AMC 12/AHSME 1990

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by djmathman, 4everwise, dft, rrusczyk

1 If $\frac{x/4}{2} = \frac{4}{x/2}$ then $x =$

(A) $\pm 1/2$ **(B)** ± 1 **(C)** ± 2 **(D)** ± 4 **(E)** ± 8

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2 $\left(\frac{1}{4}\right)^{-\frac{1}{4}} =$

(A) -16 **(B)** $-\sqrt{2}$ **(C)** $-\frac{1}{16}$ **(D)** $-\frac{1}{256}$ **(E)** $\sqrt{2}$

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3 The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is 75°, then the largest angle is

(A) 95° **(B)** 100° **(C)** 105° **(D)** 110° **(E)** 115°

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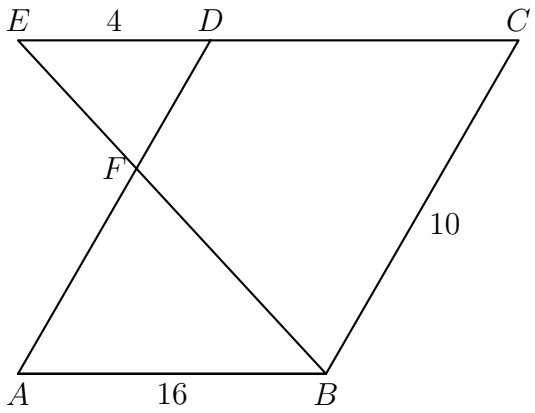
4 Let $ABCD$ be a parallelogram with $\angle ABC = 120^\circ$, $AB = 16$ and $BC = 10$. Extend $\overline{CD}$ through D to E so that $DE = 4$. If $\overline{BE}$ intersects $\overline{AD}$ at F, then FD is closest to

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5



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5 Which of these numbers is the largest?

(A) $\sqrt{\sqrt[3]{5 \cdot 6}}$ **(B)** $\sqrt{6\sqrt[3]{5}}$ **(C)** $\sqrt{5\sqrt[3]{6}}$ **(D)** $\sqrt[3]{5\sqrt{6}}$ **(E)** $\sqrt[3]{6\sqrt{5}}$

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6 Points A and B are 5 units apart. How many lines in a given plane containing A and B are 2 units from A and 3 units from B?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

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7 A triangle with integral sides has perimeter 8. The area of the triangle is

(A) $2\sqrt{2}$ **(B)** $\frac{16}{9}\sqrt{3}$ **(C)** $2\sqrt{3}$ **(D)** 4 **(E)** $4\sqrt{2}$

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8 The number of real solutions of the equation

$$|x-2|+|x-3|=1$$

is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

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9 Each edge of a cube is colored either red or black. Every face of the cube has at least one black edge. The smallest possible number of black edges is

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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10 An $11\times11\times11$ wooden cube is formed by gluing together 11^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point?

(A) 328 **(B)** 329 **(C)** 330 **(D)** 331 **(E)** 332

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11 How man y positive integers less than 50 have an odd number of positive integer divisors?
(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

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12 Let f be the function defined by $f(x)=ax^2-\sqrt{2}$ for some positive a. If $f(f(\sqrt{2}))=-\sqrt{2}$, then $a=$
(A) $\frac{2-\sqrt{2}}{2}$ (B) $\frac{1}{2}$ (C) $2-\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{2+\sqrt{2}}{2}$

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13 If the following instructions are carried out by a computer, which of X will be printed because of instruction 5?
$1.$ Start X at 3 and S at 0 $2.$ Increase the value of X by $2.$ $3.$ Increase the value of S by the value of $X.$ $4.$ If S is at least 10000, then go to instsruction 5; otherwise, go to instruction 2 and proceed from there. $5.$ Print the value of $X.$ $6.$ Stop.
(A) 19 (B) 21 (C) 23 (D) 199 (E) 201

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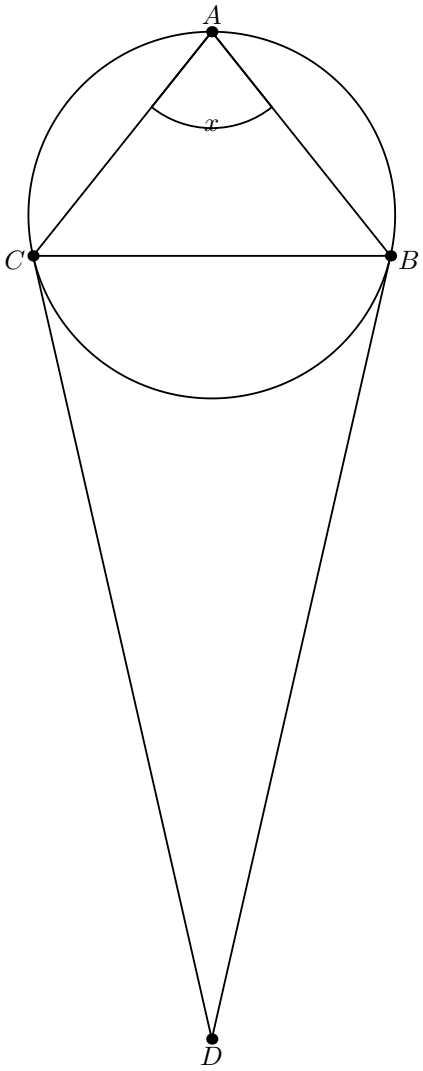
14 An acute isosceles triangle, ABC is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC=\angle ACB=2\angle D$ and x is the radian measure of $\angle A$, then $x=$



(A) $\frac{3}{7}\pi$ (B) $\frac{4}{9}\pi$ (C) $\frac{5}{11}\pi$ (D) $\frac{6}{13}\pi$ (E) $\frac{7}{15}\pi$

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15 Four whole numbers, when added three at a time, give the sums 180, 197, 208, and 222. What is the largest of the four numbers?
(A) 77 (B) 83 (C) 89 (D) 95 (E) cannot be determined

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16 At one of George Washington's parties, each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?
(A) 78 (B) 185 (C) 234 (D) 312 (E) 325

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17 How many of the numbers, $100, 101, \ldots, 999$, have three different digits in increasing order or in decreasing order?
(A) 120 (B) 168 (C) 204 (D) 216 (E) 240

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18 First a is chosen at random from the set $\{1, 2, 3, \ldots, 99, 100\}$, and then b is chosen at random from the same set. The probability that the integer $3^a + 7^b$ has units digit 8 is
(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

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19 For how many integers N between 1 and 1990 is the improper fraction $\frac{N^2+7}{N+4}$ not in lowest terms?
(A) 0 (B) 86 (C) 90 (D) 104 (E) 105

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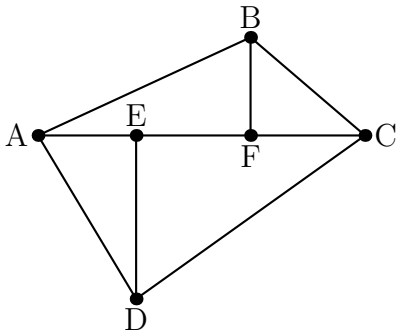
20 $ABCD$ is a quadrilateral with right angles at A and C. Points E and F are on AC, and DE and BF are perpendicular to AC. If $AE = 3$, $DE = 5$, and $CE = 7$, then $BF =$



(A) 3.6 (B) 4 (C) 4.2 (D) 4.5 (E) 5

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21 Consider a pyramid $P-ABCD$ whose base $ABCD$ is a square and whose vertex P is equidistant from A, B, C, and D. If $AB = 1$ and $\angle APD = 2\theta$ then the volume of the pyramid is
(A) $\frac{\sin\theta}{6}$ (B) $\frac{\cot\theta}{6}$ (C) $\frac{1}{6\sin\theta}$ (D) $\frac{1-\sin 2\theta}{6}$ (E) $\frac{\sqrt{\cos 2\theta}}{6\sin\theta}$

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22 If the six solutions of $x^6 = -64$ are written in the form $a + bi$, where a and b are real, then the product of those solutions with $a > 0$ is
(A) -2 (B) 0 (C) $2i$ (D) 4 (E) 16

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23 If $x, y > 0$, $\log_y x + \log_x y = \frac{10}{3}$ and $xy = 144$, then $\frac{x+y}{2} =$
(A) $12\sqrt{2}$ **(B)** $13\sqrt{3}$ **(C)** 24 **(D)** 30 **(E)** 36

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24 All students at Adams High School and at Baker High School take a certain exam. The average scores for boys, for girls, and for boys and girls combined, at Adams HS and Baker HS are

shown in the table, as is the average for boys at the two schools combined. What is the average score for the girls at the two schools combined?

	Adams	Baker	Adams and Baker
Boys:	71	81	79
Girls:	76	90	?
Boys and Girls:	74	84	

(A) 81 **(B)** 82 **(C)** 83 **(D)** 84 **(E)** 85

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25 Nine congruent spheres are packed inside a unit cube in such a way that one of them has its center at the center of the cube and each of the others is tangent to the center sphere and to three faces of the cube. What is the radius of each sphere?

(A) $1-\frac{\sqrt{3}}{2}$ **(B)** $\frac{2\sqrt{3}-3}{2}$ **(C)** $\frac{\sqrt{2}}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{\sqrt{3}(2-\sqrt{2})}{4}$

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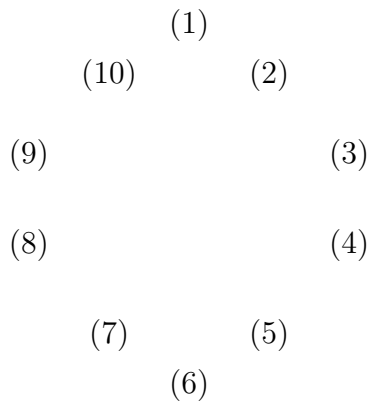
26 Ten people form a circle. Each picks a number and tells it to the two neighbors adjacent to him in the circle. Then each person computes and announces the average of the numbers of his two neighbors. The figure shows the average announced by each person (not the original number the person picked). The number picked by the person who announced the average 6 was



(A) 1 **(B)** 5 **(C)** 6 **(D)** 10 **(E)** not uniquely determined from the given information

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27 Which of these triples could not be the lengths of the three altitudes of a triangle?

(A) $1, \sqrt{3}, 2$ **(B)** $3, 4, 5$ **(C)** $5, 12, 13$ **(D)** $7, 8, \sqrt{113}$ **(E)** $8, 15, 17$

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28 A quadrilateral that has consecutive sides of lengths $70, 90, 130$ and 110 is inscribed in a circle and also has a circle inscribed in it. The point of tangency of the inscribed circle to the side of

length 130 divides that side into segments of lengths x and y. Find $|x-y|$.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

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29 A subset of the integers $1, 2, ..., 100$ has the property that none of its members is 3 times another. What is the largest number of members such a subset can have?

(A) 50 **(B)** 66 **(C)** 67 **(D)** 76 **(E)** 78

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30 If $R_n = \frac{1}{2}(a^n + b^n)$ where $a = 3 + 2\sqrt{2}$, $b = 3 - 2\sqrt{2}$, and $n = 0, 1, 2, ...$, then R_{12345} is an integer. Its units digit is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

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