# Assignment 1

## CMPSC 360

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**Question 1**:

Since there are 8 such prime numbers that are less than 20 (2, 3, 5, 7, 11, 13, 17, 19) the power set of S has a cardinality of $2^8$, or 256 subsets

**Question 2**:

a. $\{a, b, \{c, d\}, e, f, g, h\}$
b. $\{a, b, \{c, d\}, e\}$
c. $\emptyset$
d. $\{f, g, h\}$

**Question 3**:

a) $\overline{A} \cup B$ by DeMorgan's Law
therefore, $\overline{A} = \{0, 1, 4, 5, 8, 9, 12, 15\}$, B = $\{1, 4, 5, 8, 9\}$
so $\{0, 1, 4, 5, 8, 9, 12, 15\} \cup \{1, 4, 5, 8, 9\}$
$= \underline{\{0, 1, 4, 5, 8, 9, 12, 15\}}$
b) $A \times (U - A - B)$
so $U - A = \{0, 1, 4, 5, 8, 9, 12, 15\}$
therefore, $U - A - B = \{0, 12, 15\}$
so, $A \times \{0, 12, 15\}$
$= \underline{\{(2, 0), (2, 12), (2, 15), (3, 0), (3, 12), (3, 15), (6, 0), (6, 12), (6, 15), (7, 0), (7, 12), (7, 15), (10, 0), (10, 12), (10, 15)\}}$

**Question 4**:

a) $\emptyset$
b) $\{9\}$

**Question 5**:

a) $\emptyset$
b) $C \cup B = \{0, 1, \alpha, \beta\}$
$\{0, 1, \alpha, \beta\} \cup A = \{0, 1, \alpha, \beta, X, Y, Z\}$
$B \times \{0, 1, \alpha, \beta, X, Y, Z\} = \{0, 1\} \times \{0, 1, \alpha, \beta, X, Y, Z\}$
$= \underline{\{(0, 0), (0, 1), (0, \alpha), (0, \beta), (0, X), (0, Y), (0, Z), (1, 0), (1, 1), (1, \alpha), (1, \beta), (1, X), (1, Y), (1, Z)\}}$

**Question 6**:

a) 19
b) 8

**Question 7**:

$A \times C = \{\alpha, \beta, \gamma\} \times \{6, 8\} = \{(\alpha, 6), (\alpha, 8), (\beta, 6), (\beta, 8), (\gamma, 6), (\gamma, 8)\}$
$B \times C = \{(c, 6), (c, 8), (d, 6), (d, 8), (\gamma, 6), (\gamma, 8)\}$
$\{(\alpha, 6), (\alpha, 8), (\beta, 6), (\beta, 8), (\gamma, 6), (\gamma, 8)\} \times \{(c, 6), (c, 8), (d, 6), (d, 8), (\gamma, 6), (\gamma, 8)\}$
therefore $(A \times C) \cap (B \times C) = \underline{\{(\gamma, 6), (\gamma, 8)\}}$

**Question 8**:

a) This is a proposition, True
b) This is a proposition, False
c) This is a proposition, True

**Question 9**: *Collaboration with Sharon Liu and Sahil Kuwadia*

Show that $\overline{\overline{A} \cap B} = A \cup \overline{B}$
Suppose x $\in \overline{\overline{A} \cap B}$
By definition of the compliment: $x \notin (\overline{A} \cap B)$
By definition of set intersection: $x \notin (\overline{A} \text{ } and \text{ } B)$
Applying DeMorgan's Law: $x \notin \overline{A} \text{ } or \text{ } x \notin B$
By definition of the compliment: $x \in A \text{ } or \text{ } x \in \overline{B}$
By definition of set union: $x \in (A \cup \overline{B})$
so $x \in A \cup \overline{B}$
therefore, $\overline{\overline{A} \cup B} = A \cup \overline{B}$

Suppose $x \in A \cup \overline{B}$
By definition of set union: $x \in A \text{ } or \text{ } \overline{B}$
By definition of distribution: $x \in A \text{ } orx \in \overline{B}$
By definition of the compliment: $x \notin \overline{A} \text{ } or \text{ } x \notin B$
Applying DeMorgan's Law: $x \notin (\overline{A} \text{ } and \text{ } B)$
By definition of set intersection: $x \notin (\overline{A} \cap B)$
By definition of the compliment: $x \in \overline{\overline{A} \cap B}$
therefore, $A \cup \overline{B} = \overline{\overline{A} \cup B}$

**Question 10**: *Collaboration with Sharon Liu and Sahil Kuwadia*

$(B - A) \cup (C - A) = (B \cup C) - A$
Suppose $x \in (B - A) \cup (C - A)$
By definition of set union: $x \in (B - A)$ *or* $(C - A)$
By definition of set difference: $(x \in B$ *and* $x \notin A)$ *or* $(x \in C$ *and* $x \notin A)$
By definition of distribution: $(x \in B$ *or* $x \in C)$ *and* $x \notin A$
By definition of distribution: $x \in (B$ *or* $C)$ *and* $x \notin A$
By definition of set intersection and set union: $x \in (B \cup C) \cap x \notin A$
By definition of the compliment: $x \in (B \cup C) \cap x \in \overline{A}$
By definition of distribution: $x \in (B \cup C) \cap \overline{A}$
By definition of set difference: $x \in (B \cup C) - A$
therefore, $(B - A) \cup (C - A) = (B \cup C) - A$

Suppose $x \in (B \cup C) - A$
By definition of set difference: $x \in (B \cup C) \cap \overline{A}$
By definition of distribution: $x \in (B \cup C) \cap x \in \overline{A}$
By definition of the compliment: $x \in (B \cup C) \cap x \notin A$
By definition of set intersection and set union: $x \in (B$ *or* $C)$ *and* $x \notin A$
By definition of distribution: $(x \in B$ *or* $x \in C)$ *and* $x \notin A$
By definition of distribution: $(x \in B$ *and* $x \notin A)$ *or* $(x \in C$ *and* $x \notin A)$
By definition of set intersection: $(x \in B \cap x \notin A)$ *or* $(x \in C \cap x \notin A)$
By definition of set difference: $x \in (B - A)$ *or* $(C - A)$
By definition of set union $x \in (B - A) \cup (C - A)$
therefore, $(B \cup C) - A = (B - A) \cup (C - A)$