AMC 12/AHSME 1964

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by Mrdavid445, TheMaskedMagician, JSGandora, rrusczyk

1 What is the value of $[\log_{10}(5\log_{10}100)]^2$?

(A) $\log_{10}50$ **(B)** 25 **(C)** 10 **(D)** 2 **(E)** 1

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2 The graph of $x^2-4y^2=0$ is:

(A) a parabola **(B)** an ellipse **(C)** a pair of straight lines **(D)** a point **(E)** none of these

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3 When a positive integer x is divided by a positive integer y, the quotient is u and the remainder is v, where u and v are integers. What is the remainder when $x+2uy$ is divided by y?

(A) 0 **(B)** $2u$ **(C)** $3u$ **(D)** v **(E)** $2v$

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4 The expression

$$\frac{P+Q}{P-Q}-\frac{P-Q}{P+Q}$$

where $P=x+y$ and $Q=x-y$, is equivalent to:

(A) $\frac{x^2-y^2}{xy}$ **(B)** $\frac{x^2-y^2}{2xy}$ **(C)** 1 **(D)** $\frac{x^2+y^2}{xy}$ **(E)** $\frac{x^2+y^2}{2xy}$

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5 If y varies directly as x, and if $y=8$ when $x=4$, the value of y when $x=-8$ is:

(A) -16 **(B)** -4 **(C)** -2 **(D)** $4k, k=\pm1,\pm2,\dots$ **(E)** $16k, k=\pm1,\pm2,\dots$

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6 If $x, 2x+2, 3x+3, \dots$ are in geometric progression, the fourth term is:

(A) -27 **(B)** $-13\frac{1}{2}$ **(C)** 12 **(D)** $13\frac{1}{2}$ **(E)** 27

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7 Let n be the number of real values of p for which the roots of

$$x^2-px+p=0$$

are equal. Then n equals:

(A) 0 **(B)** 1 **(C)** 2 **(D)** a finite number greater than 2 **(E)** an infinitely large number

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8 The smaller root of the equation $\left(x-\frac{3}{4}\right)\left(x-\frac{3}{4}\right)+\left(x-\frac{3}{4}\right)\left(x-\frac{1}{2}\right)=0$ is:

(A) $-\frac{3}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{3}{4}$ **(E)** 1

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9 A jobber buys an article at $\$24$ less $12\frac{1}{2}\%$. He then wishes to sell the article at a gain of $33\frac{1}{3}\%$ of his cost after allowing a 20% discount on his marked price. At what price, in dollars, should the article be marked?

(A) 25.20 **(B)** 30.00 **(C)** 33.60 **(D)** 40.00 **(E)** none of these

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10 Given a square side of length s. On a diagonal as base a triangle with three unequal sides is constructed so that its area equals that of the square. The length of the altitude drawn to the base is:

(A) $s\sqrt{2}$ **(B)** $s/\sqrt{2}$ **(C)** $2s$ **(D)** $2\sqrt{s}$ **(E)** $2/\sqrt{s}$

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11 Given $2^x=8^{y+1}$ and $9^y=3^{x-9}$, find the value of $x+y$.

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

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12 Which of the following is the negation of the statement: For all x of a certain set, $x^2>0$?

(A) For all x, $x^2<0$ **(B)** For all x, $x^2\le 0$ **(C)** For no x, $x^2>0$ **(D)** For some x, $x^2>0$
(E) For some x, $x^2\le 0$

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13 A circle is inscribed in a triangle with side lengths 8, 13, and 17. Let the segments of the side of length 8, made by a point of tangency, be r and s, with $r<s$. What is the ratio $r:s$?

(A) $1:3$ **(B)** $2:5$ **(C)** $1:2$ **(D)** $2:3$ **(E)** $3:4$

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14 A farmer bought 749 sheeps. He sold 700 of them for the price paid for the 749 sheep. The remaining 49 sheep were sold at the same price per head as the other 700. Based on the cost, the percent gain on the entire transaction is:

(A) 6.5 **(B)** 6.75 **(C)** 7 **(D)** 7.5 **(E)** 8

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15 A line through the point $(-a,0)$ cuts from the second quadrant a triangular region with area T. The equation of the line is:

(A) $2Tx+a^2y+2aT=0$ **(B)** $2Tx-a^2y+2aT=0$ **(C)** $2Tx+a^2y-2aT=0$ **(D)** $2Tx-a^2y-2aT$ =

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16 Let $f(x)=x^2+3x+2$ and let S be the set of integers $\{0,1,2,\dots,25\}$. The number of members s of S such that $f(s)$ has remainder zero when divided by 6 is:

(A) 25 **(B)** 22 **(C)** 21 **(D)** 18 **(E)** 17

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17 Given the distinct points $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_1 + x_2, y_1 + y_2)$. Line segments are drawn connecting these points to each other and to the origin 0. Of the three possibilities: (1) parallelogram (2) straight line (3) trapezoid, figure $OPRQ$, depending upon the location of the points P, Q, and R, can be:

(A) (1) only **(B)** (2) only **(C)** (3) only **(D)** (1) or (2) only **(E)** all three

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18 Let n be the number of pairs of values of b and c such that $3x + by + c = 0$ and $cx - 2y + 12 = 0$ have the same graph. Then n is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** finite but more than 2 **(E)** greater than any finite number

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19 If $2x - 3y - z = 0$ and $x + 3y - 14z = 0$, $z \neq 0$, the numerical value of $\frac{x^2+3xy}{y^2+z^2}$ is:

(A) 7 **(B)** 2 **(C)** 0 **(D)** $-20/17$ **(E)** -2

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20 The sum of the numerical coefficients of all the terms in the expansion of $(x - 2y)^{18}$ is:

(A) 0 **(B)** 1 **(C)** 19 **(D)** -1 **(E)** -19

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21 If $\log_{b^2} x + \log_{x^2} b = 1, b > 0, b \neq 1, x \neq 1$, then x equals:

(A) $1/b^2$ **(B)** $1/b$ **(C)** b^2 **(D)** b **(E)** $\sqrt{b}$

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22 Given parallelogram $ABCD$ with E the midpoint of diagonal BD. Point E is connected to a point F in DA so that $DF = \frac{1}{3}DA$. What is the ratio of the area of triangle DFE to the area of quadrilateral $ABEF$?

(A) $1:2$ **(B)** $1:3$ **(C)** $1:5$ **(D)** $1:6$ **(E)** $1:7$

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23 Two numbers are such that their difference, their sum, and their product are to one another as $1:7:24$. The product of the two numbers is:

(A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 96

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24 Let $y = (x - a)^2 + (x - b)^2, a, b$ constants. For what value of x is y a minimum?

(A) $\frac{a+b}{2}$ **(B)** $a + b$ **(C)** $\sqrt{ab}$ **(D)** $\sqrt{\frac{a^2+b^2}{2}}$ **(E)** $\frac{a+b}{2ab}$

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25 The set of values of m for which $x^2+3xy+x+my-m$ has two factors, with integer coefficients, which are linear in x and y, is precisely:

(A) $0, 12, -12$ **(B)** $0, 12$ **(C)** $12, -12$ **(D)** 12 **(E)** 0

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26 In a ten-mile race First beats Second by 2 miles and First beats Third by 4 miles. If the runners maintain constant speeds throughout the race, by how many miles does Second beat Third?

(A) 2 **(B)** $2\frac{1}{4}$ **(C)** $2\frac{1}{2}$ **(D)** $2\frac{3}{4}$ **(E)** 3

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27 If x is a real number and $|x-4|+|x-3|<a$ where $a>0$, then:

(A) $0<a<.01$ **(B)** $.01<a<1$ **(C)** $0<a<1$

(D) $0<a\le 1$ **(E)** $a>1$

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28 The sum of n terms of an arithmetic progression is 153, and the common difference is 2. If the first interm is an integer, and $n>1$, then the number of possible values for n is:

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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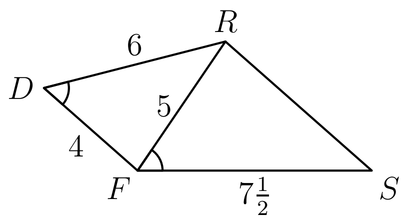
29 In this figure $\angle RFS=\angle FDR$, $FD=4$ inches, $DR=6$ inches, $FR=5$ inches, $FS=7\dfrac{1}{2}$ inches. The length of RS, in inches, is:



(A) undetermined **(B)** 4 **(C)** $5\dfrac{1}{2}$ **(D)** 6 **(E)** $6\dfrac{1}{4}$

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30 If $(7+4\sqrt{3})x^2+(2+\sqrt{3})x-2=0$, the larger root minus the smaller root is:

(A) $-2+3\sqrt{3}$ **(B)** $2-\sqrt{3}$ **(C)** $6+3\sqrt{3}$ **(D)** $6-3\sqrt{3}$ **(E)** $3\sqrt{3}+2$

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31 Let

$$f(n)=\frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n+\frac{5-3\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

Then $f(n+1)-f(n-1)$, expressed in terms of $f(n)$, equals:

(A) $\dfrac{1}{2}f(n)$ **(B)** $f(n)$ **(C)** $2f(n)+1$ **(D)** $f^2(n)$ **(E)** $\dfrac{1}{2}(f^2(n)-1)$

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32 If $\dfrac{a+b}{b+c}=\dfrac{c+d}{d+a}$, then:

(A) a must equal c **(B)** $a+b+c+d$ must equal zero

(C) either $a=c$ or $a+b+c+d=0$, or both

(D) $a+b+c+d \neq 0$ if $a=c$ **(E)** $a(b+c+d)=c(a+b+d)$

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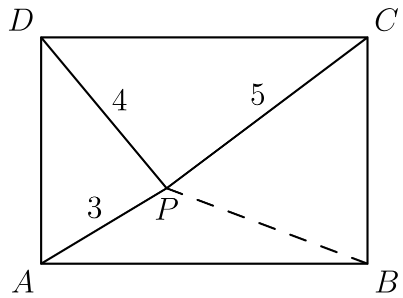
33 P is a point interior to rectangle $ABCD$ and such that $PA=3$ inches, $PD=4$ inches, and $PC=5$ inches. Then PB, in inches, equals:

(A) $2\sqrt{3}$ **(B)** $3\sqrt{2}$ **(C)** $3\sqrt{3}$ **(D)** $4\sqrt{2}$ **(E)** 2



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34 If n is a multiple of 4, the sum $s=1+2i+3i^2+...+(n+1)i^n$, where $i=\sqrt{-1}$, equals:

(A) $1+i$ **(B)** $\frac{1}{2}(n+2)$ **(C)** $\frac{1}{2}(n+2-ni)$

(D) $\frac{1}{2}[(n+1)(1-i)+2]$ **(E)** $\frac{1}{8}(n^2+8-4ni)$

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35 The sides of a triangle are of lengths 13, 14, and 15. The altitudes of the triangle meet at point H. If AD is the altitude to the side length 14, what is the ratio $HD:HA$?

(A) $3:11$ **(B)** $5:11$ **(C)** $1:2$ **(D)** $2:3$ **(E)** $25:33$

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36 In this figure the radius of the circle is equal to the altitude of the equilateral triangle ABC. The circle is made to roll along the side AB, remaining tangent to it at a variable point T and intersecting lines AC and BC in variable points M and N, respectively. Let n be the number of degrees in arc MTN. Then n, for all permissible positions of the circle:

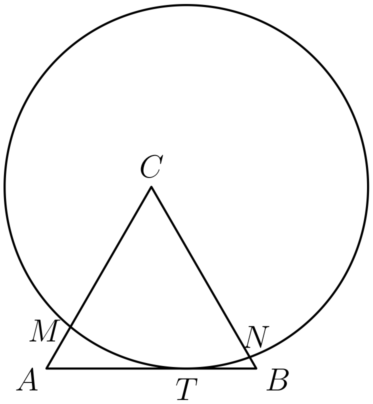
(A) varies from 30° to 90°

(B) varies from 30° to 60°

(C) varies from 60° to 90°

(D) remains constant at 30°

(E) remains constant at 60°



37 Given two positive number a, b such that $a < b$. Let A.M. be their arithmetic mean and let G.M. be their positive geometric mean. Then A.M. minus G.M. is always less than:

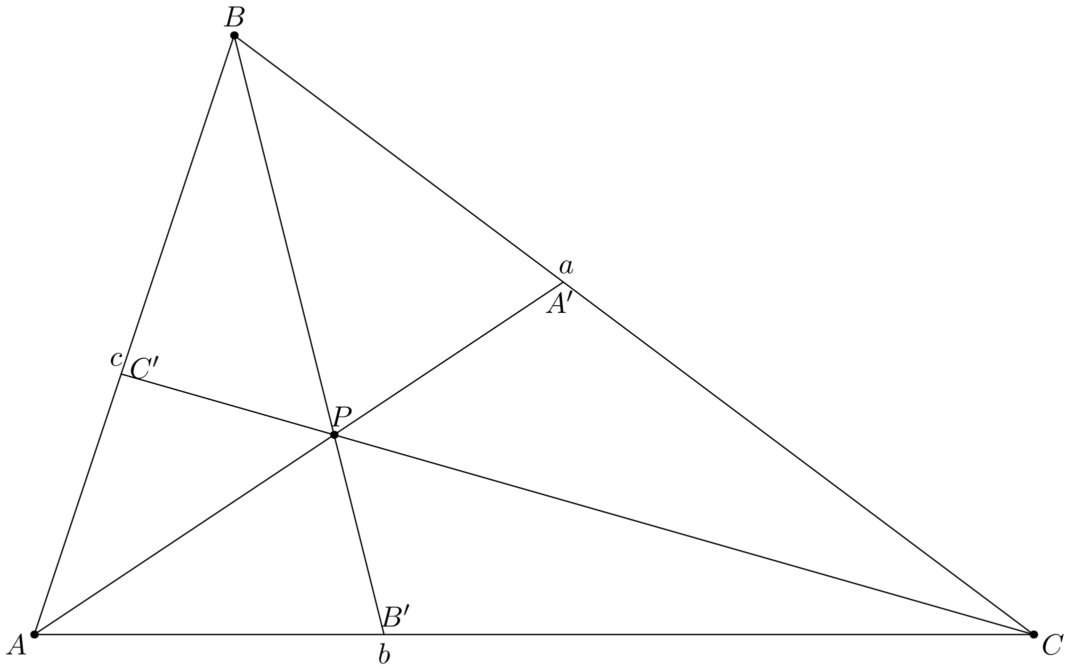
(A) $\frac{(b+a)^2}{ab}$ **(B)** $\frac{(b+a)^2}{8b}$ **(C)** $\frac{(b-a)^2}{ab}$
(D) $\frac{(b-a)^2}{8a}$ **(E)** $\frac{(b-a)^2}{8b}$

38 The sides PQ and PR of triangle PQR are respectively of lengths 4 inches, and 7 inches. The median PM is $3\frac{1}{2}$ inches. Then QR, in inches, is:

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

39 The magnitudes of the sides of triangle ABC are a, b, and c, as shown, with $c \le b \le a$. Through interior point P and the vertices A, B, C, lines are drawn meeting the opposite sides in A', B', C', respectively. Let $s = AA' + BB' + CC'$. Then, for all positions of point P, s is less than:

(A) $2a+b$ **(B)** $2a+c$ **(C)** $2b+c$ **(D)** $a+2b$ **(E)** $a+b+c$



40 A watch loses $2\frac{1}{2}$ minutes per day. It is set right at 1 P.M. on March 15. Let n be the positive correction, in minutes, to be added to the time shown by the watch at a given time. When the watch shows 9 A.M. on March 21, n equals:

(A) $14\frac{14}{23}$ **(B)** $14\frac{1}{14}$ **(C)** $13\frac{101}{115}$ **(D)** $13\frac{83}{115}$ **(E)** $13\frac{13}{23}$