# ECON3133 Midterm Exam Solution

Fall 2020, 80 minutes, 100 points

There are 4 questions.

1. (25 points) The total cost of a generic face mask firm (production line) is

$$C(q) = 100 + 4q^2,$$

where $q$ is packages of face masks. Before the pandemic, there is only one face mask firm in Hong Kong.

a. Suppose the firm is a price-taker. The price for each package of face masks is $p$. What is the firm's short-run supply function $S_i(p)$?

The firm's supply is determined by

$$MC(q) = 8q = p.$$

So the supply function is

$$S_i(p) = MC^{-1}(p) = \frac{1}{8}p$$

(The shut-down point is at $p = 0$.)

b. What is the minimum price ($p_0$) that can make this firm breakeven in the long run? What is the minimum efficient scale ($q_{min}$) of the firm?

Breakeven (zero profit) condition is determined by the average price

$$AC(q) = \frac{100}{q} + 4q$$
$$AC'(q) = -\frac{100}{q^2} + 4 = 0.$$

So the minimum of average cost, $\min_q AC(q) = 40$, when $q_{min} = 5$.

The minimum price that can make this firm breakeven is $p_0 = 40$. The minimum efficient scale is $q_{min} = 5$

c. To fight against COVID-19, Hong Kong government offers funding for firms to set up new face mask production lines (www.hkpc.org/en/our-services/additive-manufacturing/latest-information/hkpc-mask-production-support).

Suppose that there are another 15 face mask firms established. What is the industry supply function with $n = 16$ firms? All these firms have the same technology and are price-taking.

Each firm supplies

$$S(p) = n \times S_i(p) = 16 \times \frac{1}{8}p = 2p.$$

d. Continue with part (c). The market demand is $Q_D(p) = 300 - 3p$. Find the equilibrium price and quantity.

$$S(p) = 2p = Q_D(p) = 300 - 3p$$

$$5p = 300, \quad p^* = 60, \quad Q^* = 120.$$

e. Continue with part (d). If the government wants to bring the face mask price down to $p^* = 50$ per package, how many face mask firms in total need to be established?

$$S(p^*) = n \times \frac{1}{8}p^* = Q_D(p^*) = 300 - 3p^*$$

$$n \times \frac{1}{8}50 = 300 - 3 \times 50 \Rightarrow n = 24$$

So $n = 24$ face mask firms are needed.

f. In the **long run**, there is no entry barrier to the face mask industry. All firms have the same production technology as above. The market demand is $Q(p) = 300 - 3p$. What is the long-run equilibrium of this market?

The long-run equilibrium price is

$$p^{LR} = p_0 = 40.$$

The market equilibrium quantity is

$$Q^{LR} = 300 - 3 * 40 = 180.$$

The equilibrium number of firms is

$$n^{LR} = \frac{Q^{LR}}{q_{min}} = \frac{180}{5} = 36.$$

g. In the **long run**, to lower face mask price, the government provides $s = 5$ per package subsidy. Specifically, consumers pay $p_D$ per package. Firms earn $p_S$ per package. In equilibrium, $p_D + s = p_S$. Predict the total amount of subsidy the government will need to pay. The market demand is $Q(p) = 300 - 3p$.

In the long run, entry will happen as long as $p_S > p_0$. So $p_S^{LR} = 40$.

$$p_D^{LR} = p_S^{LR} - s = 40 - 5 = 35.$$

The market equilibrium quantity is

$$Q^{LR} = 300 - 3 \times p_D^{LR} = 300 - 3 \times 35 = 195.$$

The total amount of subsidy to is

$$Q^{LR} \times s = 195 \times 5 = 975.$$

2. (25 points) A monopoly firm faces a market with demand function

$$q(p) = \alpha p^{\beta}, \quad \text{where } \beta < -1.$$

The cost function of the firm is

$$C(q) = c \times q + 4,$$

so it has a constant marginal cost $c$.

a. The firm chooses quantity $q$ to maximize its profit. Express the total revenue and marginal revenue as functions of $q$.

From the demand, we first find the inverse demand

$$p(q) = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} q^{\frac{1}{\beta}}$$

$$R(q) = q \times p(q) = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} q^{\frac{1+\beta}{\beta}}$$

$$MR(q) = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} \frac{1+\beta}{\beta} q^{\frac{1}{\beta}}$$

b. Given $\alpha$, $\beta$, and $c$, find the optimal choice of quantity $q^*$ and price $p^*$.

$$\max_q R(q) - C(q)$$

$$\Rightarrow MR(q) = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} \frac{1+\beta}{\beta} q^{\frac{1}{\beta}} = MC(q) = c$$

$$\Rightarrow q^{\frac{1}{\beta}} = \alpha^{\frac{1}{\beta}} \frac{\beta}{1+\beta} c \Rightarrow q^* = \alpha \left(\frac{\beta}{1+\beta}\right)^{\beta} c^{\beta}$$

$$p^* = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} (q^*)^{\frac{1}{\beta}} = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} (\alpha)^{\frac{1}{\beta}} \left(\frac{\beta}{1+\beta}\right) c = \frac{\beta}{1+\beta} c$$

c. Let $\beta = -2$, $\alpha = 48$, $c = 2$. Compute the firm's profit $\pi$ and consumer surplus $CS$.

$$p^* = \frac{-2}{1-2} 2 = 4$$

$$q^* = 48 \times \left(\frac{-2}{1-2}\right)^{-2} \times 2^{-2} = \frac{48}{4 \times 4} = 3$$

$$\pi = p^* q^* - c q^* - 4 = 4 \times 3 - 2 \times 3 - 4 = 2$$

$$CS = \int_4^{\infty} q(p) dp = \int_4^{\infty} 48 p^{-2} dp$$

$$= 48(-1) \left[p^{-1}\right]_4^{\infty} = 48(-1)(0 - \frac{1}{4}) = 12$$

(If the student compute the producer surplus correctly, $PS = 6$, instead of profit, then only deduct 1 points.)

d. The average cost of the monopoly firm is decreasing in $q$, so this industry is a natural monopoly. Consider that the government wants to regulate the price. What is the price that maximizes total surplus? What is the lump-sum subsidy that the government should pay the monopoly to maintain a zero profit in the long run?

The government should set the price at the marginal cost $c$.

The government needs to pay a lump-sum subsidy 4 to make the monopoly have zero profit in the long run.

e. What is the problem of having a non-price-taking firm and an inelastic demand? Suppose that $-1 < \beta < 0$, show that the monopoly's profit-maximizing behavior would be strange.

The first-order condition for profit maximization is

$$MR(q) = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} \frac{1+\beta}{\beta} q^{\frac{1}{\beta}} = c$$

When $-1 < \beta < 0$, $\frac{1+\beta}{\beta} < 0$, so $MR(q) < 0$ for all $q > 0$. There is no solution for this FOC.

(The above answer is enough. Students can also use other arguments such as $p^*$ becomes negative.)

The firm's objective function is

$$\max_q \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} q^{\frac{1+\beta}{\beta}} - c \cdot q - 4,$$

which has a negative slope on $q > 0$.

3. (20 points) The supply of cigarettes is represented by $S(P, v) = 2Pv^{-2}$, where $P$ is price of cigarette, $v$ is the price of tobacco. Tobacco is a key raw material for cigarette production.

The demand of cigarette is represented by $D(P, I)$, where $I$ is the income of a representative consumer. The policymaker does not know the entire demand curve but knows that the (local) price elasticity $e_{D,P} = -0.5$ and income elasticity $e_{D,I} = 1.5$.

a. Compute $e_{S,P}$ and $e_{S,v}$. Is the demand of cigarette demand price elastic or inelastic? Cigarette is a luxury good or necessity?

$$e_{S,P} = \frac{dS}{dP}\frac{P}{S} = 1, \quad e_{S,v} = \frac{dS}{dv}\frac{v}{S} = -2$$

Because $e_{D,P} \in (-1, 0)$, so inelastic demand.

Because $e_{D,I} > 1$, so luxury good.

b. If income $I$ increases by 10%, how much will the equilibrium price of cigarette change?

$$e_{P,I} = \frac{e_{D,I}}{e_{S,P} - e_{D,p}} = \frac{1.5}{1 - (-0.5)} = 1$$

So the price of cigarette will rise by about 10%.

c. The government wants to reduce cigarette consumption by charging extra tax on the usage of the key input, tobacco. Roughly predict that, if the government charges a 12% tax on tobacco (raise $v$ by 12%), how much will the equilibrium price of cigarette rise?

$$e_{P,v} = \frac{e_{S,v}}{e_{D,p} - e_{S,P}} = \frac{-2}{-0.5 - 1} = \frac{4}{3}$$

$$e_{P,v} = \frac{\%\text{ change in } P}{\%\text{ change in } v} = \frac{\%\text{ change in } P}{12\%} = \frac{4}{3}$$

$$\Rightarrow \%\text{ change in } P = 16\%.$$

So the equilibrium price will rise by about 16%.

d. The government charges a per unit tax $t$ on cigarette consumption. Roughly predict what proportion of this tax $t$ is born by consumers.

$$P_D(t) - t = P_S(t), \text{ or } P_D(t) = P_S(t) + t$$

$$Q' = Q_D(P_D(t)) = Q_S(P_D(t) - t)$$

Differentiate both sides w.r.t. $t$,

$$\frac{dQ_D}{dP} \times \frac{dP_D}{dt} = \frac{dQ_S}{dP} \times \left(\frac{dP_D}{dt} - 1\right)$$

$$\begin{cases} \frac{dP_D}{dt} = \frac{e_{S,P}}{e_{S,P} - e_{D,P}} \\ \frac{dP_S}{dt} = \frac{e_{D,P}}{e_{S,P} - e_{D,P}} \end{cases} \Rightarrow \left|\frac{\frac{dP_S}{dt}}{\frac{dP_D}{dt}}\right| = \left|\frac{e_{D,P}}{e_{S,P}}\right| = \left|\frac{\frac{dP_S}{dt}}{\frac{dP_D}{dt}}\right| = \frac{0.5}{1} = \frac{1}{2}.$$

Therefore, the price faced by consumers change twice as much as the price faced by producers. Consumers bear $\frac{2}{3}$ (66.67%) of the tax, while producers bear $\frac{1}{3}$ (33.33%) of the tax.

e. Suppose the current equilibrium price and quantity is $P = 10$ and $Q = 100$. A per-unit tax $t = 1$ is imposed on cigarette. Predict the deadweight loss caused by this tax.

$$\begin{aligned} DW &= -0.5 \times t^2 \times \frac{e_{S,P} e_{D,P}}{e_{S,P} - e_{D,P}} \frac{Q}{P} \\ &= -0.5 \times 1^2 \times \frac{1(-0.5)}{1 - (-0.5)} \frac{100}{10} \\ &= 0.5 \times 1 \times \frac{0.5}{1 + 0.5} 10 \\ &= 0.5 \times \frac{1}{3} \times 10 = \frac{5}{3}. \end{aligned}$$

4. (20 points) There are two firms in a town, A and B. Both of them use capital (machine) $k$ and labor $l$ to produce outputs. Firm A is a toy manufacturer with the production function

$$q = f_A(k, l) = \sqrt{k}\sqrt{l}.$$

Firm B is a textile manufacturer with production function

$$y = f_B(k,l) = \sqrt{k} + \sqrt{l}.$$

$q$ and $y$ denote the quantity of toys and textiles, respectively. The output markets of toys and textiles are independent. $q$ and $y$ are given exogenously to these firms. Let $v$ denote the unit price of capital and $w$ denote the unit price of labor. Both firms are cost-minimizing.

a. What are the elasticity of substitution of firm A and firm B, $\sigma_A$ and $\sigma_B$?
Firm A's production function is Cobb-Douglas, so the elasticity of substitution is $\sigma_A = 1$.
Firm B's production function can be written as CES form

$$g(k,l) = \sqrt{k} + \sqrt{l} = (k^{\frac{1}{2}} + l^{\frac{1}{2}})^{\frac{0.5}{0.5}}, \quad \text{where } \rho = \frac{1}{2} \text{ and } \gamma = \frac{1}{2}$$

$$\sigma_B = \frac{1}{1-\rho} = 2.$$

b. The government launches an industrial policy that supports workplace automation. The policy gives subsidies to firms for using machines. As a result, the relative price of labor to capital increases by 20%. Without specifying particular values of $v, w, q$ and $y$, can you predict how will the capital-labor ratio $(k/l)$ change in the two firms? That is, report the percentage change of $k/l$ after raising $w/v$ by 20%.

By the definition of elasticity of substitution

$$\sigma = \frac{d\ln(k/l)}{d\ln(RTS)} = \frac{d\ln(k/l)}{d\ln(w/v)} = \frac{\%\text{ change of } k/l}{\%\text{ change of } w/v}$$

For firm A, capital-labor ratio rises for 20% because

$$\%\text{ change of } k/l = \sigma_A \times \%\text{ change of } w/v = 20\%.$$

For firm B, capital-labor ratio rises for 40% because

$$\%\text{ change of } k/l = \sigma_B \times \%\text{ change of } w/v = 40\%.$$

c. Let the input prices be $w = 5$ and $v = 5$. Fix the output levels at $q = 10$ and $y = 10$. Compute the (contingent) capital and labor demands of the two firms.

For firm A,

$$TRS_A = \frac{MP_l}{MP_k} = \frac{k}{l} = \frac{w}{v} \Rightarrow k = \frac{w}{v}l.$$

Plug in the constraint

$$\sqrt{\frac{w}{v}l}\sqrt{l} = q,$$

so

$$l_A(w,v,q) = q\sqrt{\frac{v}{w}} = 10\sqrt{\frac{5}{5}} = 10,$$

$$k_A(w,v,q) = q\sqrt{\frac{w}{v}} = 10\sqrt{\frac{5}{5}} = 10.$$

For firm B,

$$TRS_B = \frac{MP_l}{MP_k} = \frac{l^{-\frac{1}{2}}}{k^{-\frac{1}{2}}} = \left(\frac{k}{l}\right)^{\frac{1}{2}} = \frac{w}{v}.$$

$$\Rightarrow k^{\frac{1}{2}} = \frac{w}{v} l^{\frac{1}{2}}.$$

Plug in $\sqrt{k} + \sqrt{l} = y$, we get

$$\frac{w}{v} l^{\frac{1}{2}} + l^{\frac{1}{2}} = \frac{w+v}{v} l^{\frac{1}{2}} = y.$$

So

$$l_B(w,v,y) = y^2 \frac{v^2}{(v+w)^2} = 100 \frac{5^2}{(5+5)^2} = 25$$

$$k_A(w,v,y) = y^2 \frac{w^2}{(v+w)^2} = 100 \frac{5^2}{(5+5)^2} = 25.$$

d. Continue with part (b) and (c). Continue to fix $q = 10$ and $y = 10$. The workplace automation policy \$1 subsidy for using machines, so the new input prices are $w = 5$ and $v = 4$. Compute the (contingent) capital and labor demands of the two firms.

$$l_A(w,v,q) = q\sqrt{\frac{v}{w}} = 10\sqrt{\frac{4}{5}} = 8.94,$$

$$k_A(w,v,q) = q\sqrt{\frac{w}{v}} = 10\sqrt{\frac{4}{5}} = 11.18$$

$$l_B(w,v,y) = y^2 \frac{v^2}{(v+w)^2} = 100 \frac{4^2}{(4+5)^2} = 19.75$$

$$k_A(w,v,y) = y^2 \frac{w^2}{(v+w)^2} = 100 \frac{5^2}{(4+5)^2} = 30.86.$$

e. Based on your results above, discuss the impacts of the workplace automation policy. You can choose two aspects below for discussion.

(i) Does the policy hurts workers?
(ii) How the policy affect the two industries differently? Why?
(iii) Is the policy likely to increase inequality of the society?
(iv) Can you think about any reason that the government should promote the policy?
(v) Other impacts you can think of.

(i) Yes, total employment and total wage payment both drop.
(ii) The textile industry (firm B) is affected more because the elasticity of substitution is larger. So it is easier to switch from using labor to capital.

(iii) Yes. Capital is usually owned by a small group of people. The income of capital owners is going to rise, while the labor income drops. The inequality is likely to rise.

(iv) One important assumption we made is that the output levels are fixed. The policy lowers input prices, so the total cost function shift downward. No matter the firm is a price-taker or has market power, the output levels will increase. Therefore, it is possible that the output expansion causes both firms' profits and labor income to increase.

(v) Similar to (iv), the total cost function will drop. Both firms' profits will rise.

Another answer: if the factor market is perfectly competitive, the subsidy policy is going to cause distortion and deadweight loss.