AMC 12/AHSME 1956

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1 The value of $x + x(x^x)$ when $x = 2$ is:
(A) 10 **(B)** 16 **(C)** 18 **(D)** 36 **(E)** 64

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2 Mr. Jones sold two pipes at $\$1.20$ each. Based on the cost, his profit one was 20% and his loss on the other was 20%. On the sale of the pipes, he:
(A) broke even **(B)** lost 4 cents **(C)** gained 4 cents **(D)** lost 10 cents **(E)** gained 10 cents

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3 The distance light travels in one year is approximately $5,870,000,000,000$ miles. The distance light travels in 100 years is:
(A) $587*10^8$ miles **(B)** $587*10^{10}$ miles **(C)** $587*10^{-10}$ miles **(D)** $587*10^{12}$ miles **(E)** $587*10^{-12}$ miles

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4 A man has $\$10,000$ to invest. He invests $\$4000$ at 5% and $\$3500$ at 4%. In order to have a yearly income of $\$500$, he must invest the remainder at:
(A) 6% **(B)** 6.1% **(C)** 6.2% **(D)** 6.3% **(E)** 6.4%

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5 A nickel is placed on a table. The number of nickels which can be placed around it, each tangent to it and to two others is:
(A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 12

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6 In a group of cows and chickens, the number of legs was 14 more than twice the number of heads. The number of cows was:
(A) 5 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 14

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7 The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal if:
(A) $a = b$ **(B)** $a = bc$ **(C)** $c = a$ **(D)** $c = b$ **(E)** $c = ab$

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8 If $8 \cdot 2^x = 5^{y+8}$, then when $y = -8, x =$
(A) -4 **(B)** -3 **(C)** 0 **(D)** 4 **(E)** 8

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9 Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$; the result is:
(A) a^{16} **(B)** a^{12} **(C)** a^8 **(D)** a^4 **(E)** a^2

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10 A circle of radius 10 inches has its center at the vertex C of an equilateral triangle ABC and passes through the other two vertices. The side AC extended through C intersects the circle

at D. The number of degrees of angle ADB is:
(A) 15 **(B)** 30 **(C)** 60 **(D)** 90 **(E)** 120

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11 The expression $1-\frac{1}{1+\sqrt{3}}+\frac{1}{1-\sqrt{3}}$ equals:
(A) $1-\sqrt{3}$ **(B)** 1 **(C)** $-\sqrt{3}$ **(D)** $\sqrt{3}$ **(E)** $1+\sqrt{3}$

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12 If $x^{-1}-1$ is divided by $x-1$ the quotient is:
(A) 1 **(B)** $\frac{1}{x-1}$ **(C)** $\frac{-1}{x-1}$ **(D)** $\frac{1}{x}$ **(E)** $-\frac{1}{x}$

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13 Given two positive integers x and y with $x<y$. The percent that x is less than y is:
(A) $\frac{100(y-x)}{x}$ **(B)** $\frac{100(x-y)}{x}$ **(C)** $\frac{100(y-x)}{y}$ **(D)** $100(y-x)$ **(E)** $100(x-y)$

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14 The points A, B, C are on a circle O. The tangent line at A and the secant BC intersect at P, B lying between C and P. If $\overline{BC}=20$ and $\overline{PA}=10\sqrt{3}$, then $\overline{PB}$ equals:
(A) 5 **(B)** 10 **(C)** $10\sqrt{3}$ **(D)** 20 **(E)** 30

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15 The root(s) of $\frac{15}{x^2-4}-\frac{2}{x-2}=1$ is (are):
(A) -5 and 3 **(B)** ± 2 **(C)** 2 only **(D)** -3 and 5 **(E)** 3 only

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16 The sum of three numbers is 98. The ratio of the first to the second is $\frac{2}{3}$, and the ratio of the second to the third is $\frac{5}{8}$. The second number is:
(A) 15 **(B)** 20 **(C)** 30 **(D)** 32 **(E)** 33

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17 The fraction $\frac{5x-11}{2x^2+x-6}$ was obtained by adding the two fractions $\frac{A}{x+2}$ and $\frac{B}{2x-3}$. The values of A and B must be, respectively:
(A) $5x, -11$ **(B)** $-11, 5x$ **(C)** $-1, 3$ **(D)** $3, -1$ **(E)** $5, -11$

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18 If $10^{2y}=25$, then 10^{-y} equals:
(A) $-\frac{1}{5}$ **(B)** $\frac{1}{625}$ **(C)** $\frac{1}{50}$ **(D)** $\frac{1}{25}$ **(E)** $\frac{1}{5}$

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19 Two candles of the same height are lighted at the same time. The first is consumed in 4 hours and the second in 3 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the height of the second?
(A) $\frac{3}{4}$ **(B)** $1\frac{1}{2}$ **(C)** 2 **(D)** $2\frac{2}{5}$ **(E)** $2\frac{1}{2}$

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20 If $(0.2)^x=2$ and $\log 2=0.3010$, then the value of x to the nearest tenth is:
(A) -10.0 **(B)** -0.5 **(C)** -0.4 **(D)** -0.2 **(E)** 10.0

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21 If each of two intersecting lines intersects a hyperbola and neither line is tangent to the hyperbola, then the possible number of points of intersection with the hyperbola is:
(A) 2 **(B)** 2 or 3 **(C)** 2 or 4 **(D)** 3 or 4 **(E)** $2, 3,$ or 4

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22 Jones covered a distance of 50 miles on his first trip. On a later trip he traveled 300 miles while going three times as fast. His new time compared with the old time was:
(A) three times as much **(B)** twice as much **(C)** the same **(D)** half as much **(E)** a third as much

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23 About the equation $ax^2 - 2x\sqrt{2} + c = 0$, with a and c real constants, we are told that the discriminant is zero. The roots are necessarily:
(A) equal and integral **(B)** equal and rational **(C)** equal and real **(D)** equal and irrational **(E)** equ

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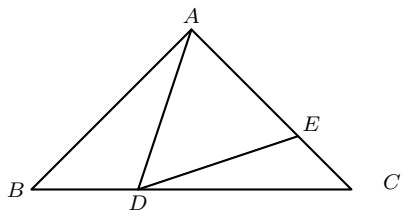
24 In the figure $\overline{AB} = \overline{AC}$, angle $BAD = 30^\circ$, and $\overline{AE} = \overline{AD}$.



Then angle CDE equals:
(A) $7\frac{1}{2}^\circ$ **(B)** 10° **(C)** $12\frac{1}{2}^\circ$ **(D)** 15° **(E)** 20°

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25 The sum of all numbers of the form $2k + 1$, where k takes on integral values from 1 to n is:
(A) n^2 **(B)** $n(n + 1)$ **(C)** $n(n + 2)$ **(D)** $(n + 1)^2$ **(E)** $(n + 1)(n + 2)$

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26 Which one of the following combinations of given parts does not determine the indicated triangle?
(A) base angle and vertex angle; isosceles triangle **(B)** vertex angle and the base; isosceles triangle
(C) the radius of the circumscribed circle; equilateral triangle **(D)** one arm and the radius of the inscribed
(E) two angles and a side opposite one of them; scalene triangle

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27 If an angle of a triangle remains unchanged but each of its two including sides is doubled, then the area is multiplied by:
(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** more than 6

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28 Mr. J left his entire estate to his wife, his daughter, his son, and the cook. His daughter and son got half the estate, sharing in the ratio of 4 to 3. His wife got twice as much as the son. If the cook received a bequest of $\$500$, then the entire estate was:
(A) $\$3500$ **(B)** $\$5500$ **(C)** $\$6500$ **(D)** $\$7000$ **(E)** $\$7500$

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29 The points of intersection of $xy = 12$ and $x^2 + y^2 = 25$ are joined in succession. The resulting

figure is:
(A) a straight line **(B)** an equilateral triangle **(C)** a parallelogram **(D)** a rectangle **(E)** a square

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30 If the altitude of an equilateral triangle is $\sqrt{6}$, then the area is:
(A) $2\sqrt{2}$ **(B)** $2\sqrt{3}$ **(C)** $3\sqrt{3}$ **(D)** $6\sqrt{2}$ **(E)** 12

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31 In our number system the base is ten. If the base were changed to four you would count as follows: $1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, \ldots$ The twentieth number would be:
(A) 20 **(B)** 38 **(C)** 44 **(D)** 104 **(E)** 110

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32 George and Henry started a race from opposite ends of the pool. After a minute and a half, they passed each other in the center of the pool. If they lost no time in turning and maintained their respective speeds, how many minutes after starting did they pass each other the second time?
(A) 3 **(B)** $4\frac{1}{2}$ **(C)** 6 **(D)** $7\frac{1}{2}$ **(E)** 9

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33 The number $\sqrt{2}$ is equal to:
(A) a rational fraction **(B)** a finite decimal **(C)** 1.41421 **(D)** an infinite repeating decimal **(E)** an in

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34 If n is any whole number, $n^2(n^2-1)$ is always divisible by
(A) 12 **(B)** 24 **(C)** any multiple of 12 **(D)** $12-n$ **(E)** 12 and 24

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35 A rhombus is formed by two radii and two chords of a circle whose radius is 16 feet. The area of the rhombus in square feet is:
(A) 128 **(B)** $128\sqrt{3}$ **(C)** 256 **(D)** 512 **(E)** $512\sqrt{3}$

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36 If the sum $1+2+3+\cdots+K$ is a perfect square N^2 and if N is less than 100, then the possible values for K are:
(A) only 1 **(B)** 1 and 8 **(C)** only 8 **(D)** 8 and 49 **(E)** $1, 8,$ and 49

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37 On a map whose scale is 400 miles to an inch and a half, a certain estate is represented by a rhombus having a 60° angle. The diagonal opposite 60° is $\frac{3}{16}$ in. The area of the estate in square miles is:
(A) $\frac{2500}{\sqrt{3}}$ **(B)** $\frac{1250}{\sqrt{3}}$ **(C)** 1250 **(D)** $\frac{5625\sqrt{3}}{2}$ **(E)** $1250\sqrt{3}$

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38 In a right triangle with sides a and b, and hypotenuse c, the altitude drawn on the hypotenuse is x. Then:
(A) $ab=x^2$ **(B)** $\frac{1}{a}+\frac{1}{b}=\frac{1}{x}$ **(C)** $a^2+b^2=2x^2$ **(D)** $\frac{1}{x^2}=\frac{1}{a^2}+\frac{1}{b^2}$ **(E)** $\frac{1}{x}=\frac{b}{a}$

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39 The hypotenuse c and one arm a of a right triangle are consecutive integers. The square of the

second arm is:
(A) ca **(B)** $\frac{c}{a}$ **(C)** $c+a$ **(D)** $c-a$ **(E)** none of these

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40 If $V=gt+V_0$ and $S=\frac{1}{2}gt^2+V_0t$, then t equals:
(A) $\frac{2S}{V+V_0}$ **(B)** $\frac{2S}{V-V_0}$ **(C)** $\frac{2S}{V_0-V}$ **(D)** $\frac{2S}{V}$ **(E)** $2S-V$

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41 The equation $3y^2+y+4=2(6x^2+y+2)$ where $y=2x$ is satisfied by:
(A) no value of x **(B)** all values of x **(C)** $x=0$ only **(D)** all integral values of x only **(E)** all rationa

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42 The equation $\sqrt{x+4}-\sqrt{x-3}+1=0$ has:
(A) no root **(B)** one real root **(C)** one real root and one imaginary root **(D)** two imaginary roots **(E)** tw

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43 The number of scalene triangles having all sides of integral lengths, and perimeter less than 13 is:
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 18

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44 If $x<a<0$ means that x and a are numbers such that x is less than a and a is less than zero, then:
(A) $x^2<ax<0$ **(B)** $x^2>ax>a^2$ **(C)** $x^2<a^2<0$ **(D)** $x^2>ax$ but $ax<0$ **(E)** $x^2>a^2$ but $a^2<0$

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45 A wheel with a rubber tire has an outside diameter of 25 in. When the radius has been decreased a quarter of an inch, the number of revolutions in one mile will:
(A) be increased about 2% **(B)** be increased about 1% **(C)** be increased about 20% **(D)** be increased

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46 For the equation $\frac{1+x}{1-x}=\frac{N+1}{N}$ to be true where N is positive, x can have:
(A) any positive value less than 1 **(B)** any value less than 1 **(C)** the value zero only **(D)** any non - ne

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47 An engineer said he could finish a highway section in 3 days with his present supply of a certain type of machine. However, with 3 more of these machines the job could be done in 2 days. If the machines all work at the same rate, how many days would it take to do the job with one machine?
(A) 6 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 36

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48 If p is a positive integer, then $\frac{3p+25}{2p-5}$ can be a positive integer, if and only if p is:
(A) at least 3 **(B)** at least 3 and no more than 35 **(C)** no more than 35 **(D)** equal to 35 **(E)** equal t

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49 Triangle PAB is formed by three tangents to circle O and $< APB = 40°$; then angle AOB equals:
(A) $45°$ **(B)** $50°$ **(C)** $55°$ **(D)** $60°$ **(E)** $70°$

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50 In triangle ABC, $\overline{CA} = \overline{CB}$. On CB square $BCDE$ is constructed away from the triangle. If x is the number of degrees in angle DAB, then
(A) x depends upon triangle ABC **(B)** x is independent of the triangle **(C)** x may equal angle CAD
(E) x is greater than $45°$ but less than $90°$

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