AMC 12/AHSME 1967

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1 The three-digit number $2a3$ is added to the number 326 to give the three-digit number $5b9$. If $5b9$ is divisible by 9, then $a+b$ equals

(A) 2 (B) 4 (C) 6 (D) 8 (E) 9

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2 An equivalent of the expression

$\left(\frac{x^2+1}{x}\right)\left(\frac{y^2+1}{y}\right)+\left(\frac{x^2-1}{y}\right)\left(\frac{y^2-1}{x}\right)$, $xy\neq 0$,

is:

(A) 1 (B) $2xy$ (C) $2x^2y^2+2$ (D) $2xy+\frac{2}{xy}$ (E) $\frac{2x}{y}+\frac{2y}{x}$

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3 The side of an equilateral triangle is s. A circle is inscribed in the triangle and a square is inscribed in the circle. The area of the square is:

(A) $\frac{s^2}{24}$ (B) $\frac{s^2}{6}$ (C) $\frac{s^2\sqrt{2}}{6}$ (D) $\frac{s^2\sqrt{3}}{6}$ (E) $\frac{s^2}{3}$

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4 Given $\frac{\log a}{p}=\frac{\log b}{q}=\frac{\log c}{r}=\log x$, all logarithms to the same base and $x\neq 1$. If $\frac{b^2}{ac}=x^y$, then y is:

(A) $\frac{q^2}{p+r}$ (B) $\frac{p+r}{2q}$ (C) $2q-p-r$ (D) $2q-pr$ (E) q^2-pr

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5 A triangle is circumscribed about a circle of radius r inches. If the perimeter of the triangle is P inches and the area is K square inches, then $\frac{P}{K}$ is:

(A)independent of the value of r (B) $\frac{\sqrt{2}}{r}$ (C) $\frac{2}{\sqrt{r}}$ (D) $\frac{2}{r}$ (E) $\frac{r}{2}$

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6 If $f(x)=4^x$ then $f(x+1)-f(x)$ equals:

(A) 4 (B) $f(x)$ (C) $2f(x)$ (D) $3f(x)$ (E) $4f(x)$

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7 If $\frac{a}{b}<-\frac{c}{d}$ where a, b, c, and d are real numbers and $bd\neq 0$, then:

(A) a must be negative (B) a must be positive (C) a must not be zero (D) a can be negative or
(E) a can be positive, negative, or zero

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8 To m ounces of a $m\%$ solution of acid, x ounces of water are added to yield a $(m-10)\%$ solution. If $m>25$, then x is

(A) $\frac{10m}{m-10}$ **(B)** $\frac{5m}{m-10}$ **(C)** $\frac{m}{m-10}$ **(D)** $\frac{5m}{m-20}$
(E) not determined by the given information

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9 Let K, in square units, be the area of a trapezoid such that the shorter base, the altitude, and the longer base, in that order, are in arithmetic progression. Then:

(A) K must be an integer **(B)** K must be a rational fraction
(C) K must be an irrational number **(D)** K must be an integer or a rational fraction **(E)** taken alon

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10 If $\frac{a}{10^x-1}+\frac{b}{10^x+2}=\frac{2\cdot 10^x+3}{(10^x-1)(10^x+2)}$ is an identity for positive rational values of x, then the value of $a-b$ is:

(A) $\frac{4}{3}$ **(B)** $\frac{5}{3}$ **(C)** 2 **(D)** $\frac{11}{4}$ **(E)** 3

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11 If the perimeter of rectangle $ABCD$ is 20 inches, the least value of diagonal $\overline{AC}$, in inches, is:

(A) 0 **(B)** $\sqrt{50}$ **(C)** 10 **(D)** $\sqrt{200}$ **(E)** none of these

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12 If the (convex) area bounded by the x-axis and the lines $y=mx+4$, $x=1$, and $x=4$ is 7, then m equals:

(A) $-\frac{1}{2}$ **(B)** $-\frac{2}{3}$ **(C)** $-\frac{3}{2}$ **(D)** -2 **(E)** none of these

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13 A triangle ABC is to be constructed given a side a (oppisite angle A). angle B, and h_c, the altitude from C. If N is the number of noncongruent solutions, then N

(A) is 1 **(B)** is 2 **(C)** must be zero **(D)** must be infinite **(E)** must be zero or infinite

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14 Let $f(t)=\frac{t}{1-t}$, $t\neq 1$. If $y=f(x)$, then x can be expressed as

(A) $f\left(\frac{1}{y}\right)$ **(B)** $-f(y)$ **(C)** $-f(-y)$ **(D)** $f(-y)$ **(E)** $f(y)$

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15 The difference in the areas of two similar triangles is 18 square feet, and the ratio of the larger area to the smaller is the square of an integer. The area of the smaller triange, in square feet, is an integer, and one of its sides is 3 feet. The corresponding side of the larger triangle, in feet, is:

(A) 12 **(B)** 9 **(C)** $6\sqrt{2}$ **(D)** 6 **(E)** $3\sqrt{2}$

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16 Let the product $(12)(15)(16)$, each factor written in base b, equal 3146 in base b. Let $s=12+15+16$, each term expressed in base b. Then s, in base b, is

(A) 43 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 47

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17 If r_1 and r_2 are the distinct real roots of $x^2 + px + 8 = 0$, then it must follow that:

(A) $|r_1 + r_2| > 4\sqrt{2}$ **(B)** $|r_1| > 3$ or $|r_2| > 3$
(C) $|r_1| > 2$ and $|r_2| > 2$ **(D)** $r_1 < 0$ and $r_2 < 0$ **(E)** $|r_1 + r_2| < 4\sqrt{2}$

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18 If $x^2 - 5x + 6 < 0$ and $P = x^2 + 5x + 6$ then

(A) P can take any real value **(B)** $20 < P < 30$
(C) $0 < P < 20$ **(D)** $P < 0$ **(E)** $P > 30$

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19 The area of a rectangle remains unchanged when it is made $2\frac{1}{2}$ inches longer and $\frac{2}{3}$ inch narrower, or when it is made $2\frac{1}{2}$ inches shorter and $\frac{4}{3}$ inch wider. Its area, in square inches, is:

(A) 30 **(B)** $\frac{80}{3}$ **(C)** 24 **(D)** $\frac{45}{2}$ **(E)** 20

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20 A circle is inscribed in a square of side m, then a square is inscribed in that circle, then a circle is inscribed in the latter square, and so on. If S_n is the sum of the areas of the first n circles so inscribed, then, as n grows beyond all bounds, S_n approaches:

(A) $\frac{\pi m^2}{2}$ **(B)** $\frac{3\pi m^2}{8}$ **(C)** $\frac{\pi m^2}{3}$ **(D)** $\frac{\pi m^2}{4}$ **(E)** $\frac{\pi m^2}{8}$

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21 In right triangle ABC the hypotenuse $\overline{AB} = 5$ and leg $\overline{AC} = 3$. The bisector of angle A meets the opposite side in A_1. A second right triangle PQR is then constructed with hypotenuse $\overline{PQ} = A_1B$ and leg $\overline{PR} = A_1C$. If the bisector of angle P meets the opposite side in P_1, the length of PP_1 is:

(A) $\frac{3\sqrt{6}}{4}$ **(B)** $\frac{3\sqrt{5}}{4}$ **(C)** $\frac{3\sqrt{3}}{4}$ **(D)** $\frac{3\sqrt{2}}{2}$ **(E)** $\frac{15\sqrt{2}}{16}$

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22 For natural numbers, when P is divided by D, the quotient is Q and the remainder is R. When Q is divided by D', the quotient is Q' and the remainder is R'. Then, when P is divided by DD', the remainder is:

(A) $R + R'D$ **(B)** $R' + RD$ **(C)** RR' **(D)** R **(E)** R'

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23 If x is real and positive and grows beyond all bounds, then $\log_3(6x - 5) - \log_3(2x + 1)$ approaches:

(A) 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** no finite number

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24 The number of solution-pairs in the positive integers of the equation $3x + 5y = 501$ is:

(A) 33 **(B)** 34 **(C)** 35 **(D)** 100 **(E)** none of these

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25 For every odd number $p > 1$ we have:

(A) $(p-1)^{\frac{1}{2}(p-1)}-1$ is divisible by $p-2$ **(B)** $(p-1)^{\frac{1}{2}(p-1)}+1$ is divisible by p
(C) $(p-1)^{\frac{1}{2}(p-1)}$ is divisible by p **(D)** $(p-1)^{\frac{1}{2}(p-1)}+1$ is divisible by $p+1$
(E) $(p-1)^{\frac{1}{2}(p-1)}-1$ is divisible by $p-1$

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26 If one uses only the tabular information $10^3 = 1000$, $10^4 = 10,000$, $2^{10} = 1024$, $2^{11} = 2048$, $2^{12} = 4096$, $2^{13} = 8192$, then the strongest statement one can make for $\log_{10} 2$ is that it lies between:

(A) $\frac{3}{10}$ and $\frac{4}{11}$ **(B)** $\frac{3}{10}$ and $\frac{4}{12}$ **(C)** $\frac{3}{10}$ and $\frac{4}{13}$ **(D)** $\frac{3}{10}$ and $\frac{40}{132}$ **(E)** $\frac{3}{11}$ and $\frac{40}{132}$

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27 Two candles of the same length are made of different materials so that one burns out completely at a uniform rate in 3 hours and the other in 4 hours. At what time P.M. should the candles be lighted so that, at 4 P.M., one stub is twice the length of the other?

(A) $1:24$ **(B)** $1:28$ **(C)** $1:36$ **(D)** $1:40$ **(E)** $1:48$

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28 Given the two hypotheses: I Some Mems are not Ens and II No Ens are Veens. If ”some” means ”at least one,” we can conclude that:

(A) Some Mems are not Veens **(B)** Some Vees are not Mems
(C) No Mem is a Vee **(D)** Some Mems are Vees
(E) Neither **(A)** nor **(B)** nor **(C)** nor **(D)** is deducible from the given statements

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29 $\overline{AB}$ is a diameter of a circle. Tangents $\overline{AD}$ and $\overline{BC}$ are drawn so that $\overline{AC}$ and $\overline{BD}$ intersect in a point on the circle. If $\overline{AD} = a$ and $\overline{BD} = b$, $a \neq b$, the diameter of the circle is:

(A) $|a-b|$ **(B)** $\frac{1}{2}(a+b)$ **(C)** $\sqrt{ab}$ **(D)** $\frac{ab}{a+b}$ **(E)** $\frac{1}{2}\frac{ab}{a+b}$

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30 A dealer bought n radios for d dollars, d a positive integer. He contributed two radios to a community bazaar at half their cost. The rest he sold at a profit of $\$8$ on each radio sold. If the overall profit was $\$72$, then the least possible value of n for the given information is:

(A) 18 **(B)** 16 **(C)** 15 **(D)** 12 **(E)** 11

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31 Let $D = a^2 + b^2 + c^2$, where a, b, are consecutive integers and $c = ab$. Then $\sqrt{D}$ is:

(A) always an even integer **(B)** sometimes an odd integer, sometimes not
(C) always an odd integer **(D)** sometimes rational, sometimes not
(E) always irrational

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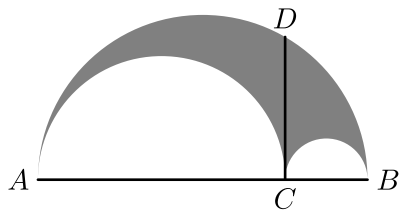
32 In quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at O, $\overline{BO} = 4$, $\overline{AO} = 8$, $\overline{OC} = 3$, and $\overline{AB} = 6$. The length of $\overline{AD}$ is:

(A) 9 **(B)** 10 **(C)** $6\sqrt{3}$ **(D)** $8\sqrt{2}$ **(E)** $\sqrt{166}$

33



In this diagram semi-circles are constructed on diameters $\overline{AB}$, $\overline{AC}$, and $\overline{CB}$, so that they are mutually tangent. If $\overline{CD} \perp \overline{AB}$, then the ratio of the shaded area to the area of a circle with $\overline{CD}$ as radius is:

(A) $1:2$ **(B)** $1:3$ **(C)** $\sqrt{3}:7$ **(D)** $1:4$ **(E)** $\sqrt{2}:6$

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34 Points D, E, F are taken respectively on sides AB, BC, and CA of triangle ABC so that $AD : DB = BE : CE = CF : FA = 1 : n$. The ratio of the area of triangle DEF to that of triangle ABC is:

(A) $\frac{n^2-n+1}{(n+1)^2}$ **(B)** $\frac{1}{(n+1)^2}$ **(C)** $\frac{2n^2}{(n+1)^2}$ **(D)** $\frac{n^2}{(n+1)^2}$ **(E)** $\frac{n(n-1)}{n+1}$

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35 The roots of $64x^3 - 144x^2 + 92x - 15 = 0$ are in arithmetic progression. The difference between the largest and smallest roots is:

(A) 2 **(B)** 1 **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{1}{4}$

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36 Given a geometric progression of five terms, each a positive integer less than 100. The sum of the five terms is 211. If S is the sum of those terms in the progression which are squares of integers, then S is:

(A) 0 **(B)** 91 **(C)** 133 **(D)** 195 **(E)** 211

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37 Segments $AD = 10$, $BE = 6$, $CF = 24$ are drawn from the vertices of triangle ABC, each perpendicular to a straight line RS, not intersecting the triangle. Points D, E, F are the intersection points of RS with the perpendiculars. If x is the length of the perpendicular segment GH drawn to RS from the intersection point G of the medians of the triangle, then x is:

(A) $\frac{40}{3}$ **(B)** 16 **(C)** $\frac{56}{3}$ **(D)** $\frac{80}{3}$ **(E)** undetermined

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38 Given a set S consisting of two undefined elements "pib" and "maa", and the four postulates: P_1: Every pib is a collection of maas, P_2: Any two distinct pibs have one and only one maa in common, P_3: Every maa belongs to two and only two pibs, P_4: There are exactly four pibs. Consider the three theorems: T_1: There are exactly six maas, T_2: There are exactly three maas in each pib, T_3: For each maa there is exactly one other maa not in the same pid with it. The theorems which are deducible from the postulates are:

(A) T_3 only **(B)** T_2 and T_3 only **(C)** T_1 and T_2 only
(D) T_1 and T_3 only **(E)** all

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39 Given the sets of consecutive integers $\{1\}$,$\{2,3\}$,$\{4,5,6\}$,$\{7,8,9,10\}$, $\cdots$, where each set contains one more element than the preceding one, and where the first element of each set is one more than the last element of the preceding set. Let S_n be the sum of the elements in the Nth set. Then S_{21} equals:

(A) 1113 **(B)** 4641 **(C)** 5082 **(D)** 53361 **(E)** none of these

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40 Located inside equilateral triangle ABC is a point P such that $PA = 8$, $PB = 6$, and $PC = 10$. To the nearest integer the area of triangle ABC is:

(A) 159 **(B)** 131 **(C)** 95 **(D)** 79 **(E)** 50

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