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EE 522 HW6

1.)

$$z = \frac{1+aw}{1-aw}$$

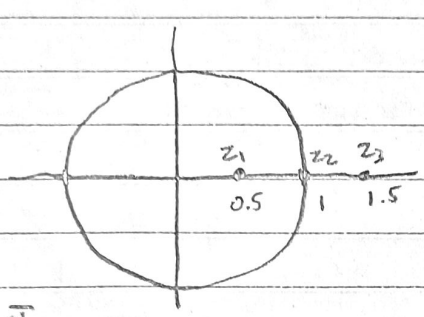
$$z(1-aw) = 1+aw$$

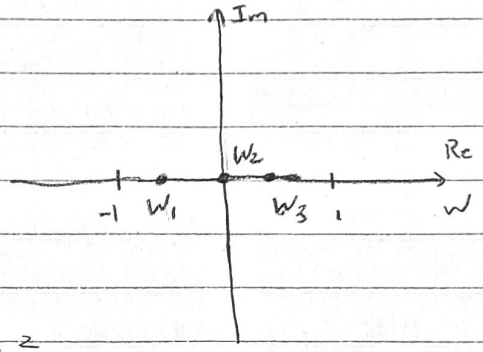
$$z - azw = 1 + aw$$

$$z - 1 = azw + aw$$

$$z - 1 = aw(z+1)$$

a) $$W = \frac{(z-1)}{a(z+1)}$$





$$a = \frac{T}{2}$$

$$w_1 = \frac{2}{T}\frac{(0.5-1)}{(0.5+1)} = -0.333\frac{2}{T}$$

$$w_2 = \frac{2}{T}\frac{(1-1)}{(1+1)} = 0$$

$$w_3 = \frac{2(1.5-1)}{T(1.5+1)} = 0.2\frac{2}{T}$$

b) $$s = jw \qquad z = e^{sT} = \qquad w = \frac{2}{T}\frac{e^{sT}-1}{e^{sT}+1} = \frac{2}{T}\frac{e^{-jwT}-1}{e^{-jwT}+1}$$

$$\frac{e^{-jwT}-1}{e^{-jwT}+1} = j\tan\left(\frac{wT}{2}\right) \qquad W = j\frac{2}{T}\tan\left(\frac{wT}{2}\right)$$

2) $D(z) = K = 1 \quad T = 0.6\,s$

$$G(z) = \frac{1.037}{z - 0.7408}$$

$$G(w) = G(z)\Big|_{z = \frac{1 + \frac{0.6}{2}w}{1 - \frac{0.6}{2}w}} = \frac{0.5217w + 0.527}{w - 0.7391}$$

a) Plot in matlab

b) Gain margin → 4.29 dB.

Phase margin → 67.1°

c) marginally stable at around $K = 1.68$

System oscillates at $5.24\ rad/s$

3) $T = 0.05$ $K = 1$

$$G(z) = \frac{0.02268z + 0.02052}{(z-1)(z-0.7408)} \qquad H(z) = 0.4$$

a) $$G_{CL}(z) = \frac{G(z)}{1 + G(z)H(z)}$$

$$G_{CL}(z) = \frac{0.02268z + 0.02052}{(z-1)(z-0.7408) + (0.02268z + 0.02052)0.4}$$

$H(z) \rightarrow -7,96\,dB$

Gain Crossover frequency → 0.999 rad/s
Phase margin → 99.11°

Phase crossover frequency → 9.799 rad/s
gain margin → -23,368 dB

b)

4) $G_{notch}(s) = \dfrac{s^2 + \omega_0^2}{s^2 + 2\delta\omega_0 s + \omega_0^2}$ $\quad \delta = \dfrac{1}{\sqrt{2}} \quad \omega = \omega_0$

$= \dfrac{s^2 + 142129}{s^2 + 533.16s + 142129}$

$\omega_0 = 2\pi 60 = 377 \text{ rad/s}$

$\omega_0^2 = 142129$

$2\delta\omega_0 = 533.16$

a) backwards difference approximation

$s \Leftrightarrow \dfrac{z-1}{Tz}$ $\quad T = \dfrac{1}{20(f_0)} = \dfrac{1}{20(60)} = \dfrac{1}{1200} \rightarrow \dfrac{1}{1000} = 0.001$

$$G_{notch}(z) = \frac{\frac{(z-1)^2}{T^2z^2} + \omega_0^2}{\frac{(z-1)^2}{T^2z^2} + 2\delta\omega_0\frac{z-1}{Tz} + \omega_0^2} = \frac{(z-1)^2 + T^2z^2\omega_0^2}{(z-1)^2 + 2\delta\omega_0(z-1)Tz + \omega_0^2T^2z^2}$$

$$= \frac{z^2 - 2z + 1 + T^2z^2\omega_0^2}{z^2 - 2z + 1 + 2\delta\omega_0Tz^2 - 2\delta\omega_0Tz + \omega_0^2T^2z^2}$$

$$= \frac{(1 + T^2\omega_0^2)z^2 - 2z + 1}{(1 + \omega_0^2T^2 + 2\delta\omega_0T)z^2 - (2 + 2\delta\omega_0T)z + 1}$$

$$= \frac{1.142z^2 - 2z + 1}{1.675z^2 - 2.533z + 1}$$

$$= \frac{0.682z^2 - 1.194z + 0.597}{z^2 - 1.512z + 0.597}$$

b) bilinear transform $T = 0.001$

$$G_{notch}(z) = \frac{0.7953z^2 - 1.481z + 0.7953}{z^2 - 1.481z + 0.5906}$$

c) bilinear tranform with prewarping

Pre warp frequency of $w = 377\ rad/s$

$$G_{notch}(z) = \frac{0.7935z^2 - 1.475z + 0.7935}{z^2 - 1.475z + 0.5869}$$

d) matched pole-zero method

$$G_{notch}(z) = \frac{0.7751z^2 - 1.441z + 0.7751}{z^2 - 1.478z + 0.5868}$$

All of the discreatized filters are very similar except the backwards diffrence approximation. This filter may not preform as well as the others

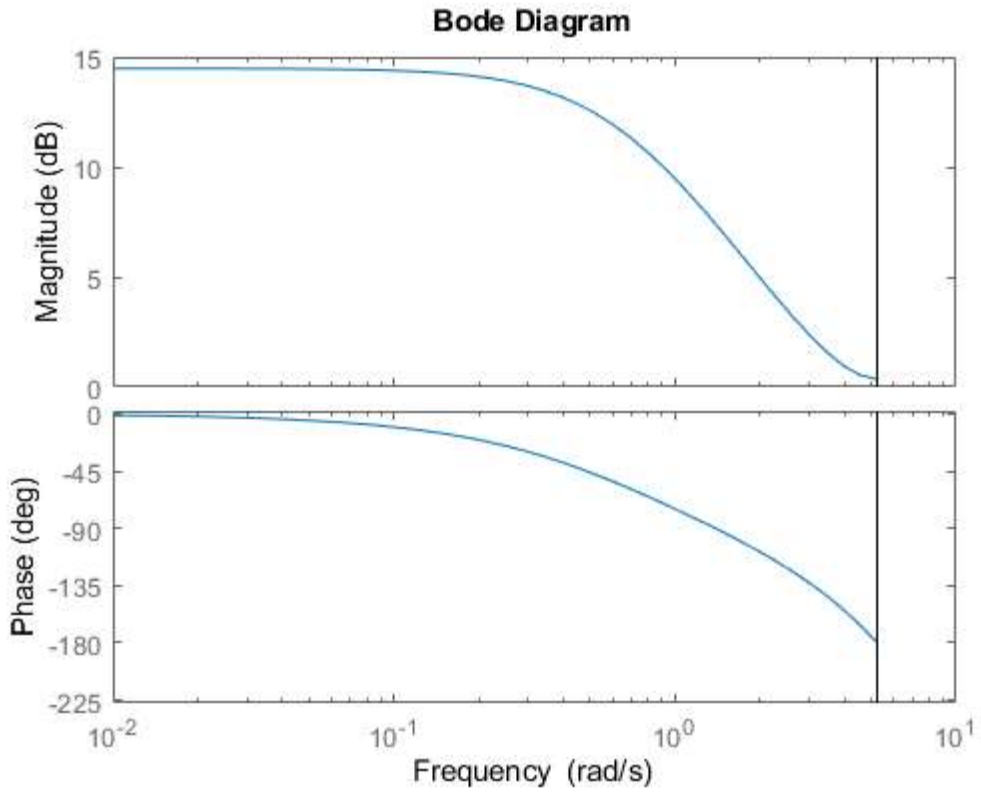
Contents


```
clear; close all; clc;
```

Problem 2

```
Ts = 0.6;
K = 1;
K = 1.68;
%K = 10;
Gs = tf(2, [1 0.5])

Gz = c2d(Gs, Ts)
Gzcl = feedback(K*Gz, 0.04)

figure(1);
hold on;
grid on;
bode(Gzcl);
hold off;
```

```
Gs =

     2
  -------
  s + 0.5

Continuous-time transfer function.


Gz =

    1.037
  ----------
  z - 0.7408

Sample time: 0.6 seconds
Discrete-time transfer function.


Gzcl =

    1.742
  ----------
  z - 0.6712

Sample time: 0.6 seconds
Discrete-time transfer function.
```



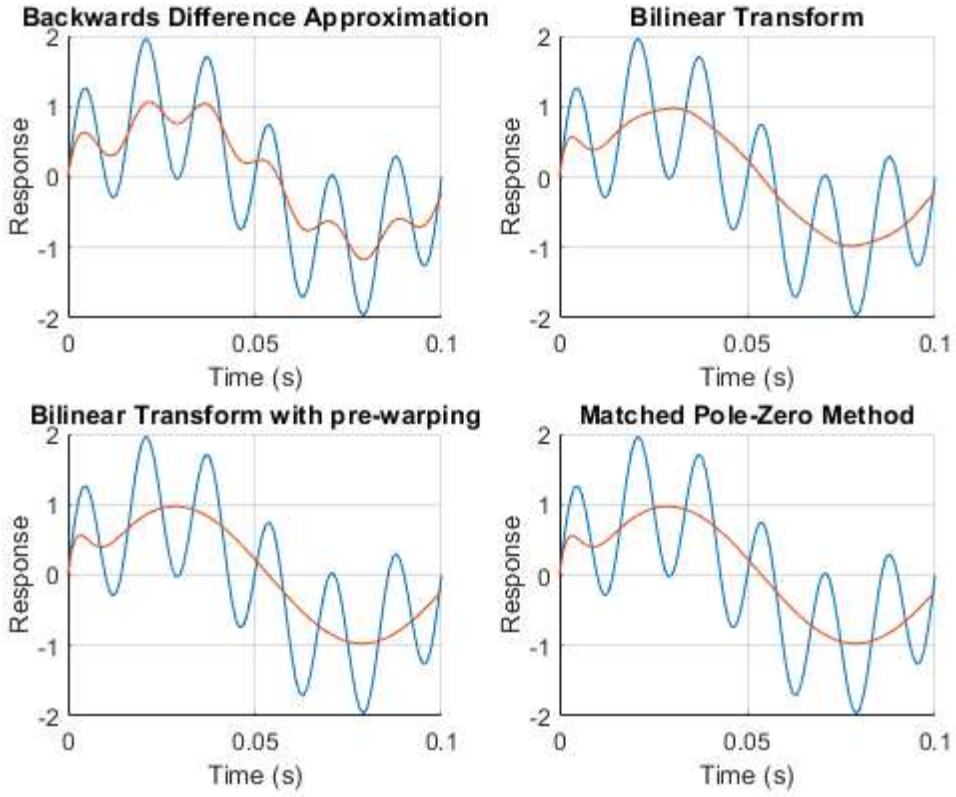
Problem 4

```
figure(3);
Ts = 0.001;
delta = 1/sqrt(2);
wo = 2*pi*60;

Gs = tf([1 0 wo^2],[1 2*delta*wo wo^2])

Gz1 = c2d(Gs, Ts, 'tustin')

opt = c2dOptions('method', 'tustin', 'PrewarpFrequency', 377)
Gz2 = c2d(Gs, Ts, opt);

Gz3 = c2d(Gs, Ts, 'matched')


Gzdiff = tf([0.682 -1.194 0.597], [1 -1.512 0.597]);
t = 0:Ts:0.1;
% plots for problem 4
sig = sin(2*pi*60*t)+sin(2*pi*10*t);

% plot a
subplot(2,2,1);
title('Backwards Difference Approximation');
xlabel('Time (s)');
ylabel('Response');
grid on;
hold on;
plot(t, sig);
[n,d] = tfdata(Gzdiff);
n = cell2mat(n);
d = cell2mat(d);
```

```
plot(t, filter(n,d, sig));
hold off;

% plot b
subplot(2,2,2);
title('Bilinear Transform');
xlabel('Time (s)');
ylabel('Response');
grid on;
hold on;
plot(t, sig);
[n,d] = tfdata(Gz1);
n = cell2mat(n);
d = cell2mat(d);
plot(t, filter(n,d, sig));
hold off;

% plot c
subplot(2,2,3);
title('Bilinear Transform with pre-warping');
xlabel('Time (s)');
ylabel('Response');
grid on;
hold on;
plot(t, sig);
[n,d] = tfdata(Gz2);
n = cell2mat(n);
d = cell2mat(d);
plot(t, filter(n,d, sig));
hold off;

% plot b
subplot(2,2,4);
title('Matched Pole-Zero Method');
xlabel('Time (s)');
ylabel('Response');
grid on;
hold on;
plot(t, sig);
[n,d] = tfdata(Gz3);
n = cell2mat(n);
d = cell2mat(d);
plot(t, filter(n,d, sig));
hold off;
```

```
Gs =

       s^2 + 1.421e05
  ------------------------
  s^2 + 533.1 s + 1.421e05

Continuous-time transfer function.


Gz1 =

  0.7953 z^2 - 1.481 z + 0.7953
  -----------------------------
     z^2 - 1.481 z + 0.5906
```

```
Sample time: 0.001 seconds
Discrete-time transfer function.


Gz3 =

  0.7751 z^2 - 1.441 z + 0.7751
  -----------------------------
     z^2 - 1.478 z + 0.5868

Sample time: 0.001 seconds
Discrete-time transfer function.
```



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