AMC 10 2010

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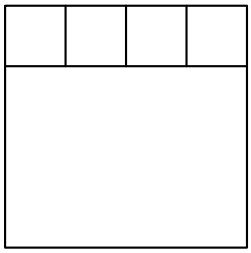
– A

– February 9th

1 Mary's top book shelf holds five books with the following widths, in centimeters: 6, $\frac{1}{2}$, 1, 2.5, and 10. What is the average book width, in centimeters?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2 Four identical squares and one rectangle are placed together to form one large square as shown. The length of the rectangle is how many times as large as its width?



(A) $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** 3

3 Tyrone had 97 marbles and Eric had 11 marbles. Tyrone then gave some of his marbles to Eric so that Tyrone ended with twice as many marbles as Eric. How many marbles did Tyrone give to Eric?

(A) 3 **(B)** 13 **(C)** 18 **(D)** 25 **(E)** 29

4 A book that is to be recorded onto compact discs takes 412 minutes to read aloud. Each disc can hold up to 56 minutes of reading. Assume that the smallest possible number of discs is used and that each disc contains the same length of reading. How many minutes of reading will each disc contain?

(A) 50.2 **(B)** 51.5 **(C)** 52.4 **(D)** 53.8 **(E)** 55.2

5 The area of a circle whose circumference is 24π is $k\pi$. What is the value of k?

(A) 6 **(B)** 12 **(C)** 24 **(D)** 36 **(E)** 144

6 For positive numbers x and y the operation $\spadesuit(x,y)$ is defined as

$$\spadesuit(x,y) = x - \frac{1}{y}$$

What is $\spadesuit(2,\spadesuit(2,2))$?
(A) $\frac{2}{3}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** $\frac{5}{3}$ **(E)** 2

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7 Crystal has a running course marked out for her daily run. She starts this run by heading due north for one mile. She then runs northeast for one mile, then southeast for one mile. The last portion of her run takes her on a straight line back to where she started. How far, in miles is this last portion of her run?
(A) 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $2\sqrt{2}$

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8 Tony works 2 hours a day and is paid $\$0.50$ per hour for each full year of his age. During a six month period Tony worked 50 days and earned $\$630$. How old was Tony at the end of the six month period?
(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

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9 A *palindrome*, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x+32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?
(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

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10 Marvin had a birthday on Tuesday, May 27 in the leap year 2008. In what year will his birthday next fall on a Saturday?
(A) 2011 **(B)** 2012 **(C)** 2013 **(D)** 2015 **(E)** 2017

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11 The length of the interval of solutions of the inequality $a \leq 2x+3 \leq b$ is 10. What is $b-a$?
(A) 6 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 30

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12 Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds $100,000$ liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

(A) 0.04 **(B)** $\frac{0.4}{\pi}$ **(C)** 0.4 **(D)** $\frac{4}{\pi}$ **(E)** 4

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13 Angelina drove at an average rate of 80 kph and then stopped 20 minutes for gas. After the stop, she drove at an average rate of 100 kph. Altogether she drove 250 km in a total trip time of 3 hours including the stop. Which equation could be used to solve for the time t in hours that she drove before her stop?
(A) $80t + 100(8/3 - t) = 250$ **(B)** $80t = 250$ **(C)** $100t = 250$
(D) $90t = 250$ **(E)** $80(8/3 - t) + 100t = 250$

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14 Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?
(A) 60° **(B)** 75° **(C)** 90° **(D)** 105° **(E)** 120°

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15 In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in the swamp, and they make the following statements:

Brian: "Mike and I are different species."
Chris: "LeRoy is a frog."
LeRoy: "Chris is a frog."
Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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16 Nondegenerate $\triangle ABC$ has integer side lengths, BD is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?
(A) 30 **(B)** 33 **(C)** 35 **(D)** 36 **(E)** 37

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17 A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?
(A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

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18 Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?
(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

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19 Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

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20 A fly trapped inside a cubical box with side length 1 meter decides to relieve its boredom by visiting each corner of the box. It will begin and end in the same corner and visit each of the other corners exactly once. To get from a corner to any other corner, it will either fly or crawl in a straight line. What is the maximum possible length, in meters, of its path?
(A) $4+4\sqrt{2}$ **(B)** $2+4\sqrt{2}+2\sqrt{3}$ **(C)** $2+3\sqrt{2}+3\sqrt{3}$ **(D)** $4\sqrt{2}+4\sqrt{3}$

(E) $3\sqrt{2}+5\sqrt{3}$

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21 The polynomial $x^3-ax^2+bx-2010$ has three positive integer zeros. What is the smallest possible value of a?
(A) 78 **(B)** 88 **(C)** 98 **(D)** 108 **(E)** 118

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22 Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?
(A) 28 **(B)** 56 **(C)** 70 **(D)** 84 **(E)** 140

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23 Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the kth position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?
(A) 45 **(B)** 63 **(C)** 64 **(D)** 201 **(E)** 1005

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24 The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?
(A) 12 **(B)** 32 **(C)** 48 **(D)** 52 **(E)** 68

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25 Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$\begin{aligned} &55\\ 55-7^2&=6\\ 6-2^2&=2\\ 2-1^2&=1\\ 1-1^2&=0 \end{aligned}$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N?
(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

– B

– February 24th

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1 What is $100(100-3)-(100\cdot 100-3)$?
(A) $-20,000$ **(B)** $-10,000$ **(C)** -297 **(D)** -6 **(E)** 0

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2 Makayla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?
(A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

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3 A drawer contains red, green, blue, and white socks with at least 2 of each color. What is the minimum number of socks that must be pulled from the drawer to guarantee a matching pair?
(A) 3 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

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4 For a real number x, define $\heartsuit(x)$ to be the average of x and x^2. What is $\heartsuit(1) + \heartsuit(2) + \heartsuit(3)$?
(A) 3 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 20

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5 A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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6 A circle is centered at O, $\overline{AB}$ is a diameter and C is a point on the circle with $\angle COB = 50^\circ$. What is the degree measure of $\angle CAB$?
(A) 20 **(B)** 25 **(C)** 45 **(D)** 50 **(E)** 65

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7 A triangle has side lengths 10, 10, and 12. A rectangle has width 4 and area equal to the area of the triangle. What is the perimeter of this rectangle?
(A) 16 **(B)** 24 **(C)** 28 **(D)** 32 **(E)** 36

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8 A ticket to a school play costs x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of $\$48$, and a group of 10th graders buys tickets costing a total of $\$64$. How many values of x are possible?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

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9 Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a - (b - (c - (d + e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincedence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?
(A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5

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10 Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?
(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

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11 A shopper plans to purchase an item that has a listed price greater than $\$100$ and can use any one of the three coupons. Coupon A gives 15% off the listed price, Coupon B gives $\$30$ the listed price, and Coupon C gives 25% off the amount by which the listed price exceeds $\$100$.

Let x and y be the smallest and largest prices, respectively, for which Coupon A saves at least as many dollars as Coupon B or C. What is $y - x$?

(A) 50 **(B)** 60 **(C)** 75 **(D)** 80 **(E)** 100

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12 At the beginning of the school year, 50% of all students in Mr. Well's math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

(A) 0 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

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13 What is the sum of all the solutions of $x = |2x - |60 - 2x||$?

(A) 32 **(B)** 60 **(C)** 92 **(D)** 120 **(E)** 124

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14 The average of the numbers $1, 2, 3, ..., 98, 99$, and x is $100x$. What is x?

(A) $\frac{49}{101}$ **(B)** $\frac{50}{101}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{51}{101}$ **(E)** $\frac{50}{99}$

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15 On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

(A) 25 **(B)** 27 **(C)** 29 **(D)** 31 **(E)** 33

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16 A square of side length 1 and a circle of radius $\sqrt{3}/3$ share the same center. What is the area inside the circle, but outside the square?

(A) $\frac{\pi}{3} - 1$ **(B)** $\frac{2\pi}{9} - \frac{\sqrt{3}}{3}$ **(C)** $\frac{\pi}{18}$ **(D)** $\frac{1}{4}$ **(E)** $2\pi/9$

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17 Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

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18 Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\frac{1}{3}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{31}{81}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{13}{27}$

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19 A circle with center O has area 156π. Triangle ABC is equilateral, $\overline{BC}$ is a chord on the circle, $OA = 4\sqrt{3}$, and point O is outside $\triangle ABC$. What is the side length of $\triangle ABC$?
(A) $2\sqrt{3}$ **(B)** 6 **(C)** $4\sqrt{3}$ **(D)** 12 **(E)** 18

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20 Two circles lie outside regular hexagon $ABCDEF$. The first is tangent to $\overline{AB}$, and the second is tangent to $\overline{DE}$. Both are tangent to lines BC and FA. What is the ratio of the area of the second circle to that of the first circle?
(A) 18 **(B)** 27 **(C)** 36 **(D)** 81 **(E)** 108

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21 A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?
(A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

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22 Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?
(A) 1930 **(B)** 1931 **(C)** 1932 **(D)** 1933 **(E)** 1934

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23 The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?
(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

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24 A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?
(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

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25 Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a, \text{ and}$$

$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?
(A) 105 **(B)** 315 **(C)** 945 **(D)** $7!$ **(E)** $8!$

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