ECON3123
Macroeconomic Theory I

Tutorial #11: The Solow Growth Model

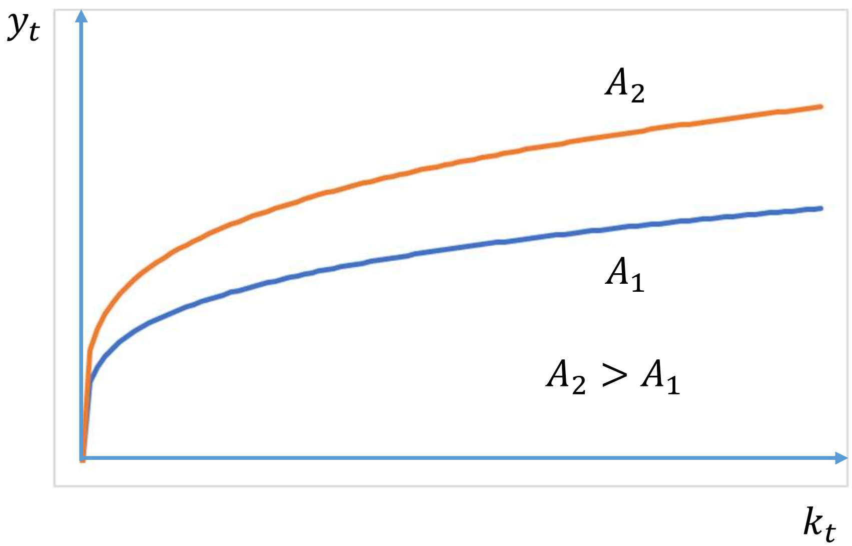
Today's tutorial

- The relationship between saving, investment and capital accumulation
- So far we have looked at the short and medium term
- We have models that help us understand short and medium term fluctuations in output, inflation and unemployment
- Now we turn to the long term
- We look at the determinants of output in the long term, and focus on saving, investment and capital accumulation

The aggregate production function

- We assume that output is produced using an aggregate constant returns to scale production function
- Production Y_t depends on capital K_t and employment N_t
  - $Y_t = F(K_t, N_t)$
  - with $\alpha Y_t = F(\alpha K_t, \alpha N_t)$ ie CRS
- We want to consider output and capital ratios to employment, so divide everything by N_t
  - $\frac{Y_t}{N_t} = F\left(\frac{K_t}{N_t}, 1\right)$
- The 1 here is always constant, so we can re-name this function f and define $y_t = \frac{Y_t}{N_t}$, $k_t = \frac{K_t}{N_t}$ to give:
  - $y_t = f(k_t)$
- This is the version of the production function that we work with

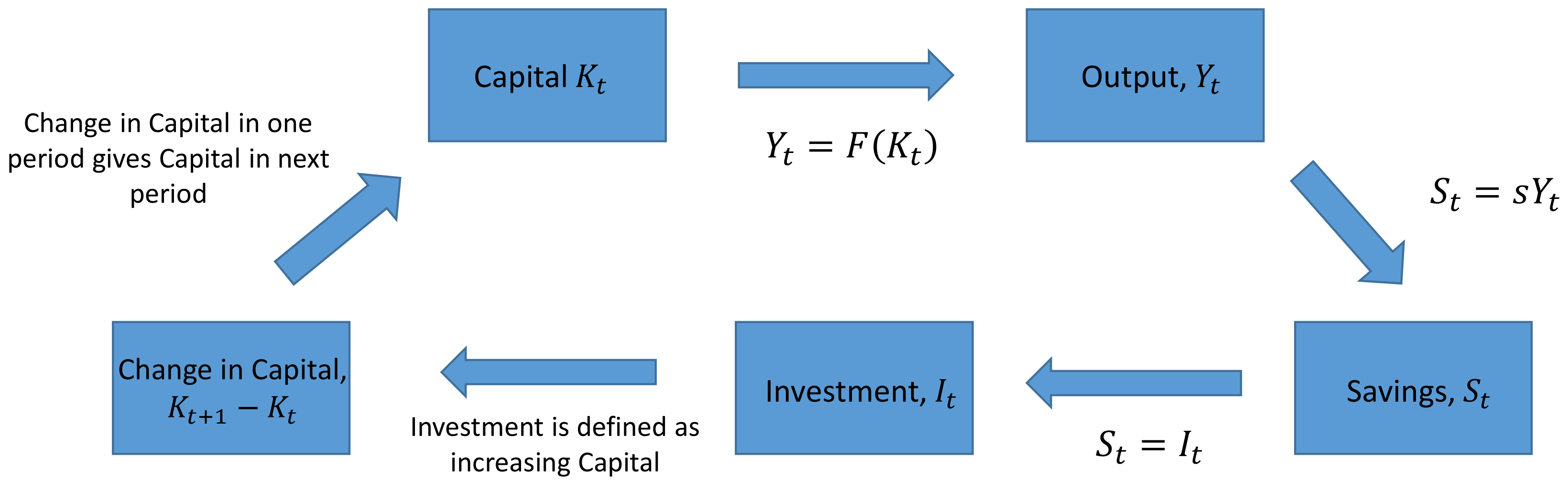
The aggregate production function



- Example:
- Consider a production function $Y_t = AK_t^{\alpha}N_t^{1-\alpha}$
  - $0 < \alpha < 1$
  - Think of A as technical progress: how much we make (Y_t) from a given amount of capital and labour depends on how advanced is our technology, A
- Then $y_t = \frac{Y_t}{N_t} = AK_t^{\alpha}N_t^{-\alpha} = A\left(\frac{K_t}{N_t}\right)^{\alpha} = Ak_t^{\alpha}$
- Note:
  - As k_t increases, y_t increases but at decreasing rate
    - y_t is concave in k_t
  - As A increases, $y_t = Ak_t^{\alpha}$ shifts upwards
- Therefore, to increase output per worker, y_t, we need either:
  - More technological progress, A
  - More capital per worker, k_t

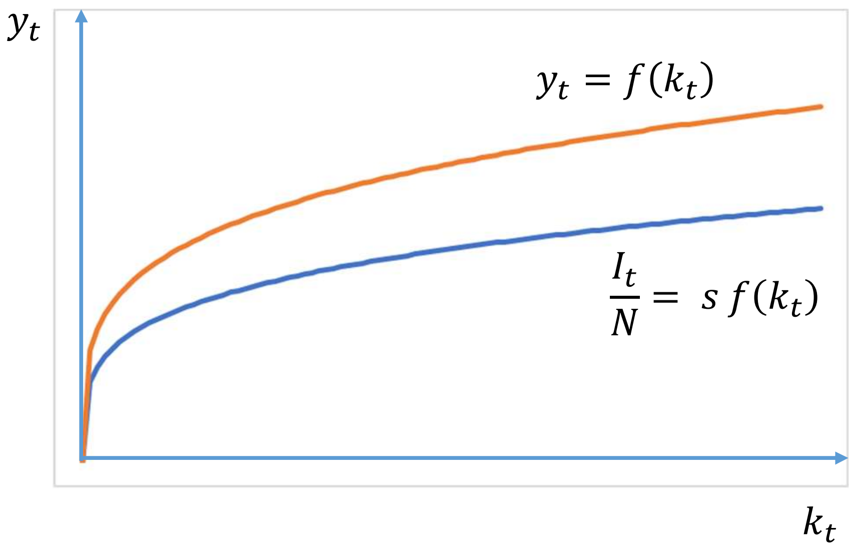
The model

- The purpose of the model is to understand what determines the long run level of output per worker
- In the model, savings, investment and increase in capital stock play key roles
- The behaviour described in the model is as follows:



- When does this process start and stop?
- Ie how do we find the equilibrium levels of output and capital?

The model: savings and investment

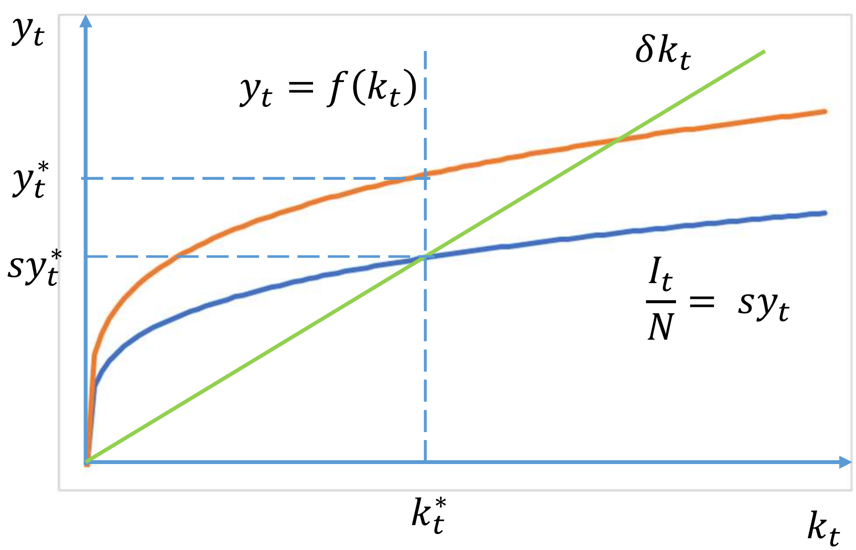


- We assume:
  - A and N fixed (so $N_t = N$)
  - A closed economy, and no budget deficit
    - $S_t - I_t = G_t - T_t$ and $G_t - T_t \Rightarrow S_t = I_t$
    - That is, savings is always equal to investment
  - Savings depends on income, and consumption is equal to income that is not saved
    - $S_t = sY_t = I_t \Rightarrow s = \frac{I_t}{Y_t}$ (with $0 < s < 1$)
    - $C_t = (1-s)Y_t$
- Dividing investment by number of workers, we have:
  - $\frac{I_t}{N} = s\, f(k_t)$

The model: how capital changes over time

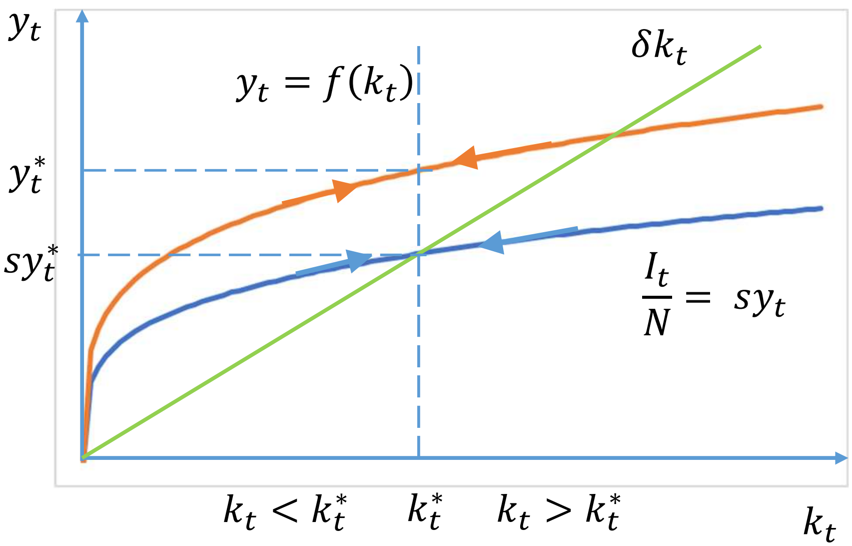
- We assume that capital at time $t+1$ is given by:
  - Capital at time t, minus
  - Depreciation during the period t, plus
  - Investment at time t
- Assume that capital depreciates at a rate of δ per period
- Then:
  - $K_{t+1} = K_t - \delta K_t + I_t$
  - $K_{t+1} = (1-\delta)K_t + I_t$
- We can divide both sides by N to give $\frac{K_{t+1}}{N} = \frac{(1-\delta)K_t}{N} + \frac{I_t}{N}$
- And since $I_t = sY_t$, and recalling the definitions of y_t and k_t we have:
  - $k_{t+1} = (1-\delta)k_t + sy_t = (1-\delta)k_t + sf(k_t)$
- This equation shows how capital changes over time

The model: the definition of the steady state



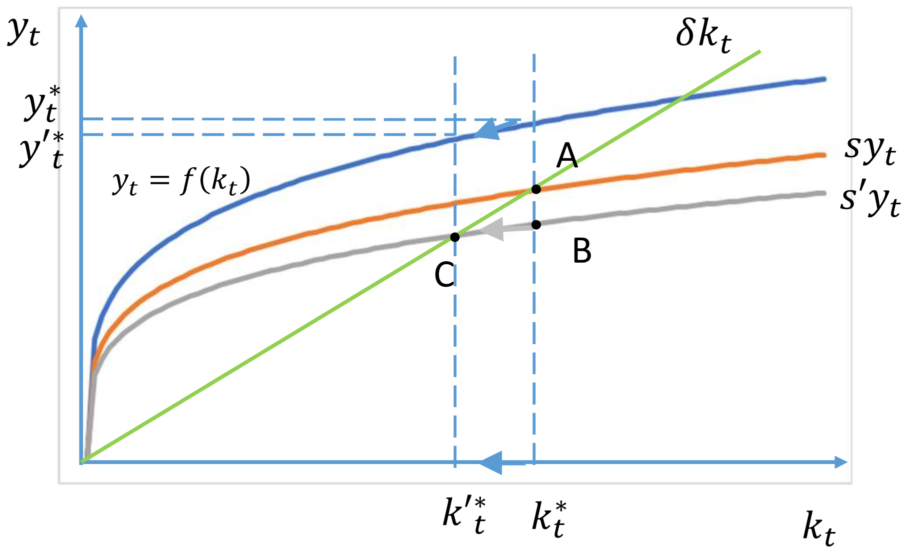
- We have:
  - $k_{t+1} = (1-\delta)k_t + sy_t$
- We may re-arrange this to give:
  - $\Delta k_{t+1} = k_{t+1} - k_t = sy_t - \delta k_t$
- This equation says that the change in capital stock is given by investment in a period minus depreciation in a period
- We may define y_t^*, k_t^* such that $\Delta k_{t+1} = 0$:
  - $sy_t^* = \delta k_t^*$
- This happens where the δk_t line crosses the sy_t line
- This point defines equilibrium in the model
- At this steady-state point, investment is replacing depleted capital

The model: model dynamics



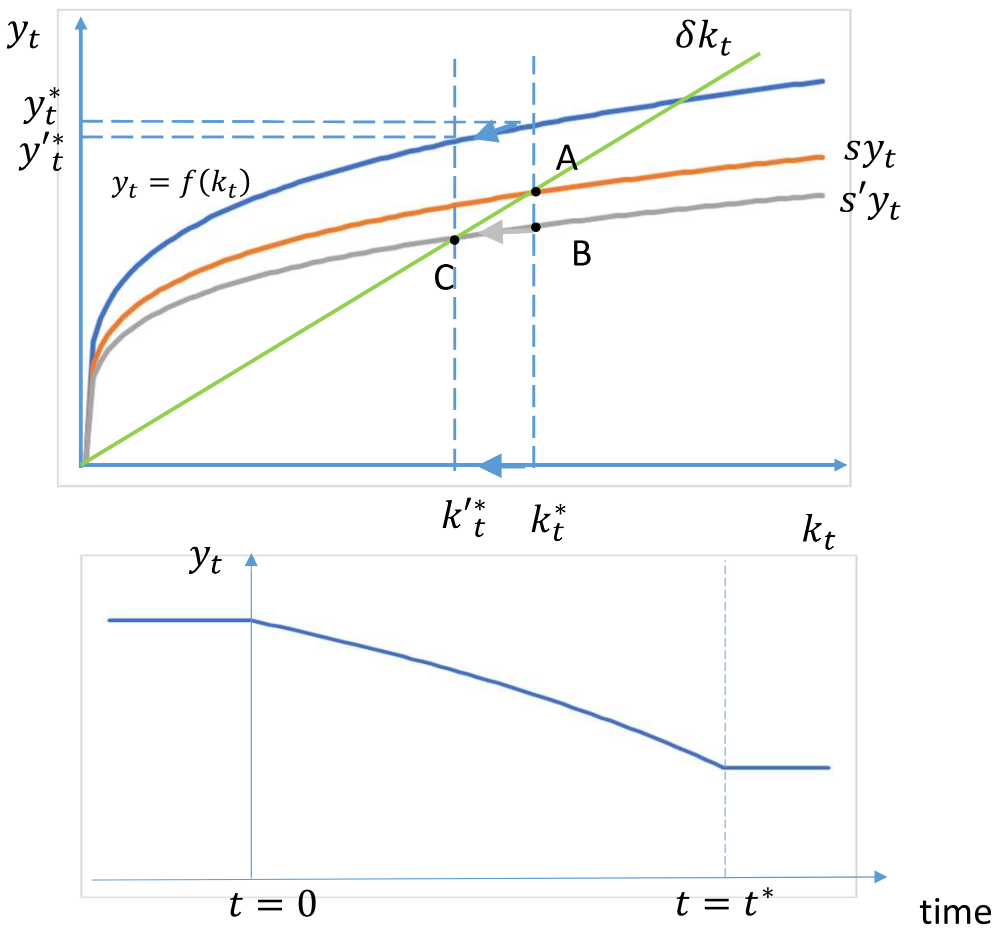
- Re-call that y_t^*, k_t^* are defined such that $\Delta k_{t+1} = 0$:
  - $\Delta k_{t+1} = k_{t+1} - k_t = sy_t - \delta k_t$
- Therefore, if $k_t < k_t^*$:
  - $\Delta k_{t+1} > 0 \Rightarrow sy_t > \delta k_t$
  - Therefore, investment at time t is adding to the capital stock at $t + 1$
  - This process continues until $k_t = k_t^*$
- Similarly, if $k_t > k_t^*$:
  - $\Delta k_{t+1} < 0 \Rightarrow sy_t < \delta k_t$
  - Therefore, investment at time t is less than depreciation, and the capital stock is falling
  - This process continues until $k_t = k_t^*$

Example: A decline in the savings rate



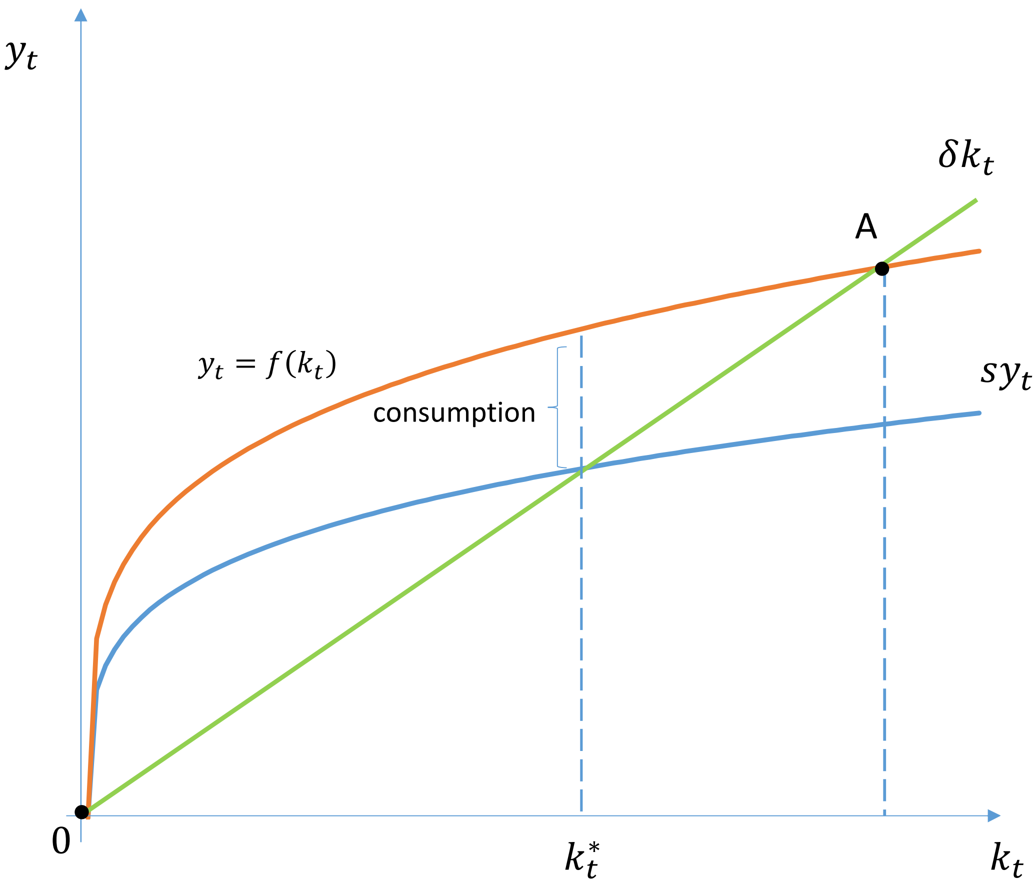
- Suppose that the economy is in a steady-state and that at $t = 0$ the savings rate falls from s to s'
- The new steady state k_t falls from k_t^* to k'^*_t and steady state y_t falls from y_t^* to y'^*_t
- At $t = 0$, the fall in savings rate causes investment to fall immediately
  - The economy moves from point A to point B
- At point B, the capital/worker ratio is too high, and so the capital stock must be allowed to fall until $k_t = k'^*_t$ at point C
  - Therefore, from $t = 1$ to $t = t^*$, investment is below the level needed to replace the depleted capital stock

Example: A decline in the savings rate (continued)



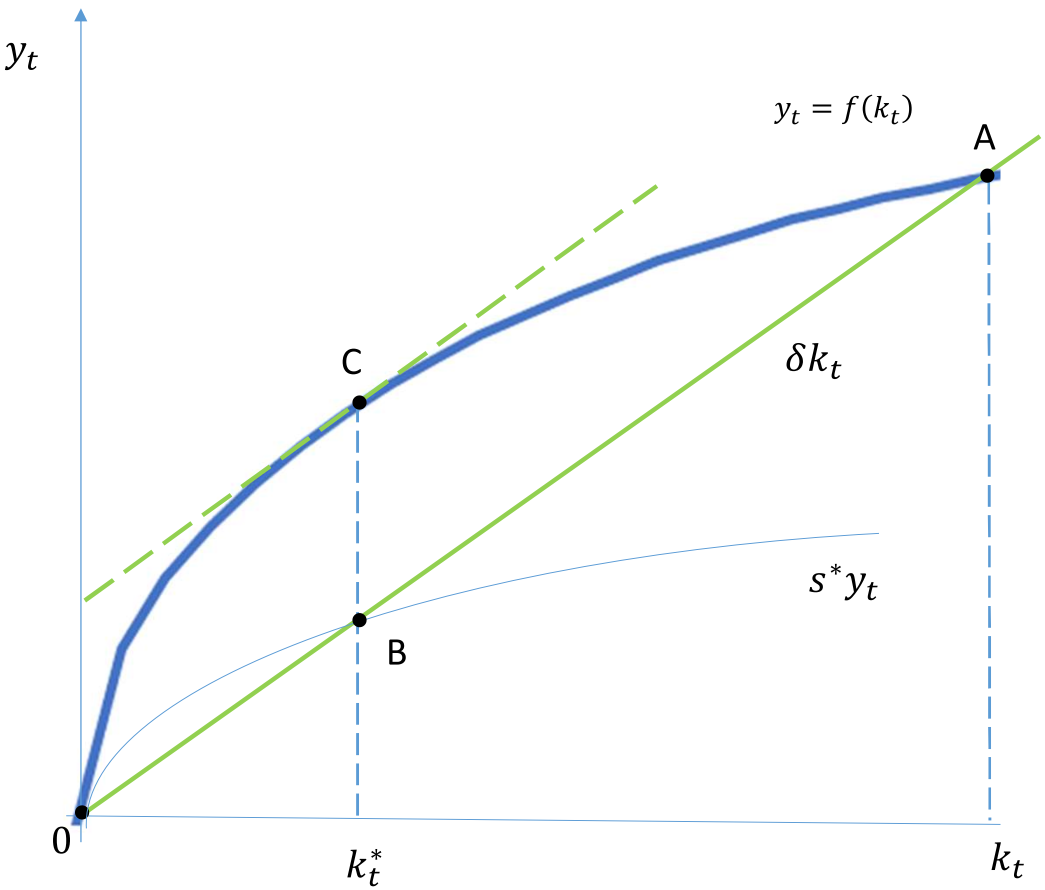
- From $t = 1$ until $t = t^*$, the growth of output is negative until the new steady state is reached
- The fall in the savings rate makes the economy permanently smaller, but only leads to a temporary recession
- Contrast with the short/medium run models that we have looked at in which a fall in the savings rate would cause a short term increase in growth

The model: Savings, consumption and the golden rule



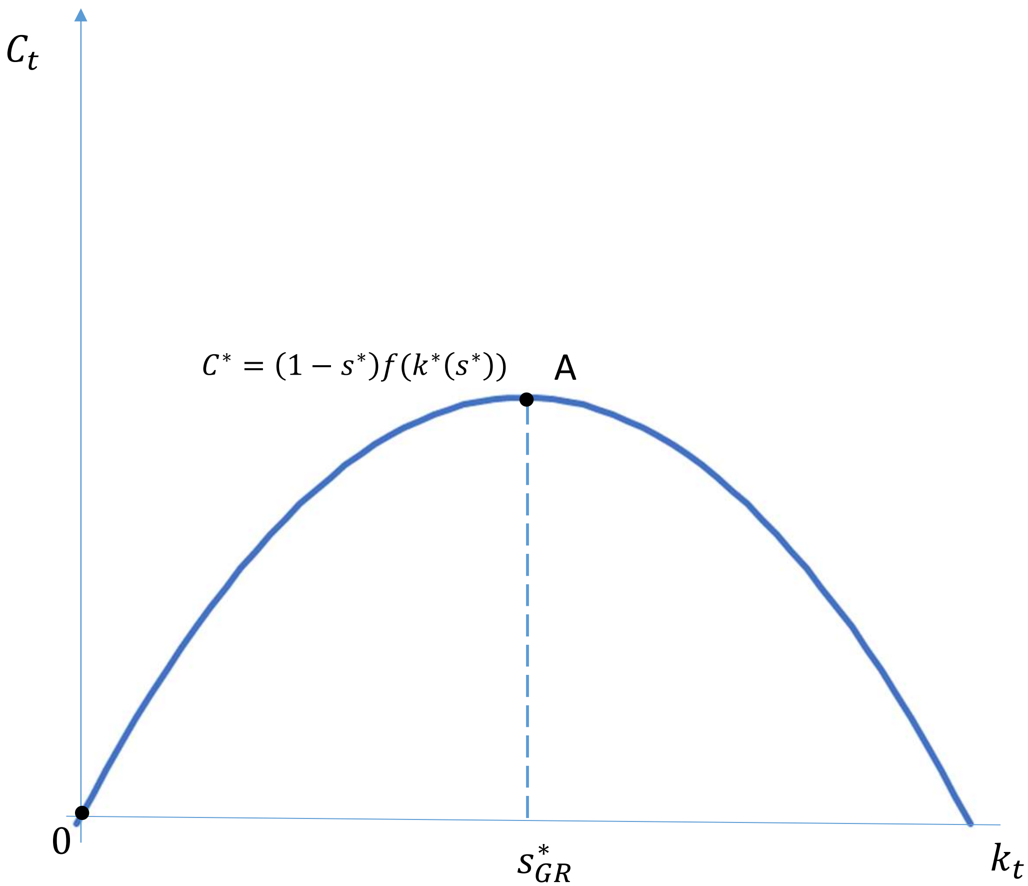
- In any economy, the investment line (ie the sy_t line) must intersect the depreciation line (ie the δk_t line) somewhere between the origin and the point A
  - At the point A, the savings rate is equal to 1 and there is no consumption in the economy
- Consumption is the difference between the output line and the investment line
- Maximising economic well-being means maximising consumption (discuss!), and from the diagram we can see that there will be a savings rate s^* and an associated k_t^* that maximises steady state consumption
- This is the so-called 'Golden Rule' level of savings and capital stock

The model: Savings, consumption and the golden rule



- How do we find the 'Golden Rule savings rate?
- And the associated level of capital?
- Consider the following:
  - At $s = 0$, $y_t = 0$ and so consumption per worker $= 0$
  - At $s = 1$, $y_t =$ investment per worker, and so consumption per worker $=$
  - Using some calculus we can show that golden rule consumption reaches a maximum between the two boundary values of zero where the slope of $y_t = f(k_t) = \delta$
- $C^* = (1 - s^*)y_t^* = (1 - s^*)f(k_t^*)$

The model: Savings, consumption and the golden rule



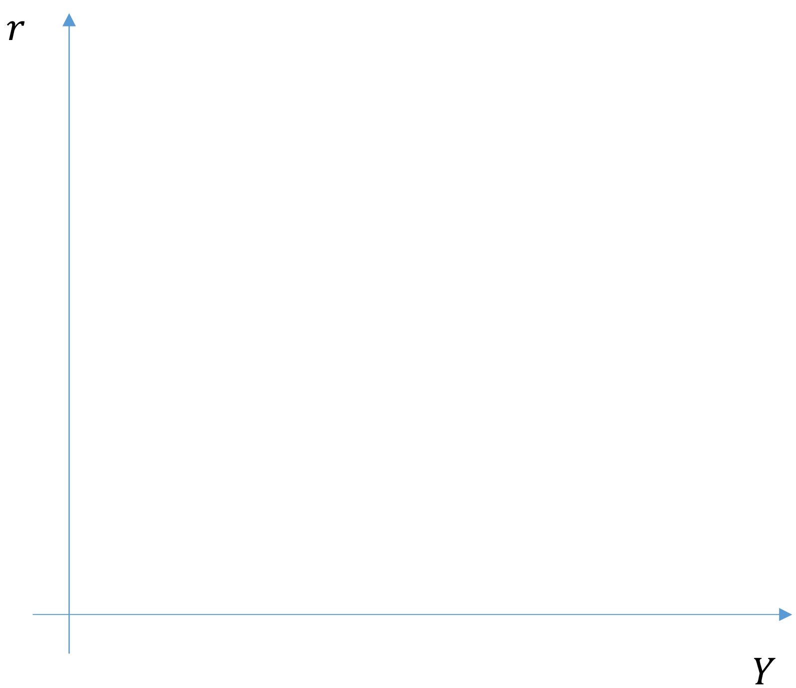
- So steady-state consumption reaches a maximum at the golden rule savings rate s^*_{GR}
- If the savings rate is below s^*_{GR}, then increasing savings increases consumption in the long run
- If the saving rate is below s^*_{GR}, then reducing savings increases consumption in the long run

Exercise 1: Blanchard ch.11 q.2

- Suppose that the head of the Finance Ministry in your country were to go on the record advocating an effort to restrain current consumption, arguing that 'lower consumption now means higher savings; and higher savings now means a permanent higher level of consumption in the future.' What would you make of such a statement?

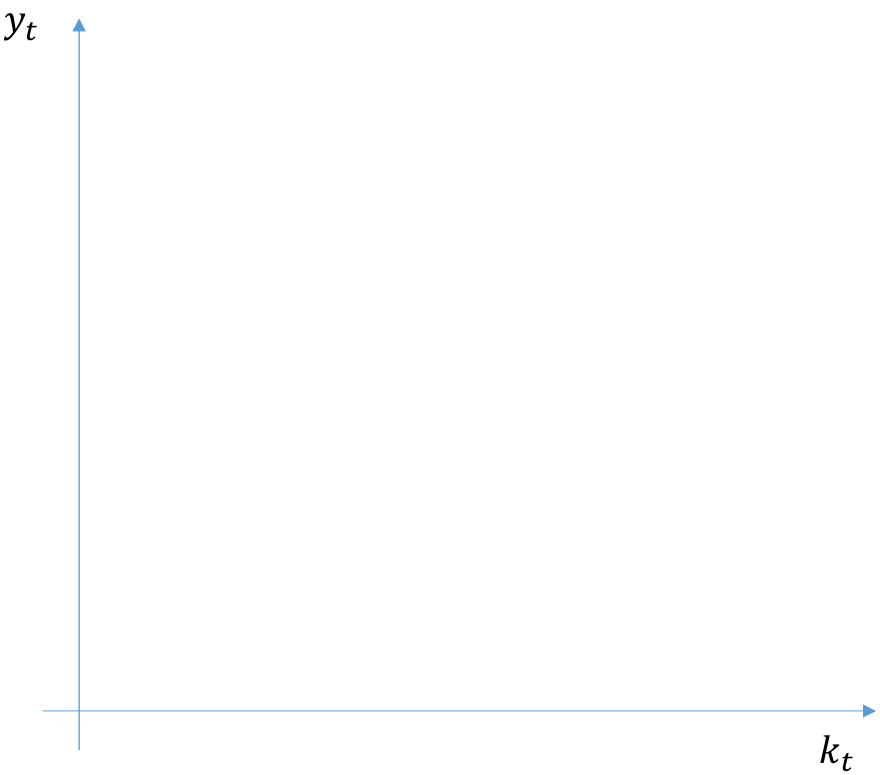
Exercise 1: Blanchard ch.11 q.2

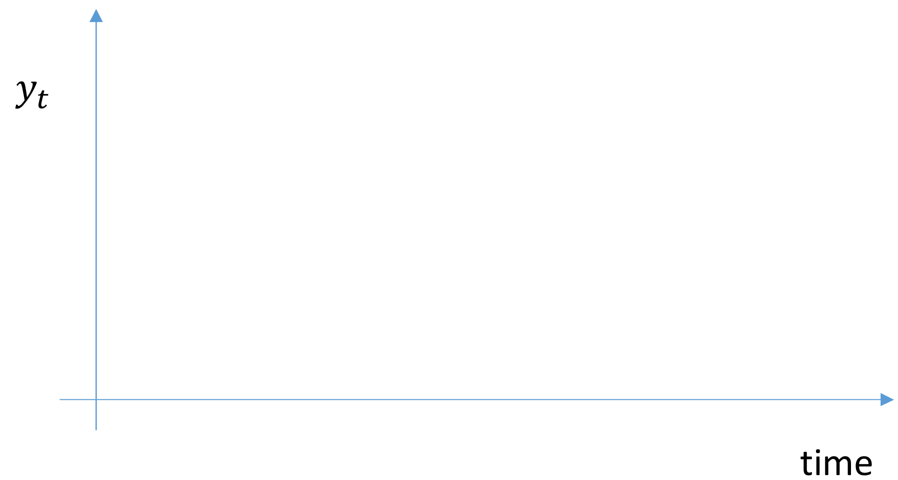
- What happens if savings increase in the short and medium term?
- How do we model increased savings in this case?



Exercise 1: Blanchard ch.11 q.2

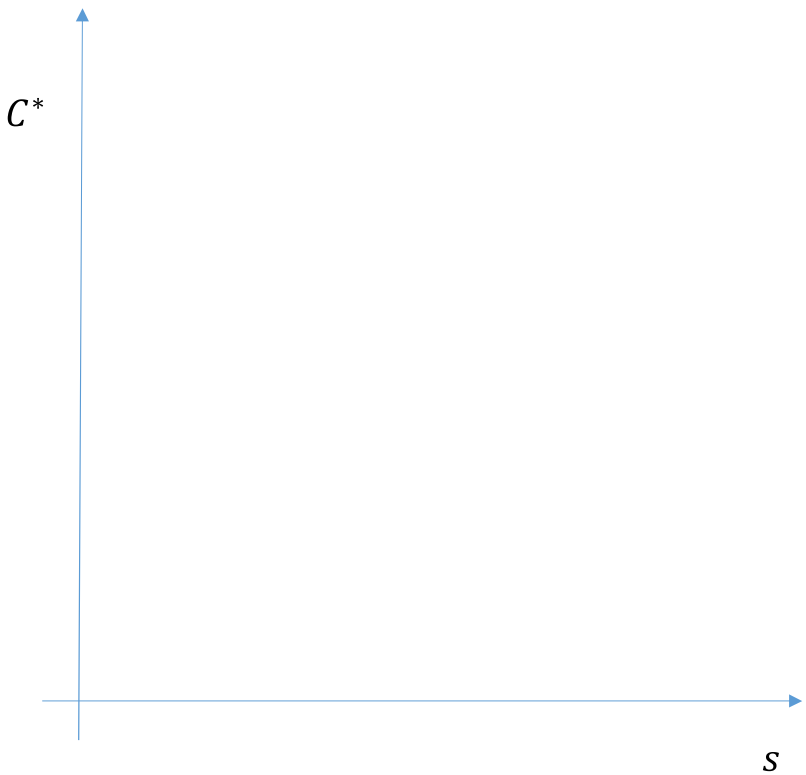
- What happens if in the long run?





Exercise 1: Blanchard ch.11 q.2

- But does this mean that the a higher savings rate will increase well-being?



Exercise 2: Blanchard ch.11 q.10

A) Consider the aggregate production function $Y_t = K_t^{0.5} N_t^{0.5}$. Express steady state capital and output per worker in terms of the savings rate s and depreciation rate δ.

Exercise 2: Blanchard ch.11 q.10

B) Using a value of 10% for the depreciation rate δ and the data below for gross private savings as a percentage of GDP, calculate the steady state capital and output per worker for the countries shown (data for 2018)

	Gross Private Savings (% GDP), s	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$
France	22.9		
Germany	29.3		
Italy	20.8		
Spain	22.3		
UK	13.4		

Exercise 2: Blanchard ch.11 q.10

B) Using a value of 10% for the depreciation rate δ and the data below for gross private savings as a percentage of GDP, calculate the steady state capital and output per worker for the countries shown (data for 2018)

	Gross Private Savings (% GDP), s	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$
France	22.9	5.2	2.3
Germany	29.3	8.6	2.9
Italy	20.8	4.3	2.1
Spain	22.3	5.0	2.2
UK	13.4	1.8	1.3

Exercise 2: Blanchard ch.11 q.10

C) So far we have assumed that a country's budget deficit is zero and therefore that $S_t = I_t$. Now assume that $S_t = I_t + G - T$. Use the data below for budget deficit as a percentage of GDP to adjust the steady state capital and output per worker estimates. Does a budget deficit increase or decrease steady state capital and output per worker?

	Gross Private Savings (% GDP), s	Budget deficit as % GDP	Total savings as % GDP	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$
France	22.9	-2.5	20.4	4.1	2.0	5.2	2.3
Germany	29.3	1.9	31.2	9.7	3.1	8.6	2.9
Italy	20.8	-2.2	18.6	3.5	1.9	4.3	2.1
Spain	22.3	-2.5	19.8	3.9	2.0	5.0	2.2
UK	13.4	-2.2	11.2	1.3	1.1	1.8	1.3

Exercise 2: Blanchard ch.11 q.10

D) Now assume that the four countries that do not have the lowest budget deficit (or highest budget surplus) have the same budget position as the lowest budget deficit country. What happens to their steady state capital and output per worker? Do you think it would be easy for these countries to achieve these results in practice?

	Gross Private Savings (% GDP), s	Budget deficit as % GDP	Total savings as % GDP	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$
France	22.9	1.9	24.8	6.1	2.5	4.1	2.0
Germany	29.3	1.9	31.2	9.7	3.1	9.7	3.1
Italy	20.8	1.9	22.7	5.1	2.3	3.5	1.9
Spain	22.3	1.9	24.2	5.8	2.4	3.9	2.0
UK	13.4	1.9	15.3	2.3	1.5	1.3	1.1