Comparing two methods of approximating the value of π in a computational context

To what extent can a method of approximation of the value π be computationally more efficient than another?

Word count:

Contents

1 Introduction

The value of π has been researched for many years, although under different names, and the amount of different approaches to reach the value is large. The value has been found through many processes, be it geometrically, algebraically or through other means.

This paper seeks to examine the extent at which two historical methods of approximation of the value π, namely the approaches suggested by the aforementioned mathematicians Madhava and Viète, differ in terms of computational efficiency and speed, and explain these differences.

2 Theoretical approach

2.1 Focus on two methods

For the sake of this paper, two different methods with similar convergence rates but different approaches have been chosen for comparison, the process for the original discovery of these methods will be explained. The two mathematicians in question are Madhava of Sangamagramma of Medieval India and the French mathematician Viète.

2.1.1 Madhava's method

Madhava,

2.1.2 Viète's method

The French mathematician, François Viète, approached the value of π from a geometric standpoint, finding the following formula:

$$\pi = 2\frac{2}{\sqrt{2}}\frac{2}{\sqrt{2+\sqrt{2}}}\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}}\cdots$$

He was able to calculate π to a place of 9 decimal points, in the year 1593 [2], using his method. His method consists of finding the area of a polygon of n sides in a circle of constant radius. As the value of n is increased, the area of the n-gon tends toward the area of a circle. The geometric origin of this formula can be found using simple right-angle trigonometry, by first finding the lengths OH and subsequently BD in Figure 1 (2.1.2).

With the radius $R = OB$,

$OH = R\cos\alpha$

and

$BD = 2BH = 2R\sin\alpha$

Since the equation for the area of a polygon is defined as $A = \frac{p \cdot a}{2}$, where p is the perimeter of the polygon and a is the apothem, in this case $BD \cdot n$ and OH respectively.

Let A_n equal the area of the polygon with n sides as such:

$A_n = \frac{OH \cdot BD \cdot n}{2}$

$= \frac{R \cos\alpha \cdot 2R \sin\alpha \cdot n}{2} = nR^2 \sin\alpha \cos\alpha$

And if n is multiplied by 2, the angle $\angle\alpha$ is divided by 2, and the new area becomes:

$A_{2n} = 2nR^2 \sin\frac{\alpha}{2} \cos\frac{\alpha}{2}$

So it can be written that, by definition,

$\frac{A_n}{A_{2n}} = \frac{nR^2 \sin\alpha \cos\alpha}{2nR^2 \sin\frac{\alpha}{2} \cos\frac{\alpha}{2}} = \frac{\sin 2\alpha}{2\sin\alpha}$

Which through the trigonometric identity... // todo

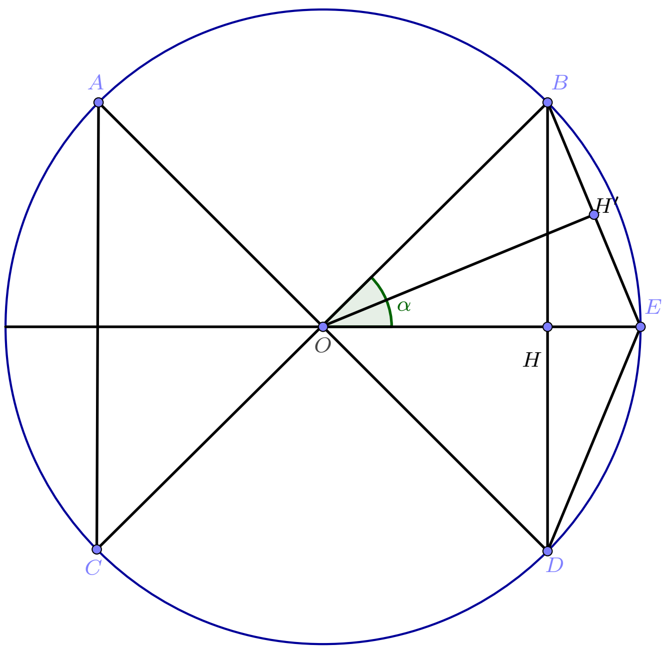

Figure 1: Circle with 1 segment from a n-gon with point H and 2 segments from an $2n$-gon, one of which on point H', inscribed in a circle of radius OB, adapted from [1]

3 Computational approach

3.1 Implementing

3.2 The variables

4 Analysis of the results

Works Cited

[1] Boris Gourévitch. *François Viete*. Apr. 2013. URL: http://www.pi314.net/fr/viete.php.

[2] Rick Kreminski. "π to Thousands of Digits from Vieta's Formula". In: *Mathematics Magazine* 81.3 (2008), p. 201. ISSN: 0025570X, 19300980. URL: http://www.jstor.org/stable/27643107.