Homework 1

CMPSC 465

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I did not work in a group

I did not consult without anyone my group member

I did not consult any non-class materials

Question 1: *In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). Give a one sentence justification for each of your answers.*

a) $f(n) = n^{1.5}$, $g(n) = n^{1.3} \rightarrow \underline{f = \Omega(g)}$ and not $O(g)$ because the exponent of f is greater than the exponent in g.

b) $f(n) = 2^{n-1}$, $g(n) = 2^n \rightarrow \underline{f = \Theta(g)}$ because $f(n) = \frac{1}{2} \cdot 2^n$, and since we disregard the coefficient, we know that asymptotically f is 2^n, which is equivalent to g.

c) $f(n) = n^{1.3 \log(n)}$, $g(n) = n^{1.5} \rightarrow \underline{f = \Omega(g)}$ because simply comparing the exponents shows that f's exponent will grow faster than g's because f's exponent grows based on n while g's is constant.

d) $f(n) = 3^n$, $g(n) = n \cdot 2^n \rightarrow \underline{f = \Omega(g)}$ because based on the rule of exponential functions for asymptotic growth, the larger the base of the exponent, the faster it will grow. Since $3 > 2$, f will grow faster than g.

e) $f(n) = (\log n)^{100}$, $g(n) = n^{0.1} \rightarrow \underline{f = O(g)}$ because $\log(n)$ grows slower than n, and since the function is dependent on the base of the exponent, g will grow faster than f.

f) $f(n) = n$, $g(n) = (\log n)^{\log \log n} \rightarrow \underline{f = O(g)}$ because g is an exponential function and f is linear, thus g will grow faster than f.

g) $f(n) = 2^n$, $g(n) = n! \rightarrow \underline{f = O(g)}$ because factorial grows much faster than any exponential function.

h) $f(n) = \log(e^n)$, $g(n) = n\log n \rightarrow \underline{f = O(g)}$ because we can simplify f down to $n\log(e)$. Since $\log(e)$ is a numerical value, we can drop it and see that f is a linear function, thus grows slower than $n\log n$.

i) $f(n) = n + \log n$, $g(n) = n + (\log n)^2 \rightarrow \underline{f = \Theta(g)}$ because $(\log n)^2$ grows slower than n, thus both f and g exhibit linear growth.

j) $f(n) = 5n + \sqrt{n}$, $g(n) = \log n + n \rightarrow \underline{f = \Theta(g)}$ because both $\sqrt{n}$ and $\log n$ grow slower than linear growth, thus, both f and g exhibit linear growth.

Question 2: