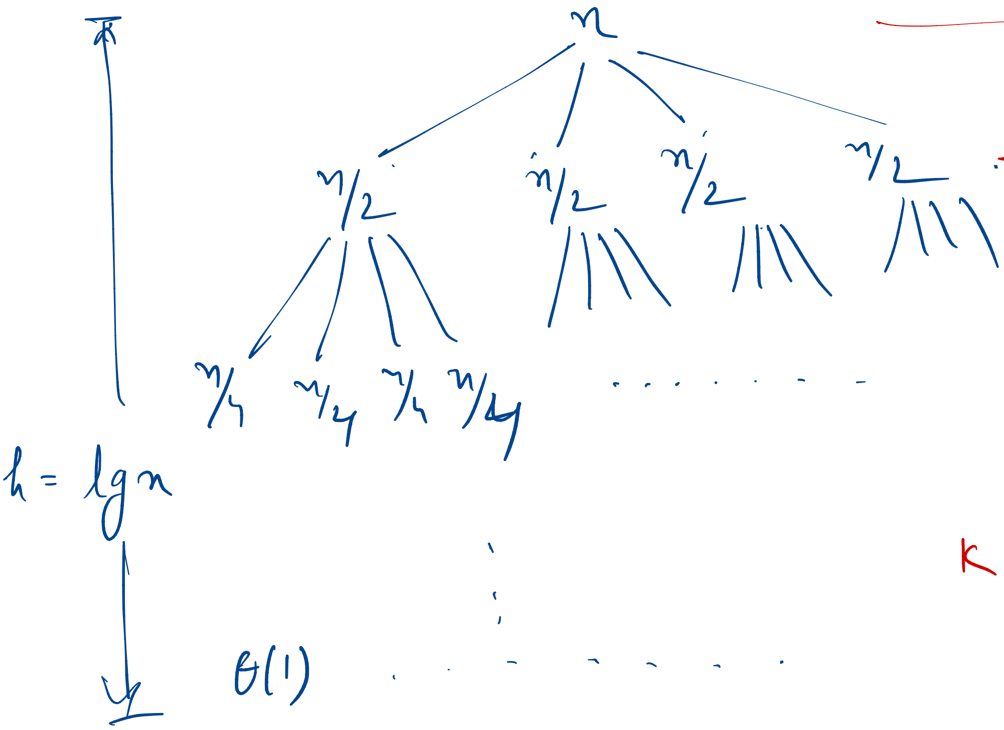
Substitution Method : Revision.

$$T(n) = 4\,T(n/2) + n$$



$\longrightarrow \; n \qquad L_0$

$\longrightarrow \; 4^1 \cdot \dfrac{n}{2^1} = 2n \qquad L_1$

$\longrightarrow \; 4^2 \cdot \dfrac{n}{2^2} = 4n$

$\longrightarrow \; 4^3 \cdot \dfrac{n}{2^3} = 8n$

$k \longrightarrow \; 4^k \cdot \dfrac{n}{2^k} = 2^k n$

$$T(n) = n + 2n + 2^2 n + 2^3 n + \dots + 2^k n + \dots 2^h$$

$$= n\underbrace{\left(1 + 2^1 + 2^2 + \dots + 2^h\right)}_{\lg n} \longrightarrow 2^{(\lg n - 1)}$$

$$= n\left(\sum_{i=0}^{\lg n - 1} 2^i\right) = n\left(2^{\lg n} - 1\right)$$

$$= n(n-1)$$

$$= O(n^2)$$

$$T(n) = 4\,T(n/2) + n$$

I. H.:
assume: $T(k) = O(n^2)$
$\Rightarrow T(k) \leq cn^2$
for $c > 0$

$$\leq 4c\left(n/2\right)^2 + n$$

$$= cn^2 + n$$

$$= \underbrace{cn^2}_{\text{desired}} - \underbrace{(-n)}_{\text{residual}}$$

$$\leq cn^2 \quad \times$$

Wrong!

$\Rightarrow$ Change the IH to include lower order terms.

IH: $\quad T(k) \leq cn^2 - dn$

$$T(n) = 4\,T(n/2) + n$$

$$\leq 4\left(c\left(\frac{n}{2}\right)^2 - d\frac{n}{2}\right) + n$$

$$= cn^2 - 2dn + n$$

$$= \underbrace{cn^2 - dn}_{\text{desired}} - \underbrace{(dn - n)}_{\text{residual}}$$

$$\leq cn^2 - dn$$

whenever, $\quad dn - n \geq 0 \Rightarrow \underline{d \geq 1}, \; c > 0$

$$\Rightarrow \quad T(n) = O(n^2)$$

Pitfall: show by substitution

$$T(n) = 2T(\lfloor n/2 \rfloor) + n$$

false assumption, $T(n) = O(n)$

IH: $T(k) = O(k)$

$\Rightarrow T(k) < ck$ for $c > 0$

Prove $T(n) \le 2(c\lfloor n/2 \rfloor) + n$

$\le cn + n$

$= O(n)$ $\times$ wrong!

$= cn - (-n)$

Desired residual