Homework 3

CMPSC 360

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Question 1:

$\exists x \in \mathbb{Z}(x^2 = 5)$ is false because $x = \pm\sqrt{5}$ which is not an integer

Question 2:

1. The sum of two negative integers is always negative $\equiv \forall x \forall y((x < 0) \land (y > 0) \to (x + y < 0))$ is false

2. Which of the two statements are equal? (C and D)
   a) $\neg\exists x(A(x)) \equiv \forall x(\neg A(x))$
   b) $\neg\forall x(\neg A(x)) \equiv \exists x(A(x))$
   c) $\neg\forall x(A(x)) \equiv \exists x(\neg A(x))$
   d) $\exists x(\neg A(x))$

3. $\neg(P(x) \lor Q(x)) \equiv \neg P(x) \lor \neg Q(x)$ is false
   $\neg P(x) \land \neg Q(x)$

4. $\neg(P \to Q) \equiv P \land \neg Q$ is true
   $\neg(\neg P \lor Q)$
   $P \land \neg Q$

Question 3:

Let $P(x, y, z) = x^2 + y^2 \geq z^2$
A) $\forall x \in (-3, 3)P(x, 4, 5) \Rightarrow$ True because $3^2 + 4^2 = 5^2$
B) $\forall w \neg P(w, w, w) \Rightarrow$ False because $1^2 + 1^2 \geq 1^2$
C) $\exists s(P(6, s, 10) \land P(s, 15, 17)) \Rightarrow$ True because $s = 100$, $6^2 + 100^2 \geq 10^2$ and $100^2 + 15^2 \geq 17^2$
D) $\forall t(P(6, t, 10) \lor P(t, 15, 17)) \Rightarrow$ False because $s = 1$, $6^2 + 1^2 \ngeq 10^2$ and $1^2 + 15^2 \ngeq 17^2$
E) $\forall \alpha(\neg P(\alpha, 1 - \alpha, 2\alpha) \lor P(\alpha, 1 - \alpha, 2\alpha)) \Rightarrow$ True because tautology identity

Question 4:

$\neg\forall x(\neg A(x) \land B(x))$
$\exists x \neg(\neg A(x) \land B(x))$ DeMorgan's Law on Universal Quantifier
$\exists x(\neg\neg A(x) \lor \neg B(x))$ DeMorgan's Law
$\exists x(A(x) \lor \neg B(x))$ Double Negation
$\exists x(B(x) \to A(x))$ Conditional Equivalence

Question 5:

Let $P(f, X) = \forall\varepsilon > 0\exists\delta > 0\forall x, y \in X|x - y| < \delta \to |f(x) - f(y)| < \varepsilon$
so $\neg P = (\neg\forall\varepsilon > 0)(\neg\exists\delta > 0)(\neg\forall x, y \in X|x - y| < \delta \to |f(x) - f(y)| < \varepsilon)$
$(\exists\neg(\varepsilon > 0))(\forall\neg(\delta > 0))(\exists x, y \notin X\neg(|x - y| < \delta \to |f(x) - f(y)| < \varepsilon))$ DeMorgan's Law
$(\exists\neg(\varepsilon > 0))(\forall\neg(\delta > 0))(\exists x, y \notin X\neg((|x-y| < \delta)\vee\neg(|f(x)-f(y)| < \varepsilon)))$ Conditional Equivalence
$(\exists(\varepsilon \leq 0))(\forall(\delta \leq 0))(\exists x, y \notin X(\neg(|x - y| < \delta) \wedge \neg\neg(|f(x) - f(y)| < \varepsilon)))$ DeMorgan's Law
$(\exists(\varepsilon \leq 0))(\forall(\delta \leq 0))(\exists x, y \notin X(\neg(|x - y| < \delta) \wedge (|f(x) - f(y)| < \varepsilon)))$ Double Negation
$\exists(\varepsilon \leq 0)\forall(\delta \leq 0)\exists x, y \notin X((|x - y| \geq \delta) \wedge (|f(x) - f(y)| \geq \varepsilon))$ DeMorgan's Law

Question 6:

Domain is $\mathbb{R}$ and $P(x, y) = x * y = 1$

a) $\exists x\exists y(P(x, y)) \Rightarrow$ True
b) $\exists x\forall y(P(x, y)) \Rightarrow$ False
c) $\forall x\forall y(P(x, y)) \Rightarrow$ False
d) $\forall x\exists y(P(x, y)) \Rightarrow$ True