

# Superconducting Integrated Circuits to Further Moore's Law

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## Abstract

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## 1. Introduction

From the first commercially available microprocessor in 1971, the Intel 4004 with 2250 transistors operating at a peak of 740 kHz [1], to Apple's M2 Ultra chip in 2022 with 134 billion transistors operating at up to 3.7 GHz [2], the semiconductor industry has seen exponential growth in computing power. With a doubling period of approximately 2 years, as postulated by Moore's Law [3]. However, as the industry approaches limitations in transistor scaling and power density, the need for alternatives to Complementary Metal Oxide Semiconductor (CMOS) becomes increasingly apparent. Superconducting electronics, particularly those based on Josephson Junctions (JJs) producing Single Flux Quantum (SFQ) logic, present a promising avenue to continue this trend.

These computational architectures leverage the incredibly low energy dissipation and high-speed switching capabilities of superconducting materials to achieve performance metrics. These overcome the current power density thermal limitations that apply to traditional CMOS based devices [4]. SFQs using JJs operate on voltage pulses with areas of magnetic flux quanta  $\Phi_0 = h/(2e) \approx 2.07 \times 10^{-15} \text{ Tm}^2$ , dramatically reducing non-cooling energy consumption [5]. In tandem the rapid voltage pulses allow for switching speeds in the order of 100s of GHz [6], far exceeding the capabilities of CMOS transistors.

Although potentially impressive, superconducting electronics still face a long road to the reach the same level of integration and manufacturability as CMOS technology. Currently maximum Superconducting Integrated Circuit (SIC) densities are in the order of  $10^6$  JJs per  $\text{cm}^2$  [7]. This puts current chipsets with JJ counts in the tens of millions, combined with the need for Helium refrigeration to 9K, makes current architectures unattractive to pursue commercially. However, the industry is beginning to reach the point where SFQ architectures are becoming well defined enough to soon

warrant investment into the kind of design automation languages that have been the boon of the CMOS industry.

## 2. Background

### 2.1 Discovery of Superconductivity

The field of superconductivity was born accidentally in 1911 when Heike Kamerlingh Onnes [8] managed to liquefy helium, opening up temperatures down to 4.2 K, the lowest of any element at normal atmospheric pressure. In a traditional conductor as described by Drude theory [9], the resistivity of a metal can be modelled by considering the the low density "gas" of conduction electrons, excited into empty states above the Fermi level  $\epsilon_F$  for a given temperature  $T$ . Where the resistivity  $\rho$  is given by

$$\rho = \frac{m}{ne^2} (\tau_{\text{imp}}^{-1} + \tau_{\text{e-e}}^{-1} + \tau_{\text{e-ph}}^{-1}) \quad (1)$$

where  $m$  the effective mass of the conduction electrons,  $n$  the density of conduction electrons,  $e$  the electron charge, and  $\tau_{\text{imp}}^{-1}$  the scattering rate due to impurities are all fixed for a given metal. While the other two scattering processes  $\tau_{\text{e-e}}^{-1}$  and  $\tau_{\text{e-ph}}^{-1}$  depend on temperature. This leaves a minimum residual resistivity at low temperatures due to impurity scattering. However, this was not what Onnes observed as evident from his illustrations shown in Figure 1. This alluded to some structural or thermodynamic transition occurring in the metal. Four decades later, the first complete microscopic understanding of this transition was achieved with the Bardeen-Cooper-Schrieffer (BCS) theory [10].

### 2.2 BCS Theory

In order to properly describe superconductivity BCS theory [10] aimed to describe all the attributes of superconductors. This included the Meissner effect of  $\mathbf{B} = 0$  inside the superconductor, the existence of an energy gap  $\Delta$  in the

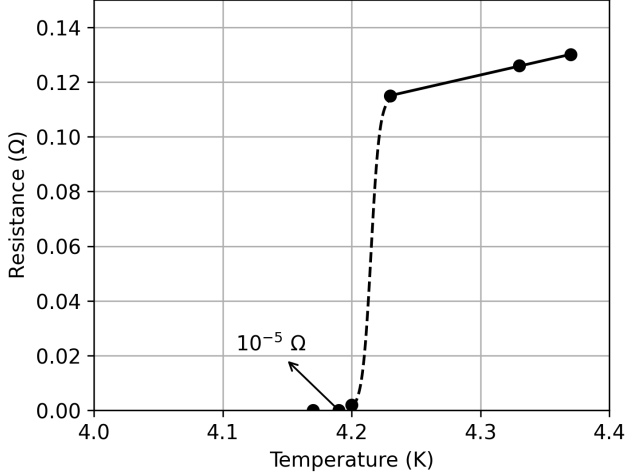


Figure 1. From this illustration of Onnes' results [8], while first cooling down a mercury sample, the resistivity suddenly drops to the lower limit of the instrumentation at a critical temperature  $T_C$  of around 4.2 K, indicating the onset of superconductivity.

electronic density of states at the Fermi level, and the seeming infinite conductivity  $E = 0$ .

The first step as described by J.F. Annett [9] in achieving this was to consider the interaction between conduction electrons and the phonons of the ionic crystal lattice. To consider these electrons can be viewed as quasiparticles of a self-consistent field of surrounding particles as described by Landau's Fermi liquid theory [11]. By considering the Coulomb interaction between quasiparticle excitations of electrons and the holes they leave behind, much of the repulsion can be screened out as

$$V_{TF}(\mathbf{r} - \mathbf{r}') = \frac{e^2}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|} e^{-|\mathbf{r} - \mathbf{r}'|/r_{TF}}, \quad (2)$$

where  $r_{TF}$  is the Thomas-Fermi screening length. This makes the effective repulsive interaction  $V_{TF}$  short ranged vanishing for larger than inter-particle spacings  $|\mathbf{r} - \mathbf{r}'| > r_{TF}$ . This is combined with the attractive interaction mediated by phonons as shown by Annett in Figure 2, to give an overall effective potential between electron quasiparticles.

Excluding the repulsive regime, where their frequencies  $\omega$  are greater than the average phonon (Debye) frequency  $\omega_D$ . Then only looking at conduction electrons within  $\epsilon_F \pm k_B T$  and in the superconducting regime  $\hbar\omega_D \gg k_B T$  (appendix AA), simplifying the interaction to

$$V_{eff}(\mathbf{q}, \omega) = -|g_{eff}|^2, \quad |\omega| < \omega_D \quad (3)$$

where  $g_{eff}$  is an effective electron-phonon coupling constant. This effective attraction enables the proposition of two electrons outside the Fermi sea to form a bound state, known as a Cooper pair [9], [10]. By solving the two-particle Schrodinger equation over all available states around the Fermi level, BCS theory shows that the binding energy of the Cooper pair is given by

$$-E = 2\hbar\omega_D e^{-1/\lambda}, \quad (4)$$

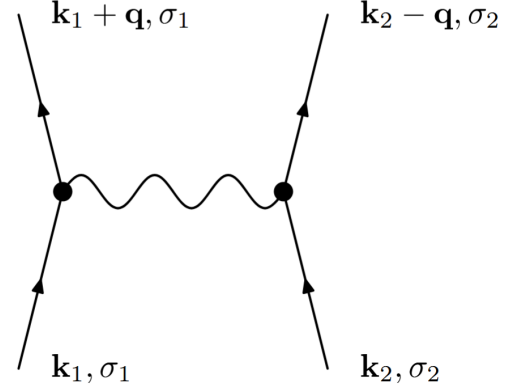


Figure 2. An excited electron scattered from a state with crystal momentum  $\mathbf{k}_{1,2}$  creates a phonon with momentum  $\pm\mathbf{q}$  which is then absorbed by another electron with momentum  $\mathbf{k}_{2,1}$ , resulting in an effective attractive transfer of momentum  $\hbar\mathbf{q}$  between the two electrons. The order of this operation is irrelevant to the interaction [9].

where  $\lambda = |g_{eff}|^2 g(\epsilon_F) \ll 1$  is the dimensionless electron-phonon coupling constant and is assumed to be small.

To find the overall ground state and energy gap  $\Delta$ , creation and annihilation operators can be defined for Cooper pairs as

$$\hat{P}_{\mathbf{k}}^\dagger = c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger \quad \& \quad \hat{P}_{\mathbf{k}} = c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \quad (5)$$

respectively, where the operators commute with themselves as long as they are for different momenta  $\mathbf{k} \neq \mathbf{k}'$ . Using these commutative operators Cooper pairs can be added sequentially to the ground state. By expanding and considering annihilation operators as available for hole Cooper pairs [9] the BCS ground state can be written as

$$|\Psi_{BCS}\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} c_{-\mathbf{k}\downarrow} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger) |0\rangle. \quad (6)$$

### 2.3 Josephson Effect

Taking a step further, Brian D. Josephson considered the tunnelling of Cooper pairs between two superconductors separated by a thin insulating barrier [12]. By considering the wavefunction on either side of the barrier

## 3. Recent Developments

### 3.1 SFQ logic stuff

- bias in JJs

### 3.2 ELASIC

### 3.3 SuperNPU

## 4. Conclusion

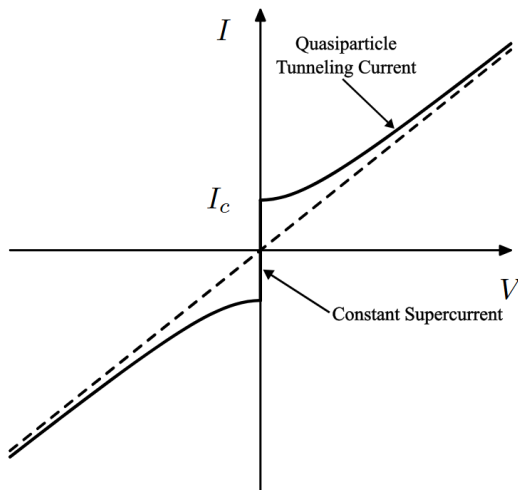


Figure 3. The IV characteristics of an overdamped Josephson junction [9]. Below the critical current  $I_c$ , a supercurrent  $I_S$  can flow with zero voltage drop  $V = 0$  or dissipation. Above  $I_c$ , quasiparticle creation and annihilation occurs across the gap resulting in a voltage drop and a phase dependent oscillating current  $I_N$ .

- Consider why cuprates are not used in superconducting electronics despite their high  $T_C$ . Manufacturing difficulty /  $I_C$  limited by inter-grain current flows due to current manufacturing processes.

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Appendix

A Trivia ..... 4

AA Isotope Effect ..... 4

A Trivia

AA Isotope Effect

This is what defines the isotope effect as the band of available states around the Fermi sea thickens with decreasing ionic mass  $M$ , as  $\omega_D \sim \sqrt{k/M}$ .