HKUST **FINA3203 Fall 2018**
School of Business **Yan Ji**

Midterm Mock Exam (solutions)

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1 (20 points) Short Questions

(Please briefly explain your answers.)

(i) (5 points) Explain briefly the role of the "initial margin" and the "maintenance margin" in the "mark-to-market" procedure in the futures market.

Answer: We need to deposit the "initial margin" into the margin account once we start a futures position. The margin account balance is adjusted daily based on the changes in daily settlement prices. If the margin account balance ever falls below the "maintenance margin", a margin call will be initiated, which requires us to deposit money to restore the balance to the "initial margin".

Note: The question is asking about the "mark-to-market" procedure. If your answer is instead about how the two types of margins help eliminate the counter-party risk exposures of the exchange, you will receive full credit as well.

(ii) (5 points) On 10/12/2012, the December Euro FX Futures are traded at the price $F = 1.2938$ USD/EUR. At the same time, the spot exchange rate between the Euro and the Dollar is $M = 1.2957$ USD/EUR. Does the fact that $F < M$ suggest that the market expects the Euro to depreciate against the dollar in the next two months?

Answer: False or it depends. The forward price F (ignoring the difference between forward price and futures price) is only the risk-neutral expectation of the exchange rate in the future, and it does not tell us (directly) what the actual expectation of the exchange rate is.

(The part below is not required for full credit.)

In fact,

$$M_0 = e^{-(\mu - r_{€})T} E[M_T], \tag{1}$$

where μ is the expected return on the foreign currency (think of foreign currency as a financial asset, e.g. a stock), and $r_€$ is the foreign interest rate that can be treated as a dividend yield. Since

$$F_0 = M_0 e^{(r_\$ - r_€)T}, \tag{2}$$

we have

$$F_0 = e^{-(\mu - r_\$)T} E[M_T]. \tag{3}$$

we can see that even if $F_0 < M_0$, if $\mu - r_\$ > 0$, it could be that $E[M_T] > M_0$.

(iii) (5 points) Explain what is a currency carry trade.

Answer: Currency carry trade requires an investor to borrow low interest rate currencies and invest in high interest rate currencies.

Note: You will also get full credit if your answer also includes trading a currency forward contract to cover the exchange rate risk.

(iv) (5 points) Explain the intuition behind the derivation of the put-call parity.

Answer: We long a put and short a call with the same strike price and maturity. The combined position gives us a "synthetic" forward contract with the forward price being the strike price.

Thus, the value of put minus call is equal to the value of this "synthetic" forward contract, which can be calculated using a standard formula.

Note: You will also get full credit if you derived the put-call parity without any explanations (then I assume that you understand the intuition)...

2 (20 points) Forwards

A stock is expected to pay a dividend of \$1.5 per share each month in the next 6 months. The current spot stock price is \$120 per share, and the annualized continuously compounded risk-free rate is 4.5%. An investor has just taken a short position in a 12-month forward contract on the stock.

(i) (5 points) What is the present value of dividends? What is the initial value of the forward contract?

Answer:

$$PV_0(D) = \$1.5 \sum_{t=1}^{6} e^{-4.5\% \times \frac{t}{12}} = \$8.88 \tag{4}$$

The initial value of the forward contract is 0.

(ii) (15 points) Six months later after the last dividend is paid, the spot stock price is \$115 per share and the risk-free rate remains at 4.5%. What is the value of the short position in the original forward contract? You need to show clearly how to use the replicating portfolio approach to derive the value, instead of using the formula directly.

Answer: Denote the value of the short position in 6 months as V.

Positions	PV at 6-month	12-month
Original short forward	V	$F_{0,12} - S_T$
long forward	0	$S_T - F_{6,12}$
Total	V	$F_{0,12} - F_{6,12}$

Now, we have to calculate the forward prices

$$\begin{aligned} F_{0,12} &= [S_0 - PV_0(D)]\, e^r \\ &= (120 - 8.88) \times e^{4.5\%} = \$116.23 \end{aligned} \tag{5}$$

$$\begin{aligned} F_{6,12} &= S_{6mth} e^{r \times \frac{6}{12}} \\ &= 115 \times e^{4.5\% \times \frac{6}{12}} = \$117.62 \end{aligned} \tag{6}$$

Thus

$$V = (F_{0,12} - F_{6,12}) e^{-r/2} = -\$1.36 \tag{7}$$

3 (20 points) Futures

The December Eurodollar futures contract is quoted as 98.40 and a company plans to borrow \$8 million for three months starting in December at LIBOR plus 0.5%.

(i) (4 points) What rate can the company lock in by using the Eurodollar futures contract?

Answer: The company can lock in a 3-month rate of $100 - 98.4 = 1.60\%$. The rate it pays is therefore locked in at $1.6 + 0.5 = 2.1\%$.

(ii) (10 points) What position should the company take in Eurodollar futures contracts to hedge the interest rate risk? Explain clearly what happens when LIBOR rates increase or decrease 3 months.

Answer: The company should sell (i.e., short) Eurodollar futures contracts.

If rates increase, the futures quote goes down and the company gains on the futures. This compensates the company's loss from higher borrowing rates.

Similarly, if rates decrease, the futures quote goes up and the company loses on the futures. But on the other hand, the company get a lower borrowing rates. The gains and losses from the two positions again offset each other.

(iii) (6 points) What position should the company take if it wants to hedge the interest rate risk using FRAs? Explain why.

Answer: The company should long FRAs.

According to the payoff formula of FRAs, the company gains when LIBOR rates increase, which compensates the company's loss from higher borrowing rates.

4 (40 points) Swaps

Suppose that some time ago a financial institution agreed to receive 1-year LIBOR and pay 3% per annum on a notional principal of $100 million. Payments are exchanged every year. The swap has a remaining life of 2.5 years. The 1-year LIBOR rates for 6-month, 18-month, and 30-month maturities are 2.8%, 3.2%, and 3.4%, respectively. The 1-year LIBOR rate at the last payment date was 2.9%. (All interest rates quoted above are simple interest rates)

(i) (10 points) The 1-year LIBOR rate for 6-month is 2.8%. The 1-year LIBOR rate at the last payment date was 2.9%. Which LIBOR rate should we use when calculating the forward rates, $f_{0,6mth,18mth}$? Which LIBOR rate should we use when calculating the floating payment in 6 months?

Answer: We should use the 1-year LIBOR rate for 6-month, 2.8%, to calculate the forward rate between the 6th month and the 18th month, because this is the forward rate observable at time 0 (now).

The last payment happened 6 months ago. We should use the 1-year LIBOR rate at the last payment date, 2.9%, to calculate the floating payment in 6 months. As mentioned in lecture notes, the convention in bond market is to use the past year's interest rate to calculate the floating payment, as this is the period during which you earn the interest.

(ii) (10 points) Calculate the continuously compounded forward rates $f_{0,6mth,18mth}$ and $f_{0,18mth,30mth}$. What are the corresponding FRA rates, $r_{\text{FRA},6mth,18mth}$ and $r_{\text{FRA},18mth,30mth}$?

Answer: The LIBOR rates we are given are simple interest rates. Thus we first calculate the corresponding continuously compounded interest rates.

$$\begin{aligned}
r_{0,6mth} &= \ln\left(1 + 0.5 \times r_{\text{LIBOR},0,6mth}\right)/0.5 = \ln\left(1 + 0.5 \times 2.8\%\right)/0.5 = 2.78\% \\
r_{0,18mth} &= \ln\left(1 + 1.5 \times r_{\text{LIBOR},0,18mth}\right)/1.5 = \ln\left(1 + 1.5 \times 3.2\%\right)/1.5 = 3.13\% \\
r_{0,30mth} &= \ln\left(1 + 2.5 \times r_{\text{LIBOR},0,30mth}\right)/2.5 = \ln\left(1 + 2.5 \times 3.4\%\right)/2.5 = 3.26\%
\end{aligned}$$

Now we use our formula to calculate the continuously compounded forward rates:

$$\begin{aligned}
f_{0,6mth,18mth} &= r_{0,18mth} \times 1.5 - r_{0,6mth} \times 0.5 = 3.13\% \times 1.5 - 2.78\% \times 0.5 = 3.305\% \\
f_{0,18mth,30mth} &= r_{0,30mth} \times 2.5 - r_{0,18mth} \times 1.5 = 3.26\% \times 2.5 - 3.13\% \times 1.5 = 3.455\%
\end{aligned}$$

The FRA rates are also in terms of simple interest rates, thus we have

$$\begin{aligned}
r_{\text{FRA},6mth,18mth} &= e^{f_{0,6mth,18mth}} - 1 = 3.36\% \\
r_{\text{FRA},18mth,30mth} &= e^{f_{0,18mth,30mth}} - 1 = 3.52\%
\end{aligned}$$

(iii) (20 points) Calculate the current value of the swap. Clearly explain and write down the payoff of each component in your portfolio.

Answer: Denote V as the value of the swap at time 0 for a notional value of \$1. The payoff table is given by:

Positions	PV	6-month	18-month
Long swap	V	$r_{\text{LIBOR},-6mth,6mth} - 3\%$	$r_{\text{LIBOR},6mth,18mth} - 3\%$
Short FRA 6-18mth		$\frac{r_{\text{FRA},6mth,18mth} - r_{\text{LIBOR},6mth,18mth}}{1+r_{\text{LIBOR},6mth,18mth}}$	
Invest in bonds		$-\frac{r_{\text{FRA},6mth,18mth} - r_{\text{LIBOR},6mth,18mth}}{1+r_{\text{LIBOR},6mth,18mth}}$	$r_{\text{FRA},6mth,18mth} - r_{\text{LIBOR},6mth,18mth}$
Short FRA 18-30mth			$\frac{r_{\text{FRA},18mth,30mth} - r_{\text{LIBOR},18mth,30mth}}{1+r_{\text{LIBOR},18mth,30mth}}$
Invest in bonds			$-\frac{r_{\text{FRA},18mth,30mth} - r_{\text{LIBOR},18mth,30mth}}{1+r_{\text{LIBOR},18mth,30mth}}$
Total	V	$r_{\text{LIBOR},-6mth,6mth} - 3\%$	$r_{FRA,6mth,18mth} - 3\%$
(put in numbers)	V	$2.9\% - 3\%$	$3.36\% - 3\%$

Positions	30-month
Long swap	$r_{\text{LIBOR},18mth,30mth} - 3\%$
Short FRA 6-18mth	
Invest in bonds	
Short FRA 18-30mth	
Invest in bonds	$r_{\text{FRA},18mth,30mth} - r_{\text{LIBOR},18mth,30mth}$
Total	$r_{\text{FRA},18mth,30mth} - 3\%$
(put in numbers)	$3.52\% - 3\%$

Now we can calculate the value of the swap contract for a notional value of \$1 as

$$V = \frac{2.9\% - 3\%}{1 + 0.5 \times r_{\text{LIBOR},0,6mth}} + \frac{3.36\% - 3\%}{1 + 1.5 \times r_{\text{LIBOR},0,18mth}} + \frac{3.52\% - 3\%}{1 + 2.5 \times r_{\text{LIBOR},0,30mth}} = 0.0072$$

The value of the swap for a notional value of \$100 million is \$0.72 million.