Homework 2

CMPSC 465

Kinner Parikh

June 29, 2023

Problem 1:

I did not work in a group
I did not consult without anyone my group member
I did not consult any non-class materials

Problem 2: *Analyzing run time.*

1. $\sum_{i=1}^{n} \frac{n-i}{5} = \frac{1}{5}\sum_{i=1}^{n} n - i = \frac{1}{5} \cdot (\sum_{i=1}^{n} n - \sum_{i=1}^{n} i) = \frac{1}{5} \cdot (n^2 - \frac{n^2+n}{2}) = \frac{n^2-n}{10} = \underline{\Theta(n^2)}$

2. $\sum_{i=1}^{n/4} n - 4i = \sum_{i=1}^{n/4} n - \sum_{i=1}^{n/4} 4i = \Theta(n^2) + \Theta(4 \cdot \frac{n/4(n/4+1)}{2}) = \Theta(\frac{33n^2+4n}{32}) = \underline{\Theta(n^2)}$

3. $\lfloor \log n \rfloor + \sum_{i=1}^{\lfloor \log n \rfloor} \frac{n}{2^i} = \lfloor \log n \rfloor + n \cdot \sum_{i=1}^{\lfloor \log n \rfloor} 2^{-i} = \underline{\Theta(n)}$

Problem 3: *Polynomials and Horner's Rule*

a) $\sum_{i=0}^{n} a_i \cdot x_0^i$
Number of Multiplications $= \frac{n(n+1)}{2}$ because each power x_0^n is $n-1$ multiplications, and you add one for multiplying a_i.
Number of Sums $= n$

b) LI $= \sum_{j=0}^{n-i} a_{n-j} x^{n-i-j}$

Initialization: At the start of the 1st iteration where $i = n$, the loop invariant states that $z = a_n$. Since this represents the coefficient for a polynomial with a maximum coefficient of 0, this is true.

Maintenance: Assume that LI holds at the start of iteration k_0. This means that the polynomial is $a_{k_0} + (a_{k_0+1}x) + ... + (a_{n-1}x^{n-k_0-1}) + (a_n x^{n-k_0})$. We need to show that at iteration k_0+1, the LI consists of the k largest polynomial powers of x with their corresponsing coefficients. In the next loop, we get $k = k_0 - 1$ where

$$z = a_{k_0-1} + (a_{k_0}x) + ... + (a_{n-1}x^{n-k_0}) + (a_n x^{n-k_0+1})$$

simplifies to: $z = a_k + (a_{k+1}x) + ... + (a_{n-1}x^{n-k-1}) + (a_n x^{n-k})$
Therefore we can say that the LI holds for every iteration of the algorithm.

Termination: We must argue that the fact that the LI holds at the start of iteration n implies the algorithm correct. When the algorithm stops, $i = 0$ so
$z = a_0 + a_1 x + ... + a_{n-1}x^{n-1} + a_n x^n$.

c) Number of Sums $= n$
Number of Multiplications $= 2n - 1$

Problem 4: *Solving recurrences*

(a) $T(n) = 2T(n/2) + \sqrt{n}$
Branching Factor = 2
Height = $\log_2 n$
Size of subproblems at depth k = $n/2^k$
Number of subproblems at depth k = 2^k
$W_k = 2^k \sqrt{n/2^k}$
$\sum_{k=0}^{\log_2 n} W_k = \sum_{k=0}^{\log_2 n} 2^k \sqrt{n/2^k} = \Theta(2^{\log_2 n} \cdot \sqrt{n/(2^{\log_2 n})}) = \underline{\Theta(n)}$

(b) $T(n) = 2T(n/3) + 1$
Branching Factor = 2
Height = $\log_3 n$
Size of subproblems at depth k = $n/3^k$
Number of subproblems at depth k = 2^k
$W_k = 2^k$
$\sum_{k=0}^{\log_3 n} W_k = \sum_{k=0}^{\log_3 n} 2^k = \underline{\Theta(n^{\log_3 2})}$

(c) $T(n) = 5T(n/4) + n$
Branching Factor = 5
Height = $\log_4 n$
Size of subproblems at depth k = $n/4^k$
Number of subproblems at depth k = 5^k
$W_k = 5^k \cdot n/4^k) = (5/4)^k \cdot n$
$\sum_{k=0}^{\log_4 n} W_k = n \cdot \sum_{k=0}^{\log_4 n} \frac{5}{4}^k = \underline{\Theta(n^{\log_4 5})}$

(d) $T(n) = 7T(n/7) + n$
Branching Factor = 7
Height = $\log_7 n$
Size of subproblems at depth k = $n/7^k$
Number of subproblems at depth k = 7^k
$W_k = 7^k \Theta(n/7^k) = \Theta(n)$
$\sum_{k=0}^{\log_7 n} W_k = \sum_{k=0}^{\log_7 n} \Theta(n) = \underline{\Theta(n \cdot \log_7 n)}$

(e) $T(n) = 9T(n/3) + n^2$
Branching Factor = 9
Height = $\log_3 n$
Size of subproblems at depth k = $n/3^k$
Number of subproblems at depth k = 9^k
$W_k = 9^k \cdot (n/3^k)^2 = n^2$
$\sum_{k=0}^{\log_3 n} W_k = \sum_{k=0}^{\log_3 n} n^2 = \underline{\Theta(n^2 \cdot \log_3 n)}$