

Homework 11

CMPSC 360

Kinner Parikh

April 21, 2022

Question 1:

- a) How many edges are there in a graph with 10 vertices, each having a degree 3? — **15**
- b) How many edges are there in a graph with 8 vertices, having a degree 1,1,2,2,3,3,3,3 respectively? — **9**
- c) How many vertices are there in a graph with 19 edges, having 3 vertices of degree 4 and all the other vertices are of degree 2? — **13**

Question 2:

With repetition: $6! = 720$

Without repetition: $\frac{6!}{2} = 360$

Question 3:

- a) $\frac{10!}{2!} = 1814400$
- b) $\frac{8!}{2!} \cdot 5! = 2419200$
- c) $\frac{10!}{2!} \cdot 2 \cdot 7 = 10! \cdot 7 = 25401600$

Question 4:

$${}_2C_1 \cdot {}_5C_2 + {}_2C_2 \cdot {}_5C_1 = 25$$

Question 5:

$$4^3 - 1 = 63$$

Question 6:

No

Question 7:

Question 8:

The five cases are: $x_2 = \{0, 1, 2, 3, 4\}$

When $x_2 = 0$

$$x_1 + x_3 + x_4 = 10, \text{ so } {}_{12}C_2$$

When $x_2 = 1$

$$x_1 + x_3 + x_4 = 9, \text{ so } {}_{11}C_2$$

When $x_2 = 2$

$$x_1 + x_3 + x_4 = 8, \text{ so } {}_{10}C_2$$

When $x_2 = 3$

$$x_1 + x_3 + x_4 = 7, \text{ so } {}_9C_2$$

When $x_2 = 4$

$$x_1 + x_3 + x_4 = 6, \text{ so } {}_8C_2$$

$$\text{So, } {}_{12}C_2 + {}_{11}C_2 + {}_{10}C_2 + {}_9C_2 + {}_8C_2 = 230$$

Question 9:

$$\begin{aligned} & {}_4C_0 \cdot 3^0 \cdot (2x)^4 + {}_4C_1 \cdot 3^1 \cdot (2x)^3 + {}_4C_2 \cdot 3^2 \cdot (2x)^2 + {}_4C_3 \cdot 3^3 \cdot (2x)^1 + {}_4C_4 \cdot 3^4 \cdot (2x)^0 \\ &= 16x^4 + 96x^3 + 216x^2 + 216x + 81 \end{aligned}$$

Question 14:

$$\begin{aligned} & \lfloor \frac{10000}{3} \rfloor + \lfloor \frac{10000}{5} \rfloor + \lfloor \frac{10000}{7} \rfloor + \lfloor \frac{10000}{11} \rfloor - \lfloor \frac{10000}{15} \rfloor - \lfloor \frac{10000}{21} \rfloor - \lfloor \frac{10000}{33} \rfloor - \lfloor \frac{10000}{35} \rfloor - \lfloor \frac{10000}{55} \rfloor - \lfloor \frac{10000}{33} \rfloor \\ & - \lfloor \frac{10000}{105} \rfloor - \lfloor \frac{10000}{165} \rfloor - \lfloor \frac{10000}{231} \rfloor - \lfloor \frac{10000}{385} \rfloor - \lfloor \frac{10000}{1155} \rfloor = 5485 \end{aligned}$$

Question 15:

$${}_{20}P_{15} = \frac{20!}{15!} = 1860480$$

Question 16:

$${}_6P_3 + {}_6P_2 = 120 + 30 = 150$$

$${}_7P_2 + 6 + 6 = 42 + 12 = 54$$

So in total, they can watch $150 + 54 = \underline{204 \text{ movies}}$