STATS 2107
Statistical Modelling and Inference II

Workshop 11:
From ANOVA to ANCOVA

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The data

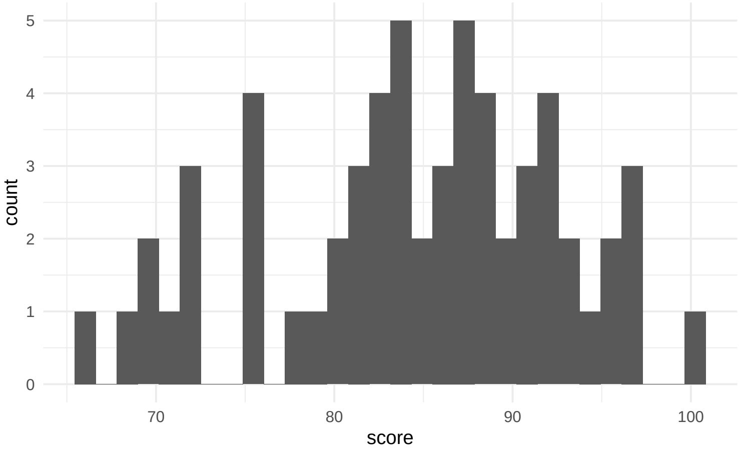
Where to get it

```
install.packages("datarium")
data("stress", package = "datarium")
stress <- as_tibble(stress) %>%
  mutate(treatment = fct_rev(treatment))
```

What do we have here?

Variable	Description	Type
id	A unique identifier	ID variable
score	Stress score out of 100	Continuous numeric (response variable)
treatment	Are they in the treatement group?	Categorical nominal
exercise	What level of exercise do they do?	Categorical nominal
age	Age of participant	Continuous numeric

score



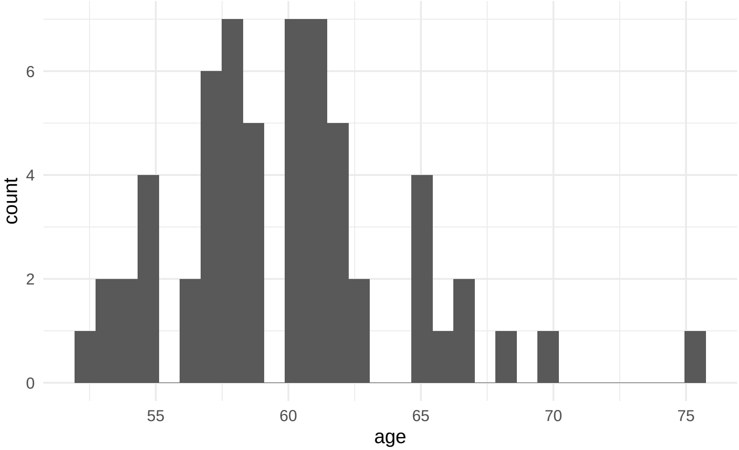
treatment

treatment	n
no	30
yes	30

exercise

exercise	n
low	20
moderate	20
high	20

age



One-way ANOVA

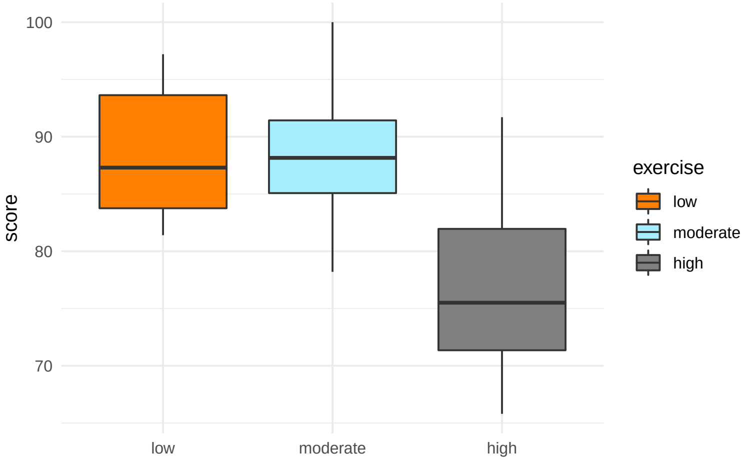
What to consider

Let's suppose that there is a relationship between stress levels and exercise:

$$\text{score}_{ik} = \mu + \alpha_i + \varepsilon_{ik}$$

where α_i is the effect of each exercise group

Is this supported by EDA



Fit it in R

We fit using the `lm` command:

```
stress_anova <- lm(score ~ exercise, data = stress)
summary(stress_anova)
```

```
##
## Call:
## lm(formula = score ~ exercise, data = stress)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -11.090  -4.674  -1.107   4.628  14.810
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)        88.725      1.361  65.186  < 2e-16 ***
## exercisemoderate   -0.610      1.925  -0.317    0.752
## exercisehigh      -11.835      1.925  -6.148 8.21e-08 ***
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 6.087 on 57 degrees of freedom
## Multiple R-squared:  0.4568, Adjusted R-squared:  0.4378
## F-statistic: 23.97 on 2 and 57 DF,  p-value: 2.791e-08
```

Do the ANOVA

```
anova(stress_anova)
```

```
## Analysis of Variance Table
##
## Response: score
##           Df Sum Sq Mean Sq F value    Pr(>F)
## exercise   2 1776.3  888.13   23.97 2.791e-08 ***
## Residuals 57 2112.0   37.05
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
```

Your turn

What to do

1. Based on the ANOVA output, would you reject or retain the null hypothesis that all exercise groups have the same mean pain score.
2. Look at the model summary. What does the intercept term represent?
3. Does this data meet the assumptions of ANOVA?

Two-way ANOVA

What to consider

Now, we know there is a treatment group, so let's suppose that there is a relationship between stress levels, exercise and treatment:

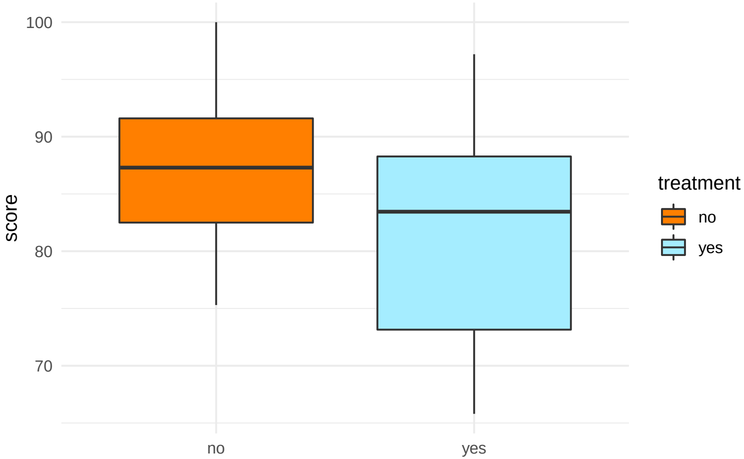
$$\text{score}_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$$

where α_i is the effect of each exercise group, β_j is the effect of each treatment group, and γ_{ij} is an interaction between the exercise and treatment.

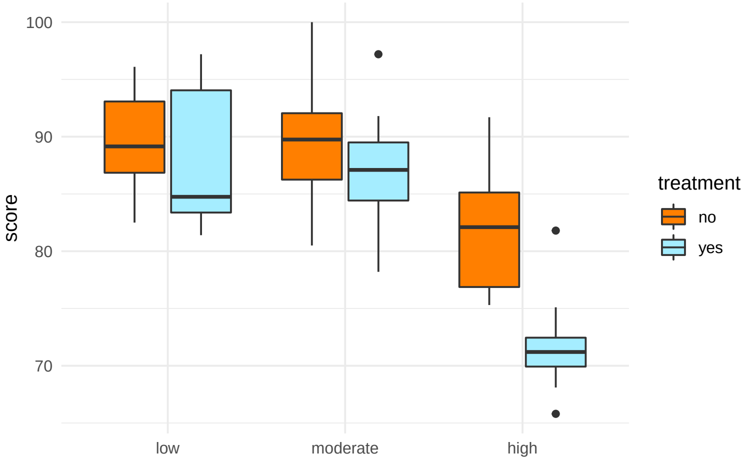
Do we have the data for an interaction?

treatment	low	moderate	high
no	10	10	10
yes	10	10	10

Is this model supported by EDA - a treatment effect



Is this model supported by EDA - an interaction



Fit it in R

We fit using the `lm` command:

```
stress_two_way_anova <- lm(score ~ exercise * treatment, data = stress)
summary(stress_two_way_anova)
```

```
##
## Call:
## lm(formula = score ~ exercise * treatment, data = stress)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -8.910 -3.797 -0.240  3.062 10.590
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     89.590      1.690  52.995  < 2e-16 ***
## exercisemoderate                -0.180      2.391  -0.075  0.94026
## exercisehigh                    -7.600      2.391  -3.179  0.00245 **
## treatmentyes                    -1.730      2.391  -0.724  0.47243
## exercisemoderate:treatmentyes   -0.860      3.381  -0.254  0.80019
## exercisehigh:treatmentyes       -8.470      3.381  -2.505  0.01529 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.346 on 54 degrees of freedom
## Multiple R-squared:  0.6031, Adjusted R-squared:  0.5663
## F-statistic: 16.41 on 5 and 54 DF,  p-value: 8.005e-10
```

Your turn

What to do

1. Look at the model summary. What does the intercept term represent?
2. Interpret the `exercisehigh` coefficient.
3. Perform the ANOVA. Is the interaction term significant? Interpret this in context.

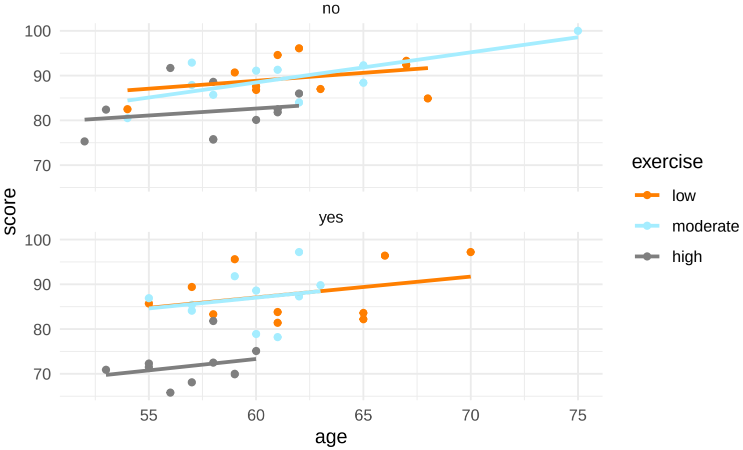
Two-way ANCOVA

What to consider

But wait, there's more! Remember we have the age covariate. It is perfectly reasonable to believe there is a relationship between age and stress, as well as some interaction with the treatment and exercise regime. Things get a little more hairy in two-way ANCOVA, so we will start big, and then select the best model. We will consider the model

```
score ∼ age * treatment * exercise
```

Is this supported by EDA



Fit it in R

We fit using the `lm` command:

```
stress_2way_ancova <- lm(score ~ age * treatment * exercise, data = stress)
summary(stress_2way_ancova)
```

```
##
## Call:
## lm(formula = score ~ age * treatment * exercise, data = stress)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.2940 -3.4969  0.3342  2.2295 10.2988
##
## Coefficients:
##                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)                        67.59947   24.92019   2.713  0.00924 **
## age                                 0.35411    0.40042   0.884  0.38091
## treatmentyes                       -8.41860   33.82489  -0.249  0.80451
## exercisemoderate                  -19.50719   30.73047  -0.635  0.52858
## exercisehigh                       -3.55340   38.80691  -0.092  0.92742
## age:treatmentyes                    0.11070    0.54501   0.203  0.83990
## age:exercisemoderate                0.31881    0.49536   0.644  0.52290
## age:exercisehigh                   -0.04420    0.65077  -0.068  0.94613
## treatmentyes:exercisemoderate      18.45149   55.34236   0.333  0.74028
## treatmentyes:exercisehigh         -12.85565   63.49342  -0.202  0.84040
## age:treatmentyes:exercisemoderate  -0.30217    0.91128  -0.332  0.74164
## age:treatmentyes:exercisehigh       0.08848    1.08428   0.082  0.93530
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.204 on 48 degrees of freedom
## Multiple R-squared:  0.6657, Adjusted R-squared:  0.5891
## F-statistic: 8.689 on 11 and 48 DF,  p-value: 3.358e-08
```

Can we simplify?

```r
drop1(stress_2way_ancova, test = "F")
```

```
## Single term deletions
##
## Model:
## score ~ age * treatment * exercise
##                        Df Sum of Sq    RSS    AIC F value Pr(>F)
## <none>                              1299.8 208.54
## age:treatment:exercise  2    3.9559 1303.8 204.72   0.073 0.9297
```

Yes we can!

```
stress_2way_ancova <- update(stress_2way_ancova, . ~ . - age:treatment:exercise)
summary(stress_2way_ancova)
```

```
##
## Call:
## lm(formula = score ~ age + treatment + exercise + age:treatment +
##     age:exercise + treatment:exercise, data = stress)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5637 -3.3982  0.4173  2.3827 10.3907
##
## Coefficients:
##                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  65.15197   21.12371   3.084  0.00332 **
## age                           0.39353    0.33916   1.160  0.25144
## treatmentyes                 -3.89781   24.16324  -0.161  0.87250
## exercisemoderate            -14.81619   24.97471  -0.593  0.55569
## exercisehigh                 -3.90658   30.22926  -0.129  0.89769
## age:treatmentyes              0.03769    0.38851   0.097  0.92311
## age:exercisemoderate          0.24286    0.40215   0.604  0.54864
## age:exercisehigh             -0.03524    0.50722  -0.069  0.94488
## treatmentyes:exercisemoderate  0.20723    3.35949   0.062  0.95106
## treatmentyes:exercisehigh    -8.12783    3.72077  -2.184  0.03365 *
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 5.106 on 50 degrees of freedom
## Multiple R-squared:  0.6647, Adjusted R-squared:  0.6043
## F-statistic: 11.01 on 9 and 50 DF,  p-value: 3.181e-09
```

Your turn

What to do

1. Finish the model selection process. What is the final model?
2. Interpret the coefficient of age in your final model.