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CS512 – Camera Calibration – Course fall 2020

Assignment - 4

Anumber → A20449343

1] forward projection:-

$p_i = M P_i$ → world points.

image point (2DH) — projection matrix

For forward projection it is important to know the intrinsic & ~~for~~ the extrinsic parameters these are which are used calculate ~~mat~~ projection matrix (M)

$M = K^* [R^* | T^*]$

K^* ⇒ intrinsic parameter

R^* → Rotation of world w.r.t camera

T^* → Transla^n of world w.r.t camera.

→ ~~for~~ Measuring translation & rotation is complicated.

2] Calibration:- Take picture of 3D object.

problem statement } → Given corresponding point $\{p_i\}_{i=n}$ in pixel in image co-ordinate and $\{P_i\}_{i=1}^{m}$ in world co-ordinate

3] Problems for of reconstruction.

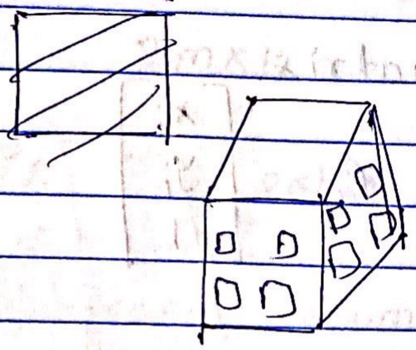
→ A scenario ~~to~~ see several view points from 4 views up to several hundreds can be taken. It is tough to match features from these number of views.

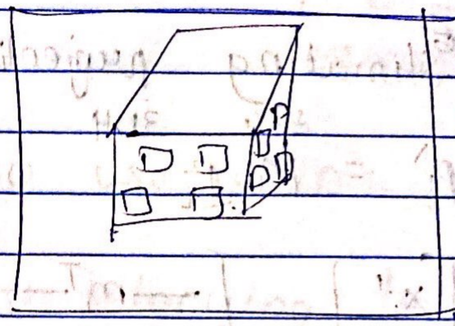
→ The easiest problem that can be solved is the calibration problem

→ The hardest one is forward project of reconstruction image, because in forward projection alot of calculations are involved in calculation of parameters & analysis of viewpoints is tough in reconstructions

Q2] Explain what is the necessary input for camera calibration.

→ Given corresponding points $\{P_i\}_{i=1}^{m} \longleftrightarrow \{P_i\}_{i=1}^{m}$ find parameters
(in pixels) (in meters)





3D object

3.2	1.7	1.8
7.2	2.2	7.8
⋮	⋮	⋮

x, y, z co-ordinates of the image

20.0	50.0
30.0	40.0
⋮	⋮

input for calibration alg.

u, v → 2D points of the object on the image. These are float numbers

→ Create a correspondence between both the measurement in 3-Dimensional real space and 2D points of image.

c) Steps in non-coplanar calibration

→ Give $\{p_i\}_{i=1} \longleftrightarrow \{\underline{P}_i\}_{i=1}^{m}$. find camera parameters

2DH. (in pixels) 3DH. (in meters)

Steps:
① find projection matrix M
② find parameter from M.

Estimating projection matrix.

$$\underset{2DH}{p_i'} = \underset{3\times4}{M}\ \underset{3DH}{\underline{P}_i} \qquad p_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \qquad \underline{P}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_i' \\ y_i' \\ w_i' \end{bmatrix} = \begin{bmatrix} — m_1^T — \\ — m_2^T — \\ — m_3^T — \end{bmatrix} \underline{P}_i \qquad x_i = \frac{x_i}{w_i} \quad \Big| \quad y_i = \frac{y_i}{w_i}$$

$$\left.\begin{aligned} x_i' &= m_1^T \underline{P}_i \\ y_i &= m_2^T \underline{P}_i \\ w_i' &= m_3^T \underline{P}_i \end{aligned}\right\} \quad x_i = \frac{m_1^T \underline{P}_i}{m_3^T \underline{P}_i} = m_1^T \underline{P}_i - x_i m_3^T \underline{P}_i = 0$$

similarly, $y_i = m_2^T \underline{P}_i - y_i m_3^T \underline{P}_i = 0$

$$\left.\begin{aligned} m_1^T \underline{P}_i - x_i m_3^T \underline{P}_i &= 0 \\ m_2^T \underline{P}_i - y_i m_3^T \underline{P}_i &= 0 \end{aligned}\right\}$$ for each point 2 equa^n with 12 unknowns. need at least 6 point pairs

→ for a single point

$$\underset{2\times12}{\begin{bmatrix} \underline{P}_i^T & 0 & -x_i \underline{P}_i^T \\ 0 & \underline{P}_i^T & -y_i \underline{P}_i^T \end{bmatrix}} \underset{12\times1}{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for m points:

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & & \\ P_m & 0 & -x_n P_m^T \\ 0 & P_m & -y_m P_m^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$2m \times 12$ $\quad$ 12×1 $\quad$ $2m \times 1$

$\boxed{Ax = 0}$

* To solve $Ax = 0$ use SVD
 $A = UDV^T$
 Solution = column of V belonging to zero singular value.

$$\hat{x} = \begin{bmatrix} \hat{m}_1 \\ \hat{m}_2 \\ \hat{m}_3 \end{bmatrix} \Rightarrow \hat{M} = \begin{bmatrix} - \hat{m}_1^T - \\ - \hat{m}_2^T - \\ - \hat{m}_3^T - \end{bmatrix}$$

Soluⁿ is not unique if $A\hat{x} = 0$
→ we have $A(\rho\hat{x}) = 0 \Rightarrow \rho\hat{x}$ is a solution

* finding parameters
→ solution for $\hat{M}$ is not unique.
→ find ρ so that:

$$M = K^* [R^* | T^*] = \rho\hat{M}$$

$$\Rightarrow [K^* R^* | K^* T^*] = \rho\hat{M}$$

$$K^* R^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} - r_1^T - \\ - r_2^T - \\ - r_3^T - \end{bmatrix}$$

$$= \left[\underbrace{\begin{bmatrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T \\ \alpha_v r_2^T + v_0 r_3^T \\ r_3^T \end{bmatrix}}_{K^* R^*} \Bigg| K^* T^* \right]$$

$$= \rho M = \rho \left[\underbrace{\begin{matrix} - a_1^T - \\ - a_2^T - \\ a_3^T - \end{matrix}}_{\text{known}} \Bigg| b \right]$$

unknown.

Extract parameters :-

u_0, v_0, α_v, s = scale parameters,

sign of ρ. $\Rightarrow$ ϵ = sign (b_z)

eg] projection matrix $= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(P_i) world point = (1, 2, 3)

find P_i = image co-ordinates of P_i

$\Rightarrow$ convert P_i to homogeneous cordinates

$\Rightarrow (1, 2, 3) \rightarrow (0.34, 0.67, 1)$

$M \Rightarrow$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} [0.34 \quad 0.67 \quad 1]$

Homogeneous co-ord of P_i

$$= \begin{matrix} 0.34 \\ 0.64 \\ 1 \end{matrix} \quad \begin{matrix} 5.82 \\ \\ 4 \end{matrix} \quad \begin{matrix} 4.48 \\ \\ 1 \end{matrix} \quad 1$$

$$= \begin{bmatrix} 0.34 & 5.82 & 8.82 & 11.765 \\ 0.64 & 0 & 4.48 & 5.98 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

d) Projection matrix : $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$$P_i = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

→ We convert world point to 3DH

$$P_i = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \therefore P$$

$$P_i = MP_i = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 4 \\ 7 \end{bmatrix} \xrightarrow[2D]{2DH} \begin{bmatrix} 18/7 \\ 4/7 \end{bmatrix}$$

e) Given corresponding world-image points $(1, 2, 3) \leftrightarrow (100, 200)$ write first two lines of matrix that needs to be formed for ~~so~~ unknown projection matrix M.

$(1, 2, 3) \longleftrightarrow (100, 200)$

$$P_i = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad p_i = \begin{bmatrix} 100 \\ 200 \\ 1 \end{bmatrix}$$

Projection matrix is given as:

$$M = \begin{bmatrix} P_i^T & 0 & -x_i & P_i^T \\ 0 & P_i^T & -y_i & P_i^T \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & -100 & 200 & -300 & -100 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 1 & -200 & -400 & -600 & -200 \end{bmatrix}$$

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f) Minimal number of points necessary to be able to find a unique solution for M. How is the solution obtained?

→ Need 2 equations with 12 unknowns, we need atleast 6 point pairs.
For single point :-

$$\underbrace{\begin{bmatrix} P_i^T & 0 & -x_i P_i^T \\ 0 & P_i^T & -y_i P_i^T \end{bmatrix}}_{2\times 12} \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{12\times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times 1}$$

$= A X = 0$

for m points :-

$$\begin{array}{l} \text{1st point} \\ \\ \\ \\ \\ m^{th}\text{ point} \end{array} \begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & & \\ \vdots & & \\ P_m & 0 & -x_n P_m^T \\ 0 & P_m^T & -y_m P_m^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \therefore Ax = 0$$

To solve $Ax = 0$ use SVD (single value decomposition)

$A = UDV^T$

Solution = column of V belonging to zero singular value

2014 ↓

$P_i = 3ML_i$ — homogenizing

The solution is not unique. if $Ax = 0$. we also have $A(\rho\hat{x}) = 0$

$\Rightarrow \rho\hat{x}$ is a solution.

(g) Principal that is used to extract unknown camera parameters from the projection matrix M.

$$M = K^* \, [R^* | T^*] = \rho \hat{M} \quad \rightarrow R, T. \quad = \rho = ?$$

find parameters and ρ from M
(K, R, T).

$$K^* [R^* | T^*] = \rho \hat{M}$$

$$\Rightarrow [K^* R^* | K^* T^*] = \rho \hat{M}$$

$$K^* R^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} - r_1^T - \\ - r_2^T - \\ - r_3^T - \end{bmatrix} = \begin{bmatrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T \\ \alpha_v r_2^T + v_0 r_3^T \\ r_3^T \end{bmatrix}$$

$$\underbrace{\left[\begin{array}{c|c} \alpha_u r_1^T + s r_2^T + u_0 r_3^T & \\ \alpha_v r_2^T + v_0 r_3^T & K^* T^* \\ r_3^T & \end{array}\right]}_{\text{unknown.}} = \underbrace{\rho \hat{M}}_{\text{unknown}} = \rho \underbrace{\left[\begin{array}{c|c} - a_1^T - & \\ - a_2^T - & b \\ - a_3^T - & \end{array}\right]}_{\text{Known } \hat{M}}$$

g) Principal used to extract the unknown camera parameters from the projection matrix M.

→ The principal of orthogonality is used to extract the unknown camera parameters

⇒ $r_1 \cdot r_2 = 0$ $\quad r_2 \cdot r_3 = 0$ $\quad r_1 \cdot r_3 = 0$

$r_1 \times r_2 = r_3$ $\quad r_2 \times r_3 = r_1$ $\quad r_3 \times r_1 = r_2$

find unknown scale :-

$$r_3^T = \rho a_3^T$$

→ $|r_3^T| = |\rho| \, |a_3^T| = \boxed{|\rho| = \frac{1}{|a_3|}}$

(h) Computation of ~~proje~~ quality of projection matrix M estimate.

→ Given $\{P_i\}_{i=1}^{m} \longleftrightarrow \{p_i\}_{i=1}^{m}$ and estimated

$$M = \begin{bmatrix} - m_1^T - \\ - m_2^T - \\ - m_3^T - \end{bmatrix}$$ assess the quality of fit using

$$E = \frac{1}{m} \sum_i \left(\left\| x_i - m_1^T P_i \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right).$$

distance betwn known & unknown predicted position

(e.g) $(7.3, 12.1, 15.3) \leftrightarrow (5, 3)$ known → Σ → error

$\qquad \to (6, 2)$ predicted using M

(i) Principal of planar camera calibration

⇒ Approach:-

① Estimate 2D homography (projective map) between calibration plane & image (for several images).

② Estimate intrinsic parameters

③ Compute extrinsic parameters for view of interest

Difference between planar & non-coplanar camera calibration.

→ Both are similar expect that the planar camera calibration has

$P_i = M P_j$

~~(2)~~ where P_i is 2D-H

P_j → real camera cordinates are 3DH

and $M = 3 \times 4$.

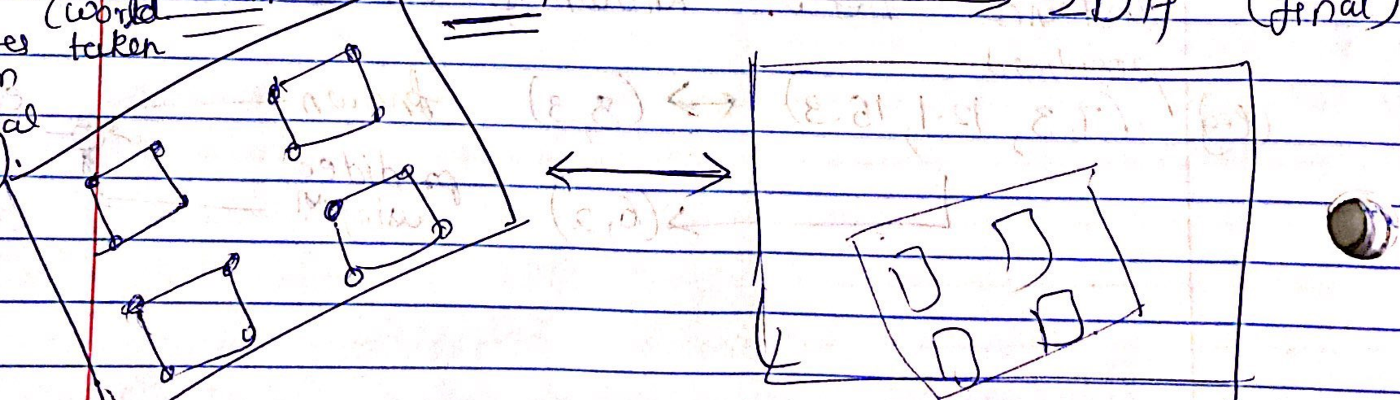
M = is 2D projection map.

(j) Difference between homography (2D projective map) H and projection matrix M. What is assumption that is used to make sure we deal with homograph matrices?

→ (initial) 2D Homography:

3DH → 2DH ⟶ 2DH (final)

(world pictures taken from several angles).



→ ~~Homography has planar calibration.~~
→ ~~It is a projection matrix, where we~~

→ Homograph has planar calibration.
→ Initially all the points are in same plane of the world co-ordinates, these points are in the 3DH form ~~can~~ and can converted into 2DH, this ~~2DH~~ intermediate case for 2DH can be mapped ~~to~~ to ~~the final camera-coordi~~ the final 2DH co-ordinates of the camera

→ The difference between a projection map and homography is that, the projection map has mapping between 3DH to 2DH, instead homography does the mapping between 2DH to 2DH.

→ for Homograph mapping has metrices:- $\hat{H}_1, \hat{H}_2, \hat{H}_3$, before for projection ~~map~~ matrix to calculate intrinsic parameters we use $\hat{M}$ a single matrix, unlike 3 for Homography

→ Here for homograph as we move the calibration target to take multiple pictures the ~~ex~~ extrinsic parameters R, and T will change with each position.

∴ Assumption for 2DH Homography:-
① Has planar calibration
② Images taken from several angles.
③ $\hat{H}_1, \hat{H}_2, \hat{H}$

Q2 Camera Calibration

a) P_i after making it homogeneous:-

$$p_i = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad P_i = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

Projection Matrix M is given as

$$M = \begin{bmatrix} P_i^T & 0 & -x_i P_i^T \\ 0 & P_i^T & -y_i P_i^T \end{bmatrix}$$

$$M = \begin{bmatrix} 3 & 4 & 5 & 1 & 0 & 0 & 0 & 0 & -3 & -4 & -5 & -1 \\ 0 & 0 & 0 & 0 & 3 & 4 & 5 & 1 & -6 & -8 & -10 & -2 \end{bmatrix}$$

... (first two lines)

b) $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \Rightarrow$ projec^n matrix

$a_1 = (1, 2, 3)$ $a_2 = (2, 3, 4)$

$a_3 = (3, 4, 5)$

$\cancel{|\rho|} = \cancel{\frac{1}{\sqrt{93}}} = \rho = \frac{1}{\sqrt{50}}$

Scanned with CamScanner

~~camara~~ camera parameters :-

$$u_0 = |\rho|^2 \; a_1 \cdot a_3$$

$$u_0 = \left(\frac{1}{\sqrt{50}}\right)^2 (3+88+15)$$

$$= \frac{1}{50}(26) = 0.52$$

$$v_0 = |\rho|^2 \cdot a_2 \cdot a_3$$

$$= \left(\frac{1}{\sqrt{50}}\right)^2 (6+12+20)$$

$$= (u_0, v_0) = (0.52, 0.76)$$

c] Image = (1, 2)   world = (3, 4, 5)

$$M = \begin{bmatrix} 1, & 2, & 3, & 4 \\ 2, & 3, & 4, & 5 \\ 3, & 4, & 5, & 6 \end{bmatrix}$$

$$\therefore P_i = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix} \qquad P_i = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$E = \frac{1}{m} \sum_i \left(\left\| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right)$$

$$\Rightarrow 1 \left(\left\| 1 - \frac{30}{56} \right\|^2 + \left\| 2 + \left(\frac{-43}{56}\right) \right\|^2 \right)$$

$$= ||0.46||^2 + ||-1.232||^2$$

$$= 0.215 + 1.518$$

$$\epsilon = 1.733$$

(d) $$\phi = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R^* = I + \phi = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^* = (1, 2, 3)$$

$$\therefore R = (R^*)^T = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translaⁿ :-

$T^* = -R^T T$

$= T^* = -(R^*)^T T^*$

$$= -\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} -6 \\ -2 \\ -3 \\ -1 \end{bmatrix}$$

(e) Image 1, 2 ; world (3, 4, 0)

$$p_i = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad P_i = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 1 \end{bmatrix} \quad P_i^* = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

First 2 rows of matrix are given as :-

$$\begin{bmatrix} P_i^{*T} & 0 & -x_i P_i^{*T} \\ 0 & P_i^{*T} & -y_i P_i^{*T} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 1 & 0 & 0 & 0 & -3 & -4 & -1 \\ 0 & 0 & 0 & 3 & 4 & 1 & -6 & -8 & -2 \end{bmatrix}$$

Q3. a) Sparse stereo matching detect keypoints in image, it also does matching to get the corresponding details.
b) It is used to calculate feature vectors of each point and match these vectors with other image.
c) It is a feature based matching

a) Dens Stereo matching takes all the points in the image and it is then matched with all the points in other images.
b) Instead of features, ~~we~~ correlation method is used to see if points ~~corre~~ correlate.
c) Advantage of Dense stereo matching is that it produces more points, hence it is more accurate when it matches all points.
d) Sparse stereo method of matching ~~to~~ handles large disparities and hence it is more robust.

e) Disadvantages of Dense stereo matching is that it is limited to very small changes in view., the points ~~corre~~ correspond to the image are difficult to distinguish.

(b) NCC and SCO.
→ Sum of Squared Distances (SSD)
→ In this we calculate difference between corresponding points of the windows and square these differences get positive values hence after which we find summation over all points in the window.

Equation:-

$$\psi(w_1, w_2) = \sum_i \left(w_1(x_i y_i) - w_2(x_i y_i)\right)^2.$$

- ~~Less the~~
- Less distance between points, hence more correlation and less error.
- When we use entire image as search space, we face a problem in uniform regions of the image.
- There could be multiple point in one ~~image~~ image that matches to points in other image.
- To reduce the search space, we can apply constraints for reducing number of candidates
- for example, we can look close to current location (neighborhood)

c) Axis Alligned Stereo.

$P_L = (100, 200)$ $P_R = (103, 200)$

$f = 10$ baseline $(T) = 100$.

we can calculate depth using the formula:-

$$Z = \frac{f \cdot T}{d}$$

where we can calculate disparity:-

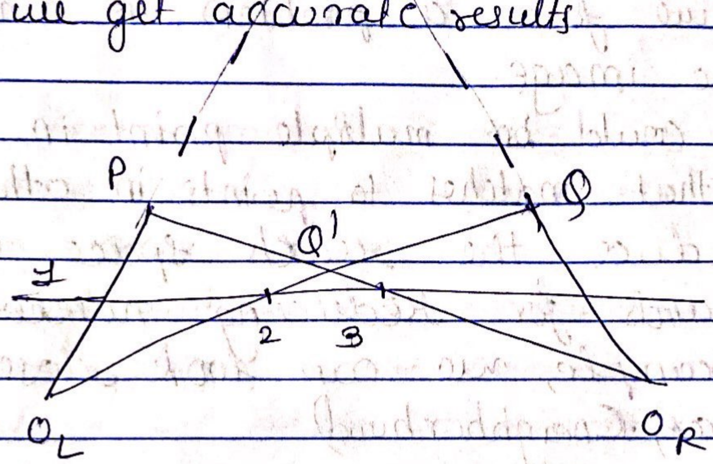
$$d = (X_R - X_L) = (103 - 100) = 3$$

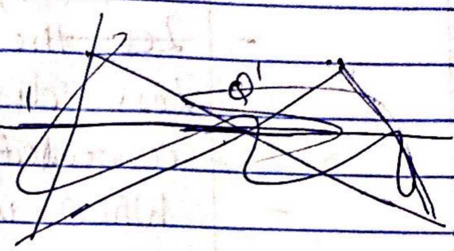
and $Z = \frac{10 \times 100}{3} = 333.34$ (Depth, Z-co-ordinate)

d) Ambiguity Problem:

- Consider the figure, we have two 3D points P and Q, O_L & O_R are optical centres of left & right image

- After projecting both points P & Q, if we correctly match the points 1,3 & 2,4 we get accurate results.





→ ~~After projecting both points P & Q, if~~

→ However, if we failed to rotate points correctly and match 1 with 4 and 2 with ~~4~~ 3, after triangulation, we get points P' & Q'. This happened due to incorrect point matching. The points P & P' and Q & Q' are far away & we get a big outlier

- This is ambiguity problem. The similarity between points 1, 2, 3 & 4 was ambiguous and as a result, we incorrectly matched the points leading to wrong triangulation & large outliers after projection.

(e) ~~Ex~~ Rotation for right camera with respect to left camera.

Equation : $R = R_l^T R_r$.

R_l and R_r ~~are~~ is rotation to left camera and rotation to right camera respectively.

Translation :-

$T_r \Rightarrow$ Translation of right camera w.r.t world

$T_l \Rightarrow$ Translation of left camera w.r.t world

Equation : $\boxed{T = R_l^T (T_r - T_l)}$

4) a) $f = 10mm$ baseline $(T) = 20mm$

$d = 30mm$.

$\therefore$ Depth of point $= f \cdot \frac{T}{(d)}$

$= 10 \cdot \left(\frac{20}{30}\right)$

$= 6.667$.

Depth of point is denoted by $z = 6.667mm$.

(b) $A = (1,2,3)$; $B = (2,3,4)$

$\therefore A \times B = [A]_\times B = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} B$

$\begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \Rightarrow$ which is $\begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$

$$[A_x] \times B = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -9 & +8 \\ 6 & +0 & -4 \\ -4 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

(c) fundamental matrix = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

Left and right points are (1, 2) & (2, 3)

Eight point's algorithm:-

$P_r^T F P_l$

$$= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & \cancel{3}\ 2 & 3 \\ 2 & \cancel{4}\ 3 & 4 \\ 3 & \cancel{8}\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 17 & 23 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= 11 + 34 + 23 = \underline{\underline{68}}$$

(d) Given left & rights points respectively as follows: (1,2) (2,3)

$$\Rightarrow \underbrace{\begin{bmatrix} x_1x_1' & x_1y_1' & x & y_1x_1' & y_1y_1' & y_1 & x_1' & y_1' & 1 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix}}_{n\times 9} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \\ \end{bmatrix}}_{9\times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{n\times 1}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 1 & 4 & 6 & 2 & 2 & 3 & 1 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$