Homework 4

CMPSC 360

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Question 1:

NEWS_KIT = I was reading the newspaper in the kitchen	H1: NEWS_KIT $\rightarrow$ GLASS_KIT
GLASS_KIT = My glasses are on the kitchen table	H2: GLASS_KIT $\rightarrow$ GLASS_BREAK
GLASS_BREAK = I saw my glasses at breakfast	H3: $\neg$ GLASS_BREAK
NEWS_LIV = I was reading the newspaper in the living room	H4: NEWS_LIV $\vee$ NEWS_KIT
GLASS_COFF = My glasses are on the coffee table	H5: NEWS_LIV $\rightarrow$ GLASS_COFF

1. GLASS_KIT $\rightarrow$ GLASS_BREAK	H2
2. $\neg$GLASS_BREAK	H3
3. $\neg$GLASS_KIT	Modus Tollens on 1 and 2
4. NEWS_KIT $\rightarrow$ GLASS_KIT	H1
5. $\neg$NEWS_KIT	Modus Tollens on 4 and 3
6. NEWS_LIV $\vee$ NEWS_KIT	H4
7. NEWS_LIV	Disjunctive Syllogism on 6 and 5
8. NEWS_LIV $\rightarrow$ GLASS_COFF	H5
9. GLASS_COFF	Modus Ponens on 8 and 7

Therefore, the glasses are at on the coffee table.

Question 2:

H1: $(\neg v \vee \neg p) \rightarrow (s \wedge z)$
H2: $s \rightarrow o$
H3: $\neg o$
C: v

1. $s \rightarrow o$	H2
2. $\neg o$	H3
3. $\neg s$	Modus Tollens on 1 and 2
4. $\neg(s \wedge z) \rightarrow \neg(\neg v \vee \neg p)$	Contrapositive of H1
5. $(\neg s \vee \neg z) \rightarrow (\neg\neg v \wedge \neg\neg p)$	DeMorgan's Law
6. $(\neg s \vee \neg z) \rightarrow (v \wedge p)$	Double Negation
7. $\neg s \vee \neg z$	Addition on 3
8. $v \wedge p$	Modus Ponens on 6 and 7
9. v	Simplification of 8

Question 3:

1. This is not a valid argument
   Ex. a^2 is positive, but a could be either $\pm$a
   Therefore, this is not a valid argument by counter example

2. P(x) = if $x^2 \neq 0$, then $x \neq 0$
   Domain: $x \in \mathbb{Z}$, $x > 0$
   $\forall$x P(x)
   $\therefore$P(a) such that a $\in$ Domain
   Therefore, this statement is true through universal instantiation

Question 4:

TAKEN(x) = x has taken CMPSC-360
CANTAKE(x) = x can take CMPSC-465
U = all students
R = {Miranda, Tom, Shilpa, Mark, and Desiree} where R $\in$ U

$\forall x$ TAKEN(x)
$\forall x$ TAKEN(x) $\rightarrow$ CANTAKE(x)
$\therefore \forall x$ CANTAKE(x)

Therefore, by universal instantiation
TAKEN(c) for an arbitrary $c \in$ U
TAKEN(c) $\rightarrow$ CANTAKE(c)
$\therefore$ CANTAKE(c) by Modus Ponens

Therefore by universal generalization,
$\forall x \in$ R TAKEN(x)
$\forall x \in$ R TAKEN(x) $\rightarrow$ CANTAKE(x)
$\therefore \forall x \in$ R CANTAKE(x)

Question 5:

Proof:
Suppose $\frac{n}{3}+\frac{n^2}{2}+\frac{n^3}{6}$, and we know that $n \in \mathbb{N}$

$$\begin{aligned}\frac{n}{3}+\frac{n^2}{2}+\frac{n^3}{6} &= \frac{2n}{6}+\frac{3n^2}{6}+\frac{n^3}{6} && \text{using algebra}\\ &= \frac{n}{6}(2+3n+n^2) && \text{simplifying the equation}\\ &= \frac{1}{6}n(n+1)(n+2) && \text{factoring}\end{aligned}$$

By definition of divides, $n(n+1)(n+2) = 6c$ where $c \in \mathbb{N}$
A natural number n can either be 1 or $x+1$ where $x \geq 1$

Case 1: $n = 1$

$$\begin{aligned}n(n+1)(n+2) &= (1)(1+1)(1+2) && \text{substitute 1 for } n\\ &= (1)(2)(3) && \text{addition}\\ &= 6 && \text{multiplication}\end{aligned}$$

therefore, $6 = 6c$, which simplifies to c = 1.
Since $1 \in \mathbb{N}$, the case $n = 1$ is validated

Case 2: $n = x+1$ where $x \in \mathbb{Z}$

$$\begin{aligned}n(n+1)(n+2) &= (x+1)\cdot[(x+1)+1]\cdot[(x+1)+2] && \text{plugging in } x+1 \text{ for } n\\ &= (x+1)(x+2)(x+3) && \text{addition}\\ &= x^3+6x^2+11x+6 && \text{algebra}\\ &= (x^3+3x^2+2x)+(3x^2+9x+6) && \text{algebra}\\ &= x(x+1)(x+2)+3(x+1)(x+2) && \text{factoring}\\ &= 6c+3(x+1)(x+2) && \text{substitute } x(x+1)(x+2) \text{ for } 6c\end{aligned}$$

Case 2a: x is even, such that $x = 2a$ for some $a \in \mathbb{N}$

$$\begin{aligned}6c+3(x+1)(x+2) &= 6c+3(2a+1)(2a+2) && \text{plugging in } 2a \text{ for } x\\ &= 6c+3(2a+1)[2(a+1)] && \text{factoring out 2}\\ &= 6c+6(2a+1)(a+1) && \text{multiplication}\\ &= 6[c+(2a+1)(a+1)] && \text{factoring out 6}\\ &= 6\cdot k_1 \text{ for some } k_1 \in \mathbb{N} \text{ where } k_1 = [c+(2a+1)(a+1)]\end{aligned}$$

Therefore, by definition of an even number, $6c+3(x+1)(x+2)$ is divisible by 6

Case 2b: x is odd, such that $x = 2a+1$ for some $a \in \mathbb{N}$

$$\begin{aligned}6c+3(x+1)(x+2) &= 6c+3(2a+1+1)(2a+1+2) && \text{plugging in } 2a+1 \text{ for } x\\ &= 6c+3(2a+2)(2a+3) && \text{addition}\\ &= 6c+3[2(a+1)](2a+3) && \text{factoring out 2}\\ &= 6c+6(a+1)(2a+3) && \text{multiplication}\\ &= 6[c+(a+1)(2a+3)] && \text{factoring out 6}\\ &= 6\cdot k_2 \text{ for some } k_2 \in \mathbb{N} \text{ where } k_2 = [c+(a+1)(2a+3)]\end{aligned}$$

Therefore, by definition of an odd number, $6c+3(x+1)(x+2)$ is divisible by 6
We notice that $n(n+1)(n+2)$ where $n = x+1$ is divisible by 6 in both cases
We notice that $n(n+1)(n+2)$ is divisible by 6 in both cases.

Therefore, for all natural numbers n, $\frac{n}{3}+\frac{n^2}{2}+\frac{n^3}{6}$ is a natural number. $\square$

Question 6:

i) The assumption is that n is an odd natural number
ii) We want to conclude that $n^2 - 1$ is always divisible by 8
Proof:
Suppose n is an odd natural number
By definition of odd, we know that $n = 2a + 1$ such that $a \in \mathbb{N}$

$$\begin{aligned}
n^2 - 1 &= (2a+1)^2 - 1 && \text{plugging in } 2a+1 \text{ for } n \\
&= 4a^2 + 4a + 1 - 1 && \text{algebra} \\
&= 4a^2 + 4a && \text{subtraction} \\
&= 4a(a+1) && \text{factoring out } 4a
\end{aligned}$$

A natural number a can be either an even or odd natural number

Case 1: Suppose a is an even natural number. By definition of even we get $a = 2x; x \in \mathbb{N}$

$$\begin{aligned}
4a(a+1) &= 4(2x)(2x+1) && \text{plugging in } 2x \text{ for } a \\
&= 8x(2x+1) && \text{multiplication} \\
&= 8 \cdot k \text{ for some } k \in \mathbb{N} \text{ where } k = x(2x+1)
\end{aligned}$$

Therefore, when a is even, the function is divisible by 8

Case 2: Suppose a is an odd natural number. By definition of odd we get $a = 2x + 1; x \in \mathbb{N}$

$$\begin{aligned}
4a(a+1) &= 4(2x+1)(2x+1+1) && \text{plugging in } 2x+1 \text{ for } a \\
&= 4(2x+1)(2x+2) && \text{addition} \\
&= 4(2x+1)[2(x+1)] && \text{factoring out } 2 \\
&= 8(2x+1)(x+1) && \text{multiplication} \\
&= 8 \cdot k \text{ for some } k \in \mathbb{N} \text{ where } k = (2x+1)(x+1)
\end{aligned}$$

Therefore, when a is odd, the function is divisible by 8

We notice that $4a(a+1)$ is divisible by 8 for both cases.
Therefore, $n^2 - 1$ is always divisible by 8 when n is an odd natural number $\square$

Question 7:

i) We know that $m \in \mathbb{Z}$
ii) Assume that m is odd
iii) Prove $3m + 7$ is even

Proof:
Suppose $m \in \mathbb{Z}$, and we know that m is odd
By definition of odd, we know that $m = 2a + 1$ such that $a \in \mathbb{Z}$

$$\begin{aligned}
3m + 7 &= 3(2a+1) + 7 && \text{plugging in } 2a+1 \text{ for } m \\
&= 6a + 3 + 7 && \text{multiplication} \\
&= 6a + 10 && \text{addition} \\
&= 2(3a+5) && \text{factoring out } 2 \\
&= 2 \cdot k \text{ for some } k \in \mathbb{Z} \text{ where } k = 3a + 5
\end{aligned}$$

Therefore, by the definition of an even number, $3m + 7$ is even when m is odd. $\square$

Question 8:

Prove that the product of any two odd integers is odd.
Proof:
Suppose $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ such that a and b are odd
By definition of odd, $a = 2x + 1$ such that $x \in \mathbb{Z}$ and $b = 2y + 1$ such that $y \in \mathbb{Z}$
so, $a \cdot b = (2x + 1)(2y + 1)$ substituting a for $2x + 1$ and b for $2y + 1$
$= 4xy + 2x + 2y + 1$ multiplication
$= 2(2xy + x + y) + 1$ factoring out 2
$= 2k + 1$ for some $k \in \mathbb{Z}$ where $k = 2xy + x + y$
Therefore, by the definition of an odd number, $a \cdot b$ is odd
Therefore, the product of any two odd integers is odd. $\square$