AMC 12/AHSME 1963

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1 Which one of the following points is not on the graph of $y = \dfrac{x}{x+1}$?

(A) $(0,0)$ **(B)** $\left(-\frac{1}{2}, -1\right)$ **(C)** $\left(\frac{1}{2}, \frac{1}{3}\right)$ **(D)** $(-1,1)$ **(E)** $(-2,2)$

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2 Let $n = x - y^{x-y}$. Find n when $x = 2$ and $y = -2$.

(A) -14 **(B)** 0 **(C)** 1 **(D)** 18 **(E)** 256

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3 If the reciprocal of $x + 1$ is $x - 1$, then x equals:

(A) 0 **(B)** 1 **(C)** -1 **(D)** ± 1 **(E)** none of these

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4 For what value(s) of k does the pair of equations $y = x^2$ and $y = 3x + k$ have two identical solutions?

(A) $\frac{4}{9}$ **(B)** $-\frac{4}{9}$ **(C)** $\frac{9}{4}$ **(D)** $-\frac{9}{4}$ **(E)** $\pm\frac{9}{4}$

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5 If x and $\log_{10} x$ are real numbers and $\log_{10} x < 0$, then:

(A) $x < 0$ **(B)** $-1 < x < 1$ **(C)** $0 < x \le 1$

(D) $-1 < x < 0$ **(E)** $0 < x < 1$

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6 Triangle BAD is right-angled at B. On AD there is a point C for which $AC = CD$ and $AB = BC$. The magnitude of angle DAB, in degrees, is:

(A) $67\frac{1}{2}$ **(B)** 60 **(C)** 45 **(D)** 30 **(E)** $22\frac{1}{2}$

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7 Given the four equations:

(1) $3y - 2x = 12$ **(2)** $-2x - 3y = 10$ **(3)** $3y + 2x = 12$ **(4)** $2y + 3x = 10$

The pair representing the perpendicular lines is:

(A) (1) and (4) **(B)** (1) and (3) **(C)** (1) and (2) **(D)** (2) and (4) **(E)** (2) and (3)

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8 The smallest positive integer x for which $1260x = N^3$, where N is an integer, is:

(A) 1050 **(B)** 1260 **(C)** 1260^2 **(D)** 7350 **(E)** 44100

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9 In the expansion of $\left(a-\dfrac{1}{\sqrt{a}}\right)^7$ the coefficient of $a^{-\frac{1}{2}}$ is:

(A) -7 **(B)** 7 **(C)** -21 **(D)** 21 **(E)** 35

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10 Point P is taken interior to a square with side-length a and such that is it equally distant from two consecutive vertices and from the side opposite these vertices. If d represents the common distance, then d equals:

(A) $\dfrac{3a}{5}$ **(B)** $\dfrac{5a}{8}$ **(C)** $\dfrac{3a}{8}$ **(D)** $\dfrac{a\sqrt{2}}{2}$ **(E)** $\dfrac{a}{2}$

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11 The arithmetic mean of a set of 50 numbers is 38. If two numbers of the set, namely 45 and 55, are discarded, the arithmetic mean of the remaining set of numbers is:

(A) 38.5 **(B)** 37.5 **(C)** 37 **(D)** 36.5 **(E)** 36

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12 Three vertices of parallelogram $PQRS$ are $P(-3,-2)$, $Q(1,-5)$, $R(9,1)$ with P and R diagonally opposite. The sum of the coordinates of vertex S is:

(A) 13 **(B)** 12 **(C)** 11 **(D)** 10 **(E)** 9

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13 If $2^a+2^b=3^c+3^d$, the number of integers a,b,c,d which can possibly be negative, is, at most:

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

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14 Given the equations $x^2+kx+6=0$ and $x^2-kx+6=0$. If, when the roots of the equation are suitably listed, each root of the second equation is 5 more than the corresponding root of the first equation, then k equals:

(A) 5 **(B)** -5 **(C)** 7 **(D)** -7 **(E)** none of these

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15 A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is:

(A) $\sqrt{3}:1$ **(B)** $\sqrt{3}:\sqrt{2}$ **(C)** $3\sqrt{3}:2$ **(D)** $3:\sqrt{2}$ **(E)** $3:2\sqrt{2}$

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16 Three numbers a,b,c, none zero, form an arithmetic progression. Increasing a by 1 or increasing c by 2 results in a geometric progression. Then b equals:

(A) 16 **(B)** 14 **(C)** 12 **(D)** 10 **(E)** 8

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17 The expression $\dfrac{\dfrac{a}{a+y}+\dfrac{y}{a-y}}{\dfrac{y}{a+y}-\dfrac{a}{a-y}}$, a real, $a\neq 0$, has the value -1 for:

(A) all but two real values of y **(B)** only two real values of y

(C) all real values of y **(D)** only one real value of y **(E)** no real values of y

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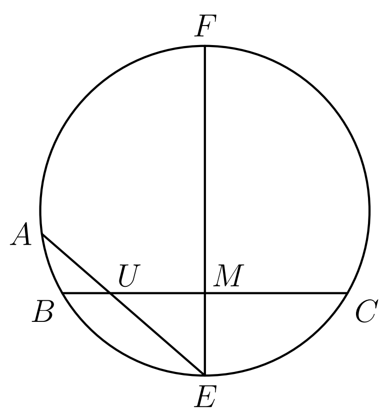
18 Chord EF is the perpendicular bisector of chord BC, intersecting it in M. Between B and M point U is taken, and EU extended meets the circle in A. Then, for any selection of U, as described, triangle EUM is similar to triangle:



(A) EFA **(B)** EFC **(C)** ABM **(D)** ABU **(E)** FMC

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19 In counting n colored balls, some red and some black, it was found that 49 of the first 50 counted were red. Thereafter, 7 out of every 8 counted were red. If, in all, 90% or more of the balls counted were red, the maximum value of n is:

(A) 225 **(B)** 210 **(C)** 200 **(D)** 180 **(E)** 175

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20 Two men at points R and S, 76 miles apart, set out at the same time to walk towards each other. The man at R walks uniformly at the rate of $4\frac{1}{2}$ miles per hour; the man at S walks at the constant rate of $3\frac{1}{4}$ miles per hour for the first hour, at $3\frac{3}{4}$ miles per hour for the second hour, and so on, in arithmetic progression. If the men meet x miles nearer R than S in an integral number of hours, then x is:

(A) 10 **(B)** 8 **(C)** 6 **(D)** 4 **(E)** 2

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21 The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has:

(A) no linear factor with integer coeficients and integer exponents

(B) the factor $-x + y + z$

(C) the factor $x - y - z + 1$

(D) the factor $x + y - z + 1$

(E) the factor $x - y + z + 1$

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22 Acute-angled triangle ABC is inscribed in a circle with center at O; $\overset{\frown}{AB} = 120$ and $\overset{\frown}{BC} = 72$. A point E is taken in minor arc AC such that OE is perpendicular to AC. Then the ratio of the magnitudes of angles OBE and BAC is:

(A) $\frac{5}{18}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{4}{9}$

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23 A gives B as many cents as B has and C as many cents as C has. Similarly, B then gives A and C as many cents as each then has. C, similarly, then gives A and B as many cents as each then has. If each finally has 16 cents, with how many cents does A start?

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

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24 Consider equations of the form $x^2 + bx + c = 0$. How many such equations have real roots and have coefficients b and c selected from the set of integers $\{1, 2, 3, 4, 5, 6\}$?

(A) 20 **(B)** 19 **(C)** 18 **(D)** 17 **(E)** 16

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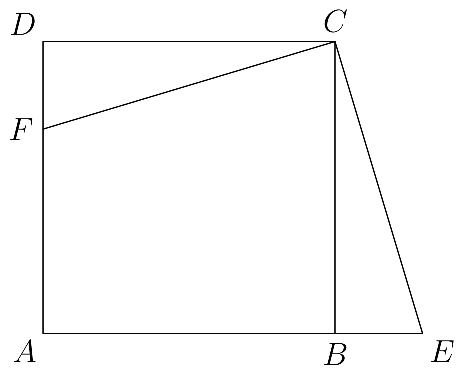
25 Point F is taken in side AD of square $ABCD$. At C a perpendicular is drawn to CF, meeting AB extended at E. The area of $ABCD$ is 256 square inches and the area of triangle CEF is 200 square inches. Then the number of inches in BE is:



(A) 12 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 20

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26 **Form 1**
Consider the statements:

(1) $p \wedge \sim q \wedge r$ **(2)** $\sim p \wedge \sim q \wedge r$ **(3)** $p \wedge \sim q \wedge \sim r$ **(4)** $\sim p \wedge q \wedge r$, where $p, q,$ and r are propositions. How many of these imply the truth of $(p \to q) \to r$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Form 2
Consider the statements (1) p and r are true and q is false (2) r is true and p and q are false (3) p is true and q and r are false (4) q and r are true and p is false. How many of these imply the truth of the statement
"r is implied by the statement that p implies q"?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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27 Six straight lines are drawn in a plane with no two parallel and no three concurrent. The number of regions into which they divide the plane is:

(A) 16 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 26

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28 Given the equation $3x^2 - 4x + k = 0$ with real roots. The value of k for which the product of the roots of the equation is a maximum is:

(A) $\frac{16}{9}$ **(B)** $\frac{16}{3}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{4}{3}$ **(E)** $-\frac{4}{3}$

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29 A particle projected vertically upward reaches, at the end of t seconds, an elevation of s feet where $s = 160t - 16t^2$. The highest elevation is:

(A) 800 **(B)** 640 **(C)** 400 **(D)** 320 **(E)** 160

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30 Let

$$F = \log \frac{1+x}{1-x}.$$

Find a new function G by replacing each x in F by

$$\frac{3x + x^3}{1 + 3x^2},$$

and simplify. The simplified expression G is equal to:

(A) $-F$ **(B)** F **(C)** $3F$ **(D)** F^3 **(E)** $F^3 - F$

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31 The number of solutions in positive integers of $2x + 3y = 763$ is:

(A) 255 **(B)** 254 **(C)** 128 **(D)** 127 **(E)** 0

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32 The dimensions of a rectangle R are a and b, $a < b$. It is required to obtain a rectangle with dimensions x and y, $x < a$, $y < a$, so that its perimeter is one-third that of R, and its area is one-third that of R. The number of such (different) rectangles is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** infinitely many

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33 Given the line $y = \frac{3}{4}x + 6$ and a line L parallel to the given line and 4 units from it. A possible equation for L is:

(A) $y = \frac{3}{4}x + 1$ **(B)** $y = \frac{3}{4}x$ **(C)** $y = \frac{3}{4}x - \frac{2}{3}$

(D) $y = \frac{3}{4}x - 1$ **(E)** $y = \frac{3}{4}x + 2$

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34 In triangle ABC, side $a = \sqrt{3}$, side $b = \sqrt{3}$, and side $c > 3$. Let x be the largest number such that the magnitude, in degrees, of the angle opposite side c exceeds x. Then x equals:

(A) 150 **(B)** 120 **(C)** 105 **(D)** 90 **(E)** 60

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35 The lengths of the sides of a triangle are integers, and its area is also an integer. One side is 21 and the perimeter is 48. The shortest side is:

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

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36 A person starting with 64 cents and making 6 bets, wins three times and loses three times, the wins and losses occurring in random order. The chance for a win is equal to the chance for a loss. If each wager is for half the money remaining at the time of the bet, then the final result is:

(A) a loss of 27 **(B)** a gain of 27 **(C)** a loss of 37

(D) neither a gain nor a loss **(E)** a gain or a loss depending upon the order in which the wins and losse

Note: Due to the lack of LaTeXpackages, the numbers in the answer choices are in cents

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37 Given points $P_1, P_2, \cdots, P_7$ on a straight line, in the order stated (not necessarily evenly spaced). Let P be an arbitrarily selected point on the line and let s be the sum of the undirected lengths

$$PP_1, PP_2, \cdots, PP_7.$$

Then s is smallest if and only if the point P is:

(A) midway between P_1 and P_7 **(B)** midway between P_2 and P_6 **(C)** midway between P_3 and P_5

(D) at P_4 **(E)** at P_1

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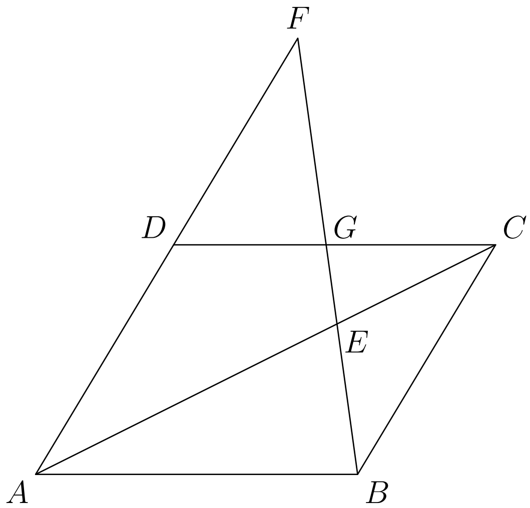
38 Point F is taken on the extension of side AD of parallelogram $ABCD$. BF intersects diagonal AC at E and side DC at G. If $EF = 32$ and $GF = 24$, then BE equals:



(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 16

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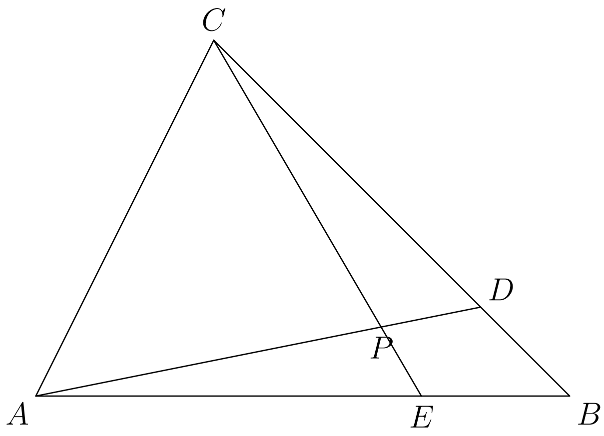
39 In triangle ABC lines CE and AD are drawn so that
$\frac{CD}{DB} = \frac{3}{1}$ and $\frac{AE}{EB} = \frac{3}{2}$. Let $r = \frac{CP}{PE}$
where P is the intersection point of CE and AD. Then r equals:



(A) 3 **(B)** $\frac{3}{2}$ **(C)** 4 **(D)** 5 **(E)** $\frac{5}{2}$

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40 If x is a number satisfying the equation $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$, then x^2 is between:

(A) 55 and 65 **(B)** 65 and 75 **(C)** 75 and 85 **(D)** 85 and 95 **(E)** 95 and 105

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