Homework 6

CMPSC 465

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November 9, 2022

Problem 1:

I worked with Sahil Kuwadia and Ethan Yeung
I did not consult without anyone my group member
I did not consult any non-class materials

Problem 2: *Let* $G = (V,E)$ *be a directed graph, with source* $s \in V$*, sink* $t \in V$ *and nonnegative edge capacities* $\{c_e\}$*. Give a polynomial time algorithm to decide whether* G *has a unique minimum s-t cut.*

We can start by finding the maximum flow through G by running the Ford-Fulkerson algorithm. We know that the maximum flow through G is the minimum cut of G, set this value to C. To find if this cut is unique, we iterate through all edges taking edge e, and set its capacity to $c_e + 1$. We can then find the maximum flow through this new graph and set this new capacity to C_e. Once we have computed the capacity, reset c_e to its original value. If $\forall C_e = C$, then the cut is not unique. If $\exists ! C_e \neq C$, then the cut is unique. Since Ford-Fulkerson runs in polynomial time, and we are running it at most $|E| + 1$ times, this algorithm runs in polynomial time.

Problem 3: *Let T be an MST of graph G. Given a connected subgraph H of G, show that $T \cap H$ is contained in some MST of H.*

The cut property states that A as a subset of edges from some MST of G. Let $(S, V - S)$ be a cut that respects A and let e be the lightest edge across the cut. Then we know that $A \cup \{e\}$ is part of some MST. Using this property, take edge e which is the lightest edge in H, thus we know that $e \in T \cap H$. We can then take e as the cut edge for H, so the cut set is $(H \cap S, H - S)$ (similar to the property above). Because $e \in T \cap H$, the same cut exists in some MST of H, proving that $T \cap H$ is contained in some MST of H.

Problem 4: *Let T be a minimum spanning tree of a graph $G = (V, E)$ and V' be a subset of V. Let T' be a subgraph of T induced by V' (i.e., an edge $(u, v) \in T$ is present in T' iff both $(u, v) \in V'$) and G' be a subgraph of G induced by V'. Show that if T' is connected, then T' is a minimum spanning tree of G'.*

We know that T is an MST of G. We also know that T' is a subgraph of T. FSOC, assume that T' is not an MST for G'. Let S be a valid MST of G'. We know that S is a subgraph of T', so S is a subgraph of T. We also know that S is a subgraph of G'. Thus, S is a subgraph of G. We know that S is a valid MST of G'. We know that T is a valid MST of G, so T is a valid MST of G. Thus, $T = S$, which is a contradiction. Thus, T' is an MST of G'.