ECON3113 Microeconomic Theory I

Mid-term solutions

Question 1

1. For each statement below, determine whether it is *true (T)* or *false (F)* or *not enough information to tell (NEIT)*. For each statement, explain your answer in **one sentence**.

   (a) Harry's utility function is $U_H(x_1, x_2) = \sqrt{x_1 x_2}$ and Ron's utility function is $U_R(x_1, x_2) = (x_1 x_2)^2$. Therefore, Ron is always happier than Harry.

   (5 points)

- False. Utility is an ordinal concept and cannot be used to make comparisons between people

Question 1

(b) Suppose Hermione always makes **coherent choices.** If presented with a menu consisting only of a *Gryffindor* and a *Slytherin* handkerchief and asked to pick one and only one, she picks the *Gryffindor* one (and strictly not *Slytherin*). Now suppose we add a *Ravenclaw* handkerchief to the menu above. With the expanded menu of *Gryffindor*, *Slytherin* and *Ravenclaw* handkerchiefs (from which she picks one), Hermione may give up the *Gryffindor* handkerchief and pick something else.

(5 points)

- True.
- Let $X = \{G, S\}$ and $c(X) = \{G\}$
- $Y = \{G, S, R\}$
- Since G, $S \in X \cap Y$ and $G \in c(X)$, $S \notin c(X)$ then coherence requires that $S \notin c(Y)$. So Hermione cannot give up G for S and remain coherent in her choices
- But there is no such restriction on R and $c(Y) = R$ is coherent. That is, Hermione could give up G for R and remain coherent in her choices

Question 1

(c) Hermione hates anything *Slytherin*. She would be strictly better off to have the *Slytherin* handkerchief removed from the menu.

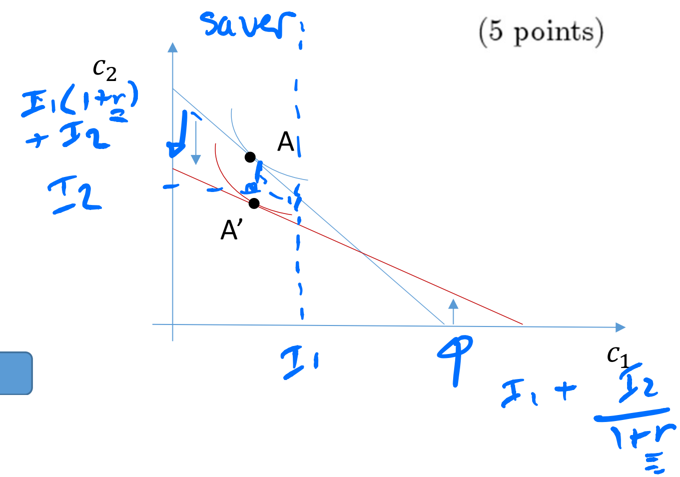
(5 points)

- False. If Hermione hate S, then she won't choose it. Its removal, therefore, has no impact on her wellbeing.

(d) Fixing a consumer's income path, a reduction in the interest rate is always a good thing for her because the present value of the income path goes up.

(5 points)

- False. For a saver, a lower interest rate reduces well-being



Question 2

2. Suppose the consumer has a utility function $U(x_1, x_2) = 3\sqrt{x_1} + \sqrt{x_2}$, and income I.

   (a) Derive the consumer's demand of good 1 as a function of prices p_1 and p_2 and income I. (Hint: you may assume without proof that the solution of utility maximization is interior.)

   · no corner points.

   (12 points)

- This can be done either using a tangency condition approach or else a Lagrangian

$$MRS = \frac{\partial u}{\partial x_1} \Big/ \frac{\partial u}{\partial x_2}.$$

$$MRS = \frac{p_1}{p_2} \Leftrightarrow \frac{\frac{3}{2}x_1^{-\frac{1}{2}}}{\frac{1}{2}x_2^{-\frac{1}{2}}} = \frac{p_1}{p_2} \Leftrightarrow \boxed{x_2 = \left(\frac{p_1}{3p_2}\right)^2 x_1}$$

Budget line:

$$p_1 x_1 + p_2 \left(\frac{p_1}{3p_2}\right)^2 x_1 = I$$

Solution:

$$x_1(p_1, p_2, I) = \frac{I}{p_1 + p_2\left(\frac{p_1}{3p_2}\right)^2}, \text{ and } \boxed{x_2(p_1, p_2, I) = \frac{I}{\frac{9}{p_1}p_2^2 + p_2}}.$$

$x_1(p_1, p_2, I)$

Question 2

(b) Suppose the price of good 1 is 20, the price of good 2 is 10, and the consumer's income is 220. What is the consumer's optimal consumption bundle? Show that the corresponding level of utility is 11.

(4 points)

- Substitution yields:
  - $x_1 = \frac{I}{P_1+P_2\left(\frac{P_1}{3P_2}\right)^2}$
  - $= \frac{220}{20+10\left(\frac{20}{3\times 10}\right)^2}$
  - $= \frac{220}{220/9} = 9$
  - $\boxed{x_2 = \frac{I}{\left(\frac{9}{P_1}\right)P_2^2+P_2}}$
  - $= \frac{220}{\left(\frac{9}{20}\right)100+}$
  - $= \frac{220}{55} = 4$
  - $U = 3\sqrt{x_1} + \sqrt{x_2}$
  - $= 3 \times 3 + 2 = 11$

$P_2 x_2^* = I - P_1 x_1^*$

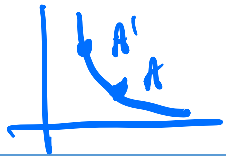
Question 2

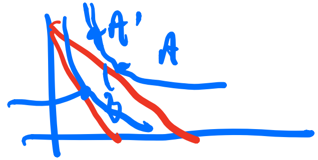
(c) What is the compensated demand function of good 1 that passes through the optimal consumption bundle identified in part (a)? (12 points)

$(9, 4)$ $\bar{u} = 11$

$x_1^c = x_1^c(p_1, p_2, \bar{u})$

- To find the compensated demand function, we use the utility function and the MRS = relative price tangency condition
- We have $\bar{U}(x_1, x_2) = 3\sqrt{x_1} + \sqrt{x_2}$, and MRS $= 3\left(\frac{x_2}{x_1}\right)^{1/2} = \frac{P_1}{P_2}$
- Fix $U = \bar{U} = 11$ and re-arrange the tangency condition to give $x_2 = x_1\left(\frac{P_1}{3P_2}\right)^2$
- Substitute this expression for x_2 into $11 = 3\sqrt{x_1} + \sqrt{x_2}$ to give:
- $11 = 3\sqrt{x_1} + \sqrt{x_1\left(\frac{P_1}{3P_2}\right)^2} = \sqrt{x_1}\left[3 + \left(\frac{P_1}{3P_2}\right)\right] = \sqrt{x_1}\left[\frac{9P_2+P_1}{3P_2}\right] \Rightarrow x_1^c = \left(\frac{33P_2}{9P_2+P_1}\right)^2$





x: at B

Question 2

from 20

(d) Suppose the price of good 1 increases to 30 (price of good 2 remains fixed at 10). Use your results in parts (a)-(c) to decompose the effect of the price increase on good 1's consumption into its substitution effect and its income effect.

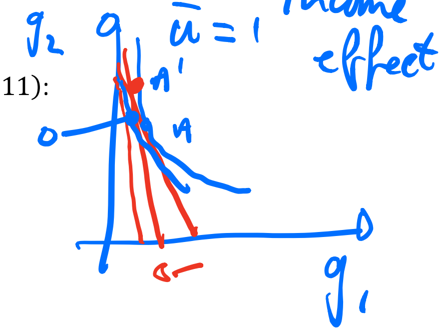
(12 points)

A to A' is subst. effect

A' to B income effect

- The substitution effect is the change in compensated demand for x_1 as a result of the price change
  - Substitution effect $= x_1^c(30{,}10{,}11) - x_1^c(20{,}10{,}11) = \left(\frac{33\times10}{9\times10+30}\right)^2 - \left(\frac{33\times10}{9\times10+20}\right)^2 = 7.5625 - 9 = -1.4375$

9



- The income effect is the change in demand for x_1 between $x_1(30{,}10{,}220)$ and $x_1^c(30{,}10{,}11)$:
  - Income effect $= x_1(30{,}10{,}220)$ - $x_1^c(30{,}10{,}11)$

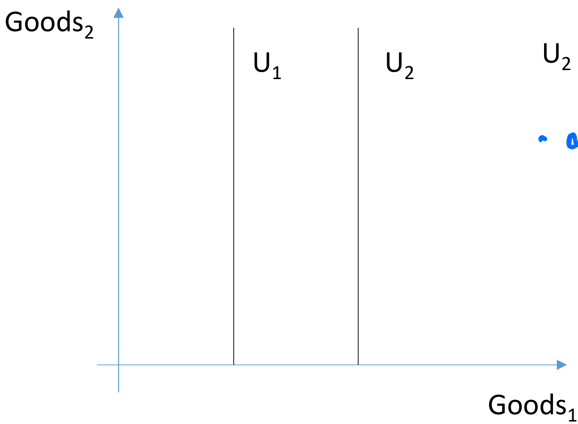
5.5 - 7.5625

  - $x_1(30{,}10{,}220) = \frac{I}{P_1+P_2\left(\frac{P_1}{3P_2}\right)^2} = \frac{220}{30+10\times(\frac{30}{30})^2} = 5.5$
  - Then income effect $= 5.5 - 7.5625 = -2.0625$
  - And total effect $=$ substitution effect $+$ income effect $= -1.4375 - 2.0625 = -3.5$

Question 3

3. Suppose a consumer has the following preference: he strictly prefers bundle (x_1, x_2) to (y_1, y_2) if $x_1 > y_1$, and he finds the two bundles indifferent if $x_1 = y_1$.

- We have that if $x_1 > y_1$ then $(x_1, x_2) \succ (y_1, y_2)$
- The indifference curves in this case are vertical lines as shown



$U_2 > U_1$

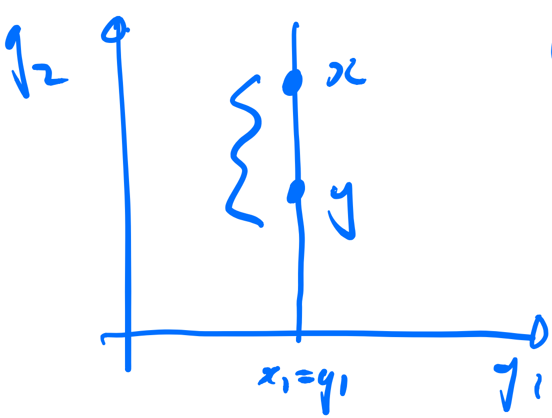
· all that matters in consumption of x_1

Question 3

(i) Strict monotonicity

(5 points)

- False. Consider a bundle (x_1, x_2) and a bundle (y_1, y_2) such that $x_1 = y_1$ and $x_2 > y_2$. Then $(x_1, x_2) > (y_1, y_2)$ but $(x_1, x_2) \sim (y_1, y_2)$. This is a violation of strict monotonicity.
- In fact the preferences satisfy monotonicity but not strict monotonicity

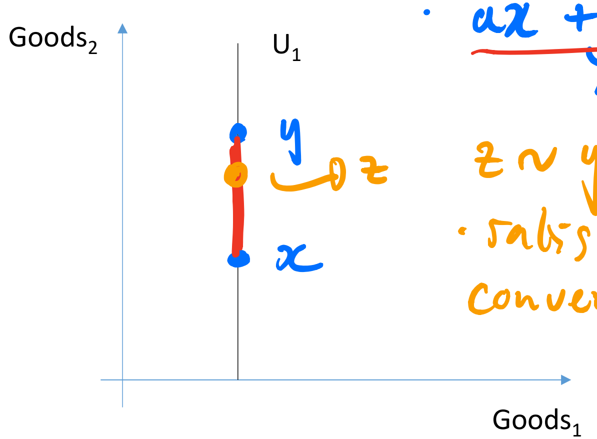


$$(x_1, x_2) > (y_1, y_2) \not\Rightarrow (x_1, x_2) \succ (y_1, y_2)$$

Question 3

(ii) Convexity

(5 points)



- any $x \sim y$.
- $\alpha x + (1-\alpha) y$
  x, y.

$z \sim y \sim x$

- satisfy convexity.

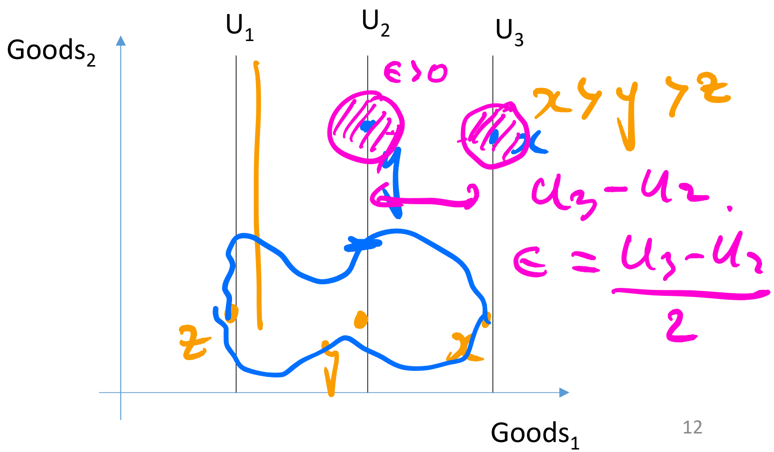
True. Consider two non-equal bundles on the same indifference curve, (x_1, x_2) and (y_1, y_2). The shape of the indifference curve means that we must have $x_1 = y_1$

- Then any line segment between (x_1, x_2) and (y_1, y_2) is given by $\alpha(x_1, x_2) + (1 - \alpha)\,(y_1, y_2)$ and is also on the same indifference curve.
- Therefore, all bundles on the line segment are at least as preferred as the two end points, which is the definition of convexity.
- Notice that the preferences are not strictly convex, which would require that non end-points of the segment were preferred to the end points, which is not the case.

Question 3

(iii) Continuity

(5 points)

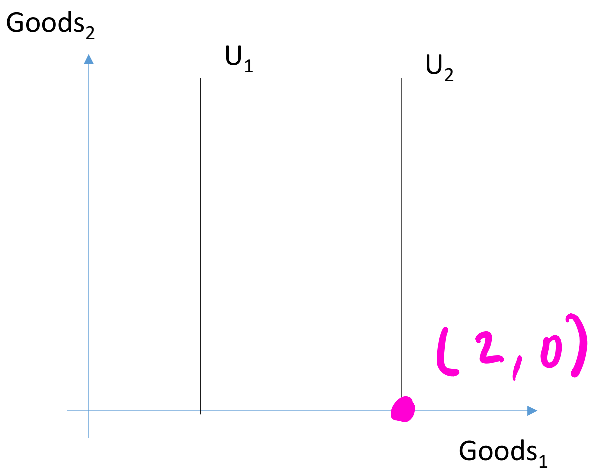


- A variety of answers:
- 1) Debreu's Theorem (Slide 25 of lecture notes 3): If the utility function is continuous, then the implied preferences are continuous
- 2) Consider bundles x, y, z such that $x \succ y \succ z$. A continuous path from x to z must cross the indifference curve of y
- 3) Consider two bundles x, y such that $x \succ y$. Then we can find a ball of radius $\varepsilon > 0$ such that everything in the ball around x is strictly preferred to everything the ball around y

Question 3

(b) Let the price of good 1 be \$50 and price of good 2 be \$30. If the consumer has an income of \$100, what is his optimal consumption bundle?

(5 points)



- Utility depends only on consumption of goods 1
- Therefore, consumption is optimised when the consumption of goods 1 is at a maximum
- This occurs when all income is spent on goods 1 and none on goods 2
- The optimal bundle is (2,0)

Question 4

4. Drake is a farmer who grows rice (good 1) and onions (good 2). The output of his farm is stable at 30 sacks of rice and 20 sacks of onions each year. Drake does not have other sources of income. Assume in this world, the size of a sack is standardized.

   His yearly utility function for rice and onion is $U(x_1, x_2) = x_1 + 2x_1x_2$, where the goods are again measured in the unit of sacks.

   (a) Let the market price of one sack of rice be \$10 and that of one sack of onion be \$40. What is the optimal consumption bundle of Drake? Is Drake a seller or buyer of rice? (Hint: you may assume without proof that the solution of utility maximization is interior.)

$I = P_1 e_1 + P_2 e_2$

- We may solve this using the MRs = price ratio tangency condition, or else by using a Lagrangian
- Using the tangency condition:
  - MRS$= \frac{1+2x_2}{2x_1} = \frac{P_1}{P_2} = \frac{10}{40} = \frac{1}{4}$

Question 4

- Then:
  - $4 + 8x_2 = 2x_1 \Rightarrow x_1 = 2 + 4x_2$
- We have income $I = P_1 \times 30 + P_2 \times 20 = 1100$, and budget constraint $1100 = 10x_1 + 40x_2$
- Substituting for x_1 gives $1100 = 20 + 40x_2 + 40x_2 \Rightarrow x_2 = 13.5$
- Then $x_1 = 2 + 4x_2 = 56$
- Given his starting endowments, Drake is a buyer of rice and a seller of onions

(30, 20)

Question 4

(b) Suppose the price of onions goes up. Would Drake benefit or suffer from the price hike? Explain.

(15 points)

- Drake is better off. The key here is to show that the original consumption bundle is still affordable after the increase in price of onions. Therefore, the solution uses ideas from Revealed Preference Theory.
- Two approaches to the solution:

<u>1) Algebraic</u>

- We have that the value of Drake's endowments of rice and onions is equal to the value of his demand for rice and onions
- Let x_1^* and x_2^* be Drake's demand for rice and onions respectively, and e_1 and e_2 be Drake's endowments of rice and onions
- Let P_1 and P_2 be the price of rice and onions respectively

Question 4

- We then have:
  - Value of demand = value of endowments
  - $P_1x_1^* + P_2x_2^* = P_1e_1 + P_2e_2 \Rightarrow P_1(x_1^* - e_1) + P_2(\underbrace{x_2^* - e_2}_{\text{Negative value}}) = 0$
  - Now suppose that the price of onions increases
    - Let $P_2' > P_2$

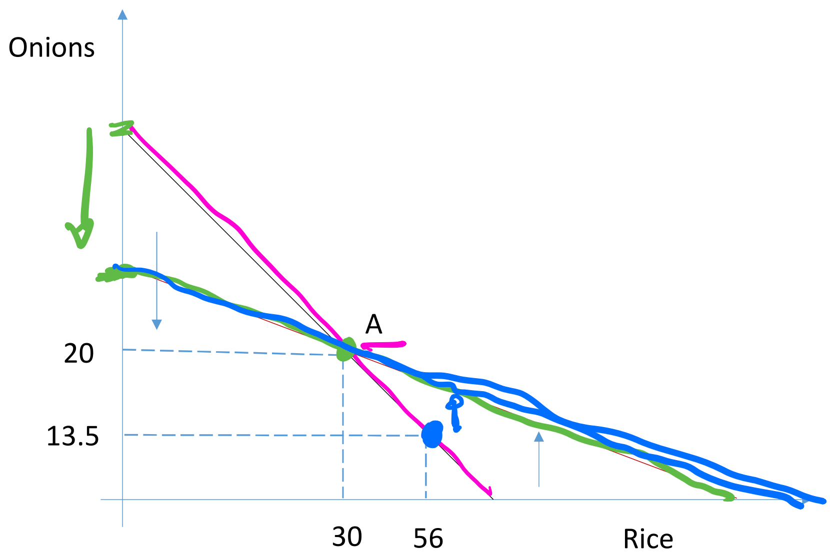
      $P_2' > P_2$.
  - As a result of the price rise, the value of Drake's endowments is now:
    - $P_1e_1 + P_2'e_2 > P_1e_1 + P_2e_2 = P_1x_1^* + P_2x_2^*$
  - Therefore Drake's original optimal consumption bundle is still affordable after the price increase

value of endowments > value of original consumption

=> original bundle is affordable.

Question 4

2) Graphical



- The initial endowment (30,20) is affordable before and after the increase in the rice of onions (point A)
- Therefore, the budget constraint passes through (30,20) in both cases
- The increase in the price of onions reduces the intercept on the onions axis
- The new budget constraint is linear and passes through (30,20)
- Therefore geometry requires that the new budget constraint passes above the point (56,13.5)
- That is, the original bundle is still affordable after the price rise
- Therefore Drake is better off after the price rise