# Hypothesis tests

# Definition 1.9

A statistical hypothesis is a statement about the unknown parameter $\theta$. The most general formulation is

$H_a, H_1$

$$H_0 : \theta \in \Theta_0$$

where

$$\Theta_0 \subset \Theta$$

and $\Theta$ is the parameter space.

$H_0: \mu = 8$

$H_0: p < 0.5$

# Scalar parameter

Usually of the form

$$H_0 \colon \underline{\theta = \theta_0}.$$

simple hypothesis

$H_0: \theta < \theta_0$

composite hypothesis

$H_0: \theta > \theta_0$

# Hypothesis testing

Hypothesis testing is formulated in terms of two hypotheses:

- $H_0$: the null hypothesis

  the default hypothesis

  initially assumed to be true

  typically we want to try to disprove this based on the observed sample

- $H_a$: the alternative hypothesis

  mutually exclusive to $H_0$

  usually the complement of $H_0$

e.g. checking if a coin is fair

let $p$ = probability of head

$H_0: p = \frac{1}{2}$

$H_0: p \neq \frac{1}{2}$

# Hypothesis test

A test of the hypothesis $H_0$ is a rule that tells us, for a given set of data $y_1, y_2, \dots, y_n$ whether we should reject or not reject $H_0$.

2 possible outcomes:

- reject $H_0$: there is enough evidence in the sample
- fail to reject $H_0$: there is insufficient evidence in the sample

# Hypothesis test

Example

Consider a jury trial. The hypotheses are:

- $H_0$: defendant is innocent
- $H_a$: defendant is guilty

$H_0$ (innocent) is rejected if $H_a$ (guilty) is supported by evidence beyond “reasonable doubt”.

Failure to reject $H_0$ (prove guilty) does not imply innocence, only that the evidence is insufficient to reject it.

# Hypothesis test

Usually a test is constructed from a test statistic, $T$, and a critical region, $C$, with the rule

- Reject $H_0$ if $T \in C$
- Fail to reject $H_0$ if $T \notin C$

test statistic: a function of the data on which our decision is based on

critical region (or rejection region): the set of all test statistic values for which $H_0$ will be rejected in favour of $H_a$

# Hypothesis testing

A statistical test is composed of these essential elements:

- Null hypothesis
- Alternative hypothesis
- Test statistic
- Critical region

# Example 1.9

If $y_1, y_2, \ldots, y_n$ are i.i.d. $N(\mu, \sigma^2)$ observations with known $\sigma^2$.

Write down the appropriate test statistic and critical region to test

$$H_0: \mu = \mu_0. \text{ vs } H_a: \mu \neq \mu_0$$

test statistic: $Z = \dfrac{\bar{Y} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

critical region: $C = \{ z : |z| \geq 1.96 \}$

(using $\alpha = 0.05$ level of significance)

# Example 1.10

A rental car company is looking for fuel additives that may increase fuel efficiency. They conducted a pilot study with 30 cars using a new additive, in a road test from Adelaide to Victor Harbour. Without the additive, the cars are known to average 10L/100km with a standard deviation of 2L/100km. It turns out, with the additive, the cars average 9L/100km. What should the company conclude?

Let $Y_i$ = fuel efficient of car $i$ in the trial, $i = 1, 2, \ldots, 30$

Assume $Y_i \sim$ iid $N(\mu, \sigma^2 = 4)$.

$H_0: \mu = 10$

$H_a: \mu < 10$

test statistic: $Z = \frac{9 - 10}{\frac{2}{\sqrt{30}}} \approx -2.74$

Critical region: $C = \{z : z < -z_\alpha\} = \{z : z < -1.64\}$ assuming $\alpha = 0.05$

Since $z$ falls within $C$, we have sufficient evidence to reject $H_0$.
It appears the additive does increase fuell efficiency.