Quiz - 2

Recursive Multiplication

1) $a = 15286347$

$b = 30548179$

Split a & b as per recursive School method

$\Rightarrow a = a_1 B^{k_1} + a_0$

$a = 1528 \times 10^4 + 6347$

$\Rightarrow$ $\underline{a_1 = 1528}$, $\underline{a_0 = 6347}$, $B = 10$, $\underline{k_1} = 4$.

$b = 3054 \times 10^4 + 8179$

$\Rightarrow$ $\underline{b_1 = 3054}$, $\underline{b_0 = 8179}$, $B = 10$, $\underline{k_1} = 4$.

Now, we have to do the same for a_1, b_1, a_0, b_0

$a_1 = 1528$

$= 15 \times 10^2 + 28$

$\Rightarrow$ $\underline{k_2} = 2$, $B = 10$, $\underline{a_{11} = 15}$, $\underline{a_{10} = 28}$

$a_0 = 6347$

$= 63 \times 10^2 + 47$

$\Rightarrow$ $\underline{a_{01} = 63}$, $\underline{a_{00} = 47}$, $B = 10$, $\underline{k_2} = 2$.

$b_1 = 3054$

$= 30 \times 10^2 + 54$

$\Rightarrow$ $\underline{b_{11} = 30}$, $\underline{b_{10} = 54}$, $B = 10$, $\underline{k_2} = 2$

$b_0 = 8179$

$\Rightarrow 81 \times 10^2 + 79$

$\Rightarrow \underline{b_{01} = 81}, \underline{b_{00} = 79}, B = 10, \underline{k = 2}$

To compute $a \times b$,

$$(a \times b) = (a_1 \times b_1) B^{2k_1} + (a_1 \times b_0 + a_0 \times b_1) B^{k_1} + a_0 \times b_0.$$

$$= (1528 \times 3054) 10^{2 \times 4} + (1528 \times 8179 + 6347 \times 3054) \times 10^4 + 6347 \times 8179$$

$(a \times b) = 46697006412113.$

Since, the question did not ask about $a \times b$, instead we need to calculate $(a_{111} \times b_{110})$

$\Rightarrow$ But, we only split the given numbers twice (i.e $a_1, b_1, a_0, b_0, a_{01}, b_{01}, a_{00}, b_{00}, a_{11} \,\&\, b_{11}$).

So, there is no third split i.e $(a_{111} \times b_{110})$

$\Rightarrow$ So, it is zero (0)

$\therefore \; a_{111} \times b_{110} = 0.$

$K_1 = 4$

$a_1 = 1528 \;,\; a_0 = 6347$

$b_1 = 3054 \;,\; b_0 = 8179$

$K_2 = 2$

$a_{11} = 15 \;,\; a_{10} = 28$

$b_{11} = 30 \;,\; b_{10} = 54$

$a_{01} = 63 \;,\; a_{00} = 47$

$b_{01} = 81 \;,\; b_{00} = 79.$

$\boxed{\text{Rest all N/A's.}}$

$a_{111} \times b_{110} = 0.$

(2) $a = 10110110_2$

$b = 11001001_2$

$B = 4$ (Base)

Sinc we have to compute $a \times b$ using recursive school method with Base $B = 4$, we need to convert the given binary numbers to base 4.

$a = (10110110)_2 = (2312)_4$

$b = (11001001)_2 = (3021)_4$

Now, we have base 4 numbers.

$a = 2312$

$b = 3021$

Split the numbers as per recursive school method.

$$a = a_1 B^{k_1} + a_0.$$

$$a = 2312 \Rightarrow 23 \times 4^2 + 12$$

$\underline{a_1 = 23}$, $\underline{a_0 = 12}$, $B = 4$, $\underline{k_1 = 2}$

$b = 3021$

$\Rightarrow 30 \times 4^2 + 21$

$\underline{b_1 = 30}$, $\underline{b_0 = 21}$, $B = 4$, $\underline{k_1 = 2}$

Now again split a_1, a_0, b_1, b_0 using recursive school method,

$a_1 = 23$

$\Rightarrow 2 \times 4^1 + 3$

$\underline{a_{11} = 2}$, $\underline{a_{10} = 3}$, $B = 4$, $\underline{k_2 = 1}$

$a_0 = 12$

$\Rightarrow 1 \times 4^1 + 2$

$\underline{a_{01} = 1}$, $\underline{a_{00} = 2}$, $B = 4$, $\underline{k_2 = 1}$

$b_1 = 3 \times 4^1 + 0$

$\underline{b_{11} = 3}$, $\underline{b_{10} = 0}$, $B = 4$, $\underline{k_2 = 1}$

$b_0 = 21$

$\Rightarrow 2 \times 4^1 + 1$

$\underline{b_{01} = 2}$, $\underline{b_{00} = 1}$, $B = 4$, $\underline{k_2 = 1}$

To compute $a \times b$ of base 4;

$(a \times b) = (a_1 \times b_1) B^{2k} + (a_1 b_0 + a_0 b_1) B^k + a_0 b_0$

Everywhere, we should do Base-4 multiplication and base-4 addition

Base-4 multiplication :

$a_1 \times b_1 = 2010 \quad (23 \times 30)$

$a_1 \times b_0 = 1203 \quad (23 \times 21)$

$a_0 \times b_1 = 1020 \quad (12 \times 30)$

$a_0 \times b_0 = 312 \quad (12 \times 21)$

$\therefore (a \times b) = (2010) \times 4^{2 \times 2} + (1203 + 1020) 4^2 + 312$

Base-4 Addition $\Rightarrow$

$$\begin{array}{r} \overset{1}{2}010 \\ 1203 \\ 1020 \\ 312 \\ \hline 20323212 \end{array}$$

[$\because$ 3+3 = 12]

$\therefore (a \times b) = 20323212$

$K_1 = 2$

$a_1 = 23$ $a_0 = 12$

$b_1 = 30$ $b_0 = 21$

$K_2 = 1$

$a_{11} = 2$ $a_{10} = 3$

$b_{11} = 3$ $b_{10} = 0$

$a_{01} = 1$ $a_{00} = 2$

$b_{01} = 2$ $b_{00} = 1$

$\boxed{\text{Rest All N/A's.}}$

Base-4) $a \times b = 20323212$

3) $a = 15286347$

$b = 30548179$

$B = 100$

Split a & b using the below mentioned formula.

$a = a_1 B^{k_1} + a_0$ & $b = b_1 B^{k_1} + b_0$

$a = 1528 \times 100^2 + 6347$

$\Rightarrow a_1 = 1528,\ a_0 = 6347,\ B = 100,\ k_1 = 2$

$b = 3054 \times 100^2 + 8179$

$\Rightarrow b_1 = 3054,\ b_0 = 8179,\ B = 100,\ k_1 = 2$

Again split a_1, a_0, b_1, b_0 using same formula.

$a_1 = 1528$

$\Rightarrow 15 \times 100^1 + 28$

$\Rightarrow a_{11} = 15,\ a_{10} = 28,\ B = 100,\ k_2 = 1$

$a_0 = 6347$

$= 63 \times 100^1 + 47$

$\Rightarrow a_{01} = 63,\ a_{00} = 47,\ k_2 = 1,\ B = 100$

$b_1 = 3054$

$\Rightarrow 30 \times 100^1 + 54$

$\Rightarrow b_{11} = 30, \; b_{10} = 54, \; B = 100, \; k_2 = 1$

$b_0 = 8179$

$\Rightarrow 81 \times 100^1 + 79$

$b_{01} = 81, \; b_{00} = 79, \; B = 100, \; k_2 = 1$

$$(a \times b) = (a_1 \times b_1) B^{2k} + (a_1 b_0 + a_0 b_1) B^k + a_0 b_0$$

In question, it is given that

$p_2 = a_1 \times b_1$

$p_x = p_1 = a_1 b_0 + a_0 b_1$

$p_0 = a_0 b_0.$

$\therefore \; p_x = a_1 \times b_0 + a_0 \times b_1$

$= (1528 \times 8179 + 6347 \times 3054)$

$$\boxed{p_x = 31881250}$$

$$\boxed{k = k_1 = 2}$$

[$\because$ Highest power i.e highest value of k]

(4) $a = 1011011011001001_2$

$b = 1100100110110110_2$

$B = 2^4$ i.e 16.

Hint: Write 179_{10} as $B3_{16}$ i.e $(11, 3)$ → value of B

↓ Separated by comma

After converting Binary to hexa, we have following

$a = B6C9$

$b = C9B6$

Now, Split a & b using the below mentioned formula.

$a = a_1 B^{k_1} + a_0$ & $b = b_1 B^{k_1} + b_0$.

$a = B6C9$

$\Rightarrow B6 \times 16^2 + C9$

$\Rightarrow \underline{a_1} = B6 = 11, 6$

$\underline{a_0} = C9 = 12, 9$

$\underline{B = 16}$, $\underline{k_1 = 2}$.

$b = C9B6$

$\Rightarrow C9 \times 16^2 + B6$

$\underline{b_1} = C9 = 12, 9$

$\underline{b_0} = B6 = 11, 6$

$\underline{K_1} = 2, \underline{B = 16}$

To calculate $a \times b$, use hexa multiplication

$$a \times b = (a_1 \times b_1) B^{2k} + ((a_1 + a_0) * (b_1 + b_0) - (a_1 \times b_1 + a_0 \times b_0)) B^{k}$$
$$+ a_0 \times b_0$$

$$\Rightarrow (B6 \times C9) 16^{2(2)} + ((B6 + C9) \times (C9 + B6) - (B6 \times C9 + C9 \times B6)) \times 16^2$$
$$+ C9 \times B6$$

$$a \times b = (8EE6) 16^4 + [(17F)(17F) - (8EE6 + 8EE6)] \times 16^2$$
$$+ 8EE6$$

$$= (8EE6) 16^4 + [23D01 - 11DCC] \times 16^2 + 8EE6$$

$$= 8EE6 \times 16^4 + 11F35 \times 16^2 + 8EE6$$

$\cancel{8EE6}$

Here 16^4, 16^2 are decimal, So we need to convert them to hexa as other terms.

$\therefore 16^4 = 10000$, $16^2 = 100$.

$\therefore a \times b = 8EE6 \times 10000 + 11F35 \times 100 + 8EE6$

$= 8EE60000 + 11F3500 + 8EE6$

$= 90053500 + 8EE6$

$a \times b = (9005C3E6)_{16}$

we need to find $(B = 2^8)$: $a \times b$.

$\therefore$ we need to convert the answer which we have in hexa to base-256.

$\therefore a \times b = (9005C3E6)_{16} = (144, 5, 195, 230)_{256}$

$k_1 = 2$

$a_1 = 11, 6$ $\quad a_0 = 12, 9$

$b_1 = 12, 9$ $\quad b_0 = 11, 6$

Rest all N/A's

$[B = 2^8]: a \times b = \boxed{144, 5, 195, 230}$