Porepressure diffusion in edge elems

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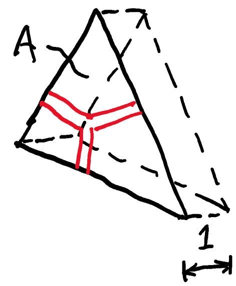
mass conservation

$\boxed{\text{Ref: Rice \& Cleary 1976}}$

Consider in a control volume $V = A \cdot 1$



$$\dot{v}_u (A_e \cdot 1) + \dot{m}_c \cdot 1 + \frac{\partial}{\partial x} (\rho_f \dot{u}_f)(A_e \cdot 1) + \frac{\partial}{\partial x} Q_c = q (A_e \cdot 1)$$

$$\dot{v}_u A_e + \dot{m}_c + \frac{\partial}{\partial x} (\rho_f \dot{u}_f) A_e + \frac{\partial}{\partial x} Q_c = q A_e$$

A_e : edge area of uncracked material
A_c : edge area of cracked material

$\Big\}$ $A = A_e + A_c$ total edge area

m_c : mass of fluid in the cracked material per unit length

v_u : mass of fluid in the uncracked material per unit volume

u_f : mass flux density through A_e

Q_c : fluid mass flux through A_c

q : mass source or sink rate

$$v_u = \rho_f \phi^*$$

ρ_f : fluid density
ϕ^* : apparent fluid volume fraction

$$\rho_f = \rho_{f_0} [1 + c_f (p - p_0)]$$

ρ_{f_0} : reference fluid density
c_f : fluid bulk modulus
p_0 : reference pressure

$$\phi^* - \phi = \underbrace{\boxed{\frac{\Delta V_f^{(1)}}{V}}}_{\text{compression / dilation of interstitial fluid}} - \underbrace{\boxed{\frac{\Delta V_f^{(2)}}{V}}}_{\text{Fluid exchange}} = -\phi c_f p + \zeta$$

ζ : increment of fluid content

$$\zeta = b \varepsilon + M^{-1} p$$

b : Biot coeffient

M : Biot modulus

$$\dot{\nu}_u = \dot{\rho}_f \phi^* + \rho_f \dot{\phi}^*$$

$$= \rho_{f_0} c_f \dot{p} \phi^* + \rho_f (-\phi c_f \dot{p} + b\dot{\varepsilon} + M^{-1}\dot{p})$$

$$\simeq \rho_{f_0} (b\dot{\varepsilon}_v + M^{-1}\dot{p})$$

$\dot{\varepsilon}_v$: volumetric strain rate

$$m_c = \rho_f A_c$$

$$\dot{m}_c = \dot{\rho}_f A_c + \rho_f \dot{A}_c = \rho_{f_0} c_f \dot{p} A_c + \rho_f \dot{A}_c$$

Darcy's flow for uncracked area:

$$u_f = -\frac{k_0}{\mu_f}\frac{\partial P}{\partial x}$$

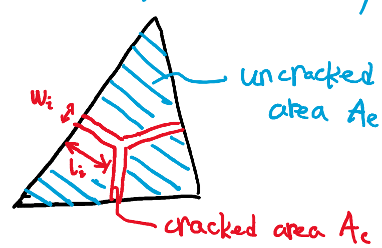
k_0 : intrinsic permeability of the porous media

μ_f : fluid dynamic viscosity

2D Poiseuille flow along crack opening :

$$Q_c = -\frac{\rho_f l_i w_i^3}{12\mu_f}\frac{\partial P}{\partial x}$$

($i = 1, 2, 3$, omitted later)



$$\rho_{f_0} (b\dot{\varepsilon}_v + M^{-1}\dot{p}) A_e + \rho_{f_0} c_f \dot{p} A_c + \rho_f \dot{A}_c + \frac{\partial}{\partial x}\left[\rho_f\left(-\frac{k_0}{\mu_f}\frac{\partial P}{\partial x}\right)\right] A_e$$

$$+ \frac{\partial}{\partial x}\left(-\frac{\rho_f}{\mu_f}\frac{l w^3}{12}\frac{\partial P}{\partial x}\right) = q A_e$$

$$\rho_{f_0} (M^{-1} A_e + c_f A_c)\dot{p} + \frac{\partial}{\partial x}\left(\rho_f A_e - \frac{k}{\mu_f} A_e \frac{\partial P}{\partial x} - \frac{\rho_f}{\mu_f}\frac{l w^3}{12}\frac{\partial p}{\partial x}\right)$$

$$= \cancel{q A_e} - \rho_{f_0} b\dot{\varepsilon} - \rho_f \dot{A}_c$$

$q = 0$

Divide by ρ_{f_0}

$$\boxed{(M^{-1} A_e + c_f A_c)\dot{p} + \frac{\partial}{\partial x}\left(\frac{\rho_f}{\rho_{f_0}} A_e - \frac{k}{\mu_f \rho_{f_0}} A_e \frac{\partial P}{\partial x} - \frac{\rho_f}{\mu_f \rho_{f_0}}\frac{l w^3}{12}\frac{\partial p}{\partial x}\right) + b\dot{\varepsilon} + \frac{\rho_f}{\rho_{f_0}}\dot{A}_c = 0}$$

$$+ b\dot{\epsilon} + \frac{\rho_f}{\rho_{f_0}} \dot{A}_c = 0$$

2D to 3D

$$M^{-1} A_e + c_f A_c = M^{-1} V_i + \frac{V_{ci}}{k_f V_i} V_i \quad (i=1,2)$$

k_f : fluid bulk modulus $\quad V_i$: total tet volume $\quad V_{ci}$: cracked volume

$\bar{\rho}_f = \rho_{f_1} g_2 + \rho_{f_2} g_1$

$$\frac{\partial}{\partial x}\left(-\frac{k_0}{\mu_f \rho_{f_0}} A_e \frac{\partial p}{\partial x} - \frac{\rho_f}{\mu_f \rho_{f_0}} \frac{L w^3}{12} \frac{\partial p}{\partial x}\right) = -\left(\frac{k_0 + k_c}{\rho_{f_0} \mu_f}\right) \frac{\partial p}{\partial x} A_e \bar{\rho}_f$$

$$k_c = \frac{1}{12 A_e}\left(\frac{g_2}{I_{c1}} + \frac{g_1}{I_{c2}}\right)^{-1}$$

$$I_{ci} = \sum_{j=1}^{3} \left(L_j S_{N_j}^{i\,3}\right) \quad (i=1,2)$$

$S_{N_j}^i$ — w_i crack opening

$$b\dot{\epsilon} + \frac{\rho_f}{\rho_{f_0}} \dot{A}_c = b\dot{\epsilon}_{v_i} V_i + \frac{\rho_f \dot{V}_{ci}}{\rho_{f_0} V_i} V_i \quad (i=1,2)$$

$$\left(M^{-1} V_i + \frac{V_{ci}}{k_f V_i} V_i\right) \dot{p} + \frac{\partial}{\partial x}\left(\frac{\rho_f}{\rho_{f_0}} A_e - \left(\frac{k_0 + k_c}{\rho_{f_0} \mu_f}\right) \frac{\partial p}{\partial x} A_e \bar{\rho}_f\right)$$

$$+ b\dot{\epsilon}_{v_i} V_i + \frac{\rho_f \dot{V}_{ci}}{\rho_{f_0} V_i} V_i = 0 \quad (i=1,2)$$

rewrite in matrix form

$$V \begin{bmatrix} g_1 C_1 & 0 \\ 0 & g_2 C_2 \end{bmatrix} \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} + \xi \frac{A_n}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + V \begin{bmatrix} g_1 S_1 \\ g_2 S_2 \end{bmatrix} = \vec{0}$$

$$C_i = M^{-1} + V_{ci} (k_f V_i)^{-1}$$

$$\xi = \frac{\bar{\rho}_f (k_0 + k_c)}{\rho_{f_0} \mu_f} \qquad k_c = \frac{1}{12 A}\left(\frac{g_2}{I_{c_1}} + \frac{g_1}{I_{c_2}}\right)^{-1} \qquad I_{ci} = \sum_{j=1}^{3} L_{f_j} (S_{N_j}^i)^3$$

$$S_i = b\dot{\varepsilon}_{v_i} + \rho_{f_i} \dot{V}_{ci} (\rho_{f_0} V_i)^{-1} \qquad \rho_{f_i} = \rho_{f_0}\left(1 + \frac{p_i - p_0}{k_f}\right)$$

Linearization by Newton-Raphson Method

$$f(\vec{P}_n) = \left\{ V g_1 \left(M_b^{-1} + \frac{V_{c1}}{k_f V_{g_1}}\right) \dot{p}_1 + \frac{A_n}{L}\left(\frac{\bar{\rho}_f (k_0 + k_c)}{\rho_0 \mu_f}\right)(p_1 - p_2) + V g_1 \left(b\dot{\epsilon}_1 + \frac{\rho_1 \dot{V}_{c_1}}{\rho_{f_0} V_{g_1}}\right) = 0 \right.$$

$$
f(\vec{P_n}) = \begin{cases} V_{g_1}\left(M_b^{-1} + \frac{V_{c1}}{k_f V_{g_1}}\right)\dot{p}_1 + \frac{A_n}{L}\left(\frac{\rho_f(k_0+k_c)}{\rho_0 \mu_f}\right)(p_1 - p_2) + V_{g_1}\left(b\dot{\varepsilon}_1 + \frac{\rho_{f1} \dot{V}_{c1}}{\rho_{f0} V_{g_1}}\right) = 0 \\ V_{g_2}\left(M_b^{-1} + \frac{V_{c2}}{k_f V_{g_2}}\right)\dot{p}_2 - \frac{A_n}{L}\left(\frac{\bar{\rho}_f(k_0+k_c)}{\rho_0 \mu_f}\right)(p_1 - p_2) + V_{g_2}\left(b\dot{\varepsilon}_2 + \frac{\rho_{f2} \dot{V}_{c2}}{\rho_{f0} V_{g_2}}\right) = 0 \end{cases}
$$

the tangent (Jacobian) matrix

$$
\frac{\partial f_1}{\partial p_1} = V_{g_1}\left(M_b^{-1} + \frac{V_{c1}}{k_f V_{g_1}}\right)\frac{1}{\Delta t} + \frac{A_n}{L}\left(\frac{\bar{\rho}_f(k_0+k_c)}{\rho_{f0}\mu_f}\right) + \frac{A_n}{L}\frac{k_0+k_c}{\mu_f} g_2 \frac{P_1 - P_2}{k_f} + \dot{V}_{c_1}\frac{1}{k_{f1}}
$$

$$
\frac{\partial f_1}{\partial p_2} = 0 - \frac{A_n}{L}\left(\frac{\bar{\rho}_f(k_0+k_c)}{\rho_{f0}\mu_f}\right) + \frac{A_n}{L}\frac{k_0+k_c}{\mu_f} g_1 \frac{P_1 - P_2}{k_f}
$$

$$
\frac{\partial f_2}{\partial p_1} = 0 - \frac{A_n}{L}\left(\frac{\bar{\rho}_f(k_0+k_c)}{\rho_{f0}\mu_f}\right) - \frac{A_n}{L}\frac{k_0+k_c}{\mu_f} g_2 \frac{P_1 - P_2}{k_f}
$$

$$
\frac{\partial f_2}{\partial p_2} = V_{g_2}\left(M_b^{-1} + \frac{V_{c2}}{k_f V_{g_2}}\right)\frac{1}{\Delta t} + \frac{A_n}{L}\left(\frac{\bar{\rho}_f(k_0+k_c)}{\rho_{f0}\mu_f}\right) - \frac{A_n}{L}\frac{k_0+k_c}{\mu_f} g_1 \frac{P_1 - P_2}{k_f} + \dot{V}_{c_2}\frac{1}{k_{f2}}
$$

Parameter list:

$[\rho_{f0},\ \mu_f,\ k_f,\ k_0,\ M_b,\ b,\ p_0,\ K]$

- ρ_{f0} → fluid ref density
- μ_f → fluid dynamic viscosity
- k_f → fluid bulk modulus
- k_0 → intrinsic permeability of porous media
- M_b → Biot modulus
- b → Biot coefficient
- p_0 → reference pressure
- K → time scaling factor