AMC 12/AHSME 1969

www.artofproblemsolving.com/community/c4833

by djmathman, btilm305, rrusczyk

1 When x is added to both the numerator and the denominator of the fraction $a/b, a \neq b, b \neq 0$, the value of the fraction is changed to c/d. Then x equals:

(A) $\frac{1}{c-d}$ **(B)** $\frac{ad-bc}{c-d}$ **(C)** $\frac{ad-bc}{c+d}$ **(D)** $\frac{bc-ad}{c-d}$ **(E)** $\frac{bc-ad}{c+d}$

---

2 If an item is sold for x dollars, there is a loss of 15% based on the cost. If, however, the same item is sold for y dollars, there is a profit of 15% based on the cost. The ratio $y:x$ is:

(A) $23:17$ **(B)** $17y:23$ **(C)** $23x:17$ **(D)** dependent upon the cost **(E)** none of these.

---

3 If N, written in base 2, is 11000, the integer immediately preceeding N, written in base 2, is:

(A) 10001 **(B)** 10010 **(C)** 10011 **(D)** 10110 **(E)** 10111

---

4 Let a binary operation $*$ on ordered pairs of integers be defined by $(a,b)*(c,d)=(a-c,b+d)$. Then, if $(3,2)*(0,0)$ and $(x,y)*(3,2)$ represent idential pairs, x equals:

(A) -3 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 6

---

5 If a number N, $N \neq 0$, diminished by four times its reciprocal, equals a given real constant R, then, for this given R, the sum of all such possible values of N is:

(A) $\frac{1}{R}$ **(B)** R **(C)** 4 **(D)** $\frac{1}{4}$ **(E)** $-R$

---

6 The area of the ring between two concentric circles is $12\frac{1}{2}\pi$ square inches. The length of a chord of the larger circle tangent to the smaller circle, in inches, is:

(A) $\frac{5}{\sqrt{2}}$ **(B)** 5 **(C)** $5\sqrt{2}$ **(D)** 10 **(E)** $10\sqrt{2}$

---

7 If the points $(1,y_1)$ and $(-1,y_2)$ lie on the graph of $y=ax^2+bx+c$, and $y_1-y_2=-6$, then b equals:

(A) -3 **(B)** 0 **(C)** 3 **(D)** $\sqrt{ac}$ **(E)** $\frac{a+c}{2}$

---

8 Triangle ABC is inscribed in a circle. The measure of the non-overlapping minor arcs AB, BC, and CA are, respectively, $x+75^\circ$, $2x+25^\circ$, $3x-22^\circ$. Then one interior angle of the triangle, in degrees, is:

(A) $57\frac{1}{2}$ **(B)** 59 **(C)** 60 **(D)** 61 **(E)** 122

---

9 The arithmetic mean (ordinary average) of the fifty-two successive positive integers beginning with 2 is:

(A) 27 **(B)** $27\frac{1}{4}$ **(C)** $27\frac{1}{2}$ **(D)** 28 **(E)** $28\frac{1}{2}$

---

10 The number of points equidistant from a circle and two parallel tangents to the circle is:

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinite

---

11 Given points $P(-1,-2)$ and $Q(4,2)$ in the xy-plane; point $R(1,m)$ is taken so that $PR+RQ$ is a minimum. Then m equals:

(A) $-\frac{3}{5}$ **(B)** $-\frac{2}{5}$ **(C)** $-\frac{1}{5}$ **(D)** $\frac{1}{5}$ **(E)** either $-\frac{1}{5}$ or $\frac{1}{5}$

---

12 Let $F=\dfrac{6x^2+16x+3m}{6}$ be the square of an expression which is linear in x. Then m has a particular value between:

(A) 3 and 4 **(B)** 4 and 5 **(C)** 5 and 6 **(D)** -4 and -3 **(E)** -6 and -5

---

13 A circle with radius r is contained within the region bounded by a circle with radius R. The area bounded by the larger circle is a/b times the area of the region outside the smaller circle and inside the larger circle. Then $R:r$ equals:

(A) $\sqrt{a}:\sqrt{b}$ **(B)** $\sqrt{a}:\sqrt{a-b}$ **(C)** $\sqrt{b}:\sqrt{a-b}$ **(D)** $a:\sqrt{a-b}$ **(E)** $b:\sqrt{a-b}$

---

14 The complete set of x-values satisfying the inequality $\dfrac{x^2-4}{x^2-1}>0$ is the set of all x such that:

(A) $x>2$ or $x<-2$ or $-1<x<1$ **(B)** $x>2$ or $x<-2$ **(C)** $x>1$ or $x<-2$ **(D)** $x>1$ or $x<-2$ **(E)** x is any real number except 1 or -1

---

15 In a circle with center at O and radius r, chord AB is drawn with length equal to r (units). From O a perpendicular to AB meets AB at M. From M a perpendicular to OA meets OA at D. In terms of r the area of triangle MDA, in appropriate square units, is:

(A) $\dfrac{3r^2}{16}$ **(B)** $\dfrac{\pi r^2}{16}$ **(C)** $\dfrac{\pi r^2\sqrt{2}}{8}$ **(D)** $\dfrac{r^2\sqrt{3}}{32}$ **(E)** $\dfrac{r^2\sqrt{6}}{48}$

---

16 When $(a-b)^n$, $n\geq 2$, $ab\neq 0$, is expanded by the binomial theorem, it is found that , when $a=kb$, where k is a positive integer, the sum of the second and third terms is zero. Then n equals:

(A) $\frac{1}{2}k(k-1)$ **(B)** $\frac{1}{2}k(k+1)$ **(C)** $2k-1$ **(D)** $2k$ **(E)** $2k+1$

---

17 The equation $2^{2x} - 8 \cdot 2^x + 12 = 0$ is satisfied by:

(A) $\log 3$ **(B)** $\frac{1}{2}\log 6$ **(C)** $1 + \log \frac{3}{4}$ **(D)** $1 + \frac{\log 3}{\log 2}$ **(E)** none of these

---

18 The number of points common to the graphs of

$$(x-y+2)(3x+y-4) = 0 \text{ and } (x+y-2)(2x-5y+7) = 0$$

is:

(A) 2 **(B)** 4 **(C)** 6 **(D)** 16 **(E)** infinite

---

19 The number of distinct ordered pairs (x,y), where x and y have positive integral values satisfying the equation $x^4y^4 - 10x^2y^2 + 9 = 0$, is:

(A) 0 **(B)** 3 **(C)** 4 **(D)** 12 **(E)** infinite

---

20 Let P equal the product of $3,659,893,456,789,325,678$ and $342,973,489,379,256$. The number of digits in P is:

(A) 36 **(B)** 35 **(C)** 34 **(D)** 33 **(E)** 32

---

21 If the graph of $x^2 + y^2 = m$ is tangent to that of $x + y = \sqrt{2m}$, then:

(A) m must equal $\frac{1}{2}$ **(B)** m must equal $\frac{1}{\sqrt{2}}$ **(C)** m must equal $\sqrt{2}$ **(D)** m must equal 2
(E) m may be any nonnegative real number

---

22 Let K be the measure of the area bounded by the x-axis, the line $x = 8$, and the curve defined by

$$f = \{(x,y) \,|\, y = x \text{ when } 0 \le x \le 5,\ y = 2x - 5 \text{ when } 5 \le x \le 8\}.$$

Then K is:

(A) 21.5 **(B)** 36.4 **(C)** 36.5 **(D)** 44 **(E)** less than 44 but arbitrarily close to it.

---

23 For any integer n greater than 1, the number of prime numbers greater than $n! + 1$ and less than $n! + n$ is:

(A) 0 **(B)** 1 **(C)** $\frac{n}{2}$ for n even, $\frac{n+1}{2}$ for n odd **(D)** $n-1$ **(E)** n

---

24 When the natural numbers P and P', with $P > P'$, are divided by the natural number D, the remainders are R and R', respectively. When PP' and RR' are divided by D, the remainders are r and r', respectively. Then:

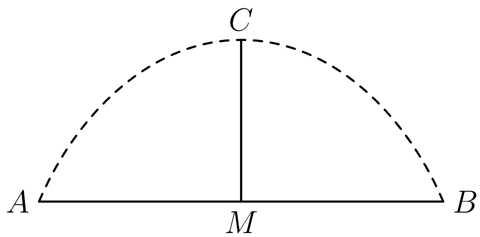
(A) $r > r'$ always **(B)** $r < r'$ always **(C)** $r > r'$ sometimes, and $r < r'$ sometimes **(D)** $r > r'$ sometimes, and $r = r'$ sometimes **(E)** $r = r'$ always

---

25 If it is known that $\log_2 a + \log_2 b \geq 6$, then the least value that can be taken on by $a + b$ is:

(A) $2\sqrt{6}$ **(B)** 6 **(C)** $8\sqrt{2}$ **(D)** 16 **(E)** none of these.

26



A parabolic arch has a height of 16 inches and a span of 40 inches. The height, in inches, of the arch at a point 5 inches from the center of M is:

(A) 1 **(B)** 15 **(C)** $15\frac{1}{3}$ **(D)** $15\frac{1}{2}$ **(E)** $15\frac{3}{4}$

---

27 A particle moves so that its speed for the second and subsequent miles varies inversely as the integral number of miles already traveled. For each subsequent mile the speed is constant. If the second mile is traversed in 2 hours, then the time, in hours, needed to traverse the nth mile is:

(A) $\dfrac{2}{n-1}$ **(B)** $\dfrac{n-1}{2}$ **(C)** $\dfrac{2}{n}$ **(D)** $2n$ **(E)** $2(n-1)$

---

28 Let n be the number of points P interior to the region bounded by a circle with radius 1, such that the sum of the squares of the distances from P to the endpoints of a given diameter is 3. Then n is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** infinite

---

29 If $x = t^{(1/(t-1))}$ and $x = t^{(t/(t-1))}$, $t > 0$, $t \neq 1$, a relation between x and y is **(A)** $y^x = x^{1/y}$ **(B)** $y^{1/x} = x^y$ **(C)** $y^x = x^y$ **(D)** $x^x = y^y$
(E) none of these

---

30 Let P be a point of hypotenuse AB (or its extension) of isosceles right triangle ABC. Let $s = AP^2 + PB^2$. Then:

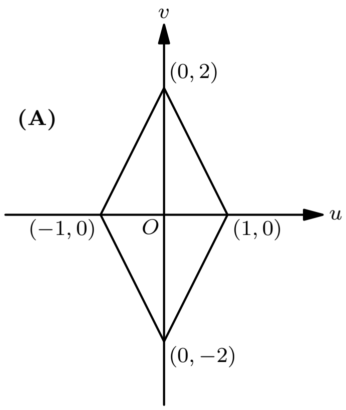
(A) $s < 2CP^2$ for a finite number of positions of P **(B)** $s < 2CP^2$ for an infinite number of positions of P

(C) $s = 2CP^2$ only if P is the midpoint of AB or an endpoint of AB **(D)** $s = 2CP^2$ always **(E)** $s > 2CP^2$ if P is a trisection point of AB

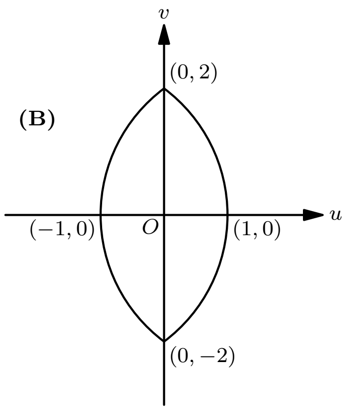
---

31 Let $OABC$ be a unit square in the xy-plane with $O(0,0), A(1,0), B(1,1)$ and $C(0,1)$. Let $u = x^2 - y^2$ and $v = 2xy$ be a transformation of the xy-plane into the uv-plane. The transform (or image) of the square is:

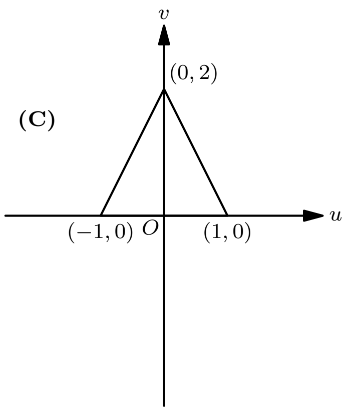
(A)



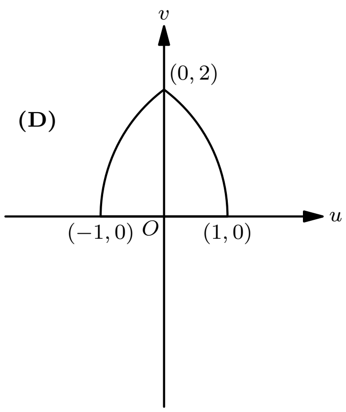
(B)



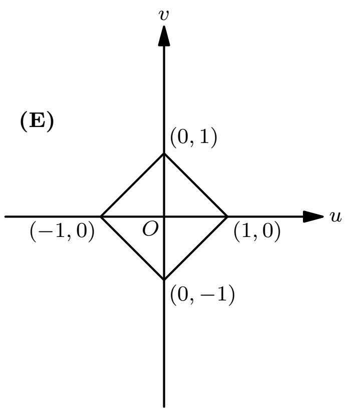
(C)



(D)



(E)



---

32 Let a sequence $\{u_n\}$ be defined by $u_1 = 5$ and the relation $u_{n+1} - u_n = 3 + 4(n-1)$, $n = 1, 2, 3, \cdots$.

If u_n is expressed as a polynomial in n, the algebraic sum of its coefficients is:

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 11

---

33 Let S_n and T_n be the respective sums of the first n terms of two arithmetic series. If $S_n : T_n = (7n+1) : (4n+27)$ for all n, the ratio of the eleventh term of the first series to the eleventh term of the second series is:

(A) $4:3$ **(B)** $3:2$ **(C)** $7:4$ **(D)** $78:71$ **(E)** undetermined

---

34 The remainder R obtained by dividing x^{100} by x^2-3x+2 is a polynomial of degree less than 2. Then R may be written as:

(A) $2^{100}-1$ **(B)** $2^{100}(x-1)-(x-2)$ **(C)** $2^{100}(x-3)$ **(D)** $x(2^{100}-1)+2(2^{99}-1)$ **(E)** $2^{100}(x+1)-(x+2)$

---

35 Let $L(m)$ be the x-coordinate of the left end point of the intersection of the graphs of $y=x^2-6$ and $y=m$, where $-6<m<6$. Let $r=[L(-m)-L(m)]/m$. Then, as m is made arbitrarily close to zero, the value of r is:

(A) arbitrarily close to zero **(B)** arbitrarily close to $\frac{1}{\sqrt{6}}$ **(C)** arbitrarily close to $\frac{2}{\sqrt{6}}$ **(D)** arbitraril
(E) undetermined

– MATHEMATICAL ASSOCIATION OF AMERICA