Homework 2: Machine numbers; Interpolation

Due Friday Jan. 25 and submitted on Canvas.

1. Floating point representation

   For the `float` data type, write a program to **empirically** (i.e., by performing tests within your program) determine the following "Machine constants":

   (a) The smallest ϵ such that $1.0 - \epsilon \neq 1.0$

   (b) The smallest ϵ such that $1.0 + \epsilon \neq 1.0$

   (c) The maximum representable number

   (d) The minimum representable positive number

   Comment on why the numbers you get are expected based on the IEEE 754 representation.

2. Roundoff error

   Numerically evaluate the expression $(1-\cos(x))/x^2$ in double precision for values of x around 10^{-7} and smaller. Explain the difference between the numerical results and the analytic limit as $x \to 0$.

3. Interpolation

   (a) Write a program to read in a two column table from a file and perform linear interpolation at an arbitrary point. You may assume that the data is evenly spaced in the independent variable (this makes it easier to determine which points to use for interpolation).

   (b) Use the program on the following input data:

x	y
-1.	0.03846154
-0.5	0.13793103
0.	1.
0.5	0.13793103
1.	0.03846154

   and provide a linear estimate of y at $x = 0.75$.

   (c) Write a program using Neville's algorithm to fit a 4th order polynomial to the above data and provide an estimate of y at $x = 0.75$. (See notes on Neville's algorithm, Ch3 in Numerical Recipes book, or e.g. Wikepedia for help.)

   (d) The actual function tabulated above is $y = \frac{1}{(1+25x^2)}$. Compare the actual value at $x = 0.75$ with the linear interpolation and the 4th order polynomial interpolation, and comment on why one is more accurate than the other.

While polynomial interpolation is a reasonable way to proceed when there are a limited number of points as above, when there are a large number of points it can have strange behavior (especially at the end points, called 'Runge's phenomenon'. As a result, the most popular way to interpolate is with cubic splines.