

Homework 2

CMPSC 360

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Question 1:

1. **This is a statement** because the truth value of the statement can be determined: Obama was the president during 2010 or not
2. **This is not a statement** because x is an arbitrary value, which means the truth value of this statement cannot be determined.
3. **This is a statement** because 15 is either an odd number or it is not
4. **This is a statement** because a natural number times two has the possibility of being an even number - the truth value can be determined
5. **This is not a statement** because it is an open ended question. There is no truth value that can be determined.

Question 2:

- a) If $1 + 1 = 3 \rightarrow F$, then dogs can fly $\rightarrow F \Rightarrow T$
- b) If $1 + 1 = 2 \rightarrow T$, then dogs can fly $\rightarrow F \Rightarrow F$
- c) If $2 + 2 = 4 \rightarrow T$, then $1 + 2 = 3 \rightarrow T \Rightarrow T$

Question 3:

1. p = the sun is shining, q = I will play tennis this afternoon $\Rightarrow p \rightarrow q$
2. p = finishing the writing of my computer program before lunch, q = tennis playing this afternoon $\Rightarrow q \rightarrow p$
3. p = low humidity, q = sunshine, r = I will play tennis this afternoon $\Rightarrow (p \wedge q) \rightarrow r$

Question 4:

- a) The election is not decided and the votes have been counted
- b) If the votes have not been counted then the election is not decided
- c) The votes have not been counted or the election is not decided and the votes have been counted

Question 5:

Table 1: $\neg(p \rightarrow q) \equiv p \wedge \neg q$

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$p \wedge \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Table 2: $p \vee \neg(p \wedge q) \equiv T$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Question 6:

- a) $r \rightarrow p$
- b) $(p \wedge \neg q) \wedge r$
- c) $(p \wedge q) \rightarrow r$
- d) $r \leftrightarrow (q \vee p)$

Question 7:

1. $\neg[(p \wedge q) \rightarrow r]$
 $\neg[\neg(p \wedge q) \vee r]$ Conditional Equivalence
 $\neg\neg(p \wedge q) \wedge \neg r$ DeMorgan's Law
 $(p \wedge q) \wedge \neg r$ Double Negation

2. $\neg[p \rightarrow (\neg q \wedge r)]$
 $\neg[\neg p \vee (\neg q \wedge r)]$ Conditional Equivalence
 $\neg\neg p \wedge \neg(\neg q \wedge r)$ DeMorgan's Law
 $p \wedge \neg(\neg q \wedge r)$ Double Negation
 $p \wedge (\neg\neg q \vee \neg r)$ DeMorgan's Law
 $p \wedge (q \vee \neg r)$ Double Negation
 $p \wedge (r \rightarrow q)$ Conditional Equivalence

3. $\neg[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$
 $\neg p \wedge \neg q \wedge \neg(\neg p \wedge \neg q \wedge r)$ DeMorgan's Law
 $\neg p \wedge \neg q \wedge (\neg\neg p \vee \neg\neg q \vee \neg r)$ DeMorgan's Rule
 $(\neg p \wedge \neg q) \wedge (\neg\neg p \vee \neg\neg q \vee \neg r)$ Associative Rule
 $(\neg p \wedge \neg q) \wedge (p \vee q \vee \neg r)$ Double Negation
 $(\neg p \wedge \neg q \wedge p) \vee (\neg p \wedge \neg q \wedge q) \vee (\neg p \wedge \neg q \wedge \neg r)$ Distributive Rule
 $[(\neg p \wedge p) \wedge \neg q] \vee [\neg p \wedge (\neg q \wedge q)] \vee (\neg p \wedge \neg q \wedge \neg r)$ Associative Rule
 $(F \wedge \neg q) \vee (\neg p \wedge F) \vee (\neg p \wedge \neg q \wedge \neg r)$ Contradiction
 $F \vee F \vee (\neg p \wedge \neg q \wedge \neg r)$ Identity Rule
 $\neg p \wedge \neg q \wedge \neg r$ Identity Rules

Question 8:

Simplify $\neg((r \rightarrow \neg p) \wedge (r \rightarrow p))$

- $$\neg(r \rightarrow \neg p) \vee \neg(r \rightarrow p)$$
- DeMorgan's Law
- $$\neg(\neg r \vee \neg p) \vee \neg(\neg r \vee p)$$
- Conditional Equivalence
- $$(\neg\neg r \wedge \neg\neg p) \vee (\neg\neg r \wedge \neg p)$$
- DeMorgan's Law
- $$(r \wedge p) \vee (r \wedge \neg p)$$
- Double Negation
- $$r \wedge (p \vee \neg p)$$
- Distribution
- $$r \wedge T$$
- Tautology
- $$r$$
- Identity Rules

Question 9:

Prove that $[p \rightarrow (q \vee r)] \equiv [(p \wedge \neg q) \rightarrow r]$

$p \rightarrow (q \vee r)$	
$\neg p \vee (q \vee r)$	Conditional Equivalence
$(\neg p \vee q) \vee r$	Associative Rule
$\neg(\neg\neg p \wedge \neg q) \vee r$	DeMorgan's Law
$\neg(p \wedge \neg q) \vee r$	Double Negation
$(p \wedge \neg q) \rightarrow r$	Conditional Equivalence

$(p \wedge \neg q) \rightarrow r$	
$\neg(p \wedge \neg q) \vee r$	Conditional Equivalence
$(\neg p \vee \neg\neg q) \vee r$	DeMorgan's Law
$(\neg p \vee q) \vee r$	Double Negation
$\neg p \vee (q \vee r)$	Associative Rule
$p \rightarrow (q \vee r)$	Conditional Equivalence

Question 10:

1. $Q(1) \Rightarrow \text{False}$
2. $\neg P(3) \Rightarrow \text{False}$
3. $P(7) \vee Q(7) = F \vee F \Rightarrow \text{False}$
4. $P(3) \wedge Q(4) = T \wedge T \Rightarrow \text{True}$