

Homework 3

CMPSC 360

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Question 1:

$\exists x \in \mathbb{Z}(x^2 = 5)$ is false because $x = \pm\sqrt{5}$ which is not an integer

Question 2:

1. The sum of two negative integers is always negative $\equiv \forall x \forall y ((x < 0) \wedge (y < 0) \rightarrow (x + y < 0))$ is **false**
2. Which of the two statements are equal? **(C and D)**
 - a) $\neg \exists x (A(x)) \equiv \forall x (\neg A(x))$
 - b) $\neg \forall x (\neg A(x)) \equiv \exists x (A(x))$
 - c) $\neg \forall x (A(x)) \equiv \exists x (\neg A(x))$
 - d) $\exists x (\neg A(x))$
3. $\neg(P(x) \vee Q(x)) \equiv \neg P(x) \vee \neg Q(x)$ is **false**
 $\neg P(x) \wedge \neg Q(x)$
4. $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$ is **true**
 $\neg(\neg P \vee Q)$
 $P \wedge \neg Q$

Question 3:

Let $P(x, y, z) = x^2 + y^2 \geq z^2$

A) $\forall x \in (-3, 3) P(x, 4, 5) \Rightarrow$ **True** because $3^2 + 4^2 = 5^2$

B) $\forall w \neg P(w, w, w) \Rightarrow$ False because $1^2 + 1^2 \geq 1^2$

C) $\exists s (P(6, s, 10) \wedge P(s, 15, 17)) \Rightarrow$ **True** because $s = 100$, $6^2 + 100^2 \geq 10^2$ and $100^2 + 15^2 \geq 17^2$

D) $\forall t (P(6, t, 10) \vee P(t, 15, 17)) \Rightarrow$ False because $s = 1$, $6^2 + 1^2 \not\geq 10^2$ and $1^2 + 15^2 \not\geq 17^2$

E) $\forall \alpha (\neg P(\alpha, 1 - \alpha, 2\alpha) \vee P(\alpha, 1 - \alpha, 2\alpha)) \Rightarrow$ **True** because tautology identity

Question 4: