Review: Coordinate geometry

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Presentation overview

Distance and midpoints

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Midpoint of line

We always find the midpoint between two coordinates by the formula:

Midpoint of line segment

$(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

Example 1

Find the midpoint of the line segment joining A(2,6) with B(-3,-4).

Distance - Here comes Pythagoras' theorem

Of course, Phythagoras' theorem always comes in handy.

Distance between two coordinates

Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 2

Calculate the distance EF if E is (-3,2) and F is (4,2).

Exercise 2A

The gradient of a straight line

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Gradient = rise over run

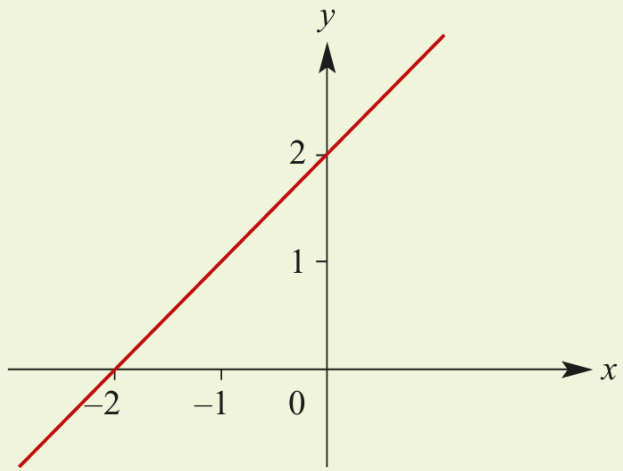
Gradient

Gradient $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

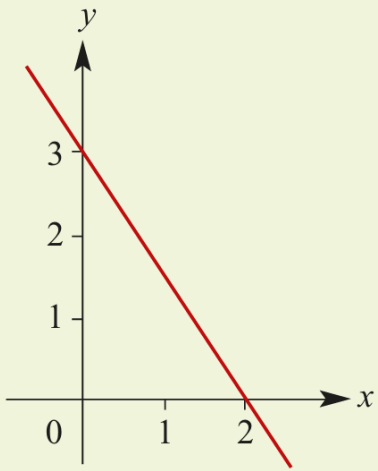
Example 3

Find the gradient of each line:

a



b



More examples

Example 4

Find the gradient of the line that passes through the points (1,6) and (-3,7).

Tangent of an angle of slope

We know:

$$tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Then, we can apply it to coordinates too!

Tangent with coordinates

$m = \pm\frac{y_2 - y_1}{x_2 - x_1}$
$m = \pm tan(\theta)$

Exercise: What is the restriction to coordinates x_2 and x_1.

Examples marching in

Example 5

Determine the gradient of the line passing through the points (3,2) and (5,7) and the angle that the line makes with the positive direction of the x-axis.

Example 6

Determine the gradient of the line passing through the points (5, 3) and (-1,5) and the angle that the line makes with the positive direction of the x-axis.

Prove of negative gradient

Exercise 2B

The equation of a straight line

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$y = mx + c$

It is actually relatively easy to understand what $y = mx + c$ is:

Equation

x, y = (x, y) coordinates
c = y-axis intercept
m = gradient we leartn previously

When the question ask to find the equation, then we just find the value of m and c.
Leave the answer as: $y = mx + c$ (E.g. $y = 4x + 5$)

Why is equation so important?

Equation shows the relation between two things with some factors with them (which are m and c).
Suppose we rent a car from GoGet (no they did not sponsor me), Let c be the fixed cost of renting a car, m be the charge for each hour, x be the number of hours, y be the total charge
Try compute it in desmos!
`https://www.desmos.com/`

Examples

Example 7

Find the gradient and y-axis intercept of the line y = 3x-4.

Example 8

Find the equation of the line with gradient -3 and y-axis intercept 5.

Example 9

State the gradient and y-axis intercept of the line 3y + 6x = 9.

Where're the coordinates?

Suppose you are living at a coordinate (x_0, y_0) (and you know this coordinate). You want to go to your friend's house at (x, y). You do not know specifically what the coordinates are but you know what speed you were going approximately, and we let this speed be m. (Given that you only walk straight, you know the distance and time [I know, I know, a lot of assumptions]).
We can actually calculate what you friend's house coordinate is at!

Point-gradient form

$y - y_0 = m(x - x_0)$
Exercise: Does this equation look familiar to you?

Examples!

Example 10

Find the equation of the line which passes through the point (-1,3) and has gradient 4.

Example 11

Find the equation of the line that passes through the point (3,2) and has a gradient of -2.

What if two points are given only?

If two points are given, we can find the gradient!
Exercise: How? (Hint: Look back at the equations we learnt) Then, after finding the gradient, we just put it in equation form by taking one of the equation into point-gradient form!

Example 12

Find the equation of the straight line passing through the points (1,-2) and (3,2).

Example... again...

Example 13

Find the equation of the straight line with y-axis intercept -3 which passes through the point with coordinates (1,10).

Two different intercepts - I swear the textbook is just giving different cases

We can derive the equation from gradient formula into:

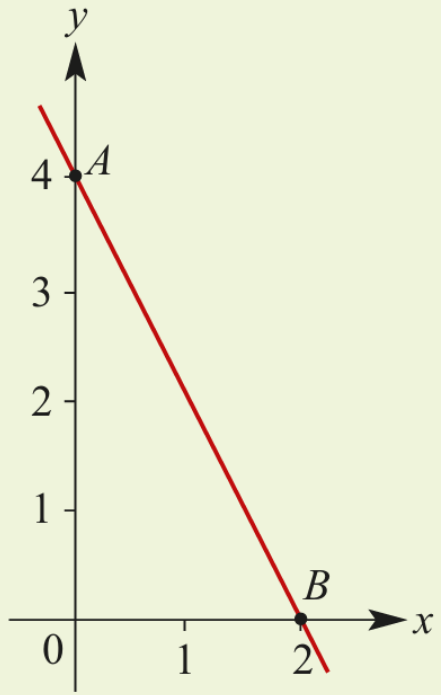
Intercept form

Two intercepts are: $(a, 0), (0, b)$ $\frac{x}{a} + \frac{y}{b} = 1$ where:
a = x-intercept
b = y-intercept

Last... example..... in this part

Example 14

Find the equation of the line shown in the graph.



Exercise 2C

Graphing straight lines

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Sketching time!

Before we start: (Sketch vs draw)
Sketch: Coordinates at axis, shape of graph, turning point (some questions only)
Draw: Find all x and y value requested

Example 15 and 16

1. Sketch the graph $2x + 4y = 10$
2. Sketch the graph $y = 2x - 6$

Angle of the slope

Example 17

For each of the following lines, find the magnitude of the angle θ (2.d.p.) that the line makes with the positive direction of the x-axis:

1. $y = 2x + 3$
2. $3y = 3x - 6$
3. $y = -0.3x + 1.5$

Exercise 2D

Parallel and perpendicular lines

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Parallel vs perpendicular lines

Parallel: same m, θ, others can be different Perpendicular:
Given two straight line equations:$y = m_1x + c_1$ and $y = m_2x + c_2$
IF $m_1m_2 = -1$, then both the equations are perpendicular to each other.
Also: θ = 90°

Examples

Example 18

Find the equation of the straight line which passes through (1,2) and is:

1. parallel to the line with equation $2x - y = 4$
2. perpendicular to the line with equation $2x - y = 4$.

Another one

Example 19

The coordinates of the vertices of a triangle ABC are A(0,-1), B(2,3) and C(3,-$2\frac{1}{2}$) Show that the side AB is perpendicular to the side AC.

Exercise 2E

Families of straight line

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WE ARE FAMILY

As long as we have one parameter (not variables), then it has a family.
E.g.

$y = mx$

$y = 3x + c$

$y = mx + 2$

Examples

Example 20

Find the value of m if the line $y = mx + 2$ passes through the point (3,11).

Example 21

A family of lines have equations of the form $y = mx + 2$, where m is a negative number.

1. Find the x-axis intercept of a line in this family in terms of m.
2. For which values of m is the x-axis intercept greater than 3?
3. Find the equation of the line perpendicular to the line $y = mx + 2$ at the point (0,2)

Exercise 2F

Linear models

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Finally, some real-life applications

Example 22

A historical site charges a tour company for priority entrance to the site. The charge consists of a monthly fee of $\$200$ plus $\$3.50$ for each tourist brought to the site. Construct a cost function that describes the monthly charge and sketch the linear graph for this.

Another one

Example 23

A car starts from point A on a highway $10km$ past the Wangaratta post office. The car travels at a constant speed of $90km/h$ towards picnic stop B, which is $120km$ further on from A. Let t hours be the time after the car leaves point A.

1. Find an expression for the distance d_1 of the car from the post office at time t hours.
2. Find an expression for the distance d_2 of the car from point B at the time t hours.
3. On seperate sets of axes, sketch the graphs of d_1 against t and d_2 against t and state the gradient of each graph.

Exercise 2G

Simultaneous linear equations

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Geometry of simultaneous equations

If unique solution: only one point being intersecting
If ∞ solutions: they are lines being coincide
If no solution: lines are parallel

Describe it

Example 24

Explain why the simultaneous equations $2x + 3y = 6$ and $4x + 6y = 24$ have no solution.

Example 25

The simultaneous equations $2x + 3y = 6$ and $4x + 6y = 12$ have infinitely many solutions. Describe these solutions through the use of a parameter.

More problems

Example 26

The family of lines $y = mx + 2$ with varying gradient m all pass through the point (0,2).

1. For what values of m does the line $y = mx + 2$ not intersect the line $y = 5x - 3$?
2. For what values of m does the line $y = mx + 2$ intersect the line $y = 5x - 3$?
3. If the line $y = mx + 2$ intersects the line $y = 5x - 3$ at the point (5,22), find the value of m.

Bruh... why so many problems

Example 27

The lines $y = x + k$ and y = mx+4intersect at (1,3). Find the values of m and k.

Example 28

The lines $(m-2)x + y = 2$ and $mx + 2y = k$ intersect at (2,8). Find the values of m and k.

Are you done yet?

Example 29

Consider the simultaneous linear equations $(m-2)x+y=2$ and $mx+2y=k$. Find the values of m and k such that the system of equations has:

1. no solution
2. infinitely many solutions
3. a unique solution

Application... really?

Example 30

There are two possible methods for paying gas bills:
Method A: A fixed charge of $\$25$ per quarter + 50c per unit of gas used
Method B: A fixed charge of $\$50$ per quarter + 25c per unit of gas used
Determine the number of units which must be used before method B becomes cheaper than method A.

Example 31

Robyn and Cheryl race over 100 metres. Robyn runs so that it takes a seconds to run 1 metre, and Cheryl runs so that it takes b seconds to run 1 metre. Cheryl wins the race by 1 second. The next day they again race over 100 metres but Cheryl gives Robyn a 5-metre start so that Robyn runs 95 metres. Cheryl wins this race by 0.4 seconds. Find the values of a and b and the speed at which Robyn runs.

Exercise 2H