Sampling distributions

Definitions 2.1

- A random variable (RV) is discrete if it takes on a finite or countably infinite number of distinct values.

- A RV is continuous if it can take on any values in an interval.

- The probability function (PF) of a discrete random variable X is denoted by
$$p(x) = P(X = x)$$

- The probability density function (PDF) of a continuous random variable X is denoted by $f(x)$. It uses area to represent probability:
$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

Properties of PF and PDF

- A probability function (PF) must satisfy the following properties:
  - The probabilities are non-negative $p(x) \geq 0$ for all x
  - The probabilities sum to one $\sum_x p(x) = 1$

- A probability density function (PDF) must satisfy the following properties:
  - $f(x) \geq 0$ for all x
  - $\int_{-\infty}^{\infty} f(x)\, dx = 1$

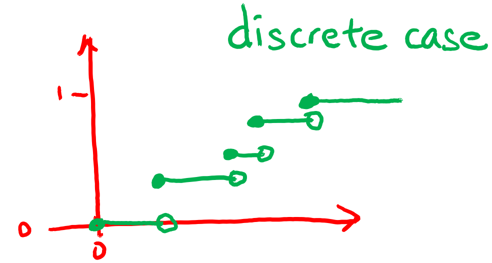
Definitions 2.2

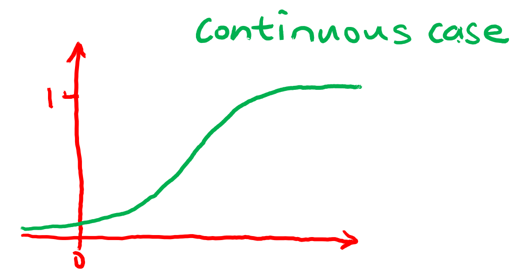
The cumulative distribution function (CDF) is defined for both discrete and continuous RV by

$$\underline{F(x) = P(X \leq x)}$$

discrete case: $F(x) = \sum_{x_i \leq x} F(x_i)$

continuous case: $F(x) = \int_{-\infty}^{x} f(t)\, dt$





It satisfies the following properties:

- $F(-\infty) = 0$ $= \lim_{x \to -\infty} F(x)$
- $F(\infty) = 1$ $= \lim_{x \to \infty} F(x)$
- F is monotonically non-decreasing $F(b) \geq F(a)$ if $b > a$
- $P(a \leq X \leq b) = F(b) - F(a)$
- $\dfrac{dF(x)}{dx} = f(x)$

Commonly used distributions

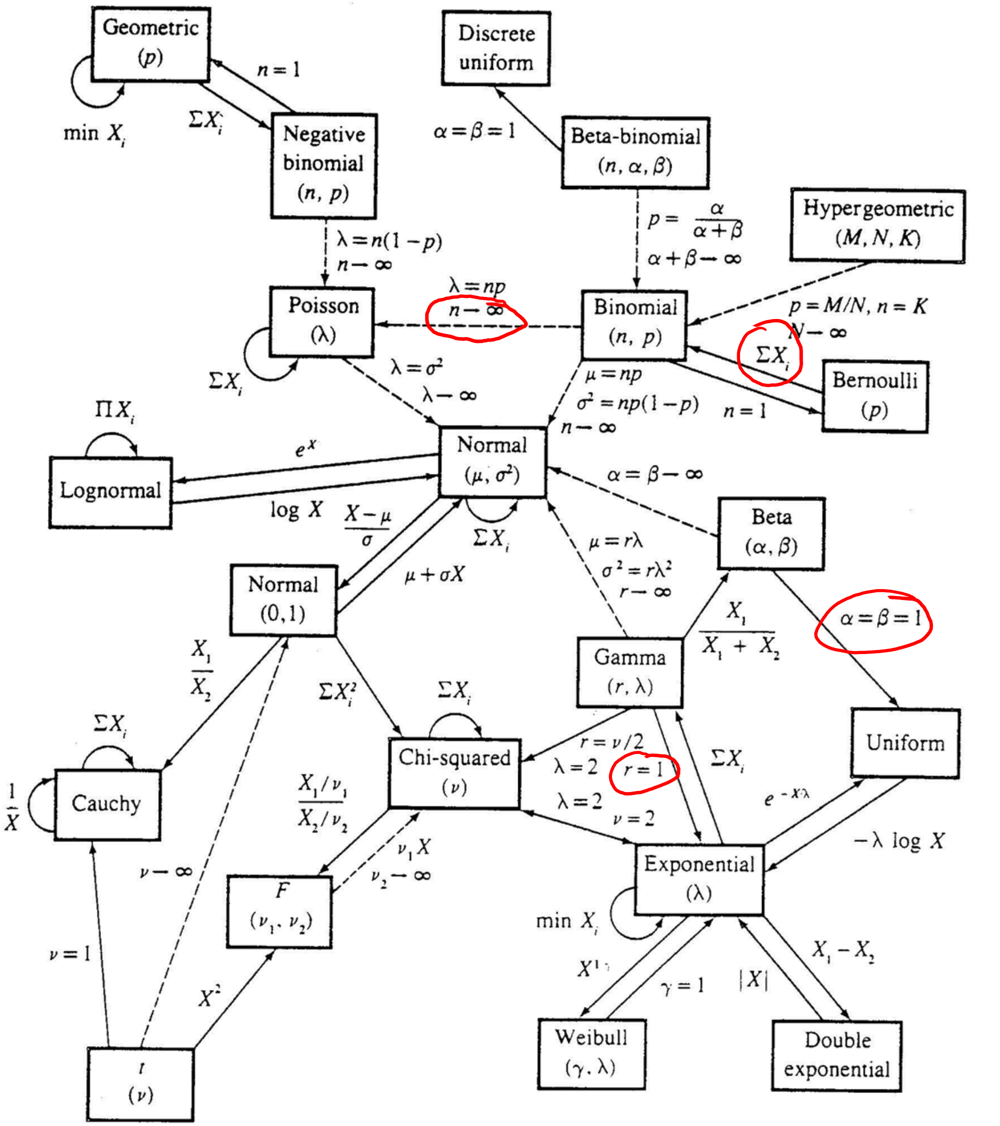
	Distribution	Probability mass function / probability density function	Expectation	Variance
discrete	Binomial	$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \ldots, n$	np	$np(1-p)$
	Geometric	$p(x) = p(1-p)^{x-1}$ for $x = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
	Poisson	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$	λ	λ
continuous	Uniform	$f(x) = \frac{1}{b-a}$ for $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
	Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
	Gamma	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$ for $x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
	Normal	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(1/2\sigma^2)(x-\mu)^2}$ for $-\infty < x < \infty$	μ	σ^2
	Beta	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}$ for $0 < x < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

t

χ^2

F

Relationships between common distributions



Relationships among common distributions. Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).

Computing integrals related to PDF

Gamma	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$ for $x > 0$
Normal	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(1/2\sigma^2)(x-\mu)^2}$ for $-\infty < x < \infty$
Beta	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}$ for $0 < x < 1$

$$\int_0^\infty x^{\alpha-1} e^{-\lambda x}\,dx = \frac{\Gamma(\alpha)}{\lambda^\alpha}$$

$$\int_{-\infty}^\infty e^{-\frac{1}{2\sigma^2}x^2}\,dx = \sigma\sqrt{2\pi}$$

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$