USAMO 1973

www.artofproblemsolving.com/community/c4471

by Brut3Forc3, rrusczyk

1 Two points P and Q lie in the interior of a regular tetrahedron $ABCD$. Prove that angle $PAQ < 60^\circ$.

---

2 Let $\{X_n\}$ and $\{Y_n\}$ denote two sequences of integers defined as follows:

$$X_0 = 1,\ X_1 = 1,\ X_{n+1} = X_n + 2X_{n-1} \quad (n = 1, 2, 3, \ldots),$$
$$Y_0 = 1,\ Y_1 = 7,\ Y_{n+1} = 2Y_n + 3Y_{n-1} \quad (n = 1, 2, 3, \ldots).$$

Prove that, except for the "1", there is no term which occurs in both sequences.

---

3 Three distinct vertices are chosen at random from the vertices of a given regular polygon of $(2n+1)$ sides. If all such choices are equally likely, what is the probability that the center of the given polygon lies in the interior of the triangle determined by the three chosen random points?

---

4 Determine all roots, real or complex, of the system of simultaneous equations

$$x + y + z = 3,$$
$$x^2 + y^2 + z^2 = 3,$$
$$x^3 + y^3 + z^3 = 3.$$

---

5 Show that the cube roots of three distinct prime numbers cannot be three terms (not necessarily consecutive) of an arithmetic progression.

– MATHEMATICAL ASSOCIATION OF AMERICA