Data assimilation in an elastic friction model

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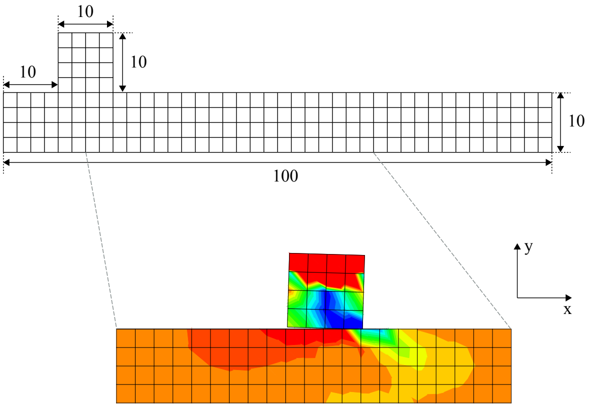
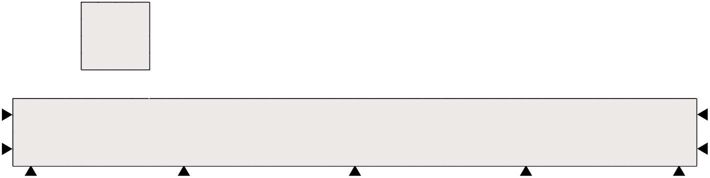
Question

- Is it possible to estimate weakly known parameters from a simple elastic friction model by using the tools of data assimilation?



Model, 2D

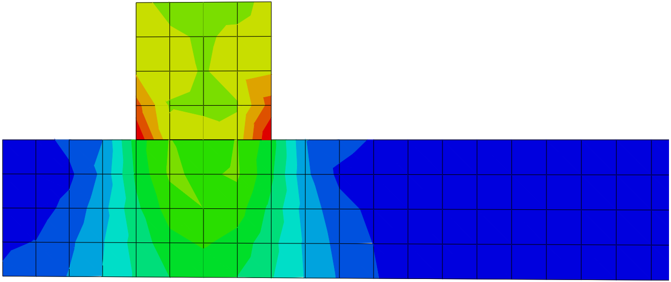
- Initial setting: Block, slab
- Boundary conditions



- Abaqus/Standard 6.12-1, simulations on CSC

Steps of the simulation

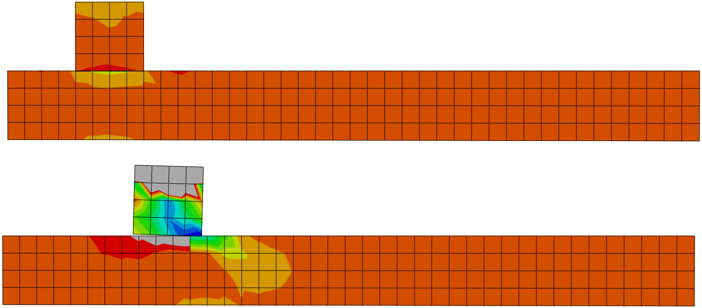
- Step 1: 5 kN force acting downwards



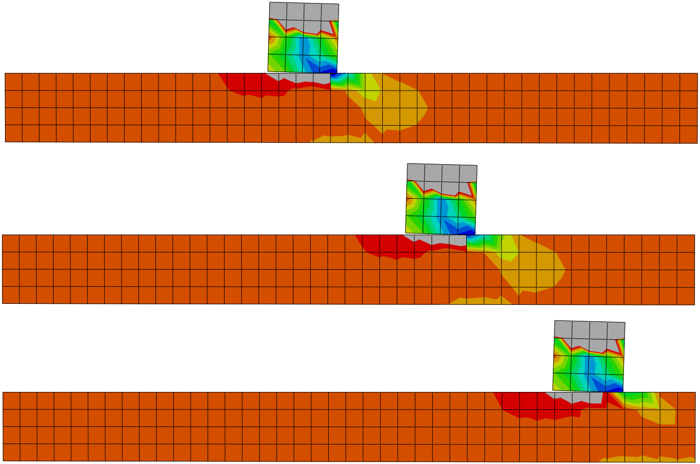
- Step 2: Displacement of block's upper boundary by 70 cm to the right

Steps of the simulation: displacement of the block

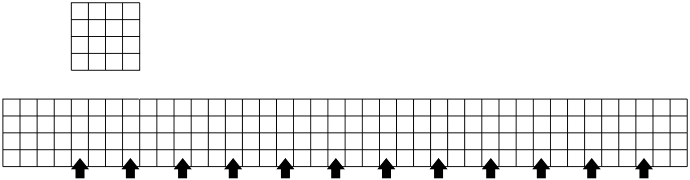
- Done by using boundary conditions. Thus, a "slow displacement"



Steps of the simulation: displacement of the block, 2



- Attempting to estimate friction coefficient μ
- As *a priori* data: x directional stress in chosen measurement points
  ($\sim$ strain gauge)



Data assimilation

- Gives an answer to the problem: How to merge measurement and model data in an optimal way?
- Traditional applications: Weather prediction, oceanography
- Multiple different methods: 3D- and 4DVar, the family of Kalman filters, ...
- In this work *Ensemble Kalman Filter*

Ensemble Kalman Filter

- System is characterized by a bunch of state vectors $\boldsymbol{\psi} \in \mathbb{R}^n$
- They estimate the same *true* state and their deviation characterizes the uncertainty
- This group of states is known as *ensemble*
- Each member of the ensemble is integrated forwards in time to the next measurement point
- After the time integration each state is merged with the measurement using

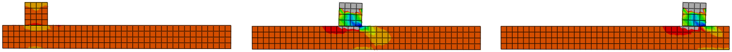
$$\boldsymbol{\psi}^a = \boldsymbol{\psi}^f + \boldsymbol{\Sigma}_\psi \mathbf{M}^{\mathrm{T}} \left(\boldsymbol{\Sigma}_d + \mathbf{M}\boldsymbol{\Sigma}_\psi \mathbf{M}^{\mathrm{T}}\right)^{-1} \left(\boldsymbol{d} - \mathbf{M}\boldsymbol{\psi}^f\right)$$

Ensemble Kalman Filter, estimating parameters

- Using the same procedure as previously but extend each state vector with the unknown parameters
- Formulation of the method guarantees that the unknown parameters are found
- Note! There must be some kind of correlation between the state and the unknown parameters

Back to the problem

- Model



- Estimated parameter μ
- Define the state as

$$\psi = (\sigma_x^1, \sigma_x^2, \sigma_x^3, \ldots, \sigma_x^N, \mu)^{\mathrm{T}}$$

- No contact in the beginning $\Rightarrow$ all the stress components are zero
- Therefore, initial state is specified by μ_0

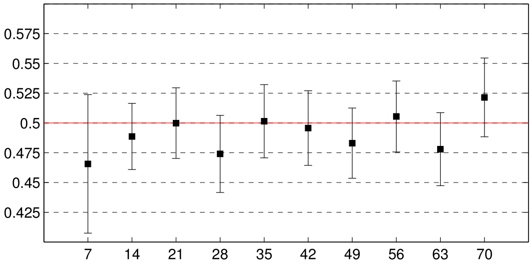
Back to the problem 2

- What we need is
  - First-guess $\mu_0 = 0{,}6$
  - Deviation for first-guess $\sigma_0 = 0{,}1$
  - Ensemble size $n = 200$
  - Initial ensemble taken from distribution $\mathcal{N}(\mu_0, \sigma_0^2)$
- A single state of the initial ensemble

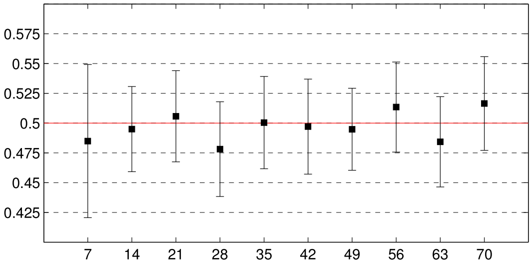
$$\psi_j = (\underbrace{0, 0, \ldots, 0, 0,}_{N \text{ kpl}} \mu_0 + \epsilon)^{\mathrm{T}}, \; j = 1, \ldots, n$$

- Measurement data when block's displacement is $\Delta x = 7, 14, 21, \ldots, 70$ (cm)
- Measurement data is synthetic

Some results, $n = 200$



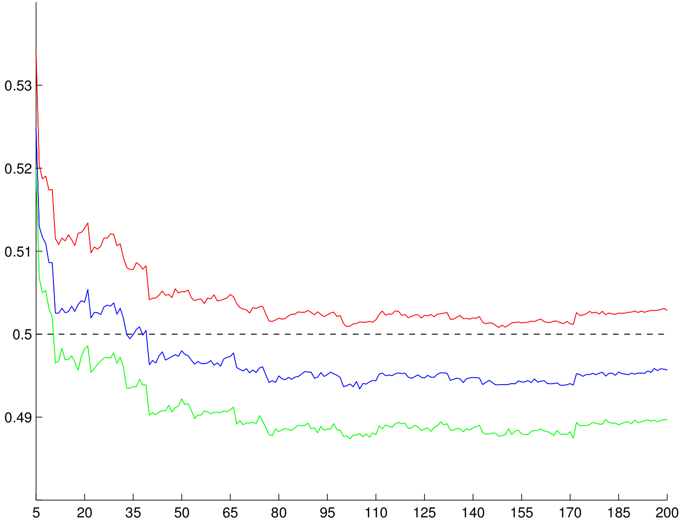
$\mathbf{Q} = \mathbf{0}$



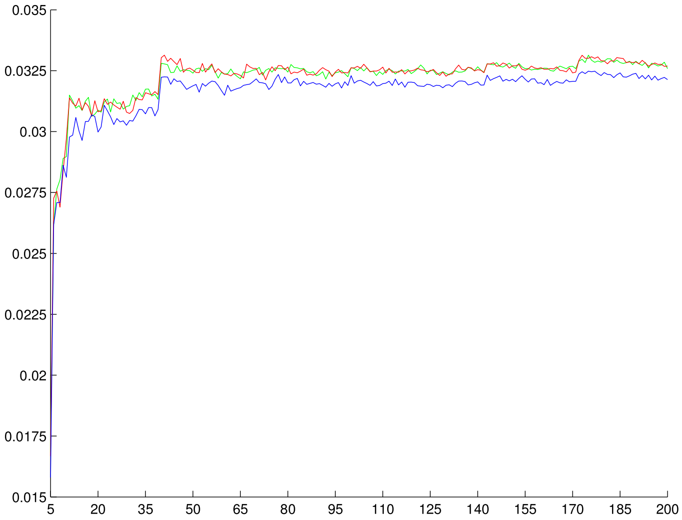
$\mathbf{Q} = \mathbf{I}\sigma$

Influence of ensemble size, mean

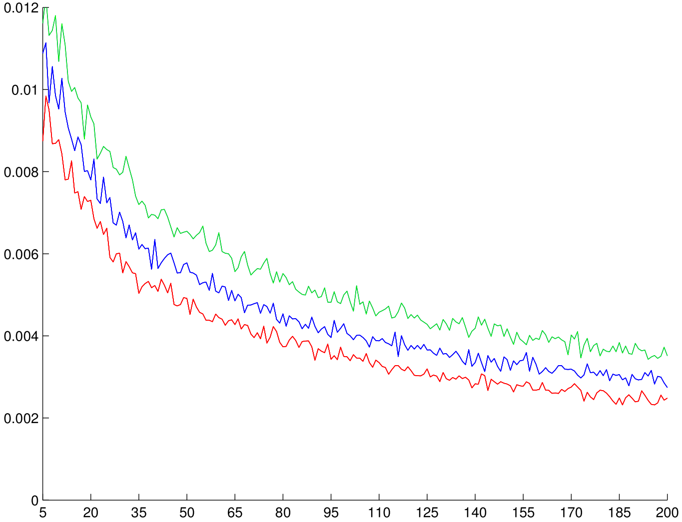
Red: $\Delta x = 70$, blue: $\Delta x = 42$, green: $\Delta x = 14$



Standard deviation of the analysed ensemble



Variance between consecutive analyses



Any questions?