AMC 12/AHSME 1958

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1 The value of $[2-3(2-3)^{-1}]^{-1}$ is:
(A) 5 **(B)** -5 **(C)** $\frac{1}{5}$ **(D)** $-\frac{1}{5}$ **(E)** $\frac{5}{3}$

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2 If $\frac{1}{x}-\frac{1}{y}=\frac{1}{z}$, then z equals:
(A) $y-x$ **(B)** $x-y$ **(C)** $\frac{y-x}{xy}$ **(D)** $\frac{xy}{y-x}$ **(E)** $\frac{xy}{x-y}$

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3 Of the following expressions the one equal to $\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}}$ is:
(A) $\frac{a^2b^2}{b^2-a^2}$ **(B)** $\frac{a^2b^2}{b^3-a^3}$ **(C)** $\frac{ab}{b^3-a^3}$ **(D)** $\frac{a^3-b^3}{ab}$ **(E)** $\frac{a^2b^2}{a-b}$

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4 In the expression $\frac{x+1}{x-1}$ each x is replaced by $\frac{x+1}{x-1}$. The resulting expression, evaluated for $x=\frac{1}{2}$, equals:
(A) 3 **(B)** -3 **(C)** 1 **(D)** -1 **(E)** none of these

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5 The expression $2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}$ equals:
(A) 2 **(B)** $2-\sqrt{2}$ **(C)** $2+\sqrt{2}$ **(D)** $2\sqrt{2}$ **(E)** $\frac{\sqrt{2}}{2}$

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6 The arithmetic mean between $\frac{x+a}{x}$ and $\frac{x-a}{x}$, when $x\neq 0$, is:
(A) 2, if $a\neq 0$ **(B)** 1 **(C)** 1, only if $a=0$ **(D)** $\frac{a}{x}$ **(E)** x

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7 A straight line joins the points $(-1,1)$ and $(3,9)$. Its x-intercept is:
(A) $-\frac{3}{2}$ **(B)** $-\frac{2}{3}$ **(C)** $\frac{2}{5}$ **(D)** 2 **(E)** 3

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8 Which of these four numbers $\sqrt{\pi^2}$, $\sqrt[3]{.8}$, $\sqrt[4]{.00016}$, $\sqrt[3]{-1}\cdot\sqrt{(.09)^{-1}}$, is (are) rational:
(A) none **(B)** all **(C)** the first and fourth **(D)** only the fourth **(E)** only the first

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9 A value of x satisfying the equation $x^2+b^2=(a-x)^2$ is:
(A) $\frac{b^2+a^2}{2a}$ **(B)** $\frac{b^2-a^2}{2a}$ **(C)** $\frac{a^2-b^2}{2a}$ **(D)** $\frac{a-b}{2}$ **(E)** $\frac{a^2-b^2}{2}$

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10 For what real values of k, other than $k=0$, does the equation $x^2+kx+k^2=0$ have real roots?
(A) $k<0$ **(B)** $k>0$ **(C)** $k\geq 1$ **(D)** all values of k **(E)** no values of k

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11 The number of roots satisfying the equation $\sqrt{5-x}=x\sqrt{5-x}$ is:
(A) unlimited **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

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12 If $P = \frac{s}{(1+k)^n}$ then n equals:
(A) $\frac{\log\left(\frac{s}{P}\right)}{\log(1+k)}$ **(B)** $\log\left(\frac{s}{P(1+k)}\right)$ **(C)** $\log\left(\frac{s-P}{1+k}\right)$
(D) $\log\left(\frac{s}{P}\right) + \log(1+k)$ **(E)** $\frac{\log(s)}{\log(P(1+k))}$

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13 The sum of two numbers is 10; their product is 20. The sum of their reciprocals is:
(A) $\frac{1}{10}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

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14 At a dance party a group of boys and girls exchange dances as follows: one boy dances with 5 girls, a second boy dances with 6 girls, and so on, the last boy dancing with all the girls. If b represents the number of boys and g the number of girls, then:
(A) $b = g$ **(B)** $b = \frac{g}{5}$ **(C)** $b = g - 4$ **(D)** $b = g - 5$
(E) It is impossible to determine a relation between b and g without knowing $b + g$.

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15 A quadrilateral is inscribed in a circle. If an angle is inscribed into each of the four segments outside the quadrilateral, the sum of these four angles, expressed in degrees, is:
(A) 1080 **(B)** 900 **(C)** 720 **(D)** 540 **(E)** 360

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16 The area of a circle inscribed in a regular hexagon is 100π. The area of hexagon is:
(A) 600 **(B)** 300 **(C)** $200\sqrt{2}$ **(D)** $200\sqrt{3}$ **(E)** $120\sqrt{5}$

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17 If x is positive and $\log x \geq \log 2 + \frac{1}{2}\log x$, then:
(A) x has no minimum or maximum value
(B) the maximum value of x is 1
(C) the minimum value of x is 1
(D) the maximum value of x is 4
(E) the minimum value of x is 4

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18 The area of a circle is doubled when its radius r is increased by n. Then r equals:
(A) $n(\sqrt{2}+1)$ **(B)** $n(\sqrt{2}-1)$ **(C)** n **(D)** $n(2-\sqrt{2})$ **(E)** $\frac{n\pi}{\sqrt{2}+1}$

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19 The sides of a right triangle are a and b and the hypotenuse is c. A perpendicular from the vertex divides c into segments r and s, adjacent respectively to a and b. If $a : b = 1 : 3$, then the ratio of r to s is:
(A) $1:3$ **(B)** $1:9$ **(C)** $1:10$ **(D)** $3:10$ **(E)** $1:\sqrt{10}$

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20 If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals:
(A) $5\sqrt{5}$ **(B)** $\sqrt{5}$ **(C)** $25\sqrt{5}$ **(D)** 125 **(E)** 25

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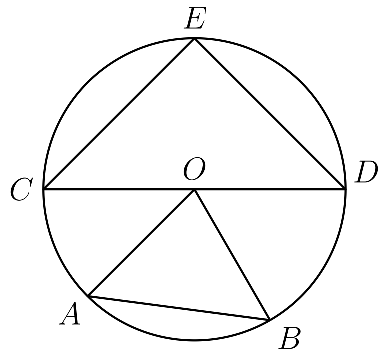
21 In the accompanying figure $\overline{CE}$ and $\overline{DE}$ are equal chords of a circle with center O. Arc AB is a quarter-circle. Then the ratio of the area of triangle CED to the area of triangle AOB is:



(A) $\sqrt{2} : 1$ **(B)** $\sqrt{3} : 1$ **(C)** $4 : 1$ **(D)** $3 : 1$ **(E)** $2 : 1$

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22 A particle is placed on the parabola $y = x^2 - x - 6$ at a point P whose y-coordinate is 6. It is allowed to roll along the parabola until it reaches the nearest point Q whose y-coordinate is -6. The horizontal distance traveled by the particle (the numerical value of the difference in the x-coordinates of P and Q) is:
(A) 5 **(B)** 4 **(C)** 3 **(D)** 2 **(E)** 1

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23 If, in the expression $x^2 - 3$, x increases or decreases by a positive amount of a, the expression changes by an amount:
(A) $\pm 2ax + a^2$ **(B)** $2ax \pm a^2$ **(C)** $\pm a^2 - 3$ **(D)** $(x + a)^2 - 3$
(E) $(x - a)^2 - 3$

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24 A man travels m feet due north at 2 minutes per mile. He returns due south to his starting point at 2 miles per minute. The average rate in miles per hour for the entire trip is:
(A) 75 **(B)** 48 **(C)** 45 **(D)** 24
(E) impossible to determine without knowing the value of m

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25 If $\log_k x \cdot \log_5 k = 3$, then x equals:
(A) k^6 **(B)** $5k^3$ **(C)** k^3 **(D)** 243 **(E)** 125

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26 A set of n numbers has the sum s. Each number of the set is increased by 20, then multiplied by 5, and then decreased by 20. The sum of the numbers in the new set thus obtained is:
(A) $s + 20n$ **(B)** $5s + 80n$ **(C)** s **(D)** $5s$ **(E)** $5s + 4n$

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27 The points $(2, -3)$, $(4, 3)$, and $(5, k/2)$ are on the same straight line. The value(s) of k is (are):
(A) 12 **(B)** -12 **(C)** ± 12 **(D)** 12 or 6 **(E)** 6 or $6\frac{2}{3}$

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28 A 16-quart radiator is filled with water. Four quarts are removed and replaced with pure antifreeze liquid. Then four quarts of the mixture are removed and replaced with pure antifreeze.

This is done a third and a fourth time. The fractional part of the final mixture that is water is:
(A) $\frac{1}{4}$ **(B)** $\frac{81}{256}$ **(C)** $\frac{27}{64}$ **(D)** $\frac{37}{64}$ **(E)** $\frac{175}{256}$

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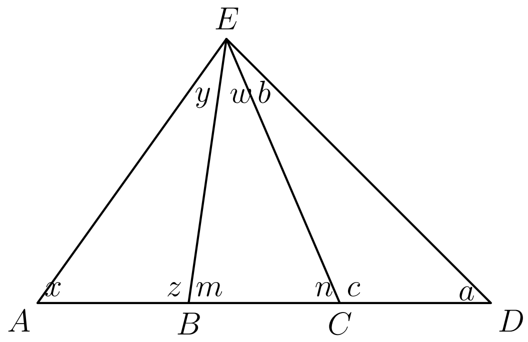
29 In a general triangle ADE (as shown) lines $\overline{EB}$ and $\overline{EC}$ are drawn. Which of the following angle relations is true?



(A) $x + z = a + b$ **(B)** $y + z = a + b$ **(C)** $m + x = w + n$
(D) $x + z + n = w + c + m$ **(E)** $x + y + n = a + b + m$

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30 If $xy = b$ and $\frac{1}{x^2} + \frac{1}{y^2} = a$, then $(x + y)^2$ equals:
(A) $(a + 2b)^2$ **(B)** $a^2 + b^2$ **(C)** $b(ab + 2)$ **(D)** $ab(b + 2)$ **(E)** $\frac{1}{a} + 2b$

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31 The altitude drawn to the base of an isosceles triangle is 8, and the perimeter 32. The area of the triangle is:
(A) 56 **(B)** 48 **(C)** 40 **(D)** 32 **(E)** 24

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32 With $\$1000$ a rancher is to buy steers at $\$25$ each and cows at $\$26$ each. If the number of steers s and the number of cows c are both positive integers, then:
(A) this problem has no solution
(B) there are two solutions with s exceeding c
(C) there are two solutions with c exceeding s
(D) there is one solution with s exceeding c
(E) there is one solution with c exceeding s

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33 For one root of $ax^2 + bx + c = 0$ to be double the other, the coefficients $a,\ b,\ c$ must be related as follows:
(A) $4b^2 = 9c$ **(B)** $2b^2 = 9ac$ **(C)** $2b^2 = 9a$
(D) $b^2 - 8ac = 0$ **(E)** $9b^2 = 2ac$

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34 The numerator of a fraction is $6x + 1$, then denominator is $7 - 4x$, and x can have any value

between -2 and 2, both included. The values of x for which the numerator is greater than the denominator are:
(A) $\frac{3}{5} < x \le 2$ **(B)** $\frac{3}{5} \le x \le 2$ **(C)** $0 < x \le 2$
(D) $0 \le x \le 2$ **(E)** $-2 \le x \le 2$

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35 A triangle is formed by joining three points whose coordinates are integers. If the x-coordinate and the y-coordinate each have a value of 1, then the area of the triangle, in square units:
(A) must be an integer **(B)** may be irrational **(C)** must be irrational **(D)** must be rational
(E) will be an integer only if the triangle is equilateral.

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36 The sides of a triangle are 30, 70, and 80 units. If an altitude is dropped upon the side of length 80, the larger segment cut off on this side is:
(A) 62 **(B)** 63 **(C)** 64 **(D)** 65 **(E)** 66

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37 The first term of an arithmetic series of consecutive integers is k^2+1. The sum of $2k+1$ terms of this series may be expressed as:
(A) $k^3+(k+1)^3$ **(B)** $(k-1)^3+k^3$ **(C)** $(k+1)^3$
(D) $(k+1)^2$ **(E)** $(2k+1)(k+1)^2$

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38 Let r be the distance from the origion to a point P with coordinates x and y. Designate the ratio $\frac{y}{r}$ by s and the ratio $\frac{x}{r}$ by c. Then the values of s^2-c^2 are limited to the numbers:
(A) less than -1 are greater than $+1$, both excluded
(B) less than -1 are greater than $+1$, both included
(C) between -1 and $+1$, both excluded
(D) between -1 and $+1$, both included
(E) -1 and $+1$ only

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39 We may say concerning the solution of

$$|x|^2+|x|-6=0$$

that:
(A) there is only one root **(B)** the sum of the roots is $+1$ **(C)** the sum of the roots is 0
(D) the product of the roots is $+4$ **(E)** the product of the roots is -6

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40 Given $a_0=1$, $a_1=3$, and the general relation $a_n^2-a_{n-1}a_{n+1}=(-1)^n$ for $n\ge 1$. Then a_3 equals:
(A) $\frac{13}{27}$ **(B)** 33 **(C)** 21 **(D)** 10 **(E)** -17

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41 The roots of $Ax^2+Bx+C=0$ are r and s. For the roots of

$$x^2+px+q=0$$

to be r^2 and s^2, p must equal:
(A) $\frac{B^2-4AC}{A^2}$ **(B)** $\frac{B^2-2AC}{A^2}$ **(C)** $\frac{2AC-B^2}{A^2}$
(D) B^2-2C **(E)** $2C-B^2$

42 In a circle with center O, chord $\overline{AB}$ equals chord $\overline{AC}$. Chord $\overline{AD}$ cuts $\overline{BC}$ in E. If $AC = 12$ and $AE = 8$, then AD equals:
(A) 27 **(B)** 24 **(C)** 21 **(D)** 20 **(E)** 18

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43 $\overline{AB}$ is the hypotenuse of a right triangle ABC. Median $\overline{AD}$ has length 7 and median $\overline{BE}$ has length 4. The length of $\overline{AB}$ is:
(A) 10 **(B)** $5\sqrt{3}$ **(C)** $5\sqrt{2}$ **(D)** $2\sqrt{13}$ **(E)** $2\sqrt{15}$

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44 Given the true statements: (1) If a is greater than b, then c is greater than d (2) If c is less than d, then e is greater than f. A valid conclusion is:
(A) If a is less than b, then e is greater than f
(B) If e is greater than f, then a is less than b
(C) If e is less than f, then a is greater than b
(D) If a is greater than b, then e is less than f
(E) none of these

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45 A check is written for x dollars and y cents, x and y both two-digit numbers. In error it is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by $\$17.82$. Then:
(A) x cannot exceed 70
(B) y can equal $2x$
(C) the amount of the check cannot be a multiple of 5
(D) the incorrect amount can equal twice the correct amount
(E) the sum of the digits of the correct amount is divisible by 9

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46 For values of x less than 1 but greater than -4, the expression

$$\frac{x^2 - 2x + 2}{2x - 2}$$

has:
(A) no maximum or minimum value
(B) a minimum value of $+1$
(C) a maximum value of $+1$
(D) a minimum value of -1
(E) a maximum value of -1

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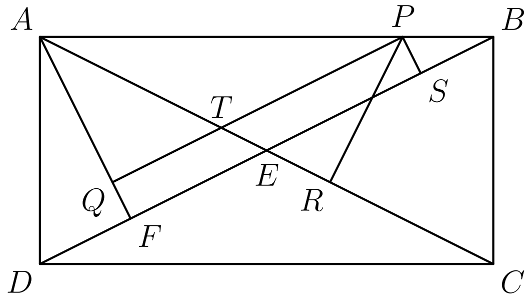
47 $ABCD$ is a rectangle (see the accompanying diagram) with P any point on $\overline{AB}$. $\overline{PS} \perp \overline{BD}$ and $\overline{PR} \perp \overline{AC}$. $\overline{AF} \perp \overline{BD}$ and $\overline{PQ} \perp \overline{AF}$. Then $PR + PS$ is equal to:



(A) PQ **(B)** AE **(C)** $PT + AT$ **(D)** AF **(E)** EF

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48 Diameter $\overline{AB}$ of a circle with center O is 10 units. C is a point 4 units from A, and on $\overline{AB}$. D is a point 4 units from B, and on $\overline{AB}$. P is any point on the circle. Then the broken-line path from C to P to D:

(A) has the same length for all positions of P
(B) exceeds 10 units for all positions of P
(C) cannot exceed 10 units
(D) is shortest when $\triangle CPD$ is a right triangle
(E) is longest when P is equidistant from C and D.

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49 In the expansion of $(a+b)^n$ there are $n+1$ dissimilar terms. The number of dissimilar terms in the expansion of $(a+b+c)^{10}$ is:
(A) 11 **(B)** 33 **(C)** 55 **(D)** 66 **(E)** 132

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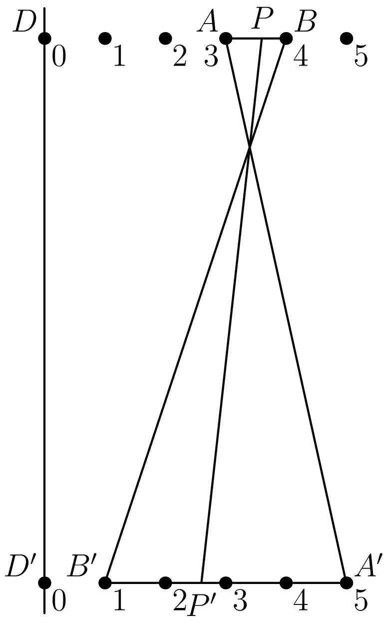
50 In this diagram a scheme is indicated for associating all the points of segment $\overline{AB}$ with those of segment $\overline{A'B'}$, and reciprocally. To described this association scheme analytically, let x be the distance from a point P on $\overline{AB}$ to D and let y be the distance from the associated point P' of $\overline{A'B'}$ to D'. Then for any pair of associated points, if $x = a,\ x + y$ equals:



(A) $13a$ **(B)** $17a - 51$ **(C)** $17 - 3a$ **(D)** $\frac{17-3a}{4}$ **(E)** $12a - 34$

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