Exercise 4 (Assignment 2) solution

1. $g(x,y) = f(x+y)$

   $g'_x = f'(x+y), g'_y = f'(x+y), g''_{xx} = f''(x+y), g''_{xy} = f''(x+y), g''_{yy} = f''(x+y)$

2. $g(x,y) = f(xy)$

   $g'_x = yf'(xy), g'_y = xf'(xy), g''_{xx} = y^2 f''(xy), g''_{xy} = f'(xy) + xyf''(xy), g''_{yy} = x^2 f''(xy)$

3. $g(x,y) = f(x+y, x-y)$

   $g'_x = f'_1(x+y, x-y) + f'_2(x+y, x-y)$; $g'_y = f'_1(x+y, x-y) - f'_2(x+y, x-y)$

   $$\begin{aligned} g''_{xx} &= f''_{11}(x+y, x-y) + f''_{12}(x+y, x-y) + f''_{21}(x+y, x-y) + f''_{22}(x+y, x-y) \\ &= f''_{11}(x+y, x-y) + 2f''_{12}(x+y, x-y) + f''_{22}(x+y, x-y) \end{aligned}$$

   $$\begin{aligned} g''_{xy} &= f''_{11}(x+y, x-y) - f''_{12}(x+y, x-y) + f''_{21}(x+y, x-y) - f''_{22}(x+y, x-y) \\ &= f''_{11}(x+y, x-y) - f''_{22}(x+y, x-y) \end{aligned}$$

   $$\begin{aligned} g''_{yy} &= f''_{11}(x+y, x-y) - f''_{12}(x+y, x-y) - f''_{21}(x+y, x-y) + f''_{22}(x+y, x-y) \\ &= f''_{11}(x+y, x-y) - 2f''_{12}(x+y, x-y) + f''_{22}(x+y, x-y) \end{aligned}$$

4. Let $f \in C^2$ and $z = f(x,y)$, where $x = a+bt, y = c+et$, find $\frac{dz}{dt}$ and $\frac{d^2z}{dt^2}$

   $\frac{dz}{dt} = bf_1(x,y) + ef_2(x,y)$

   $\frac{d^2z}{dt^2} = b^2 f_{11}(x,y) + 2bef_{12}(x,y) + e^2 f_{22}(x,y)$