AMC 12/AHSME 1979

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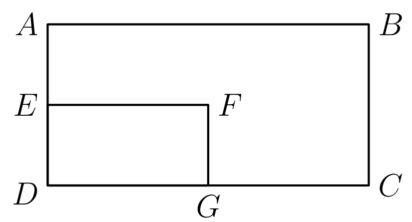

If rectangle $ABCD$ has area 72 square meters and E and G are the midpoints of sides AD and CD, respectively, then the area of rectangle $DEFG$ in square meters is

(A) 8 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

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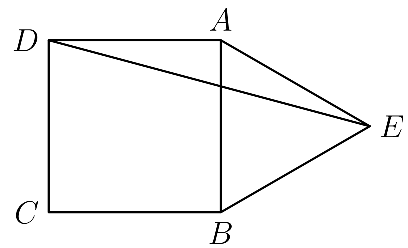
2 For all non-zero real numbers x and y such that $x - y = xy$, $\frac{1}{x} - \frac{1}{y}$ equals

(A) $\frac{1}{xy}$ **(B)** $\frac{1}{x-y}$ **(C)** 0 **(D)** -1 **(E)** $y - x$

3



In the adjoining figure, $ABCD$ is a square, ABE is an equilateral triangle and point E is outside square $ABCD$. What is the measure of $\measuredangle AED$ in degrees?

(A) 10 **(B)** 12.5 **(C)** 15 **(D)** 20 **(E)** 25

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4 For all real numbers x, $x[x\{x(2-x)-4\}+10]+1=$

(A) $-x^4+2x^3+4x^2+10x+1$

(B) $-x^4 - 2x^3 + 4x^2 + 10x + 1$

(C) $-x^4 - 2x^3 - 4x^2 + 10x + 1$

(D) $-x^4 - 2x^3 - 4x^2 - 10x + 1$

(E) $-x^4 + 2x^3 - 4x^2 + 10x + 1$

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5 Find the sum of the digits of the largest even three digit number (in base ten representation) which is not changed when its units and hundreds digits are interchanged.

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

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6 $\frac{3}{2} + \frac{5}{4} + \frac{9}{8} + \frac{17}{16} + \frac{33}{32} + \frac{65}{64} - 7 =$

(A) $-\frac{1}{64}$ **(B)** $-\frac{1}{16}$ **(C)** 0 **(D)** $\frac{1}{16}$ **(E)** $\frac{1}{64}$

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7 The square of an integer is called a *perfect square*. If x is a perfect square, the next larger perfect square is

(A) $x + 1$ **(B)** $x^2 + 1$ **(C)** $x^2 + 2x + 1$ **(D)** $x^2 + x$ **(E)** $x + 2\sqrt{x} + 1$

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8 Find the area of the smallest region bounded by the graphs of $y = |x|$ and $x^2 + y^2 = 4$.

(A) $\frac{\pi}{4}$ **(B)** $\frac{3\pi}{4}$ **(C)** π **(D)** $\frac{3\pi}{2}$ **(E)** 2π

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9 The product of $\sqrt[3]{4}$ and $\sqrt[4]{8}$ equals

(A) $\sqrt[7]{12}$ **(B)** $2\sqrt[7]{12}$ **(C)** $\sqrt[7]{32}$ **(D)** $\sqrt[12]{32}$ **(E)** $2\sqrt[12]{32}$

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10 If $P_1P_2P_3P_4P_5P_6$ is a regular hexagon whose apothem (distance from the center to midpoint of a side) is 2, and Q_i is the midpoint of side P_iP_{i+1} for $i = 1, 2, 3, 4$, then the area of quadrilateral $Q_1Q_2Q_3Q_4$ is

(A) 6 **(B)** $2\sqrt{6}$ **(C)** $\frac{8\sqrt{3}}{3}$ **(D)** $3\sqrt{3}$ **(E)** $4\sqrt{3}$

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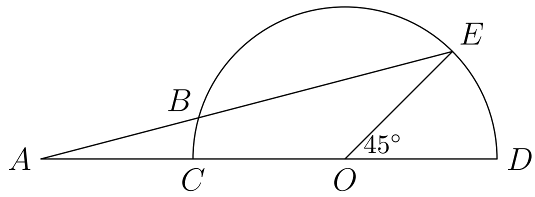
11 Find a positive integral solution to the equation

$$\frac{1 + 3 + 5 + \cdots + (2n - 1)}{2 + 4 + 6 + \cdots + 2n} = \frac{115}{116}$$

(A) 110 **(B)** 115 **(C)** 116 **(D)** 231 **(E)** The equation has no positive integral solutions.

12



In the adjoining figure, CD is the diameter of a semi-circle with center O. Point A lies on the extension of DC past C; point E lies on the semi-circle, and B is the point of intersection (distinct from E) of line segment AE with the semi-circle. If length AB equals length OD, and the measure of $\measuredangle EOD$ is 45°, then the
measure of $\measuredangle BAO$ is

(A) 10° **(B)** 15° **(C)** 20° **(D)** 25° **(E)** 30°

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13 The inequality $y - x < \sqrt{x^2}$ is satisfied if and only if

(A) $y < 0$ or $y < 2x$ (or both inequalities hold)

(B) $y > 0$ or $y < 2x$ (or both inequalities hold)

(C) $y^2 < 2xy$ **(D)** $y < 0$ **(E)** $x > 0$ and $y < 2x$

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14 In a certain sequence of numbers, the first number is 1, and, for all $n \ge 2$, the product of the first n numbers in the sequence is n^2. The sum of the third and the fifth numbers in the sequence is

(A) $\frac{25}{9}$ **(B)** $\frac{31}{15}$ **(C)** $\frac{61}{16}$ **(D)** $\frac{576}{225}$ **(E)** 34

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15 Two identical jars are filled with alcohol solutions, the ratio of the volume of alcohol to the volume of water being $p : 1$ in one jar and $q : 1$ in the other jar. If the entire contents of the two jars are mixed together, the ratio of the volume of alcohol to the volume of water in the mixture is

(A) $\frac{p+q}{2}$ **(B)** $\frac{p^2+q^2}{p+q}$ **(C)** $\frac{2pq}{p+q}$ **(D)** $\frac{2(p^2+pq+q^2)}{3(p+q)}$ **(E)** $\frac{p+q+2pq}{p+q+2}$

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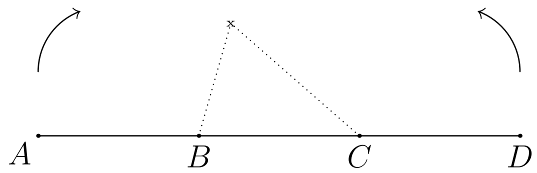
16 A circle with area A_1 is contained in the interior of a larger circle with area $A_1 + A_2$. If the radius of the larger circle is 3, and if $A_1, A_2, A_1 + A_2$ is an arithmetic progression, then the radius of the smaller circle is

(A) $\frac{\sqrt{3}}{2}$ **(B)** 1 **(C)** $\frac{2}{\sqrt{3}}$ **(D)** $\frac{3}{2}$ **(E)** $\sqrt{3}$

17



Points A, B, C, and D are distinct and lie, in the given order, on a straight line. Line segments AB, AC, and AD have lengths x, y, and z , respectively. If line segments AB and CD may be rotated about points B and C, respectively, so that points A and D coincide, to form a triangle with positive area, then which of the following three inequalities must be satisfied?

I. $x < \frac{z}{2}$ **II.** $y < x + \frac{z}{2}$ **III.** $y < \frac{z}{2}$

(A) **I.** only **(B)** **II.** only

(C) **I.** and **II.** only **(D)** **II.** and **III.** only **(E)** **I.** , **II.** , and **III.**

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18 To the nearest thousandth, $\log_{10} 2$ is $.301$ and $\log_{10} 3$ is $.477$. Which of the following is the best approximation of $\log_5 10$?

(A) $\frac{8}{7}$ **(B)** $\frac{9}{7}$ **(C)** $\frac{10}{7}$ **(D)** $\frac{11}{7}$ **(E)** $\frac{12}{7}$

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19 Find the sum of the squares of all real numbers satisfying the equation

$$x^{256} - 256^{32} = 0.$$

(A) 8 **(B)** 128 **(C)** 512 **(D)** $65,536$ **(E)** $2(256^{32})$

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20 If $a = \frac{1}{2}$ and $(a + 1)(b + 1) = 2$ then the radian measure of $\arctan a + \arctan b$ equals

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{5}$ **(E)** $\frac{\pi}{6}$

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21 The length of the hypotenuse of a right triangle is h , and the radius of the inscribed circle is r. The ratio of the area of the
circle to the area of the triangle is

(A) $\frac{\pi r}{h+2r}$ **(B)** $\frac{\pi r}{h+r}$ **(C)** $\frac{\pi}{2h+r}$ **(D)** $\frac{\pi r^2}{r^2+h^2}$ **(E)** none of these

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22 Find the number of pairs (m, n) of integers which satisfy the equation $m^3 + 6m^2 + 5m = 27n^3 + 9n^2 + 9n + 1$.

(A) 0 **(B)** 1 **(C)** 3 **(D)** 9 **(E)** infinitely many

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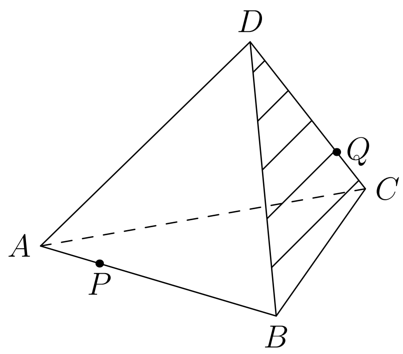
23 The edges of a regular tetrahedron with vertices $A,\ B,\ C$, and D each have length one. Find the least possible distance between a pair of points P and Q, where P is on edge AB and Q is on edge CD.

(A) $\frac{1}{2}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{3}}{2}$ **(E)** $\frac{\sqrt{3}}{3}$



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24 Sides $AB,\ BC$, and CD of (simple*) quadrilateral $ABCD$ have lengths $4,\ 5$, and 20, respectively. If vertex angles B and C are obtuse and $\sin C = -\cos B = \frac{3}{5}$, then side AD has length

(A) 24 **(B)** 24.5 **(C)** 24.6 **(D)** 24.8 **(E)** 25

*A polygon is called simple if it is not self intersecting.

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25 If $q_1(x)$ and r_1 are the quotient and remainder, respectively, when the polynomial x^8 is divided by $x+\frac{1}{2}$, and if $q_2(x)$ and r_2 are the quotient and remainder, respectively, when $q_1(x)$ is divided by $x+\frac{1}{2}$, then r_2 equals

(A) $\frac{1}{256}$ **(B)** $-\frac{1}{16}$ **(C)** 1 **(D)** -16 **(E)** 256

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26 The function f satisfies the functional equation

$$f(x)+f(y)=f(x+y)-xy-1$$

for every pair $x,\ y$ of real numbers. If $f(1)=1$, then the number of integers $n\neq 1$ for which $f(n)=n$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinite

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27 An ordered pair (b,c) of integers, each of which has absolute value less than or equal to five, is chosen at random, with each
such ordered pair having an equal likelihood of being chosen. What is the probability that the equation $x^2+bx+c=0$ will
not have distinct positive real roots?

(A) $\frac{106}{121}$ **(B)** $\frac{108}{121}$ **(C)** $\frac{110}{121}$ **(D)** $\frac{112}{121}$ **(E)** none of these

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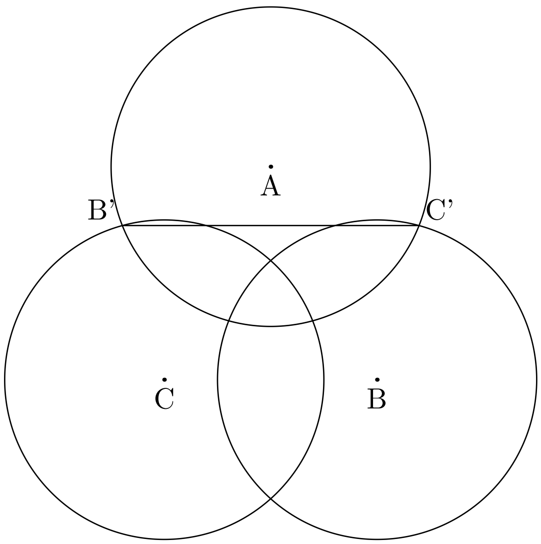
28 Circles with centers $A,\ B$, and C each have radius r, where $1 < r < 2$. The distance between each pair of centers is 2. If B' is the point of intersection of circle A and circle C which is outside circle B, and if C' is the point of intersection of circle A and circle B which is outside circle C, then length $B'C'$ equals

(A) $3r-2$ **(B)** r^2 **(C)** $r+\sqrt{3(r-1)}$

(D) $1+\sqrt{3(r^2-1)}$ **(E)** none of these



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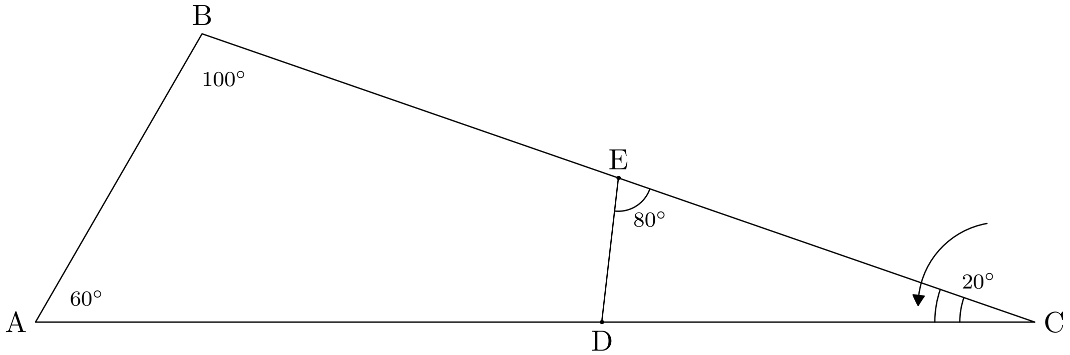
29 For each positive number x, let

$$f(x)=\frac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+\left(x^3+\frac{1}{x^3}\right)}.$$

The minimum value of $f(x)$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

30



In $\triangle ABC$, E is the midpoint of side BC and D is on side AC. If the length of AC is 1 and $\measuredangle BAC = 60^\circ$, $\measuredangle ABC = 100^\circ$, $\measuredangle ACB = 20^\circ$ and $\measuredangle DEC = 80^\circ$, then the area of $\triangle ABC$ plus twice the area of $\triangle CDE$ equals

(A) $\frac{1}{4}\cos 10^\circ$ **(B)** $\frac{\sqrt{3}}{8}$ **(C)** $\frac{1}{4}\cos 40^\circ$ **(D)** $\frac{1}{4}\cos 50^\circ$ **(E)** $\frac{1}{8}$

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