Additional algebra

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Presentation overview

Polynomial identities

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What is a polynomial?

Polynomial function is written with the form of:
$P(x) = a_n x^n + an - 1x^{n-1} + \cdots + a_1 x + a_0$
where n is the natural number or zero, and the coefficients $a_0, \ldots, a_n$ are real numbers with $a_n \neq 0$.
Leading term = $a_n x^n$
Degree of polynomial = n
Monic polynomial = Polynomial whose leading term has coefficient 1
Constant term = Term of index 0.
Note: $P(x) = 0$ is **zero polynomial**; degree is undefined

Example 1

If $(a+2b)x^2-(a-b)x+8=3x^2-6x+8$ for all x, find the values of a and b.

Example 2

Express x^2 in the form $c(x-3)^2 + a(x-3) + d$.

Example 3

Find the values of a, b, c and d such that $x^3 = a(x+2)^3 + b(x+1)^2 + cx + d$ for all x

Example 4

Show that $2x^3 - 5x^2 + 4x + 1$ cannot be expressed in the form $a(x+b)^3 + c$.

Exercise 4A

Quadratic equations

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Polynomial (Quadratic)

We say quadratic function is known as a polynomial function of degree 2. Hence, the general quadratic function can be written as:
$P(x) = ax^2 + bx + c$, where $a \neq 0$, $P(x) \in \mathcal{P}^2$

To solve $0 = ax^2 + bx + c$, we can factorise, completing the square, or the general formula below (which is derived from completing the square)

General quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 5

Solve the following quadratic equations for x:

1. $2x^2 + 5x = 12$
2. $3x^2 + 4x = 2$
3. $9x^2 + 6x + 1 = 0$

What is discriminant?

Discriminant is used to determine the number of solutions to a quadratic. It is within the general formula.
$b^2 - 4ac = \Delta$

Real solution

1. If $\Delta > 0$, there are two real solutions
2. If $\Delta = 0$, there are one real solutions
3. If $\Delta < 0$, there are no real solutions (but two complex solutions)

Rational solution

If $a, b, c \in \mathbb{Q}$

1. If $\Delta \neq 0$ AND Δ is perfect square, it has two rational solutions
2. If $\Delta = 0$, then one rational solution
3. If $\Delta > 0$ AND Δ is not a perfect square, it has two irrational solutions.

Example 6

Consider the quadratic equation $x^2 - 4x = t$. Make x the subject and give the values of t for which real solution(s) to the equation can be found.

Example 7

1. Find the discriminant of quadratic $x^2 + px - \frac{25}{4}$ in terms of p.
2. Solve the quadratic equation $x^2 + px - \frac{25}{4} = 0$ in terms of p.
3. Prove that there are two solutions for all values of p.
4. Find the values of p, where p is a non-negative integer, for which the quadratic equation has rational solutions.

Example 8

A rectangle has an area of $288cm^2$. If the width is decreased by $1cm$ and the length increased by $1cm$, the area would be decreased by $3cm^2$. Find the original dimensions of the rectangle.

Example 9

Solve the equation $x - 4\sqrt{x} - 12 = 0$ for x

Exercise 4B

Applying quadratic equations to rate problems

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Rate

Before going to the application of calculation, we must understand what rate is.
A rate describes how a certain quantity changes with respect to the change in another quantity (often time).
Examples of rate: Speed, flow, acceleration
E.g. In speed, it is how fast it is to travel from start to end
If time is higher, then the speed(rate) is lower, else vice versa

Example 10

1. Express $\frac{6}{x} + \frac{6}{x+8}$ as a single fraction.
2. Solve the equation $\frac{6}{x} + \frac{6}{x+8} = 2$ for x

Example 11

A car travels 500km at a constant speed. If it had travelled at a speed of 10km/h less, it would have taken 1 hour more to travel the distance. Find the speed the car.

Example 12

A tank is filled by two pipes. The smaller pipe alone will take 24 minutes longer than the larger pipe alone, and 32 minutes longer than when both pipes are used. How long will each pipe take to fill the tank alone? How long will it take for both pipes used together to fill the tank?

Exercise 4C

Partial fractions

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Partial fractions

A rational function is the quotient of two polynomials. If $P(x)$ and $Q(x)$ are polynomials, then $f(x) = \frac{P(x)}{Q(x)}$ is a rational function
If the degree of $P(x)$ is greater than $Q(x)$, it is known as improper fraction, else it is proper fraction.

Example 13

Resolve $\frac{3x+5}{(x-1)(x+3)}$ into partial fractions.

Example 14

Resolve $\frac{2x+10}{(x+1)(x-1)^2}$ into partial fractions.

Example 15

Resolve $\frac{x^2+6x+5}{(x-2)(x^2+x+1)}$ into partial fractions.

Example 16

Resolve $\frac{x^5+2}{x^2-1}$ into partial fractions.

Exercise 4D

Simultaneous equations - Advanced

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Example 17

Find the coordinates of the points of intersection of the parabola with equation $y = x^2 - 2x - 2$ and the straight line with equation $y = x + 4$

Example 18

Find the points of intersection of the circle with equation $(x-4)^2+y^2=16$ and the line with equation $x-y=0$.

Example 19

Find the points of contact of the straight line with equation $\frac{1}{9}x + y = \frac{2}{3}$ and the curve with equation $xy = 1$

Example 20

Find the coordinates of the points of intersection of the graphs of $y = -3x^2 - 4x + 1$ and $y = 2x^2 - x - 1$.

Exercise 4E