STATS 2107
Statistical Modelling and Inference II
Tutorial 1

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1. Suppose $Y_1, Y_2, \dots, Y_n$ are i.i.d. $N(\mu, \sigma^2)$ random variables and let $\bar{Y}$ denote the sample mean. Using moment generating functions (MGFs), prove that $\bar{Y} \sim N(\mu, \sigma^2/n)$.
2. Let $Z_1, Z_2, \dots, Z_p$ be i.i.d. $N(0,1)$ random variables.

a. Show that the moment generating function $M_{Z_i^2}(t)$ of Z_i^2 is given by $(1-2t)^{-\frac{1}{2}}$ for each $i = 1, 2, \dots, p$.

b. Hence, show that the moment generating function of $X = \sum_{i=1}^{p} Z_i^2$ is

$$M_X(t) = \frac{1}{(1-2t)^{p/2}}\,.$$

3. Let $Y \sim Bin(n, p)$ and consider two estimators for p; namely

$$\hat{p}_1 = \frac{Y}{n} \text{ and } \hat{p}_2 = \frac{Y+1}{n+2}.$$

a. Show that $\hat{p}_1$ is unbiased for p.

b. Derive the bias of $\hat{p}_2$.

c. Find $MSE_{\hat{p}_1}(p)$ and $MSE_{\hat{p}_2}(p)$.

d. If $p = 0.5$, which estimator has the largest MSE?