Homework 3

CMPSC 360

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Question 1:

$\exists x \in \mathbb{Z}(x^2 = 5)$ is false because $x = \pm\sqrt{5}$ which is not an integer

Question 2:

1. The sum of two negative integers is always negative $\equiv \forall x \forall y((x < 0) \wedge (y > 0) \rightarrow (x + y < 0))$ is false

2. Which of the two statements are equal? (C and D)
   a) $\neg\exists x(A(x)) \equiv \forall x(\neg A(x))$
   b) $\neg\forall x(\neg A(x)) \equiv \exists x(A(x))$
   c) $\neg\forall x(A(x)) \equiv \exists x(\neg A(x))$
   d) $\exists x(\neg A(x))$

3. $\neg(P(x) \vee Q(x)) \equiv \neg P(x) \vee \neg Q(x)$ is false
   $\neg P(x) \wedge \neg Q(x)$

4. $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$ is true
   $\neg(\neg P \vee Q)$
   $P \wedge \neg Q$

Question 3:

Let $P(x, y, z) = x^2 + y^2 \geq z^2$
A) $\forall x \in (-3, 3) P(x, 4, 5) \Rightarrow$ True because $3^2 + 4^2 = 5^2$
B) $\forall w \neg P(w, w, w) \Rightarrow$ False because $1^2 + 1^2 \geq 1^2$
C) $\exists s(P(6, s, 10) \wedge P(s, 15, 17)) \Rightarrow$ True because $s = 100$, $6^2 + 100^2 \geq 10^2$ and $100^2 + 15^2 \geq 17^2$
D) $\forall t(P(6, t, 10) \vee P(t, 15, 17)) \Rightarrow$ False because $s = 1$, $6^2 + 1^2 \ngeq 10^2$ and $1^2 + 15^2 \ngeq 17^2$
E) $\forall \alpha(\neg P(\alpha, 1 - \alpha, 2\alpha) \vee P(\alpha, 1 - \alpha, 2\alpha)) \Rightarrow$ True because tautology identity

Question 4:

$\neg\forall x(\neg A(x) \wedge B(x))$
$\exists x \neg(\neg A(x) \wedge B(x))$ DeMorgan's Law on Universal Quantifier
$\exists x(\neg\neg A(x) \vee \neg B(x))$ DeMorgan's Law
$\exists x(A(x) \vee \neg B(x))$ Double Negation
$\exists x(B(x) \rightarrow A(x))$ Conditional Equivalence

Question 5:

Let $P(f, X) = \forall \varepsilon > 0 \exists \delta > 0 \forall x, y \in X |x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon$
so $\neg P = (\neg \forall \varepsilon > 0)(\neg \exists \delta > 0)(\neg \forall x, y \in X |x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon)$
$(\exists \neg(\varepsilon > 0))(\forall \neg(\delta > 0))(\exists x, y \notin X \neg(|x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon))$ DeMorgan's Law
$(\exists \neg(\varepsilon > 0))(\forall \neg(\delta > 0))(\exists x, y \notin X \neg((|x-y| < \delta) \vee \neg(|f(x)-f(y)| < \varepsilon)))$ Conditional Equivalence
$(\exists(\varepsilon \leq 0))(\forall(\delta \leq 0))(\exists x, y \notin X(\neg(|x - y| < \delta) \wedge \neg\neg(|f(x) - f(y)| < \varepsilon)))$ DeMorgan's Law
$(\exists(\varepsilon \leq 0))(\forall(\delta \leq 0))(\exists x, y \notin X(\neg(|x - y| < \delta) \wedge (|f(x) - f(y)| < \varepsilon)))$ Double Negation
$\exists(\varepsilon \leq 0)\forall(\delta \leq 0)\exists x, y \notin X((|x - y| \geq \delta) \wedge (|f(x) - f(y)| \geq \varepsilon))$ DeMorgan's Law

Question 6:

Domain is $\mathbb{R}$ and $P(x, y) = x * y = 1$

a) $\exists x \exists y(P(x, y)) \Rightarrow$ True
b) $\exists x \forall y(P(x, y)) \Rightarrow$ False
c) $\forall x \forall y(P(x, y)) \Rightarrow$ False
d) $\forall x \exists y(P(x, y)) \Rightarrow$ True

Question 7:

a) $\forall x < 0\ \forall y > 0\ (x * y < 0)$
b) $\forall x \in \mathbb{Z}(x$ is even $\vee\ x$ is odd$)$
c) $\forall x(P(x) \wedge R(x)) \rightarrow Q(x)$
d) $\neg \forall x A(x) \rightarrow (B(x) \vee C(x))$

Question 8:

a) Some animals in the world do not live in a forest
b) For all real numbers x, there is some real number y that satisfies the condition $y^3 = x$
c) For all integers x and y, the product of x and y will be an integer.
d) For all real numbers x, $x - x = 0$

Question 9:

a) For all y, there is some x for which x is less than y. — True
b) For all x greater than 0, there is some prime number y which is greater than x. — True
c) For all x and some y, x will be less than y and y will be prime. — True

Question 10:

$$\begin{array}{l} p \vee q \\ \underline{(p \wedge r) \to q} \\ r \end{array}$$

$(p \wedge r) \to q$
$(p \wedge r) \vee \neg q$
$(p \vee \neg q) \wedge (r \wedge \neg q)$

$(p \vee q) \wedge (p \vee \neg q) \wedge (r \vee \neg q)$