Chapter 7: Normalization

Database System Concepts, 7th Ed.

Outline

- Features of Good Relational Design
- Functional Dependencies
- Decomposition Using Functional Dependencies
- Normal Forms
- Functional Dependency Theory
- Algorithms for Decomposition using Functional Dependencies
- Decomposition Using Multivalued Dependencies
- More Normal Form
- Atomic Domains and First Normal Form
- Database-Design Process
- Modeling Temporal Data

Overview of Normalization

Can you see the problems in design of this table?

- Suppose *Emp_Dept* relation as follows:

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

- There is repetition of information
- If we add a new department with no instructors – ???
  - Need to use null values

Can you see the problems in design of this table?

- Suppose *Emp_Dept* relation as follows:

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
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83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

- Deletion anomaly
  - Deleting an employee ‘Mozart’ leads to delete “Music’ dept info.
- Insertion anomaly
  - Cannot insert a new dept unless an employee is assigned to.
- Update anomaly
  - Changing the dept_building from ‘Painter’ to ‘Gary’ may cause this update to be made for all employess working on ‘Painter’ dept.

Relational DB Design guidelines (informal)

- 4 informal guidelines for good relational design
  - Making sure that the semantics of the attributes is clear in the schema
  - Reducing the redundant information in tuples
  - Reducing the NULL values in tuples
  - Disallowing the possibility of generating spurious tuples

Decomposition

- The only way to avoid the repetition-of-information problem in the *in_dep* schema is to decompose it into two schemas – instructor and *department* schemas.
- Not all decompositions are good. Suppose we decompose

  employee(ID, name, street, city, salary)

  into

  employee1 (*ID*, *name*)

  employee2 (*name*, *street, city, salary*)

  The problem arises when we have two employees with the same name
- The next slide shows how we lose information -- we cannot reconstruct the original *employee* relation -- and so, this is a **lossy decomposition**.

A Lossy Decomposition

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

employee

ID	name
⋮	
57766	Kim
98776	Kim
⋮	

name	street	
⋮		
Kim	Main	Pe
Kim	North	H
⋮		

natural join

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
57766	Kim	North	Hampton	67000
98776	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

Lossless Decomposition

- Let R be a relation schema and let R_1 and R_2 form a decomposition of R . That is R = R_1 U R_2
- We say that the decomposition is a **lossless decomposition** if there is no loss of information by replacing R with the two relation schemas R_1 U R_2
- Formally,

$$\prod_{R1}(r) \bowtie \prod_{R2}(r) = r$$

- And, conversely a decomposition is lossy if

$$r \subset \prod_{R1}(r) \bowtie \prod_{R2}(r) = r$$

Example of Lossless Decomposition

- Decomposition of *R = (A, B, C)*
  $R_1 = (A, B)$ $R_2 = (B, C)$

A	*B*	*C*
α	1	A
β	2	B

r

A	*B*
α	*1*
β	*2*

$\Pi_{A,B}(r)$

B	*C*
1	A
2	B

$\Pi_{B,C}(r)$

$\Pi_A (r) \bowtie \Pi_B (r)$

A	*B*	*C*
α	1	A
β	2	B

Normalization Theory

- Decide whether a particular relation *R* is in “good” form.
- In the case that a relation *R* is not in “good” form, decompose it into set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - Each relation is in good form
  - The decomposition is a lossless decomposition
- Our theory is based on:
  - Functional dependencies
  - Multivalued dependencies

Functional Dependencies

- There are usually a variety of constraints (rules) on the data in the real world.
- For example, some of the constraints that are expected to hold in a university database are:
  - Students and instructors are uniquely identified by their ID.
  - Each student and instructor has only one name.
  - Each instructor and student is (primarily) associated with only one department.
  - Each department has only one value for its budget, and only one associated building.

Functional Dependencies (Cont.)

- Functional dependencies is a constraint between 2 sets of attributes
- An instance of a relation that satisfies all such real-world constraints is called a **legal instance** of the relation;
- A legal instance of a database is one where all the relation instances are legal instances
- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key.*

Functional Dependencies Definition

- Let R be a relation schema

  $\alpha \subseteq R$ *and* $\beta \subseteq R$

- The **functional dependency**

  $\boldsymbol{\alpha \rightarrow \beta}$

  holds on R if and only if for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

  $$t_1[\alpha] = t_2[\alpha] \;\Rightarrow\; t_1[\beta] = t_2[\beta]$$

- Example: Consider r(A,B) with the following instance of r.

1	4
1	5
3	7

- On this instance, $B \rightarrow A$ hold; $A \rightarrow B$ does **NOT** hold,

Closure of a Set of Functional Dependencies

- Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*. For example,
  - If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow$ C

- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by ***F*⁺**.

Closure of a Set of Functional Dependencies

- We can compute F^+, the closure of F, by repeatedly applying **Armstrong's Axioms:**
  - **Reflexive rule:** if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
  - **Augmentation rule:** if $\alpha \rightarrow \beta$, then $\gamma\, \alpha \rightarrow \gamma\, \beta$
  - **Transitivity rule:** if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
- These rules are
  - **Sound** -- generate only functional dependencies that actually hold, and
  - **Complete** -- generate all functional dependencies that hold.

Example of F^+

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- Some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$
      and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,
      and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
      and then transitivity

Closure of Functional Dependencies (Cont.)

- Additional rules:
  - **Union rule**: If $\alpha \rightarrow \beta$ holds *and* $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds.
  - **Decomposition rule**: If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds.
  - **Pseudotransitivity rule**:If $\alpha \rightarrow \beta$ holds *and* $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds.
- The above rules can be inferred from Armstrong's axioms.

Procedure for Computing F^+

- To compute the closure of a set of functional dependencies F:

  $F^+ = F$
  repeat
  for each functional dependency *f* in F^+
  apply reflexivity and augmentation rules on *f*
  add the resulting functional dependencies to F^+
  for each pair of functional dependencies f_1 and f_2 in F^+
  if f_1 and f_2 can be combined using transitivity
  then add the resulting functional dependency to F^+
  until F^+ does not change any further

- **NOTE**: We shall see an alternative procedure for this task later

Keys and Functional Dependencies

- *K* is a superkey for relation schema *R* if and only if $K \rightarrow R$
- *K* is a candidate key for *R* if and only if
  - $K \rightarrow R$, and
  - for no $\alpha \subset K$, $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

  Emp_dep (<u>*ID*</u>, *name, salary,* <u>*dept_name*</u>*, building, budget*).

  We expect these functional dependencies to hold:

  dept_name$\rightarrow$ *building*

  ID □ *name*

  but would not expect the following to hold:

  dept_name $\rightarrow$ *salary*

  ID □ *building*

Use of Functional Dependencies

- We use functional dependencies to:
  - To test relations to see if they are legal under a given set of functional dependencies.
    - If a relation *r* is legal under a set *F* of functional dependencies, we say that *r* **satisfies** *F.*
  - To specify constraints on the set of legal relations
    - We say that *F* **holds on** *R* if all legal relations on *R* satisfy the set of functional dependencies *F.*
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
  - For example, a specific instance of *instructor* may, by chance, satisfy

    $name \rightarrow ID.$

Trivial Functional Dependencies

- *A* functional dependency is **trivial** if it is satisfied by all instances of a relation
- Example:
  - *ID, name* → *ID*
  - *name* → *name*
- In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

Lossless Decomposition

- We can use functional dependencies to show when certain decomposition are lossless.
- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

  $$r = \prod_{R1}(r) \bowtie \prod_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless decomposition if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
- $R_1 = (A, B)$, $R_2 = (B, C)$
  - Lossless decomposition:

    $$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$
- $R_1 = (A, B)$, $R_2 = (A, C)$
  - Lossless decomposition:

    $$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$
- *Note:*
  - $B \rightarrow BC$

    is a shorthand notation for
  - $B \rightarrow \{B, C\}$

Dependency Preservation

- Testing functional dependency constraints each time the database is updated can be costly
- It is useful to design the database in a way that constraints can be tested efficiently.
- If testing a functional dependency can be done by considering just one relation, then the cost of testing this constraint is low
- When decomposing a relation it is possible that it is no longer possible to do the testing without having to perform a Cartesian Produced.
- A decomposition that makes it computationally hard to enforce functional dependency is said to be NOT **dependency preserving**.

Dependency Preservation Example

- Consider a schema:

  dept_advisor(s_ID, i_ID, department_name)

- With function dependencies:

  $i\_ID \rightarrow dept\_name$

  $s\_ID, dept\_name \rightarrow i\_ID$

- In the above design we are forced to repeat the department name once for each time an instructor participates in a *dept_advisor* relationship.
- To fix this, we need to decompose *dept_advisor*
- Any decomposition will not include all the attributes in

  $s\_ID, dept\_name \rightarrow i\_ID$

- Thus, the decomposition NOT be dependency preserving

Normal Forms

Normalization of Relations

- It is a process of analyzing a given relation schemas based on their FDs and PKs to achieve minimum redundancy and update anomalies

- **Normalization**: The process of decomposing unsatisfactory "bad" relations by breaking up their attributes into smaller relations

- **Normal form**: Condition using keys and FDs of a relation to certify whether a relation schema is in a particular normal form
  - 2NF, 3NF, BCNF based on keys and FDs of a relation schema
  - 4NF based on keys, multi-valued dependencies : MVDs; 5NF based on keys, join dependencies : JDs

- Additional properties may be needed to ensure a good relational design (lossless join, dependency preservation)

Practical Use of Normal Forms

- **Normalization** is carried out in practice so that the resulting designs are of high quality and meet the desirable properties
- The practical utility of these normal forms becomes questionable when the constraints on which they are based are **hard to understand** or to **detect**
- The database designers ***need not*** normalize to the highest possible normal form. (usually up to 3NF, BCNF or 4NF)
- **Denormalization:** the process of storing the join of higher normal form relations as a base relation—which is in a lower normal form

Definitions of Keys and Attributes Participating in Keys

- A **superkey** of a relation schema $R = \{A_1, A_2, ..., A_n\}$ is a set of attributes S *subset-of* R with the property that **no** two tuples t_1 and t_2 in any legal relation state r of R will have $t_1[S] = t_2[S]$

- A **key** K is a superkey with the *additional property* that removal of any attribute from K will cause K not to be a superkey any more.

- If a relation schema has more than one key, each is called a **candidate key.** One of the candidate keys is *arbitrarily* designated to be the **primary key,** and the others are called *secondary keys*.

- A **Prime attribute** must be a member of *some candidate key*

- A **Nonprime attribute** is not a prime attribute—that is, it is not a member of any candidate key.

First Normal Form (1NF)

- Disallows composite attributes, multivalued attributes, and **nested relations**; attributes whose values *for an individual tuple* are non-atomic

- Considered to be part of the formal definition of a relation in the basic relational model

Normalization into 1NF

(a)

DEPARTMENT

Dname	Dnumber	Dmgr_ssn	Dlocations

(b)

DEPARTMENT

Dname	Dnumber	Dmgr_ssn	Dlocations
Research	5	333445555	{Bellaire, Sugarland, Houston}
Administration	4	987654321	{Stafford}
Headquarters	1	888665555	{Houston}

(c)

DEPARTMENT

Dname	Dnumber	Dmgr_ssn	Dlocation
Research	5	333445555	Bellaire
Research	5	333445555	Sugarland
Research	5	333445555	Houston
Administration	4	987654321	Stafford
Headquarters	1	888665555	Houston

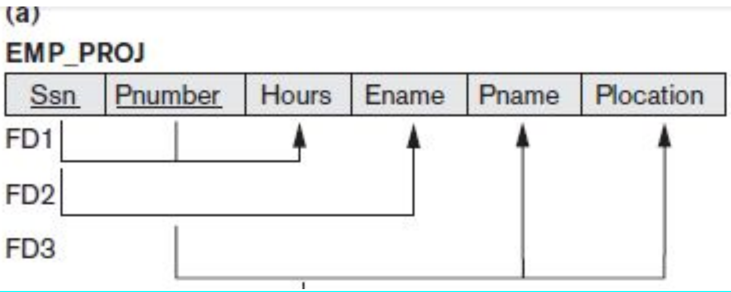
Figure 15.9
Normalization into 1NF. (a) A relation schema that is not in 1NF. (b) Sample state of relation DEPARTMENT. (c) 1NF version of the same relation with redundancy.

Second Normal Form (2NF)

- A relation schema R is in **second normal form (2NF)** if every non-prime attribute A in R is fully functionally dependent on the primary key

- R can be decomposed into 2NF relations via the process of 2NF normalization

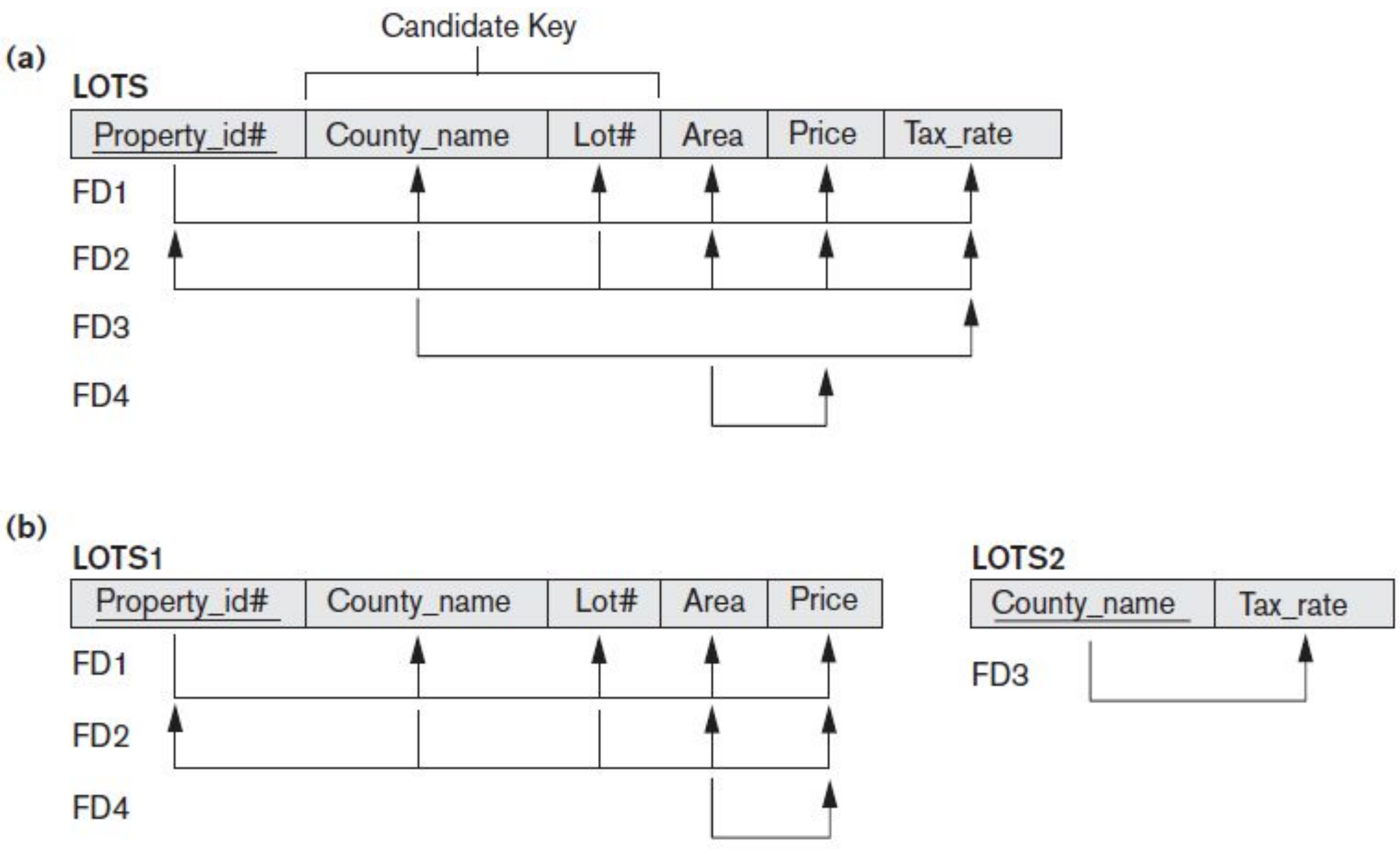
Normalizing into 2NF



- Ename violates 2NF because of FD2,
- Pname and Plocation because of FD3 also violates 2NF.
- The relation schema can be *second normalized* or *2NF normalized* into a number of 2NF relations. FD1, FD2, and FD3 lead to the decomposition of EMP_PROJ into the three relation schemas EP1, EP2, and EP3, each of which is in 2NF.

General Definition of 2NF

Definition. A relation schema *R* is in **second normal form** (2NF) if every nonprime attribute *A* in *R* is not partially dependent on *any* key of *R*.[11]



Third Normal Form (3NF)

- A relation schema R is in **third normal form (3NF)** if it is in 2NF *and* no non-prime attribute A in R is transitively dependent on the primary key
- Alternatively, a relation schema R is in **third normal form (3NF)** if whenever a FD X **->** A holds in R, then either:
  (a) X is a superkey of R, or
  (b) A is a prime attribute of R

- R can be decomposed into 3NF relations via the process of 3NF normalization

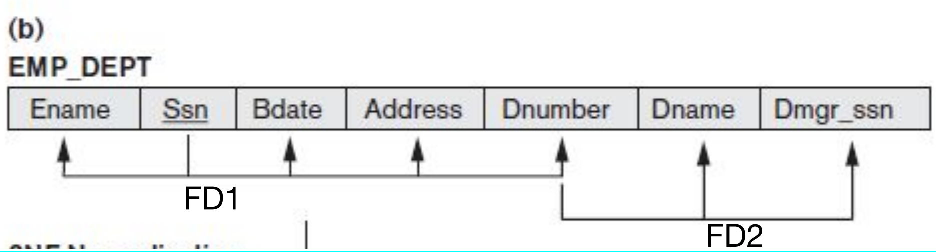
NOTE:

In X -> Y and Y -> Z, with X as the primary key, we consider this a problem only if Y is <u>not</u> a candidate key. When Y is a candidate key, there is no problem with the transitive dependency .

E.g., Consider EMP (SSN, Emp#, Salary).

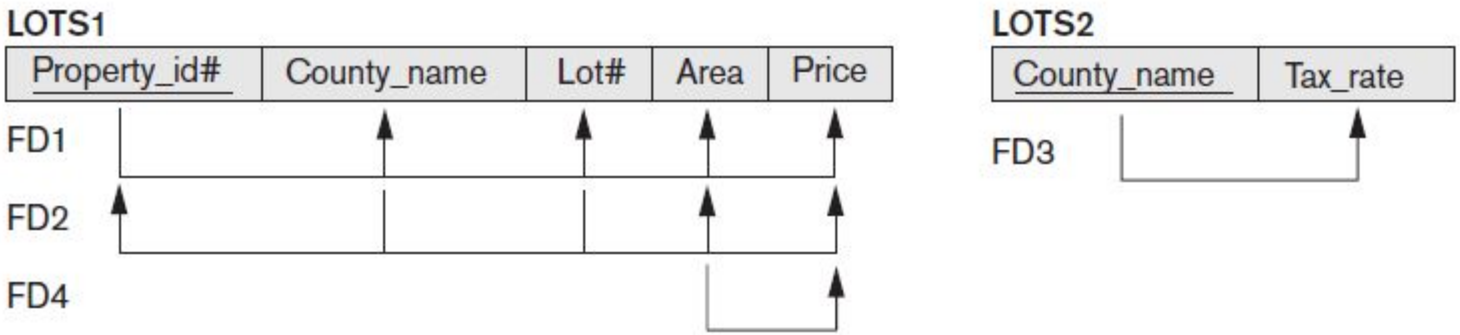
Here, SSN -> Emp# -> Salary and Emp# is a candidate key.

Normalizing into 3NF

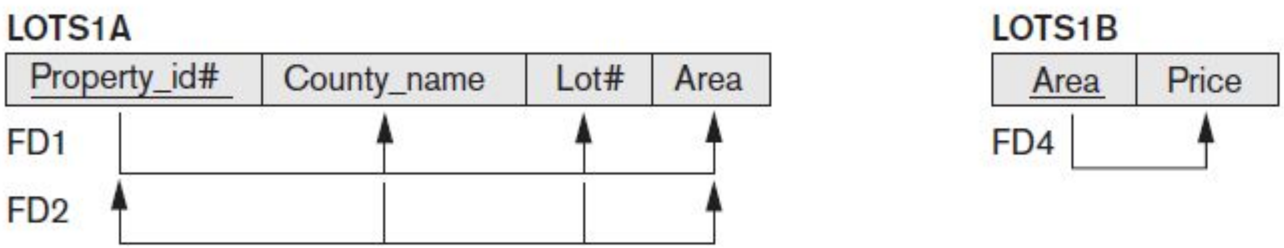


- Dname or Dmgr_ssn violates 3NF because of Ssn as PK in FD1,
- The relation schema can be *normalized* into 3NF relations. FD1, FD2 lead to the decomposition of EMP_DEPT into the two relation schemas ED1,and ED3, each of which is in 3NF.

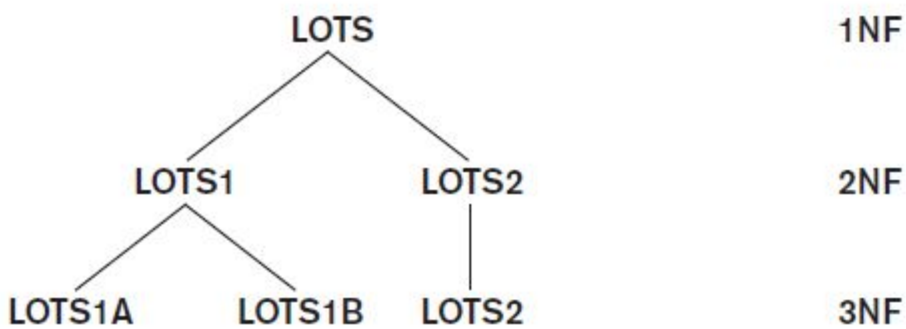
(b)


LOTS1
Property_id#
County_name
Lot#
Area
Price
FD1
FD2
FD4
LOTS2
County_name
Tax_rate
FD3

(c)


LOTS1A
Property_id#
County_name
Lot#
Area
FD1
FD2
LOTS1B
Area
Price
FD4

(d)


LOTS
1NF
LOTS1
LOTS2
2NF
LOTS1A
LOTS1B
LOTS2
3NF

General Definitions of Second and Third Normal Forms

Table 15.1 Summary of Normal Forms Based on Primary Keys and Corresponding Normalization

Normal Form	Test	Remedy (Normalization)
First (1NF)	Relation should have no multivalued attributes or nested relations.	Form new relations for each multivalued attribute or nested relation.
Second (2NF)	For relations where primary key contains multiple attributes, no nonkey attribute should be functionally dependent on a part of the primary key.	Decompose and set up a new relation for each partial key with its dependent attribute(s). Make sure to keep a relation with the original primary key and any attributes that are fully functionally dependent on it.
Third (3NF)	Relation should not have a nonkey attribute functionally determined by another nonkey attribute (or by a set of nonkey attributes). That is, there should be no transitive dependency of a nonkey attribute on the primary key.	Decompose and set up a relation that includes the nonkey attribute(s) that functionally determine(s) other nonkey attribute(s).

Boyce-Codd Normal Form (BCNF)

- A relation schema R is in **Boyce-Codd Normal Form** (**BCNF**) if whenever an FD X **->** A holds in R, then X is a superkey of R
- Each normal form is strictly stronger than the previous one
  - Every 2NF relation is in 1NF
  - Every 3NF relation is in 2NF
  - Every BCNF relation is in 3NF
- There exist relations that are in 3NF but not in BCNF
- Most relation schemas that are in 3NF are also in BCNF
- The goal is to have each relation in BCNF (or 3NF)

Boyce-Codd normal form – An example

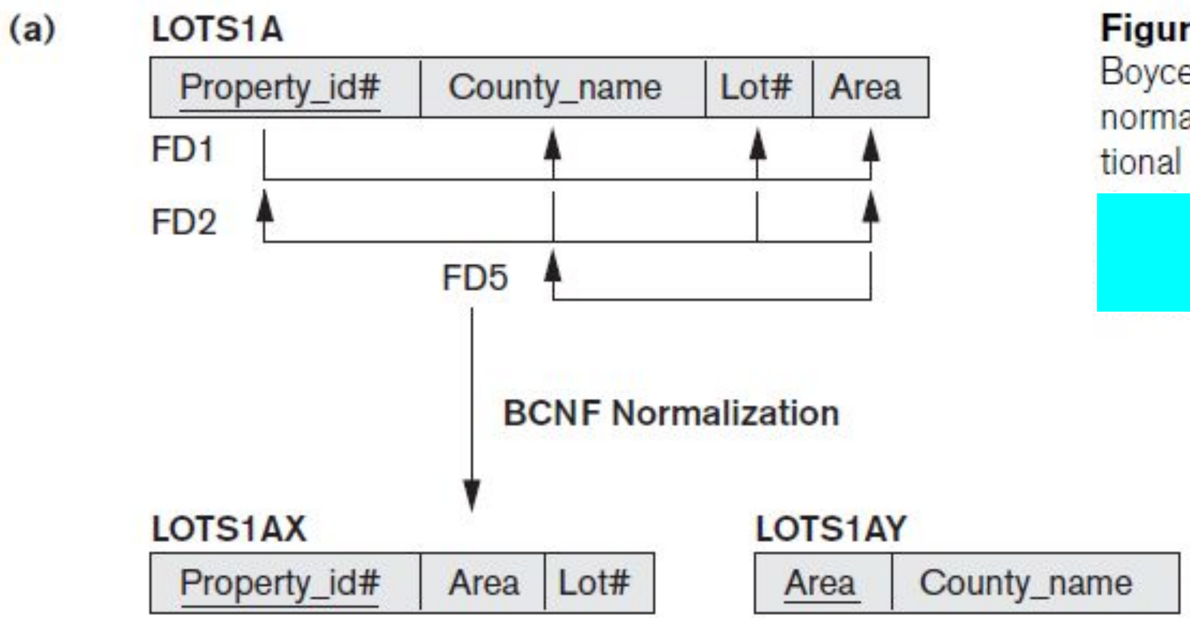


Figure 15.13
Boyce-Codd normal form. (a) BCNF normalization of LOTS1A with the functional dependency FD2 being lost in

Backup

Normal Forms

Boyce-Codd Normal Form

- A relation schema R is in BCNF with respect to a set F of functional dependencies if for all functional dependencies in F^+ of the form

  $$\alpha \rightarrow \beta$$

  where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:
  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
  - α is a superkey for R

Boyce-Codd Normal Form (Cont.)

- Example schema that is ***not*** in BCNF:

  in_dep (<u>*ID*,</u> *name, salary*<u>, *dept_name*, </u>*building, budget*)

  because :

  - *dept_name*→ *building, budget*
    - holds on *in_dep*
    - but
  - *dept_name* is not a superkey
- When decompose *in_dept* into *instructor* and *department*
  - *instructor* is in BCNF
  - *department* is in BCNF

Decomposing a Schema into BCNF

- Let R be a schema *R* that is not in BCNF. Let $\alpha \rightarrow \beta$ be the FD that causes a violation of BCNF.
- We decompose *R* into:
  - (α U β)
  - (*R* - (β - α))
- In our example of *in_dep*,
  - α = *dept_name*
  - β = *building, budget*

  and *in_dep* is replaced by
  - (α U β) = (*dept_name, building, budget*)
  - (*R* - (β - α)) = (*ID, name, dept_name, salary*)

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition:

    $R_1 \cap R_2 = \{B\}$ and $B \rightarrow BC$
  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition:

    $R_1 \cap R_2 = \{A\}$ and $A \rightarrow AB$
  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

BCNF and Dependency Preservation

- It is not always possible to achieve both BCNF and dependency preservation
- Consider a schema:

  dept_advisor(s_ID, i_ID, department_name)
- With function dependencies:

  $i\_ID \rightarrow dept\_name$

  $s\_ID, dept\_name \rightarrow i\_ID$
- *dept_advisor* is not in BCNF
  - *i_ID* is not a superkey.
- Any decomposition of *dept_advisor* will not include all the attributes in

  $s\_ID, dept\_name \rightarrow i\_ID$
- Thus, the composition is NOT be dependency preserving

Third Normal Form

- A relation schema R is in **third normal form (3NF)** if for all:

  $\alpha \rightarrow \beta$ in F^+

  at least one of the following holds:
  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
  - α is a superkey for R
  - Each attribute A in $\beta - \alpha$ is contained in a candidate key for R.

  (**NOTE***:* each attribute may be in a different candidate key)
- If a relation is in BCNF it is in 3NF (since in BCNF one of the first two conditions above must hold).
- Third condition is a minimal relaxation of BCNF to ensure dependency preservation (will see why later).

3NF Example

- Consider a schema:

  dept_advisor(s_ID, i_ID, dept_name)
- With function dependencies:

  i_ID → *dept_name*

  s_ID, dept_name → *i_ID*
- Two candidate keys = {*s_ID, dept_name*}, {*s_ID, i_ID* }
- We have seen before that *dept_advisor* is not in BCNF
- *R,* however, is in 3NF
  - *s_ID, dept_name* is a superkey
  - *i_ID* → *dept_name* *and* *i_ID* is NOT a superkey, but:
    - { *dept_name*} – {*i_ID* } = {*dept_name* } and
    - *dept_name* is contained in a candidate key

Redundancy in 3NF

- Consider the schema R below, which is in 3NF
  - $R = (J, K, L)$
  - $F = \{JK \rightarrow L, L \rightarrow K\}$
  - And an instance table:

J	L	K
j_1	l_1	k_1
j_2	l_1	k_1
j_3	l_1	k_1
null	l_2	k_2

- What is wrong with the table?
  - Repetition of information
  - Need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J)

Comparison of BCNF and 3NF

- Advantages to 3NF over BCNF. It is always possible to obtain a 3NF design without sacrificing losslessness or dependency preservation.
- Disadvantages to 3NF.
  - We may have to use null values to represent some of the possible meaningful relationships among data items.
  - There is the problem of repetition of information.

Goals of Normalization

- Let R be a relation scheme with a set F of functional dependencies.
- Decide whether a relation scheme R is in "good" form.
- In the case that a relation scheme R is not in "good" form, need to decompose it into a set of relation scheme $\{R_1, R_2, ..., R_n\}$ such that:
  - Each relation scheme is in good form
  - The decomposition is a lossless decomposition
  - Preferably, the decomposition should be dependency preserving.

How good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a relation

  inst_info (ID, child_name, phone)

  - where an instructor may have more than one phone and can have multiple children
  - Instance of *inst_info*

ID	*child_name*	*phone*
99999	David	512-555-1234
99999	David	512-555-4321
99999	William	512-555-1234
99999	William	512-555-4321

How good is BCNF? (Cont.)

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if we add a phone 981-992-3443 to 99999, we need to add two tuples

  (99999, David, 981-992-3443)
  (99999, William, 981-992-3443)

Higher Normal Forms

- It is better to decompose *inst_info* into:
    - *inst_child*:

ID	*child_name*
99999	David
99999	William

    - *inst_phone:*

ID	*phone*
99999	512-555-1234
99999	512-555-4321

- This suggests the need for higher normal forms, such as Fourth Normal Form (4NF), which we shall see later

Functional-Dependency Theory

Functional-Dependency Theory Roadmap

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving

Closure of a Set of Functional Dependencies

- Given a set F set of functional dependencies, there are certain other functional dependencies that are logically implied by F.
  - If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow$ C
  - etc.
- The set of **all** functional dependencies logically implied by F is the **closure** of F.
- We denote the *closure* of F by $\boldsymbol{F^+}$.

Closure of a Set of Functional Dependencies

- We can compute F^+, the closure of F, by repeatedly applying **Armstrong's Axioms:**
  - **Reflexive rule:** if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
  - **Augmentation rule:** if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$
  - **Transitivity rule:** if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
- These rules are
  - **Sound** -- generate only functional dependencies that actually hold, and
  - **Complete** -- generate all functional dependencies that hold.

Example of F^+

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- Some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$
      and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,
      and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
      and then transitivity

Closure of Functional Dependencies (Cont.)

- Additional rules:
  - **Union rule**: If α → *β* holds and α → γ holds, then α → *β* γ holds.
  - **Decomposition rule**: If α → *β* γ holds, then α → *β* holds and α → γ holds.
  - **Pseudotransitivity rule**:If α → *β* holds and γ *β* → δ holds, then α γ → δ holds.
- The above rules can be inferred from Armstrong's axioms.

Procedure for Computing F^+

- To compute the closure of a set of functional dependencies F:

```
F + = F
 repeat
for each functional dependency f in F+
        apply reflexivity and augmentation rules on f
        add the resulting functional dependencies to F +
for each pair of functional dependencies f1 and f2 in F +
        if f1 and f2 can be combined using transitivity
               then add the resulting functional dependency to F +
  until F + does not change any further
```

- **NOTE**: We shall see an alternative procedure for this task later

Closure of Attribute Sets

- Given a set of attributes α, define the ***closure*** of α **under** *F* (denoted by α^+) as the set of attributes that are functionally determined by α under *F*
- Algorithm to compute α^+, the closure of α under *F*

result := α;
while (changes to *result*) **do**
    for each $\beta \rightarrow \gamma$ **in** *F* **do**
        begin
            if $\beta \subseteq$ *result* **then** *result* := *result* $\cup\ \gamma$
        end

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- $(AG)^+$
  1. *result* = AG
  2. *result* = $ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. *result* = $ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. *result* = $ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is R $\supseteq (AG)^+$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is R $\supseteq (A)^+$
     2. Does $G \rightarrow R$? == Is R $\supseteq (G)^+$
     3. In general: check for each subset of size *n-1*

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute $\alpha^{+,}$ and check if α^{+} contains all attributes of *R*.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^{+}), just check if $\beta \subseteq \alpha^{+}$.
  - That is, we compute α^{+} by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^{+}, and for each $S \subseteq \gamma^{+}$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover

- Suppose that we have a set of functional dependencies *F* on a relation schema. Whenever a user performs an update on the relation, the database system must ensure that the update does not violate any functional dependencies; that is, all the functional dependencies in *F* are satisfied in the new database state.
- If an update violates any functional dependencies in the set *F,* the system must roll back the update.
- We can reduce the effort spent in checking for violations by testing a simplified set of functional dependencies that has the same closure as the given set.
- This simplified set is termed the **canonical cover**
- To define canonical cover we must first define **extraneous attributes.**
  - An attribute of a functional dependency in *F* is **extraneous** if we can remove it without changing F^+

Extraneous Attributes

- Removing an attribute from the left side of a functional dependency could make it a stronger constraint.
  - For example, if we have AB → C and remove B, we get the possibly stronger result A → C. It may be stronger because A → C logically implies AB → C, but AB → C does not, on its own, logically imply A → C
- But, depending on what our set F of functional dependencies happens to be, we may be able to remove B from AB → C safely.
  - For example, suppose that
  - F = {AB → C, A → D, D → C}
  - Then we can show that F logically implies A → C, making extraneous in AB → C.

Extraneous Attributes (Cont.)

- Removing an attribute from the right side of a functional dependency could make it a weaker constraint.
  - For example, if we have $AB \rightarrow CD$ and remove C, we get the possibly weaker result $AB \rightarrow D$. It may be weaker because using just $AB \rightarrow D$, we can no longer infer $AB \rightarrow C$.
- But, depending on what our set F of functional dependencies happens to be, we may be able to remove C from $AB \rightarrow CD$ safely.
  - For example, suppose that

    $F = \{ AB \rightarrow CD, A \rightarrow C.$
  - Then we can show that even after replacing $AB \rightarrow CD$ by $AB \rightarrow D$, we can still infer $\$AB \rightarrow C$ and thus $AB \rightarrow CD$.

Extraneous Attributes

- An attribute of a functional dependency in F is **extraneous** if we can remove it without changing F^+
- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
  - **Remove from the left side**: Attribute A is **extraneous** in α if
    - $A \in \alpha$ and
    - F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - **Remove from the right side**: Attribute A is **extraneous** in β if
    - $A \in \beta$ and
    - The set of functional dependencies

      $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one

Testing if an Attribute is Extraneous

- Let R be a relation schema and let F be a set of functional dependencies that hold on R. Consider an attribute in the functional dependency $\alpha \rightarrow \beta$.
- To test if attribute $A \in \beta$ is extraneous in β
  - Consider the set:

    $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$

  - check that α^+ contains A; if it does, A is extraneous in β
- To test if attribute $A \in \alpha$ is extraneous in α
  - Let $\gamma = \alpha - \{A\}$. Check if $\gamma \rightarrow \beta$ can be inferred from F.
    - Compute γ^+ using the dependencies in F
    - If γ^+ includes all attributes in β then, A is extraneous in α

Examples of Extraneous Attributes

- Let $F = \{AB \rightarrow CD, A \rightarrow E, E \rightarrow C\}$
- To check if C is extraneous in $AB \rightarrow CD$, we:
  - Compute the attribute closure of AB under $F' = \{AB \rightarrow D, A \rightarrow E, E \rightarrow C\}$
  - The closure is *ABCDE,* which includes *CD*
  - This implies that *C* is extraneous

Canonical Cover

A **canonical cover** for F is a set of dependencies F_c such that

- F logically implies all dependencies in F_c , and
- F_c logically implies all dependencies in F, and
- No functional dependency in F_c contains an extraneous attribute, and
- Each left side of functional dependency in F_c is unique. That is, there are no two dependencies in F_c
  - $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ such that
  - $\alpha_1 = \alpha_2$

Canonical Cover

- To compute a canonical cover for *F*:

  repeat

  Use the union rule to replace any dependencies in *F* of the form

  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$

  Find a functional dependency $\alpha \rightarrow \beta$ in F_c with an extraneous attribute either in α or in β

  /* Note: test for extraneous attributes done using $F_{c,}$ not F*/

  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$

  until (F_c not change

- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Example: Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
      - Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$

Dependency Preservation

- Let F_i be the set of dependencies F^+ that include only attributes in R_i.
  - A decomposition is **dependency preserving**, if

    $$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$

- Using the above definition, testing for dependency preservation take exponential time.
- Not that if a decomposition is NOT dependency preserving then checking updates for violation of functional dependencies may require computing joins, which is expensive.

Dependency Preservation (Cont.)

- Let F be the set of dependencies on schema R and let R_1, R_2, .., R_n be a decomposition of R.
- The restriction of F to R_i is the set F_i of all functional dependencies in F^+ that include **only** attributes of R_i.
- Since all functional dependencies in a restriction involve attributes of only one relation schema, it is possible to test such a dependency for satisfaction by checking only one relation.
- Note that the definition of restriction uses all dependencies in in F^+, not just those in F.
- The set of restrictions F_1, F_2, .. , F_n is the set of functional dependencies that can be checked efficiently.

Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into $R_1, R_2, \ldots, R_n$, we apply the following test (with attribute closure done with respect to F)
  - *result* = α
    repeat
    for each R_i in the decomposition
    $t = (result \cap R_i)^+ \cap R_i$
    $result = result \cup t$
    until (*result* does not change)
  - If *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B$
  $B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$
  - R_1 and R_2 in BCNF
  - Lossless-join decomposition
  - Dependency preserving

Algorithm for Decomposition Using Functional Dependencies

Testing for BCNF

- To check if a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF
  1. compute α^+ (the attribute closure of α), and
  2. verify that it includes all attributes of *R*, that is, it is a superkey of *R*.
- **Simplified test**: To check if a relation schema *R* is in BCNF, it suffices to check only the dependencies in the given set *F* for violation of BCNF, rather than checking all dependencies in F^+.
  - If none of the dependencies in *F* causes a violation of BCNF, then none of the dependencies in F^+ will cause a violation of BCNF either.
- However, **simplified test** using only F is incorrect when testing a relation in a decomposition of R
  - Consider $R = (A, B, C, D, E)$, with $F = \{ A \rightarrow B, BC \rightarrow D\}$
    - Decompose *R* into $R_1 = (A,B)$ and $R_2 = (A,C,D, E)$
    - Neither of the dependencies in *F* contain only attributes from (A,C,D,E) so we might be mislead into thinking R_2 satisfies BCNF.
    - In fact, dependency $AC \rightarrow D$ in F^+ shows R_2 is not in BCNF.

Testing Decomposition for BCNF

To check if a relation R_i in a decomposition of R is in BCNF

- Either test R_i for BCNF with respect to the **restriction** of F^+ to R_i (that is, all FDs in F^+ that contain only attributes from R_i)
- Or use the original set of dependencies F that hold on R, but with the following test:
  - for every set of attributes $\alpha \subseteq R_i$, check that α^+ (the attribute closure of α) either includes no attribute of $R_i - \alpha$, or includes all attributes of R_i.
  - If the condition is violated by some $\alpha \rightarrow \beta$ in F^+, the dependency

    $\alpha \rightarrow (\alpha^+ - \alpha) \cap R_i$

    can be shown to hold on R_i, and R_i violates BCNF.
  - We use above dependency to decompose R_i

BCNF Decomposition Algorithm

result := $\{R\}$;
done := false;
compute F^+;
while (not *done*) **do**
  if (there is a schema R_i in *result* that is not in BCNF)
    then begin
      let $\alpha \rightarrow \beta$ be a nontrivial functional dependency that
        holds on R_i such that $\alpha \rightarrow R_i$ is not in F^+,
          and $\alpha \cap \beta = \varnothing$;
      result := $(result - R_i) \cup (R_i - \beta) \cup (\alpha, \beta)$;
    end
    else *done* := **true;**

Note: each R_i is in BCNF, and decomposition is lossless-join.

Example of BCNF Decomposition

- *class* (*course_id*, *title*, *dept_name*, *credits*, *sec_id*, *semester*, *year*, *building*, *room_number*, *capacity*, *time_slot_id*)
- Functional dependencies:
  - *course_id*→ *title*, *dept_name*, *credits*
  - *building*, *room_number*→*capacity*
  - *course_id*, *sec_id*, *semester*, *year*→*building*, *room_number*, *time_slot_id*
- A candidate key {*course_id*, *sec_id*, *semester*, *year*}.
- BCNF Decomposition:
  - *course_id*→ *title*, *dept_name*, *credits* holds
    - but *course_id* is not a superkey.
  - We replace *class* by:
    - *course*(*course_id*, *title*, *dept_name*, *credits*)
    - *class-1* (*course_id*, *sec_id*, *semester*, *year*, *building*, *room_number*, *capacity*, *time_slot_id*)

BCNF Decomposition (Cont.)

- *course* is in BCNF
  - How do we know this?
- *building*, *room_number*→*capacity* holds on *class-1*
  - but {*building*, *room_number*} is not a superkey for *class-1*.
  - We replace *class-1* by:
    - *classroom* (*building*, *room_number*, *capacity*)
    - *section* (*course_id*, *sec_id*, *semester*, *year*, *building*, *room_number*, *time_slot_id*)
- *classroom* and *section* are in BCNF.

Third Normal Form

- There are some situations where
  - BCNF is not dependency preserving, and
  - efficient checking for FD violation on updates is important
- Solution: define a weaker normal form, called Third Normal Form (3NF)
  - Allows some redundancy (with resultant problems; we will see examples later)
  - But functional dependencies can be checked on individual relations without computing a join.
  - There is always a lossless-join, dependency-preserving decomposition into 3NF.

3NF Example -- Relation *dept_advisor*

- *dept_advisor (s_ID, i_ID, dept_name)*
  $F = \{s\_ID, dept\_name \rightarrow i\_ID,\ \ i\_ID \rightarrow dept\_name\}$
- Two candidate keys: *s_ID, dept_name,* and *i_ID, s_ID*
- *R* is in 3NF
  - $s\_ID, dept\_name \rightarrow i\_ID\ \ \ s\_ID$
    - *dept_name* is a superkey
  - $i\_ID \rightarrow dept\_name$
    - *dept_name* is contained in a candidate key

Testing for 3NF

- Need to check only FDs in *F*, need not check all FDs in F^+.
- Use attribute closure to check for each dependency $\alpha \rightarrow \beta$, if α is a superkey.
- If α is not a superkey, we have to verify if each attribute in β is contained in a candidate key of *R*
  - This test is rather more expensive, since it involve finding candidate keys
  - Testing for 3NF has been shown to be NP-hard
  - Interestingly, decomposition into third normal form (described shortly) can be done in polynomial time

3NF Decomposition Algorithm

Let F_c be a canonical cover for F;
$i := 0$;
for each functional dependency $\alpha \rightarrow \beta$ in F_c **do**
  if none of the schemas R_j, $1 \leq j \leq i$ contains $\alpha\,\beta$
    then begin
      $i := i + 1$;
      $R_i := \alpha\,\beta$
    end
if none of the schemas R_j, $1 \leq j \leq i$ contains a candidate key for R
  then begin
    $i := i + 1$;
    R_i := any candidate key for R;
  end
/* Optionally, remove redundant relations */
repeat
if any schema R_j is contained in another schema R_k
  then /* delete R_j ***/**
    $R_j = R_i$;
    $i = i - 1$;
return $(R_1, R_2, ..., R_i)$

3NF Decomposition Algorithm (Cont.)

Above algorithm ensures

- Each relation schema R_i is in 3NF
- Decomposition is dependency preserving and lossless-join
- Proof of correctness is at end of this presentation (click here)

3NF Decomposition: An Example

- Relation schema:

  cust_banker_branch = (*customer_id, employee_id*, *branch_name, type*)

- The functional dependencies for this relation schema are:
  - *customer_id, employee_id* → *branch_name, type*
  - *employee_id* → *branch_name*
  - *customer_id, branch_name* → *employee_id*
- We first compute a canonical cover
  - *branch_name* is extraneous in the r.h.s. of the 1st dependency
  - No other attribute is extraneous, so we get F_C =

    customer_id, employee_id → *type*
    employee_id → *branch_name*
    customer_id, branch_name → *employee_id*

3NF Decompsition Example (Cont.)

- The **for** loop generates following 3NF schema:

  (*customer_id, employee_id, type*)

  (<u>*employee_id*</u>, *branch_name*)

  (*customer_id, branch_name, employee_id)*

  - Observe that (*customer_id, employee_id, type*) contains a candidate key of the original schema, so no further relation schema needs be added

- At end of for loop, detect and delete schemas, such as (<u>*employee_id*</u>*, branch_name*), which are subsets of other schemas

  - result will not depend on the order in which FDs are considered

- The resultant simplified 3NF schema is:

  (*customer_id, employee_id, type*)

  (*customer_id, branch_name, employee_id)*

Comparison of BCNF and 3NF

- It is always possible to decompose a relation into a set of relations that are in 3NF such that:
  - The decomposition is lossless
  - The dependencies are preserved
- It is always possible to decompose a relation into a set of relations that are in BCNF such that:
  - The decomposition is lossless
  - It may not be possible to preserve dependencies.

Design Goals

- Goal for a relational database design is:
  - BCNF.
  - Lossless join.
  - Dependency preservation.
- If we cannot achieve this, we accept one of
  - Lack of dependency preservation
  - Redundancy due to use of 3NF
- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys.

  Can specify FDs using assertions, but they are expensive to test, (and currently not supported by any of the widely used databases!)
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left hand side is not a key.

Multivalued Dependencies

Multivalued Dependencies (MVDs)

- Suppose we record names of children, and phone numbers for instructors:
  - *inst_child*(*ID*, *child_name*)
  - *inst_phone*(*ID*, *phone_number*)
- If we were to combine these schemas to get
  - *inst_info*(*ID*, *child_name*, *phone_number*)
  - Example data:
    (99999, David, 512-555-1234)
    (99999, David, 512-555-4321)
    (99999, William, 512-555-1234)
    (99999, William, 512-555-4321)
- This relation is in BCNF
  - Why?

Multivalued Dependencies

- Let R be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The **multivalued dependency**

  $$\alpha \rightarrow\rightarrow \beta$$

  holds on R if in any legal relation $r(R)$, for all pairs for tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that:

  $$\begin{aligned} t_1[\alpha] = t_2[\alpha] &= t_3[\alpha] = t_4[\alpha] \\ t_3[\beta] &= t_1[\beta] \\ t_3[R - \beta] &= t_2[R - \beta] \\ t_4[\beta] &= t_2[\beta] \\ t_4[R - \beta] &= t_1[R - \beta] \end{aligned}$$

MVD -- Tabular representation

- Tabular representation of $\alpha \rightarrow\rightarrow \beta$

	α	β	R
t_1	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	a_j
t_2	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	b_j
t_3	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	b_j
t_4	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	a_j

MVD (Cont.)

- Let R be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

  Y, Z, W

- We say that $Y \rightarrow\rightarrow Z$ (Y **multidetermines** Z) if and only if for all possible relations $r(R)$

  $< y_1, z_1, w_1 > \in r$ and $< y_1, z_2, w_2 > \in r$

  then

  $< y_1, z_1, w_2 > \in r$ and $< y_1, z_2, w_1 > \in r$

- Note that since the behavior of Z and W are identical it follows that

  $Y \rightarrow\rightarrow Z$ if $Y \rightarrow\rightarrow W$

Example

- In our example:

  $ID \rightarrow\rightarrow$ *child_name*
  $ID \rightarrow\rightarrow$ *phone_number*

- The above formal definition is supposed to formalize the notion that given a particular value of Y (*ID*) it has associated with it a set of values of Z *(child_name)* and a set of values of W *(phone_number)*, and these two sets are in some sense independent of each other.

- Note:
  - If $Y \rightarrow Z$ then $Y \rightarrow\rightarrow Z$
  - Indeed we have (in above notation) $Z_1 = Z_2$
    The claim follows.

Use of Multivalued Dependencies

- We use multivalued dependencies in two ways:
  1. To test relations to **determine** whether they are legal under a given set of functional and multivalued dependencies
  2. To specify **constraints** on the set of legal relations. We shall concern ourselves *only* with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation *r* fails to satisfy a given multivalued dependency, we can construct a relations *r′* that does satisfy the multivalued dependency by adding tuples to *r.*

Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
  - If $\alpha \rightarrow \beta$, then $\alpha \rightarrow\rightarrow \beta$

  That is, every functional dependency is also a multivalued dependency
- The **closure** D^+ of *D* is the set of all functional and multivalued dependencies logically implied by *D*.
  - We can compute D^+ from *D*, using the formal definitions of functional dependencies and multivalued dependencies.
  - We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice
  - For complex dependencies, it is better to reason about sets of dependencies using a system of inference rules (Appendix C).

Fourth Normal Form

- A relation schema *R* is in **4NF** with respect to a set *D* of functional and multivalued dependencies if for all multivalued dependencies in D^+ of the form $\alpha \rightarrow\rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
  - $\alpha \rightarrow\rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$ or $\alpha \cup \beta = R$)
  - α is a superkey for schema *R*
- If a relation is in 4NF it is in BCNF

Restriction of Multivalued Dependencies

- The restriction of D to R_i is the set D_i consisting of
  - All functional dependencies in D^+ that include only attributes of R_i
  - All multivalued dependencies of the form

    $$\alpha \rightarrow\rightarrow (\beta \cap R_i)$$

    where $\alpha \subseteq R_i$ and $\alpha \rightarrow\rightarrow \beta$ is in D^+

4NF Decomposition Algorithm

result: = {*R*};
done := false;
compute D^+;
Let D_i denote the restriction of D^+ to R_i

while (**not** *done*)
    if (there is a schema $\mathbf{R}_i$ in *result* that is not in 4NF) **then**
        begin
    let $\alpha \rightarrow\rightarrow \beta$ be a nontrivial multivalued dependency that holds
        on R_i such that $\alpha \rightarrow R_i$ is not in D_i, and $\alpha \cap \beta = \phi$;
        result := (*result* - R_i) $\cup$ (R_i - β) $\cup$ (α, β);
        end
    else *done*:= true;

Note: each R_i is in 4NF, and decomposition is lossless-join

$\rightarrow\rightarrow$

Example

- $R = (A, B, C, G, H, I)$

  $F = \{ A \rightarrow\rightarrow B$

  $B \rightarrow\rightarrow HI$

  $CG \rightarrow\rightarrow H \}$
- R is not in 4NF since $A \rightarrow\rightarrow B$ and A is not a superkey for R
- Decomposition

  a) $R_1 = (A, B)$ (R_1 is in 4NF)

  b) $R_2 = (A, C, G, H, I)$ (R_2 is not in 4NF, decompose into R_3 and R_4)

  c) $R_3 = (C, G, H)$ (R_3 is in 4NF)

  d) $R_4 = (A, C, G, I)$ (R_4 is not in 4NF, decompose into R_5 and R_6)

  - $A \rightarrow\rightarrow B$ and $B \rightarrow\rightarrow HI$ □ $A \rightarrow\rightarrow HI$, (MVD transitivity), and
  - and hence $A \rightarrow\rightarrow I$ *(MVD restriction to R_4)*

  e) $R_5 = (A, I)$ (R_5 is in 4NF)

  f) $R_6 = (A, C, G)$ (R_6 is in 4NF)

Additional issues

Further Normal Forms

- **Join dependencies** generalize multivalued dependencies
  - lead to **project-join normal form (PJNF)** (also called **fifth normal form**)
- A class of even more general constraints, leads to a normal form called **domain-key normal form**.
- Problem with these generalized constraints: are hard to reason with, and no set of sound and complete set of inference rules exists.
- Hence rarely used

Overall Database Design Process

We have assumed schema *R* is given

- *R* could have been generated when converting E-R diagram to a set of tables.
- *R* could have been a single relation containing *all* attributes that are of interest (called **universal relation**).
- Normalization breaks *R* into smaller relations.
- *R* could have been the result of some ad hoc design of relations, which we then test/convert to normal form.

ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization.
- However, in a real (imperfect) design, there can be functional dependencies from non-key attributes of an entity to other attributes of the entity
  - Example: an *employee* entity with
    - attributes
      department_name and *building*,
    - functional dependency
      department_name $\rightarrow$ *building*
    - Good design would have made department an entity
- Functional dependencies from non-key attributes of a relationship set possible, but rare --- most relationships are binary

Denormalization for Performance

- May want to use non-normalized schema for performance
- For example, displaying *prereqs* along with *course_id*, and *title* requires join of *course* with *prereq*
- Alternative 1: Use denormalized relation containing attributes of *course* as well as *prereq* with all above attributes
  - faster lookup
  - extra space and extra execution time for updates
  - extra coding work for programmer and possibility of error in extra code
- Alternative 2: use a materialized view defined a *course* $\bowtie$ *prereq*
  - Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors

Other Design Issues

- Some aspects of database design are not caught by normalization
- Examples of bad database design, to be avoided:

  Instead of *earnings* (*company_id, year, amount*), use
  - *earnings_2004, earnings_2005, earnings_2006*, etc., all on the schema (*company_id, earnings*).
    - Above are in BCNF, but make querying across years difficult and needs new table each year
  - *company_year* (*company_id, earnings_2004, earnings_2005, earnings_2006*)
    - Also in BCNF, but also makes querying across years difficult and requires new attribute each year.
    - Is an example of a **crosstab**, where values for one attribute become column names
    - Used in spreadsheets, and in data analysis tools

Modeling Temporal Data

- **Temporal data** have an association time interval during which the data are *valid.*
- A **snapshot** is the value of the data at a particular point in time
- Several proposals to extend ER model by adding valid time to
  - attributes, e.g., address of an instructor at different points in time
  - entities, e.g., time duration when a student entity exists
  - relationships, e.g., time during which an instructor was associated with a student as an advisor.
- But no accepted standard
- Adding a temporal component results in functional dependencies like

  $ID \rightarrow street, city$

  not holding, because the address varies over time
- A **temporal functional dependency** X → Y holds on schema *R* if the functional dependency X □ Y holds on all snapshots for all legal instances r (*R*).

Modeling Temporal Data (Cont.)

- In practice, database designers may add start and end time attributes to relations
  - E.g., *course*(*course_id, course_title*) is replaced by

    course(*course_id, course_title, start, end*)
  - Constraint: no two tuples can have overlapping valid times
    - Hard to enforce efficiently
- Foreign key references may be to current version of data, or to data at a point in time
  - E.g., student transcript should refer to course information at the time the course was taken

End of Chapter 7

Proof of Correctness of 3NF Decomposition Algorithm

Correctness of 3NF Decomposition Algorithm

- 3NF decomposition algorithm is dependency preserving (since there is a relation for every FD in F_c)
- Decomposition is lossless
  - A candidate key (C) is in one of the relations R_i in decomposition
  - Closure of candidate key under F_c must contain all attributes in R.
  - Follow the steps of attribute closure algorithm to show there is only one tuple in the join result for each tuple in R_i

Correctness of 3NF Decomposition Algorithm (Cont.)

- Claim: if a relation R_i is in the decomposition generated by the above algorithm, then R_i satisfies 3NF.
- Proof:
  - Let R_i be generated from the dependency $\alpha \rightarrow \beta$
  - Let $\gamma \rightarrow B$ be any non-trivial functional dependency on R_i. (We need only consider FDs whose right-hand side is a single attribute.)
  - Now, *B* can be in either β or α but not in both. Consider each case separately.

Correctness of 3NF Decomposition (Cont.)

- Case 1: If B in β:
  - If γ is a superkey, the 2nd condition of 3NF is satisfied
  - Otherwise α must contain some attribute not in γ
  - Since $\gamma \rightarrow B$ is in F^+ it must be derivable from F_c, by using attribute closure on γ.
  - Attribute closure not have used $\alpha \rightarrow \beta$. If it had been used, α must be contained in the attribute closure of γ, which is not possible, since we assumed γ is not a superkey.
  - Now, using $\alpha \rightarrow (\beta - \{B\})$ and $\gamma \rightarrow B$, we can derive $\alpha \rightarrow B$

    (since $\gamma \subseteq \alpha\beta$, and $B \notin \gamma$ since $\gamma \rightarrow B$ is non-trivial)
  - Then, B is extraneous in the right-hand side of $\alpha \rightarrow \beta$; which is not possible since $\alpha \rightarrow \beta$ is in F_c.
  - Thus, if B is in β then γ must be a superkey, and the second condition of 3NF must be satisfied.

Correctness of 3NF Decomposition (Cont.)

- Case 2: *B* is in α.
  - Since α is a candidate key, the third alternative in the definition of 3NF is trivially satisfied.
  - In fact, we cannot show that γ is a superkey.
  - This shows exactly why the third alternative is present in the definition of 3NF.

Q.E.D.

First Normal Form

- Domain is **atomic** if its elements are considered to be indivisible units
  - Examples of non-atomic domains:
    - Set of names, composite attributes
    - Identification numbers like CS101 that can be broken up into parts
- A relational schema R is in **first normal form** if the domains of all attributes of R are atomic
- Non-atomic values complicate storage and encourage redundant (repeated) storage of data
  - Example: Set of accounts stored with each customer, and set of owners stored with each account
  - We assume all relations are in first normal form (and revisit this in Chapter 22: Object Based Databases)

First Normal Form (Cont.)

- Atomicity is actually a property of how the elements of the domain are used.
  - Example: Strings would normally be considered indivisible
  - Suppose that students are given roll numbers which are strings of the form *CS0012* or *EE1127*
  - If the first two characters are extracted to find the department, the domain of roll numbers is not atomic.
  - Doing so is a bad idea: leads to encoding of information in application program rather than in the database.