

# Homework 4

## CMPSC 360

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### Question 1:

|   |   |
|---|---|
| NEWS.KIT = I was reading the newspaper in the kitchen     | H1: NEWS.KIT $\rightarrow$ GLASS.KIT    |
| GLASS.KIT = My glasses are on the kitchen table           | H2: GLASS.KIT $\rightarrow$ GLASS.BREAK |
| GLASS.BREAK = I saw my glasses at breakfast               | H3: $\neg$ GLASS.BREAK                  |
| NEWS.LIV = I was reading the newspaper in the living room | H4: NEWS.LIV $\vee$ NEWS.KIT            |
| GLASS.COFF = My glasses are on the coffee table           | H5: NEWS.LIV $\rightarrow$ GLASS.COFF   |

|  |                                  |
|--|----------------------------------|
| 1. GLASS.KIT $\rightarrow$ GLASS.BREAK | H2                               |
| 2. $\neg$ GLASS.BREAK                  | H3                               |
| 3. $\neg$ GLASS.KIT                    | Modus Tollens on 1 and 2         |
| 4. NEWS.KIT $\rightarrow$ GLASS.KIT    | H1                               |
| 5. $\neg$ NEWS.KIT                     | Modus Tollens on 4 and 3         |
| 6. NEWS.LIV $\vee$ NEWS.KIT            | H4                               |
| 7. NEWS.LIV                            | Disjunctive Syllogism on 6 and 5 |
| 8. NEWS.LIV $\rightarrow$ GLASS.COFF   | H5                               |
| 9. GLASS.COFF                          | Modus Ponens on 8 and 7          |

Therefore, the glasses are at on the coffee table.

### Question 2:

H1:  $(\neg v \vee \neg p) \rightarrow (s \wedge z)$   
H2:  $s \rightarrow o$   
H3:  $\neg o$   
C:  $v$

|  |                          |
|--|--------------------------|
| 1. $s \rightarrow o$   | H2                       |
| 2. $\neg o$  | H3                       |
| 3. $\neg s$  | Modus Tollens on 1 and 2 |
| 4. $\neg(s \wedge z) \rightarrow \neg(\neg v \vee \neg p)$             | Contrapositive of H1     |
| 5. $(\neg s \vee \neg z) \rightarrow (\neg \neg v \wedge \neg \neg p)$ | DeMorgan's Law           |
| 6. $(\neg s \vee \neg z) \rightarrow (v \wedge p)$                     | Double Negation          |
| 7. $\neg s \vee \neg z$  | Addition on 3            |
| 8. $v \wedge p$  | Modus Ponens on 6 and 7  |
| 9. $v$   | Simplification of 8      |

**Question 3:**

1. This is not a valid argument ( $a^2$  is positive, but  $a$  could be  $\pm a$ )
2. This is a valid argument (the only solution for  $\sqrt{0^2}$  is 0)

**Question 4:**

TAKEN( $x$ ) =  $x$  has taken CMPSC-360  
CANTAKE( $x$ ) =  $x$  can take CMPSC-465  
 $U$  = all students

$\forall x$  TAKEN( $x$ )  
 $\forall x$  TAKEN( $x$ )  $\rightarrow$  CANTAKE( $x$ )  
 $\therefore \forall x$  CANTAKE( $x$ )

Therefore, by universal instantiation  
TAKEN( $c$ ) for an arbitrary  $c \in U$   
TAKEN( $c$ )  $\rightarrow$  CANTAKE( $c$ )  
 $\therefore$  CANTAKE( $c$ ) by Modus Tollens

so by universal generalization, we can conclude that someone who has taken CMPSC-360, can take CMPSC-465. Therefore, since all five roommates have taken CMPSC-360, they can all take CMPSC-465

**Question 5:**

Proof:

Suppose  $\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6}$ , and we know that  $n \in \mathbb{N}$

$$\begin{aligned}\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6} &= \frac{2n}{6} + \frac{3n^2}{6} + \frac{n^3}{6} && \text{using algebra} \\ &= \frac{n}{6}(2 + 3n + n^2) && \text{simplifying the equation} \\ &= \frac{1}{6}n(n+1)(n+2) && \text{factoring}\end{aligned}$$

By definition of divides,  $n(n+1)(n+2) = 6c$  where  $c \in \mathbb{N}$

A natural number  $n$  can either be 1 or  $x+1$  where  $x \geq 1$

**Case 1:**  $n = 1$

$$\begin{aligned}n(n+1)(n+2) &= (1)(1+1)(1+2) && \text{substitute 1 for } n \\ &= (1)(2)(3) && \text{addition} \\ &= 6 && \text{multiplication}\end{aligned}$$

therefore,  $6 = 6c$ , which simplifies to  $c = 1$ .

Since  $1 \in \mathbb{N}$ , the case  $n = 1$  is validated

**Case 2:**  $n = x+1$  where  $x \in \mathbb{Z}$

$$\begin{aligned}n(n+1)(n+2) &= (x+1) \cdot [(x+1)+1] \cdot [(x+1)+2] && \text{plugging in } x+1 \text{ for } n \\ &= (x+1)(x+2)(x+3) && \text{addition} \\ &= x^3 + 6x^2 + 11x + 6 && \text{algebra} \\ &= (x^3 + 3x^2 + 2x) + (3x^2 + 9x + 6) && \text{algebra} \\ &= x(x+1)(x+2) + 3(x+1)(x+2) && \text{factoring} \\ &= 6c + 3(x+1)(x+2) && \text{substitute } x(x+1)(x+2) \text{ for } 6c\end{aligned}$$

**Case 2a:**  $x$  is even, such that  $x = 2a$  for some  $a \in \mathbb{N}$

$$\begin{aligned}6c + 3(x+1)(x+2) &= 6c + 3(2a+1)(2a+2) && \text{plugging in } 2a \text{ for } x \\ &= 6c + 3(2a+1)[2(a+1)] && \text{factoring out 2} \\ &= 6c + 6(2a+1)(a+1) && \text{multiplication} \\ &= 6[c + (2a+1)(a+1)] && \text{factoring out 6} \\ &= 6 \cdot k_1 \text{ for some } k_1 \in \mathbb{N} \text{ where } k_1 = [c + (2a+1)(a+1)]\end{aligned}$$

Therefore, by definition of an even number,  $6c + 3(x+1)(x+2)$  is divisible by 6

**Case 2b:**  $x$  is odd, such that  $x = 2a+1$  for some  $a \in \mathbb{N}$

$$\begin{aligned}6c + 3(x+1)(x+2) &= 6c + 3(2a+1+1)(2a+1+2) && \text{plugging in } 2a+1 \text{ for } x \\ &= 6c + 3(2a+2)(2a+3) && \text{addition} \\ &= 6c + 3[2(a+1)](2a+3) && \text{factoring out 2} \\ &= 6c + 6(a+1)(2a+3) && \text{multiplication} \\ &= 6[c + (a+1)(2a+3)] && \text{factoring out 6} \\ &= 6 \cdot k_2 \text{ for some } k_2 \in \mathbb{N} \text{ where } k_2 = [c + (a+1)(2a+3)]\end{aligned}$$

Therefore, by definition of an odd number,  $6c + 3(x+1)(x+2)$  is divisible by 6

We notice that  $n(n+1)(n+2)$  where  $n = x+1$  is divisible by 6 in both cases

We notice that  $n(n+1)(n+2)$  is divisible by 6 in both cases.

Therefore, for all natural numbers  $n$ ,  $\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6}$  is a natural number.  $\square$

**Question 6:**

- i) The assumption is that  $n$  is an odd natural number
- ii) We want to conclude that  $n^2 - 1$  is always divisible by 8

Proof:

Suppose  $n$  is an odd natural number

By definition of odd, we know that  $n = 2a + 1$  such that  $a \in \mathbb{N}$

$$\begin{aligned} n^2 - 1 &= (2a + 1)^2 - 1 && \text{plugging in } 2a + 1 \text{ for } n \\ &= 4a^2 + 4a + 1 - 1 && \text{algebra} \\ &= 4a^2 + 4a && \text{subtraction} \\ &= 4a(a + 1) && \text{factoring out } 4a \end{aligned}$$

A natural number  $a$  can be either an even or odd natural number

**Case 1:** Suppose  $a$  is an even natural number. By definition of even we get  $a = 2x; x \in \mathbb{N}$

$$\begin{aligned} 4a(a + 1) &= 4(2x)(2x + 1) && \text{plugging in } 2x \text{ for } a \\ &= 8x(2x + 1) && \text{multiplication} \\ &= 8 \cdot k \text{ for some } k \in \mathbb{N} \text{ where } k = x(2x + 1) \end{aligned}$$

Therefore, when  $a$  is even, the function is divisible by 8

**Case 2:** Suppose  $a$  is an odd natural number. By definition of odd we get  $a = 2x + 1; x \in \mathbb{N}$

$$\begin{aligned} 4a(a + 1) &= 4(2x + 1)(2x + 1 + 1) && \text{plugging in } 2x + 1 \text{ for } a \\ &= 4(2x + 1)(2x + 2) && \text{addition} \\ &= 4(2x + 1)[2(x + 1)] && \text{factoring out } 2 \\ &= 8(2x + 1)(x + 1) && \text{multiplication} \\ &= 8 \cdot k \text{ for some } k \in \mathbb{N} \text{ where } k = (2x + 1)(x + 1) \end{aligned}$$

Therefore, when  $a$  is odd, the function is divisible by 8

We notice that  $4a(a + 1)$  is divisible by 8 for both cases.

Therefore,  $n^2 - 1$  is always divisible by 8 when  $n$  is an odd natural number  $\square$

**Question 7:**

- i) We know that  $m \in \mathbb{Z}$
- ii) Assume that  $m$  is odd
- iii) Prove  $3m + 7$  is even

Proof:

Suppose  $m \in \mathbb{Z}$ , and we know that  $m$  is odd

By definition of odd, we know that  $m = 2a + 1$  such that  $a \in \mathbb{Z}$

$$\begin{aligned} 3m + 7 &= 3(2a + 1) + 7 && \text{plugging in } 2a + 1 \text{ for } m \\ &= 6a + 3 + 7 && \text{multiplication} \\ &= 6a + 10 && \text{addition} \\ &= 2(3a + 5) && \text{factoring out } 2 \\ &= 2 \cdot k \text{ for some } k \in \mathbb{Z} \text{ where } k = 3a + 5 \end{aligned}$$

Therefore, by the definition of an even number,  $3m + 7$  is even when  $m$  is odd.  $\square$

**Question 8:**

Prove that the product of any two odd integers is odd.

Proof:

Suppose  $a \in \mathbb{Z}$  and  $b \in \mathbb{Z}$  such that  $a$  and  $b$  are odd

By definition of odd,  $a = 2x + 1$  such that  $x \in \mathbb{Z}$  and  $b = 2y + 1$  such that  $y \in \mathbb{Z}$

so,  $a \cdot b = (2x + 1)(2y + 1)$  substituting  $a$  for  $2x + 1$  and  $b$  for  $2y + 1$

$$= 4xy + 2x + 2y + 1 \text{ multiplication}$$

$$= 2(2xy + x + y) + 1 \text{ factoring out } 2$$

$$= 2k + 1 \text{ for some } k \in \mathbb{Z} \text{ where } k = 2xy + x + y$$

Therefore, by the definition of an odd number,  $a \cdot b$  is odd

Therefore, the product of any two odd integers is odd.  $\square$