AMC 12/AHSME 1959

www.artofproblemsolving.com/community/c4823

by Binomial-theorem, rrusczyk

---

1 Each edge of a cube is increased by 50%. The percent of increase of the surface area of the cube is: **(A)** 50 **(B)** 125 **(C)** 150 **(D)** 300 **(E)** 750

---

2 Through a point P inside the triangle ABC a line is drawn parallel to the base AB, dividing the triangle into two equal areas. If the altitude to AB has a length of 1, then the distance from P to AB is: **(A)** $\frac{1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{2}$ **(D)** $\frac{2-\sqrt{2}}{2}$ **(E)** $\frac{2+\sqrt{2}}{8}$

---

3 If the diagonals of a quadrilateral are perpendicular to each other, the figure would always be included under the general classification: **(A)** rhombus **(B)** rectangles **(C)** square **(D)** isosceles tr

---

4 If 78 is divided into three parts which are proportional to $1, \frac{1}{3}, \frac{1}{6}$, the middle part is: **(A)** $9\frac{1}{3}$ **(B)** 13 **(C)** 1

---

5 The value of $(256)^{.16}(256)^{.09}$ is: **(A)** 4 **(B)** 16 **(C)** 64 **(D)** 256.25 **(E)** -16

---

6 Given the true statement: If a quadrilateral is a square, then it is a rectangle. It follows that, of the converse and the inverse of this true statement is: **(A)** only the converse is true **(B)** only the inverse **(D)** neither is true **(E)** the inverse is true, but the converse is sometimes true

---

7 The sides of a right triangle are $a, a+d$, and $a+2d$, with a and d both positive. The ratio of a to d is: **(A)** $1:3$ **(B)** $1:4$ **(C)** $2:1$ **(D)** $3:1$ **(E)** $3:4$

---

8 The value of $x^2-6x+13$ can never be less than: **(A)** 4 **(B)** 4.5 **(C)** 5 **(D)** 7 **(E)** 13

---

9 A farmer divides his herd of n cows among his four sons so that one son gets one-half the herd, a second son, one-fourth, a third son, one-fifth, and the fourth son, 7 cows. Then n is: **(A)** 80 **(B)** 100 **(C)** 140 **(D)** 180 **(E)** 240

---

10 In triangle ABC with $\overline{AB}=\overline{AC}=3.6$, a point D is taken on AB at a distance 1.2 from A. Point D is joined to E in the prolongation of AC so that triangle AED is equal in area to ABC. Then $\overline{AE}$ is: **(A)** 4.8 **(B)** 5.4 **(C)** 7.2 **(D)** 10.8 **(E)** 12.6

---

11 The logarithm of $.0625$ to the base 2 is: **(A)** $.025$ **(B)** $.25$ **(C)** 5 **(D)** -4 **(E)** -2

---

12 By adding the same constant to $20, 50, 100$ a geometric progression results. The common ratio is: **(A)** $\frac{5}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{1}{3}$

13 The arithmetic mean (average) of a set of 50 numbers is 38. If two numbers, namely, 45 and 55, are discarded, the mean of the remaining set of numbers is: **(A)** 36.5 **(B)** 37 **(C)** 37.2 **(D)** 37.5 **(**

14 Given the set S whose elements are zero and the even integers, positive and negative. Of the five operations applied to any pair of elements: (1) addition (2) subtraction (3) multiplication (4) division (5) finding the arithmetic mean (average), those elements that only yield elements of S are: **(A)** all **(B)** $1,2,3,4$ **(C)** $1,2,3,5$ **(D)** $1,2,3$ **(E)** $1,3,5$

15 In a right triangle the square of the hypotenuse is equal to twice the product of the legs. One of the acute angles of the triangle is: **(A)** 15° **(B)** 30° **(C)** 45° **(D)** 60° **(E)** 75°

16 The expression $\frac{x^2-3x+2}{x^2-5x+6} \div \frac{x^2-5x+4}{x^2-7x+12}$, when simplified is: **(A)** $\frac{(x-1)(x-6)}{(x-3)(x-4)}$ **(B)** $\frac{x+3}{x-3}$ **(C)** $\frac{x+1}{x-1}$ **(D)** 1 **(E**

17 If $y = a + \frac{b}{x}$, where a and b are constants, and if $y = 1$ when $x = -1$, and $y = 5$ when $x = -5$, then $a + b$ equals: **(A)** -1 **(B)** 0 **(C)** 1 **(D)** 10 **(E)** 11

18 The arithmetic mean (average) of the first n positive integers is: **(A)** $\frac{n}{2}$ **(B)** $\frac{n^2}{2}$ **(C)** n **(D)** $\frac{n-1}{2}$ **(E**

19 With the use of three different weights, namely 1 lb., 3 lb., and 9 lb., how many objects of different weights can be weighed, if the objects is to be weighed and the given weights may be placed in either pan of the scale? **(A)** 15 **(B)** 13 **(C)** 11 **(D)** 9 **(E)** 7

20 It is given that x varies directly as y and inversely as the square of z, and that $x = 10$ when $y = 4$ and $z = 14$. Then, when $y = 16$ and $z = 7$, x equals: **(A)** 180 **(B)** 160 **(C)** 154 **(D)** 140 **(E)** 120

21 If p is the perimeter of an equilateral triangle inscribed in a circle, the area of the circle is: **(A)** $\frac{\pi p^2}{3}$ **(B)** $\frac{\pi p^2}{9}$ **(C)** $\frac{\pi p^2}{27}$ **(D)** $\frac{\pi p^2}{81}$ **(E)** $\frac{\pi p^2\sqrt{3}}{27}$

22 The line joining the midpoints of the diagonals of a trapezoid has length 3. If the longer base is 97, then the shorter base is: **(A)** 94 **(B)** 92 **(C)** 91 **(D)** 90 **(E)** 89

23 The set of solutions of the equation $\log_{10}\left(a^2 - 15a\right) = 2$ consists of **(A)** two integers **(B)** one integer ar **(C)** two irrational numbers **(D)** two non-real numbers **(E)** no numbers, that is, the empty set

24 A chemist has m ounces of salt that is $m\%$ salt. How many ounces of salt must he add to make a solution that is $2m\%$ salt?

(A) $\frac{m}{100+m}$ **(B)** $\frac{2m}{100-2m}$ **(C)** $\frac{m^2}{100-2m}$ **(D)** $\frac{m^2}{100+2m}$ **(E)** $\frac{2m}{100+2m}$

---

25 The symbol $|a|$ means $+a$ if a is greater than or equal to zero, and $-a$ if a is less than or equal to zero; the symbol $<$ means "less than"; the symbol $>$ means "greater than."
The set of values x satisfying the inequality $|3-x|<4$ consists of all x such that: **(A)** $x^2 < 49$ **(B)** $x^2 > 1$ **(C)** $1 < x^2 < 49$ **(D)** $-1 < x < 7$ **(E)** $-7 < x < 1$

---

26 The base of an isosceles triangle is $\sqrt{2}$. The medians to the leg intersect each other at right angles. The area of the triangle is: **(A)** 1.5 **(B)** 2 **(C)** 2.5 **(D)** 3.5 **(E)** 4

---

27 Which one of the following is *not* true for the equation

$$ix^2 - x + 2i = 0,$$

where $i = \sqrt{-1}$? **(A)** The sum of the roots is 2 **(B)** The discriminant is 9 **(C)** The roots are imagina
(D) The roots can be found using the quadratic formula **(E)** The roots can be found by factoring, usin

---

28 In triangle ABC, AL bisects angle A and CM bisects angle C. Points L and M are on BC and AB, respectively. The sides of triangle ABC are $a, b,$ and c. Then $\frac{\overline{AM}}{MB} = k\frac{\overline{CL}}{LB}$ where k is: **(A)** 1 **(B)** $\frac{bc}{a^2}$ **(C)** $\frac{a^2}{bc}$ **(D)** $\frac{c}{b}$ **(E)** $\frac{c}{a}$

---

29 On a examination of n questions a student answers correctly 15 of the first 20. Of the remaining questions he answers one third correctly. All the questions have the same credit. If the student's mark is 50%, how many different values of n can there be? **(A)** 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** th

---

30 A can run around a circular track in 40 seconds. B, running in the opposite direction, meets A every 15 seconds. What is B's time to run around the track, expressed in seconds? **(A)** $12\frac{1}{2}$ **(B)** 24 **(C**

---

31 A square, with an area of 40, is inscribed in a semicircle. The area of a square that could be inscribed in the entire circle with the same radius, is: **(A)** 80 **(B)** 100 **(C)** 120 **(D)** 160 **(E)** 200

---

32 The length l of a tangent, drawn from a point A to a circle, is $\frac{4}{3}$ of the radius r. The (shortest) distance from A to the circle is: **(A)** $\frac{1}{2}r$ **(B)** r **(C)** $\frac{1}{2}l$ **(D)** $\frac{2}{3}l$ **(E)** a value between r and l.

---

33 A harmonic progression is a sequence of numbers such that their reciprocals are in arithmetic progression.
Let S_n represent the sum of the first n terms of the harmonic progression; for example S_3 represents the sum of the first three terms. If the first three terms of a harmonic progression are $3, 4, 6$, then: **(A)** $S_4 = 20$ **(B)** $S_4 = 25$ **(C)** $S_5 = 49$ **(D)** $S_6 = 49$ **(E)** $S_2 = \frac{1}{2}S_4$

34 Let the roots of $x^2 - 3x + 1 = 0$ be r and s. Then the expression $r^2 + s^2$ is:

(A) a positive integer **(B)** a positive fraction greater than 1 **(C)** a positive fraction less than 1 **(D)** an irrational number **(E)** an imaginary number

35 The symbol $\geq$ means "greater than or equal to"; the symbol $\leq$ means "less than or equal to". In the equation $(x-m)^2 - (x-n)^2 = (m-n)^2$; m is a fixed positive number, and n is a fixed negative number. The set of values x satisfying the equation is: **(A)** $x \geq 0$ **(B)** $x \leq n$ **(C)** $x = 0$ **(D)** the set of all real numbers **(E)** none of these

36 The base of a triangle is 80, and one side of the base angle is 60°. The sum of the lengths of the other two sides is 90. The shortest side is: **(A)** 45 **(B)** 40 **(C)** 36 **(D)** 17 **(E)** 12

37 When simplified the product $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{n}\right)$ becomes: **(A)** $\frac{1}{n}$ **(B)** $\frac{2}{n}$ **(C)** $\frac{2(n-1)}{n}$

38 If $4x + \sqrt{2x} = 1$, then x: **(A)** is an integer **(B)** is fractional **(C)** is irrational **(D)** is imaginary **(E)**

39 Let S be the sum of the first nine terms of the sequence

$$x + a, x^2 + 2a, x^3 + 3a, \cdots.$$

Then S equals: **(A)** $\frac{50a+x+x^8}{x+1}$ **(B)** $50a - \frac{x+x^{10}}{x-1}$ **(C)** $\frac{x^9-1}{x+1} + 45a$ **(D)** $\frac{x^{10}-x}{x-1} + 45a$ **(E)** $\frac{x^{11}-x}{x-1} + 45a$

40 In triangle ABC, BD is a median. CF intersects BD at E so that $\overline{BE} = \overline{ED}$. Point F is on AB. Then, if $\overline{BF} = 5$, $\overline{BA}$ equals: **(A)** 10 **(B)** 12 **(C)** 15 **(D)** 20 **(E)** none of these

41 On the same side of a straight line three circles are drawn as follows: a circle with a radius of 4 inches is tangent to the line, the other two circles are equal, and each is tangent to the line and to the other two circles. The radius of the equal circles is: **(A)** 24 **(B)** 20 **(C)** 18 **(D)** 16 **(E)** 12

42 Given three positive integers $a, b,$ and c. Their greatest common divisor is D; their least common multiple is m. Then, which two of the following statements are true? (1) the product MD cannot be les
(2) the product MD cannot be greater than abc (3) MD equals abc if and only if a,b,c are each prime
(4) MD equals abc if and only if a,b,c are each relatively prime in pairs (This means: no two have a comm
(A) $1, 2$ **(B)** $1, 3$ **(C)** $1, 4$ **(D)** $2, 3$ **(E)** $2, 4$

43 The sides of a triangle are $25, 39,$ and 40. The diameter of the circumscribed circle is: **(A)** $\frac{133}{3}$ **(B)** $\frac{125}{3}$

44 The roots of $x^2 + bx + c = 0$ are both real and greater than 1. Let $s = b + c + 1$. Then s :
(A) may be less than zero **(B)** may be equal to zero **(C)** must be greater than zero **(D)** must be
(E) must be between -1 and +1

---

45 If $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$, then y equals: **(A)** $\frac{9}{2}$ **(B)** 9 **(C)** 18 **(D)** 27 **(E)** 81

---

46 A student on vacation for d days observed that (1) it rained 7 times, morning or afternoon (2) when it rained in the afternoon, it was clear in the morning (3) there were five clear afternoons (4) there were six clear mornings. Then d equals: **(A)** 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

---

47 Assume that the following three statements are true: I. All freshmen are human. II. All students are human. III. Some students think.
Given the following four statements: **(1)** All freshmen are students. **(2)** Some humans think.
(3) No freshmen think. **(4)** Some humans who think are not students.
Those which are logical consequences of I,II, and III are: **(A)** 2 **(B)** 4 **(C)** $2, 3$ **(D)** $2, 4$ **(E)** $1, 2$

---

48 Given the polynomial $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$, where n is a positive integer or zero, and a_0 is a positive integer. The remaining a's are integers or zero. Set $h = n + a_0 + |a_1| + |a_2| + \cdots + |a_n|$. [See example 25 for the meaning of $|x|$.] The number of polynomials with $h = 3$ is:
(A) 3 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 9

---

49 For the infinite series $1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} - \cdots$ let S be the (limiting) sum. Then S equals: **(A)** 0 **(B)** $\frac{2}{7}$ **(C)** $\frac{6}{7}$ **(D)** $\frac{9}{32}$ **(E)** $\frac{27}{32}$

---

50 A club with x members is organized into four committees in accordance with these two rules:
(1) Each member belongs to two and only two committees
(2) Each pair of committees has one and only one member in common
Then x: **(A)** cannont be determined **(B)** has a single value between 8 and 16 **(C)** has two values b
(D) has a single value between 4 and 8 **(E)** has two values between 4 and 8

–