Homework 9

CMPSC 360

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Question 1: Show that if x is an odd integer, then x^2 has the form $8k+1$, for some $k \in \mathbb{Z}$

Question 2: Solve for $23^3 \pmod{30}$

Question 3: Show that if an integer n is not divisible by 3, then $n^2 - 1$ is always divisible by 3. Similarly, show that if an integer n is not divisible by 3, then $n^3 - 1 \equiv 0$

Question 4: Find GCD of 2947 and 3997 using Euclidean Theorem.

$$\begin{aligned}
3997 &= 2947(1) + 1050 \\
2947 &= 1050(2) + 847 \\
1050 &= 847(1) \quad + 203 \\
847 &= 203(4) \quad + 35 \\
203 &= 35(5) \quad + 28 \\
35 &= 28(1) \quad + 7 \\
28 &= 7(4) \quad + 0
\end{aligned}$$

So, $gcd(2947, 3997) = 7$

Question 5: Express $gcd(128469, 12818)$ as a linear combination of 128469 and 12818 using extended Euclid algorithm.

Applying Euclid's algorithm:

$$\begin{aligned}
128469 &= 12818(10) + 289 \\
12818 &= 289(44) \quad + 102 \\
289 &= 102(2) \quad + 85 \\
102 &= 85(1) \quad + 17 \\
85 &= 17(5) \quad + 0
\end{aligned}$$

i	r_i	r_{i+1}	q_{i+1}	r_{i+2}	s_i	t_i
0	128469	12818	10	289	1	0
1	12818	289	44	102	0	1
2	289	102	2	85	1	-10
3	102	85	1	17	-44	441
4	85	17	5	0	90	-891
5					-224	2223

-224 * 128469 + 2223 * 12818

Question 6: Prove that if $a \mid bc$ with $gcd(a, b) = 1$, then $a \mid c$

Question 7: Prove that $gcd(a^2, b^2) = gcd(a, b)^2$ using Bezout's identity.

Question 8: For Z_{11}, find out:

a) $3 \oplus 7$
b) $3 \otimes 7$
c) $10 \ominus 7$
d) $10 \oslash 7$

Question 9: Determine whether every element a of Z_n has an inverse for $n = 5, 6$ and $7, 11$

Question 10: Write the following decimal string 334_{10} to senary (base 6) showing work

$$\begin{aligned} 334 \div 6 &= 55 \;\; R\ 4 \\ 55 \div 6 &= 9 \quad R\ 1 \\ 9 \div 6 &= 1 \quad R\ 3 \\ 1 \div 6 &= 0 \quad R\ 1 \end{aligned}$$

So $334_{10} = 1314_6$.