AMC 12/AHSME 1962

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1 The expression $\frac{1^{4y-1}}{5^{-1}+3^{-1}}$ is equal to:
(A) $\frac{4y-1}{8}$ **(B)** 8 **(C)** $\frac{15}{2}$ **(D)** $\frac{15}{8}$ **(E)** $\frac{1}{8}$

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2 The expression $\sqrt{\frac{4}{3}}-\sqrt{\frac{3}{4}}$ is equal to:
(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{-\sqrt{3}}{6}$ **(C)** $\frac{\sqrt{-3}}{6}$ **(D)** $\frac{5\sqrt{3}}{6}$ **(E)** 1

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3 The first three terms of an arithmetic progression are $x-1, x+1, 2x+3,$ in the order shown. The value of x is:
(A) -2 **(B)** 0 **(C)** 2 **(D)** 4 **(E)** undetermined

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4 If $8^x = 32$, then x equals:
(A) 4 **(B)** $\frac{5}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{1}{4}$

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5 If the radius of a circle is increased by 1 unit, the ratio of the new circumference to the new diameter is:
(A) $\pi+2$ **(B)** $\frac{2\pi+1}{2}$ **(C)** π **(D)** $\frac{2\pi-1}{2}$ **(E)** $\pi-2$

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6 A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ square inches. Expressed in inches the diagonal of the square is:
(A) $\frac{9}{2}$ **(B)** $2\sqrt{5}$ **(C)** $4\sqrt{2}$ **(D)** $\frac{9\sqrt{2}}{2}$ **(E)** none of these

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7 Let the bisectors of the exterior angles at B and C of triangle ABC meet at D. Then, if all measurements are in degrees, angle BDC equals:
(A) $\frac{1}{2}(90-A)$ **(B)** $90-A$ **(C)** $\frac{1}{2}(180-A)$ **(D)** $180-A$ **(E)** $180-2A$

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8 Given the set of n numbers; $n>1$, of which one is $1-\frac{1}{n}$ and all the others are 1. The arithmetic mean of the n numbers is:
(A) 1 **(B)** $n-\frac{1}{n}$ **(C)** $n-\frac{1}{n^2}$ **(D)** $1-\frac{1}{n^2}$ **(E)** $1-\frac{1}{n}-\frac{1}{n^2}$

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9 When x^9-x is factored as completely as possible into polynomials and monomials with integral coefficients, the number of factors is:
(A) more than 5 **(B)** 5 **(C)** 4 **(D)** 3 **(E)** 2

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10 A man drives 150 miles to the seashore in 3 hours and 20 minutes. He returns from the shore to the starting point in 4 hours and 10 minutes. Let r be the average rate for the entire trip.

Then the average rate for the trip going exceeds r in miles per hour, by:
(A) 5 **(B)** $4\frac{1}{2}$ **(C)** 4 **(D)** 2 **(E)** 1

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11 The difference between the larger root and the smaller root of $x^2 - px + (p^2 - 1)/4 = 0$ is:
(A) 0 **(B)** 1 **(C)** 2 **(D)** p **(E)** $p+1$

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12 When $\left(1 - \frac{1}{a}\right)^6$ is expanded the sum of the last three coefficients is:
(A) 22 **(B)** 11 **(C)** 10 **(D)** -10 **(E)** -11

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13 R varies directly as S and inverse as T. When $R = \frac{4}{3}$ and $T = \frac{9}{14}$, $S = \frac{3}{7}$. Find S when $R = \sqrt{48}$ and $T = \sqrt{75}$.
(A) 28 **(B)** 30 **(C)** 40 **(D)** 42 **(E)** 60

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14 Let s be the limiting sum of the geometric series $4 - \frac{8}{3} + \frac{16}{9} - \dots$, as the number of terms increases without bound. Then s equals:
(A) a number between 0 and 1 **(B)** 2.4 **(C)** 2.5 **(D)** 3.6 **(E)** 12

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15 Given triangle ABC with base AB fixed in length and position. As the vertex C moves on a straight line, the intersection point of the three medians moves on:
(A) a circle **(B)** a parabola **(C)** an ellipse **(D)** a straight line **(E)** a curve here not listed

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16 Given rectangle R_1 with one side 2 inches and area 12 square inches. Rectangle R_2 with diagonal 15 inches is similar to R_1. Expressed in square inches the area of R_2 is:
(A) $\frac{9}{2}$ **(B)** 36 **(C)** $\frac{135}{2}$ **(D)** $9\sqrt{10}$ **(E)** $\frac{27\sqrt{10}}{4}$

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17 If $a = \log_8 225$ and $b = \log_2 15$, then a, in terms of b, is:
(A) $\frac{b}{2}$ **(B)** $\frac{2b}{3}$ **(C)** b **(D)** $\frac{3b}{2}$ **(E)** $2b$

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18 A regular dodecagon (12 sides) is inscribed in a circle with radius r inches. The area of the dodecagon, in square inches, is:
(A) $3r^2$ **(B)** $2r^2$ **(C)** $\frac{3r^2\sqrt{3}}{4}$ **(D)** $r^2\sqrt{3}$ **(E)** $3r^2\sqrt{3}$

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19 If the parabola $y = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$, and $(2, -3)$, the value of $a + b + c$ is:
(A) -4 **(B)** -2 **(C)** 0 **(D)** 1 **(E)** 2

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20 The angles of a pentagon are in arithmetic progression. One of the angles in degrees, must be:
(A) 108 **(B)** 90 **(C)** 72 **(D)** 54 **(E)** 36

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21 It is given that one root of $2x^2 + rx + s = 0$, with r and s real numbers, is $3 + 2i (i = \sqrt{-1})$. The value of s is:

(A) undetermined **(B)** 5 **(C)** 6 **(D)** -13 **(E)** 26

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22 The number 121_b, written in the integral base b, is the square of an integer, for
(A) $b = 10,$ only **(B)** $b = 10$ and $b = 5,$ only **(C)** $2 \le b \le 10$ **(D)** $b > 2$ **(E)** no value of b

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23 In triangle ABC, CD is the altitude to AB and AE is the altitude to BC. If the lengths of AB, CD, and AE are known, the length of DB is:
(A) not determined by the information given **(B)** determined only if A is an acute angle
(C) determined only if B is an acute angle **(D)** determined only in ABC is an acute triangle
(E) none of these is correct

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24 Three machines P, Q, and R, working together, can do a job in x hours. When working alone, P needs an additional 6 hours to do the job; Q, one additional hour; and R, x additional hours. The value of x is:
(A) $\frac{2}{3}$ **(B)** $\frac{11}{12}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** 3

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25 Given square $ABCD$ with side 8 feet. A circle is drawn through vertices A and D and tangent to side BC. The radius of the circle, in feet, is:
(A) 4 **(B)** $4\sqrt{2}$ **(C)** 5 **(D)** $5\sqrt{2}$ **(E)** 6

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26 For any real value of x the maximum value of $8x - 3x^2$ is:
(A) 0 **(B)** $\frac{8}{3}$ **(C)** 4 **(D)** 5 **(E)** $\frac{16}{3}$

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27 Let $a@b$ represent the operation on two numbers, a and b, which selects the larger of the two numbers, with $a@a = a$. Let $a!b$ represent the operator which selects the smaller of the two numbers, with $a!a = a$. Which of the following three rules is (are) correct?
(1) $a@b = b@a$ **(2)** $a@(b@c) = (a@b)@c$ **(3)** $a!(b@c) = (a!b)@(a!c)$
(A) (1) only **(B)** (2) only **(C)** (1) and (2) only **(D)** (1) and (3) only **(E)** all three

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28 The set of x-values satisfying the equation $x^{\log_{10} x} = \frac{x^3}{100}$ consists of:
(A) $\frac{1}{10}$ **(B)** 10, only **(C)** 100, only **(D)** 10 or 100, only **(E)** more than two real numbers.

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29 Which of the following sets of x-values satisfy the inequality $2x^2 + x < 6$?
(A) $-2 < x < \frac{3}{2}$ **(B)** $x > \frac{3}{2}$ or $x < -2$ **(C)** $x < \frac{3}{2}$ **(D)** $\frac{3}{2} < x < 2$ **(E)** $x < -2$

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30 Consider the statements:
(1) p and q are both true **(2)** p is true and q is false **(3)** p is false and q is true **(4)** p is false and

How many of these imply the negative of the statement "p and q are both true?"
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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31 The ratio of the interior angles of two regular polygons with sides of unit length is $3:2$. How many such pairs are there?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

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32 If $x_{k+1} = x_k + \frac{1}{2}$ for $k = 1, 2, \dots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \dots + x_n$.
(A) $\frac{n+1}{2}$ **(B)** $\frac{n+3}{2}$ **(C)** $\frac{n^2-1}{2}$ **(D)** $\frac{n^2+n}{4}$ **(E)** $\frac{n^2+3n}{4}$

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33 The set of x-values satisfying the inequality $2 \le |x-1| \le 5$ is:
(A) $-4 \le x \le -1$ or $3 \le x \le 6$ **(B)** $3 \le x \le 6$ or $-6 \le x \le -3$ **(C)** $x \le -1$ or $x \ge 3$ **(D)** $-1 \le x \le 3$ **(E)** $-4 \le x \le 6$

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34 For what real values of K does $x = K^2(x-1)(x-2)$ have real roots?
(A) none **(B)** $-2 < K < 1$ **(C)** $-2\sqrt{2} < K < 2\sqrt{2}$ **(D)** $K > 1$ or $K < -2$ **(E)** all

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35 A man on his way to dinner short after $6:00$ p.m. observes that the hands of his watch form an angle of 110°. Returning before $7:00$ p.m. he notices that again the hands of his watch form an angle of 110°. The number of minutes that he has been away is:
(A) $36\frac{2}{3}$ **(B)** 40 **(C)** 42 **(D)** 42.4 **(E)** 45

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36 If both x and y are both integers, how many pairs of solutions are there of the equation $(x-8)(x-10) = 2^y$?
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

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37 $ABCD$ is a square with side of unit length. Points E and F are taken respectively on sides AB and AD so that $AE = AF$ and the quadrilateral $CDFE$ has maximum area. In square units this maximum area is:
(A) $\frac{1}{2}$ **(B)** $\frac{9}{16}$ **(C)** $\frac{19}{32}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{2}{3}$

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38 The population of Nosuch Junction at one time was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square.

The original population is a multiple of:
(A) 3 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 17

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39 Two medians of a triangle with unequal sides are 3 inches and 6 inches. Its area is $3\sqrt{15}$ square inches. The length of the third median in inches, is:
(A) 4 **(B)** $3\sqrt{3}$ **(C)** $3\sqrt{6}$ **(D)** $6\sqrt{3}$ **(E)** $6\sqrt{6}$

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40 The limiting sum of the infinite series, $\frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \dots$ whose nth term is $\frac{n}{10^n}$ is:
(A) $\frac{1}{9}$ **(B)** $\frac{10}{81}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{17}{72}$ **(E)** larger than any finite quantity

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