Group 1
Presentation

Robust Portfolio Optimization

December 1, 2020

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Introduction & Problem Setting

Team Member Biographies

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Introduction & Problem Setting

Objective & Problem Formulation

- Objective: To develop a method for generating optimal portfolios and an appropriate decision criterion for choosing the single best "robust" alternative among the available options
  - **Robustness**: A robust portfolio is one that best mitigates potential drawdowns of investor capital, while still providing exposure to a selected basket of securities
  - **Desired Outcome**: The final product of this process should be *one set of portfolio weights*, spread among the basket of securities, that offers the best expected downside risk protection
  - **Constraint Considerations**: Should portfolios permit the use of "short" positions or leverage?

Three Distinct Phases of Work:

- Phase I: Obtaining Data
  - Dow Jones Industrial Avg. components chosen for scrutiny
    - DJIA offers liquid securities and a manageable number of securities for simulation
  - Daily log returns calculated
    - Additivity property and log normality of returns dist.
  - Determine if returns are approx. normally distributed

- Phase II: Optimal Portfolios & Simulations
  - Determine a method of generating optimal portfolios
  - Simulate returns chosen basket of securities for 1,000 periods
  - Calculate the optimal portfolio for each of the 1,000 simulated sets of returns

- Phase III: Cross-Validate Optimal Portfolios
  - Determine expected portfolio returns for each of the 1,000 optimal portfolios across the 1,000 simulations
    - Store each set of expected returns as a tuple
  - Determine single optimal robust portfolio using minimax regret decision criterion

Introduction & Problem Setting

Data Analysis: Structure of dataset

1) Use adjusted daily closing prices to calculate daily log returns

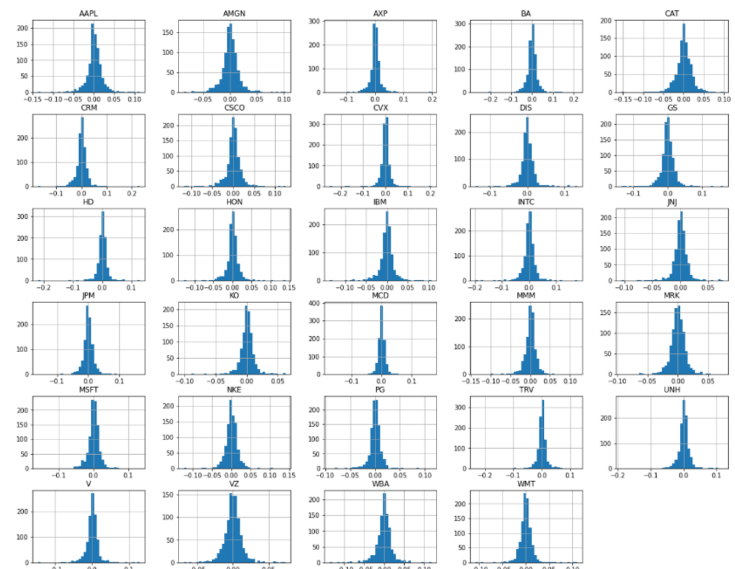
Date	AAPL	AMGN	AXP	BA	CAT	CRM	CSCO	CVX	DIS	GS	...	MRK	MSFT
2017-01-03	27.277639	134.517654	70.771912	145.533676	84.248009	70.540001	26.905218	99.251274	101.584358	226.374527	...	53.849270	58.673244
2017-01-04	27.247108	136.427490	71.932106	147.063477	83.871544	72.800003	26.745281	99.226013	102.886719	227.836365	...	53.831364	58.410725
2017-01-05	27.385668	136.525650	71.045456	147.146927	83.360641	72.790001	26.807476	98.796509	102.829262	226.140228	...	53.813457	58.410725
2017-01-06	27.690971	139.916946	71.186943	147.508514	83.396454	73.800003	26.860790	98.400665	104.361458	229.495026	...	53.956696	58.917015
2017-01-09	27.944603	141.755325	71.554817	146.785309	82.795929	73.959999	26.816360	97.558479	103.767731	227.611450	...	54.699753	58.729496

Date	AAPL	AMGN	AXP	BA	CAT	CRM	CSCO	CVX	DIS	GS	...	MRK	MSFT	NKE	
2017-01-03	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	...	NaN	NaN	NaN	
2017-01-04	-0.001120	0.014098	0.016261	0.010457	-0.004479	0.031536	-0.005962	-0.000255	0.012739	0.006437	...	-0.000333	-0.004484	0.020753	0.0
2017-01-05	0.005072	0.000719	-0.012403	0.000567	-0.006110	-0.000137	0.002323	-0.004338	-0.000559	-0.007472	...	-0.000333	0.000000	-0.000188	0.0
2017-01-06	0.011087	0.024536	0.001990	0.002454	0.000430	0.013780	0.001987	-0.004015	0.014790	0.014726	...	0.002658	0.008630	0.015893	-0.0
2017-01-09	0.009118	0.013054	0.005154	-0.004915	-0.007227	0.002166	-0.001655	-0.008596	-0.005705	-0.008241	...	0.013677	-0.003188	-0.009880	-0.0

2) Generate covariance matrix of daily log returns

	AAPL	AMGN	AXP	BA	CAT	CRM	CSCO	CVX	DIS	GS	...
AAPL	0.101573	0.042846	0.056560	0.070682	0.051836	0.067699	0.056430	0.049725	0.044311	0.060307	...
AMGN	0.042846	0.070083	0.038411	0.035555	0.038586	0.041456	0.041288	0.036262	0.030646	0.039716	...
AXP	0.056560	0.038411	0.127626	0.121672	0.072500	0.055739	0.058728	0.090443	0.070712	0.092278	...
BA	0.070682	0.035555	0.121672	0.248474	0.088851	0.069501	0.065910	0.108134	0.080726	0.100850	...
CAT	0.051836	0.038586	0.072500	0.088851	0.108961	0.048234	0.055461	0.067987	0.050802	0.070588	...
CRM	0.067699	0.041456	0.055739	0.069501	0.048234	0.119047	0.055087	0.045785	0.043109	0.055039	...
CSCO	0.056430	0.041288	0.058728	0.065910	0.055461	0.055087	0.084935	0.050589	0.046466	0.055988	...
CVX	0.049725	0.036262	0.090443	0.108134	0.067987	0.045785	0.050589	0.122194	0.056553	0.077550	...
DIS	0.044311	0.030646	0.070712	0.080726	0.050802	0.043109	0.046466	0.056553	0.084521	0.060583	...
GS	0.060307	0.039716	0.092278	0.100850	0.070588	0.055039	0.055988	0.077550	0.060583	0.109542	...
HD	0.051839	0.036103	0.059636	0.076210	0.048224	0.054233	0.046913	0.057516	0.044549	0.057841	...
HON	0.045964	0.032754	0.076369	0.090864	0.061845	0.040620	0.047657	0.066622	0.051622	0.064471	...
IBM	0.044294	0.034297	0.060405	0.073705	0.052757	0.044166	0.050208	0.054235	0.043934	0.055054	...
INTC	0.067406	0.045428	0.064307	0.078854	0.057626	0.060960	0.063802	0.059024	0.047157	0.063024	...
JNJ	0.030321	0.031701	0.035888	0.039808	0.031457	0.027369	0.034604	0.033627	0.024906	0.031751	...
JPM	0.051262	0.038183	0.095967	0.103705	0.070175	0.047428	0.052358	0.081274	0.060662	0.090502	...
KO	0.028168	0.024352	0.046048	0.056010	0.034975	0.025843	0.027869	0.040093	0.033266	0.036818	...
MCD	0.036871	0.025014	0.054216	0.062281	0.035407	0.038335	0.035240	0.050248	0.038034	0.044777	...
MMM	0.041937	0.032721	0.055770	0.061686	0.061127	0.036375	0.046532	0.048755	0.040288	0.054384	...
MRK	0.030073	0.035398	0.034946	0.037497	0.028930	0.032288	0.032466	0.037586	0.024427	0.032921	...

3) Scrutinize returns for approximate normality



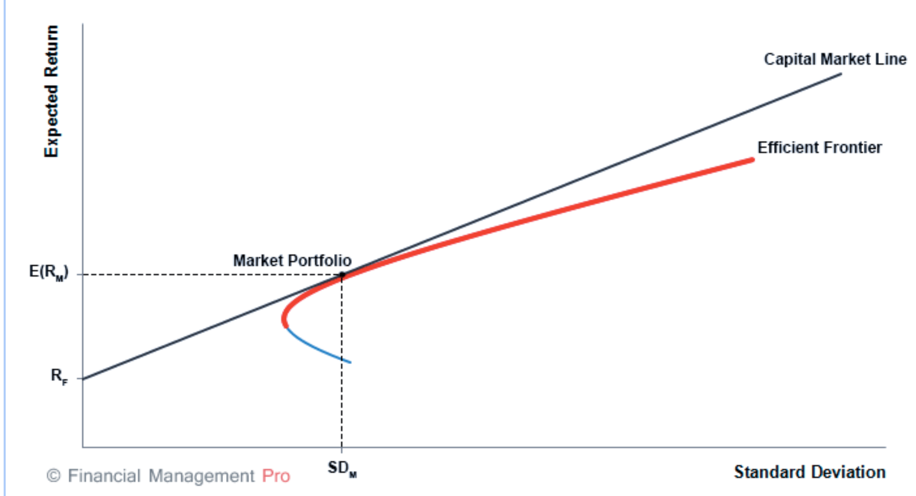
Literature Review

Literature Review

Mean Variance Optimization

- Foundation of portfolio theory laid in Markowitz's 1952 paper, *Portfolio Selection*
  - *Established the mathematical relationship between risk (expected portfolio volatility) and return (expected portfolio return)*
- The parameters of the MVO model are deceptively simple
  - Expected portfolio return = E = $\sum_{i=1}^{N} X_i \, \mu_i$
  - Expected portfolio variance = V = $\sum_{i=1}^{N} \sum_{i=1}^{N} X_i \, Xj \, \sigma_{ij}$
  - Sum of the security allocations = $\sum_{i=1}^{N} X_i = 1$
  - Minimum security allocation: $X_i \geq 0$
- Quadratic optimization methods are used (notably, Markowitz's Critical Line Algorithm)

The Efficient Frontier



MVO Advantages

- Uses known optimization methods (quadratic programming) to reach an optimal solution
  - Produces a convex solution space, meaning that optimality is global, not local
- Permits investors to express their desired level of risk and solve for the highest attainable expected return

MVO Disadvantages

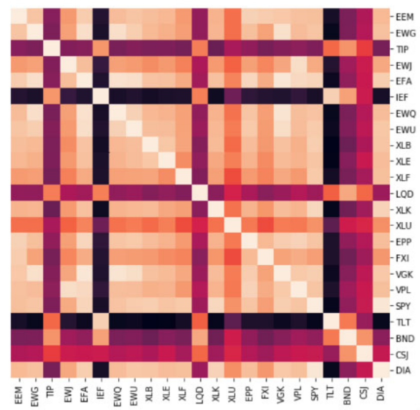
- Assumes expected returns and covariance for securities are known quantities
- It has been shown that small deviations in expected returns can cause significantly different optimal portfolio allocations
  - High recommended concentration in few securities often results, for minimal advances in "risk" reduction
- Covariance matrices are frequently unstable as the number of securities increases, rendering the results unreliable

Literature Review

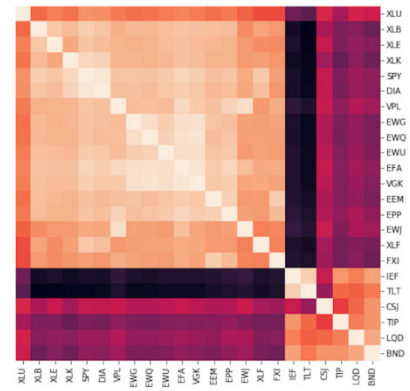
Hierarchical Risk Parity

- A risk-based portfolio optimization algorithm, which has been shown to generate diversified portfolios with robust performance in out-of-sample returns scenarios
  - Generates minimum-variance portfolios
- Switches from classical mathematics of traditional portfolio opt. (geometry, linear algebra, calculus) to more modern methods (graph theory and machine learning)
  - Imposes a hierarchical structure on a basket of securities, using correlations among securities

Correlation Matrices



Unclustered Correlations



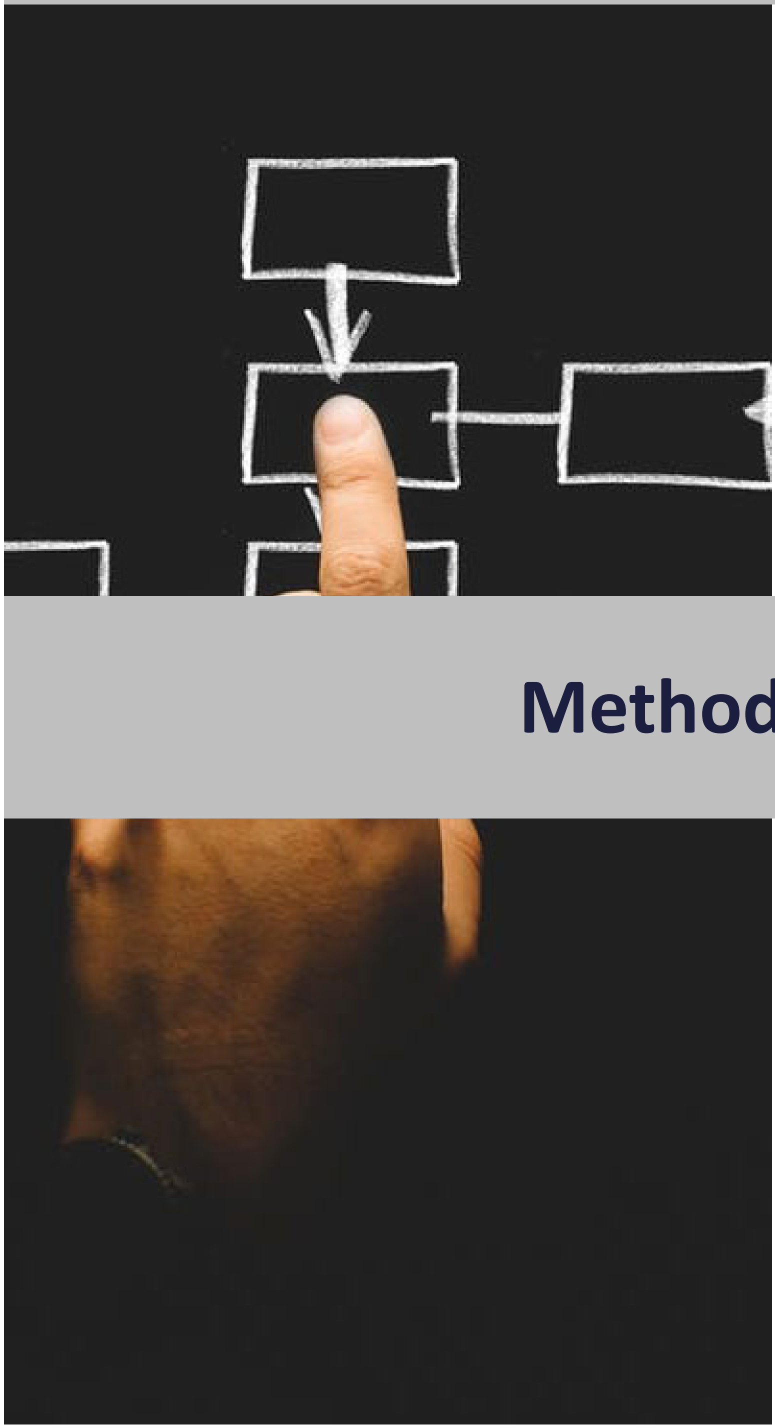
Clustered Correlations

HRP Advantages

- The hierarchy introduced by the clustering algorithm means that securities aren't viewed as perfect substitutes
  - Tends to generate clusters of "like" securities in intuitively appealing manner
- Does not require covariance matrix inversion, leading to greater stability
- Recommended allocations do not vary dramatically based on reasonable changes in inputs

HRP Disadvantages

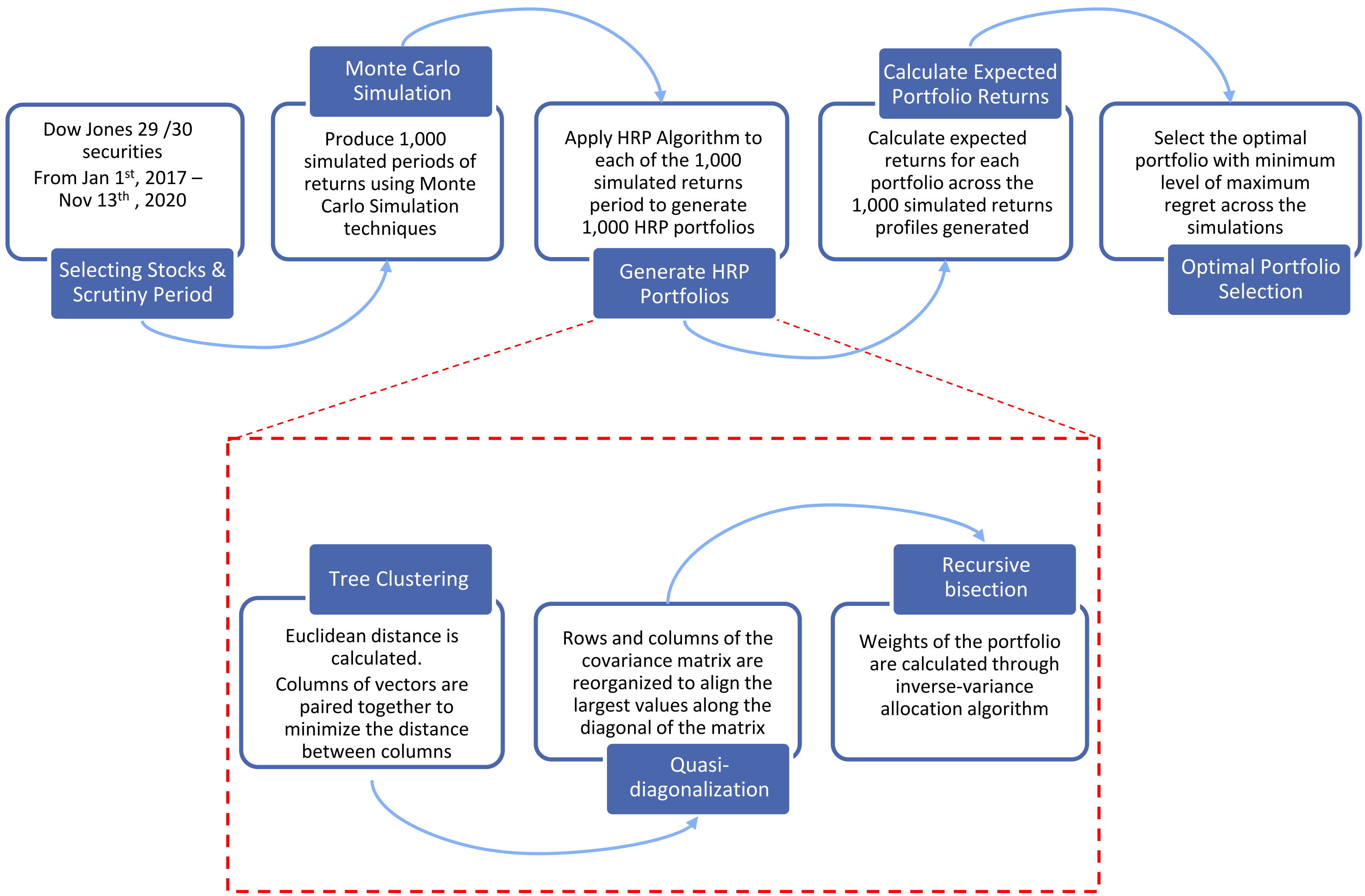
- Does not lend itself to <u>targeting</u> of specified levels of risk or return
- Naïve bi-section of subsets of securities in the algorithm could lead to the separation of some highly correlated assets



Methodology

Methodology

Project Methodology Diagram



Methodology

Hierarchical Risk Parity Algorithm & Decision Criterion

- Objective Function, Decision Variables & Constraints:
  - Objective Function: MINIMIZE V = $\sum_{i=1}^{N} \sum_{i=1}^{N} w_i \, w_j \, \sigma_{ij}$
  - Decision Variables: w_i = the percentage of the investors assets to invest in the i^{th} security
  - Constraints: $\sum_{n=1}^{N} w_i = 1$; $0 \leq w_i \leq 1$
  - Decision Criterion: Minimax Regret – minimize the maximum regret

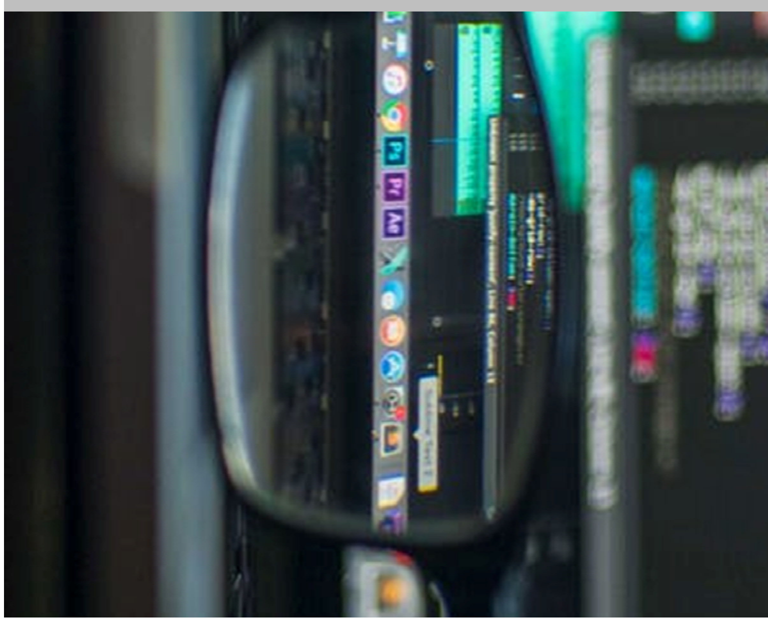
HRP Algorithm Steps

- Step (1) Tree Clustering
  - Convert the correlation matrix into a distance matrix
  - Euclidean Distance = $\tilde{d}_{ij} = \sqrt{\sum_{n=1}^{N} (d_{n,1} - d_{n,j})^2}$
- Step (2) Quasi-Diagonalization
  - Reorganize the covariance matrix, using the clusters generated by the distance matrix in the previous step, to align the largest values along the diagonal
- Step (3) Recursive Bisection
  - Determine the weights for the portfolio by applying an inverse-variance allocation algorithm
  - This stage guarantees $0 \leq w_i \leq 1$ and $\sum_{n=1}^{N} w_i = 1$

Decision Criterion

- Step (4) Minimax Regret Decision Criterion
  - Determine expected returns for all HRP portfolios and then select the "optimal" portfolio as that which minimizes the maximum expected drawdown across the simulations

Results & Recommendations



Results & Recommendations

HRP Allocations – Descriptive Statistics

- Distribution of HRP Allocations:
  - Distribution characteristics for the security allocations from 1,000 HRP portfolios are listed below
  - Relative stability of allocations across simulations is an attractive feature
    - This is in stark contrast the MVO optimizations where small changes in inputs can lead to dramatic changes in recommended allocations
  - No security receives a recommended allocation of 0% of investor assets, leading to a broadly diversified portfolio within and across clusters

	AAPL	AMGN	AXP	BA	CAT	CRM
count	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
mean	0.0207	0.0376	0.0145	0.0116	0.0224	0.0216
std	0.0032	0.0042	0.0016	0.0024	0.0044	0.0049
min	0.0159	0.0308	0.0117	0.0073	0.0163	0.0145
25%	0.0187	0.0353	0.0135	0.0101	0.0196	0.0186
50%	0.0201	0.0368	0.0142	0.0110	0.0210	0.0203
75%	0.0217	0.0387	0.0150	0.0119	0.0233	0.0226
max	0.0406	0.0630	0.0306	0.0199	0.0385	0.0401

	CSCO	CVX	DIS	GS	HD	HON
count	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
mean	0.0285	0.0179	0.0318	0.0167	0.0316	0.0268
std	0.0056	0.0034	0.0053	0.0019	0.0058	0.0031
min	0.0207	0.0132	0.0225	0.0131	0.0224	0.0215
25%	0.0252	0.0160	0.0285	0.0155	0.0280	0.0248
50%	0.0268	0.0172	0.0308	0.0163	0.0296	0.0260
75%	0.0289	0.0186	0.0333	0.0175	0.0324	0.0282
max	0.0491	0.0371	0.0518	0.0357	0.0505	0.0493

	IBM	INTC	JNJ	JPM	KO	MCD
count	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
mean	0.0344	0.0218	0.0558	0.0193	0.0588	0.0441
std	0.0063	0.0041	0.0088	0.0029	0.0091	0.0068
min	0.0240	0.0151	0.0444	0.0144	0.0449	0.0300
25%	0.0303	0.0193	0.0512	0.0170	0.0529	0.0401
50%	0.0330	0.0211	0.0535	0.0188	0.0566	0.0428
75%	0.0359	0.0229	0.0563	0.0212	0.0612	0.0463
max	0.0527	0.0371	0.0946	0.0302	0.0977	0.0675

	MMM	MRK	MSFT	NKE	PG	TRV
count	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
mean	0.0339	0.0544	0.0239	0.0344	0.0573	0.0304
std	0.0066	0.0127	0.0033	0.0058	0.0098	0.0061
min	0.0218	0.0412	0.0179	0.0251	0.0440	0.0218
25%	0.0299	0.0465	0.0216	0.0303	0.0520	0.0265
50%	0.0319	0.0491	0.0233	0.0321	0.0548	0.0284
75%	0.0347	0.0540	0.0254	0.0386	0.0581	0.0314
max	0.0600	0.0917	0.0455	0.0567	0.0976	0.0498

	UNH	V	VZ	WBA	WMT
count	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
mean	0.0296	0.0269	0.0811	0.0440	0.0682
std	0.0043	0.0034	0.0098	0.0059	0.0089
min	0.0220	0.0207	0.0578	0.0278	0.0444
25%	0.0273	0.0247	0.0727	0.0431	0.0628
50%	0.0286	0.0263	0.0836	0.0459	0.0658
75%	0.0301	0.0284	0.0885	0.0478	0.0695
max	0.0505	0.0495	0.1154	0.0570	0.0945

Results & Recommendations

Optimal Allocations

- The optimal HRP, which demonstrated a minimum, maximum drawdown across simulations held the allocations shown to the right
  - Note that the HRP algorithm has developed a diversified portfolio without overwhelming allocations dedicated to any single security
- For illustrative purposes, the optimal MVO portfolio is displayed as well
  - Note that the MVO algorithm allocated all resources between 5 securities, neglecting 80%+ of the available securities

HRP Optimal Allocations

AAPL	0.019458	JPM	0.017191
AMGN	0.030779	KO	0.054128
AXP	0.012800	MCD	0.063747
BA	0.007579	MMM	0.032134
CAT	0.021180	MRK	0.043471
CRM	0.024155	MSFT	0.028325
CSCO	0.045413	NKE	0.032350
CVX	0.014886	PG	0.060319
DIS	0.025349	TRV	0.029940
GS	0.016498	UNH	0.024800
HD	0.031235	V	0.026028
HON	0.022065	VZ	0.089743
IBM	0.037715	WBA	0.033189
INTC	0.019804	WMT	0.089956
JNJ	0.045761		

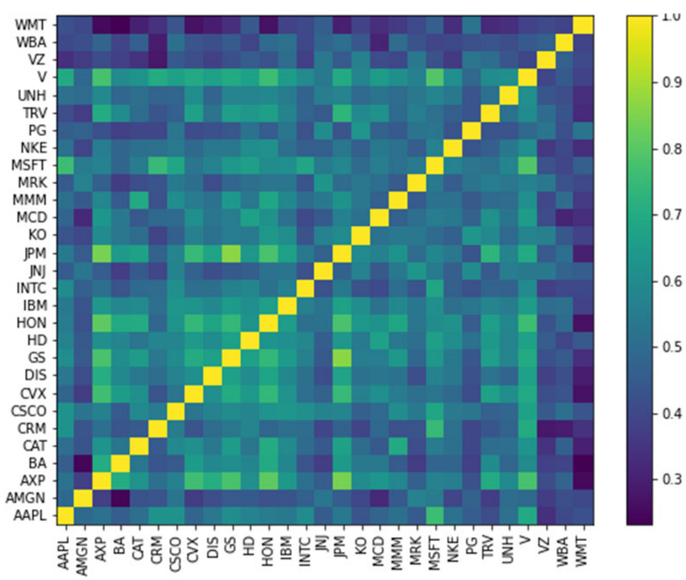
Expected Portfolio Std. Dev. = .1878
Expected Portfolio Return = .1334
Sharpe Ratio = 0.71

MVO Optimal Allocations

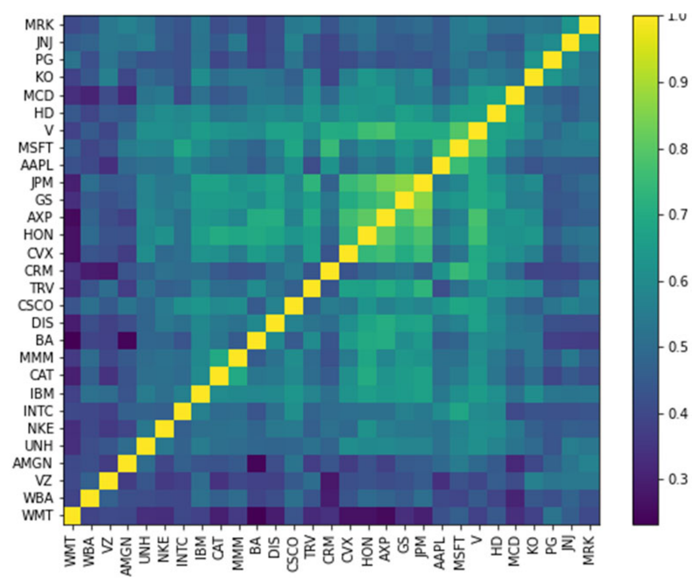
AAPL	0	0.3191	JPM	0	0.0000
AMGN	0	0.0000	KO	0	0.0000
AXP	0	0.0000	MCD	0	0.0000
BA	0	0.0000	MMM	0	0.0000
CAT	0	0.0000	MRK	0	0.0000
CRM	0	0.0151	MSFT	0	0.0000
CSCO	0	0.0000	NKE	0	0.2242
CVX	0	0.0000	PG	0	0.0075
DIS	0	0.0000	TRV	0	0.0000
GS	0	0.0000	UNH	0	0.0000
HD	0	0.0000	V	0	0.0000
HON	0	0.0000	VZ	0	0.0000
IBM	0	0.0000	WBA	0	0.0000
INTC	0	0.0000	WMT	0	0.4340
JNJ	0	0.0000			

Expected Portfolio Std. Dev. = .2127
Expected Portfolio Return = .2808
Sharpe Ratio = 1.32

Non-optimized Correlation Matrix



HRP Optimized Correlation Matrix



Next Steps

Next Steps

- The analysis we detailed today was conducted as of a snapshot in time (November 13, 2020), and does not represent a time-series analysis of the HRP method
  - Further studies could/should be undertaken to determine the efficacy of HRP methods for inter- and intra-day portfolio rebalancing

- Expanding the basket of securities to scrutinize the applicability of the HRP method across asset classes would significantly bolster the potential range of applications of the method

Thank You For Your Time.

Questions?

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