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A. Let  $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ;  $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ ;  $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ , find -

i)  $2A - B$

$$= 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

2)  $\|A\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

and the angle of  $A$  relative to the

positive  $X$ -axis.

Angle of  $A$  relative to the positive  $X$ -axis:-

Hence,  
Positive  $X$ -axis  $= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Angle between  $\|A\|$  &  $X$  is  $\cos \theta = \frac{A \cdot x}{\|A\| \|x\|}$

$$= \cos^{-1} \left( \frac{(1, 2, 3) \cdot (1, 0, 0)}{(\sqrt{14}) \cdot 1} \right)$$

$$= \cos^{-1} \left( \frac{\frac{1}{\sqrt{14}}}{\sqrt{14}} \right)$$

3)  $\hat{A}$ , a unit vector in direction of  $A$ .

$$\hat{A} = \frac{A}{|A|} = \frac{1}{\sqrt{14}} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

4) The direction cosines of  $A$ .

$$\cos(A_x) = \frac{A_x}{|A|} = \frac{1}{\sqrt{14}}$$

$$\cos(A_y) = \frac{A_y}{|A|} = \frac{2}{\sqrt{14}}$$

$$\cos(A_z) = \frac{A_z}{|A|} = \frac{3}{\sqrt{14}}$$

5)  $A \cdot B$  and  $B \cdot A$

$$A \cdot B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 \\ = 4 + 10 + 18 = 32$$

$$B \cdot A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 4 \times 1 + 5 \times 2 + 6 \times 3 \\ = 4 + 10 + 18 = 32$$

6) the angle between  $A$  and  $B$

$$\cos \theta = \frac{A \cdot B}{|A||B|} = (A \cdot B) \text{ from previous problem.}$$

$\hookrightarrow \text{I}$

$$|A| = \sqrt{1^2 + 2^2 + 3^2}; |B| = \sqrt{4^2 + 5^2 + 6^2}.$$

$$|A| = \sqrt{14}; |B| = \sqrt{77}.$$

$\hookrightarrow \text{II}$        $\hookrightarrow \text{III}$

∴ to

$$\therefore \cos \theta = \frac{32}{(\sqrt{14})(\sqrt{77})} = \frac{32}{\sqrt{1078}}.$$

$$\therefore \cos \theta = \cos^{-1} \left( \frac{32}{\sqrt{1078}} \right)$$

$\theta = 12.93^\circ$

7). A vector which is perpendicular to  $A$ .

Consider  $y$  be the vector perpendicular to  $A$ .

$\hookrightarrow$  Also, a vector which is perpendicular to  $A$  the dot product will be 0.

$$\therefore A \cdot x = 0.$$

$$= 1x_1 + 2x_2 + 3x_3 = 0$$

Hence, choose values of

$x_1, x_2$ , and  $x_3$  such that it satisfies

condition  $[A \cdot x = 0]$

$$A = \begin{bmatrix} & \\ & \end{bmatrix}$$

Hence possible values could be :-

$$(1)(2) + (2)(2) + (3)(-2) = 0. \quad \underline{\text{or}}$$

$$(1)(-1) + (2)(-4) + (3)(3) = 0.$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} \quad \underline{\text{OR}} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ 3 \end{bmatrix}$$

8)  $A \times B$  and  $B \times A$

$$A \times B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2(6) - 3(5) \\ 1(6) - (4)(3) \\ 1(5) - 2(4) \end{bmatrix}$$

$$\equiv \begin{bmatrix} 12 - 15 \\ 6 - 12 \\ 5 - 18 \end{bmatrix} \equiv \begin{bmatrix} -3 \\ -6 \\ -3 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5(3) - 6(2) \\ 4(3) - 6(1) \\ 4(2) - 5(1) \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$$

9) a vector which is perpendicular to both A and B such that

$$\vec{V} = \begin{vmatrix} a & b & c \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = -\frac{1}{6}(\vec{a} + \vec{b} + \vec{c})$$

~~-3~~  
~~+12~~  
~~-9~~  
~~-24~~  
~~+45~~  
~~-36~~
-6 + 9 - 6  
3 - 6

$$= -6\hat{a} + 9\hat{b} - 6\hat{c}$$

Now :-  $A \cdot V = -6 \times 4 + 5 \times 9 - 6 \times 6$

$$\vec{V} = \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = -3\hat{a} + 6\hat{b} - 3\hat{c}$$

$$= -3\hat{a} + 6\hat{b} - 3\hat{c}$$

Now,  $A \cdot V = 1 \times (-3) + 2 \times 6 + 3 \times (-3) = 0$

$B \cdot V = 4 \times (-3) + 5 \times 6 + 6 \times (-3) = 0$

$\therefore \vec{V}(-3, 6, -3)$  is perpendicular to A and B

10) The linear dependency between A, B, C

$$A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= [4 \times 1 + 5 \times 2 + 6 \times 3]$$

$$= 4 + 10 + 18 = 32$$

$$AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = 4 + 10 + 18 = 32$$

$$= \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix} =$$

Q(B)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix};$$

C

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}.$$

j)  $2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & 1 \end{bmatrix}$$

2)  $AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4+9 & 2+4+6 & 1+(-8)+3 \\ 4-4+9 & 8-2-6 & 4+8-3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix} = \begin{bmatrix} 14 & 10 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 6 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6+1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-8+0 & 6+4+5 & 9+6+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3 & 10 \\ 6 & -18 & 13 \\ -5 & 15 & 16 \end{bmatrix}$$

3)  $(AB)^T$  ans:-

$$(AB) = \begin{bmatrix} 14 & 12 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix} = \begin{bmatrix} 14 & 9 & 7 \\ 12 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 & 6 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1+4+9 & 4+4+9 & 0+10+3 \\ 2+2-6 & 8+2+6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0+20-1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1x1 + 2x2 + 3x3 & 1x1 - 2x(-2) + 3x3 & 0x1 + 2x5 + 3x(-1) \\ 2x1 + 2x2 + 3x(-2) & 4x2 - 2x1 + 3x(-2) & 0x2 + 5x1 + 2x(-1) \\ 1x1 + 2x4 + 3x1 & 4x1 - 2x4 + 3x1 & 0x2 - 4x5 - 1x1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 4-4+9 & 0+10-3 \\ 2+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0-20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$4) |A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & 1 \end{vmatrix} = 1(2-15) - 2(-4+6) \\ = +3(20-0) \\ = -13 + 18 + 60 = 55$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix} = 1(15-6) - 2(12-6) \\ = +3(4+5) \\ = 9 - 2(6) + 3(9)$$

$$\Rightarrow 9 - 12 + 18 = 15$$

Ques 5) the matrix (A, B, or C) in which the row vectors form an orthogonal set.

Condition of orthogonality :-

Dot product of rows should be zero.

For A,

$$(1, 2, 3) \cdot (4, 2, 3) = 1 \times 4 - 2 \times 2 + 3 \times 3 = 9 \neq 0$$

$$(4, -2, 3) \cdot (0, 5, -1) = (0 \times 4 - 2 \times 5 - 3 \times 1) =$$

$$= 0 - 10 - 3 = -13 \neq 0$$

Hence A is not orthogonal.

B)  $(1, 2, 1) \cdot (2, 1, -4) =$   
 $2 \times 1 + 2 \times 1 - 4 \times 1 = 0$   
 $2 + 2 - 4 = 0 \Rightarrow 0 = 0$

$$(2, 1, -4) \cdot (3, -2, 1) \\ = 2 \times 3 + (-2) \times 1 - 4 \times 1 \\ = 6 - 2 - 4 \\ = 0 = 0$$

∴ B is orthogonal.

C)  $(1, 2, 3) \cdot (4, 5, 6) \neq 0$   
 $1 \times 4 + 2 \times 5 + 3 \times 6 \neq 0$

$$(4, 3, 6) \cdot (-1, 1, 3)$$

$$4 \times (-1) + 3 \times 1 + 6 \times 3$$

$$-4 + 3 + 18 = 19 \neq 0$$

C is not orthogonal.

$$G). \quad A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -3 & 3 \\ 6 & 5 & -1 \end{bmatrix}$$

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$$B^{-1} = \frac{\text{Adj}(B)}{|B|}$$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ -3 & 2 & 1 \end{bmatrix}$$

$$\text{Adj } A \Rightarrow \begin{bmatrix} 2-15 & -(4-0) & 20-0 \\ 2+15 & -1-0 & -5 \\ 12 & -9 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 2 & 9 & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} 13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$|A| = |(2-15) - 2(-4-0) + 3(20-6)| \\ = -13 + 8 + 6 = 55$$

$$A^{-1} = \frac{1}{55} \begin{bmatrix} 13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

55

$$B^{-1} = \frac{\text{Adj}(B)}{|B|} = \begin{bmatrix} 1-8 & -2-12 & -7 \\ -2-2 & 1-3 & 8 \\ 8-1 & 4+2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -14 & -7 \\ -4 & -2 & 8 \\ -9 & 6 & -3 \end{bmatrix}$$

$$|(-7) - 28 + 1|$$

$$\text{Adj } (B) = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix} \Rightarrow B \Rightarrow \\ = +7/2 - 42 \\ = 4$$

$$B^{-1} = \frac{1}{42} \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

Q3]  $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$ , find.

7) Eigen Values and corresponding Eigen Vectors

formula:-  $|A - I\lambda| = 0$ .  $I$  = Identity matrix

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \left| \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0.$$

$$= \begin{vmatrix} 1-\lambda & 2-0 \\ 3-0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(2-\lambda) - 3(2) = 0$$

$$= 1(2-\lambda) - 1(2-\lambda) - 3(2) = 0$$

$$= \underline{2-\lambda} - \underline{2\lambda} + \lambda^2 - 6 = 0$$

$$= 2 - 3\lambda + \lambda^2 - 6 = 0$$

$$= \lambda^2 - 3\lambda - 4 = 0$$

$$\Rightarrow (A-4)(\lambda+1) = 0$$

$$\begin{array}{r} -3 \\ -4 \\ +1 \\ \hline = 4 \end{array}$$

$$\therefore \underline{\lambda = 4} \text{ or } \underline{\lambda = -1}$$

Case 1:  $\lambda = 4$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 4 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \begin{cases} xx_1 + 2xy = 4x \\ 3xx + 2xy = 4y \end{cases}$$

$$\Rightarrow \begin{pmatrix} x+2y \\ 3x+2y \end{pmatrix} = \begin{pmatrix} 4x \\ 4y \end{pmatrix}$$

$$\therefore x+2y=4x \Rightarrow 2y=4x-x \therefore 2y=3x \\ \Rightarrow \boxed{x = \frac{2y}{3}}$$

$$\therefore 3x+2y=4y \Rightarrow 3x+2y-4y=0 \\ = 3x-2y=0 \\ = \boxed{x = \frac{2y}{3}}$$

$$\Rightarrow \text{Eigenvector } \vec{v}_3 = \begin{pmatrix} 2/3 \\ 1 \end{pmatrix}$$

$$\text{Case 2) } \lambda=1 \Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} xx_1+2xy \\ 3xx+2xy \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix} \Rightarrow \begin{bmatrix} x+2y \\ 3x+2y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$\Rightarrow x+2y=-x \Rightarrow 2x+2y=0; x+y=0 \quad \boxed{x=-y}$$

$$\Rightarrow 3x+2y=-y \Rightarrow 3x+3y=0 \quad \therefore \boxed{x=-y}$$

$$\text{Eigenvector} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

2] The matrix  $V^{-1}AV$  where  $V$  is composed of eigen vectors of  $A$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \Rightarrow V = \begin{bmatrix} -1 & 2/3 \\ -1 & 1 \end{bmatrix}$$

$$\therefore V^{-1} = \frac{1}{1-2/3} \times \begin{bmatrix} 1 & 2/3 \\ -1 & -1 \end{bmatrix} = \frac{1}{\frac{-1}{3}} = -3$$

$$= -3 \begin{bmatrix} 1 & 2/3 \\ -1 & -1 \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix}$$

$$V^{-1}(A) = \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \times 1 + 2 \times 3 & 3 \times 2 + 2 \times 2 \\ -3 \times 1 + -3 \times 3 & 2 \times 2 + (-3) \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+6 & 6+4 \\ -3-9 & 4-6 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ -12 & -2 \end{bmatrix}$$

$$\therefore V^{-1}(A) = \begin{bmatrix} 9 & 10 \\ -12 & -2 \end{bmatrix}$$

3) Dot product of Eigen Vectors of A

$$\begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 2/3 \\ 1 \end{bmatrix} = -1 \times \frac{2}{3} + (-1)(1) \\ = \frac{-2}{3} - 1 = \frac{-2 - 1 \times 3}{3} = \frac{-2 - 3}{3}$$

$$\boxed{\begin{bmatrix} -5 \\ 3 \end{bmatrix}}$$

4) Dot product of Eigen vectors of B.

$$|B - \lambda I| = 0$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \begin{vmatrix} 2 - \lambda & -2 \\ -2 & 5 - \lambda \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 2 - \lambda & -2 \\ -2 & 5 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (2 - \lambda)(5 - \lambda) - (-2)(-2) = 0$$

$$\therefore 10 - 7\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 1)(\lambda - 6) = 0$$

$$\therefore \boxed{\lambda = 1} \text{ or } \boxed{\lambda = 6}$$

Case 1

for  $\lambda = 1$ ,

$$\begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1 \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} 2x - 2xy \\ -2x + 5xy \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x - 2y \\ -2x + 5y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

L

$$2x - 2y = x$$

$$2x - x - 2y = 0$$

$$2x - 2y = 0$$

$$\left\{ \begin{array}{l} 2x = 2y \\ x = y \end{array} \right.$$

$$\text{EigenVector} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$-2x + 5y = y$$

$$-2x + 5y - y = 0$$

$$-2x + 4y = 0$$

$$\therefore \boxed{2x = 2y}$$

(Case 2)  $\lambda = 6$

$$\begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 6 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2x - 2xy \\ -2x + 5xy \end{bmatrix} = \begin{bmatrix} 6x \\ 6y \end{bmatrix} \Rightarrow \begin{bmatrix} 2x - 2y \\ -2x + 5y \end{bmatrix} = \begin{bmatrix} 6x \\ 6y \end{bmatrix}$$

$$\therefore 2x - 2y = 6x$$

$$\Rightarrow +2x - 6x - 2y = 0$$

$$-4x - 2y = 0$$

$$\therefore -2x - y = 0$$

$$\therefore -2x + 4$$

$$= -2x = y$$

$$\therefore \boxed{x = \frac{-y}{2}}$$

$$-2x + 5y = 6y$$

$$-2x + 5y - 6y = 0$$

$$-2x - y = 0$$

$$-2x = y$$

$$\therefore$$

$$\boxed{x = -y}$$

$$\text{Dot Product: } \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$$2x - \frac{1}{2} + 1x = [-1 + 1] \\ = 0$$

All Eigen Vectors of B are +

5)

The property of eigenvectors of B :-

$\Rightarrow$  The property is that the eigenvector of B have dot product as zero, hence they are the same.

$\Rightarrow$  The dot product of Eigenvector is zero  
hence eigenvectors are perpendicular.

Q 29]

$$\text{Let } f(x) = x^2 + 3; \quad g(x, y) = x^2 + y^2$$

i). The first and second derivatives of  $f(x)$  with respect to  $x$ :

$$f'(x), \text{ and } f''(x).$$

$$\text{i) } \frac{d(x)}{dx} = \frac{d}{dx}(x^2 + 3)$$

$$\left[ \frac{d(x)}{dx} = \frac{2x}{=} \right] \rightarrow \text{first derivative.}$$

$$\text{i) } \frac{d}{dx} \left[ \frac{d(x)}{dx} \right] - \frac{d}{dx}(2x) \\ = 2 \rightarrow \text{2nd derivative.}$$

2) Partial Derivatives:-

$$\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2)$$

$$\frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2)$$

$$\therefore \frac{\partial g}{\partial y} = 2y$$

$$\text{Now!- } \frac{\partial g}{\partial y} = \frac{\partial}{\partial y}$$

3. The Gradient Vector  $\nabla g(x, y)$

$$\nabla g(x, y) = \left( \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right) = (2x, 2y)$$

4. The probability density function (pdf) of a univariate Gaussian (normal) distribution.

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Substitute values:-  $\sigma=1$ ;  $t=0$ ;  $\mu=0$ .

$$\therefore = \frac{1}{\sqrt{2\pi\sigma^2}}$$