

# **Homework 9**

CMPSC 465

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November 17, 2022

**Problem 1:**

I worked with Sahil Kuwadia and Ethan Yeung  
I did not consult without anyone my group member  
I did not consult any non-class materials

**Problem 2:** Suppose we have an optimal prefix code on a set  $C = \{0, 1, \dots, n - 1\}$  of characters and we wish to transmit this code using as few bits as possible. Show how to represent any optimal prefix code on  $C$  using only  $2n - 1 + n\lceil \log n \rceil$  bits.

Since there are  $n$  characters, there are  $n$  leaves in the tree. Thus, there will be  $n - 1$  vertices within the graph, so the entire graph will contain  $2n - 1$  total vertices, thus  $2n - 1$  bits. The height of a full binary tree for  $n$  characters is  $\lceil \log n \rceil$ .

We can say that to associate the members of  $C$  with the leaves of the tree,  $\lceil \log n \rceil$  bits will be enough to represent all members. We know that no delimiters are required if each character is represented by a unique prefix. So, with  $n$  leaves, it requires  $n\lceil \log n \rceil$  bits to represent all characters.

Thus, we can say that the total number of bits required to represent the optimal prefix code is  $2n - 1 + n\lceil \log n \rceil$ .

**Problem 3:** *Generalize Huffman's algorithm to ternary codewords (i.e., codewords using the symbols 0, 1, and 2), and prove that it yields optimal ternary codes.*

**Problem 4:**