SAT Solving in Interactive Configuration

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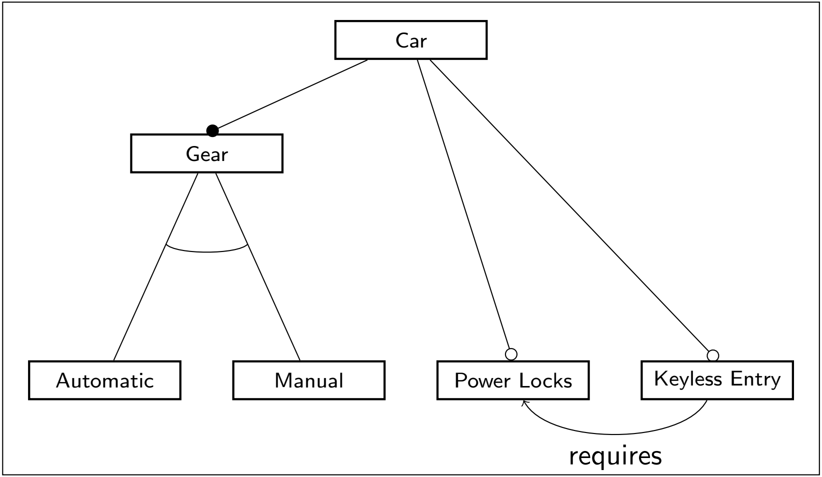
Feature Diagrams



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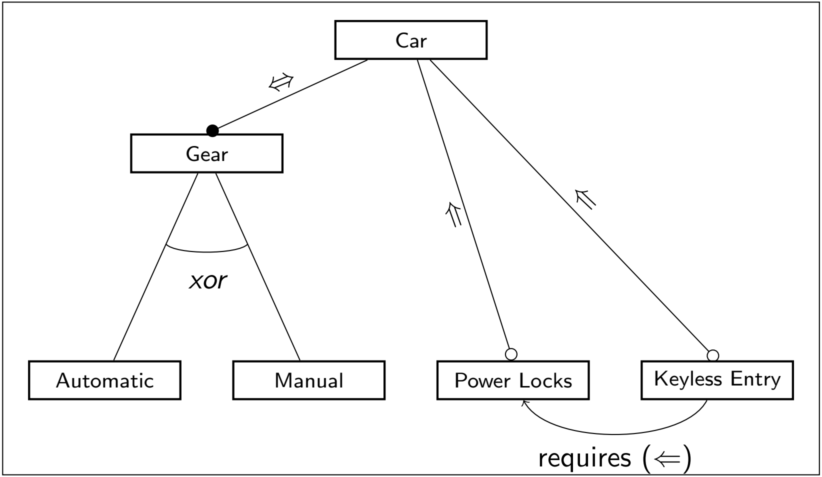

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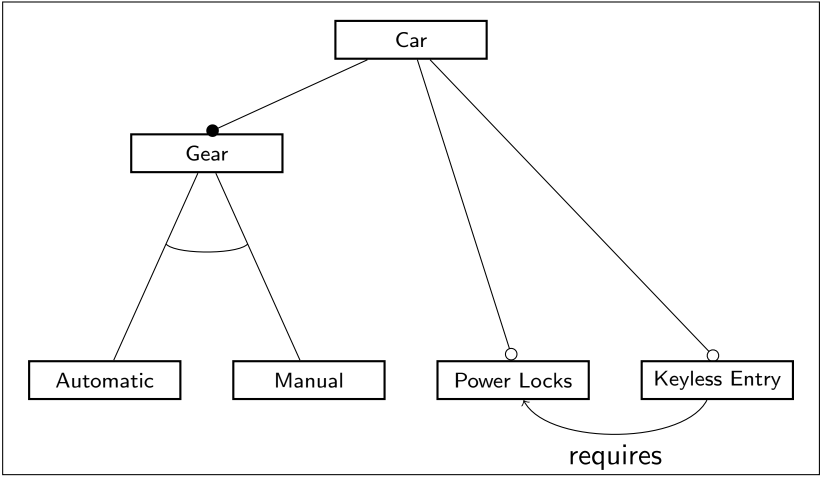

Legend

Selection Eliminate Autoselect Autoeliminate

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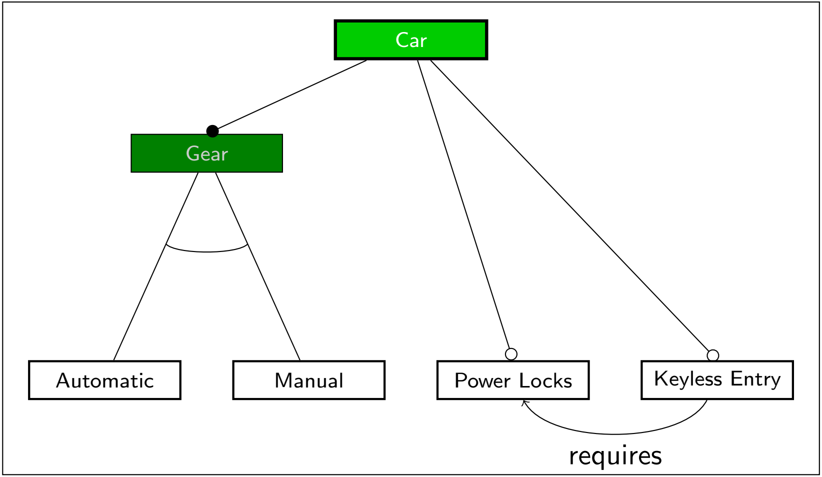

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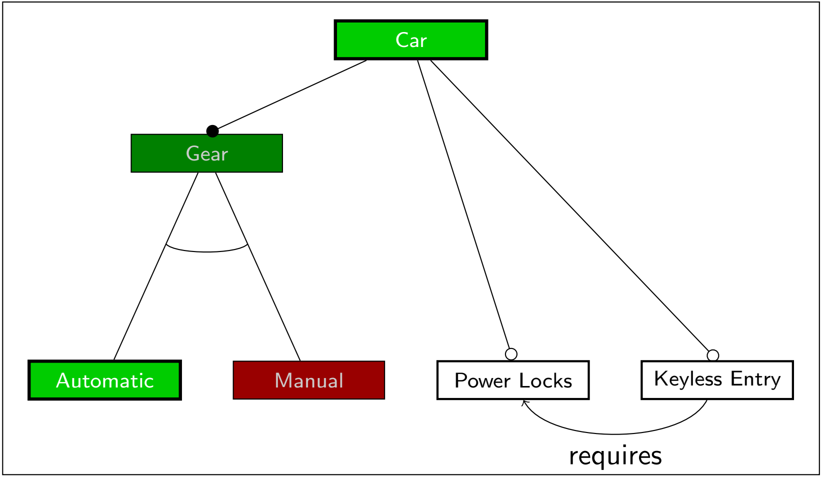

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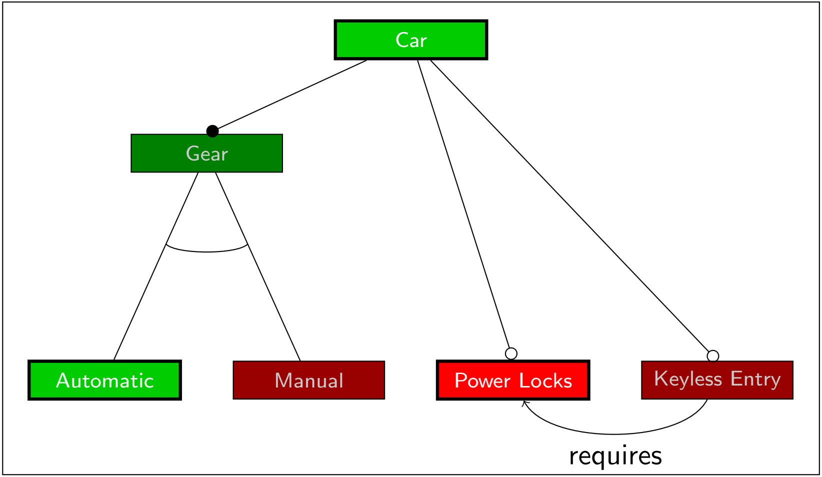

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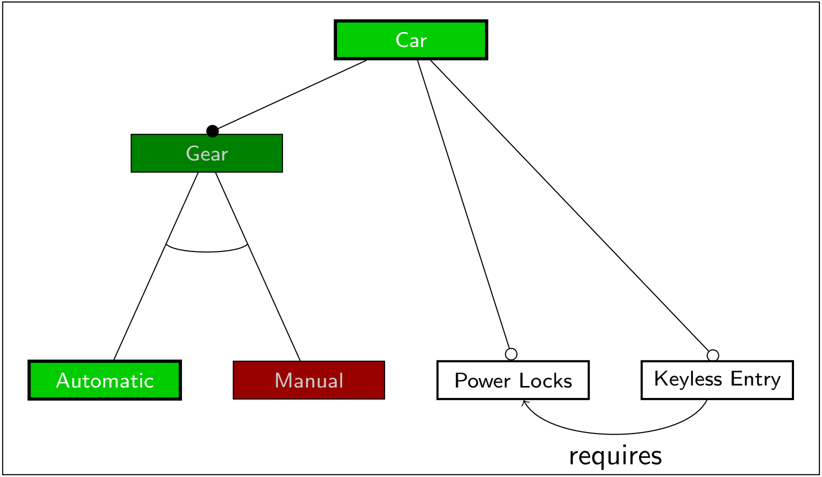

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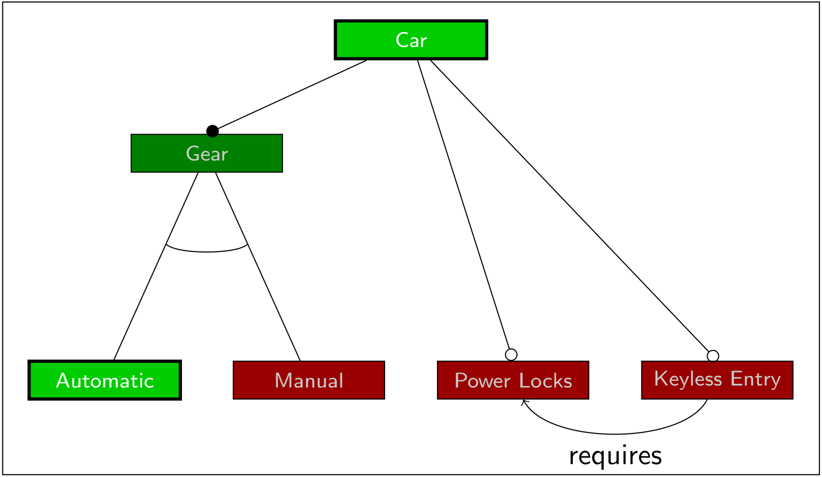

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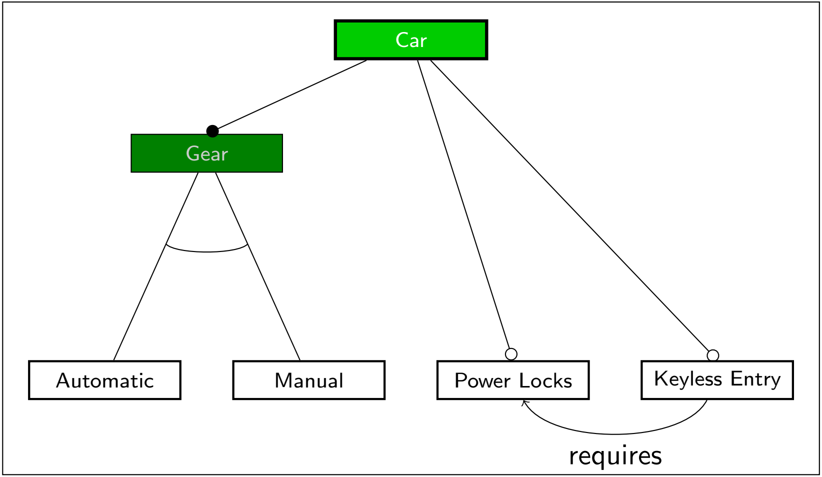

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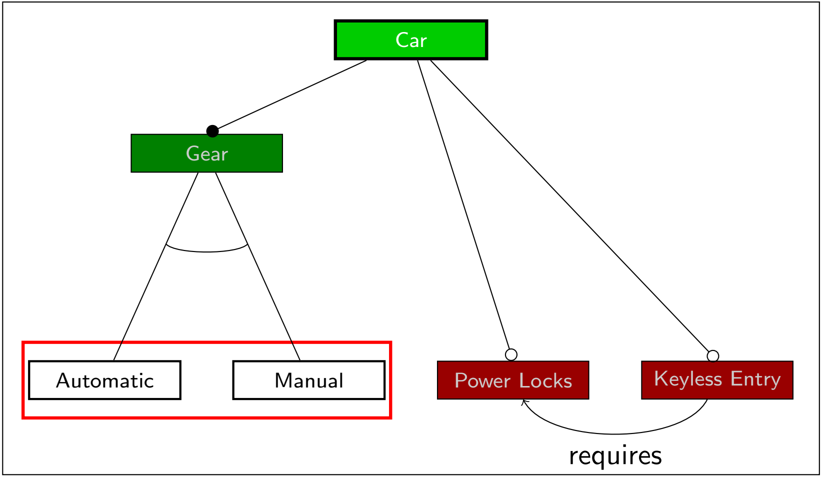

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The General Problem

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Provide assistance during interactive configuration.

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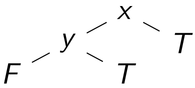
- **applicability** – a means to personalization of products and systematic reuse of software components and services
- **constraints** reduce the risk of incorrect usage
- **interactive** – increases comfort for the user

Binary Decision Diagrams (BDDs)



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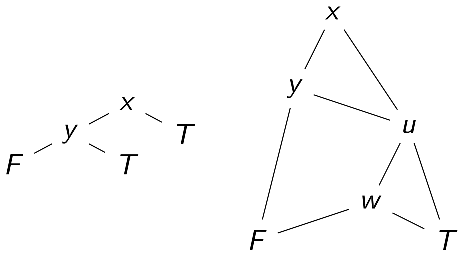

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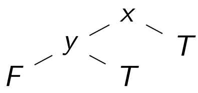

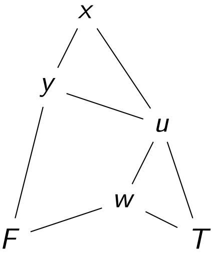

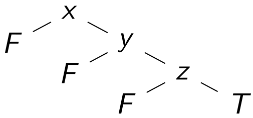

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- the number of models of a formula grows exponentially
- the ordering of the variables matters
- the way of constructing the BDD matters
- heuristics are needed for the two above

Specific Problem

Thesis

Are modern SAT solvers suitable means for implementing assistance during interactive configuration?

. . . where the suitability is measured by the response time.

Why SAT solvers?

The problem solved is computationally challenging

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SAT Solvers are neat

- SAT solvers are means to deciding an NP-complete problem (formula satisfiability).
- Modern SAT solvers became very efficient.
- The tools can be used off-the-shelf.

Novelty

Existing Alternative Approaches to Assistance

- ad hoc
- precompilation
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Syntactic Optimization

- Presented optimization is known but the focus on the proof reconstruction is novel.

Research Methodology

Constructive Research

- review of related work
- formulation of hypothesis
- formalization of the problem
- development of the algorithms
- an empirical evaluation of the implemented algorithms

A Case for the Thesis

Contributions

- study of a lazy approach to interactive configuration,
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- precompilation may fail or may require specific expertise to be carried out,
- more informative explanations can be provided.

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Contributions

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Conclusions from the Evaluation

- the time of feedback during configuration was satisfactory,
- the runtime off the algorithm for configuration completion was satisfactory in majority of the cases but timed out in some.

How Do We Go About This?

Use a SAT Solver

- determines the satisfiability of a given Boolean formula
- operates on Conjunctive Normal Form (CNF)
- a certification of the response is produced
- nowadays SAT solvers are *very* efficient

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Assumptions

- constraints encoded in a CNF
- decisions so far encoded as a conjunction of literals

$$\phi \equiv f_1 \wedge \neg f_8 \wedge \dots$$

Towards Satisfiability

Proposition

for a formula ϕ and a literal l

$$\phi \models l \quad \textit{iff} \quad (\phi \wedge \neg l) \text{ is unsatisfiable}$$

Example

for $\phi = (x \Rightarrow y) \wedge x$

$$\phi \models y$$

$$\phi \wedge \neg y \text{ is unsatisfiable}$$

SAT Solver for Configuration

Testing all unassigned variables after each user's decision

```
TEST-VARS()
foreach $x$ that was not assigned to by the user
    do $CanBeTrue \leftarrow \mathrm{TEST\text{-}SAT}(\phi, x)$
       $CanBeFalse \leftarrow \mathrm{TEST\text{-}SAT}(\phi, \neg x)$
       if $\neg CanBeTrue \wedge \neg CanBeFalse$
          then error "Unsatisfiable constraint!"
       if $\neg CanBeTrue$ then SET($x$, FALSE)
       if $\neg CanBeFalse$ then SET($x$, TRUE)
       if $CanBeTrue \wedge CanBeFalse$
          then RESET($x$)
               UNLOCK($x$)
          else LOCK($x$)
```

Can We Improve This?

SAT

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Example

if we have

$$SAT((x \Rightarrow y) \wedge y) = \{x, y\}$$

then

$$((x \Rightarrow y) \wedge x)$$

is satisfiable

Can We Improve This?

UNSAT

- Can a negative response of the solver help in the future?
- Example

$$\left.\begin{array}{l} f_1 => f_2 \\ \neg f_2 \\ \ldots \end{array}\right\} \neg f_1$$

- Recording disabled values *may* help with further queries.

Satisfiability with Caching

- *KnownValues* represent values known to be SAT
- *DisabledValues* represent values known to be UNSAT

```
$\textsc{Test-SAT}(\phi\colon \mathit{Formula}, l\colon \mathit{Literal}) : \mathit{Boolean}$
if $l \in \mathit{KnownValues}$ then return $\textsc{True}$
if $l \in \mathit{DisabledValues}$ then return $\textsc{False}$
$L \leftarrow \mathrm{SAT}(\phi \wedge l \wedge \bigwedge_{k \in \mathit{DisabledValues}} \neg k)$
if $L \neq$ null
    then $\mathit{KnownValues} \leftarrow \mathit{KnownValues} \cup L$
    else $\mathit{DisabledValues} \leftarrow \mathit{DisabledValues} \cup \{l\}$
return $L \neq$ null
```

Explanations

- The solver produces a unsatisfiable subset of given formulas.
- This may not be minimal, several techniques how to minimize.
- In the tool an iterative technique by Zhang and Malik.

How is a configuration process completed?

Conditions

- All variables must have a value.
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Scenarios

- **manual** — user fills in everything
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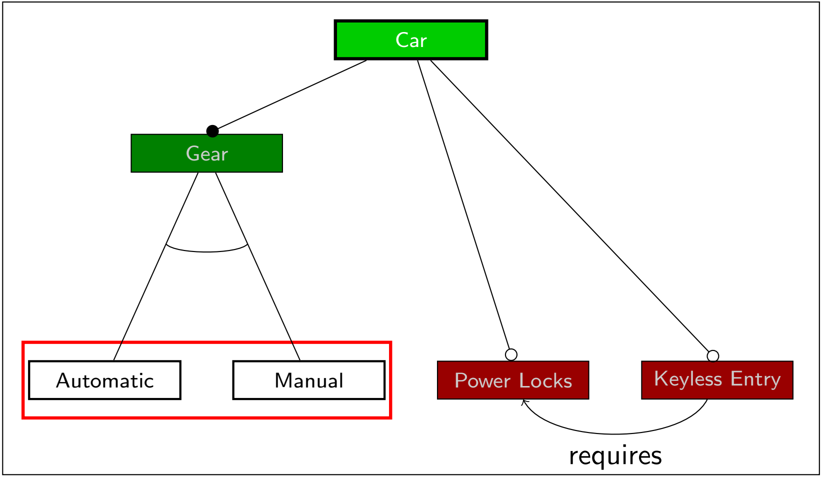
- **smart completion** — automated tool fills as much as possible *without making decisions* for the user

Smart Completion



Car

Gear

Automatic

Manual

Power Locks

Keyless Entry

requires

Legend

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Decisions and Dispensable Variables

- A set of variables is eliminable *iff* they can be all eliminated all at once.

$$\mathcal{D}(\phi, X) \stackrel{\text{def}}{=} SAT\left(\phi \wedge \bigwedge_{v \in X} \neg v\right)$$

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- A variable is dispensable *iff* it does not belong to any set that must be decided.

Examples

$$\mathbf{x} \vee \mathbf{y} \vee \mathbf{z}$$

- Deselectable: $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x\}$, $\{y\}$, $\{z\}$, and $\emptyset$
- Not eliminable: $\{x, y, z\}$
- $\{x, y, z\}$ must be decided and none of the variables are dispensable.

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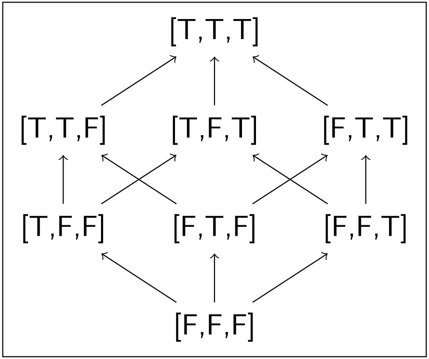

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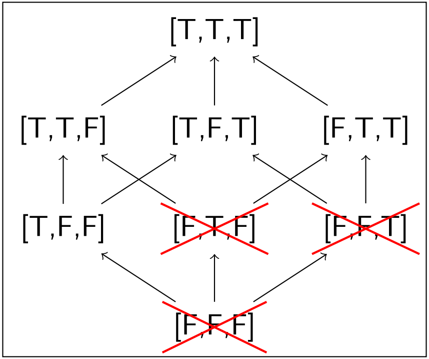

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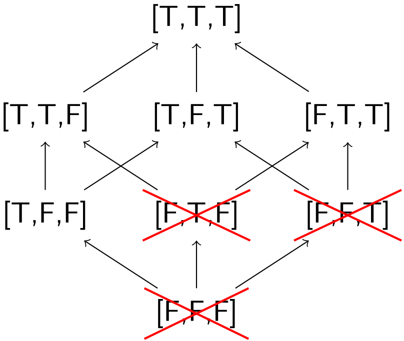

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Enumerating All Minimal Models

schema

```
while ($\phi$ is satisfiable)
    do $M \leftarrow$ some minimal model of $\phi$
       remove from $\phi$ all models that
           are greater or equal to $M$
```

Other Relations

Non-monotonic Reasoning

- Propositional Circumscription

$$\phi \models_{\min} \psi$$

- Generalized Closed World Assumption (GCWA)

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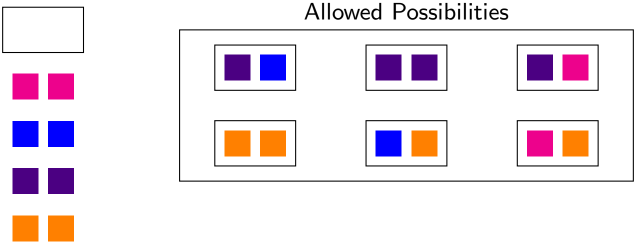

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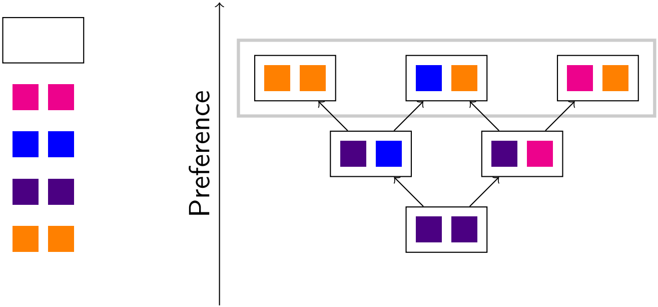

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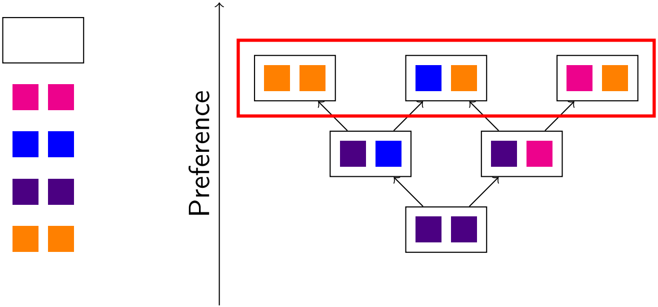

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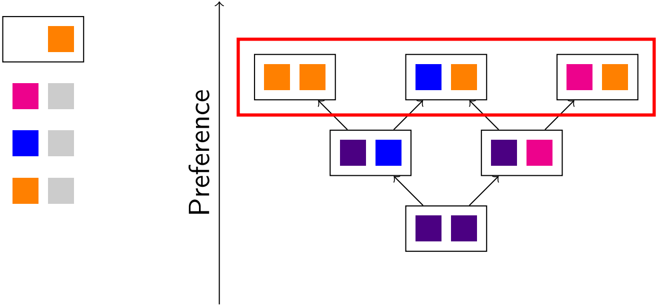

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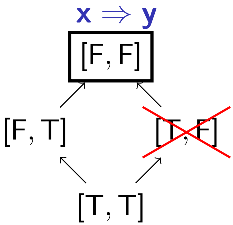
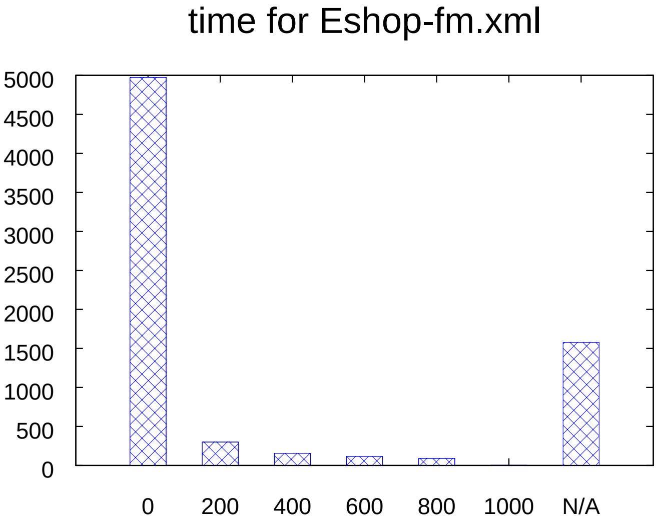


Experimental Results

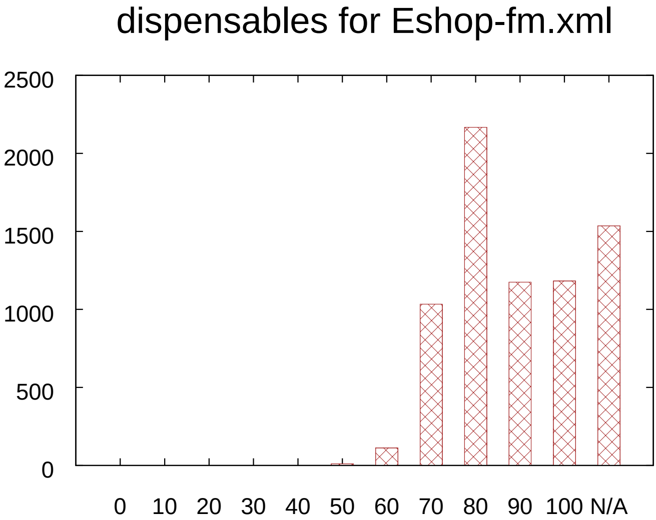
Name	#variables	#clauses
E-shop	287	420
Violet	170	341
Berkeley	94	183
T-shirt	16	40
3blocks	283	9690
rocket	351	2398

Name	Length	#decisions	noop	BIS	Cat	Ord	BCO
E-shop	144.3	7215	53	42	53	9	7
Violet	50.34	2517	16	15	16	4	4
Berkeley	25.12	1256	14	11	14	7	5
T-shirt	4.7	235	8	7	8	5	5
3blocks	6.7	335	655	616	557	3418	2671
rocket	7.52	376	166	163	158	274	260

Times for Minimal Model Enumeration



Percentages of Dispensable Variables



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- To provide smart completion we introduce dispensable variables.
- Dispensable variables are closely related to CWA.
- In non-propositional case we needed preference.
- Dispensable variables can be seen as a *preference for deselecting*.
- Analogously CWA as a preference for False.

- SAT solvers are fast enough to be used as oracles in more challenging problems