**AMC 12/AHSME 1954**

www.artofproblemsolving.com/community/c4818

by AIME15, rrusczyk

**1** The square of $5-\sqrt{y^2-25}$ is:
**(A)** $y^2-5\sqrt{y^2-25}$ **(B)** $-y^2$ **(C)** $y^2$
**(D)** $(5-y)^2$ **(E)** $y^2-10\sqrt{y^2-25}$

---

**2** The equation $\frac{2x^2}{x-1}-\frac{2x+7}{3}+\frac{4-6x}{x-1}+1=0$ can be transformed by eliminating fractions to the equation $x^2-5x+4=0$. The roots of the latter equation are $4$ and $1$. Then the roots of the first equation are:
**(A)** $4$ and $1$ **(B)** only $1$ **(C)** only $4$ **(D)** neither 4 nor 1 **(E)** 4 and some other root

---

**3** If $x$ varies as the cube of $y$, and $y$ varies as the fifth root of $z$, then $x$ varies as the $n$th power of $z$, where $n$ is:
**(A)** $\frac{1}{15}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{3}{5}$ **(D)** $15$ **(E)** $8$

---

**4** If the Highest Common Divisor of $6432$ and $132$ is diminished by $8$, it will equal:
**(A)** $-6$ **(B)** $6$ **(C)** $-2$ **(D)** $3$ **(E)** $4$

---

**5** A regular hexagon is inscribed in a circle of radius $10$ inches. Its area is:
**(A)** $150\sqrt{3}$ sq. in. **(B)** 150 sq. in. **(C)** $25\sqrt{3}$ sq. in. **(D)** 600 sq. in. **(E)** $300\sqrt{3}$ sq. in.

---

**6** The value of $\frac{1}{16}a^0+\left(\frac{1}{16a}\right)^0-\left(64^{-\frac{1}{2}}\right)-(-32)^{-\frac{4}{5}}$ is:
**(A)** $1\frac{13}{16}$ **(B)** $1\frac{3}{16}$ **(C)** $1$ **(D)** $\frac{7}{8}$ **(E)** $\frac{1}{16}$

---

**7** A housewife saved $\$2.50$ in buying a dress on sale. If she spent $\$25$ for the dress, she saved about:
**(A)** $8\%$ **(B)** $9\%$ **(C)** $10\%$ **(D)** $11\%$ **(E)** $12\%$

---

**8** The base of a triangle is twice as long as a side of a square and their areas are the same. Then the ratio of the altitude of the triangle to the side of the square is:
**(A)** $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $1$ **(D)** $2$ **(E)** $4$

---

**9** A point $P$ is outside a circle and is $13$ inches from the center. A secant from $P$ cuts the circle at $Q$ and $R$ so that the external segment of the secant $PQ$ is $9$ inches and $QR$ is $7$ inches. The radius of the circle is:
**(A)** $3$" **(B)** $4$" **(C)** $5$" **(D)** $6$" **(E)** $7$"

---

**10** The sum of the numerical coefficients in the expansion of the binomial $(a+b)^8$ is:
**(A)** $32$ **(B)** $16$ **(C)** $64$ **(D)** $48$ **(E)** $7$

---

**11** A merchant placed on display some dresses, each with a marked price. He then posted a sign $\frac{1}{3}$ off on these dresses. The cost of the dresses was $\frac{3}{4}$ of the price at which he actually sold them. Then the ratio of the cost to the marked price was:
**(A)** $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

---

**12** The solution of the equations

$$2x-3y=7$$
$$4x-6y=20$$

is:

**(A)** $x=18, y=12$ **(B)** $x=0, y=0$ **(C)** There is no solution
**(D)** There are an unlimited number of solutions **(E)** $x=8, y=5$

---

**13** A quadrilateral is inscribed in a circle. If angles are inscribed in the four arcs cut off by the sides of the quadrilateral, their sum will be:
**(A)** $180^\circ$ **(B)** $540^\circ$ **(C)** $360^\circ$ **(D)** $450^\circ$ **(E)** $1080^\circ$

---

**14** When simplified $\sqrt{1+\left(\frac{x^4-1}{2x^2}\right)^2}$ equals:
**(A)** $\frac{x^4+2x^2-1}{2x^2}$ **(B)** $\frac{x^4-1}{2x^2}$ **(C)** $\frac{\sqrt{x^2+1}}{2}$
**(D)** $\frac{x^2}{\sqrt{2}}$ **(E)** $\frac{x^2}{2}+\frac{1}{2x^2}$

---

**15** $\log 125$ equals:
**(A)** $100\log 1.25$ **(B)** $5\log 3$ **(C)** $3\log 25$ **(D)** $3-3\log 2$ **(E)** $(\log 25)(\log 5)$

---

**16** If $f(x)=5x^2-2x-1$, then $f(x+h)-f(x)$ equals:
**(A)** $5h^2-2h$ **(B)** $10xh-4x+2$ **(C)** $10xh-2x-2$
**(D)** $h(10x+5h-2)$ **(E)** $3h$

---

**17** The graph of the function $f(x)=2x^3-7$ goes:
**(A)** up to the right and down to the left
**(B)** down to the right and up to the left
**(C)** up to the right and up to the left
**(D)** down to the right and down to the left
**(E)** none of these ways.

---

**18** Of the following sets, the one that includes all values of $x$ which will satisfy $2x-3>7-x$ is:
**(A)** $x>4$ **(B)** $x<\frac{10}{3}$ **(C)** $x=\frac{10}{3}$ **(D)** $x>\frac{10}{3}$ **(E)** $x<0$

---

**19** If the three points of contact of a circle inscribed in a triangle are joined, the angles of the resulting triangle:
**(A)** are always equal to $60^\circ$
**(B)** are always one obtuse angle and two unequal acute angles
**(C)** are always one obtuse angle and two equal acute angles
**(D)** are always acute angles
**(E)** are always unequal to each other

---

**20** The equation $x^3+6x^2+11x+6=0$ has:
**(A)** no negative real roots **(B)** no positive real roots **(C)** no real roots
**(D)** 1 positive and 2 negative roots **(E)** 2 positive and 1 negative root

---

**21** The roots of the equation $2\sqrt{x}+2x^{-\frac{1}{2}}=5$ can be found by solving:
**(A)** $16x^2-92x+1=0$ **(B)** $4x^2-25x+4=0$ **(C)** $4x^2-17x+4=0$
**(D)** $2x^2-21x+2=0$ **(E)** $4x^2-25x-4=0$

---

**22** The expression $\frac{2x^2-x}{(x+1)(x-2)}-\frac{4+x}{(x+1)(x-2)}$ cannot be evaluated for $x=-1$ or $x=2$, since division by zero is not allowed. For other values of $x$:

**(A)** The expression takes on many different values.
**(B)** The expression has only the value 2.
**(C)** The expression has only the value 1.
**(D)** The expression always has a value between $-1$ and $+2$.
**(E)** The expression has a value greater than 2 or less than $-1$.

---

**23** If the margin made on an article costing $C$ dollars and selling for $S$ dollars is $M=\frac{1}{n}C$, then the margin is given by:
**(A)** $M=\frac{1}{n-1}S$ **(B)** $M=\frac{1}{n}S$ **(C)** $M=\frac{n}{n+1}S$
**(D)** $M=\frac{1}{n+1}S$ **(E)** $M=\frac{n}{n-1}S$

---

**24** The values of $k$ for which the equation $2x^2-kx+x+8=0$ will have real and equal roots are:
**(A)** $9$ and $-7$ **(B)** only $-7$ **(C)** 9 and 7
**(D)** $-9$ and $-7$ **(E)** only 9

---

**25** The two roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are $1$ and:
**(A)** $\frac{b(c-a)}{a(b-c)}$ **(B)** $\frac{a(b-c)}{c(a-b)}$ **(C)** $\frac{a(b-c)}{b(c-a)}$ **(D)** $\frac{c(a-b)}{a(b-c)}$ **(E)** $\frac{c(a-b)}{b(c-a)}$

---

**26** The straight line $\overline{AB}$ is divided at $C$ so that $AC=3CB$. Circles are described on $\overline{AC}$ and $\overline{CB}$

as diameters and a common tangent meets $AB$ produced at $D$. Then $BD$ equals:
**(A)** diameter of the smaller circle
**(B)** radius of the smaller circle
**(C)** radius of the larger circle
**(D)** $CB\sqrt{3}$
**(E)** the difference of the two radii

---

**27** A right circular cone has for its base a circle having the same radius as a given sphere. The volume of the cone is one-half that of the sphere. The ratio of the altitude of the cone to the radius of its base is:
**(A)** $\frac{1}{1}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{2}{1}$ **(E)** $\sqrt{\frac{5}{4}}$

---

**28** If $\frac{m}{n}=\frac{4}{3}$ and $\frac{r}{t}=\frac{9}{14}$, the value of $\frac{3mr-nt}{4nt-7mr}$ is:
**(A)** $-5\frac{1}{2}$ **(B)** $-\frac{11}{14}$ **(C)** $-1\frac{1}{4}$ **(D)** $\frac{11}{14}$ **(E)** $-\frac{2}{3}$

---

**29** If the ratio of the legs of a right triangle is $1:2$, then the ratio of the corresponding segments of the hypotenuse made by a perpendicular upon it from the vertex is:
**(A)** $1:4$ **(B)** $1:\sqrt{2}$ **(C)** $1:2$ **(D)** $1:\sqrt{5}$ **(E)** $1:5$

---

**30** $A$ and $B$ together can do a job in $2$ days; $B$ and $C$ can do it in four days; and $A$ and $C$ in $2\frac{2}{5}$ days. The number of days required for $A$ to do the job alone is:
**(A)** $1$ **(B)** $3$ **(C)** $6$ **(D)** $12$ **(E)** $2.8$

---

**31** In triangle $ABC$, $AB=AC$, $\angle A=40^\circ$. Point $O$ is within the triangle with $\angle OBC\cong\angle OCA$. The number of degrees in angle $BOC$ is:
**(A)** $110$ **(B)** $35$ **(C)** $140$ **(D)** $55$ **(E)** $70$

---

**32** The factors of $x^4+64$ are:
**(A)** $(x^2+8)^2$ **(B)** $(x^2+8)(x^2-8)$ **(C)** $(x^2+2x+4)(x^2-8x+16)$
**(D)** $(x^2-4x+8)(x^2-4x-8)$ **(E)** $(x^2-4x+8)(x^2+4x+8)$

---

**33** A bank charges $\$6$ for a loan of $\$120$. The borrower receives $\$114$ and repays the loan in $12$ installments of $\$10$ a month. The interest rate is approximately:
**(A)** $5\%$ **(B)** $6\%$ **(C)** $7\%$ **(D)** $9\%$ **(E)** $15\%$

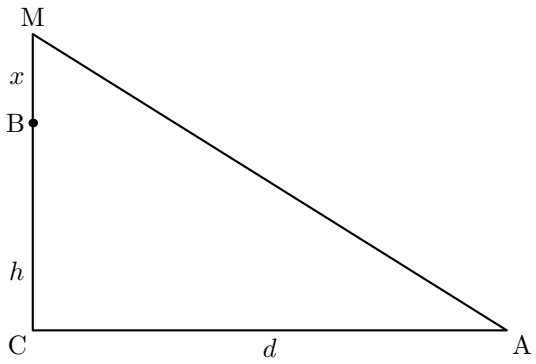
---

**34** The fraction $\frac{1}{3}$:
**(A)** equals 0.33333333 **(B)** is less than 0.33333333 by $\frac{1}{3\cdot10^8}$
**(C)** is less than 0.33333333 by $\frac{1}{3\cdot10^9}$
**(D)** is greater than 0.33333333 by $\frac{1}{3\cdot10^8}$
**(E)** is greater than 0.33333333 by $\frac{1}{3\cdot10^9}$

---

**35** In the right triangle shown the sum of the distances $BM$ and $MA$ is equal to the sum of the distances $BC$ and $CA$. If $MB = x$, $CB = h$, and $CA = d$, then $x$ equals:



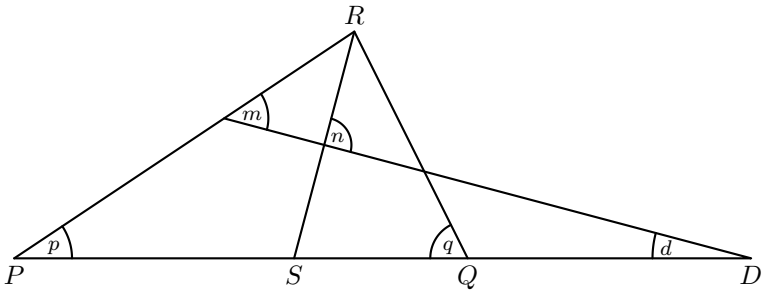

**(A)** $\frac{hd}{2h+d}$ **(B)** $d-h$ **(C)** $\frac{1}{2}d$ **(D)** $h+d-\sqrt{2d}$ **(E)** $\sqrt{h^2+d^2}-h$

---

**36** A boat has a speed of $15$ mph in still water. In a stream that has a current of $5$ mph it travels a certain distance downstream and returns. The ratio of the average speed for the round trip to the speed in still water is:
**(A)** $\frac{5}{4}$ **(B)** $\frac{1}{1}$ **(C)** $\frac{8}{9}$ **(D)** $\frac{7}{8}$ **(E)** $\frac{9}{8}$

---

**37** Given triangle $PQR$ with $\overline{RS}$ bisecting $\angle R$, $PQ$ extended to $D$ and $\angle n$ a right angle, then:




**(A)** $\angle m = \frac{1}{2}(\angle p - \angle q)$ **(B)** $\angle m = \frac{1}{2}(\angle p + \angle q)$ **(C)** $\angle d = \frac{1}{2}(\angle q + \angle p)$ **(D)** $\angle d = \frac{1}{2}\angle m$ **(E)** none of these

---

**38** If $\log 2 = .3010$ and $\log 3 = .4771$, the value of $x$ when $3^{x+3} = 135$ is approximately:
**(A)** $5$ **(B)** $1.47$ **(C)** $1.67$ **(D)** $1.78$ **(E)** $1.63$

---

**39** The locus of the midpoint of a line segment that is drawn from a given external point $P$ to a

given circle with center $O$ and radius $r$, is:
**(A)** a straight line perpendicular to $\overline{PO}$
**(B)** a straight line parallel to $\overline{PO}$
**(C)** a circle with center $P$ and radius $r$
**(D)** a circle with center at the midpoint of $\overline{PO}$ and radius $2r$
**(E)** a circle with center at the midpoint $\overline{PO}$ and radius $\frac{1}{2}r$

---

**40** If $\left(a+\frac{1}{a}\right)^2=3$, then $a^3+\frac{1}{a^3}$ equals:
**(A)** $\frac{10\sqrt{3}}{3}$ **(B)** $3\sqrt{3}$ **(C)** $0$ **(D)** $7\sqrt{7}$ **(E)** $6\sqrt{3}$

---

**41** The sum of all the roots of $4x^3-8x^2-63x-9=0$ is:
**(A)** $8$ **(B)** $2$ **(C)** $-8$ **(D)** $-2$ **(E)** $0$

---

**42** Consider the graphs of (1): $y=x^2-\frac{1}{2}x+2$ and (2) $y=x^2+\frac{1}{2}x+2$ on the same set of axis. These parabolas are exactly the same shape. Then:
**(A)** the graphs coincide.
**(B)** the graph of (1) is lower than the graph of (2).
**(C)** the graph of (1) is to the left of the graph of (2).
**(D)** the graph of (1) is to the right of the graph of (2).
**(E)** the graph of (1) is higher than the graph of (2).

---

**43** The hypotenuse of a right triangle is $10$ inches and the radius of the inscribed circle is $1$ inch. The perimeter of the triangle in inches is:
**(A)** $15$ **(B)** $22$ **(C)** $24$ **(D)** $26$ **(E)** $30$

---

**44** A man born in the first half of the nineteenth century was $x$ years old in the year $x^2$. He was born in:
**(A)** $1849$ **(B)** $1825$ **(C)** $1812$ **(D)** $1836$ **(E)** $1806$

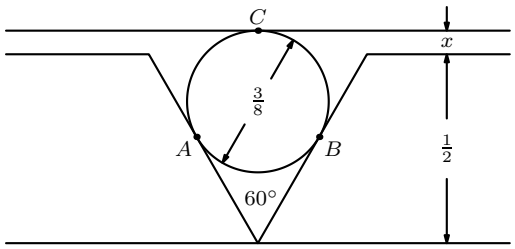
---

**45** In a rhombus, $ABCD$, line segments are drawn within the rhombus, parallel to diagonal $BD$, and terminated in the sides of the rhombus. A graph is drawn showing the length of a segment as a function of its distance from vertex $A$. The graph is:
**(A)** A straight line passing through the origin.
**(B)** A straight line cutting across the upper right quadrant.
**(C)** Two line segments forming an upright V.
**(D)** Two line segments forming an inverted V.
**(E)** None of these.

---

**46** In the diagram, if points $A$, $B$ and $C$ are points of tangency, then $x$ equals:




**(A)** $\frac{3}{16}$" **(B)** $\frac{1}{8}$" **(C)** $\frac{1}{32}$" **(D)** $\frac{3}{32}$" **(E)** $\frac{1}{16}$"

---

**47** At the midpoint of line segment $AB$ which is $p$ units long, a perpendicular $MR$ is erected with length $q$ units. An arc is described from $R$ with a radius equal to $\frac{1}{2}AB$, meeting $AB$ at $T$. Then $AT$ and $TB$ are the roots of:

**(A)** $x^2 + px + q^2 = 0$
**(B)** $x^2 - px + q^2 = 0$
**(C)** $x^2 + px - q^2 = 0$
**(D)** $x^2 - px - q^2 = 0$
**(E)** $x^2 - px + q = 0$

---

**48** A train, an hour after starting, meets with an accident which detains it a half hour, after which it proceeds at $\frac{3}{4}$ of its former rate and arrives $3\frac{1}{2}$ hours late. Had the accident happened $90$ miles farther along the line, it would have arrived only $3$ hours late. The length of the trip in miles was:

**(A)** $400$ **(B)** $465$ **(C)** $600$ **(D)** $640$ **(E)** $550$

---

**49** The difference of the squares of two odd numbers is always divisible by $8$. If $a > b$, and $2a + 1$ and $2b + 1$ are the odd numbers, to prove the given statement we put the difference of the squares in the form:

**(A)** $(2a + 1)^2 - (2b + 1)^2$
**(B)** $4a^2 - 4b^2 + 4a - 4b$
**(C)** $4[a(a + 1) - b(b + 1)]$
**(D)** $4(a - b)(a + b + 1)$
**(E)** $4(a^2 + a - b^2 - b)$

---

**50** The times between $7$ and $8$ o'clock, correct to the nearest minute, when the hands of a clock will form an angle of $84$ degrees are:

**(A)** 7: 23 and 7: 53 **(B)** 7: 20 and 7: 50 **(C)** 7: 22 and 7: 53
**(D)** 7: 23 and 7: 52 **(E)** 7: 21 and 7: 49

---