STATS 2107
Statistical Modelling and Inference II
Solutions
Workshop 7:
Assumptions in simple linear regression.

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Contents

Simple linear regression

The model

Suppose you have data $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ where $x_i, y_i \in \mathbb{R}$ for each i.

THE MODEL:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \,,$$

where $\varepsilon_i \sim N(0, \sigma^2)$ independently for each $i = 1, 2, \ldots, n$.

What are the assumptions?

- **Linearity**: $\mathrm{E}[\varepsilon_i] = 0$
- **Homoscedasticity**: $\mathrm{Var}(\varepsilon_i) = \sigma^2$
- **Normality**: $\varepsilon_i \sim N$
- **Independence**: Design assumption

How do we check the assumptions?

We look at the residuals $\hat{e}_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)$:

- Use the `plot` command in R
  - **Linearity**: The Residuals vs Fitted plot
  - **Homoscedasticity**: The Residuals vs Fitted plot
  - **Normality**: The Normal QQ plot
  - **Independence**: Check the design/data collection

5-point check

When checking assumptions, answer:

- **What?**
- **Where?**
- **What do you expect?**

- **What do you see?**
- **What do you conclude?**

Your turn

What to do

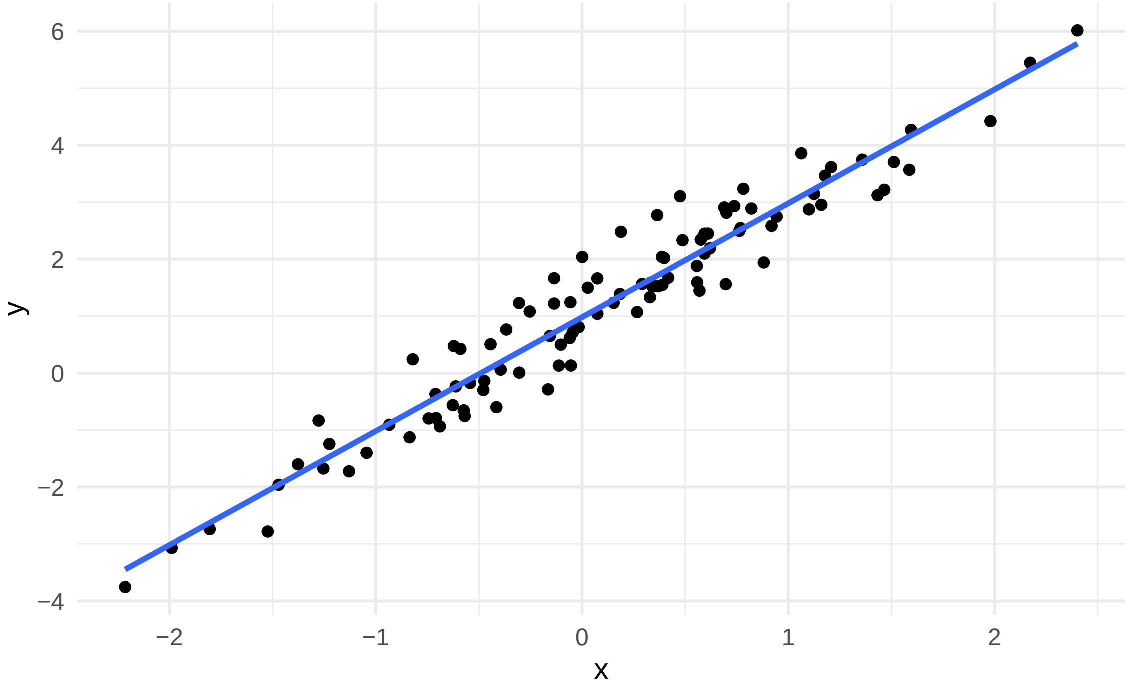
1. Load the `ceddar.csv` dataset. Fit the simple linear regression of `taste` on either `acetic`, `h2s`, or `lactic`.

Solutions:
For example, we could fit the model `taste ~ lactic`:

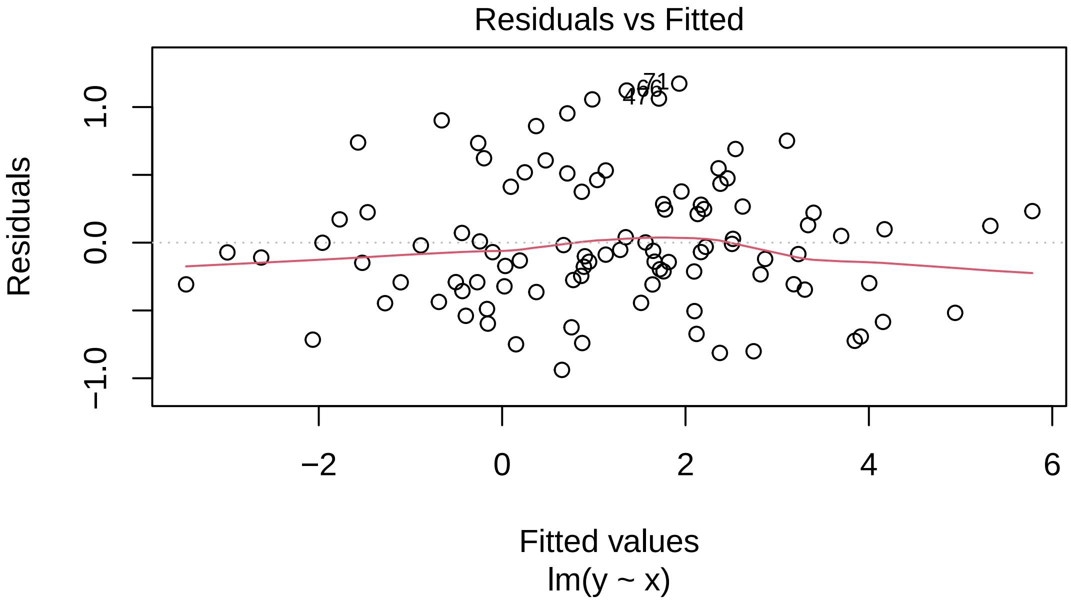
```
cheddar <- read_csv("../data/cheddar.csv", col_types = cols())
cheddar_lm <- lm(taste ~ lactic, data = cheddar)
```

Linearity

A linear model



The residual vs fitted plot

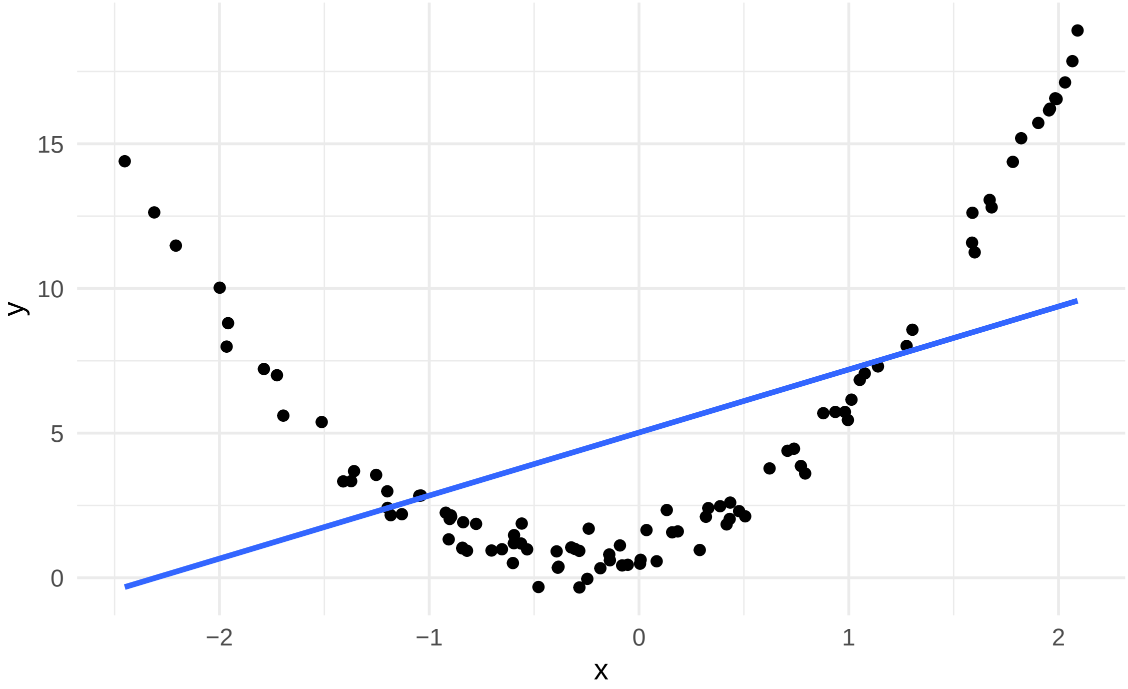


The true model

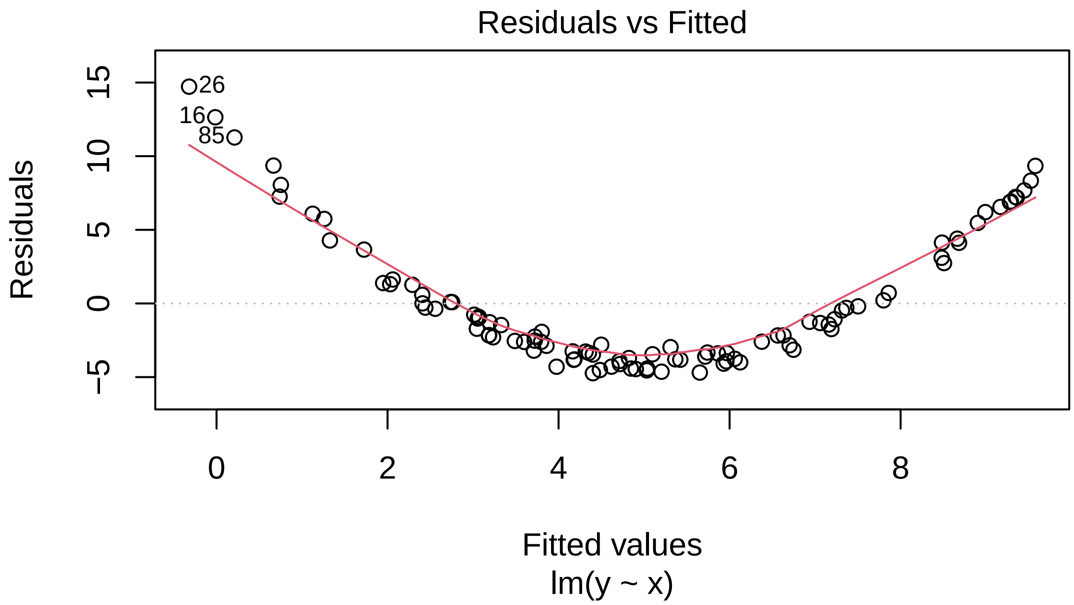
$$y_i = 1 + 2x_i + \varepsilon_i$$

where $\varepsilon_i \sim N(0, 0.5^2)$.

A non-linear model



The residual vs fitted plot



The true model

$$y_i = 1 + 2x_i + 3x_i^2 + \varepsilon_i$$

where $\varepsilon_i \sim N(0, 0.5^2)$.

Your turn

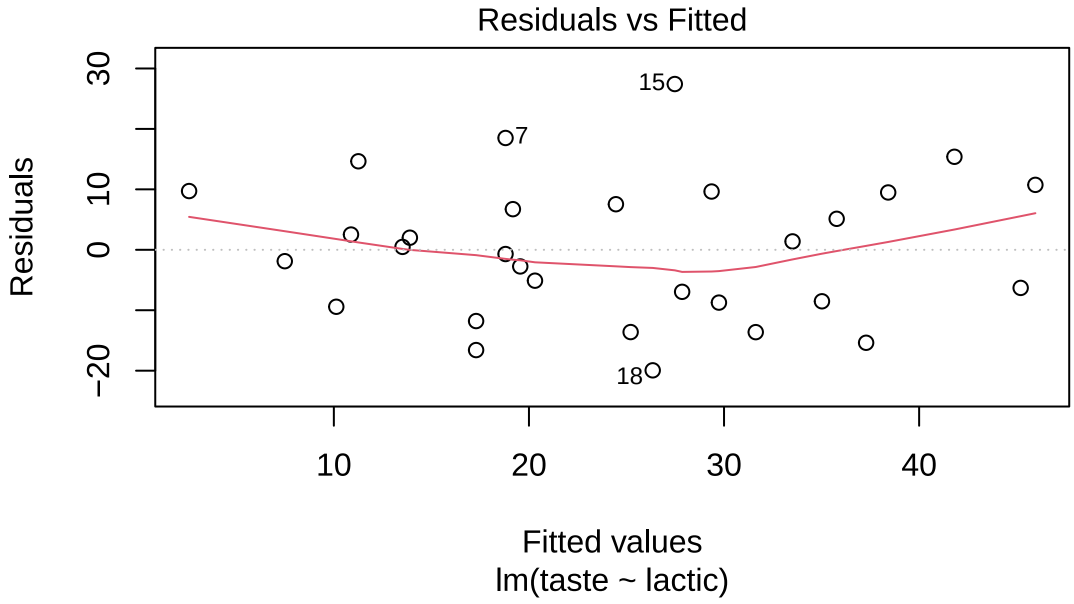
What to do

1. For you model fitted previously, test the linearity assumption.

Solutions:
See the plot below

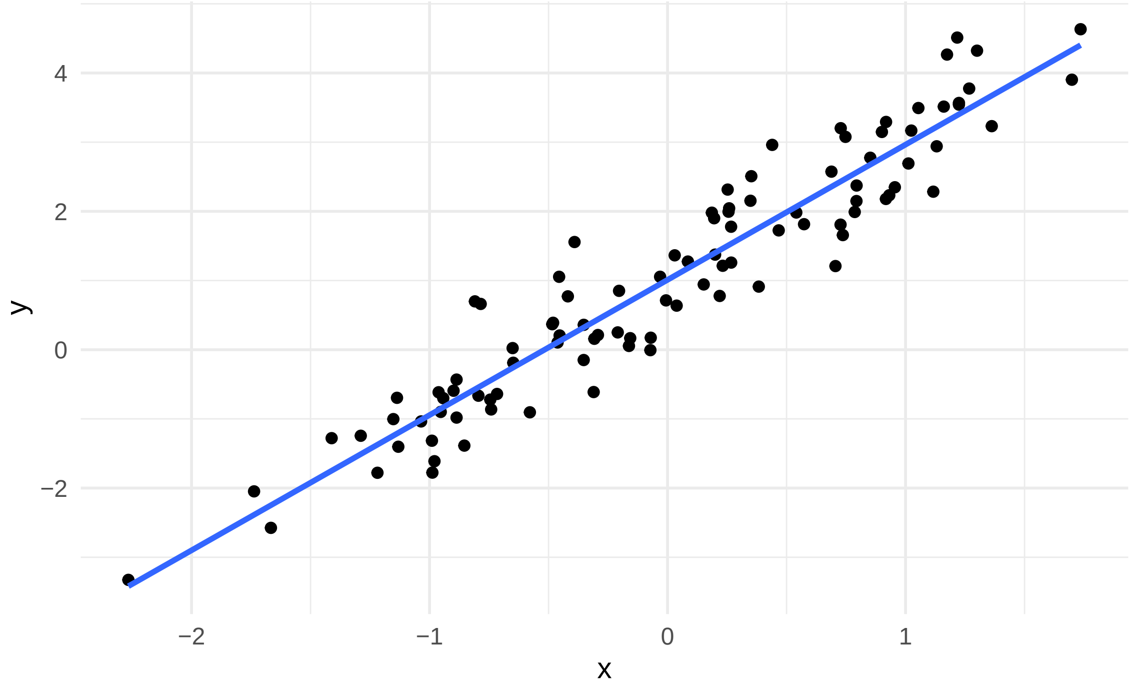
- **What?** Linearity.
- **Where?** Residual vs fitted plot.
- **What do you expect?** Random scatter above and below 0.
- **What do you see?** Random scatter above and below 0.
- **What do you conclude?** Linearity reasonable.

```
plot(cheddar_lm, which = 1)
```

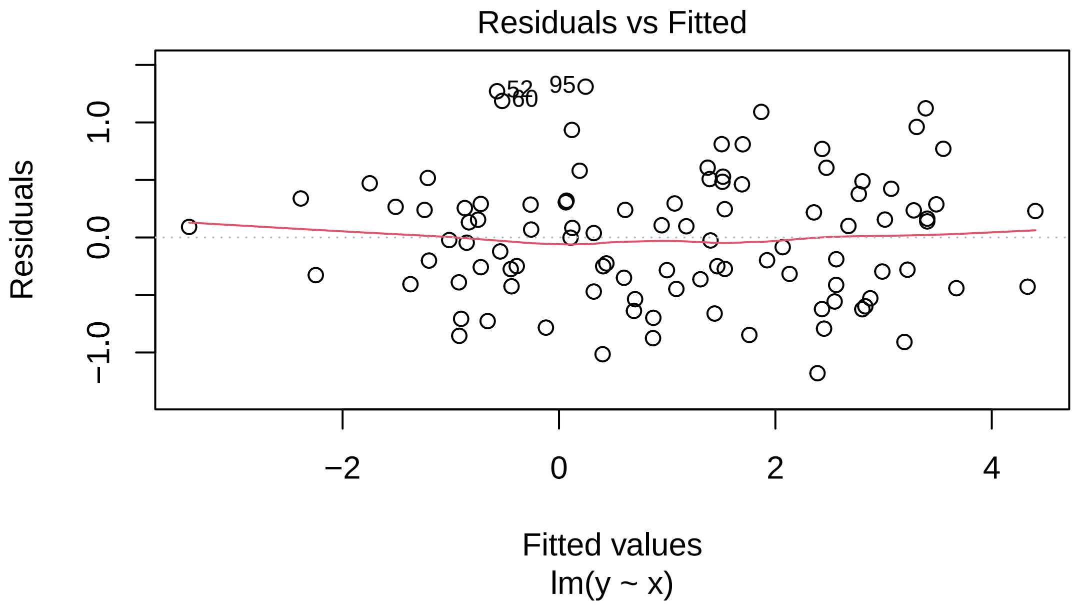


Homoscedasticity

A homoscedastic model



The residual vs fitted plot

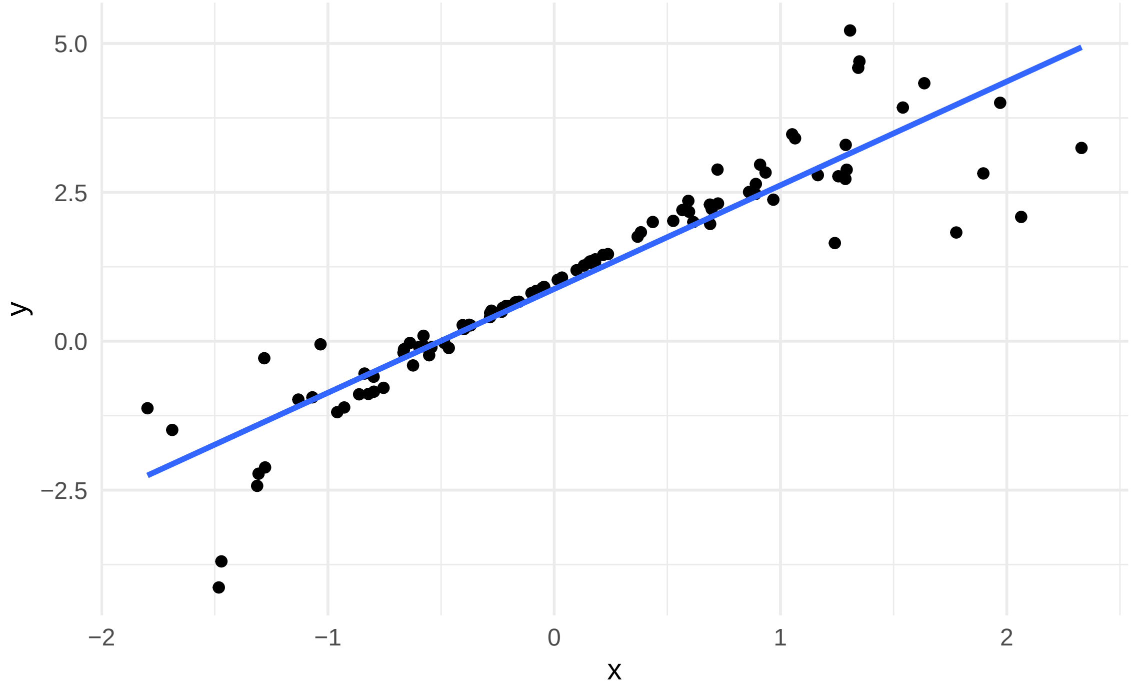


The true model

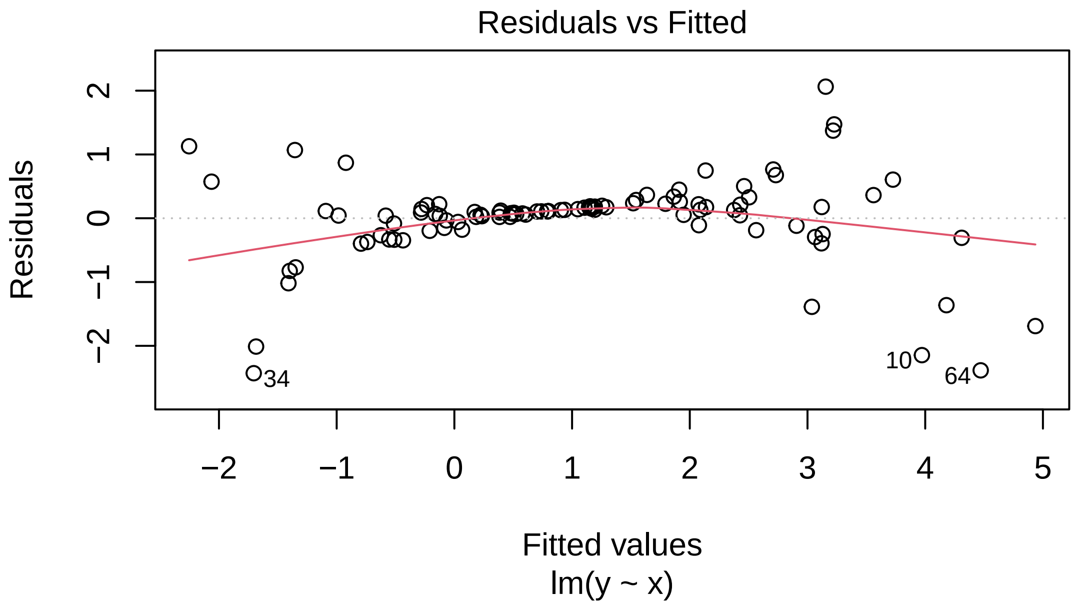
$$y_i = 1 + 2x_i + \varepsilon_i$$

where $\varepsilon_i \sim N(0, 0.5^2)$.

A heteroscedastic model



The residual vs fitted plot



The true model

$$y_i = 1 + 2x_i + \varepsilon_i$$

where $\varepsilon_i \sim N(0, 0.5^2 x_i^4)$.

Your turn

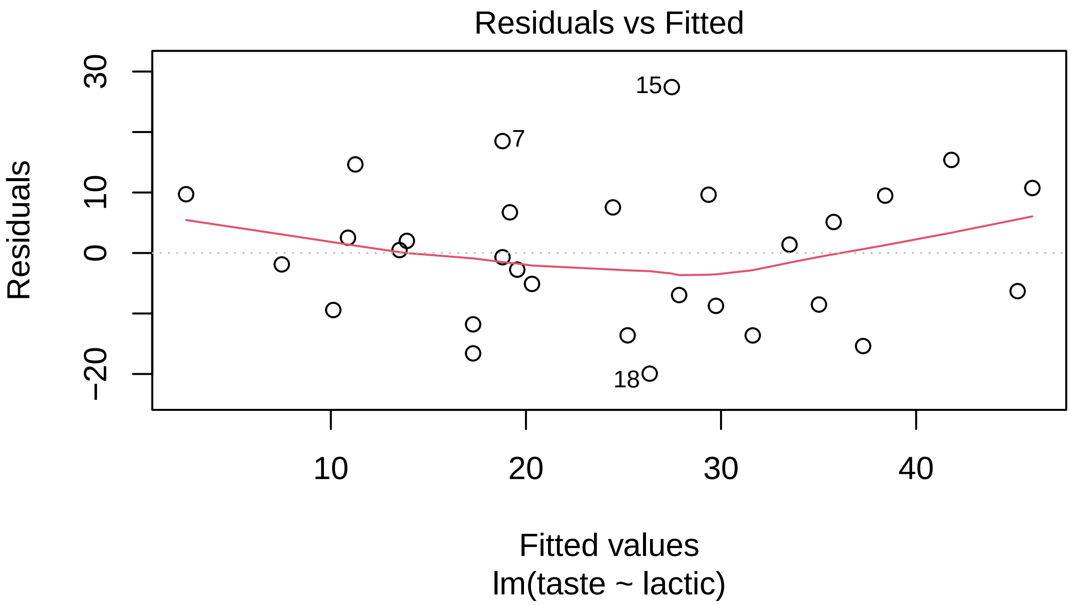
What to do

1. For you model fitted previously, test the homoscedasticity assumption.

Solutions:
See the plot below

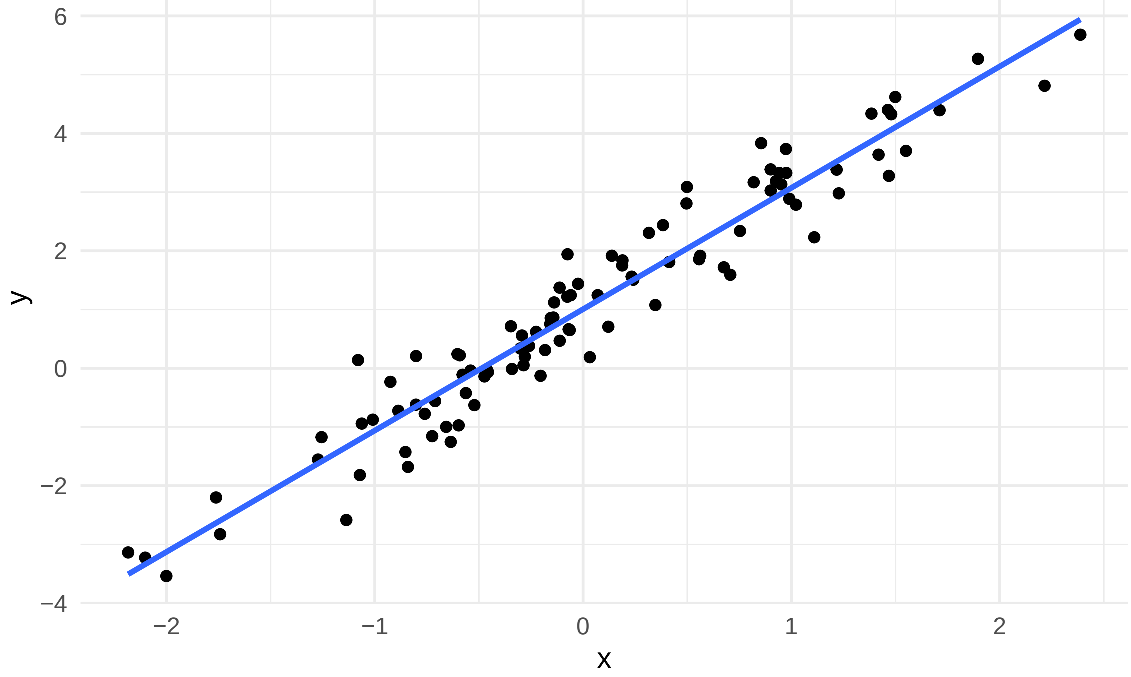
- **What?** homoscedasticity
- **Where?** Residual vs fitted plot.
- **What do you expect?** Random scatter from left to right.
- **What do you see?** Random scatter from left to right, given the data size
- **What do you conclude?** homoscedasticity reasonable.

```
plot(cheddar_lm, which = 1)
```

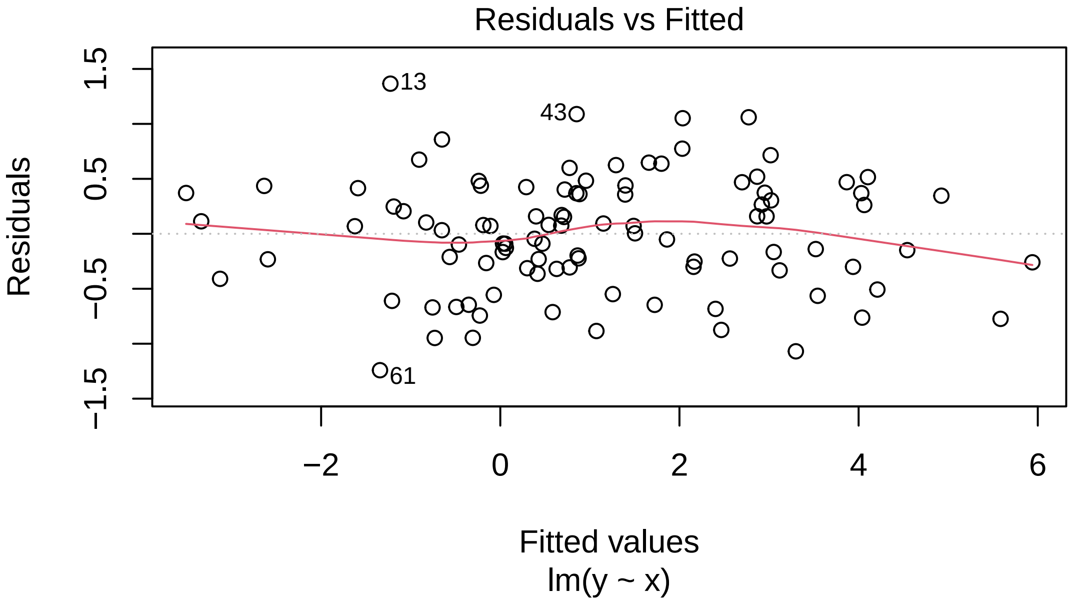


Normality

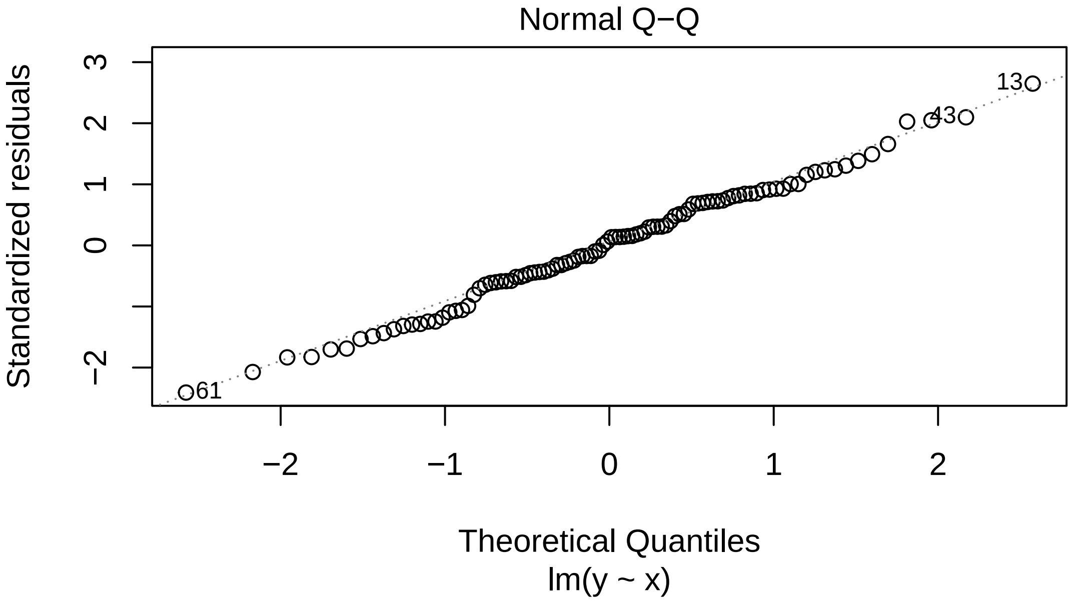
A normal model



The residual vs fitted plot



The residual QQ plot

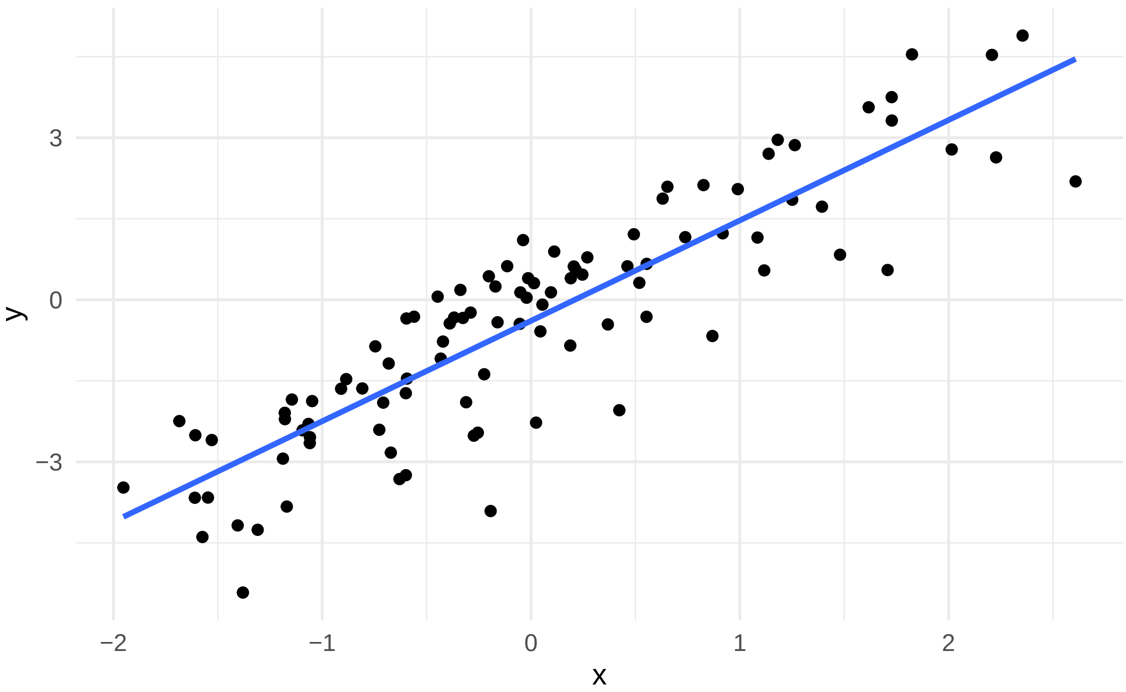


The true model

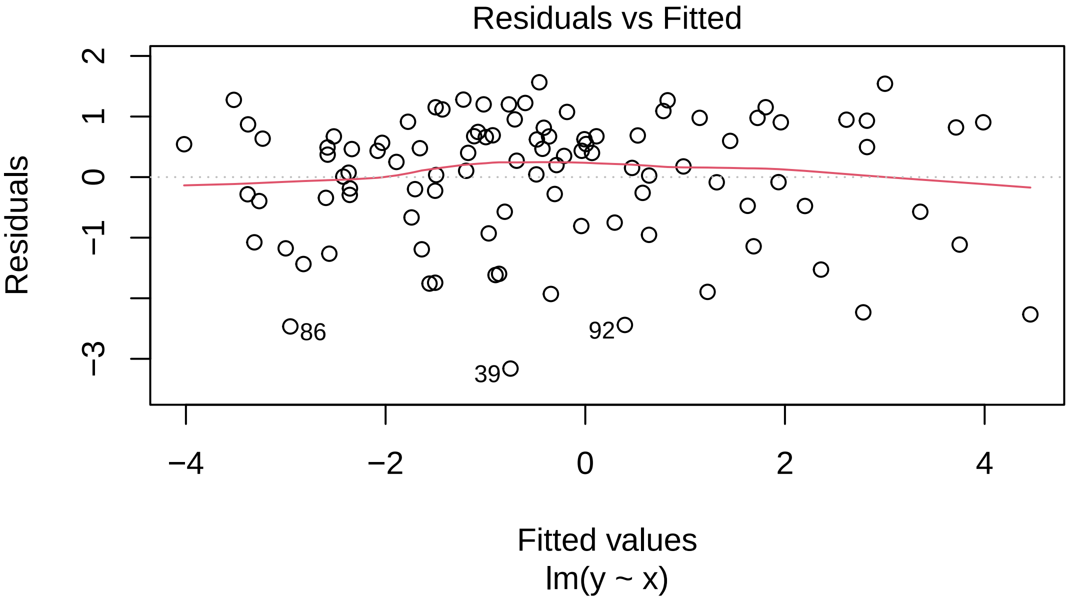
$$y_i = 1 + 2x_i + \varepsilon_i$$

where $\varepsilon_i \sim N(0, 0.5^2)$.

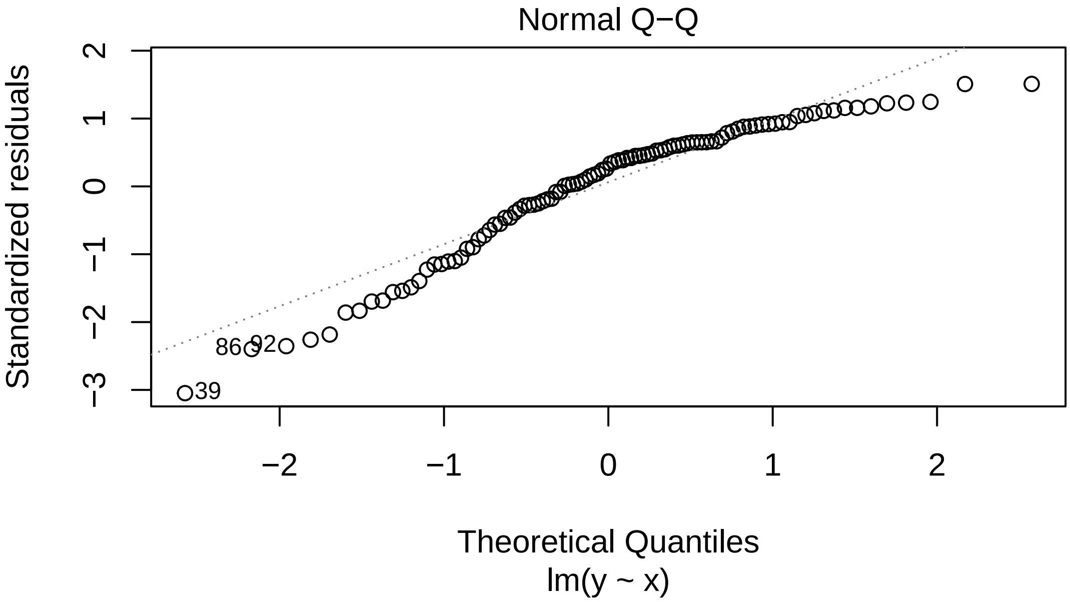
A non-normal model



The residual vs fitted plot



The residual QQ plot



The true model

$$y_i = 1 + 2x_i + \varepsilon_i$$

where $\varepsilon_i \sim \log \left| N(0, 0.5^2) \right|$.

Your turn

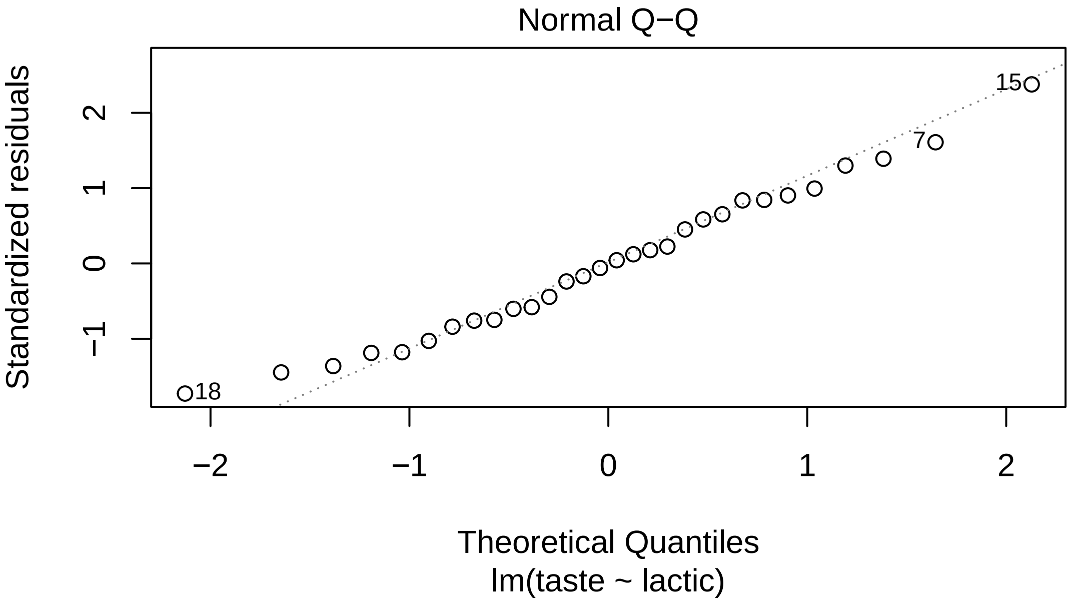
What to do

1. For you model fitted previously, test the normality assumption.

Solutions:
See the plot below

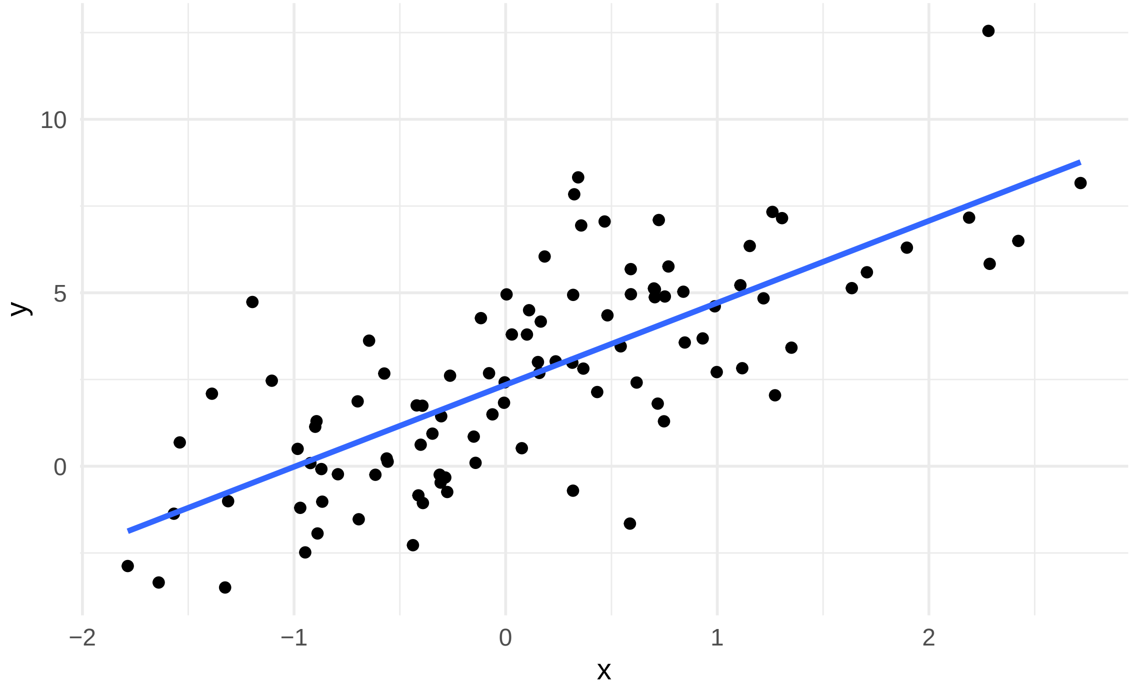
- **What?** normality
- **Where?** normal QQ plot
- **What do you expect?** A relative straight line
- **What do you see?** A relative straight line
- **What do you conclude?** normality reasonable.

```
plot(cheddar_lm, which = 2)
```

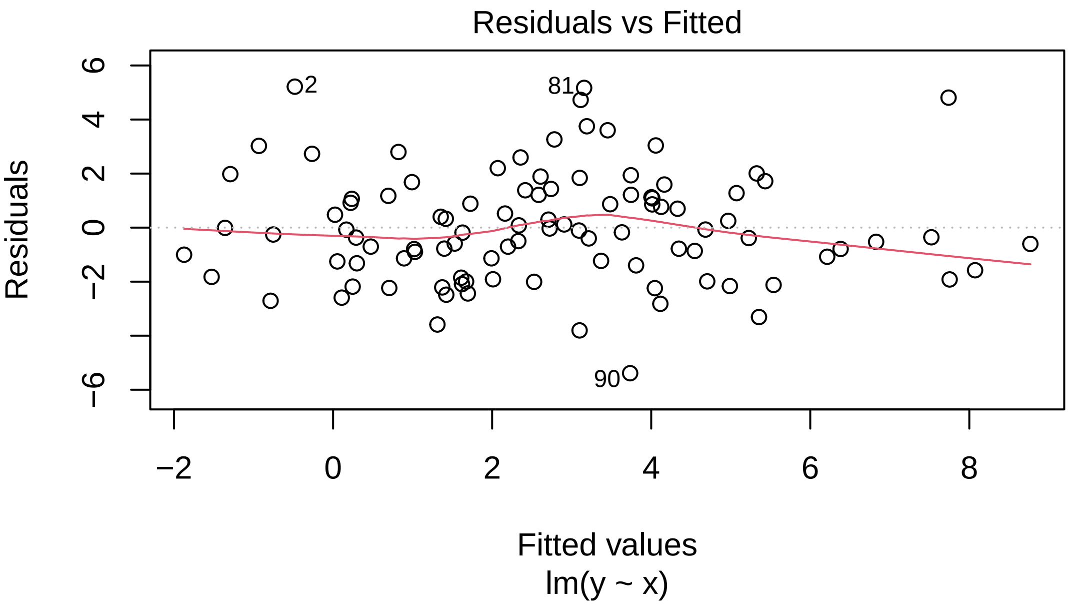


One more assumption

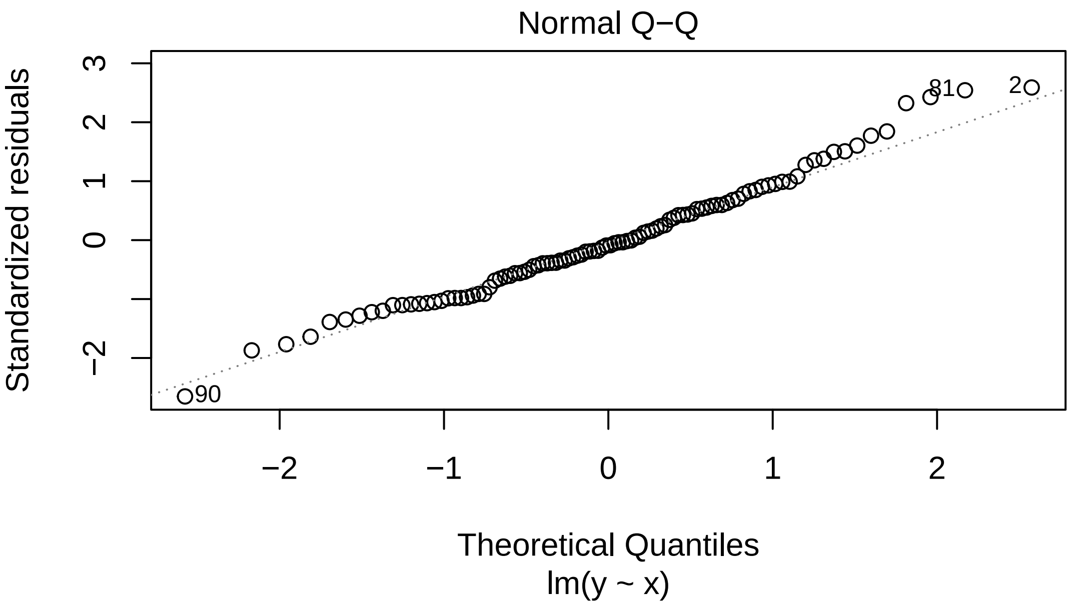
The plot



The residual vs fitted plot



The redisual QQ plot



The true model

$$y_i = 1 + 2x_i + y_{i-1} + \varepsilon_i$$

where $\varepsilon_i \sim N(0, 0.5^2)$.

Your turn

What to do

1. For you model fitted previously, test the independence assumption.

Solutions:
Must check data collection

- **What?** independence
- **Where?** experiment design
- **What do you expect?** random collections
- **What do you see?** Overall taste scores were combined from several testers
- **What do you conclude?** This is iffy. I would say not independent, we are averaging out over multiple tested, but we must assume that the same testers tasted each cheese.