\frac{a}{1-a}}$ is:

(A) a if $a \neq 0$ **(B)** 1 **(C)** a if $a \neq -1$ **(D)** $1-a$ with not restriction on a **(E)** a if $a \neq 1$

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4 The first step in finding the product $(3x+2)(x-5)$ by use of the distributive property in the form $a(b+c) = ab + ac$ is:

(A) $3x^2 - 13x - 10$ **(B)** $3x(x-5) + 2(x-5)$
(C) $(3x+2)x + (3x+2)(-5)$ **(D)** $3x^2 - 17x - 10$ **(E)** $3x^2 + 2x - 15x - 10$

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5 Through the use of theorems on logarithms

$$\log\frac{a}{b} + \log\frac{b}{c} + \log\frac{c}{d} - \log\frac{ay}{dx}$$

can be reduced to:

(A) $\log\frac{y}{x}$ **(B)** $\log\frac{x}{y}$ **(C)** 1 **(D)** 0 **(E)** $\log\frac{a^2y}{d^2x}$

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6 An open box is constructed by starting with a rectangular sheet of metal 10 in. by 14 in. and cutting a square of side x inches from each corner. The resulting projections are folded up and the seams welded. The volume of the resulting box is:

(A) $140x - 48x^2 + 4x^3$ **(B)** $140x + 48x^2 + 4x^3$
(C) $140x + 24x^2 + x^3$ **(D)** $140x - 24x^2 + x^3$ **(E)** none of these

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7 The area of a circle inscribed in an equilateral triangle is 48π. The perimeter of this triangle is:

(A) $72\sqrt{3}$ **(B)** $48\sqrt{3}$ **(C)** 36 **(D)** 24 **(E)** 72

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8 The numbers $x,\ y,\ z$ are proportional to $2,\ 3,\ 5$. The sum of x, y, and z is 100. The number y is given by the equation $y = ax - 10$. Then a is:

(A) 2 **(B)** $\frac{3}{2}$ **(C)** 3 **(D)** $\frac{5}{2}$ **(E)** 4

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9 The value of $x - y^{x-y}$ when $x = 2$ and $y = -2$ is:
(A) -18 **(B)** -14 **(C)** 14 **(D)** 18 **(E)** 256

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10 The graph of $y = 2x^2 + 4x + 3$ has its:
(A) lowest point at $(-1, 9)$ **(B)** lowest point at $(1, 1)$
(C) lowest point at $(-1, 1)$ **(D)** highest point at $(-1, 9)$
(E) highest point at $(-1, 1)$

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11 The angle formed by the hands of a clock at $2:15$ is:
(A) 30° **(B)** $27\frac{1}{2}^\circ$ **(C)** $157\frac{1}{2}^\circ$ **(D)** $172\frac{1}{2}^\circ$ **(E)** none of these

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12 Comparing the numbers 10^{-49} and $2 \cdot 10^{-50}$ we may say:
(A) the first exceeds the second by $8 \cdot 10^{-1}$
(B) the first exceeds the second by $2 \cdot 10^{-1}$
(C) the first exceeds the second by $8 \cdot 10^{-50}$
(D) the second is five times the first
(E) the first exceeds the second by 5

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13 A rational number between $\sqrt{2}$ and $\sqrt{3}$ is:
(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ **(B)** $\frac{\sqrt{2}\cdot\sqrt{3}}{2}$ **(C)** 1.5 **(D)** 1.8 **(E)** 1.4

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14 If $y = \sqrt{x^2 - 2x + 1} + \sqrt{x^2 + 2x + 1}$, then y is:
(A) $2x$ **(B)** $2(x + 1)$ **(C)** 0 **(D)** $|x - 1| + |x + 1|$ **(E)** none of these

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15 The table below shows the distance s in feet a ball rolls down an inclined plane in t seconds.

t	0	1	2	3	4	5
s	0	10	40	90	160	250

The distance s for $t = 2.5$ is:
(A) 45 **(B)** 62.5 **(C)** 70 **(D)** 75 **(E)** 82.5

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16 Goldfish are sold at 15 cents each. The rectangular coordinate graph showing the cost of 1 to 12 goldfish is:
(A) a straight line segment
(B) a set of horizontal parallel line segments
(C) a set of vertical parallel line segments
(D) a finite set of distinct points **(E)** a straight line

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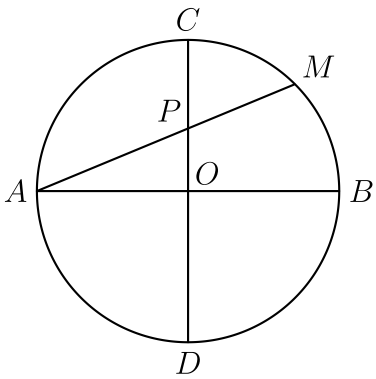
17 A cube is made by soldering twelve 3-inch lengths of wire properly at the vertices of the cube. If a fly alights at one of the vertices and then walks along the edges, the greatest distance it could travel before coming to any vertex a second time, without retracing any distance, is:
(A) 24 in. **(B)** 12 in. **(C)** 30 in. **(D)** 18 in. **(E)** 36 in.

18 Circle O has diameters AB and CD perpendicular to each other. AM is any chord intersecting CD at P. Then $AP \cdot AM$ is equal to:



(A) $AO \cdot OB$ **(B)** $AO \cdot AB$ **(C)** $CP \cdot CD$ **(D)** $CP \cdot PD$ **(E)** $CO \cdot OP$

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19 The base of the decimal number system is ten, meaning, for example, that $123 = 1 \cdot 10^2 + 2 \cdot 10 + 3$. In the binary system, which has base two, the first five positive integers are $1,\ 10,\ 11,\ 100,\ 101$. The numeral 10011 in the binary system would then be written in the decimal system as:
(A) 19 **(B)** 40 **(C)** 10011 **(D)** 11 **(E)** 7

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20 A man makes a trip by automobile at an average speed of 50 mph. He returns over the same route at an average speed of 45 mph. His average speed for the entire trip is:
(A) $47\frac{7}{19}$ **(B)** $47\frac{1}{4}$ **(C)** $47\frac{1}{2}$ **(D)** $47\frac{11}{19}$ **(E)** none of these

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21 Start with the theorem "If two angles of a triangle are equal, the triangle is isosceles," and the following four statements:

1. If two angles of a triangle are not equal, the triangle is not isosceles.
2. The base angles of an isosceles triangle are equal.
3. If a triangle is not isosceles, then two of its angles are not equal.
4. A necessary condition that two angles of a triangle be equal is that the triangle be isosceles.

Which combination of statements contains only those which are logically equivalent to the given theorem?
(A) $1,\ 2,\ 3,\ 4$ **(B)** $1,\ 2,\ 3$ **(C)** $2,\ 3,\ 4$ **(D)** $1,\ 2$ **(E)** $3,\ 4$

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22 If $\sqrt{x-1} - \sqrt{x+1} + 1 = 0$, then $4x$ equals:
(A) 5 **(B)** $4\sqrt{-1}$ **(C)** 0 **(D)** $1\frac{1}{4}$ **(E)** no real value

23 The graph of $x^2+y=10$ and the graph of $x+y=10$ meet in two points. The distance between these two points is:
(A) less than 1 **(B)** 1 **(C)** $\sqrt{2}$ **(D)** 2 **(E)** more than 2

24 If the square of a number of two digits is decreased by the square of the number formed by reversing the digits, then the result is not always divisible by:
(A) 9 **(B)** the product of the digits **(C)** the sum of the digits **(D)** the difference of the digits

25 The vertices of triangle PQR have coordinates as follows: $P(0,a),\ Q(b,0),\ R(c,d)$, where $a,\ b,\ c$ and d are positive. The origin and point R lie on opposite sides of PQ. The area of triangle PQR may be found from the expression:
(A) $\frac{ab+ac+bc+cd}{2}$ **(B)** $\frac{ac+bd-ab}{2}$ **(C)** $\frac{ab-ac-bd}{2}$ **(D)** $\frac{ac+bd+ab}{2}$ **(E)** $\frac{ac+bd-ab-cd}{2}$

26 From a point within a triangle, line segments are drawn to the vertices. A necessary and sufficient condition that the three triangles thus formed have equal areas is that the point be:
(A) the center of the inscribed circle
(B) the center of the circumscribed circle
(C) such that the three angles fromed at the point each be 120°
(D) the intersection of the altitudes of the triangle
(E) the intersection of the medians of the triangle

27 The sum of the reciprocals of the roots of the equation $x^2+px+q=0$ is:
(A) $-\frac{p}{q}$ **(B)** $\frac{q}{p}$ **(C)** $\frac{p}{q}$ **(D)** $-\frac{q}{p}$ **(E)** pq

28 If a and b are positive and $a\neq 1,\ b\neq 1$, then the value of $b^{\log_b a}$ is:
(A) dependent upon b **(B)** dependent upon a **(C)** dependent upon a and b **(D)** zero **(E)** one

29 The relation $x^2(x^2-1)\geq 0$ is true only for:
(A) $x\geq 1$ **(B)** $-1\leq x\leq 1$ **(C)** $x=0,\ x=1,\ x=-1$
(D) $x=0,\ x\leq -1,\ x\geq 1$ **(E)** $x\geq 0$

30 The sum of the squares of the first n positive integers is given by the expression $\frac{n(n+c)(2n+k)}{6}$, if c and k are, respectively:
(A) 1 and 2 **(B)** 3 and 5 **(C)** 2 and 2 **(D)** 1 and 1 **(E)** 2 and 1

31 A regular octagon is to be formed by cutting equal isosceles right triangles from the corners of a square. If the square has sides of one unit, the leg of each of the triangles has length:
(A) $\frac{2+\sqrt{2}}{3}$ **(B)** $\frac{2-\sqrt{2}}{2}$ **(C)** $\frac{1+\sqrt{2}}{2}$ **(D)** $\frac{1+\sqrt{2}}{3}$ **(E)** $\frac{2-\sqrt{2}}{3}$

32 The largest of the following integers which divides each of the numbers of the sequence $1^5-1,\,2^5-2,\,3^5-3,\,\cdots,n^5-n,\cdots$ is:

(A) 1 **(B)** 60 **(C)** 15 **(D)** 120 **(E)** 30

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33 If $9^{x+2}=240+9^x$, then the value of x is:
(A) 0.1 **(B)** 0.2 **(C)** 0.3 **(D)** 0.4 **(E)** 0.5

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34 The points that satisfy the system $x+y=1,\,x^2+y^2<25,$ constitute the following set:
(A) only two points
(B) an arc of a circle
(C) a straight line segment not including the end-points
(D) a straight line segment including the end-points
(E) a single point

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35 Side AC of right triangle ABC is divide into 8 equal parts. Seven line segments parallel to BC are drawn to AB from the points of division. If $BC=10$, then the sum of the lengths of the seven line segments:
(A) cannot be found from the given information **(B)** is 33 **(C)** is 34 **(D)** is 35 **(E)** is 45

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36 If $x+y=1$, then the largest value of xy is:
(A) 1 **(B)** 0.5 **(C)** an irrational number about 0.4 **(D)** 0.25 **(E)** 0

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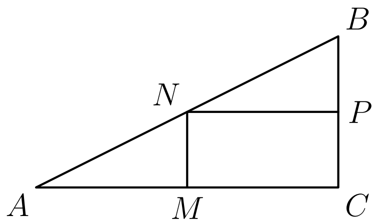
37 In right triangle ABC, $BC=5$, $AC=12$, and $AM=x$; $\overline{MN}\perp\overline{AC}$, $\overline{NP}\perp\overline{BC}$; N is on AB. If $y=MN+NP$, one-half the perimeter of rectangle $MCPN$, then:



(A) $y=\frac{1}{2}(5+12)$ **(B)** $y=\frac{5x}{12}+\frac{12}{5}$ **(C)** $y=\frac{144-7x}{12}$ **(D)** $y=12$ **(E)** $y=\frac{5x}{12}+6$

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38 From a two-digit number N we subtract the number with the digits reversed and find that the result is a positive perfect cube. Then:
(A) N cannot end in 5
(B) N can end in any digit other than 5

(C) N does not exist
(D) there are exactly 7 values for N
(E) there are exactly 10 values for N

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39 Two men set out at the same time to walk towards each other from M and N, 72 miles apart. The first man walks at the rate of 4 mph. The second man walks 2 miles the first hour, $2\frac{1}{2}$ miles the second hour, 3 miles the third hour, and so on in arithmetic progression. Then the men will meet:
(A) in 7 hours **(B)** in $8\frac{1}{4}$ hours **(C)** nearer M than N
(D) nearer N than M **(E)** midway between M and N

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40 If the parabola $y = -x^2 + bx - 8$ has its vertex on the x-axis, then b must be:
(A) a positive integer
(B) a positive or a negative rational number
(C) a positive rational number
(D) a positive or a negative irrational number
(E) a negative irrational number

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41 Given the system of equations

$$ax + (a-1)y = 1(a+1)x - ay = 1.$$

For which one of the following values of a is there no solution x and y?
(A) 1 **(B)** 0 **(C)** -1 **(D)** $\frac{\pm\sqrt{2}}{2}$ **(E)** $\pm\sqrt{2}$

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42 If $S = i^n + i^{-n}$, where $i = \sqrt{-1}$ and n is an integer, then the total number of possible distinct values for S is:
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

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43 We define a lattice point as a point whose coordinates are integers, zero admitted. Then the number of lattice points on the boundary and inside the region bounded by the x-axis, the line $x = 4$, and the parabola $y = x^2$ is:
(A) 24 **(B)** 35 **(C)** 34 **(D)** 30 **(E)** not finite

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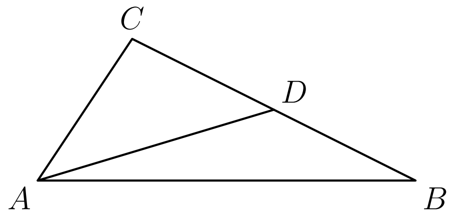
44 In triangle ABC, $AC = CD$ and $\angle CAB - \angle ABC = 30^\circ$. Then $\angle BAD$ is:



(A) 30° **(B)** 20° **(C)** $22\frac{1}{2}^\circ$ **(D)** 10° **(E)** 15°

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45 If two real numbers x and y satisfy the equation $\frac{x}{y} = x - y$, then: **(A)** $x \geq 4$ and $x \leq 0$
(B) y can equal 1
(C) both x and y must be irrational
(D) x and y cannot both be integers
(E) both x and y must be rational

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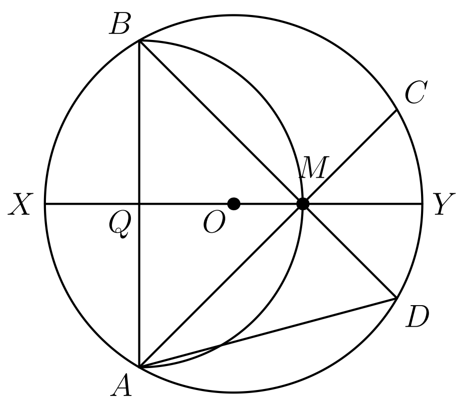
47 In circle O, the midpoint of radius OX is Q; at Q, $\overline{AB} \perp \overline{XY}$. The semi-circle with $\overline{AB}$ as diameter intersects $\overline{XY}$ in M. Line $\overline{AM}$ intersects circle O in C, and line $\overline{BM}$ intersects circle O in D. Line $\overline{AD}$ is drawn. Then, if the radius of circle O is r, AD is:



(A) $r\sqrt{2}$ **(B)** r **(C)** not a side of an inscribed regular polygon **(D)** $\frac{r\sqrt{3}}{2}$ **(E)** $r\sqrt{3}$

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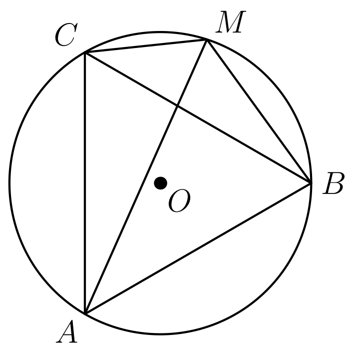
48 Let ABC be an equilateral triangle inscribed in circle O. M is a point on arc BC. Lines $\overline{AM}$, $\overline{BM}$, and $\overline{CM}$ are drawn. Then AM is:



(A) equal to $BM + CM$ **(B)** less than $BM + CM$ **(C)** greater than $BM + CM$ **(D)** equal, less t
(E) none of these

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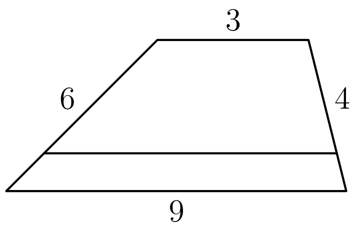
49 The parallel sides of a trapezoid are 3 and 9. The non-parallel sides are 4 and 6. A line parallel to the bases divides the trapezoid into two trapezoids of equal perimeters. The ratio in which each of the non-parallel sides is divided is:



(A) $4 : 3$ **(B)** $3 : 2$ **(C)** $4 : 1$ **(D)** $3 : 1$ **(E)** $6 : 1$

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50 In circle O, G is a moving point on diameter $\overline{AB}$. $\overline{AA'}$ is drawn perpendicular to $\overline{AB}$ and equal to $\overline{AG}$. $\overline{BB'}$ is drawn perpendicular to $\overline{AB}$, on the same side of diameter $\overline{AB}$ as $\overline{AA'}$, and equal to BG. Let O' be the midpoint of $\overline{A'B'}$. Then, as G moves from A to B, point O':
(A) moves on a straight line parallel to AB
(B) remains stationary
(C) moves on a straight line perpendicular to AB
(D) moves in a small circle intersecting the given circle
(E) follows a path which is neither a circle nor a straight line

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