HKUST **FINA3203 Spring 2020**
School of Business **Yan Ji**

Final Exam

May, 2020

1 (20 points) Short Questions

(Please briefly explain your answers.)

(i) (5 points) An American call option on a foreign currency should never be exercised before maturity. Explain your answer.

(ii) (5 points) Since a European call on the S&P500 has a lot of upside (when the stock price rises) but limited downside (when the stock price falls), it is a safer investment than the S&P500 index and should offer lower expected return than the S&P500 does. Explain your answer.

(iii) (5 points) The Theta (Θ) of a put option is always positive because shrinking the time to maturity (due to a rise in t) leads to less discounting on the strike. Explain your answer.

(iv) (5 points) Describe concisely the procedure for forecasting the probability of default for a publicly traded company using the Merton model. State clearly what information you need for the forecast.

2 (20 points) Long Question 1

A two-month European put option on a non-dividend-paying stock is currently selling for \$2. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6% (annualized, continuous compounding).

(i) (13 points) What opportunities are there for an arbitrageur? Please explain.

(ii) (7 points) Would the above market prices still provide an arbitrage opportunity if the stock will pay a dividend of \$2/per share in 1 month?

3 (30 points) Long Question 2

Suppose that stock XYZ, whose current price is $S_0 = \$100$, can either increase by $u = 1.1$ or decrease by $d = 1/1.1$ per year in the next 2 years. The annual interest rate is zero.

(i) (5 points) Use the risk-neutral pricing methodology to compute the price of a European at-the-money 2-year call option based on a 2-step binomial tree.

(ii) (10 points) Suppose you sold this option to a client. Devise a dynamic trading strategy to hedge your position. Describe the strategy in detail, including the changes in stock and bond position over time.

(iii) (8 points) A client would like to enter into an option, which gives the client the right (but not the obligation) to buy the option described in question (i) at time $T = 1$ for a strike price $K = 5$. What is the price of this option? (This type of options is called compound options. It is an option on options.)

(iv) (7 points) You sold the compound option in question (iii) to the client. You decide to hedge against this short position by going long the underlying, that is, the option in question (i). What is your hedging position at time 0? Make sure to describe both the position in the underlying and in bonds.

4 (30 points) Long Question 3

Consider a firm whose value of asset is $V_0 = 100$ at $t = 0$. The firm issues a 2-year zero-coupon bond with face value $F = 90$ at $t = 0$. The continuously-compounded, annualized risk-free rate is $r = 3\%$, and the continuously-compounded, annualized expected return on assets is $\mu = 6\%$. The daily volatility of the firm's assets' value is $\sigma = 0.9\%$.

(i) (5 points) What is the annualized volatility of the firm's assets' value? Assume there are 252 business days in a year.

(ii) (10 points) What is the probability of default for the firm at $t = 2$ when the zero coupon bond matures. Compute this probability using the Merton model.

Note: The cumulative density function of the normal distribution, i.e. $N(\cdot)$ is provided on the last page. In this table, the label for rows contains the integer part and the first decimal place of z. The label for columns contains the second decimal place of z. The values within the table are the probabilities corresponding to the table type. For example, to find $z = -0.55$, one would look down the rows to find -0.5 and then across the columns to 0.05 which would yield a probability of $N(-0.55) = 0.2912$. **Please do not use excel to do the calculation of $N(\cdot)$ as the numbers may not exactly match the numbers in the table due to rounding errors.**

(iii) (10 points) What is the price of the 2-year zero coupon bond at $t = 0$? Use the table on the last page as needed.

(iv) (5 points) Consider a 1-year CDS contract on the name with the risk-neutral default rate $d^* = 7.3\%$. Suppose that the recovery rate is 40%. Using traders' "short-cut" formula to compute the CDS spread. Suppose you use the actual default rate to compute the CDS spread, will you get a higher or lower CDS spread?

5 (30 points) Long Question 4

This question is more difficult compared to other questions. The purpose here is to test whether you understand the logic/intuition of option pricing or just merely memorize the formulas in lecture notes.

The exchange option is a contract that gives the holder the right to exchange asset B for asset A. Hence the payoff of the European exchange call option can be written as

$$\begin{aligned} C_{\text{exchange}} &= \max\left\{0, S^A(T) - S^B(T)\right\} \\ &= S^B(T)\max\left\{0, \frac{S^A(T)}{S^B(T)} - 1\right\} \end{aligned}$$

The way to price these exchange options is by modeling the dynamics of the ratio of the two prices instead of the dynamics of each stock price. For example, consider a 1-period binomial tree. The initial ratio is given by

$$\frac{S_0^A}{S_0^B}.$$

Next period the ratio can go up to $u \times \frac{S_0^A}{S_0^B}$ or down to $d \times \frac{S_0^A}{S_0^B}$. Let the data of this one-period tree be given by

$$\begin{aligned} S_0^A &= 100 \\ S_0^B &= 125 \\ u &= 2 \\ d &= 0.5 \end{aligned}$$

Assume no dividend payments for stock A or B.

(i) (20 points) Compute a replicating portfolio for the 1-period European exchange call option that invests in stock A and stock B. How many shares of stock A and stock B do you want to buy? Use this replicating portfolio to determine the price of the call. (HINT: No need to use bonds.)

(ii) (5 points) Price the American exchange call (which can be exercised immediately).

(iii) (5 points) Use the price of the exchange option to find the price of the call that pays the maximum of the two stocks, that is, the one with a final payoff:

$$\max\left\{S^A(T), S^B(T)\right\}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952