Homework 3

CMPSC 360

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Question 1:

$\exists x \in \mathbb{Z}(x^2 = 5)$ is false because $x = \pm\sqrt{5}$ which is not an integer

Question 2:

1. The sum of two negative integers is always negative $\equiv \forall x \forall y((x < 0) \wedge (y > 0) \rightarrow (x + y < 0))$ is false

2. Which of the two statements are equal? (C and D)
   a) $\neg\exists x(A(x)) \equiv \forall x(\neg A(x))$
   b) $\neg\forall x(\neg A(x)) \equiv \exists x(A(x))$
   c) $\neg\forall x(A(x)) \equiv \exists x(\neg A(x))$
   d) $\exists x(\neg A(x))$

3. $\neg(P(x) \vee Q(x)) \equiv \neg P(x) \vee \neg Q(x)$ is false
   $\neg P(x) \wedge \neg Q(x)$

4. $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$ is true
   $\neg(\neg P \vee Q)$
   $P \wedge \neg Q$

Question 3:

Let $P(x, y, z) = x^2 + y^2 \geq z^2$
A) $\forall x \in (-3, 3)P(x, 4, 5) \Rightarrow$ False because $1^2 + 4^2 < 5^2$
B) $\forall w \neg P(w, w, w) \Rightarrow$ False because $1^2 + 1^2 \geq 1^2$
C) $\exists s(P(6, s, 10) \wedge P(s, 15, 17)) \Rightarrow$ True because $s = 100$, $6^2 + 100^2 \geq 10^2$ and $100^2 + 15^2 \geq 17^2$
D) $\forall t(P(6, t, 10) \vee P(t, 15, 17)) \Rightarrow$ False because $s = 1$, $6^2 + 1^2 \ngeq 10^2$ and $1^2 + 15^2 \ngeq 17^2$
E) $\forall \alpha(\neg P(\alpha, 1 - \alpha, 2\alpha) \vee P(\alpha, 1 - \alpha, 2\alpha)) \Rightarrow$ True because tautology identity

Question 4:

$\neg\forall x(\neg A(x) \wedge B(x))$	
$\exists x \neg(\neg A(x) \wedge B(x))$	DeMorgan's Law on Universal Quantifier
$\exists x(\neg\neg A(x) \vee \neg B(x))$	DeMorgan's Law
$\exists x(A(x) \vee \neg B(x))$	Double Negation
$\exists x(B(x) \rightarrow A(x))$	Conditional Equivalence

Question 5:

Let $P(f, X) = \forall \varepsilon > 0 \exists \delta > 0 \forall x, y \in X |x - y| < \delta \to |f(x) - f(y)| < \varepsilon$
so $\neg P = \neg(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x, y \in X |x - y| < \delta \to |f(x) - f(y)| < \varepsilon)$

$\exists \varepsilon > 0\ (\neg \exists \delta > 0)(\forall x, y \in X |x - y| < \delta \to |f(x) - f(y)| < \varepsilon)$	DeMorgan's Law
$\exists \varepsilon > 0\ \forall \delta > 0\ (\neg \forall x, y \in X |x - y| < \delta \to |f(x) - f(y)| < \varepsilon)$	DeMorgan's Law
$\exists \varepsilon > 0\ \forall \delta > 0\ \exists x, y \in X\ \neg[(|x - y| < \delta) \to (|f(x) - f(y)| < \varepsilon)]$	DeMorgan's Law
$\exists \varepsilon > 0\ \forall \delta > 0\ \exists x, y \in X\ \neg[\neg(|x - y| < \delta) \vee (|f(x) - f(y)| < \varepsilon)]$	Conditional Equivalence
$\exists \varepsilon > 0\ \forall \delta > 0\ \exists x, y \in X\ [\neg\neg(|x - y| < \delta) \wedge \neg(|f(x) - f(y)| < \varepsilon)]$	DeMorgan's Law
$\exists \varepsilon > 0\ \forall \delta > 0\ \exists x, y \in X\ [(|x - y| < \delta) \wedge \neg(|f(x) - f(y)| < \varepsilon)]$	Double Negation
$\exists \varepsilon > 0\ \forall \delta > 0\ \exists x, y \in X\ [(|x - y| < \delta) \wedge (|f(x) - f(y)| \geq \varepsilon)]$	DeMorgan's Law

Question 6:

Domain is $\mathbb{R}$ and $P(x, y) = x * y = 1$

a) $\exists x \exists y\ [P(x, y)] \Rightarrow$ True
b) $\exists x \forall y\ [P(x, y)] \Rightarrow$ False
c) $\forall x \forall y\ [P(x, y)] \Rightarrow$ False
d) $\forall x \exists y\ [P(x, y)] \Rightarrow$ True

Question 7:

a) $\forall x \in \mathbb{Z}, x < 0\ \forall y \in \mathbb{Z}, y > 0\ (x * y < 0)$
b) $E(x) = x$ is even, $O(x) = x$ is odd
$\forall x \in \mathbb{Z}\ (E(x) \vee O(x))$
c) $\forall x\ [P(x) \wedge R(x)] \to Q(x)$
d) $\neg \forall x\ A(x) \to [B(x) \vee C(x)]$

Question 8:

a) Some animals in the world do not live in a forest
b) For all real numbers x, there is some real number y such that $y^3 = x$
c) For all integers x and y, the product of x and y will be an integer.
d) For all real numbers x, the difference of x and x is 0.

Question 9:

a) For all integer y, there is some integer x such that x is less than y. — True
b) For all integer x and some integer y, such that if x is positive, then y is a prime number and x is less than y — True
c) For all integer x, there is some integer y such that x will be less than y and y will be prime. — True

Question 10:

$$\begin{array}{c} p \lor q \\ \underline{(p \land r) \to q} \\ \therefore r \end{array}$$

p	q	r	$p \lor q$	$p \land r$	$(p \land r) \to q$	$(p \lor q) \land [(p \land r) \to q]$	$(p \lor q) \land [(p \land r) \to q] \to r$
F	F	F	F	F	T	F	T
F	F	T	F	F	T	F	T
F	T	F	T	F	T	T	F
F	T	T	T	F	T	T	T
T	F	F	T	F	T	T	F
T	F	T	T	T	F	F	T
T	T	F	T	F	T	T	F
T	T	T	T	T	T	T	T

This is not a valid argument because it is not a tautology. We can prove this using the truth table. The lines with the red highlights show the truth values where the argument is not valid.