AMC 12/AHSME 1991

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1 If for any three distinct numbers a, b and c we define

$$\boxed{a, b, c} = \frac{c+a}{c-b},$$

then $\boxed{1, -2, -3} =$

(A) -2 **(B)** $-\frac{2}{5}$ **(C)** $-\frac{1}{4}$ **(D)** $\frac{2}{5}$ **(E)** 2

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2 $|3-\pi| =$

(A) $\frac{1}{7}$ **(B)** 0.14 **(C)** $3-\pi$ **(D)** $3+\pi$ **(E)** $\pi-3$

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3 $(4^{-1}-3^{-1})^{-1} =$

(A) -12 **(B)** -1 **(C)** $\frac{1}{12}$ **(D)** 1 **(E)** 12

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4 Which of the following triangles cannot exist?

(A) An acute isosceles triangle **(B)** An isosceles right triangle **(C)** An obtuse right triangle **(D)** A scalene rig
(E) A scalene obtuse triangle

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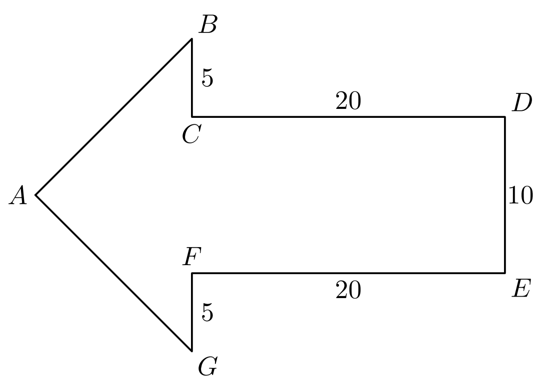
5 In the arrow-shaped polygon [see figure], the angles at vertices A, C, D, E and F are right angles, $BC = FG = 5$, $CD = FE = 20$, $DE = 10$, and $AB = AG$. The area of the polygon is closest to



(A) 288 **(B)** 291 **(C)** 294 **(D)** 297 **(E)** 300

6 If $x \geq 0$, then $\sqrt{x\sqrt{x\sqrt{x}}} =$

(A) $x\sqrt{x}$ **(B)** $x\sqrt[4]{x}$ **(C)** $\sqrt[8]{x}$ **(D)** $\sqrt[8]{x^3}$ **(E)** $\sqrt[8]{x^7}$

7 If $x = \frac{a}{b}$, $a \neq b$ and $b \neq 0$, then $\frac{a+b}{a-b} =$

(A) $\frac{x}{x+1}$ **(B)** $\frac{x+1}{x-1}$ **(C)** 1 **(D)** $x - \frac{1}{x}$ **(E)** $x + \frac{1}{x}$

8 Liquid X does not mix with water. Unless obstructed, it spreads out on the surface of water to form a circular film 0.1 cm thick. A rectangular box measuring 6 cm by 3 cm by 12 cm is filled with liquid X. Its contents are poured onto a large body of water. What will be the radius, in centimeters, of the resulting circular film?

(A) $\frac{\sqrt{216}}{\pi}$ **(B)** $\sqrt{\frac{216}{\pi}}$ **(C)** $\sqrt{\frac{2160}{\pi}}$ **(D)** $\frac{216}{\pi}$ **(E)** $\frac{2160}{\pi}$

9 From time $t = 0$ to time $t = 1$ a population increased by $i\%$, and from time $t = 1$ to time $t = 2$ the population increased by $j\%$. Therefore, from time $t = 0$ to time $t = 2$ the population increased by

(A) $(i + j)\%$ **(B)** $ij\%$ **(C)** $(i + ij)\%$ **(D)** $\left(i + j + \frac{ij}{100}\right)\%$ **(E)** $\left(i + j + \frac{i+j}{100}\right)\%$

10 Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 29

11 Jack and Jill run 10 kilometers. They start at the same point, run 5 kilometers up a hill, and return to the starting point by the same route. Jack has a 10 minute head start and runs at the rate of 15 km/hr uphill and 20 km/hr downhill. Jill runs 16 km/hr uphill and 22 km/hr downhill. How far from the top of the hill are they when they pass going in opposite directions?

(A) $\frac{5}{4}\ km$ **(B)** $\frac{35}{27}\ km$ **(C)** $\frac{27}{20}\ km$ **(D)** $\frac{7}{3}\ km$ **(E)** $\frac{28}{9}\ km$

12 The measures (in degrees) of the interior angles of a convex hexagon form an arithmetic sequence of positive integers. Let m° be the measure of the largest interior angle of the hexagon. The largest possible value of m° is

(A) 165° **(B)** 167° **(C)** 170° **(D)** 175° **(E)** 179°

13 Horses X, Y and Z are entered in a three-horse race in which ties are not possible. If the odds against X winning are $3 - to - 1$ and the odds against Y winning are $2 - to - 3$, what are the odds against Z winning? (By "*odds against H winning are p-to-q*" we mean that probability of H winning the race is $\frac{q}{p+q}$.)

(A) $3-to-20$ **(B)** $5-to-6$ **(C)** $8-to-5$ **(D)** $17-to-3$ **(E)** $20-to-3$

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14 If x is the cube of a positive integer and d is the number of positive integers that are divisors of x, then d could be

(A) 200 **(B)** 201 **(C)** 202 **(D)** 203 **(E)** 204

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15 A circular table has exactly 60 chairs around it. There are N people seated at this table in such a way that the next person to be seated must sit next to someone. The smallest possible value of N is

(A) 15 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 58

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16 One hundred students at Century High School participated in the AHSME last year, and their mean score was 100. The number of non-seniors taking the AHSME was 50% more than the number of seniors, and the mean score of the seniors was 50% higher than that of the non-seniors. What was the mean score of the seniors?

(A) 100 **(B)** 112.5 **(C)** 120 **(D)** 125 **(E)** 150

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17 A positive integer N is a *palindrome* if the integer obtained by reversing the sequence of digits of N is equal to N. The year 1991 is the only year in the current century with the following two properties:
(a) It is a palindrome
(b) It factors as a product of a 2-digit prime palindrome and a 3-digit prime palindrome.
How many years in the millennium between 1000 and 2000 (including the year 1991) have properties (a) and (b)?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

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18 If S is the set of points z in the complex plane such that $(3+4i)z$ is a real number, then S is a

(A) right triangle **(B)** circle **(C)** hyperbola **(D)** line **(E)** parabola

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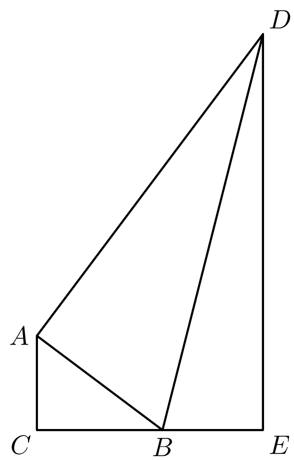
19 Triangle ABC has a right angle at C, $AC=3$ and $BC=4$. Triangle ABD has a right angle at A and $AD=12$. Points C and D are on opposite sides of $\overline{AB}$. The line through D parallel to $\overline{AC}$ meets $\overline{CB}$ extended at E. If $\frac{DE}{DB}=\frac{m}{n}$, where m and n are relatively prime positive integers, then $m+n=$



(A) 25 **(B)** 128 **(C)** 153 **(D)** 243 **(E)** 256

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20 The sum of all real x such that $(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3$ is

(A) $3/2$ **(B)** 2 **(C)** $5/2$ **(D)** 3 **(E)** $7/2$

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21 If $f\left(\frac{x}{x-1}\right) = \frac{1}{x}$ for all $x \neq 0, 1$ and $0 < \theta < \frac{\pi}{2}$, then $f(\sec^2\theta) =$

(A) $\sin^2\theta$ **(B)** $\cos^2\theta$ **(C)** $\tan^2\theta$ **(D)** $\cot^2\theta$ **(E)** $\csc^2\theta$

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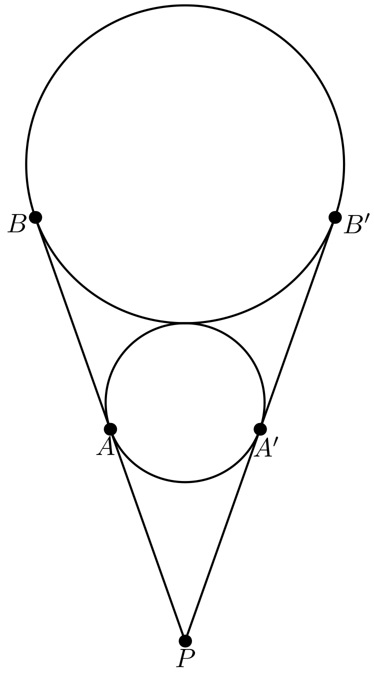
22 Two circles are externally tangent. Lines $\overline{PAB}$ and $\overline{PA'B'}$ are common tangents with A and A' on the smaller circle and B and B' on the larger circle. If $PA = AB = 4$, then the area of the smaller circle is



(A) 1.44π **(B)** 2π **(C)** 2.56π **(D)** $\sqrt{8}\pi$ **(E)** 4π

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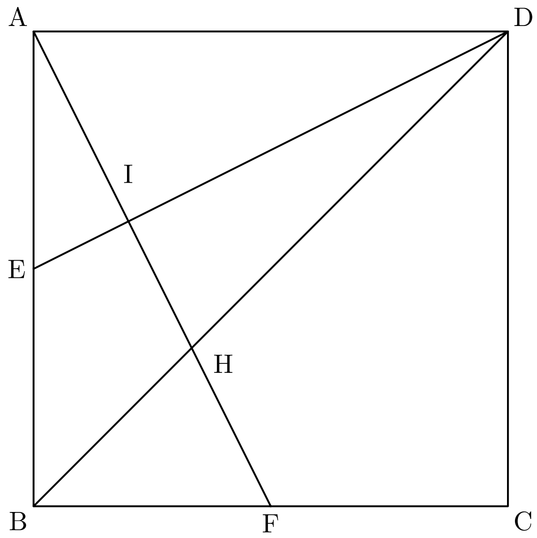
23 If $ABCD$ is a 2 X 2 square, E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$, $\overline{AF}$ and $\overline{DE}$ intersect at I, and $\overline{BD}$ and $\overline{AF}$ intersect at H, then the area of quadrilateral $BEIH$ is



(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{7}{15}$ **(D)** $\frac{8}{15}$ **(E)** $\frac{3}{5}$

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24 The graph, G of $y = \log_{10} x$ is rotated 90° counter-clockwise about the origin to obtain a new graph G'. Which of the following is an equation for G'?

(A) $y = \log_{10}\left(\frac{x+90}{9}\right)$ **(B)** $y = \log_x 10$ **(C)** $y = \frac{1}{x+1}$ **(D)** $y = 10^{-x}$ **(E)** $y = 10^x$

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25 If $T_n = 1 + 2 + 3 + \ldots + n$ and

$$P_n = \frac{T_2}{T_2 - 1} \cdot \frac{T_3}{T_3 - 1} \cdot \frac{T_4}{T_4 - 1} \cdot \cdots \cdot \frac{T_n}{T_n - 1} \quad \text{for } n = 2, 3, 4, \ldots,$$

then P_{1991} is closest to which of the following numbers?

(A) 2.0 **(B)** 2.3 **(C)** 2.6 **(D)** 2.9 **(E)** 3.2

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26 An n-digit positive integer is *cute* if its n digits are an arrangement of the set $\{1, 2, \ldots, n\}$ and its first k digits form an integer that is divisible by k, for $k = 1, 2, \ldots, n$. For example 321 is a cute 3-digit integer because 1 divides 3, 2 divides 32, and 3 divides 321. How many cute 6-digit integers are there?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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27 If $x + \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2-1}} = 20$ then $x^2 + \sqrt{x^4 - 1} + \frac{1}{x^2 + \sqrt{x^4-1}} =$

(A) 5.05 **(B)** 20 **(C)** 51.005 **(D)** 61.25 **(E)** 400

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28 Initially an urn contains 100 black marbles and 100 white marbles. Repeatedly, three marbles are removed from the urn and replaced from a pile outside the urn as follows:

MARBLES REMOVED	REPLACED WITH
3 black	1 black
2 black, 1 white	1 black, 1 white
1 black, 2 white	2 white
3 white	1 black, 1 white

Which of the following sets of marbles could be the contents of the urn after repeated applications of this procedure?

(A) 2 black marbles **(B)** 2 white marbles **(C)** 1 black marble **(D)** 1 black and 1 white marble **(E)** 1 white marb

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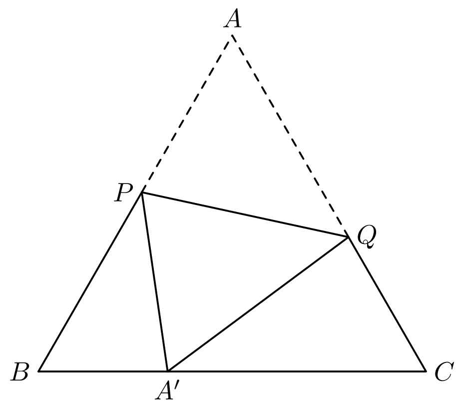
29 Equilateral triangle ABC has been creased and folded so that vertex A now rests at A' on $\overline{BC}$ as shown. If $BA' = 1$ and $A'C = 2$ then the length of crease $\overline{PQ}$ is



(A) $\frac{8}{5}$ **(B)** $\frac{7}{20}\sqrt{21}$ **(C)** $\frac{1+\sqrt{5}}{2}$ **(D)** $\frac{13}{8}$ **(E)** $\sqrt{3}$

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30 For any set S, let $|S|$ denote the number of elements in S, and let $n(S)$ be the number of subsets of S, including the empty set and the set S itself. If A, B and C are sets for which

$$n(A)+n(B)+n(C)=n(A\cup B\cup C) \quad \text{and} \quad |A|=|B|=100,$$

then what is the minimum possible value of $|A\cap B\cap C|$?

(A) 96 **(B)** 97 **(C)** 98 **(D)** 99 **(E)** 100

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