ECON 3113 Microeconomic Theory I
Lecture 13: Signaling

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Introduction

- We have seen that asymmetric information can cause market inefficiency.
- Sellers with peaches have strong incentives to make it known to buyers.
- But **talk is cheap**: it is costless for any seller to claim she has a peach.
- Action speaks louder than words!
- To **credibly convince** buyers, sellers with peaches must take **observable action that lemon sellers are unwilling (or unable) to.**
    - e.g. warranties: if the car broke down within a year, I will pay the repair bill.

Introduction

- Spence (1973): Why do students spend so much effort in studying while the course materials do not seem to be "useful"?
- You know you are smart (high productivity) but your **future employers do not know**.
- One credible way to convince them is to take an action that **people with lower productivity are unwilling or incapable of** doing, e.g., getting a degree from a prestigious university.
- The degree must be difficult enough so that people with lower productivity find it too costly to acquire, even if they understand that it gives future employers a good impression.
- Even if education does not increase your productivity, employers are willing to hire and pay you well after you get the degree, because they **understand that a degree-holder must have a high productivity**.

Signaling is Everywhere

Model of Education Signaling

- There is a large number of workers and even more firms.
- There are **two possible types** of workers: their productivity is either high or low. They create a respective value of 60000 and 30000 to their employers.
- The proportion of high-productivity and low-productivity are θ and $1-\theta$ respectively.
- Each firm needs only one worker. It can only offer a fixed wage for a worker's labour service.
- **Workers know their own productivities.**

Model of Education Signaling

- **Firms do not know a worker's productivity at the time of hiring.**
- As there are more firms than workers, firms will bid with each other until the wage equals the worker's expected productivity.
- Without any means of signaling productivities, firms do not know anything about a worker's productivity, so they believe that he/she has a high productivity with probability θ.
- Therefore, *in the absence of any possibility for signaling*, the equilibrium wage offer is

$$60000\theta + 30000\left(1-\theta\right).$$

Model of Education Signaling

- Now we introduce **education choice** by workers.
- Before entering the labour market, each worker can acquire e years of education at a cost of
  - $2000e$ if he/she has high productivity, or
  - $7500e$ if he/she has low productivity.
- **Marginal cost of education is lower for workers with a high productivity.**
- We make the simplifying (and not necessarily realistic) assumption that education does not improve productivity.
- After seeing the worker's education choice e, each firm offers a wage for the worker's service.
- Because of perfect competition among firms, the wage is bid up to a worker's expected productivity conditional on the observsed education choice.

Separating Equilibrium

- Our goal is to look for a **separating equilibrium**.
- In a separating equilibrium, **each type of worker chooses a different action**, so firms can infer worker's productivity from the education level chosen.
- A separating equilibrium consists of a description of
  1. Workers' strategy: what e each type of worker chooses?
     - Here, it is a pair of different numbers e_H and e_L.
  2. Firms' strategy: what wage w each firm offers to workers with e years of education?
     - Here, it is a wage function $w(e)$.

Separating Equilibrium

- A separating equilibrium $\{e_H, e_L, w(e)\}$ must satisfy the following 3 conditions:

1. Each type of worker chooses a different action, i.e., $e_H \neq e_L$.
2. Given the wage function $w(e)$, each type of worker finds it optimal to choose e_H and e_L respectively.
3. Given the worker's strategy (e_H, e_L), the wage function $w(e)$ is perfectly competitive, i.e., each firm receives a zero expected profit.

Separating Equilibrium

- Let's make a guess that a separating equilibrium looks as follows:
- High-productivity worker chooses $e_H = 4$
- Low-productivity worker chooses $e_L = 0$
- The wage function is

$$w(e) = \begin{cases} 60000 & \text{if } e \geq 4 \\ 30000 & \text{if } e < 4 \end{cases}.$$

Separating Equilibrium

- Given the specified wage function, do high-productivity workers find it optimal to take $e = e_H = 4$?
- To get a high wage, $e = 4$ is the minimum requirement. The payoff is $60000 - 4 \times (2000) = 52000$.
- The only relevant alternative is choosing $e = 0$, giving a low wage. The payoff is $30000 - 0 \times (2000) = 30000$.
- Thus, choosing $e = 4$ is optimal for high-productivity workers.

Separating Equilibrium

- Given the specified wage function, do low-productivity workers find it optimal to take $e = e_L = 0$?
- Following the strategy, low-productivity worker ends up with a low wage. The payoff is $30000 - 0 \times (7500) = 30000$.
- To get a high wage, $e = 4$ is the minimum requirement. The payoff is $60000 - 4 \times (7500) = 30000$.
- Thus, choosing $e = 0$ is optimal for low-productivity workers.

Separating Equilibrium

- Given the worker's strategy (e_H, e_L), is $w(e)$ perfectly competitive?
- Conditional on $e = 4$, the worker has a high productivity with probability 1:

$$\begin{aligned}
\Pr(\text{high} | e = 4) &= \frac{\Pr(\text{high } \& \ e = 4)}{\Pr(e = 4)} \\
&= \frac{\Pr(\text{high } \& \ e = 4)}{\Pr(\text{high } \& \ e = 4) + \Pr(\text{low } \& \ e = 4)} \\
&= \frac{\theta}{\theta + 0} = 1.
\end{aligned}$$

- Thus, the worker's expected productivity (conditional on $e = 4$) is 60000.

Separating Equilibrium

- Similarly, conditional on $e = 0$, the worker has a low productivity with probability 1. Thus, the worker's expected productivity (conditional on $e = 0$) is 30000.
- In both cases, each firm's (expected) profit is zero, as the offered wage is equal to the worker's expected productivity conditional on the observable choice of e.

Separating Equilibrium: Summary

- Our guess is correct!
- Getting 4 years of education is **much less costly for high-productivity worker** than for low-productivity worker.
- High-productivity workers can **separate/distinguish** themselves from low-productivity workers by getting 4 years of education. Low-productivity workers **do not find it worthwhile to mimic** high-productivity worker because they find 4 years of education is too costly.
- Spence's theory of education suggests that we are willing to go to school *even if (i) education does not improve productivity; (ii) test scores do not reveal ability.*
- It is the worker's **willingness to take challenging courses** that reveals his/her productivity.

Separating Equilibrium: Efficiency

- In this example, education does not increase productivity. *From the society's perspective, education is wasteful.*
  - Social welfare increases if the government bans education.
- The conclusion can be reversed if education increases productivity or serves other good purposes.
- In other settings, signaling improves social welfare by mitigating information asymmetry (recall the lemons market).

Pooling Equilibrium

- There is another class of equilibrium, known as the **pooling equilibrium**. It consists of a description of
  1. Workers' strategy: what common e each type of worker chooses?
     - Here, it is a single number $\bar{e}$.
  2. Firms' strategy: what wage w each firm offers to workers with e years of education?
     - Again, it is a wage function $w(e)$.
- A pooling equilibrium $\{\bar{e}, w(e)\}$ must satisfy the following 3 conditions:
  - Each type of worker chooses the same action, i.e., $e = \bar{e}$ for both types.
  - Given the wage function $w(e)$, each type of worker finds it optimal to choose $\bar{e}$.
  - Given the worker's strategy $\bar{e}$, the wage function $w(e)$ is perfectly competitive, i.e., each firm receives a zero profit.

Pooling Equilibrium

- Let's guess the following is a pooling equilibrium.
  - $\bar{e} = 0$.
  - Wage function is $w(e) = 60000\theta + 30000(1-\theta)$.
- Does each type of worker finds it optimal to choose $\bar{e}$?
  - Yes. Choosing a positive e is costly but doesn't improve wage.
- Given the worker's strategy $\bar{e}$, is the wage function $w(e)$ perfectly competitive?
  - Yes. Conditional on $e = 0$, the probability that the worker has high productivity is

$$\Pr(\text{high}|e=0) = \frac{\Pr(\text{high } \& \ e=0)}{\Pr(e=0)} = \frac{\Pr(\text{high})}{1} = \theta.$$

  - Thus, the worker's expected productivity (conditional on $e = 0$) is $60000\theta + 30000(1-\theta)$, which coincides with the wage $w(0)$.

Discussion on Multiple Equilibria

- A general signaling model admits multiple equilibria. Which one is more reasonable?
- To sharpen prediction, economists have propose different notions of *equilibrium refinement*.
- Under a widely accepted refinement notion, the only "surviving" equilibrium is the separating equilibrium described above.
- This equilibrium is often referred to as the **least-costly separating equilibrium**, as it involves the minimum level of education for both types that is required for separation.
- Another (possibly) reasonable criterion of picking equilibrium is the social norm or culture.

Now you should be able use the signaling theory to explain....

- Why do firms spend huge sum of money on uninformative advertisement?
- Why do ladies spend so much money and effort in dressing/ cosmetics/ skin-care products?
- Why do gentlemen send valuable gifts to their girlfriends?
- Why does the price of a stock react negatively to equity offering?
- Why do good leaders often lead by examples?
- Why does evolution "lead" to the magnificent tail feathers of a peacock, at the cost of inhibiting its movement?

Summary

- Mechanism of signaling: whenever there is **some observable action that is much less costly (or much more beneficial) for one type** to undertake than the other, the former type can **take the action in order to separate herself** from the latter type.
- Efficiency: signaling may or may not be welfare-enhancing. It depends on whether the cost of sending a signal is more or less than the benefit involved in mitigating information asymmetry.
- A signaling model typically has multiple equilibria. Among them, the least-costly separating equilibrium is reasonable and intuitive.