

Homework 2

Due on Oct 06

Choose the best answer

1. The opportunity cost of producing a bicycle refers to the
 - a. out of pocket payments made to produce the bicycle.
 - b. value of the goods that were given up to produce the bicycle.
 - c. bicycle's retail price.
 - d. marginal cost of the last bicycle produced.
2. Which of the following production functions exhibits a constant elasticity of substitution?
 - a. $q = 3k + 2l$.
 - b. $q = k^{0.5}l^{0.5}$.
 - c. $q = 1 - \frac{1}{k} - \frac{1}{l}$.
 - d. All of the above have a constant elasticity of substitution.
3. The shape of a firm's long run average cost curve is determined by
 - a. the degree to which each input encounters diminishing marginal productivity.
 - b. the underlying nature of the firm's production function when all inputs are able to be varied.
 - c. how much the firm decides to produce.
 - d. the way in which the firm's expansion path reacts to changes in the rental rate on capital.
4. The input demand functions that can be derived from cost functions are referred to as "conditional" demand functions because the functions:
 - a. assume input costs are constant.
 - b. express input demand as a function of output.
 - c. depend on the assumption of profit maximization.
 - d. assume constant returns to scale in production.

Analytical questions

1. A firm has production function

$$f(k, l) = (k^\rho + l^\rho)^{\frac{\gamma}{\rho}}, \quad \rho \in (0, 1),$$

Denote the price of output by p , capital price v , and labor price w .

- a. Find the RTS between two inputs
- b. Solve the cost minimization problem

$$\min_{k, l} vk + wl, \quad \text{s.t. } q = (k^\rho + l^\rho)^{\frac{\gamma}{\rho}}.$$

- c. Find the cost function $C(w, v, q)$.
- d. Verify Shepard's lemma (on labor).

$$\frac{\partial C(w, v, q)}{\partial w} = l(w, v, q)$$

e.* Solve the cost minimization problem with a new production function:

$$\min_{k, l} vk + wl, \quad \text{s.t. } q = [(\alpha k)^\rho + (\beta l)^\rho]^{\frac{\gamma}{\rho}}.$$

[Hint: use change of variable to help you find the answer.]

2. A price-taking firm need three inputs, capital (k), labor (l), and raw material (m), for production of output (q). The production function is

$$q = f(k, l, m) = k^\alpha l^\beta m^\gamma.$$

The input price for k , l , and m are denoted by v , w , and r , respectively. α , β , γ are positive parameters.

- a. Characterize conditions of α that make the production function exhibit diminishing marginal product of capital.
- b. Characterize conditions of α , β , γ that make the production function exhibit increasing return to scale.
- c. In the short run, suppose that only raw material can be adjusted. Capital and labor are fixed at k_1 and l_1 , respectively. Compute the corresponding cost function $C^{SR}(q)$.
- d. Continue with part (c), let $\alpha = \beta = \gamma = 0.5$, $v = w = r = 2$, $k_1 = l_1 = 4$, and output price be p . Find the firm's supply function $S(p)$.
- e. In the "median" run, suppose that both labor and raw material can be adjusted. Capital is fixed at k_1 . Compute the input demands $l^{MR}(w, v, r, q, k_1)$, $m^{MR}(w, v, r, q, k_1)$ and the corresponding cost function $C^{MR}(q)$.

3. Textbook exercise 10.1
4. Textbook exercise 10.3
5. Textbook exercise 10.4
6. Textbook exercise 10.7