Neuroprothetics Exercise 1
Programming Basics

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1 Signal generation

The first part of the exercise consists of the generation of two signals each composed of superimposed sinusoidal signals. For generating the signals, a time array containing equally spaced numbers whose spacing corresponds to the sampling rate. Starting with that time array, for each time, the corresponding amplitude is calculated using the following equation:

$$f(t) = A_0 + \sum_{i=1}^{n} A_i \cdot \sin\left(2\pi F_i \cdot t\right) \, . \tag{1}$$

Where A_0 is the DC offset, A_i the amplitude and F_i the frequency of the respective signal component i. For each value of the time array t, the corresponding signal amplitude of the superimposed signals is calculated.

1.1 Signal plotting of f_1 and f_2

The first signal f_1 is plotted consists of the frequencies 50, 500 and 5000 kHz with the corresponding amplitudes 2, 4 and 2 in an arbitrary unit. There is no DC offset, the signal is generated for 1 s and the sampling rate is 12 kHz.

The second signal f_2 is plotted consists of the frequencies 0, 1 and 10 kHz with the corresponding amplitudes 3, 5 and 3 in an arbitrary unit. Note that the frequency of 0 Hz corresponds to a DC offset of 3. The signal is generated for 1 second and the sampling rate is 12 kHz.

1.2 Results signal generation

The generated signals f_1 and f_2 are depicted in Figure 1. Since the signal is periodic, a time window of 50 ms was chosen to ensure good visibility of the signal. In the plot of the signal f_2 a horizontal line where the amplitude is 0 arb. unit was inserted to show the DC offset.

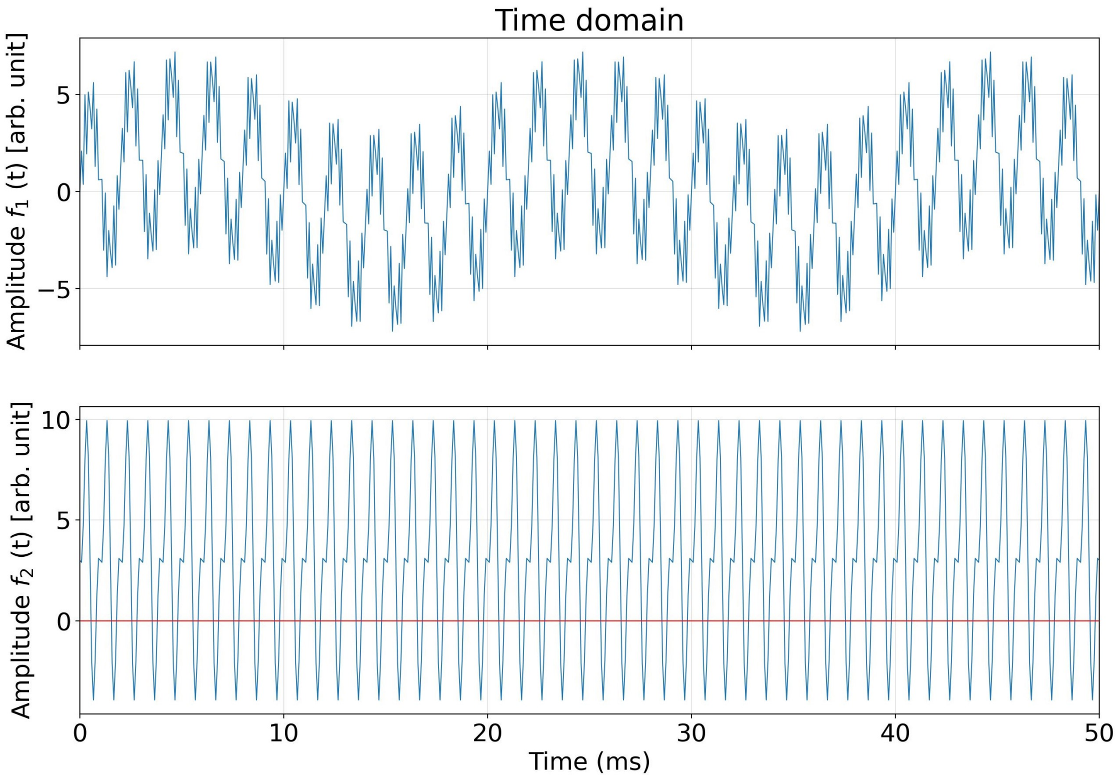

Figure 1: Generated signals f_1 and f_2. The signals are only shown in a time window of 50 ms to ensure a good view of the signals. Red line at $A = 0$ arb. unit to show DC offset.

2 Spectrum plotting

The second task was to analyse the generated signals by their frequency components. For that, a fast Fourier transform (FFT) was applied using the python package numpy. Besides cutting the DC component in half like it has to be done to get the correct result, the rest of the frequency intensities were doubled, since the spectrum is symmetric and the negative frequencies only are to be considered in a complex analysis. The resulting frequency spectra are shown in Figure 2.

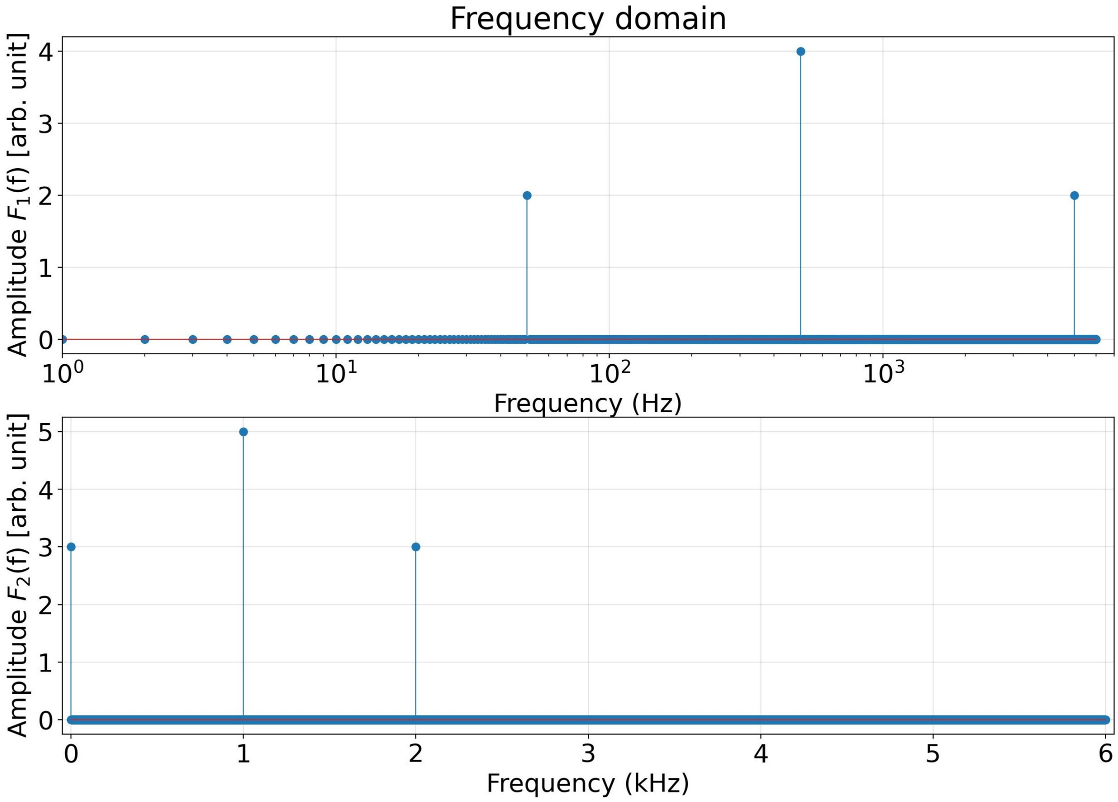

Figure 2: Signals from section 1 in the frequency domain. F_1 is shown in a semi-logarithmic plot and F_2 in a linear one.

3 Spectrum interpretation

3.1 Content of spectral analysis

The plots in the frequency domain only partwise show correctly what you would expect from the input signals generated as described in section 1. The frequencies and corresponding amplitude can be easily read out of the F_1 plot. In the plot of F_2 however, a frequency of 2 kHz is shown rather than the expected 10 kHz. The reason for that is, that the frequency of 10 kHz gets rather close to the sampling rate of 12 kHz. That makes it impossible to correctly analyse the signal. The Nyquist–Shannon sampling theorem states, that a correct single interpretation, where no information is lost during the sampling process, requires a sampling frequency that is twice as large as the maximal signal frequency [1]. In the case of F_2 this theorem is violated and therefore the information of the largest frequency is not correctly interpreted by the FFT. Only frequencies > 6 kHz can be correctly analysed with the given sampling rate of 12 kHz.

3.2 Choice of axis scaling

The scaling of the axis for F_1 was chosen to be semi-logarithmic. This allows us to show all frequency components equally spaced since they differ by a factor of 10. For the plot of F_2, linear scaling was used, since in a semi-logarithmic plot, the DC offset could not be shown since it would require an infinitely large plot to show a frequency of 0 Hz on a logarithmic scale.

3.3 Avoiding artefacts

Assuming we know the sampling frequency, we can apply a low-pass filter to filter out the frequencies that violate the Nyquist–Shannon sampling theorem. We still don't get the information about the high frequency, but we avoid artefacts that can disturb our interpretation of the true nature of the signal. From F_2 of Figure 2, for example, you could assume that there is a frequency component at 2 kHz, which is simply wrong.

References

[1] Emiel Por, Maaike van Kooten, and Vanja Sarkovic. Nyquist–shannon sampling theorem. *Leiden University*, 1(1), 2019.