**AIME Problems 1998**

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by joml88, towersfreak2006, Hokkage, minsoens, 1234567890, calculuslover800, white_horse_king88, 4everwise, rrusczyk

**1** For how many values of $k$ is $12^{12}$ the least common multiple of the positive integers $6^6, 8^8$, and $k$?

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**2** Find the number of ordered pairs $(x, y)$ of positive integers that satisfy $x \leq 2y \leq 60$ and $y \leq 2x \leq 60$.

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**3** The graph of $y^2 + 2xy + 40|x| = 400$ partitions the plane into several regions. What is the area of the bounded region?

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**4** Nine tiles are numbered $1, 2, 3, \ldots, 9$, respectively. Each of three players randomly selects and keeps three of the tile, and sums those three values. The probability that all three players obtain an odd sum is $m/n$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

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**5** Given that $A_k = \frac{k(k-1)}{2} \cos \frac{k(k-1)\pi}{2}$, find $|A_{19} + A_{20} + \cdots + A_{98}|$.

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**6** Let $ABCD$ be a parallelogram. Extend $\overline{DA}$ through $A$ to a point $P$, and let $\overline{PC}$ meet $\overline{AB}$ at $Q$ and $\overline{DB}$ at $R$. Given that $PQ = 735$ and $QR = 112$, find $RC$.

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**7** Let $n$ be the number of ordered quadruples $(x_1, x_2, x_3, x_4)$ of positive odd integers that satisfy $\sum_{i=1}^4 x_i = 98$. Find $\frac{n}{100}$.

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**8** Except for the first two terms, each term of the sequence $1000, x, 1000 - x, \ldots$ is obtained by subtracting the preceding term from the one before that. The last term of the sequence is the first negative term encounted. What positive integer $x$ produces a sequence of maximum length?

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**9** Two mathematicians take a morning coffee break each day. They arrive at the cafeteria independently, at random times between 9 a.m. and 10 a.m., and stay for exactly $m$ mintues. The probability that either one arrives while the other is in the cafeteria is $40\%$, and $m = a - b\sqrt{c}$, where $a, b$, and $c$ are positive integers, and $c$ is not divisible by the square of any prime. Find $a + b + c$.

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**10** Eight spheres of radius 100 are placed on a flat surface so that each sphere is tangent to two others and their centers are the vertices of a regular octagon. A ninth sphere is placed on the flat surface so that it is tangent to each of the other eight spheres. The radius of this last sphere is $a+b\sqrt{c}$, where $a, b$, and $c$ are positive integers, and $c$ is not divisible by the square of any prime. Find $a+b+c$.

**11** Three of the edges of a cube are $\overline{AB}, \overline{BC}$, and $\overline{CD}$, and $\overline{AD}$ is an interior diagonal. Points $P, Q$, and $R$ are on $\overline{AB}, \overline{BC}$, and $\overline{CD}$, respectively, so that $AP=5, PB=15, BQ=15$, and $CR=10$. What is the area of the polygon that is the intersection of plane $PQR$ and the cube?

**12** Let $ABC$ be equilateral, and $D, E$, and $F$ be the midpoints of $\overline{BC}, \overline{CA}$, and $\overline{AB}$, respectively. There exist points $P, Q$, and $R$ on $\overline{DE}, \overline{EF}$, and $\overline{FD}$, respectively, with the property that $P$ is on $\overline{CQ}$, $Q$ is on $\overline{AR}$, and $R$ is on $\overline{BP}$. The ratio of the area of triangle $ABC$ to the area of triangle $PQR$ is $a+b\sqrt{c}$, where $a, b$ and $c$ are integers, and $c$ is not divisible by the square of any prime. What is $a^2+b^2+c^2$?

**13** If $\{a_1, a_2, a_3, \ldots, a_n\}$ is a set of real numbers, indexed so that $a_1 < a_2 < a_3 < \cdots < a_n$, its complex power sum is defined to be $a_1 i + a_2 i^2 + a_3 i^3 + \cdots + a_n i^n$, where $i^2 = -1$. Let $S_n$ be the sum of the complex power sums of all nonempty subsets of $\{1, 2, \ldots, n\}$. Given that $S_8 = -176 - 64i$ and $S_9 = p + qi$, were $p$ and $q$ are integers, find $|p| + |q|$.

**14** An $m \times n \times p$ rectangular box has half the volume of an $(m+2) \times (n+2) \times (p+2)$ rectangular box, where $m, n$, and $p$ are integers, and $m \le n \le p$. What is the largest possible value of $p$?

**15** Define a domino to be an ordered pair of distinct positive integers. A proper sequence of dominos is a list of distinct dominos in which the first coordinate of each pair after the first equals the second coordinate of the immediately preceding pair, and in which $(i, j)$ and $(j, i)$ do not both appear for any $i$ and $j$. Let $D_{40}$ be the set of all dominos whose coordinates are no larger than 40. Find the length of the longest proper sequence of dominos that can be formed using the dominos of $D_{40}$.