Homework 5

CMPSC 465

Kinner Parikh

June 29, 2023

Problem 1:

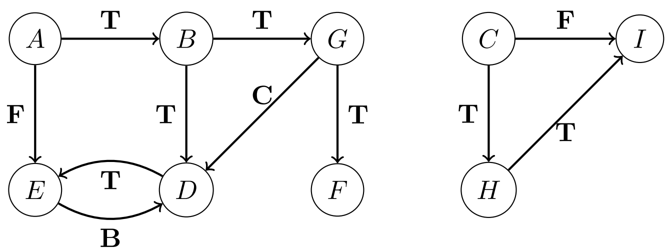
I did not work in a group
I did not consult without anyone my group member
I did not consult any non-class materials

Problem 2: *DFS Basics*

a) $A,\ B,\ D,\ E,\ G,\ F,\ C,\ H,\ I$

b) $A(1,12), B(2,11), D(3,6), E(4,5), G(7,10), F(8,9), C(13,18), H(14,17), I(15,16)$

c) Tree Edges: $\{(A,B),(B,D),(D,E),(B,G),(G,F),(C,H),(H,I)\}$
Back Edges: $\{(E,D)\}$
Forward Edges: $\{(A,E),(C,I)\}$
Cross Edges: $\{(G,D)\}$



Problem 3: *Pre and Post Processing*

a)
Claim: if $\{u, v\}$ is an edge in an undirected graph, and during depth-first search $\texttt{post}(u) < \texttt{post}(v)$, then v is an ancestor of u in the DFS tree.

Proof:
We know there is an edge between u and v. Given a DFS tree, there are two cases to consider given that $\texttt{post}(u) < \texttt{post}(v)$. Furthermore, we must follow the property that for node n, $\texttt{pre}(n) < \texttt{post}(n)$.

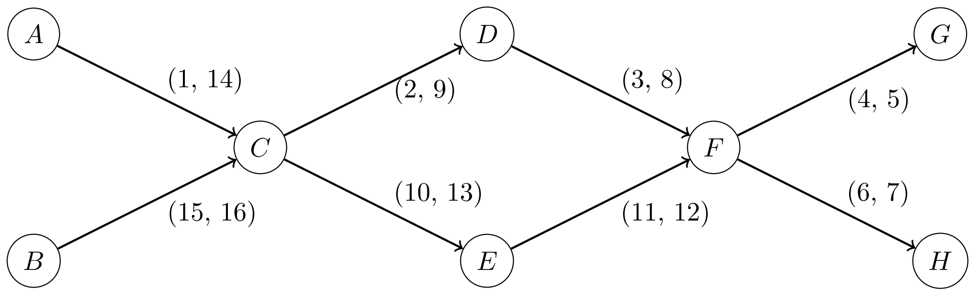
Case 1: $\texttt{pre}(v) < \texttt{pre}(u) < \underline{\texttt{post}(u) < \texttt{post}(v)}$
This represents a forward (tree) edge in the graph. According to the DFS algorithm, once v has been visited, we must explore the neighbors of v, and after all of v's ancestors have been explored, will we assign v's post number. Thus, we can confidently say that v is an ancestor of u.

Case 2: $\texttt{pre}(u) < \underline{\texttt{post}(u) < \texttt{pre}(v) < \texttt{post}(v)}$
According to the DFS algorithm, we know that once u has been visited, all of its neighbors and descendants must be visited before looking at v. This means that there exists no edge $\{u, v\}$ ↯
Therefore, we can say that if $\texttt{post}(u) < \texttt{post}(v)$, then v is an ancestor of u. $\square$

b)
We can preprocess the graph by running a DFS algorithm on the graph and assigning pre and post numbers to each of the nodes and storing them in a data structure that has constant lookup time. In this case, the data structure can be a hash map where the key is the value of the node and the value is the pre and post numbers stored in an array. Then, when we want to find if u is an ancestor of v, we can simply refer to the data structure and see if $\texttt{post}(v) < \texttt{post}(u)$, and if so, based on the proof above, we can say that u is an ancestor of v.

Problem 4: *Linearization Basics*

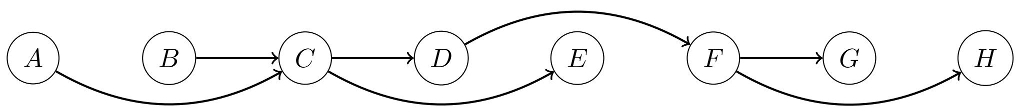
a) (pre, post)



A: (1, 14) B: (15, 16) C: (2, 9) D: (10, 13) E: (3, 8) F: (11, 12) G: (4, 5) H: (6, 7)

b) Source Nodes: A, B
Sink Nodes: G, H

c)



d) This graph will have 8 total linearizations because at node C and node F, we can go in two different directions. Furthermore, we could either start from A or B, so that gives us 2 more options. Thus, $2 \cdot 2 \cdot 2 = 8$.