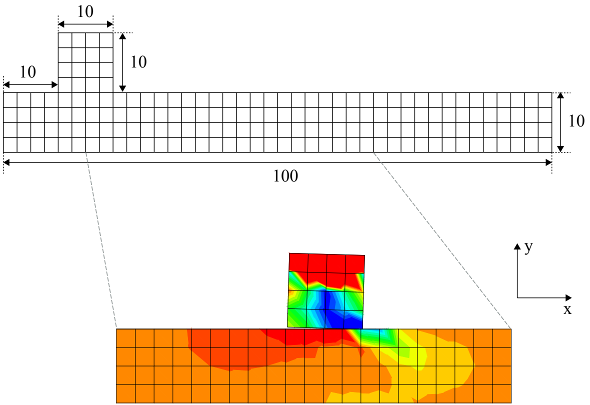
Data assimilation in an elastic friction model

Tom Gustafsson

20. September 2012

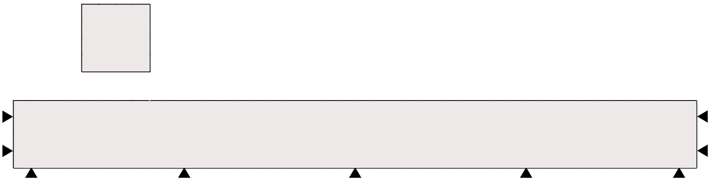
Question

- Is it possible to estimate weakly known parameters from a simple elastic friction model by using the tools of data assimilation?



Model, 2D

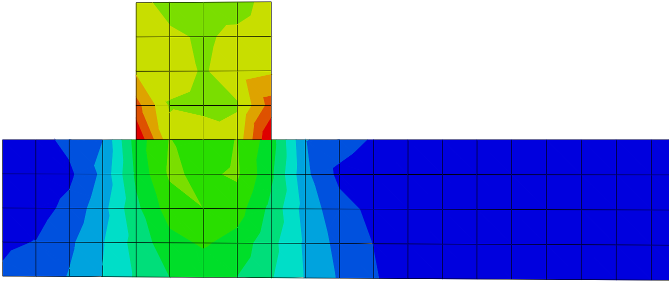
- Initial setting: Block, slab
- Boundary conditions



- Abaqus/Standard 6.12-1, simulations on CSC

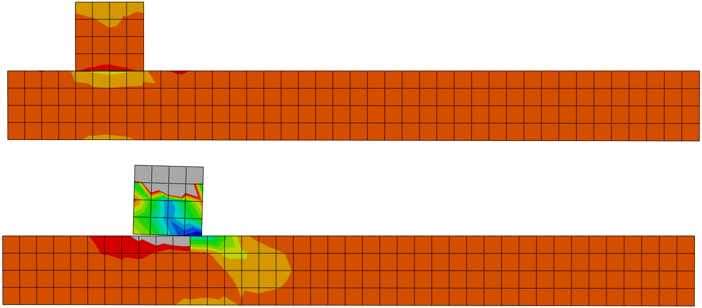
Steps of the simulation

- Step 1: 5 kN force acting downwards

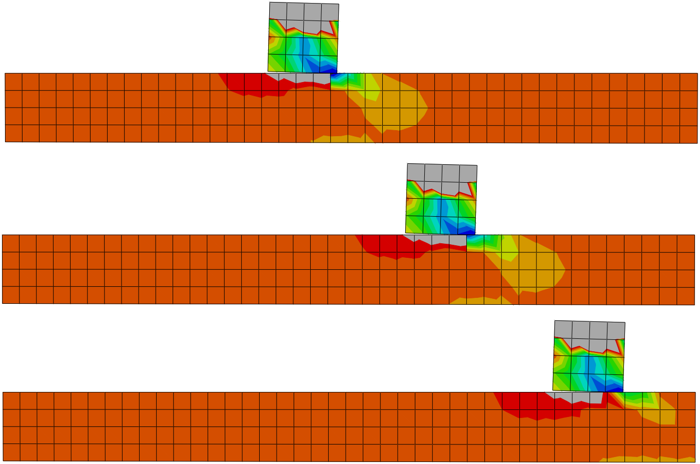


- Step 2: Displacement of block's upper boundary by 70 cm to the right

- Done by using boundary conditions. Thus, a "slow displacement"

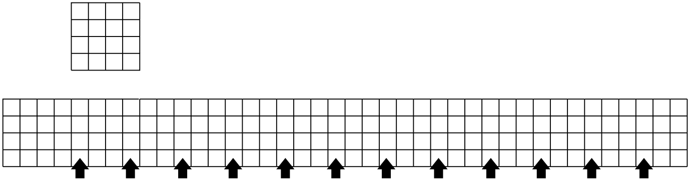


Steps of the simulation: Displacement of the block, 2



Inversion problem

- Attempting to estimate friction coefficient μ
- As *a priori* data: x directional stress in chosen measurement points
  ($\sim$ strain gauge)



Data assimilation

- Gives an answer to the problem: How to merge measurement and model data in an optimal way?
- Traditional applications: Weather prediction, oceanography
- Multiple different methods: 3D- and 4DVar, the family of Kalman filters, ...
- In this work *Ensemble Kalman Filter*

Ensemble Kalman Filter

- System is characterized by a bunch of state vectors $\psi \in \mathbb{R}^n$
- They estimate the same *true* state and their deviation characterizes the uncertainty
- This group of states is known as *ensemble*
- Each member of the ensemble is integrated forwards in time to the next measurement point
- After the time integration each state is merged with the measurement using

$$\psi^a = \psi^f + \mathbf{\Sigma}_\psi \mathbf{M}^{\mathrm{T}} \left(\mathbf{\Sigma}_d + \mathbf{M}\mathbf{\Sigma}_\psi \mathbf{M}^{\mathrm{T}}\right)^{-1} \left(\boldsymbol{d} - \mathbf{M}\psi^f\right)$$

Ensemble Kalman Filter, parametrien estimointi

- Nyt malli on

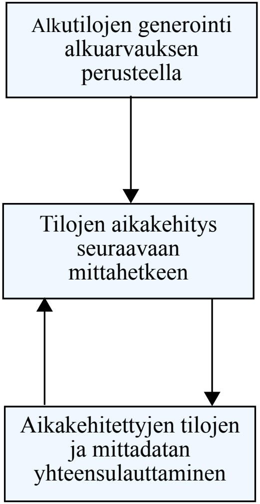
$$\dot{\psi} = \boldsymbol{G}(\psi, t; \boldsymbol{\alpha})$$

- Käytännössä jatketaan tilaa parametreilla $\boldsymbol{\alpha}$

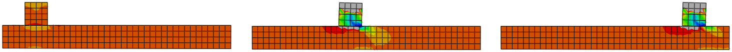
$$\hat{\psi}^{f} = \left(\psi^{f},\ \boldsymbol{\alpha}\right)^{\mathrm{T}}$$

- Karsitaan lisätyt parametrit vertailuista mitattujen arvojen kanssa muokkaamalla mittamatriisia
- $\rightarrow$ Estimoitavat parametrit loksahtavat kohdalleen ratkaistaessa analyysiongelma

Ensemble Kalman Filter, yhteenveto



- Malli



- Estimoitava parametri μ
- Määritellään tilaksi

$$\psi = (\sigma_x^1, \sigma_x^2, \sigma_x^3, \ldots, \sigma_x^N, \mu)^{\mathrm{T}}$$

- Alussa ei kosketusta $\Rightarrow$ jännitykset nollia
- Alkutilan määrää ainoastaan siis μ_0

Takaisin ongelmaan 2

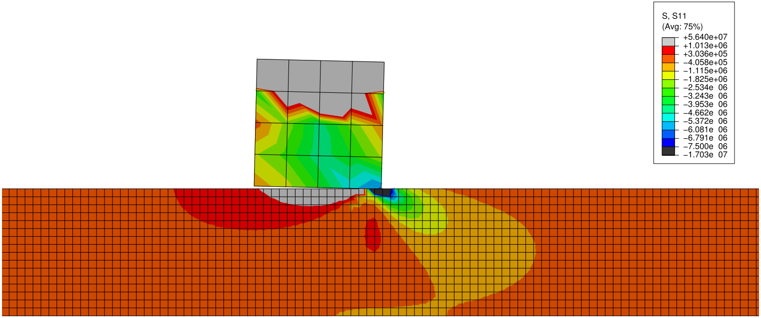
- Tarvitaan
  - Alkuarvaus $\mu_0 = 0{,}6$
  - Alkuarvauksen virhe $\sigma_0 = 0{,}1$
  - Kokoelman koko $n = 200$
  - Alkukokoelma jakaumasta $\mathcal{N}(\mu_0, \sigma_0^2)$
- Alkukokoelman yksittäinen tila on siis muotoa

$$\psi_j = (\underbrace{0, 0, \dots, 0, 0,}_{N \text{ kpl}} \mu_0 + \epsilon)^{\mathrm{T}}, \; j = 1, \dots, n$$

- Mitta"hetket": Yläreunan siirtymät $\Delta x = 7, 14, 21, \dots, 70$
- Mittadata synteettisesti

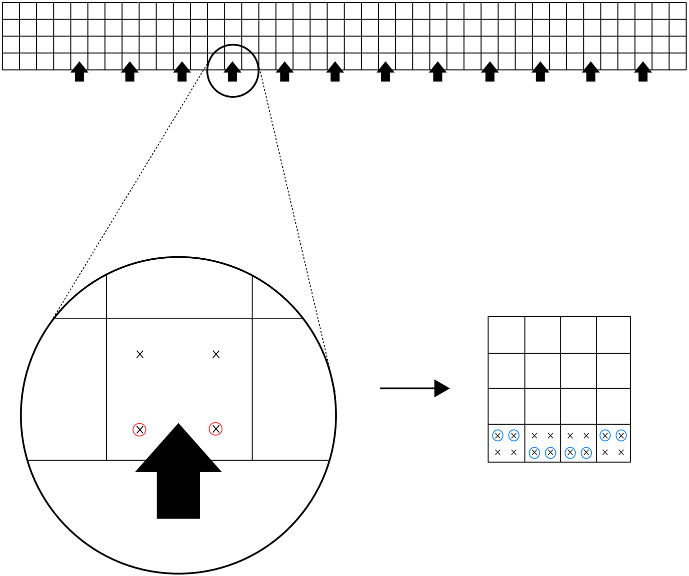
Synteettisen mittadatan generointi

- Minimoidaan inversiorikosta $\rightarrow$ mittadata tiheämmästä verkosta

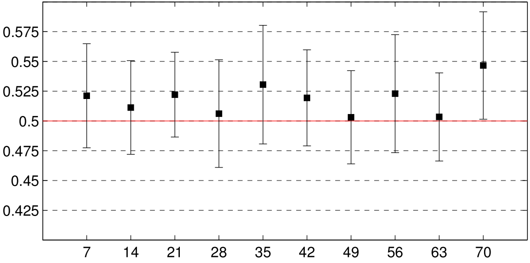


- Miten verrata tiheämmän ja harvemman verkon antamia jännityksiä?

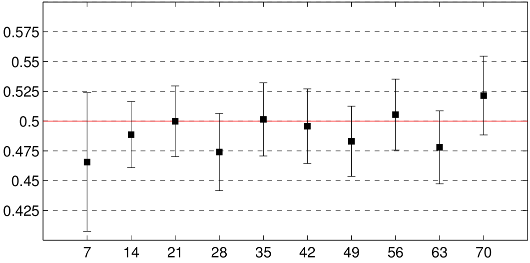
Synteettisen mittadatan generointi 2



Tuloksia, $Q = 0$

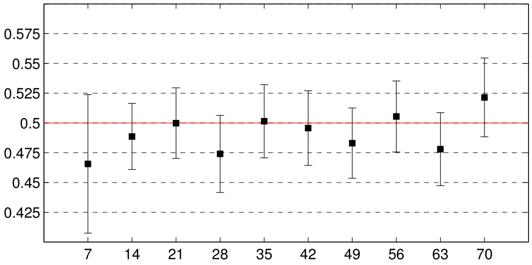


$n = 20$

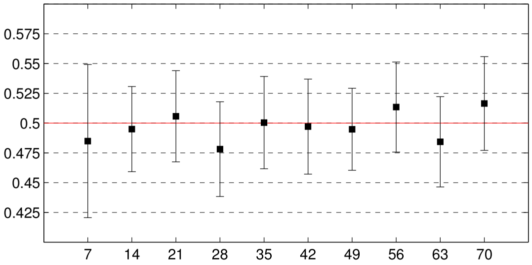


$n = 200$

Tuloksia, mallivirheen vaikutus, $n = 200$



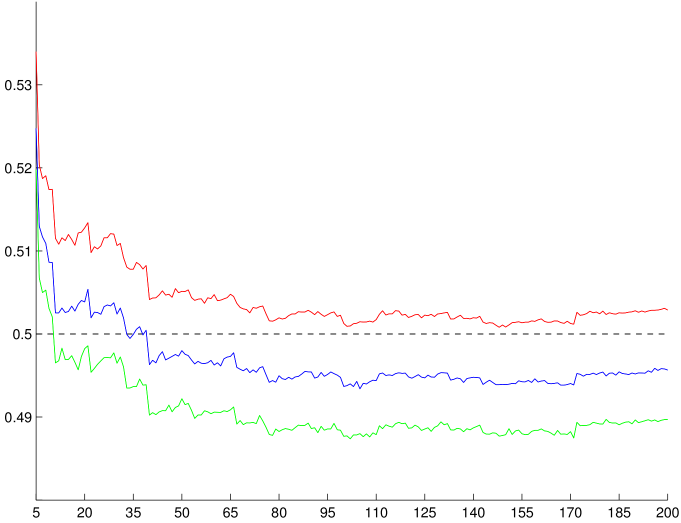
$\mathbf{Q} = \mathbf{0}$



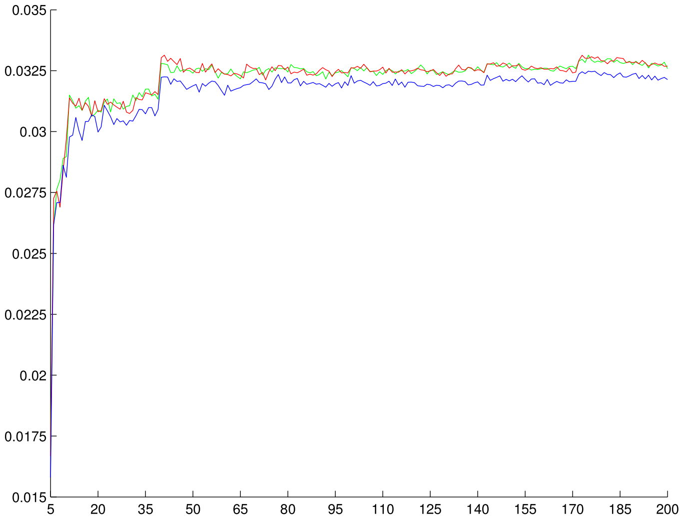
$\mathbf{Q} = \mathbf{I}\sigma$

Kokoelman koon vaikutus, keskiarvo

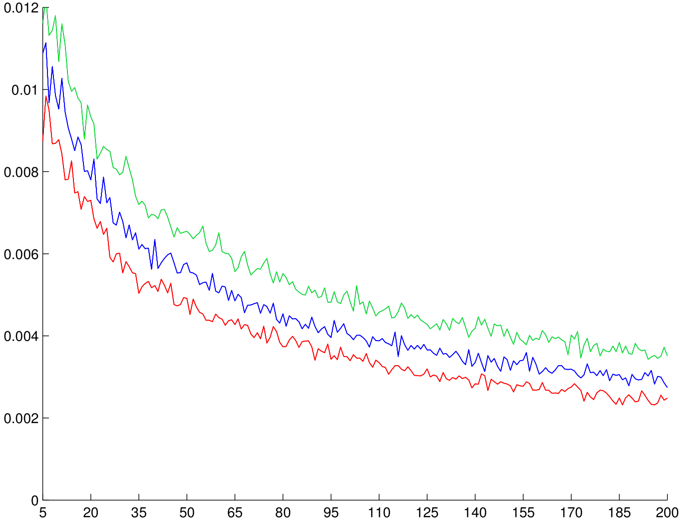
Punainen: $\Delta x = 70$, sininen: $\Delta x = 42$, vihreä: $\Delta x = 14$



Kokoelman hajonta



Peräkkäisten analyysien varianssi



Kysymyksiä?