

Homework 3

CMPSC 360

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Question 1:

$\exists x \in \mathbb{Z}(x^2 = 5)$ is false because $x = \pm\sqrt{5}$ which is not an integer

Question 2:

- The sum of two negative integers is always negative $\equiv \forall x \forall y ((x < 0) \wedge (y < 0) \rightarrow (x + y < 0))$ is **false**
- Which of the two statements are equal? **(C and D)**
 - $\neg \exists x (A(x)) \equiv \forall x (\neg A(x))$
 - $\neg \forall x (\neg A(x)) \equiv \exists x (A(x))$
 - $\neg \forall x (A(x)) \equiv \exists x (\neg A(x))$
 - $\exists x (\neg A(x))$
- $\neg(P(x) \vee Q(x)) \equiv \neg P(x) \wedge \neg Q(x)$ is **false**
 $\neg P(x) \wedge \neg Q(x)$
- $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$ is **true**
 $\neg(\neg P \vee Q)$
 $P \wedge \neg Q$

Question 3:

Let $P(x, y, z) = x^2 + y^2 \geq z^2$

A) $\forall x \in (-3, 3) P(x, 4, 5) \Rightarrow$ False because $1^2 + 4^2 < 5^2$

B) $\forall w \neg P(w, w, w) \Rightarrow$ False because $1^2 + 1^2 \geq 1^2$

C) $\exists s (P(6, s, 10) \wedge P(s, 15, 17)) \Rightarrow$ **True** because $s = 100$, $6^2 + 100^2 \geq 10^2$ and $100^2 + 15^2 \geq 17^2$

D) $\forall t (P(6, t, 10) \vee P(t, 15, 17)) \Rightarrow$ False because $s = 1$, $6^2 + 1^2 \not\geq 10^2$ and $1^2 + 15^2 \not\geq 17^2$

E) $\forall \alpha (\neg P(\alpha, 1 - \alpha, 2\alpha) \vee P(\alpha, 1 - \alpha, 2\alpha)) \Rightarrow$ **True** because tautology identity

Question 4:

$\neg \forall x (\neg A(x) \wedge B(x))$

$\exists x \neg (\neg A(x) \wedge B(x))$

$\exists x (\neg \neg A(x) \vee \neg B(x))$

$\exists x (A(x) \vee \neg B(x))$

$\exists x (B(x) \rightarrow A(x))$

DeMorgan's Law on Universal Quantifier

DeMorgan's Law

Double Negation

Conditional Equivalence

Question 5:

Let $P(f, X) = \forall \varepsilon > 0 \exists \delta > 0 \forall x, y \in X x - y < \delta \rightarrow f(x) - f(y) < \varepsilon$	
so $\neg P = \neg(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x, y \in X x - y < \delta \rightarrow f(x) - f(y) < \varepsilon)$	
$\exists \varepsilon > 0 (\neg \exists \delta > 0)(\forall x, y \in X x - y < \delta \rightarrow f(x) - f(y) < \varepsilon)$	DeMorgan's Law
$\exists \varepsilon > 0 \forall \delta > 0 (\neg \forall x, y \in X x - y < \delta \rightarrow f(x) - f(y) < \varepsilon)$	DeMorgan's Law
$\exists \varepsilon > 0 \forall \delta > 0 \exists x, y \in X \neg[(x - y < \delta) \rightarrow (f(x) - f(y) < \varepsilon)]$	DeMorgan's Law
$\exists \varepsilon > 0 \forall \delta > 0 \exists x, y \in X \neg[(x - y < \delta) \vee \neg(f(x) - f(y) < \varepsilon)]$	Conditional Equivalence
$\exists \varepsilon > 0 \forall \delta > 0 \exists x, y \in X [\neg(x - y < \delta) \wedge \neg \neg(f(x) - f(y) < \varepsilon)]$	DeMorgan's Law
$\exists \varepsilon > 0 \forall \delta > 0 \exists x, y \in X [\neg(x - y < \delta) \wedge (f(x) - f(y) < \varepsilon)]$	Double Negation
$\exists \varepsilon > 0 \forall \delta > 0 \exists x, y \in X [(x - y \geq \delta) \wedge (f(x) - f(y) < \varepsilon)]$	DeMorgan's Law

Question 6:

Domain is \mathbb{R} and $P(x, y) = x * y = 1$

- a) $\exists x \exists y [P(x, y)] \Rightarrow \text{True}$
- b) $\exists x \forall y [P(x, y)] \Rightarrow \text{False}$
- c) $\forall x \forall y [P(x, y)] \Rightarrow \text{False}$
- d) $\forall x \exists y [P(x, y)] \Rightarrow \text{True}$

Question 7:

- a) $\forall x < 0 \forall y > 0 (x * y < 0)$
- b) $\forall x \in \mathbb{Z} (x \text{ is even} \vee x \text{ is odd})$
- c) $\forall x [P(x) \wedge R(x)] \rightarrow Q(x)$
- d) $\neg \forall x A(x) \rightarrow [B(x) \vee C(x)]$

Question 8:

- a) Some animals in the world do not live in a forest
- b) For all real numbers x , there is some real number y such that $y^3 = x$
- c) For all integers x and y , the product of x and y will be an integer.
- d) For all real numbers x , the difference of x and x is 0.

Question 9:

- a) For all integer y , there is some integer x such that x is less than y . — True
- b) For all integer x and some integer y , such that if x is positive, then y is a prime number and x is less than y — True
- c) For all integer x , there is some integer y such that x will be less than y and y will be prime. — True

Question 10:

$$\frac{p \vee q}{(p \wedge r) \rightarrow q} \therefore r$$

p	q	r	$p \vee q$	$p \wedge r$	$(p \wedge r) \rightarrow q$	$(p \vee q) \wedge [(p \wedge r) \rightarrow q]$	$(p \vee q) \wedge [(p \wedge r) \rightarrow q] \rightarrow r$
F	F	F	F	F	T	F	T
F	F	T	F	F	T	F	F
F	T	F	T	F	F	F	T
F	T	T	T	F	F	F	F
T	F	F	T	F	T	T	T
T	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

This is not a valid argument because it is not a tautology. We can prove this using the truth table. The lines with the red highlights show the truth values where the argument is not valid.