

¹ RingsPy: A Python package for Voronoi mesh generation of cellular solids with radial growth pattern

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⁵ Summary

⁶ RingsPy is an open-source package entirely written in Python that generates 3D meshes
⁷ of prismatic cellular solids with tunable radial growth rules featured by many natural or
⁸ architected cellular solids with a radial, differential cell growth pattern. Typical examples of
⁹ this type of materials are microstructures of wood ([Tonn & Greb, 2017](#)), skeletal muscle fibers
¹⁰ ([Jorgenson et al., 2020](#)), and anisotropic polymeric foams ([Martinez et al., 2016](#)). The 2D
¹¹ geometry of the cellular structure of the solid is first constructed with a 2D Voronoi tessellation,
¹² and then the 2D Voronoi cells are extruded in the longitudinal (parallel-to-grain) direction
¹³ with a certain grain angle around the longitudinal axis. This process is supposed to mimic
¹⁴ the morphology and dynamic growth of natural or additively manufactured cellular materials.
¹⁵ The package is dependent only on numpy ([Harris et al., 2020](#)) and scipy ([Virtanen et al.,
2020](#)) for core implementation, however, regular cells (e.g., hexagonal honeycomb structures)
¹⁶ option is also provided, with the help of Python package hexalattice. The visualization of the
¹⁷ generated geometry is implemented using Python package matplotlib for 2D cross-sectional
¹⁸ images, and for 3D models using the VTK or STL format files (which can be then used in
¹⁹ scientific visualization tools such as Paraview and for 3D printing).
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²¹ Statement of need

²² Natural and bioinspired materials are often characterized by irregular and heterogeneous cellular
²³ microstructures, they include materials such as wood, nacre, trabecular bone, or 3D printed
²⁴ biomimetic composites ([Bhate et al., 2019](#); [Gibson & Ashby, 1997](#); [Tekog et al., 2011](#)).

²⁵ Cellular yet irregular morphology of materials is a natural outcome of local cell growth that
²⁶ follows simple intrinsic physical rules, among these, the Voronoi diagram, has been proposed
²⁷ to be one of the best approximations of cell interfaces at the steady-state by the equilibrium
²⁸ of intracellular pressures and the intercellular surface tension ([Sánchez-Gutiérrez et al., 2016](#);
²⁹ [Saye & Sethian, 2011](#)). Besides, there is also an underlying process, which governs the global
³⁰ growth of cells to form the radial pattern in plane and the growth of either straight or helical
³¹ grains in the perpendicular direction to the plane ([Hartmann et al., 2017](#); [Sozio & Yavari,
2019](#)). Understanding the role of locally irregular and globally radial morphology in determining
³² material properties offers a new path to unveil the behavior of natural materials and engineer
³³ materials with superior functionalities, such as anisotropic programmable material properties,
³⁴ stress redirection, and oriented impact energy dissipation.
³⁵

³⁶ While the laboratory and in-site imaging approaches (e.g., 3D X-ray tomography) can be
³⁷ reliable for reproducing the most geometrically accurate material morphology, however, these
³⁸ approaches are still costly and time-consuming in some highly repetitive scenarios, such as
³⁹ prototyping and probabilistic analyses. Given that, an algorithm-based, stochastic generator of
⁴⁰ cellular geometries is at a new level of need.

⁴¹ We propose a fundamental, growth-inspired program that evokes the formation of cellular
⁴² architectures with radial and longitudinal growth patterns in natural systems. This virtual
⁴³ growth program imposes both local and global generation rules on a limited category of basic
⁴⁴ elements. It generates meshes that exhibit stochastic geometrical properties, starting from very
⁴⁵ limited initial constraints, which echoes the diversity of biological and biomimetic systems.

⁴⁶ RingsPy workflow

⁴⁷ RingsPy has a basic workflow illustrated as follows (Figure 1):

- ⁴⁸ 1. The users first assign a radial growth pattern of the cellular solid with a couple of input
⁴⁹ parameters. This process will create 2D concentric ring-like regions for Voronoi cell
⁵⁰ nuclei placement. Each placement region has a specific generation rule (for example, for
⁵¹ wood, the cell nuclei will be assigned larger cell sizes in earlywood regions and smaller
⁵² ones in latewood regions; this pattern is called binary wood). These nuclei with given
⁵³ cell sizes will be used to pack many non-overlapping circles in each region.
- ⁵⁴ 2. Once the nuclei are properly placed, an associated Voronoi diagram is then generated
⁵⁵ by calling `scipy.spatial.Voronoi`. The Voronoi diagram automatically tessellates
⁵⁶ the region into Voronoi cells. The ridges of Voronoi cells will be treated as the cell
⁵⁷ walls/interfaces. A user-defined clipping box will constrain the Voronoi mesh into the
⁵⁸ desired shape, and the data structure of the Voronoi mesh is correspondingly rebuilt.
- ⁵⁹ 3. The 2D Voronoi cells are then extruded in the longitudinal (or parallel-to-grain) direction
⁶⁰ with a pre-twisting angle. Fibrils (a.k.a., grains or beams) will be formed during the
⁶¹ extrusion process at Voronoi vertices (joints of Voronoi ridges). The connectivity of
⁶² 2D Voronoi ridges will remain at every extrusion layer. A 3D prismatic cellular mesh is
⁶³ constructed, accompanied by data files.

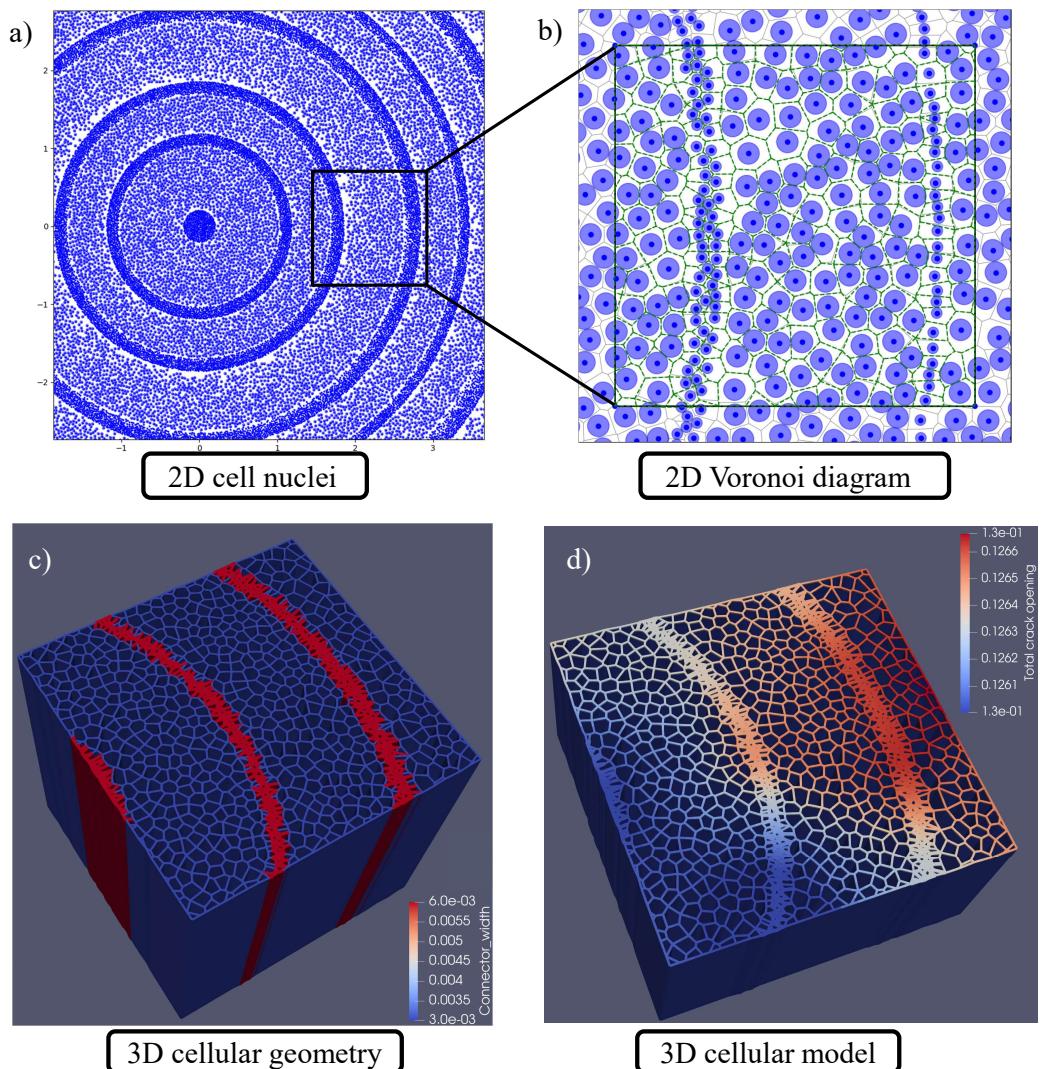


Figure 1: Pipeline of RingsPy – an example of the microstructure generation of a cubic spruce wood specimen: a) cell nuclei of a spruce wood log are firstly generated by a binary wood (abrupt earlywood-latewood transition) radial growth rule, b) cell nuclei are used as Voronoi sites to generate the Voronoi diagram, and the geometric boundaries (clipping box) for a cubic specimen then cut and trim Voronoi diagram into the desired shape, c) 2D Voronoi cells are then extruded in the longitudinal direction to form the 3D prismatic cellular geometry, 3D meshes are constructed, they can be used for volumetric rendering of the cubic specimen in Paraview, the color map shows the thicknesses of cell walls/interfaces, and d) the 3D meshes can be used for other purposes, such as mechanical strength analysis, the color map shows the simulation results of cell wall crack openings associated with the mechanical tensile tests of the specimen

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