Homework 1
CMPSC 360

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Question 1:

Given statement: x is an integer
Assumption: x is an odd integer
Conclusion: $x^2 + 5x + 7$ is odd

Proof:
Suppose x is odd such that $x \in \mathbb{Z}$
By definition of odd, $x = 2a + 1$ where $a \in \mathbb{Z}$

$$
\begin{aligned}
x^2 + 5x + 7 &= (2a+1)^2 + 5(2a+1) + 7 && \text{plugging in } 2a+1 \text{ for } x \\
&= 4a^2 + 4a + 1 + 10a + 5 + 7 && \text{algebra} \\
&= 4a^2 + 14a + 12 + 1 && \text{addition} \\
&= 2(a^2 + 7a + 6) + 1 && \text{factoring out 2} \\
&= 2c + 1 \text{ from some } c \in \mathbb{Z} \text{ where } c = a^2 + 7a + 6
\end{aligned}
$$

so, by definition of odd, $x^2 + 5x + 7$ is odd
therefore, when x is an odd integer, $x^2 + 5x + 7$ is odd $\square$

Question 2:

i) $\forall x \in \mathbb{Z}, \, x < 0 \, (x^3 < 0)$
ii) $\exists! x \in \mathbb{R} \, \forall y \in \mathbb{R} \, (x \cdot y = y)$

Question 3:

H1: A $\rightarrow$ B
H2: $\neg$C $\rightarrow$ $\neg$B
H3: A
C: C

1. A $\rightarrow$ B	[H1]	
2. $\neg$C $\rightarrow$ $\neg$B	[H2]	
3. $\neg\neg$B $\rightarrow$ $\neg\neg$C	[Contrapositive of 2]	
4. B $\rightarrow$ C	[Double negation on 3]	
5. A $\rightarrow$ C	[Hypothetical syllogism on 1 and 4]	
6. A	[H3]	
7. C	[Modus Ponens on 5 and 6]	