Homework 2

CMPSC 360

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Question 1:

1. This is a statement because the truth value of the statement can be determined: Obama was the president during 2010 or not
2. This is not a statement becuase x is an arbitrary value, which means the truth value of this statement cannot be determined.
3. This is a statement because 15 is either an odd number or it is not
4. This is a statement becuase a natural number times two has the possibility of being an even number - the truth value can be determined
5. This is not a statement because it is an open ended question. There is no truth value that can be determined.

Question 2:

a) If $1 + 1 = 3 \rightarrow$ F, then dogs can fly $\rightarrow$ F $\Rightarrow$ T
b) If $1 + 1 = 2 \rightarrow$ T, then dogs can fly $\rightarrow$ F $\Rightarrow$ F
c) If $2 + 2 = 4 \rightarrow$ T, then $1 + 2 = 3 \rightarrow$ T $\Rightarrow$ T

Question 3:

1. p = the sun is shining, q = I will play tennis this afternoon $\Rightarrow$ $p \rightarrow q$
2. p = finishing the writing of my computer program before lunch, q = tennis playing this afternoon $\Rightarrow$ $q \rightarrow p$
3. p = low humidity, q = sunshine, r = I will play tennis this afternoon $\Rightarrow$ $(p \wedge q) \rightarrow r$

Question 4:

a) The election is not decided and the votes have been counted
b) If the votes have not been counted then the election is not decided
c) The votes have not been counted or the election is not decided and the votes have been counted

Question 5:

Table 1: $\neg(p \to q) \equiv p \wedge \neg q$

p	q	$p \to q$	$\neg(p \to q)$	$p \wedge \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Table 2: $p \vee \neg(p \wedge q) \equiv \mathrm{T}$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q))$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Question 6:

a) $r \to p$
b) $(p \wedge \neg q) \wedge r$
c) $(p \wedge q) \to r$
d) $r \leftrightarrow (q \vee p)$

Question 7:

1. $\neg[(p \wedge q) \rightarrow r]$

$\neg[\neg(p \wedge q) \vee r]$	Conditional Equivalence
$\neg\neg(p \wedge q) \wedge \neg r$	DeMorgan's Law
$(p \wedge q) \wedge \neg r$	Double Negation

2. $\neg[p \rightarrow (\neg q \wedge r)]$

$\neg[\neg p \vee (\neg q \wedge r)]$	Conditional Equivalence
$\neg\neg p \wedge \neg(\neg q \wedge r)$	DeMorgan's Law
$p \wedge \neg(\neg q \wedge r)$	Double Negation
$p \wedge (\neg\neg q \vee \neg r)$	DeMorgan's Law
$p \wedge (q \vee \neg r)$	Double Negation
$p \wedge (r \rightarrow q)$	Conditional Equivalence

3. $\neg[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$

$\neg p \wedge \neg q \wedge \neg(\neg p \wedge \neg q \wedge r)$	DeMorgan's Law
$\neg p \wedge \neg q \wedge (\neg\neg p \vee \neg\neg q \vee \neg r)$	DeMorgan's Rule
$(\neg p \wedge \neg q) \wedge (\neg\neg p \vee \neg\neg q \vee \neg r)$	Associative Rule
$(\neg p \wedge \neg q) \wedge (p \vee q \vee \neg r)$	Double Negation
$(\neg p \wedge \neg q \wedge p) \vee (\neg p \wedge \neg q \wedge q) \vee (\neg p \wedge \neg q \wedge \neg r)$	Distributive Rule
$[(\neg p \wedge p) \wedge \neg q] \vee [\neg p \wedge (\neg q \wedge q)] \vee (\neg p \wedge \neg q \wedge \neg r)$	Associative Rule
$(F \wedge \neg q) \vee (\neg p \wedge F) \vee (\neg p \wedge \neg q \wedge \neg r)$	Contradiction
$F \vee F \vee (\neg p \wedge \neg q \wedge \neg r)$	Identity Rule
$\neg p \wedge \neg q \wedge \neg r$	Identity Rules

Question 8:

Simplify $\neg((r \rightarrow \neg p) \wedge (r \rightarrow p))$

$\neg(r \rightarrow \neg p) \vee \neg(r \rightarrow p)$	DeMorgan's Law
$\neg(\neg r \vee \neg p) \vee \neg(\neg r \vee p)$	Identity Rule
$(\neg\neg r \wedge \neg\neg p) \vee (\neg\neg r \wedge \neg p)$	DeMorgan's Law
$(r \wedge p) \vee (r \wedge \neg p)$	Double Negation
$r \wedge (p \vee \neg p)$	Distribution
$r \wedge T$	Tautology
r	Identity Rules

Question 9:

Prove that $[p \to (q \vee r)] \equiv [(p \wedge \neg q) \to r]$

$p \to (q \vee r)$	
$\neg p \vee (q \vee r)$	Identity Rule
$(\neg p \vee q) \vee r$	Associative Rule
$\neg(\neg\neg p \wedge \neg q) \vee r$	DeMorgan's Law
$\neg(p \wedge \neg q) \vee r$	Double Negation
$(p \wedge \neg q) \to r$	Identity Rule

$(p \wedge \neg q) \to r$	
$\neg(p \wedge \neg q) \vee r$	Identity Rule
$(\neg p \vee \neg\neg q) \vee r$	DeMorgan's Law
$(\neg p \vee q) \vee r$	Double Negation
$\neg p \vee (q \vee r)$	Associative Rule
$p \to (q \vee r)$	Identity Rule

Question 10:

1. Q(1) $\Rightarrow$ False
2. $\neg$P(3) $\Rightarrow$ False
3. P(7) $\vee$ Q(7) = F $\vee$ F $\Rightarrow$ False
4. P(3) $\wedge$ Q(4) = T $\wedge$ T $\Rightarrow$ True