Simultaneous calibration of odometry and sensor parameters for mobile robots

1. 问题描述：

机器人坐标系看作odom坐标系， 传感器坐标系是激光坐标系 。

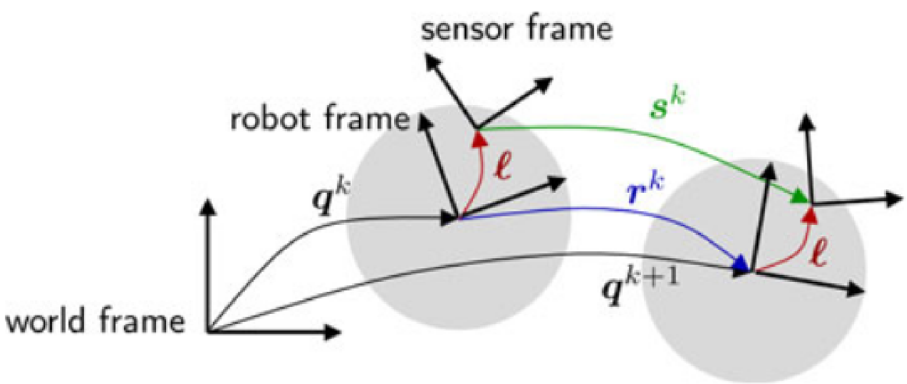

Fig. 1. Robot pose is $q^k \in \mathrm{SE}(2)$ with respect to the world frame; the sensor pose is $\ell \in \mathrm{SE}(2)$ with respect to the robot frame; $r^k \in \mathrm{SE}(2)$ is the robot displacement between poses; and $s^k \in \mathrm{SE}(2)$ is the displacement seen by the sensor in its own reference frame.

物理量表示含义

Calibration parameters to be estimated	
$r_{\mathrm{R}}, r_{\mathrm{L}}$	wheel radii
b	distance between wheels
$\boldsymbol{\ell}$	sensor pose relative to robot frame
Robot kinematics	
$\boldsymbol{q}$	robot pose relative to world frame
$\omega_{\mathrm{L}}, \omega_{\mathrm{R}}$	left/right wheel velocity
v, ω	driving/steering robot velocities
$\boldsymbol{J}$	linear map between wheel and robot velocities
Sensing process	
$\boldsymbol{m}^k$	exteroceptive measurements, available at time t_k
$\boldsymbol{r}^k$	robot displacement in the k-th interval $[t_k, t_{k+1}]$
$\boldsymbol{s}^k$	sensor displacement in the k-th interval
$\hat{\boldsymbol{s}}^k$	sensor displacement estimated from $\boldsymbol{m}^k$ and $\boldsymbol{m}^{k+1}$
$\boldsymbol{\nu}$	sensor velocity in the sensor frame
Other symbols	

2. 外参标定描述：

标定参数为左右轮半径 r_l r_r ，轮间距b， 和激光相对轮子的外参（x,y,yaw）

输入： 左右轮转度（rad/s）　　激光里程计数据

输出： 标定参数

2.1 运动模型： 左右轮线速度转化为基体的线速度与角速度

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \boldsymbol{J} \begin{pmatrix} \omega_{\mathrm{L}} \\ \omega_{\mathrm{R}} \end{pmatrix}. \qquad (2)$$

$$\boldsymbol{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} +r_{\mathrm{L}}/2 & +r_{\mathrm{R}}/2 \\ -r_{\mathrm{L}}/b & +r_{\mathrm{R}}/b \end{pmatrix} \qquad (3)$$

$$\boldsymbol{s}^k = \ominus\left(\boldsymbol{q}^k \oplus \boldsymbol{\ell}\right) \oplus \left(\boldsymbol{q}^{k+1} \oplus \boldsymbol{\ell}\right)$$

$$\boldsymbol{r}^k = \ominus \boldsymbol{q}^k \oplus \boldsymbol{q}^{k+1} \longrightarrow \boldsymbol{s}^k = \ominus \boldsymbol{\ell} \oplus \boldsymbol{r}^k \oplus \boldsymbol{\ell}$$

其中，$\boldsymbol{r}^k = \boldsymbol{r}^k(r_{\mathrm{L}}, r_{\mathrm{R}}, b).$

2.2 SE 2 计算公式：

$\oplus, \ominus$ "$\oplus$" is the group operation on SE(2):

$$\begin{pmatrix} a_x \\ a_y \\ a_\theta \end{pmatrix} \oplus \begin{pmatrix} b_x \\ b_y \\ b_\theta \end{pmatrix} = \begin{pmatrix} a_x + b_x \cos(a_\theta) - b_y \sin(a_\theta) \\ a_y + b_x \sin(a_\theta) + b_y \cos(a_\theta) \\ a_\theta + b_\theta \end{pmatrix}$$

"$\ominus$" is the group inverse:

$$\ominus \begin{pmatrix} a_x \\ a_y \\ a_\theta \end{pmatrix} = \begin{pmatrix} -a_x \cos(a_\theta) - a_y \sin(a_\theta) \\ +a_x \sin(a_\theta) - a_y \cos(a_\theta) \\ -a_\theta \end{pmatrix}$$

SE2坐标变换的关系

$$R_z(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

同一坐标系下 2D. $(a_x\ a_y\ a_\theta)^T$ $(b_x\ b_y\ b_\theta)^T$

$$\begin{pmatrix} a_x \\ a_y \\ a_\theta \end{pmatrix} \oplus \begin{pmatrix} b_x \\ b_y \\ b_\theta \end{pmatrix} = \left\{ \begin{array}{c} \begin{bmatrix} \cos a_\theta & -\sin a_\theta & a_x \\ \sin a_\theta & \cos a_\theta & a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ 1 \end{bmatrix} \\ a_\theta + b_\theta \end{array} \right\}$$

$$= \begin{pmatrix} a_x + b_x \cdot \cos a_\theta - b_y \cdot \sin a_\theta \\ a_y + b_x \cdot \sin a_\theta + b_y \cdot \cos a_\theta \\ a_\theta + b_\theta \end{pmatrix}$$

李群SE2逆 ∵若 $A \oplus B = \mathcal{E}$ (幺元)

则 $B = A^{-1}$

则令 $\left\{ \begin{array}{l} a_x + b_x \cdot \cos a_\theta - b_y \cdot \sin a_\theta = 0 \\ a_y + b_x \cdot \sin a_\theta + b_y \cos a_\theta = 0 \\ a_\theta + b_\theta = 0 \end{array} \right. \xrightarrow[\text{求}B]{\text{知}A} \begin{Bmatrix} b_x \\ b_y \\ b_\theta \end{Bmatrix}$

$$\ominus \begin{pmatrix} a_x \\ a_y \\ a_\theta \end{pmatrix} = \begin{pmatrix} -a_x \cdot \cos a_\theta - a_y \cdot \sin a_\theta \\ a_x \cdot \sin a_\theta - a_y \cos a_\theta \\ -a_\theta \end{pmatrix}$$

2.3 优化方程

$$\mathcal{J} = -\frac{1}{2} \sum_{k=1}^{n} \| \hat{\boldsymbol{s}}^k - \ominus \boldsymbol{\ell} \oplus \boldsymbol{r}^k(r_{\mathrm{L}}, r_{\mathrm{R}}, b) \oplus \boldsymbol{\ell} \|^2_{\boldsymbol{\Sigma}_k^{-1}}$$

2.4 求解方法：

3种通用求解方案：

1）基于模型的控制理论：连续时间

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x})$$

2） 静态分析：
y离散可观， 确定性的的输入x， 加上足够的约束一起去求解X.

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}),$$

3) 统计分析：Fisher Information Matrix (FIM)： 线性系统
u选择适当的情况下如果FIM中X是满秩的，系统可观

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) + \boldsymbol{\epsilon}$$

==》 本校正系统选用方法二的静态分析（离散的，非线性系统）

前提约束条件：

1) 全局参数变量的二义性： ==》 b>0 来区分

Proposition 2: The two sets of calibration parameters $(r_{\mathrm{L}}, r_{\mathrm{R}}, b, \ell_x, \ell_y, \ell_\theta)$ and $(-r_{\mathrm{L}}, -r_{\mathrm{R}}, -b, -\ell_x, -\ell_y, \ell_\theta + \pi)$ are indistinguishable.

2） 标定参数可观的条件： **左右轮独立， 机器人观察区间内移动需要有旋转**

Proposition 3: All calibration parameters are observable if and only if the dataset contains at least one pair of trajectories that satisfy the following conditions.

1) The vectors $(\Delta_L^1\ \Delta_R^1)^T$ and $(\Delta_L^2\ \Delta_R^2)^T$ are linearly independent.
2) The motions $\boldsymbol{r}^1$ and $\boldsymbol{r}^2$ are "independent," in the sense that there exists no $\gamma \in \mathbb{R}$ such that

$$\log(\boldsymbol{r}^1) = \gamma \log(\boldsymbol{r}^2). \quad (7)$$

3) The motions $\boldsymbol{r}^1$ and $\boldsymbol{r}^2$ are not both pure translations.

3）轨迹速度与轮子转速： 线性独立关系

Corollary 4: In the case of trajectories of constant velocity, the parameters are observable if and only if *the vectors containing the constant wheel velocities* $(\omega_L^1\ \omega_R^1)^T$ and $(\omega_L^2\ \omega_R^2)^T$ are linearly independent (for example, a pure rotation and a pure forward translation).

3. 标定系统问题

Problem 5 (Simultaneous calibration, maximum-likelihood formulation): Maximize (9) with respect to $r_{\mathrm{L}}, r_{\mathrm{R}}, b, \ell_x, \ell_y, \ell_\theta$.

The log-likelihood $\mathcal{J} = \log p(\{\hat{\boldsymbol{s}}^k\}|r_{\mathrm{L}}, r_{\mathrm{R}}, b, \boldsymbol{\ell})$ is

$$\mathcal{J} = -\frac{1}{2}\sum_{k=1}^{n} \|\hat{\boldsymbol{s}}^k - \ominus\boldsymbol{\ell} \oplus \boldsymbol{r}^k(r_{\mathrm{L}}, r_{\mathrm{R}}, b) \oplus \boldsymbol{\ell}\|^2_{\boldsymbol{\Sigma}_k^{-1}} \tag{9}$$

非凸优化问题》 通常的数值优化问题并不能有效解决。》 closed form

本标定算法是建立在大前提假设：

Assumption 1: The covariance $\boldsymbol{\Sigma}_k$ of the estimate $\hat{\boldsymbol{s}}^k$ is diagonal and isotropic in the x- and y-directions

$$\boldsymbol{\Sigma}_k = \mathrm{diag}((\sigma^k_{xy})^2, (\sigma^k_{xy})^2, (\sigma^k_\theta)^2).$$

补充： 假设一不成立就得考虑 ==》covariance inflation

》 旋转和平移的相互独立性，不相关。》 =》 系统分解为线性部分和非线性部分

3.1 线性估计 J_{21} J_{22} ：旋转的一致和可观性

$J_{21} = -r_{\mathrm{L}}/b, J_{22} = r_{\mathrm{R}}/b$

$\hat{s}^k_\theta == s^k_\theta$,且与 J_{21} J_{22} 线性相关

$$r^k_\theta = \boldsymbol{L}_k \begin{pmatrix} J_{21} \\ J_{22} \end{pmatrix}$$

其中，

$$\boldsymbol{L}_k = \left(\int_{t_k}^{t_{k+1}} \omega_{\mathrm{L}}(t)\,dt \quad \int_{t_k}^{t_{k+1}} \omega_{\mathrm{R}}(t)\,dt \right). \tag{10}$$

==> 线性系统 =》 最新二乘问题 连续系统离散化处理 ==》估计 得到参数 $\hat{J}_{21}$ $\hat{J}_{22}$ 也就是 $-r_L/b, r_R/b$

$$\begin{pmatrix} \hat{J}_{21} \\ \hat{J}_{22} \end{pmatrix} = \left[\sum_k \frac{\boldsymbol{L}_k^T \boldsymbol{L}_k}{(\sigma^k_\theta)^2} \right]^{-1} \sum_k \frac{\boldsymbol{L}_k^T}{(\sigma^k_\theta)^2} \hat{s}^k_\theta. \tag{11}$$

3.2 非线性估计其他参数 b, L_x, L_y, L_θ

参差项转化：（不同坐标系下参差表示）

$$\|\boldsymbol{s}^k - \ominus\boldsymbol{\ell} \oplus \boldsymbol{r}^k \oplus \boldsymbol{\ell}\|_2 = \|\boldsymbol{\ell} \oplus \boldsymbol{s}^k - \boldsymbol{r}^k \oplus \boldsymbol{\ell}\|_2.$$

优化问题转化为：

$$\mathcal{J} = -\frac{1}{2}\sum_{k} \left\| \boldsymbol{\ell} \oplus \hat{\boldsymbol{s}}^{k} - \boldsymbol{r}^{k} \oplus \boldsymbol{\ell} \right\|^{2}_{\boldsymbol{\Sigma}_{k}^{-1}}. \qquad (12)$$

问题求解：分解

$$\dot{\boldsymbol{r}} = \begin{pmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{r}_\theta \end{pmatrix} = \begin{pmatrix} v \ \cos r_\theta \\ v \ \sin r_\theta \\ \omega \end{pmatrix}, \quad \boldsymbol{r}(t_k) = \mathbf{0}. \qquad (13)$$

$$r_\theta(t) = \int_{t_k}^{t} \left(J_{21}\omega_{\mathrm{L}}(\tau) + J_{22}\omega_{\mathrm{R}}(\tau) \right) d\tau. \qquad (14)$$

1）里程计运动平移部分的表示为：

$$r_x(t) = \int_{t_k}^{t} v(\tau) \ \cos \ r_\theta(\tau) d\tau \qquad (15)$$

$$r_y(t) = \int_{t_k}^{t} v(\tau) \sin \ r_\theta(\tau) d\tau. \qquad (16)$$

其中，

$$v = J_{11}\omega_{\mathrm{L}} + J_{12}\omega_{\mathrm{R}} = b\left(-\frac{1}{2} J_{21}\omega_{\mathrm{L}} + \frac{1}{2} J_{22}\omega_{\mathrm{R}} \right) \qquad (17)$$

平移运动简化为： $r_x^k = c_x^k\, b, \quad r_y^k = c_y^k\, b$

其中：

$$c_x^k = \frac{1}{2}\int_{t_k}^{t_{k+1}} \left(-J_{21}\omega_{\mathrm{L}}(\tau) + J_{22}\omega_{\mathrm{R}}(\tau) \right) \cos r_\theta^k(\tau) d\tau \qquad (19)$$

$$c_y^k = \frac{1}{2}\int_{t_k}^{t_{k+1}} \left(-J_{21}\omega_{\mathrm{L}}(\tau) + J_{22}\omega_{\mathrm{R}}(\tau) \right) \sin r_\theta^k(\tau) d\tau. \qquad (20)$$

公式18,19中的 r_θ^k 参考公式13的离散化

2） 系统转化为二次型系统问题：

参数：

$$\boldsymbol{\varphi} = \begin{pmatrix} b & \ell_x & \ell_y & \cos\ell_\theta & \sin\ell_\theta \end{pmatrix}^T \tag{21}$$

转移矩阵：

$$\boldsymbol{Q}_k = \frac{1}{\sigma_{xy}^k}\begin{pmatrix} -c_x^k & 1-\cos\hat{r}_\theta^k & +\sin\hat{r}_\theta^k & +\hat{s}_x^k & -\hat{s}_y^k \\ -c_y^k & -\sin\hat{r}_\theta^k & 1-\cos\hat{r}_\theta^k & +\hat{s}_y^k & +\hat{s}_x^k \end{pmatrix} \tag{22}$$

原优化问题等价的矩阵表示为：$-\frac{1}{2}\boldsymbol{\varphi}^T\boldsymbol{M}\boldsymbol{\varphi}$，其中 $\boldsymbol{M} = \sum_k \boldsymbol{Q}_k^T\boldsymbol{Q}_k$

极大似然问题转化为二次型问题 表示为：

$$\min \quad \boldsymbol{\varphi}^T\boldsymbol{M}\boldsymbol{\varphi} \tag{23}$$

$$\text{subject to} \quad \varphi_4^2 + \varphi_5^2 = 1. \tag{24}$$

$$\varphi_1 \geq 0. \tag{25}$$

具体推到表示如下：

$|\boldsymbol{\ell} \oplus \boldsymbol{s}^k$

$$\| l \oplus s^k - r^k \oplus l \|$$

$$l \oplus s^k = \begin{bmatrix} \cos l_\theta & -\sin l_\theta & l_x \\ \sin l_\theta & \cos l_\theta & l_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \hat{s}_x^k \\ s_y^k \\ 1 \end{bmatrix} =$$

① $\tilde{s}_x^k \cos l_\theta - \tilde{s}_y^k \sin l_\theta + l_x$

② $\tilde{s}_x^k \sin l_\theta + \tilde{s}_y^k \cos l_\theta + l_y.$

$\boldsymbol{r}^k \oplus \boldsymbol{\ell}|$

$$r^k \oplus l = \begin{bmatrix} \cos r_\theta & -\sin r_\theta & r_x \\ \sin r_\theta & \cos r_\theta & r_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_x \\ l_y \\ 1 \end{bmatrix}$$

③ $l_x \cdot \cos r_\theta - l_y \sin r_\theta + r_x \to b \cdot c_x^k$

④ $l_x \cdot \sin r_\theta + l_y \cdot \cos r_\theta + l_y \to b \cdot c_y^k$

$$\boldsymbol{\ell} \oplus \boldsymbol{s}^k - \boldsymbol{r}^k \oplus \boldsymbol{\ell}$$

① − ③

② − ④

$$\begin{bmatrix} -c_x^k & 1-\cos r_\theta & \sin r_\theta & \tilde{s}_x^k & -\tilde{s}_y^k \\ -c_y^k & -\sin r_\theta & 1-\cos r_\theta & \tilde{s}_y^k & \tilde{s}_x^k \end{bmatrix} \begin{bmatrix} b \\ l_x \\ l_y \\ \cos l_\theta \\ \sin l_\theta \end{bmatrix}$$

3） 带约束的最小二乘问题：

$$\boldsymbol{\varphi}^T \boldsymbol{W} \boldsymbol{\varphi} = 1, \quad \text{with } \boldsymbol{W} = \begin{pmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times2} \\ \mathbf{0}_{2\times3} & \boldsymbol{I}_{2\times2} \end{pmatrix}. \qquad (26)$$

$$\mathcal{L} = \boldsymbol{\varphi}^T \boldsymbol{M} \boldsymbol{\varphi} + \lambda(\boldsymbol{\varphi}^T \boldsymbol{W} \boldsymbol{\varphi} - 1).$$

Consider the Lagrangian $\mathcal{L} = \boldsymbol{\varphi}^T \boldsymbol{M} \boldsymbol{\varphi} + \lambda(\boldsymbol{\varphi}^T \boldsymbol{W} \boldsymbol{\varphi} - 1)$. In this problem, Slater's condition holds; thus the Karush–Kuhn–Tucker conditions are necessary for optimality

$$\frac{\partial \mathcal{L}^T}{\partial \boldsymbol{\varphi}} = 2\left(\boldsymbol{M} + \lambda \boldsymbol{W}\right) \boldsymbol{\varphi} = \mathbf{0}. \qquad (27)$$

对应存在λ 使得$M + \lambda W$为奇异矩阵， 即

$$\det\left(\boldsymbol{M} + \lambda \boldsymbol{W}\right) = 0.$$

其中

$$
\boldsymbol{M} = \begin{pmatrix} m_{11} & 0 & m_{13} & m_{14} & m_{15} \\ & m_{22} & 0 & m_{35} & -m_{34} \\ & & m_{22} & m_{34} & m_{35} \\ & & & m_{44} & 0 \\ \text{(symmetric)} & & & & m_{44} \end{pmatrix}
$$

（a）可得到关于λ的二阶项，项系数为： $a_2\,\lambda^2 + a_1\,\lambda + a_0$；

求得系数 $\lambda^{(1)}\,\lambda^{(2)}$

$$
a_2 = m_{11}{m_{22}}^2 - m_{22}{m_{13}}^2 \tag{28}
$$

$$
\begin{aligned}
a_1 = {} & 2m_{13}m_{22}m_{35}m_{15} - {m_{22}}^2{m_{15}}^2 \\
& + 2m_{13}m_{22}m_{34}m_{14} - 2m_{22}{m_{13}}^2m_{44} - {m_{22}}^2{m_{14}}^2 \\
& + 2m_{11}{m_{22}}^2m_{44} + {m_{13}}^2{m_{35}}^2 - 2m_{11}m_{22}{m_{34}}^2 \\
& + {m_{13}}^2{m_{34}}^2 - 2m_{11}m_{22}{m_{35}}^2
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
a_0 = {} & -2m_{13}m_{35}^3m_{15} - m_{22}m_{13}^2m_{44}^2 + m_{13}^2m_{35}^2m_{44} \\
& + 2m_{13}m_{22}m_{34}m_{14}m_{44} + 2m_{13}m_{22}m_{35}m_{15}m_{44} \\
& + {m_{13}}^2{m_{34}}^2m_{44} - 2m_{11}m_{22}{m_{34}}^2m_{44} \\
& - 2m_{13}{m_{34}}^3m_{14} - 2m_{11}m_{22}{m_{35}}^2m_{44} \\
& + 2m_{11}{m_{35}}^2{m_{34}}^2 + m_{22}{m_{14}}^2{m_{35}}^2 \\
& - 2m_{13}{m_{35}}^2m_{34}m_{14} - 2m_{13}{m_{34}}^2m_{35}m_{15} \\
& + m_{11}{m_{34}}^4 + m_{22}{m_{15}}^2{m_{34}}^2 + m_{22}{m_{35}}^2{m_{15}}^2 \\
& + m_{11}{m_{35}}^4 + m_{11}{m_{22}}^2{m_{44}}^2 + m_{22}{m_{34}}^2{m_{14}}^2 \\
& - {m_{22}}^2{m_{15}}^2m_{44} - {m_{22}}^2{m_{14}}^2m_{44}.
\end{aligned} \tag{30}
$$

(b) 求解参数$\varphi^{(i)}$

$\gamma^{(i)}$ 是$M+\lambda^{(i)}W$ 核的非零向量， 也就是$M+\lambda^{(i)}W=0$的通解。 同时依据公式24的约束归一化求得$\varphi^{(i)}$ ：

$$\boldsymbol{\varphi}^{(i)} = \frac{\operatorname{sign}(\gamma_1^{(i)})}{\|(\gamma_4^{(i)} \ \gamma_5^{(i)})^T\|}\boldsymbol{\gamma}^{(i)}. \qquad (31)$$

（c）通过具体的约束关系 公式25，从$\varphi^{(1)}$ $\varphi^{(2)}$ 找到正确的 $\hat{\varphi}$

4） 系统参数的解

$$\begin{aligned}
\hat{b} &= \hat{\varphi}_1 \\
\hat{r}_{\mathrm{L}} &= -\hat{\varphi}_1 \hat{J}_{21}, \quad \hat{r}_{\mathrm{R}} = +\hat{\varphi}_1 \hat{J}_{22} \\
\hat{\boldsymbol{\ell}} &= (\hat{\varphi}_2, \hat{\varphi}_3, \operatorname{arctan2}(\hat{\varphi}_5, \hat{\varphi}_4)). \qquad (32)
\end{aligned}$$

4 异常剔除

轮子打滑， 传感器平移估计出错, 以及数据同步异常等原因，需要剔除异常值

利用估计的模型参数计算里程计增量在传感器坐标系下的表示，比较其与传感器计算的观测增量，偏差比较大则为异常值，要剔除。

1) Run the calibration procedure with the current samples.
2) Compute the χ-value of each sample as

$$\chi^k = \|\hat{\boldsymbol{s}}^k - \ominus\hat{\boldsymbol{\ell}} \oplus \boldsymbol{r}^k(\hat{r}_{\mathrm{L}}, \hat{r}_{\mathrm{R}}, \hat{b}) \oplus \hat{\boldsymbol{\ell}}\|_{\boldsymbol{\Sigma}_k^{-1}}. \qquad (33)$$

3) Discard a fraction α of samples with highest values of χ^k.

The empirical distribution of the residual errors

$$\boldsymbol{e}^k = \hat{\boldsymbol{s}}^k - \ominus\hat{\boldsymbol{\ell}} \oplus \boldsymbol{r}^k(\hat{r}_{\mathrm{L}}, \hat{r}_{\mathrm{R}}, \hat{b}) \oplus \hat{\boldsymbol{\ell}} \qquad (34)$$

5. 不确定性分析： 协方差估计

. 协方差估计：协方差的估计直接采用信息矩阵的逆来计算。如果给定一个模型 $\boldsymbol{y}_k = \boldsymbol{f}_k(\boldsymbol{x}) + \boldsymbol{\epsilon}_k$ 其中 $\boldsymbol{f}$ 是可导函数，$\boldsymbol{\epsilon}_k$ 是协方差为 $\boldsymbol{\Sigma}_k$ 的高斯噪声。协方差的下界 Cramer-Rao bound，$\operatorname{cov}(\hat{\boldsymbol{x}}) \geq \mathcal{I}(\boldsymbol{x})^{-1}$. 在这个应用中，$\boldsymbol{x} = (r_{\mathrm{R}}, r_{\mathrm{L}}, b, \ell_x, \ell_y, \ell_\theta)$，$\boldsymbol{y}_k = \hat{\boldsymbol{s}}^k$ ，观测函数 $\boldsymbol{f}_k$ 为公式（2）所示。这个模型最小二乘估计的参数 $\boldsymbol{x}$ 的信息矩阵为

$$\mathcal{I}(\boldsymbol{x}) = \sum_k \frac{\partial \boldsymbol{f}_k}{\partial \boldsymbol{x}} \boldsymbol{\Sigma}_k^{-1} \frac{\partial \boldsymbol{f}_k}{\partial \boldsymbol{x}}$$

6. 离散化 常速度简化形式

$$\boldsymbol{r}(t) = \begin{pmatrix} (v_0/\omega_0)\sin(\omega_0 t) \\ (v_0/\omega_0)\ (1-\cos(\omega_0 t)) \\ \omega_0 t \end{pmatrix}. \tag{35}$$

$$T^k = t_{k+1} - t_k$$

$$\boldsymbol{L}_k = (\, T^k\omega_{\mathrm{L}}^k \quad T^k\omega_{\mathrm{R}}^k \,) \tag{36}$$

$$r_\theta^k = J_{21}T^k\omega_{\mathrm{L}}^k + J_{22}T^k\omega_{\mathrm{R}}^k \tag{37}$$

$$c_x^k = \tfrac{1}{2}T^k(-J_{21}\omega_{\mathrm{L}}^k + J_{22}\omega_{\mathrm{R}}^k)\frac{\sin(r_\theta^k)}{r_\theta^k} \tag{38}$$

$$c_y^k = \tfrac{1}{2}T^k(-J_{21}\omega_{\mathrm{L}}^k + J_{22}\omega_{\mathrm{R}}^k)\frac{1-\cos(r_\theta^k)}{r_\theta^k}. \tag{39}$$

Bounding the Approximation Error 区间常速度的限制：

$$\bar{\omega}_{\mathrm{L}} = \frac{1}{T}\int_0^T \omega_{\mathrm{L}}(\tau)d\tau, \quad \bar{\omega}_{\mathrm{R}} = \frac{1}{T}\int_0^T \omega_{\mathrm{R}}(\tau)d\tau.$$

$$|\omega_{\mathrm{L}}(t) - \bar{\omega}_{\mathrm{L}}| \leq \epsilon, \quad |\omega_{\mathrm{R}}(t) - \bar{\omega}_{\mathrm{R}}| \leq \epsilon. \tag{40}$$

7. 实验参考

7.1 数据量越多越好

7.2 小环境测量更佳

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With minimal tuning of the maximum velocities and the interval length, one can
make the robot stay in a small region.
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7.3 分段输入

7.4 相对低速运动： 低速且不要很低

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Choose commands that lead to relatively low speeds. This minimizes the
possibility of slipping and ensures that the sensor data are not perturbed by
the robot motion. However, do not choose speeds so low that the nonlinear
effects of the dynamics become relevant, especially if using the constant speed
assumption (usually robots with DC motors are commanded in velocities via
voltage, but the platform does not attain constant velocity instantaneously).
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7.5 时间区间T的选择

T太短，标定会过于敏感，T越长关于参数的信息会越丰富；但T不能过长，不然激光计算的数据的相关性会降低（匹配精度会损失）

作者实验条件参考：T＝0.8s　　w_max=　0.5 rad/s
==》　T内平移 1 cm　旋转 20°

laser：5 H Z　 取4帧

```
In our setting, we first chose the maximum wheel speed to be •0.5 rad/s (•30 ◦
/s),
which made sure that the robot does not slip on the particular terrain. We
recorded range-finder readings at 5 Hz as well as dense odometry readings (at
•100 Hz). Then, we used only one in four range readings, which corresponds to
choosing an interval of T • 0.8 s, such that the robot travels approximately 1
cm (in translation) and 20 ◦ (in rotation) per interval.
```

四组运动实验：直线运动， 纯旋转， 朝左右运动

four pairs of “canonical” inputs $(\omega_L, \omega_R) = \pm\omega_{max}(+1,+1)$, $\pm\omega_{max}(+1,-1)$, $\pm\omega_{max}(+1,0)$, $\pm\omega_{max}(0,+1)$. The nominal

github代码daima建议：

建议你采集一小段距离（运动 1 分钟 左右）就可以了，距离越长，outlier 越多，需要调整参数，才能有好的标定结果。标定路径不需要闭环，简单跑一下弯曲的轨迹就行，迭代次数可以修改，论文中采用过 4-8之间，效果差不多。

补充： 拉格朗日乘子方法求解带约束的优化问题 [如何理解拉格朗日乘子法？]

$$\begin{gathered} max\, f(P) = d(M,P) + d(P,C) \\ subject\, to\,:\, g(P) = 0. \end{gathered}$$

我们知道在多元微积分中如果想求一个函数的极值一般的做法是把 $\nabla f(\boldsymbol{P}) = 0$，如何把这个公式和我们的约束条件 $\nabla g(\boldsymbol{P}) = 0$ 统一在一起呢?

答案是：引入 λ 并且定义一个新的函数： $F(\boldsymbol{P}, \lambda) = f(\boldsymbol{P}) - \lambda g(\boldsymbol{P})$，

令： $\nabla F(\boldsymbol{P}, \lambda) = \nabla F(x, y, \lambda) = \begin{pmatrix} F_x \\ F_y \\ F_\lambda \end{pmatrix} = \mathbf{0}$ 与我们要求解的优化问题是 等价的：

因为: $F_\lambda = g(\boldsymbol{P}) = g(x, y) = 0$ 与约束条件等价，而且此时 $F(\boldsymbol{P}, \lambda) = f(\boldsymbol{P}) - \lambda 0 = f(\boldsymbol{P})$ 即拉格朗日函数 $F(\boldsymbol{P})$ 与我们的目标函数 $f(\boldsymbol{P})$ 取相同值。 用拉格朗日函数把目标函数和约束条件统一在了一起。

实际上这种方法与上面的几何方法是完全等价的：

$$\begin{gathered} F_x = 0 \Rightarrow f_x - \lambda g_x = 0 \Rightarrow f_x = \lambda g_x \ (1) \\ F_y = 0 \Rightarrow f_y - \lambda g_y = 0 \Rightarrow f_y = \lambda g_y \ (2) \\ g(x, y) = 0 \ (3) \end{gathered}$$

8. 算法伪代码

Algorithm 1 Simultaneous calibration of odometry and sensor parameters

1) Passively collect measurements over any sufficiently exciting trajectory.
2) For each interval, run the sensor displacement algorithm to obtain the estimates $\hat{\boldsymbol{s}}^k$.
   Each interval thus contributes the data sample

$$\langle \hat{\boldsymbol{s}}^k, \omega_{\mathrm{L}}(t), \omega_{\mathrm{R}}(t) \rangle, \qquad t \in [t_k, t_{k+1}].$$

3) Repeat N times (for outlier rejection):
   Linear estimation of J_{21}, J_{22}:
   a) For all samples, compute the matrix $\boldsymbol{L}_k$ using (10).
   b) Form the matrix $\sum_k \boldsymbol{L}_k^T \boldsymbol{L}_k$. If the condition number of this matrix is over a threshold, declare the problem underconstrained and stop.
   c) Compute J_{21}, J_{22} using (11).

   Nonlinear estimation of the calibration parameters:
   d) For all samples, compute c_x^k, c_y^k (19–20) and $\boldsymbol{Q}_k$ using (22).
   e) Let $\boldsymbol{M} = \sum_k \boldsymbol{Q}_k^T \boldsymbol{Q}_k$.
   f) Compute the coefficients a, b, c using to (28–30) and find the two candidates $\lambda^{(1)}, \lambda^{(2)}$.
   g) For each $\lambda^{(i)}$:
      i) Compute the 5×5 matrix $\boldsymbol{N}^{(i)} = \boldsymbol{M} - \lambda^{(i)} \boldsymbol{W}$.
      ii) If the rank of $\boldsymbol{N}^{(i)}$ is less than 4, declare the problem underconstrained and stop.
      iii) Find a vector $\gamma^{(i)}$ in the kernel of $\boldsymbol{N}^i$.
      iv) Compute $\varphi^{(i)}$ using (31).
   h) Choose the optimal φ between $\varphi^{(1)}$ and $\varphi^{(2)}$ by computing the objective function.
   i) Compute the other parameters using (32).

   Outlier rejection:
   j) Compute the χ-value of each sample using (33).
   k) Discard a fraction α of samples with the highest χ.

9. 参考

论文： As paper [Simultaneous Calibration of Odometry and Sensor Parameters for Mobile Robots]

论文对应代码: https://github.com/AndreaCensi/calibration

中文参考博客：[2d Laser 和 Odomter 内外参数标定工具原理及使用方法]()

中文参考博客对应代码：https://github.com/MegviiRobot/OdomLaserCalibraTool

10 ys_astrid实验数据

	1	2	3	4	aver	ref-measure
Axle between wheels	0.602922	0.538422	0.549336	0.546807	0.5449	0.53575
LiDAR-odom x	0.119317	0.123547	0.139148	0.138092	0.1336	0.14
LiDAR-odom y	0.0435663	0.00500196	0.00582043	0.0092771	0.0067	0.0
LiDAR-odom yaw	0.00382949	0.00594876	0.000741958	0.00450296	0.0037	0.0
Left wheel radius	0.083863	0.0834078	0.0841896	0.0837314	0.0838	0.0845
Right wheel radius	0.0871739	0.0834341	0.0838561	0.0834509	0.0836	0.0845
数据是否可靠	N	y	y	y		