ADSA Integer Airthmetic Quiz

Q1) Base 2 :

2 | 12345
2 | 6172 − 1
2 | 3086 − 0
2 | 1543 − 0
2 | 771 − 1
2 | 385 − 1
2 | 192 − 1
2 | 96 − 0
2 | 48 − 0
2 | 24 − 0
2 | 12 − 0
2 | 6 − 0
2 | 3 − 0
2 | 1 − 1

11000000111001

14 digits

Base 3:

$$
\begin{array}{r|l}
3 & 12345 \\
3 & 4115 - 0 \\
3 & 1371 - 2 \\
3 & 457 - 0 \\
3 & 152 - 1 \\
3 & 50 - 2 \\
3 & 16 - 2 \\
3 & 5 - 1 \\
 & 1 - 2
\end{array}
$$

121221020

9 digits

Base-4

4 | 12345
4 | 3086 - 1
4 | 771 - 2
4 | 192 - 3
4 | 48 - 0
4 | 12 - 0
3 - 0

3000321

7 digits

Base-5

5 | 12345
5 | 2469 - 0
5 | 493 - 4
5 | 98 - 3
5 | 19 - 3
3 - 4

343340

6 digits

Similarly do it for the remaing bases

Base 6 : 133053 has 6 digits

Base 7 : 50664 has 5 digits

Base 8 : 30071 has 5 digits

Base 9 : 17836 has 5 digits

Base 11 : 9303 has 4 digits

Base 12 : 7189 has 4 digits

Base 13 : 5808 has 4 digits

Base 14 : 46DB has 4 digits

Base 15 : 39D0 has 4 digits

Base 16 : 3039 has 4 digits

~~Base 16~~ :

Question 2:

$$(FC5)_{16} + (ADD5)_{15} + (AD5A)_{14} = ?$$

in Base 12

First Compute Base 10

$$(FC5)_{16} = 15 \times 16^2 + 12 \times 16^1 + 5 \times 16^0$$

$$= (4037)_{10}$$

$$(ADD5)_{15} = 10 \times 15^3 + 13 \times 15^2 + 13 \times 15^1 + 5 \times 15^0$$

$$= (36875)_{10}$$

$$(AD5A) = 10 \times 14^3 + 13 \times 14^2 + 5 \times 14^1 + 10 \times 14^0$$

$$= (30068)_{10}$$

Adding them

$$\hookrightarrow 4037 + 36875 + 30068$$

$$= (70980)_{10}$$

Now Compute Base 12 of 70980

$$
\begin{array}{r|l}
12 & 70980 \\
12 & 5915 \; - 0 \\
12 & 492 \; - 11 \\
12 & 41 \; - 0 \\
 & 3 \; - 5
\end{array}
$$

$\therefore (FC5)_{16} + (ADD5)_{15} + (AD5A)_{14}$

is Base 12 $= \underline{\underline{350B0}}$

Question 3:

Add $2n$-digit with n-digit

compute $A+B$

where $A = 2n$-digit

$B = n$ digit

$S = A+B$

$= (2n\text{-digit} + n\text{-digit})$

$\therefore$ The first operation performed is $2n$.

↳ Hence primitive operation is $\underline{2n}$

$2n$-digit + n-digit will always result in $2n+1$ digit.

For ex: $2n$-digit $= 99$

n-digit $= 9$

$99+9 = 108$

i.e. a $\underline{2n+1 \text{ digit}}$

Hence output digit $= \underline{2n+1}$

Question 4 :

Compute $max(A+B, C+D) + E$

where $A, C = n$ digits

$B, D = 3n$ digits

$\therefore$ The primitive operations for

$A+B = 3n$

Similarly for $C+D$ we need another $3n$ primitive operations.

we know that,

$3n + n$ will lead to $3n+1$ digit output

Since we take $Max(3n+n, 3n+n)$ this will essentially result in $3n+1$ digit output

$\therefore$ Adding this number to $2n$ number (E) will require $(3n+1)$ primitive operations

$\therefore$ Total primitive operations $= 3n + 3n + (3n+1)$

$= 9n+1$

We now know that we get $3n+1$ digit from $\max(A+B, C+D) + E$ — (1)

$\therefore$ Output digit $= \underline{3n+1}$

For ex: let $A = 9$

$B = 999$

& $E = 99$

$A + B + E = 1107$

so 1107 is a 4 digit,

It is impossible to have $3n+2$ digits in equation (1)

Hence the output digit of eq^n (1) : $\underline{3n+1}$

Question 5: Multiply $2n$-digit by n-digits

For each partial product (P_j) we multiply $2n$ number with a single digit.

↳ implies $2n$ multiplications

& $2n+1$ additionals (carry + alignment)

Total operation of a partial product $= 2n + 2n + 1$

$= 4n + 1$

We repeat it for n times

$\therefore$ Total operations for partial product $= (4n+1)n$

$= 4n^2 + n$

Similarly we consider the additionals ($2n+1$) for each partial prod.

$\therefore$ for $n \rightarrow n(2n+1)$

$= 2n^2 + n$

$\therefore$ Total operation $= 4n^2 + n + 2n^2 + n$

$\therefore$ Total Primitive operations $= \underline{6n^2 + 2n}$

Multiplying: $2n$-digit with n-digit will result in $3n$-digit output

$\therefore$ The output digit $= \underline{3n}$