ECON3113
Microeconomic Theory I

Tutorial #1: The foundations of consumer choice

Your TA

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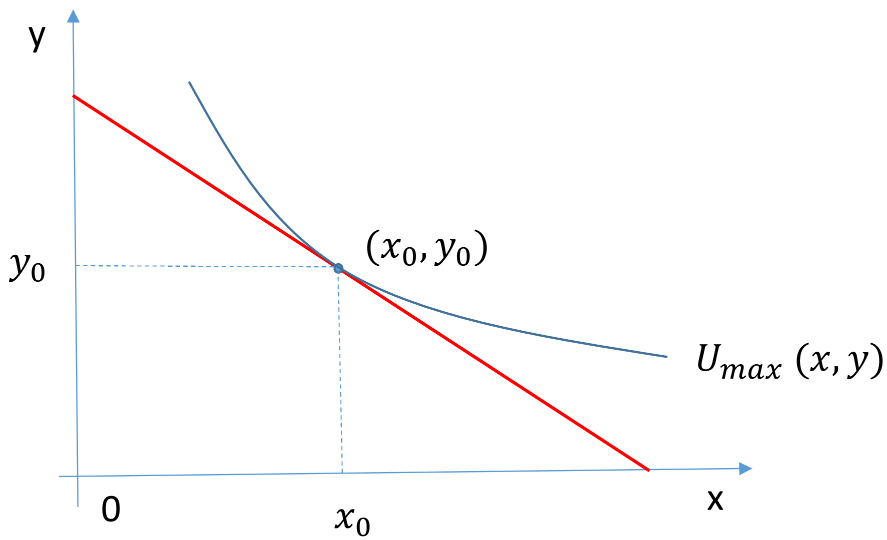
Today’s tutorial: the foundations of consumer choice

- Limitations of the ‘traditional’ approach
- A modern approach: the building blocks of consumer choice
    - Coherence
    - Preference relations
    - Choice function
- Exercises
- Postscript: Is Transitivity ever violated?

- Limitations of the 'traditional' approach to utility and consumer choice



- The traditional approach to consumer choice in economics starts with a utility function eg
  - $U = x^{1/2}y^{1/2}$
- And a budget constraint
  - $I = \ P_x x + P_y y$
- And maximises U subject to $P_x x + P_y y \leq I$
- But, we do not know U for individuals, who might not behave in this way in any case

- Can we build a theory of consumer choice that does not rely on knowing an individual's utility function?

The building blocks of a modern theory of consumer choice

- We will use one key concept and three building blocks:
  - Key concept: Coherence
  - Building blocks:
    - Choice function
    - Preference relation
    - Utility function
- <u>Our goal is to show how these building blocks are linked</u>

Building block #1: the Choice Function

- Assume that a consumer has a set of feasible alternatives, $A = \{x_1, x_2, \dots, x_n\}$, and will choose one of them (ie not making a choice is not an option)
- Then we define $\mathrm{c}(\mathrm{A})$ as the Choice Function ie the choices made by the consumer

Example:

- Assume that a consumer may choose from the following drinks in the morning:

$A = \{$tea, espresso, latte$\}$

- If the consumer chooses tea, then we may write $\mathrm{c}(\mathrm{A}) = \{tea\}$
- If the consumer chooses tea and latte, then we may write $\mathrm{c}(\mathrm{A}) = \{$tea, latte$\}$
    - Note that the consumer may choose more than one alternative
    - Note that $\mathrm{c}(\mathrm{A})$ is a set

Building block #1: the Choice Function

- Assume that a consumer has a set of feasible alternatives, $A = \{x_1, x_2, \ldots, x_n\}$, and will choose one of them (ie not making a choice is not an option)
- Then we define $\mathrm{c(A)}$ as the Choice Function ie the choices made by the consumer

Example:

- Assume that a consumer may choose from the following drinks in the morning:

$A = \{$tea, espresso, latte$\}$

- If the consumer always chooses tea, then we may write $\mathrm{c(A)} = \{$tea$\}$
- If the consumer always chooses either tea and latte, we may write $\mathrm{c(A)} = \{$tea, latte$\}$
  - Note that the consumer may choose more than one alternative (and could choose the whole set A)

Key Concept: Coherence

- Assume that our consumer visits her/his favourite coffee shop and has the choice of:

$$A = \{\text{tea, espresso, latte}\}$$

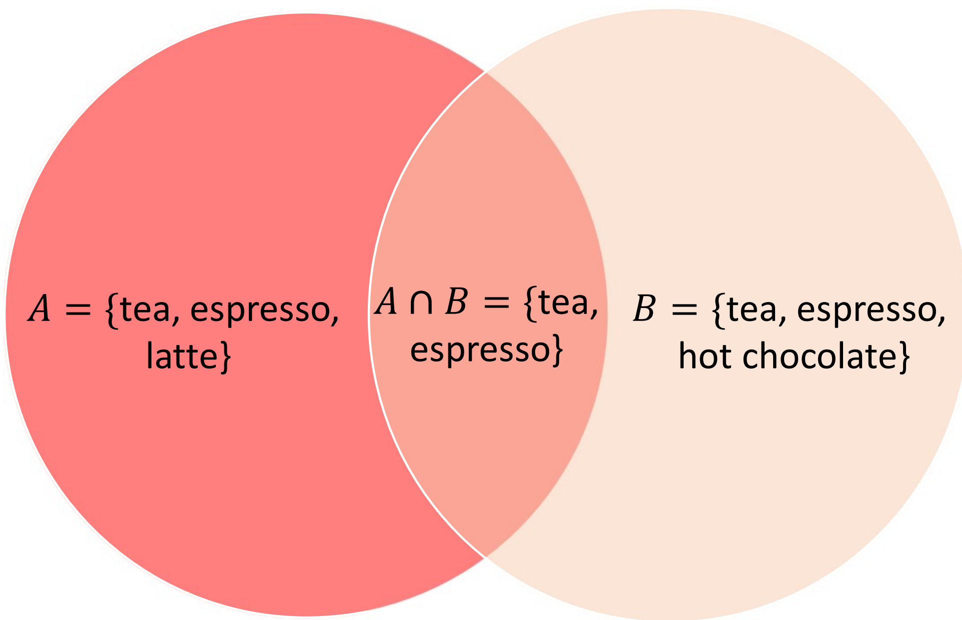
- Assume that she/he always chooses tea ie $\mathrm{c(A)} = \{\text{tea}\}$
- Now suppose that an employee says that in fact there is a new product range, and that the consumer may now choose from:

$$B = \{\text{tea, espresso, hot chocolate}\}$$

- Is it Coherent behavior:
  - Still to choose tea?
  - To switch to hot chocolate?
  - To switch instead to espresso?

Key Concept: Coherence

- Coherence in symbols:



Coherence:

- When offered set A, the consumer chooses tea and rejects espresso
- When offered set B, both tea and espresso are still available
- If the consumer switches to espresso, this is not Coherent behavior
- Coherent behavior is to choose tea or hot chocolate

Building block #2: The Preference Relation

- Given a set of alternatives, X, and any pair of alternatives, x and y:

$$x \geq y$$

means that x is at least as good as y

- $x \geq y$ is read as 'x is <u>weakly preferred</u> to y'
- $x > y$ is read as 'x is <u>strongly preferred</u> to y'

- We assume that the pairwise comparison can be done for all x and y in X

Building block #2: The Preference Relation

- For any x and y in X, we have the following possibilities:

Possibility	Notation	Read: 'the consumer…
$x \geq y$ and $y \geq x$	$x \sim y$	Is indifferent between x and y'
$x \geq y$ but not $y \geq x$	$x > y$	strictly prefers x to y'
$y \geq x$ but not $x \geq y$	$y > x$	strictly prefers y to x'
Neither $x \geq y$ nor $y \geq x$	Not defined	cannot compare x and y'

- Therefore, $x \geq y$ is equivalent to saying either $x > y$ or $x \sim y$
- The analysis here focuses on weak preference

Building block #2: The Preference Relation – Completeness, Reflexivity, and Transitivity

- **Completeness**: The preference relation is complete if a comparison exists for each x and y in X
  - ie $x \geq y$, or $y \geq x$ or both
  - Includes comparing x to itself
- **Reflexivity**: The relation is reflexive if $x \geq x$ (ie the item is at least as good as itself)
- **Transitivity**: If x is preferred to y and y is preferred to z, then x is preferred to z:

$$\text{If } x \geq y \text{ and } y \geq z, \text{ then } x \geq z$$

Building block #2: The Preference Relation – Completeness, Reflexivity, and Transitivity

- Why is Transitivity important?
- Example: The Money Pump Scheme
- Assume that Person 2 has preferences:
  - $x \geq y$, $y \geq \$10$, $\$8 \geq x$
- Then Person 1 could extract all Person 2's wealth as follows:

Step #	Step	Person 1	Person 2
1	Person 1 sells y to Person 2 for $10	-y +$10	+y -$10
2	Person 1 exchanges x for y from Person 2	-y +$10 -x +y	+y -$10 +x -y
3	Person 1 buys x from Person 2 for $8	-y +$10 -x +y -$8 +x	+y -$10 +x -y +$8 -x
4	Repeat	Earns $2 each time	Loses $2 each time

A Preference Relation implies a Choice Function

- Assume that we have a set of alternatives, A
- Also assume that we have a preference relation that satisfies Completeness, Reflexivity and Transitivity
- Can our preference relation generate a choice function?
- That is, given A, can we find $\mathrm{c}(\mathrm{A})$?

- The answer is 'yes' – here's how…

A Preference Relation implies a Choice Function

- Assume that we have a set of alternatives, A, and a preference relation that satisfies Completeness, Reflexivity and Transitivity
- Assume that x' and y' are in A
- Two points:
  - If y' $\geq$ x' and <u>not</u> x' $\geq$ y', then we would expect x' <u>not</u> to be chosen
  - If x' is chosen, we cannot find a y' in A that x' is <u>not</u> weakly preferred to
- Implication:
  - Given a set of alternatives, A, all choices are those members of A that are weakly preferred to everything in A
- This allows us to derive a Choice Function

A Preference Relation implies a Choice Function

- Assume that $A = \{\text{tea, espresso, latte}\}$
- Assume that we have the following preferences:
  - tea $\geq$ espresso; not espresso $\geq$ tea (ie tea strongly preferred to espresso)
  - tea $\geq$ latte; latte $\geq$ tea (ie indifferent between tea and latte)
  - latte $\geq$ espresso; not espresso $\geq$ latte (ie latte strongly preferred to espresso)

- We have the following preference table (row heading □ column heading):

	tea	espresso	latte
tea	$\sim$	>	$\sim$
espresso	<	$\sim$	
latte	$\sim$	>	$\sim$

A Preference Relation implies a Choice Function

	tea	espresso	latte
tea	$\sim$	$>$	$\sim$
espresso	$<$	$\sim$	
latte	$\sim$	$>$	$\sim$

- To find the members of $\mathrm{c}(\mathrm{A})$, look at the rows of the table
  - If the row header is (at least) weakly preferred to everything in the columns, then the row header is in $\mathrm{c}(\mathrm{A})$
- So in this case $\mathrm{c}(\mathrm{A}) = \{\text{tea, latte}\}$
- <u>Therefore, our preference relation implies a choice function:</u>
  - $A \rightarrow c(A)$
  - $\{\text{tea, espresso, latte}\} \rightarrow \{\text{tea, latte}\}$

Progress so far…

- So we have shown (but not proved!) that a well-defined preference relation that obeys Completeness, Reflexivity and Transitivity generates a choice function
- Next step:
  - To show that a utility function generates a preference relation and a choice function

Exercise #1

- We may denote a binary relation R on a set A
- For $x, y \in A, xRy$ means that x obeys the relation R with respect to y
- Are the following binary relations Complete, Reflexive and Transitive on the given sets?

Set	Binary relation	Complete	Reflexive	Transitive
All students at HKUST	'Takes the same course as'			

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Set	Binary relation	Complete	Reflexive	Transitive
All people in the world	'Is an ancestor of'			

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Set	Binary relation	Complete	Reflexive	Transitive
Rock, paper, scissors	'x beats y'			

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- For $x, y \in A$, xRy means that x obeys the relation R with respect to y
- Are the following binary relations Complete, Reflexive and Transitive on the given sets?

Set	Binary relation	Complete	Reflexive	Transitive
Real numbers	'is a factor of'			

Exercise #1

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- For $x, y \in A, xRy$ means that x obeys the relation R with respect to y
- Are the following binary relations Complete, Reflexive and Transitive on the given sets?

Set	Binary relation	Complete	Reflexive	Transitive
Words in an English dictionary	'has the same meaning as'			

Exercise #2

- Given the following set $A = \{x_1, x_2, x_3, x_4\}$ and the given preferences, is the preference relation Complete, Reflexive and Transitive?
- If it fails one of the criteria, change it so that it meets them all

	x_1	x_2	x_3	x_4
x_1	$\sim$	$>$	$\sim$	$<$
x_2	$<$	$\sim$	$\sim$	$<$
x_3	$<$	$>$	$\sim$	$<$
x_4	$>$	$\sim$	$>$	$\sim$

Exercise #2

- Given the following set $A = \{x_1, x_2, x_3, x_4\}$ and the given preferences, is the preference relation Complete, Reflexive and Transitive?
- If it fails one of the criteria, change it so that it meets them all

	x_1	x_2	x_3	x_4
x_1	$\sim$	$>$	$\sim$	$<$
x_2	$<$	$\sim$	$\sim$ $<$	$<$
x_3	$<$ $\sim$	$>$	$\sim$	$<$
x_4	$>$	$\sim$ $>$	$>$	$\sim$

Exercise #2

- Use the correct preference relation to derive the choice function in this case

	x_1	x_2	x_3	x_4
x_1	$\sim$	$>$	$\sim$	$<$
x_2	$<$	$\sim$	$<$	$<$
x_3	$\sim$	$>$	$\sim$	$<$
x_4	$>$	$>$	$>$	$\sim$

- To find the choice function, look across each row
- If an element is weakly preferred to all others, then it is in $\mathrm{c}(\mathrm{A})$

- $A \to c(A)$

- $\{x_1, x_2, x_3, x_4\} \to \{ \qquad \}$

Progress so far…

- So we have shown (but not proved!) that a well-defined preference relation that obeys Completeness, Reflexivity and Transitivity generates a choice function
- Next step:
  - To show that a utility function generates a preference relation and a choice function

Postscript: Is Transitivity ever violated?

- A major topic of research and debate over the past 40 years
- A classic finding: 'Loss Aversion'
  - People hate losing much more than they like winning
  - Can lead to apparent violations of Transitivity between preferences
- Note: This requires a world of uncertainty, so different to the model that we have been considering

Postscript: Is Transitivity ever violated?

- Suppose that an individual is offered 3 possibilities:

#	Win	P(Win)	Lose	P(Lose)	E(Gain)
A	$1,000,000	75%	$200,000	25%	$700,000
B	$200,000	75%	$50,000	25%	$137,500
C	$10,000	75%	$10,000	25%	$5,000

Kahneman, Daniel (2011). Thinking, Fast and Slow. Farrar, Straus and Giroux. ISBN 978-1-4299-6935-2. Retrieved March 10, 2016.

*Kahneman, Daniel; Tversky, Amos (1979). "Prospect Theory: An Analysis of Decision under Risk" (PDF). Econometrica. **47** (2): 263–291. CiteSeerX 10.1.1.407.1910. doi:10.2307/1914185. ISSN 0012-9682. JSTOR 1914185.*

- Ranking according to expected gain ought to give preferences:
  - A > B > C, therefore A > C
- But if the individual strongly does not want to lose a lot of money, we could have preferences:
  - A > B , B > C, but C < A
  - An apparent violation of Transitivity
  - But doesn't this mean that we shouldn't measure behaviour just on Expected Gain?

Postscript: Prospect Theory - references

*Kahneman, Daniel; Tversky, Amos (1979). "Prospect Theory: An Analysis of Decision under Risk" (PDF). Econometrica. **47** (2): 263–291. CiteSeerX 10.1.1.407.1910. doi:10.2307/1914185. ISSN 0012-9682. JSTOR 1914185.*

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