AMC 12/AHSME 1992

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by jeffq, rrusczyk

1 $6^6+6^6+6^6+6^6+6^6+6^6=$

(A) 6^6 **(B)** 6^7 **(C)** 36^6 **(D)** 6^{36} **(E)** 36^{36}

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2 If $3(4x+5\pi)=P$, then $6(8x+10\pi)=$

(A) $2P$ **(B)** $4P$ **(C)** $6P$ **(D)** $8P$ **(E)** $18P$

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3 An urn is filled with coins and beads, all of which are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

(A) 40% **(B)** 48% **(C)** 52% **(D)** 60% **(E)** 80%

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4 If $m>0$ and the points $(m,3)$ and $(1,m)$ lie on a line with slope m, then $m=$

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\sqrt{5}$

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5 If a, b and c are positive integers and a and b are odd, then $3^a+(b-1)^2c$ is

(A) odd for all choices of c **(B)** even for all choices of c **(C)** odd, if c is even; even, if c is odd **(D)** odd, if c is
(E) odd, if c is not a multiple of 3; even if c is a multiple of 3

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6 If $x>y>0$, then $\frac{x^y y^x}{y^y x^x}=$

(A) $(x-y)^{y/x}$ **(B)** $\left(\frac{x}{y}\right)^{x-y}$ **(C)** 1 **(D)** $\left(\frac{x}{y}\right)^{y-x}$ **(E)** $(x-y)^{x/y}$

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7 The ratio of w to x is $4:3$, of y to z is $3:2$ and of z to x is $1:6$. What is the ratio of w to y?

(A) $1:3$ **(B)** $16:3$ **(C)** $20:3$ **(D)** $27:4$ **(E)** $12:1$

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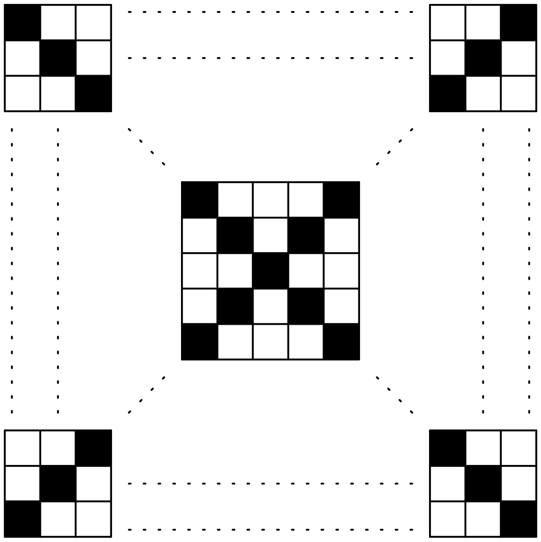
8 A square floor is tiled with congruent square tiles. The tiles on the two diagonals of the floor are black. The rest of the tiles are white. If there are 101 black tiles, then the total number of tiles is



(A) 121 **(B)** 625 **(C)** 676 **(D)** 2500 **(E)** 2601

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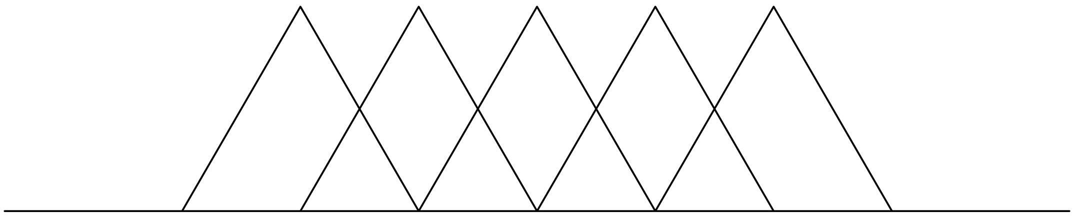
9 Five equilateral triangles, each with side $2\sqrt{3}$, are arranged so they are all on the same side of a line containing one side of each. Along this line, the midpoint of the base of one triangle is a vertex of the next. The area of the region of the plane that is covered by the union of the five triangular regions is



(A) 10 **(B)** 12 **(C)** 15 **(D)** $10\sqrt{3}$ **(E)** $12\sqrt{3}$

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10 The number of positive integers k for which the equation $kx - 12 = 3k$ has an integer solution for x is

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

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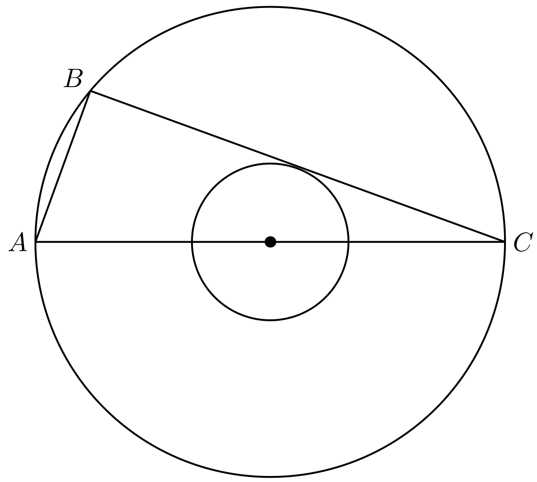
11 The ratio of the radii of two concentric circles is $1:3$. If $\overline{AC}$ is a diameter of the larger circle, $\overline{BC}$ is a chord of the larger circle that is tangent to the smaller circle, and $AB = 12$, then the radius of the larger circle is



(A) 13 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 26

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12 Let $y = mx + b$ be the image when the line $x - 3y + 11 = 0$ is reflected across the x-axis. The value of $m + b$ is

(A) -6 **(B)** -5 **(C)** -4 **(D)** -3 **(E)** -2

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13 How many pairs of positive integers (a, b) with $a + b \leq 100$ satisfy the equation $\frac{a+b^{-1}}{a^{-1}+b} = 13$?

(A) 1 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 13

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14 Which of the following equations have the same graph?

$I. \, y = x - 2 \; II. \, y = \frac{x^2-4}{x+2} \; III. \, (x+2)y = x^2 - 4$

(A) I and II only **(B)** I and III only **(C)** II and III only **(D)** I, II and III **(E)** None. All equations have different graph

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15 Let $i = \sqrt{-1}$. Define a sequence of complex numbers by $z_1 = 0, z_{n+1} = z_n^2 + i$ for $n \geq 1$. In the complex plane, how far from the origin is z_{111}?

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** $\sqrt{110}$ **(E)** $\sqrt{2^{55}}$

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16 If $\frac{y}{x-z} = \frac{x+y}{z} = \frac{x}{y}$ for three positive numbers x, y and z, all different, then $\frac{x}{y} =$

(A) $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{5}{3}$ **(E)** 2

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17 The two digit integers from 19 to 92 are written consecutively to form the larger integer $N = 19202122 \ldots 909192$. If 3^k is the highest power of 3 that is a factor of N, then $k =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

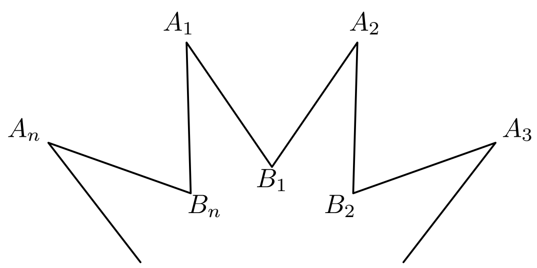
18 The increasing sequence of positive integers $a_1, a_2, a_3, \ldots$ has the property that $a_{n+2} = a_n + a_{n+1}$ for all $n \geq 1$. If $a_7 = 120$, then a_8 is

(A) 128 **(B)** 168 **(C)** 193 **(D)** 194 **(E)** 210

19 For each vertex of a solid cube, consider the tetrahedron determined by the vertex and the midpoints of the three edges that meet at that vertex. The portion of the cube that remains when these eight tetrahedra are cut away is called a *cuboctahedron*. The ratio of the volume of the cuboctahedron to the volume of the original cube is closest to which of these?

(A) 75% **(B)** 78% **(C)** 81% **(D)** 84% **(E)** 87%

20 Part of an "n-pointed regular star" is shown. It is a simple closed polygon in which all $2n$ edges are congruent, angles A_1, A_2, ..., A_n are congruent and angles B_1, B_2, ..., B_n are congruent. If the acute angle at A_1 is 10° less than the acute angle at B_1, then $n =$



(A) 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 60

21 For a finite sequence $A = (a_1, a_2, \ldots, a_n)$ of numbers, the *Cesaro sum* of A is defined to be

$$\frac{S_1 + S_2 + \cdots + S_n}{n}$$

where $S_k = a_1 + a_2 + \cdots + a_k \quad (1 \leq k \leq n)$. If the Cesaro sum of the 99-term sequence $(a_1, a_2, \ldots, a_{99})$ is 1000, what is the Cesaro sum of the 100-term sequence $(1, a_1, a_2, \ldots, a_{99})$?

(A) 991 **(B)** 999 **(C)** 1000 **(D)** 1001 **(E)** 1009

22 Ten points are selected on the positive x-axis, X^+, and fives points are selected on the positive y-axis, Y^+. The fifty segments connecting the ten selected points on X^+ to the five selected points on Y^+ are drawn. What is the maximum possible number of points of intersection of these fifty segments that could lie in the interior of the first quadrant?

(A) 250 **(B)** 450 **(C)** 500 **(D)** 1250 **(E)** 2500

23 What is the size of the largest subset, S, of $\{1, 2, 3, \ldots, 50\}$ such that no pair of distinct elements of S has a sum divisible by 7?

(A) 6 **(B)** 7 **(C)** 14 **(D)** 22 **(E)** 23

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24 Let $ABCD$ be a parallelogram of area 10 with $AB = 3$ and $BC = 5$. Locate E, F and G on segments $\overline{AB}$, $\overline{BC}$ and $\overline{AD}$, respectively, with $AE = BF = AG = 2$. Let the line through G parallel to $\overline{EF}$ intersect $\overline{CD}$ at H. The area of the quadrilateral $EFHG$ is

(A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

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25 In triangle ABC, $\angle ABC = 120^\circ$, $AB = 3$ and $BC = 4$. If perpendiculars constructed to $\overline{AB}$ at A and to $\overline{BC}$ at C meet at D, then $CD =$

(A) 3 **(B)** $\frac{8}{\sqrt{3}}$ **(C)** 5 **(D)** $\frac{11}{2}$ **(E)** $\frac{10}{\sqrt{3}}$

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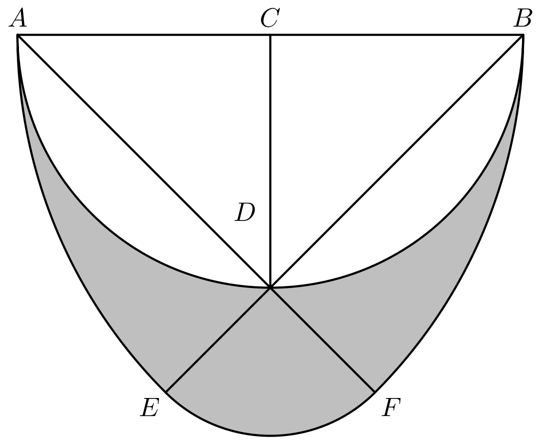
26 Semicircle $\widehat{AB}$ has center C and radius 1. Point D is on $\widehat{AB}$ and $\overline{CD} \perp \overline{AB}$. Extend $\overline{BD}$ and $\overline{AD}$ to E and F, respectively, so that circular arcs $\widehat{AE}$ and $\widehat{BF}$ have B and A as their respective centers. Circular arc $\widehat{EF}$ has center D. The area of the shaded "smile", $AEFBDA$, is



(A) $(2-\sqrt{2})\pi$ **(B)** $2\pi - \pi\sqrt{2} - 1$ **(C)** $\left(1 - \frac{\sqrt{2}}{2}\right)\pi$ **(D)** $\frac{5\pi}{2} - \pi\sqrt{2} - 1$ **(E)** $(3-2\sqrt{2})\pi$

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27 A circle of radius r has chords $\overline{AB}$ of length 10 and $\overline{CD}$ of length 7. When $\overline{AB}$ and $\overline{CD}$ are extended through B and C, respectively, they intersect at P, which is outside the circle. If $\angle APD = 60^\circ$ and $BP = 8$, then $r^2 =$

(A) 70 **(B)** 71 **(C)** 72 **(D)** 73 **(E)** 74

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28 Let $i = \sqrt{-1}$. The product of the real parts of the roots of $z^2 - z = 5 - 5i$ is

(A) -25 **(B)** -6 **(C)** -5 **(D)** $\frac{1}{4}$ **(E)** 25

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29 An "unfair" coin has a $2/3$ probability of turning up heads. If this coin is tossed 50 times, what is the probability that the total number of heads is even?

(A) $25\left(\frac{2}{3}\right)^{50}$ **(B)** $\frac{1}{2}\left(1-\frac{1}{3^{50}}\right)$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{2}\left(1+\frac{1}{3^{50}}\right)$ **(E)** $\frac{2}{3}$

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30 Let $ABCD$ be an isosceles trapezoid with bases $AB=92$ and $CD=19$. Suppose $AD=BC=x$ and a circle with center on $\overline{AB}$ is tangent to segments $\overline{AD}$ and $\overline{BC}$. If m is the smallest possible value of x, then $m^2=$

(A) 1369 **(B)** 1679 **(C)** 1748 **(D)** 2109 **(E)** 8825

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