Contrasts

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A linear combination of variables whose coefficients sums to zero is called a contrast.

If

$$\boldsymbol{\lambda}^\top \mathbf{1} = \mathbf{0}$$

then $\boldsymbol{\lambda}$ is called a contrast vector.

e.g. $\lambda = (1, 1, -1, -1)$,
$\lambda = (0, 1, -\frac{1}{2}, -\frac{1}{2})$
are contrast vectors, but
$\lambda = (0, 1, -\frac{1}{2}, -1)$
is not.

Contrasts are useful for comparing different group means:

$$\boldsymbol{\lambda}^\top \boldsymbol{\mu}$$

where $\boldsymbol{\lambda}^\top \mathbf{1} = \mathbf{0}$.

μ is a vector of population mean.
It can be expressed in terms of β.

Example 3.6

An experiment was conducted to compare the effectiveness of 6 different feed supplements on the growth of chickens. 71 newly hatched chickens were randomly allocated to be fed one of the 6 supplements. Their weights were recorded after 6 weeks. The data is available from `data(chickwts)` in R.

a) Fit a linear regression of weights on feed type.

b) Is there a difference in mean weight between the feeds casein and meat meal?

c) Is there a difference in mean weight between natural plant feeds (horse bean, linseed, soybean, sunflower) vs other feeds (casein, meat meal)?

Example 3.6 Solution

```
data("chickwts")
head(chickwts)
```

```
##   weight      feed
## 1    179 horsebean
## 2    160 horsebean
## 3    136 horsebean
```

a)

```
chickwts_lm <- lm(weight ~ feed, data = chickwts)
summary(chickwts_lm)
```

```
##
## Call:
## lm(formula = weight ~ feed, data = chickwts)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -123.909  -34.413    1.571   38.170  103.091
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    323.583  β0 15.834  20.436  < 2e-16 ***
## feedhorsebean -163.383  β1 23.485  -6.957 2.07e-09 ***
## feedlinseed   -104.833  β2 22.393  -4.682 1.49e-05 ***
## feedmeatmeal   -46.674  β3 22.896  -2.039 0.045567 *
## feedsoybean    -77.155  β4 21.578  -3.576 0.000665 ***
## feedsunflower    5.333  β5 22.393   0.238 0.812495
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 54.85 on 65 degrees of freedom
## Multiple R-squared:  0.5417, Adjusted R-squared:  0.5064
## F-statistic: 15.36 on 5 and 65 DF,  p-value: 5.936e-10
```

casein

meat meal

Se

casein was used as the baseline

Example 3.6 Solution

b)

```
#install.packages("emmeans")
library(emmeans)
chickwts_em <- emmeans(chickwts_lm, "feed")
```

$\mu = (\mu_{casein}, \mu_{horsefeed}, \mu_{linseed}, \mu_{meatmeal}, \mu_{soybean}, \mu_{sunflower})$

We want $\lambda^T \mu = \mu_{casein} - \mu_{meatmeal}$.

Set $\lambda = (1, 0, 0, -1, 0, 0)$.

$H_0: \mu_{casein} - \mu_{meatmeal} = 0$ vs $H_a: \mu_{casein} - \mu_{meatmeal} \neq 0$.

```
chickwts_em_c1 <- contrast(chickwts_em, method=list("casein vs meatmeal"=c(1,0,0,-1,0,0)))
chickwts_em_c1
```

```
##  contrast           estimate   SE df t.ratio p.value
##  casein vs meatmeal     46.7 22.9 65 2.039   0.0456
```

There is evidence to reject H_0 at 5% significance level.

```
confint(chickwts_em_c1)
```

```
##  contrast           estimate   SE df lower.CL upper.CL
##  casein vs meatmeal     46.7 22.9 65    0.948     92.4
##
## Confidence level used: 0.95
```

The CI for $\mu_{casein} - \mu_{meatmeal}$ is $(0.948, 92.4)$.

Example 3.6 Solution

c) Set $\lambda = (\frac{1}{2}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{4}, -\frac{1}{4})$.

H_0: $\frac{1}{2}\mu_{casein} + \frac{1}{2}\mu_{meatmeal} = \frac{1}{4}\mu_{horsebean} + \frac{1}{4}\mu_{linseed} + \frac{1}{4}\mu_{soybean} + \frac{1}{4}\mu_{sunflower}$

```
chickwts_em_c2 <- contrast(chickwts_em, method=list("plant vs other"=c(1/2,-1/4,-1/4,1/2,-1/4,-1/4)))
chickwts_em_c2
```

```
##  contrast       estimate SE df t.ratio p.value
##  plant vs other     61.7 14 65 4.421   <.0001
```

There is strong evidence to reject H_0.

```
confint(chickwts_em_c2)
```

```
##  contrast       estimate SE df lower.CL upper.CL
##  plant vs other     61.7 14 65    33.8     89.5
##
## Confidence level used: 0.95
```