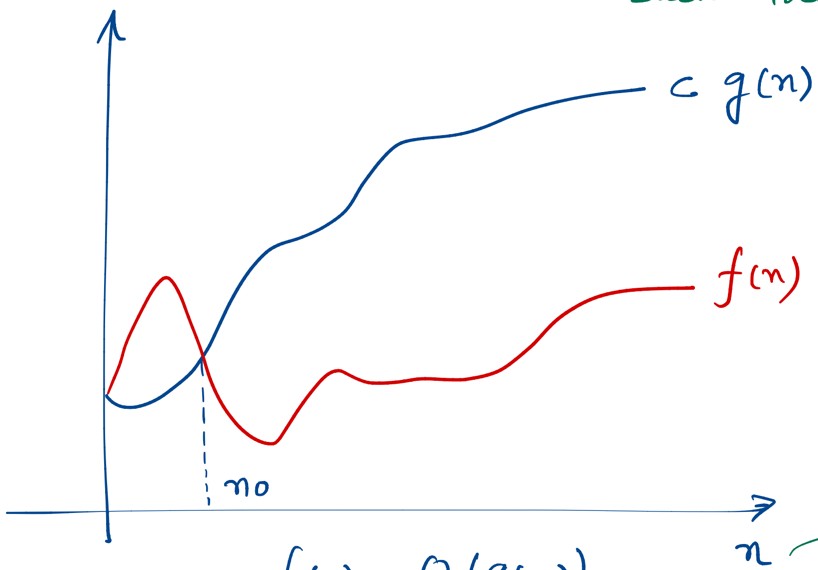
① $f(n) = O(g(n))$: Asymptotic upper bound

$$O(g(n)) = \{ f(n) : \underbrace{\exists}_{\text{There exists}} \text{ constants } c > 0,\ n_0 > 0, \ \underbrace{\text{s.t.}}_{\text{such that}} \ 0 \le f(n) \le c\,g(n) \ \underbrace{\forall}_{\text{for all}}\, n > n_0 \}$$

cool



$f(n) = O(g(n))$

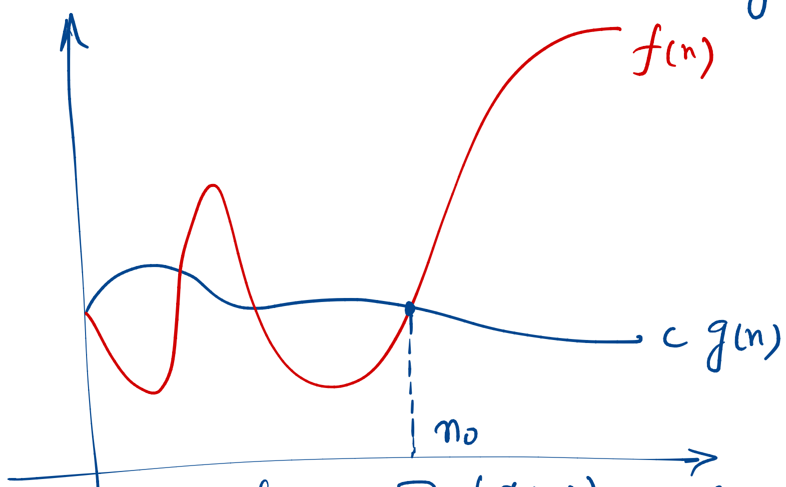
$n \to$ input size

symbolic

$f(n) \le g(n)$

② $f(n) = \Omega(g(n))$: Asymptotic lower bound

$$\Omega(g(n)) = \{ f(n) : \exists \text{ constants } c > 0,\ n_0 > 0 \text{ s.t. } 0 \le c\,g(n) \le f(n) \ \forall\, n > n_0 \}$$



$f(n) = \Omega(g(n))$

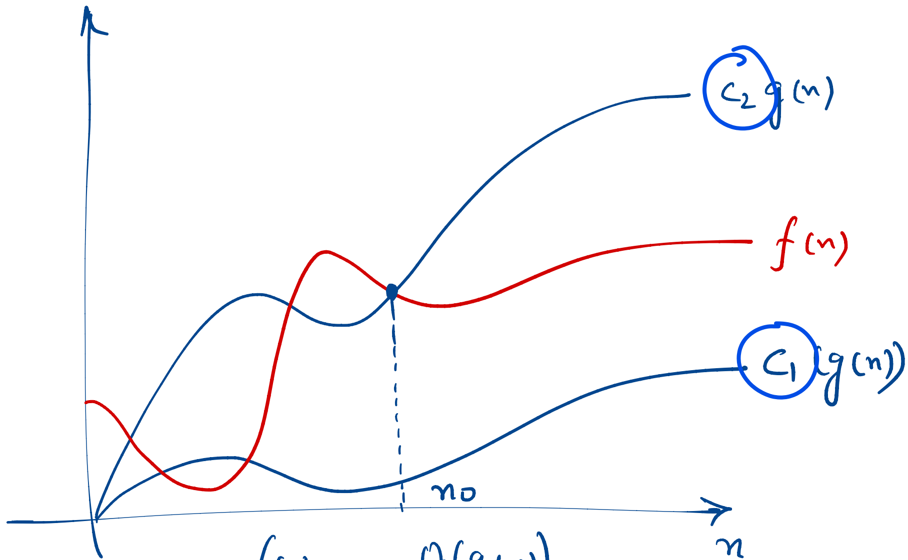
symbolically

$f(n) \ge g(n)$

(3) $f(n) = \Theta(g(n))$: Asymptotic tight bound

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$

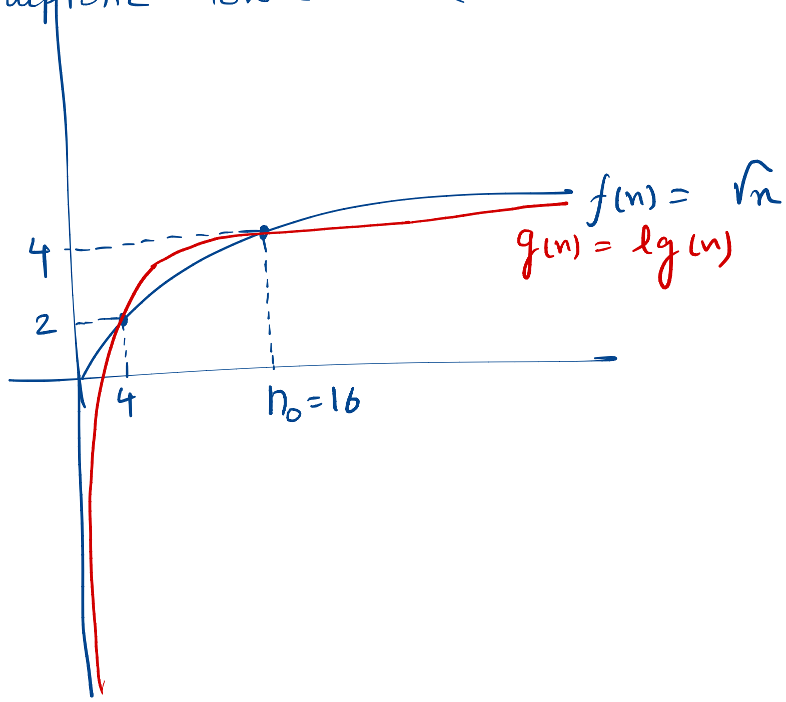
$$\Theta(g(n)) = \{ f(n) : \exists \text{ constants } c_1, c_2, n_0 > 0, \text{ s.t. } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \ \forall n \ge n_0 \}$$



$f(n) = \Theta(g(n))$

$\sqrt{n} = \Omega(\lg(n))$ $\quad c = 1, \ n_0 = 16$

Ex. Asymptotic lower bound : Ω



Ex. Asymptotic tighter bound : Θ

$f(n) = \frac{1}{2}n^2 - 2n$ $\qquad g(n) = n^2$ $\qquad c_2 = 3/4$

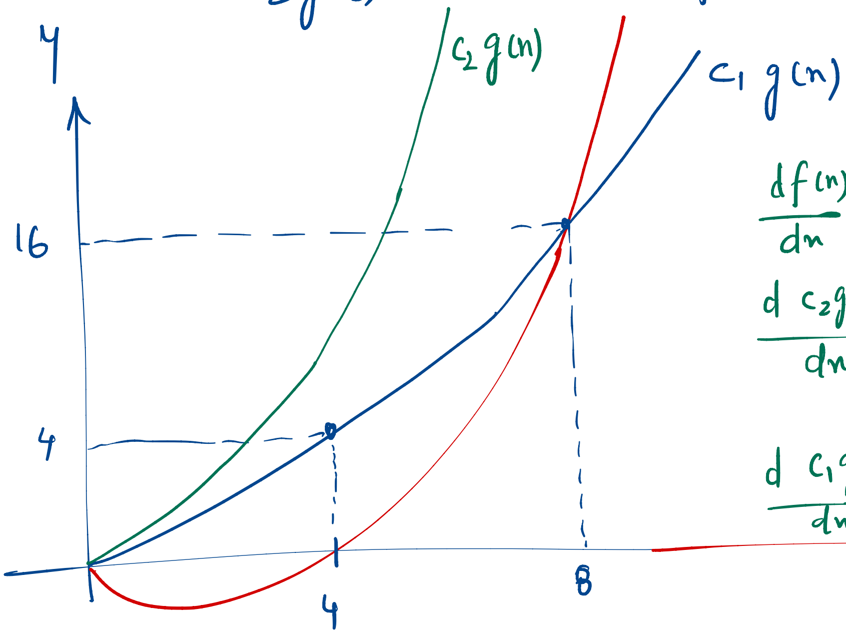
$n\left(\frac{n}{2} - 2\right)$ $\qquad c_1 = 1/4$

$c_1\, g(n) = n^2/4$, $\quad c_2\, g(n) = \frac{3n^2}{4}$ $\qquad n_0 = 8$

$\downarrow \qquad \downarrow$

$c_1\, g(n_0) = 16$ $\qquad c_2\, g(n_0) = 48$ $\qquad f(n_0) = 32 - 16 = 16$

$c_1\, g(4) = 4$



$\frac{df(n)}{dn} = n - 2\Big|_{n=8} = 6$

$\frac{d\, c_2 g(n)}{dn} = \frac{3n}{2}\Big|_{n=8} = 12$

$\frac{d\, c_1 g(n)}{dn} = \frac{n}{2}\Big|_{n=8} = 4$

Solving Recurrence Relations

3 methods:

1. Substitution method
2. Recursion tree method
3. master method