

Number systems and sets

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February 20, 2024

Presentation overview

- ① 2A: Set notation
- ② 2B: Sets of numbers
- ③ 2C: Surds
- ④ 2D: Natural numbers
- ⑤ 2E: Problems involving sets

Set notation

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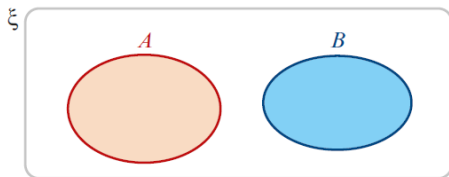
Before we begin, we must understand the following Mathematical Notations:

- ① \mathbb{N} - Natural number (1 to ∞)
- ② \mathbb{Z} - Integers (No decimals)
- ③ \mathbb{Q} - Rational number (Recurring or non-infinity decimals)
- ④ \mathbb{R} - Real numbers (Numbers that includes rational and irrationals)
- ⑤ \mathbb{C} - Complex number (Numbers involves with i)

Set notation

One of the simplest way to represent a whole set of number is by Venn diagram! It pictures out the summary of the case.

The case is known as **disjoint**, such that in A and B have no elements in common.



We also create the sets like the following statement: E.g. $A = -3, 3 = x : x^2 = 9$

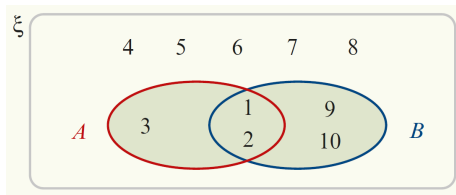
This example, $x : \dots$ is read as 'the set of all x such that...'

Set notation

- ① \in - is an element of ($3 \in \text{Odd number}$)
- ② \notin - not an element of
- ③ $=$ - if they contain same elements (if $A = 1, 2, 3$, and also $B = 1, 2, 3$, then $A = B$)
- ④ \emptyset - if a set has no element
- ⑤ ξ - contains all elements which are being considered
- ⑥ \subset - when some elements are contained in one other set ($B = a, b, c$, $A = a, b$, $\therefore A \subset B$)

Union (\cup), intersect (\cap) and complement (')

Find $A \cup B$, $A \cap B$, A' , B' , $A' \cup B$, $A \cap B'$.



Finite and infinite sets

The things mentioned previously were all finite sets, it ends at a point.
However, there can be infinite sets too, where it is endless!
(A = set of real number, \mathbb{R})

Exercise 2A

Sets of numbers

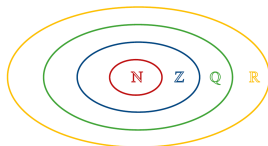
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Sets of numbers

As mentioned, there are infinite sets, such that it can be represented in \mathbb{R} , \mathbb{Z} , and etc.

It is clear that $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$



There are other examples, such as:

- ① $x : 0 < x < 1$
- ② $x : x \geq 3$
- ③ $2n : n \in \mathbb{Z}$

The set of all ordered pairs of real numbers is denoted by \mathbb{R}^2 . That is,
 $\mathbb{R}^2 = (x, y) : x, y \in \mathbb{R}$

This set is known as the **Catersian product** of \mathbb{R} with itself.

Theorem

Every rational number can be written as a terminating or recurring decimal.

Proof:

Theorem

A real number has a terminating decimal representation if and only if it can be written as

$$\frac{m}{2^\alpha \times 5^\beta}$$

for some $m \in \mathbb{Z}$ and some $\alpha, \beta \in \mathbb{N} \cup 0$.

Proof:

Rational number

Example

Write each of the following in the form $\frac{m}{n}$, where m and n are integers:

a 0.05

b $0.\dot{4}28571\dot{1}$

Real numbers

Made up of two important subsets: **algebraic numbers**, **transcendental numbers**. Algebraic number - a solution to a polynomial equation of the form

$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$, where a_0, a_1, \dots, a_n are integers

All rational numbers is algebraic!

The irrational number $\sqrt{2}$ is algebraic, as it is a solution of the equation $x^2 - 2 = 0$.

But π is not! It is a transcendental number. Proof is too difficult to be given here.

Interval notation

$$(a, b) = \{ x : a < x < b \} \qquad [a, b] = \{ x : a \leq x \leq b \}$$

$$(a, b] = \{ x : a < x \leq b \} \qquad [a, b) = \{ x : a \leq x < b \}$$

$$(a, \infty) = \{ x : a < x \} \qquad [a, \infty) = \{ x : a \leq x \}$$

$$(-\infty, b) = \{ x : x < b \} \qquad (-\infty, b] = \{ x : x \leq b \}$$

The following are subsets of the real numbers for which we have special notation:

- ① $\mathbb{R}^+ = \{ x : x > 0 \}$
- ② $\mathbb{R}^- = \{ x : x < 0 \}$
- ③ Real numbers excluding zero: $\mathbb{R} \setminus \{0\}$

Exercise 2B

Surds

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Before introducing surds...

We need to understand the properties of square roots:

Properties of square roots

① $\sqrt{ab} = \sqrt{a}\sqrt{b}$

② $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Intro. to surds!

What is a surd?

In general, a **surd of order n** is a number of the form $\sqrt[n]{a}$, where a is a rational number which is **not** a perfect n th power.

But we are mainly focusing on quadratic surd, where $n = 2$.

Why is surd important?

We can expect to see surds as an exact solution (without converting to decimals), such as in trig.

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}, \quad \sin(15^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Example

Write each of the following in simplest form:

① $\sqrt{72}$

② $\sqrt{28}$

③ $\sqrt{\frac{700}{117}}$

④ $\sqrt{\frac{99}{64}}$

Like surds

Surds which have the same 'irrational factor' are **like surds**.

E.g. $3\sqrt{7}$, $2\sqrt{7}$, $\sqrt{7}$ have factor of $\sqrt{7}$, \therefore it is like surds.

Properties of like surds

$$\textcircled{1} \quad m\sqrt{p} + n\sqrt{p} = (m + n)\sqrt{p}$$

$$\textcircled{2} \quad m\sqrt{p} - n\sqrt{p} = (m - n)\sqrt{p}$$

Example

- ① $\sqrt{147} + \sqrt{108} - \sqrt{363}$
- ② $\sqrt{3} + \sqrt{5} + \sqrt{20} + \sqrt{27} - \sqrt{45} - \sqrt{48}$
- ③ $\sqrt{50} + \sqrt{2} - 2\sqrt{18} + \sqrt{8}$

Rationalising the denominator

Rationalising - a method to keep the surds clean and tidy

Example

① $\frac{1}{2\sqrt{7}}$

② $\frac{1}{2-\sqrt{3}}$

③ $\frac{1}{\sqrt{3}-\sqrt{6}}$

④ $\frac{3+\sqrt{8}}{3-\sqrt{8}}$

Exercise 2C

Natural numbers

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Factors and composites

Factors:

Let a be natural number, and it is a factor of number b , then there exists a natural number k , where $b = ak$.

E.g. factor of 8: 1, 2, 4, 8

Composite:

Let m be composite number if it can be written as a product of $m = a \times b$, where a, b are natural numbers greater than 1 and less than m

E.g. 12 is a composite number because: $12 = 4 \times 3 = 2^2 \times 3$

What are their differences?

In short:

Factor: natural number(s) that can divide the number you are investigating

Composites: natural number that can change to a group of numbers multiplying

Prime decomposition

Prime number: natural number greater than 1 and if its only factor of itself and 1.

Basically, we break down the composite number into group of prime numbers multiplying.

Fundamental theorem of arithmetic

Fundamental Theorem of Arithmetic states that every integer greater than 1 is either a prime number or can be expressed in the form of primes. In other words, all the natural numbers can be expressed in the form of the product of its prime factors.

Example

Give the prime decomposition of 17248, and hence list the factors of this number.

Highest common factor(HCF) vs Lowest common multiple(LCM)

HCF or Greater common divisor (GCF)

The highest common factor of two natural numbers a and b is the largest natural number that is a factor of both a and b . It is denoted by $\text{HCF}(a, b)$.

LCM

- 1 A natural number a is a multiple of a natural number b if there exists a natural number k such that $a = kb$.
- 2 The lowest common multiple of two natural numbers a and b is the smallest natural number that is a multiple of both a and b . It is denoted by $\text{LCM}(a, b)$.

Example

- 1 Find the highest common factor of 528 and 3168.
- 2 Find the highest common factor of 3696 and 3744.

Exercise 2D

Problems involving sets

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Number of elements

Number of elements

Let A be a finite set, then the number of elements in A is denoted by $|A|$.

Example

Two hundred and eighty students each travel to school by either train or tram or both. Of these students, 150 travel by train, and 60 travel by both train and tram.

- ① Show this information on a Venn diagram
- ② Hence find the number of students who travel by:
 - ① tram
 - ② train but not tram
 - ③ just one of these modes of transport

Example

An athletics team has 18 members. Each member competes in at least one of three events: sprints (S), jumps (J) or hurdles (H). Every hurdler also jumps or sprints. The following additional information is available:

$$|S| = 11 \quad |J| = 10 \quad |J \cap H' \cap S'| = 5 \quad |J \cap H' \cap S| = 5 \quad |J \cap H'| = 7$$

① Draw a Venn diagram

② Find:

① $|H|$

② $|S \cap H \cap J|$

③ $|S \cup J|$

④ $|S \cap J \cap H'|$

Exercise 2E