Homework 4

CMPSC 360

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February 7, 2022

Question 1:

NEWS_KIT = I was reading the newspaper in the kitchen	H1: NEWS_KIT $\rightarrow$ GLASS_KIT
GLASS_KIT = My glasses are on the kitchen table	H2: GLASS_KIT $\rightarrow$ GLASS_BREAK
GLASS_BREAK = I saw my glasses at breakfast	H3: $\neg$ GLASS_BREAK
NEWS_LIV = I was reading the newspaper in the living room	H4: NEWS_LIV $\vee$ NEWS_KIT
GLASS_COFF = My glasses are on the coffee table	H5: NEWS_LIV $\rightarrow$ GLASS_COFF

1. GLASS_KIT $\rightarrow$ GLASS_BREAK	H2
2. $\neg$GLASS_BREAK	H3
3. $\neg$GLASS_KIT	Modus Tollens on 1 and 2
4. NEWS_KIT $\rightarrow$ GLASS_KIT	H1
5. $\neg$NEWS_KIT	Modus Tollens on 4 and 3
6. NEWS_LIV $\vee$ NEWS_KIT	H4
7. NEWS_LIV	Disjunctive Syllogism on 6 and 5
8. NEWS_LIV $\rightarrow$ GLASS_COFF	H5
9. GLASS_COFF	Modus Ponens on 8 and 7

Therefore, the glasses are at on the coffee table.

Question 2:

H1: $(\neg v \vee \neg p) \rightarrow (s \wedge z)$
H2: $s \rightarrow o$
H3: $\neg o$
C: v

1. $s \rightarrow o$	H2
2. $\neg o$	H3
3. $\neg s$	Modus Tollens on 1 and 2
4. $(\neg v \vee \neg p) \rightarrow (s \wedge z)$	H1
5. $\neg v \rightarrow (s \wedge z)$	Additive rule on 4
6. $\neg v \rightarrow s$	Simplification of 5
7. $\neg\neg v$	Modus Tollens of 6 and 3
8. v	Double negation on 7

Question 3:

1. This is not a valid argument (a^2 is positive, but a could be $\pm a$)
2. This is a valid argument (the only solution for $\sqrt{0^2}$ is 0)

Question 4:

$\mathrm{P}(x)$ = if x has taken CMPSC-360, then they can take CMPSC-465 next semester

$$\frac{\forall x \mathrm{P}(x)}{\therefore \mathrm{P}(c) \text{ if } c \in \mathrm{U}}$$

The argument is valid because of universal instantiation.

Question 5:

Proof:
Suppose $\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6}$, and we know that $n \in \mathbb{N}$

$$\begin{aligned}
\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6} &= \frac{2n}{6} + \frac{3n^2}{6} + \frac{n^3}{6} && \text{using algebra} \\
&= \frac{n}{6}(2 + 3n + n^2) && \text{simplifying the equation} \\
&= \frac{1}{6}n(n+1)(n+2) && \text{factoring}
\end{aligned}$$

By definition of divides, $n(n+1)(n+2) = 6c$ where $c \in \mathbb{N}$
A natural number n can either be 1 or $n + 1$ where $n \geq 1$

Case 1: $n = 1$

$$\begin{aligned}
n(n+1)(n+2) &= (1)(1+1)(1+2) && \text{substitute 1 for } n \\
&= (1)(2)(3) && \text{addition} \\
&= 6 && \text{multiplication}
\end{aligned}$$

therefore, $6 = 6c$, which simplifies to c = 1.
Since $1 \in \mathbb{N}$, the case $n = 1$ is validated

Case 2: $n = n + 1$

$$\begin{aligned}
n(n+1)(n+2) &= (n+1) \cdot [(n+1)+1] \cdot [(n+1)+2] && \text{plugging in } n+1 \text{ for } n \\
&= (n+1)(n+2)(n+3) && \text{addition} \\
&= n^3 + 6n^2 + 11n + 6 && \text{algebra} \\
&= (n^3 + 3n^2 + 2n) + (3n^2 + 9n + 6) && \text{algebra} \\
&= n(n+1)(n+2) + 3(n+1)(n+2) && \text{factoring} \\
&= 6c + 3(n+1)(n+2) && \text{substitute } n(n+1)(n+2) \text{ for } 6c
\end{aligned}$$

Case 2a: n is even, such that $n = 2a$ for some $a \in \mathbb{N}$

$$\begin{aligned}
6c + 3(n+1)(n+2) &= 6c + 3(2a+1)(2a+2) && \text{plugging in } 2a \text{ for } n \\
&= 6c + 3(2a+1)[2(a+1)] && \text{factoring out 2} \\
&= 6c + 6(2a+1)(a+1) && \text{multiplication} \\
&= 6[c + (2a+1)(a+1)] && \text{factoring out 6} \\
&= 6 \cdot k_1 \text{ for some } k_1 \in \mathbb{N} \text{ where } k_1 = [c + (2a+1)(a+1)]
\end{aligned}$$

Therefore, by definition of an even number, $6c + 3(n+1)(n+2)$ is divisible by 6

Case 2b: n is odd, such that $n = 2a + 1$ for some $a \in \mathbb{N}$

$$\begin{aligned}
6c + 3(n+1)(n+2) &= 6c + 3(2a+1+1)(2a+1+2) && \text{plugging in } 2a+1 \text{ for } n \\
&= 6c + 3(2a+2)(2a+3) && \text{addition} \\
&= 6c + 3[2(a+1)](2a+3) && \text{factoring out 2} \\
&= 6c + 6(a+1)(2a+3) && \text{multiplication} \\
&= 6[c + (a+1)(2a+3)] && \text{factoring out 6} \\
&= 6 \cdot k_2 \text{ for some } k_2 \in \mathbb{N} \text{ where } k_2 = [c + (a+1)(2a+3)]
\end{aligned}$$

Therefore, by definition of an odd number, $6c + 3(n+1)(n+2)$ is divisible by 6

We notice that $n(n+1)(n+2)$ where $n = n + 1$ is divisible by 6 in both cases

We notice that $n(n+1)(n+2)$ is divisible by 6 in both cases.
Therefore, for all natural numbers n, $\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6}$ is a natural number. $\square$

Question 6:

i) The assumption is that n is an odd natural number
ii) We want to conclude that $n^2 - 1$ is always divisible by 8
Proof:
Suppose $n^2 - 1$, and n is an odd natural number
By definition of odd, we know that $n = 2a + 1$ such that $a \in \mathbb{N}$

$$\begin{aligned} n^2 - 1 &= (2a+1)^2 - 1 && \text{plugging in } 2a+1 \text{ for } n \\ &= 4a^2 + 4a + 1 - 1 && \text{algebra} \\ &= 4a^2 + 4a && \text{subtraction} \\ &= 4a(a+1) && \text{factoring out } 4a \end{aligned}$$

A natural number a can be either an even or odd natural number

Case 1: Suppose a is an even natural number. By definition of even we get $a = 2x; x \in \mathbb{N}$

$$\begin{aligned} 4a(a+1) &= 4(2x)(2x+1) && \text{plugging in } 2x \text{ for } a \\ &= 8x(2x+1) && \text{multiplication} \\ &= 8 \cdot k \text{ for some } k \in \mathbb{N} \text{ where } k = x(2x+1) \end{aligned}$$

Therefore, when a is even, the function is divisible by 8

Case 2: Suppose a is an odd natural number. By definition of odd we get $a = 2x + 1; x \in \mathbb{N}$

$$\begin{aligned} 4a(a+1) &= 4(2x+1)(2x+1+1) && \text{plugging in } 2x+1 \text{ for } a \\ &= 4(2x+1)(2x+2) && \text{addition} \\ &= 4(2x+1)[2(x+1)] && \text{addition} \\ &= 8(2x+1)(x+1) && \text{multiplication} \\ &= 8 \cdot k \text{ for some } k \in \mathbb{N} \text{ where } k = (2x+1)(x+1) \end{aligned}$$

Therefore, when a is odd, the function is divisible by 8

We notice that $4a(a + 1)$ is divisible by 8 for both cases.
Therefore, $n^2 - 1$ is always divisible by 8 when n is an odd natural number $\square$

Question 7:

i) We know that $m \in \mathbb{Z}$
ii) Assume that m is odd
iii) Prove $3m + 7$ is even

Proof:
Suppose $m \in \mathbb{Z}$, and we know that m is odd
By definition of odd, we know that $m = 2a + 1$ such that $a \in \mathbb{Z}$

$$\begin{aligned} 3m + 7 &= 3(2a+1) + 7 && \text{plugging in } 2a+1 \text{ for } m \\ &= 6a + 3 + 7 && \text{multiplication} \\ &= 6a + 10 && \text{addition} \\ &= 2(3a+5) && \text{factoring out } 2 \\ &= 2 \cdot k \text{ for some } k \in \mathbb{Z} \text{ where } k = 3a + 5 \end{aligned}$$

Therefore, by the definition of an even number, $3m + 7$ is even when m is odd. $\square$

Question 8:

Prove that the product of any two odd integers is odd.
Proof:
Suppose $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ such that a and b are odd
By definition of odd, $a = 2x + 1$ such that $x \in \mathbb{Z}$ and $b = 2y + 1$ such that $y \in \mathbb{Z}$
so, $a \cdot b = (2x + 1)(2y + 1)$ substituting a for $2x + 1$ and b for $2y + 1$
$= 4xy + 2x + 2y + 1$ multiplication
$= 2(2xy + x + y) + 1$ factoring out 2
$= 2k + 1$ for some $k \in \mathbb{Z}$ where $k = 2xy + x + y$
Therefore, by the definition of an odd number, $a \cdot b$ is odd
Therefore, the product of any two odd integers is odd. $\square$