Homework 2

CMPSC 360

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Question 1:

1. This is a statement because the truth value of the statement can be determined: Obama was the president during 2010 or not
2. This is a statement becuase the quantity x + 3 could be a positive integer or not, which means the truth value can be determined
3. This is a statement because 15 is either an odd number or it is not
4. This is a statement becuase a natural number times two has the possibility of being an even number - the truth value can be determined
5. This is not a statement because it is an open ended question. There is no truth value that can be determined.

Question 2:

a) If $1 + 1 = 3 \rightarrow$ F, then dogs can fly $\rightarrow$ F $\Rightarrow$ T
b) If $1 + 1 = 2 \rightarrow$ T, then dogs can fly $\rightarrow$ F $\Rightarrow$ F
c) If $2 + 2 = 4 \rightarrow$ T, then $1 + 2 = 3 \rightarrow$ T $\Rightarrow$ T

Question 3:

1. p = the sun is shining, q = I will play tennis this afternoon $\therefore p \rightarrow q$
2. p = finishing the writing of my computer program before lunch, q = tennis playing this afternoon $\Rightarrow p \rightarrow q$
3. p = low humidity, q = sunshine, r = I will play tennis this afternoon $\Rightarrow (p \wedge q) \rightarrow r$

Question 4:

a) The election is not decided or the votes have been counted
b) If the votes have not been counted then the election is not decided
c) The election is not decided or the election is not decided and the votes have been counted

Question 5:

Table 1: $\neg(p \to q) \equiv p \wedge \neg q$

p	q	$p \to q$	$\neg(p \to q)$	$p \wedge \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Table 2: $p \vee \neg(p \wedge q) \equiv \mathrm{T}$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q))$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Question 6:

a) $p \leftrightarrow q$
b) $(p \wedge q)$
c) $(p \wedge q) \to r$
d) $r \leftrightarrow (q \vee p)$

Question 7:

1. $\neg[(p \wedge q) \to r]$
   $\neg[\neg(p \wedge q) \vee r]$ Identity Rule
   $\neg\neg(p \wedge q) \wedge \neg r$ DeMorgan's Law
   $(p \wedge q) \wedge \neg r$ Double Negation

2. $\neg[p \to (\neg q \wedge r)]$
   $\neg[\neg p \vee (\neg q \wedge r)]$ Identity Rule
   $\neg\neg p \wedge \neg(\neg q \wedge r)$ DeMorgan's Law
   $p \wedge \neg(\neg q \wedge r)$ Double Negation
   $p \wedge (\neg\neg q \vee \neg r)$ DeMorgan's Law
   $p \wedge (q \vee \neg r)$ Double Negation
   $p \wedge (r \to q)$ Identity Rule

3. $\neg[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$
   $\neg p \wedge \neg q \wedge \neg(\neg p \wedge \neg q \wedge r)$ DeMorgan's Law
   $\neg p \wedge \neg q \wedge (\neg\neg p \vee \neg\neg q \vee \neg r)$ DeMorgan's Law
   $\neg p \wedge \neg q \wedge (p \vee q \vee \neg r)$ Double Negation
   $\neg(p \vee q) \wedge (p \vee q \vee \neg r)$ DeMorgan's Law
   NOT DONE

Question 8:

Simplify $\neg((r \to \neg p) \wedge (r \to p))$

$\neg(r \to \neg p) \vee \neg(r \to p)$	DeMorgan's Law
$\neg(\neg r \vee \neg p) \vee \neg(\neg r \vee p)$	Identity Rule
$(\neg\neg r \wedge \neg\neg p) \vee (\neg\neg r \wedge \neg p)$	DeMorgan's Law
$(r \wedge p) \vee (r \wedge \neg p)$	Double Negation
$r \wedge (p \vee \neg p)$	Distribution
$r \wedge T$	Tautology
r	Identity Rules

Question 9:

Prove that $[p \to (q \vee r)] \equiv [(p \wedge \neg q) \to r]$

$p \to (q \vee r)$	
$\neg p \vee (q \vee r)$	Identity Rule
$(\neg p \vee q) \vee r$	Associative Rule
$\neg(\neg\neg p \wedge \neg q) \vee r$	DeMorgan's Law
$\neg(p \wedge \neg q) \vee r$	Double Negation
$(p \wedge \neg q) \to r$	Identity Rule

$(p \wedge \neg q) \to r$	
$\neg(p \wedge \neg q) \vee r$	Identity Rule
$(\neg p \vee \neg\neg q) \vee r$	DeMorgan's Law
$(\neg p \vee q) \vee r$	Double Negation
$\neg p \vee (q \vee r)$	Associative Rule
$p \to (q \vee r)$	Identity Rule

Question 10:

1. Q(1) $\Rightarrow$ False
2. $\neg$P(3) $\Rightarrow$ False
3. P(7) $\vee$ Q(7) = F $\vee$ F $\Rightarrow$ False
4. P(3) $\wedge$ Q(4) = T $\wedge$ T $\Rightarrow$ True