Homework 2

CMPSC 360

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January 24, 2022

Question 1:

1. This is a statement because the truth value of the statement can be determined: Obama was the president during 2010 or not
2. This is a statement becuase the quantity x + 3 could be a positive integer or not, which means the truth value can be determined
3. This is a statement because 15 is either an odd number or it is not
4. This is a statement becuase a natural number times two has the possibility of being an even number - the truth value can be determined
5. This is not a statement because it is an open ended question. There is no truth value that can be determined.

Question 2:

a) If $1 + 1 = 3 \rightarrow$ F, then dogs can fly $\rightarrow$ F $\Rightarrow$ T
b) If $1 + 1 = 2 \rightarrow$ T, then dogs can fly $\rightarrow$ F $\Rightarrow$ F
c) If $2 + 2 = 4 \rightarrow$ T, then $1 + 2 = 3 \rightarrow$ T $\Rightarrow$ T

Question 3:

1. a

Question 4:

Question 5:

Table 1: $\neg (p \rightarrow q) \equiv p \wedge \neg q$

p	q	$p \rightarrow q$	$\neg (p \rightarrow q)$	$p \wedge \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Table 2: $p \vee \neg(p \wedge q) \equiv T$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Question 6:

Question 7:

1. $\neg[(p \wedge q) \rightarrow r]$
   $\neg[\neg(p \wedge q) \vee r]$ Identity Rule
   $\neg\neg(p \wedge q) \wedge \neg r$ DeMorgan's Law
   $(p \wedge q) \wedge \neg r$ Double Negation

2. $\neg[p \rightarrow (\neg q \wedge r)]$
   $\neg[\neg p \vee (\neg q \wedge r)]$ Identity Rule
   $\neg\neg p \wedge \neg(\neg q \wedge r)$ DeMorgan's Law
   $p \wedge \neg(\neg q \wedge r)$ Double Negation
   $p \wedge (\neg\neg q \vee \neg r)$ DeMorgan's Law
   $p \wedge (q \vee \neg r)$ Double Negation
   $p \wedge (r \rightarrow q)$ Identity Rule

3. $\neg[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$
   $\neg p \wedge \neg q \wedge \neg(\neg p \wedge \neg q \wedge r)$ DeMorgan's Law
   $\neg p \wedge \neg q \wedge (\neg\neg p \vee \neg\neg q \vee \neg r)$ DeMorgan's Law
   $\neg p \wedge \neg q \wedge (p \vee q \vee \neg r)$ Double Negation
   $\neg(p \vee q) \wedge (p \vee q \vee \neg r)$ DeMorgan's Law
   NOT DONE

Question 8:

Simplify $\neg((r \rightarrow \neg p) \wedge (r \rightarrow p))$

$\neg(r \rightarrow \neg p) \vee \neg(r \rightarrow p)$ DeMorgan's Law
$\neg(\neg r \vee \neg p) \vee \neg(\neg r \vee p)$ Identity Rule
$(\neg\neg r \wedge \neg\neg p) \vee (\neg\neg r \wedge \neg p)$ DeMorgan's Law
$(r \wedge p) \vee (r \wedge \neg p)$ Double Negation
$r \vee (p \wedge \neg p)$ Distribution
$r \vee F$ Contradiction
r Identity Rules

Question 9:

Prove that $[p \to (q \vee r)] \equiv [(p \wedge \neg q) \to r]$

$p \to (q \vee r)$	
$\neg p \vee (q \vee r)$	Identity Rule
$(\neg p \vee q) \vee r$	Associative Rule
$\neg(\neg\neg p \wedge \neg q) \vee r$	DeMorgan's Law
$\neg(p \wedge \neg q) \vee r$	Double Negation
$(p \wedge \neg q) \to r$	Identity Rule

$(p \wedge \neg q) \to r$	
$\neg(p \wedge \neg q) \vee r$	Identity Rule
$(\neg p \vee \neg\neg q) \vee r$	DeMorgan's Law
$(\neg p \vee q) \vee r$	Double Negation
$\neg p \vee (q \vee r)$	Associative Rule
$p \to (q \vee r)$	Identity Rule

Question 10:

1. Q(1) $\Rightarrow$ False
2. $\neg$P(3) $\Rightarrow$ False
3. P(7) $\vee$ Q(7) = F $\vee$ F $\Rightarrow$ False
4. P(3) $\wedge$ Q(4) = T $\wedge$ T $\Rightarrow$ True