AMC 12/AHSME 1966

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1 Given that the ratio of $3x-4$ to $y+15$ is constant, and $y=3$ when $x=2$, then, when $y=12$, x equals:
(A) $\frac{1}{8}$ (B) $\frac{7}{3}$ (C) $\frac{7}{8}$ (D) $\frac{7}{2}$ (E) 8

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2 When the base of a triangle is increased 10% and the altitude to this base is decreased 10%, the change in area is
(A) 1% increase (B) $\frac{1}{2}\%$ increase (C) 0% (D) $\frac{1}{2}\%$ decrease (E) 1% decrease

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3 If the arithmetic mean of two numbers is 6 and thier geometric mean is 10, then an equation with the given two numbers as roots is:
(A) $x^2+12x+100=0$ (B) $x^2+6x+100=0$ (C) $x^2-12x-10=0$ (D) $x^2-12x+100=0$ (E) $x^2-6x+100=0$

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4 Circle I is circumscribed about a given square and circle II is inscribed in the given square. If r is the ratio of the area of circle I to that of circle II, then r equals:
(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) $2\sqrt{2}$ (E) $2\sqrt{3}$

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5 The number of values of x satisfying the equation

$$\frac{2x^2-10x}{x^2-5x}=x-3$$

is:
(A) zero (B) one (C) two (D) three (E) an integer greater than 3

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6 AB is the diameter of a circle centered at O. C is a point on the circle such that angle BOC is 60°. If the diameter of the circle is 5 inches, the length of chord AC, expressed in inches, is:
(A) 3 (B) $\frac{5\sqrt{2}}{2}$ (C)$\frac{5\sqrt{3}}{2}$ (D) $3\sqrt{3}$ (E) none of these

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7 Let $\frac{35x-29}{x^2-3x+2}=\frac{N_1}{x-1}+\frac{N_2}{x-2}$ be an identity in x. The numerical value of N_1N_2 is:
(A) -246 (B) -210 (C) -29 (D) 210 (E) 246

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8 The length of the common chord of two intersecting circles is 16 feet. If the radii are 10 feet and 17 feet, a possible value for the distance between the centers of teh circles, expressed in feet, is:
(A) 27 (B) 21 (C) $\sqrt{389}$ (D) 15 (E) undetermined

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9 If $x = (\log_8 2)^{(\log_2 8)}$, then $\log_3 x$ equals:
(A) -3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 3 (E) 9

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10 If the sum of two numbers is 1 and their product is 1, then the sum of their cubes is:
(A) 2 (B) $-2-\frac{3i\sqrt{3}}{4}$ (C) 0 (D) $-\frac{3i\sqrt{3}}{4}$ (E) -2

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11 The sides of triangle BAC are in the ratio $2:3:4$. BD is the angle-bisector drawn to the shortest side AC, dividing it into segments AD and CD. If the length of AC is 10, then the length of the longer segment of AC is:
(A) $3\frac{1}{2}$ (B) 5 (C) $5\frac{5}{7}$ (D) 6 (E) $7\frac{1}{2}$

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12 The number of real values of x that satisfy the equation

$$(2^{6x+3})(4^{3x+6}) = 8^{4x+5}$$

is:
(A) 0 (B) 1 (C) 2 (D) 3 (E) greater than 3

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13 The number of points with positive rational coordinates selected from the set of points in the xy-plane such that $x+y \le 5$, is:
(A) 9 (B) 10 (C) 14 (D) 15 (E) infinite

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14 The length of rectangle $ABCD$ is 5 inches and its width is 3 inches. Diagonal AC is dibided into three equal segments by points E and F. The area of triangle BEF, expressed in square inches, is:
(A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{5}{2}$ (D) $\frac{1}{3}\sqrt{34}$ (E) $\frac{1}{3}\sqrt{68}$

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15 If $x-y>x$ and $x+y<y$, then
(A) $y<x$ (B) $x<y$ (C) $x<y<0$ (D) $x<0, y<0$ (E) $x<0, y>0$

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16 If $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$, x and y are real numbers, then xy equals:
(A) $\frac{12}{5}$ (B) 4 (C) 6 (D) 12 (E) -4

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17 The number of distinct points common to the curves $x^2+4y^2=1$ and $4x^2+y^2=5$ is:
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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18 In a given arithmetic sequence the first term is 2, the last term is 29, and the sum of all the terms is 155. The common difference is:
(A) 3 (B) 2 (C) $\frac{27}{19}$ (D) $\frac{13}{9}$ (E) $\frac{23}{38}$

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19 Let s_1 be the sum of the first n terms of the arithmetic sequence $8, 12, \cdots$ and let s_2 be the sum of the first n terms of the arithmetic sequence $17, 19 \cdots$. Assume $n \neq 0$. Then $s_1 = s_2$ for:

(A) no value of n (B) one value of n (C) two values of n (D) four values of n (E) more than four v

20 The negation of the proposition "For all pairs of real numbers a, b, if $a = 0$, then $ab = 0$" is: There are real numbers a, b such that
(A) $a \neq 0, ab \neq 0$ (B) $a \neq 0, ab = 0$ (C) $a = 0, ab \neq 0$ (D) $ab \neq 0, a \neq 0$ (E) $ab = 0, a \neq 0$

21 An "n-pointed star" is formed as follows: the sides of a convex polygon are numbered consecutively $1, 2, \cdots, k, \cdots, n$, $n \geq 5$; for all n values of k, sides k and $k+2$ are non-parallel, sides $n+1$ and $n+2$ being respectively identical with sides 1 and 2; prolong the n pairs of sides numbered k and $k+2$ until they meet. (A figure is shown for the case $n = 5$)

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Let S be the degree-sum of the interior angles at the n points of the star; then S equals:
(A) 180 (B) 360 (C) $180(n+2)$ (D) $180(n-2)$ (E) $180(n-4)$

22 Consider the statements:

(I) $\sqrt{a^2+b^2} = 0$ (II) $\sqrt{a^2+b^2} = ab$ (III) $\sqrt{a^2+b^2} = a+b$ (IV) $\sqrt{a^2+b^2} = a-b$,

where we allow a and b to be real or complex numbers. Those statements for which there exist solutions other than $a = 0$ and $b = 0$ are:

(A) (I), (II), (III), (IV) (B) (II), (III), (IV) (C) (I), (III), (IV) (D) (III), (IV) (E) (I)

23 If x is a real and $4y^2 + 4xy + x + 6 = 0$, then the complete set of values of x for which y is real, is:

(A) $x \leq -2$ or $x \geq 3$ (B) $x \leq 2$ or $x \geq 3$ (C) $x \leq -3$ or $x \geq 2$ (D) $-3 \leq x \leq 2$ (E) $-2 \leq x \leq 3$

24 If $\log_M N = \log_N M$, $M \neq N$, $MN > 0$, $M \neq 1$, $N \neq 1$, then MN equals:
(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 10 (E) a number greater than 2 and less than 10

25 If $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, \ldots$, and $F(1) = 2$, then $F(101)$ equals:
(A) 49 (B) 50 (C) 51 (D) 52 (E) 53

26 Let m be a positive integer and let the lines $13x + 11y = 700$ and $y = mx - 1$ intersect in a point whose coordinates are integers. Then m is:
(A) 4 (B) 5 (C) 6 (D) 7 (E) one of the integers $4, 5, 6, 7$ and one other positive integer

27 At his usual rate a man rows 15 miles downstream in five hours less time than it takes him to return. If he doubles his usual rate, the time downstream is only one hour less than the time

upstream. In miles per hour, the rate of the stream's current is:
(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) 4

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28 Five points O, A, B, C, D are taken in order on a straight line with distances $OA = a$, $OB = b$, $OC = c$, and $OD = d$. P is a point on the line between B and C and such that $AP : PD = BP : PC$. Then OP equals:

(A) $\dfrac{b^2 - bc}{a - b + c - d}$ (B) $\dfrac{ac - b}{a - b + c - d}$ (C) $-\dfrac{bd + c}{a - b + c - d}$
(D) $\dfrac{bc + ad}{a + b + c + d}$ (E) $\dfrac{ac - bd}{a + b + c + d}$

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29 The number of postive integers less than 1000 divisible by neither 5 nor 7 is:

(A) 688 (B) 686 (C) 684 (D) 658 (E) 630

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30 If three of the roots of $x^4 + ax^2 + bx + c = 0$ are 1, 2, and 3, then the value of $a + c$ is:

(A) 35 (B) 24 (C) -12 (D) -61 (E) -63

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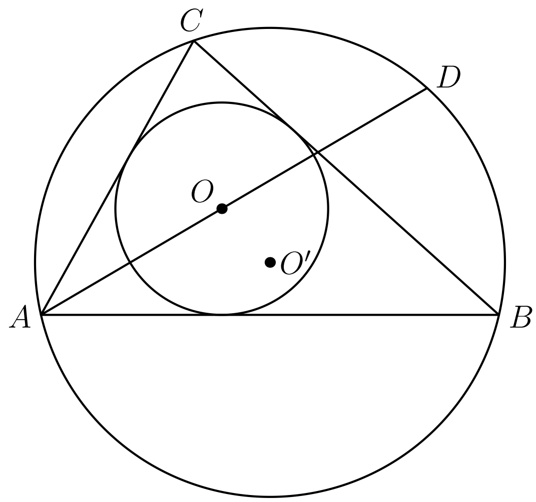
31 Triangle ABC is inscribed in a circle with center O'. A circle with center O is inscribed in triangle ABC. AO is drawn, and extended to intersect the larger circle in D. Then, we must have:

(A) $CD = BD = O'D$ (B) $AO = CO = OD$ (C) $CD = CO = BD$
(D) $CD = OD = BD$ (E) $O'B = O'C = OD$



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32 Let M be the midpoint of side AB of the triangle ABC. Let$P$ be a point on AB between A and M, and let MD be drawn parallel to PC and intersecting BC at D. If the ratio of the area of the triangle BPD to that of triangle ABC is denoted by r, then

(A) $\frac{1}{2} < r < 1$ depending upon the position of P
(B) $r = \frac{1}{2}$ independent of the position of P
(C) $\frac{1}{2} \le r < 1$ depending upon the position of P
(D) $\frac{1}{3} < r < \frac{2}{3}$ depending upon the position of P
(E) $r = \frac{1}{3}$ independent of the position of P

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33 If $ab \neq 0$ and $|a| \neq |b|$ the number of distinct values of x satisfying the equation

$$\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$$

is: (A) zero (B) one (C) two (D) three (E) four

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34 Let r be the speed in miles per hour at which a wheel, 11 feet in circumference, travels. If the time for a complete rotation of the wheel is shortened by $\frac{1}{4}$ of a second, the speed r is increased by 5 miles per hour. The r is:

(A) 9 (B) 10 (C) $10\frac{1}{2}$ (D) 11 (E) 12

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35 Let O be an interior point of triangle ABC, and let $s_1 = OA+OB+OC$. If $s_2 = AB+AC+CA$, then

(A) for every triangle $s_2 > 2s_1, s_1 \le s_2$
(B) for every triangle $s_2 \ge 2s_1, s_1 < s_2$
(C) for every triangle $s_1 > \frac{1}{2}s_2, s_1 < s_2$
(D) for every triangle $s_2 \ge 2s_1, s_1 \le s_2$
(E) neither (A) nor (B) nor (C) nor (D) applies to every triangle

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36 Let $(1+x+x^2)^n = a_0+a_1x+a_2x^2+...+a_{2n}x^{2n}$ be an identity in x. If we lt $s = a_0+a_2+a_4+...+a_{2n}$, then s equals:

(A) 2^n (B) 2^n+1 (C) $\frac{3^n-1}{2}$ (D) $\frac{3^n}{2}$ (E) $\frac{3^n+1}{2}$

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37 Three men, Alpha, Beta, and Gamma, working together, do a job in 6 hours less time than Alpha alone, in 1 hour less time than Beta alone, and in one-half the time needed by Gamma when working alone. Let h be the number of hours needed by Alpha and Beta, working together to do the job. Then h equals:

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) $\frac{5}{4}$ (E) $\frac{3}{4}$

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38 In triangle ABC the medians AM and CN to sides BC and AB, respectively, intersect in point O. P is the midpoint of side AC, and MP intersects CN in Q. If the area of triangle OMQ is n, then the area of triangle ABC is:

(A) $16n$ (B) $18n$ (C) $21n$ (D) $24n$ (E) $27n$

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39 In base R_1 the expanded fraction F_1 becomes $0.373737...$, and the expanded fraction F_2 becomes $0.737373....$ In base R_2 fraction F_1, when expanded, becomes $0.252525...$, while fraction F_2 becomes $0.525252....$ The sum of R_1 and R_2, each written in base ten is:

(A) 24 (B) 22 (C) 21 (D) 20 (E) 19

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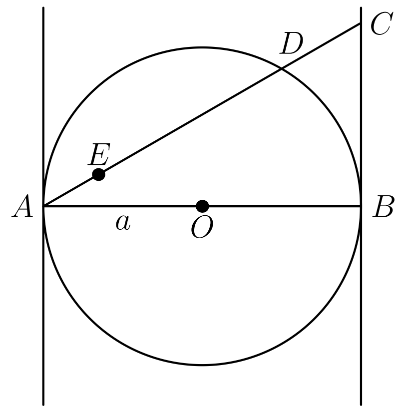

In this figure AB is a diameter of a circle, centered at O, with radius a. A chord AD is drawn and extended to meet the tangent to the circle at B in point C. Point E is taken on AC so that $AE = DC$. Denoting the distances of E from the tangent through A and from the diameter AB by x and y, respectively, we can deduce the relation:

(A) $y^2 = \frac{x^3}{2a-x}$ (B) $y^2 = \frac{x^3}{2a+x}$ (C) $y^4 = \frac{x^2}{2-x}$

(D) $x^2 = \frac{y^2}{2a-x}$ (E) $x^2 = \frac{y^2}{2a+x}$

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