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- 1 Determine all non-negative integral solutions $(n_1, n_2, \dots, n_{14})$ if any, apart from permutations, of the Diophantine Equation

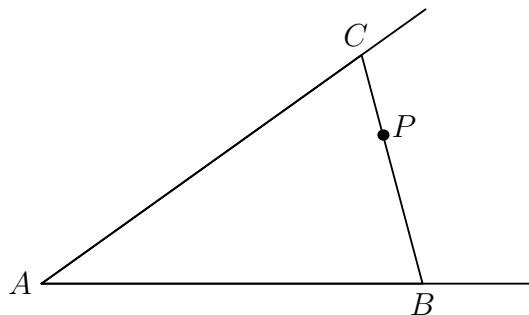
$$n_1^4 + n_2^4 + \dots + n_{14}^4 = 1,599.$$

- 2 Let S be a great circle with pole P . On any great circle through P , two points A and B are chosen equidistant from P . For any *spherical triangle* ABC (the sides are great circles arcs), where C is on S , prove that the great circle arc CP is the angle bisector of angle C .

Note. A great circle on a sphere is one whose center is the center of the sphere. A pole of the great circle S is a point P on the sphere such that the diameter through P is perpendicular to the plane of S .

- 3 Given three identical n -faced dice whose corresponding faces are identically numbered with arbitrary integers. Prove that if they are tossed at random, the probability that the sum of the bottom three face numbers is divisible by three is greater than or equal to $\frac{1}{4}$.

- 4 Show how to construct a chord BPC of a given angle A through a given point P such that $\frac{1}{BP} + \frac{1}{PC}$ is a maximum.



- 5 A certain organization has n members, and it has $n + 1$ three-member committees, no two of which have identical membership. Prove that there are two committees which share exactly one member.



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