Homework 6

Due on Nov 24

Choose the best answer

1. Which statement is **true** of a typical Battle of sexes game?
    a. The game has a unique Nash equilibrium.
    b. A player may receive lower expected payoff in the mixed-strategy Nash equilibrium than some other equilibrium.
    c. The follower has an advantage in the sequential version.
    d. None of the above.

2. In a mixed-strategy Nash equilibrium, a player is willing to randomize because
    a. this confuses opponents.
    b. he or she is indifferent between the actions in equilibrium.
    c. the actions provide the same payoffs regardless of what the other player does.
    d. he or she does not know what the other player is doing.

3. In the mixed-strategy Nash equilibrium of the following game in which players randomize between B and C and do not play A at all, what is the probability that player 1 plays B?

		2		
		A	B	C
	A	4, 4	0, 5	0, 0
1	B	5, 0	1, 1	0, 0
	C	0, 0	0, 0	3, 3

    a. 3/4.
    b. 1/2.
    c. 1/4.
    d. 1/3.

4. The pure-strategy Nash equilibrium is social optimal in the situation of
    a. Prisoner dilemma.
    b. Battle of sexes.
    c. Cournot duopoly competition.
    d. None of above.

Analytical questions

1. Find all pure strategy Nash equilibrium of the following games

<div align="center">

2

	C1	C2	C3	C4
R1	2, 6	5, 7	2, 5	6, 9
R2	5, 5	3, 2	5, 6	2, 4
R3	4, 4	4, 9	3, 4	7, 2
R4	6, 8	4, 5	4, 2	6, 3

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(with **1** labeling the rows)

2. Consider the following Battle of sexes game:

<div align="center">

Woman

|		Action [c]	Drama
Man	Action [r]	4, 2	1, 1
	Drama	0, 0	2, 4

</div>

Both the man and the woman would like to spend their time together at the cinema, but they have different preference on the type of movie.

a. Find all Nash equilibrium (including mixed strategy).

b. If the man arrive the cinema first and decides to purchase tickets before the woman arrives, what will be the equilibrium (SPE)? Finish the game tree (extensive form).
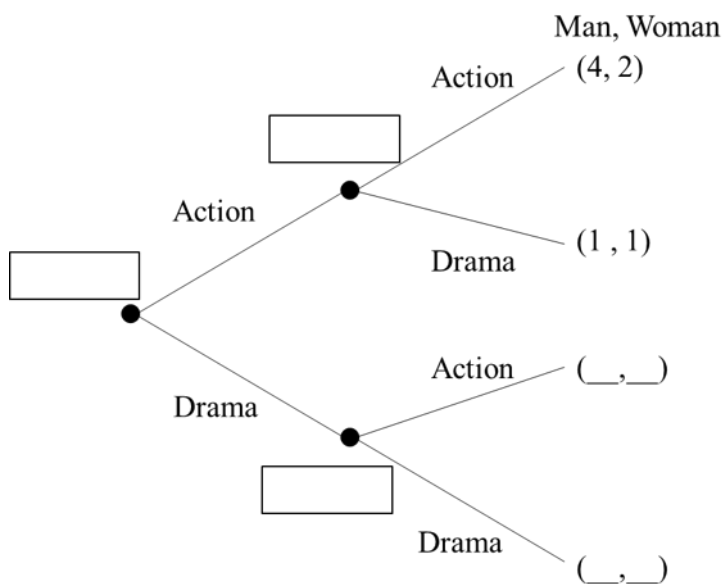
c. If the woman arrive the cinema first and decides to purchase tickets before the man arrives. Draw the complete extensive form of the game and find the subgame perfect equilibrium.
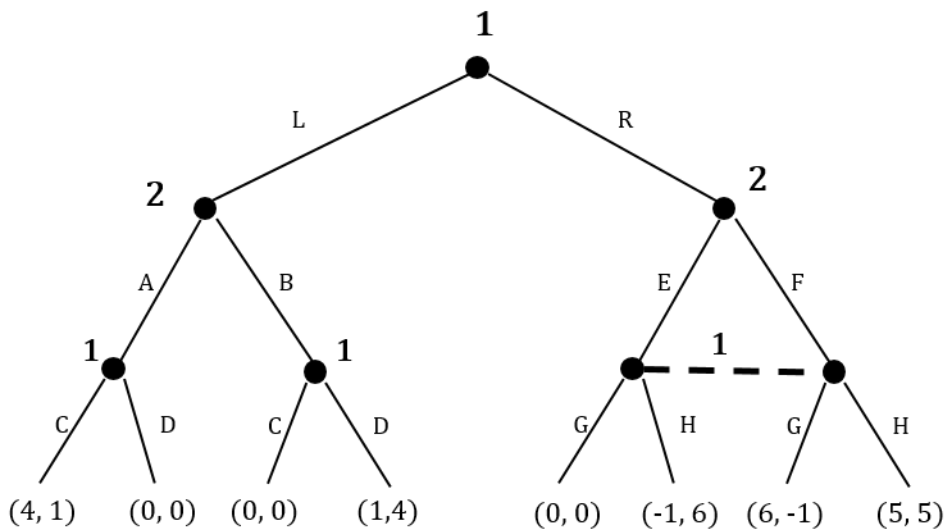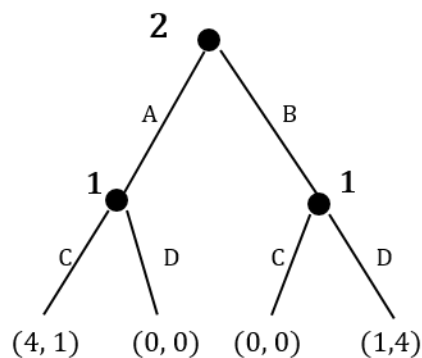
3. Consider the following game
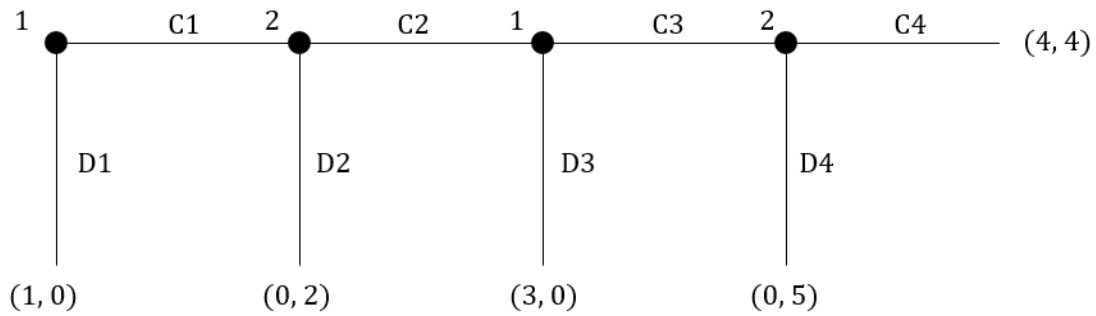
a. Predict the outcome of this game.

b. Can you revise only one payoff in the game so that the outcome becomes (C1,C2,C3,C4)?

4. Solve the following game following steps (a) (b) (c). Note that for each payoff (u_1, u_2), the left-hand side number is for player 1; and the right-hand side number is for player 2.

a. Predict the outcome if player 1 selects L by analyzing the following game.

b. Predict the outcome if player 1 selects R by analyzing the following game. To do so, first turn this game into a normal form. Then, find the Nash equilibrium.
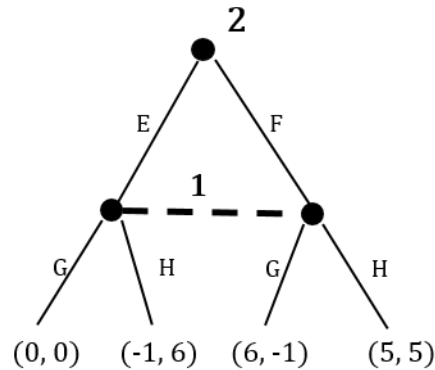
		2	
		E	F
1	G	0, 0	—, —
	H	—, —	—, —

c. Predict the outcome of the entire sequential game.

5. Consider the following game in which two countries choose between Peace and War.

		Country 2	
		Peace	War
Country 1	Peace	2, 2	-3, x
	War	x, -3	-2, -2

a. Find all pure-strategy NE when $x = 1$.

b. Find all pure-strategy NE when $x = 3$.

c. Continue with part (b) with $x = 3$. Suppose that the two countries play this game with infinitely number of times with discount factor $\delta \in (0, 1)$. What is the minimum discount factor δ that can induce the two countries to stay peace based on trigger strategies? Write down the trigger strategy for $t = 0, 1, 2.....$

d. Continue with part (c). If x increases, is it easier or harder to maintain peace between these two countries? Prove it.

6. A firm has hired a worker to implement a project at wage w. If the project is successful, it will earn the firm $v > 0$ dollars of revenue. The project will be successful if the worker *works* and unsuccessful if the worker *shirks*. The cost of effort for the worker is g dollars if he works and nothing if he shirks. The worker's effort is unobservable unless the firm chooses to monitor the worker, so the firm will have to pay the worker unless it monitors and the worker shirks. If the firm monitors the worker it incurs a monitoring cost of h dollars. In the first stage of the project, the firm chooses the wage $w \geq 0$ it will pay the worker. The worker chooses whether to work on the project or shirk, and the firm simultaneously chooses whether or not to monitor the worker. Let $w > g > h > 0$ and v is large enough.

a. Complete the normal form game expression and find all NE of this game.

		Firm	
		Monitor [m]	Not
	Shirk [p]	$0, -h$	$w, \underline{\quad}$
Worker	Work	$\underline{\quad}, v - w - h$	$\underline{\quad}, \underline{\quad}$

b. What is the expected payoff of the firm in the equilibrium you obtain in (a)? If the firm can choose w before they player the game, what's the optimal w?

6. Textbook exercise 8.2

7. Textbook exercise 8.4

8. Textbook exercise 8.6