Problem Set 2[1]
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Question 1: Basic Concepts on Forward Curves (2/10)

(1) Define what a forward curve is. (1 sentence)

(2) Define what contango is. (1 sentence)

(3) Define what backwardation is. (1 sentence)

(4) Define what convenience yield is. (1 sentence)

Question 2: Basic Concepts on Currency Forward Contracts (2/10)

(1) Define what forward exchange rate is. (1 sentence)

(2) Define what currency carry trade is. (1 sentence)

(3) Define what covered interest rate parity is.

(4) Define what uncovered interest rate parity is.

(Optional) Question 3: Non-Arbitrage and Transaction Costs   (This question is designed to help you understand the meaning of the bid-ask spread and how it affects your trading profits. **This question is very difficult and will not be tested in exams.** Please see my notes below to see the intuitions (not in any textbooks!))

(1) The current bid and ask prices of a share in the firm NoDividends (NDV) are $20 and $20.10 respectively. The 6-month T-Bill rate (lending rate) and the LIBOR rate (borrowing rate) are respectively 5% and 6% (both in continuously compounded terms). Assume that there are no other transactions costs. If the current bid and ask prices for a 6-month forward on NDV stock are $20.35 and $20.45 respectively, is there an arbitrage? If so, what is it? If not, explain why not.

(2) How would your answer change if there are additional transactions costs associated with shorting NDV stock? (You do not necessarily need to do any further calculations. Just clearly state your reasoning.)

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[1]Note: optional questions are for your practice only. They are not counted toward your grades.

(3) Re-do part (1) for the case of current bid and ask forward prices of $20.81 and $20.91 respectively.

(4) How would your answer to part (3) change if there are additional transactions costs associated with shorting NDV stock? (Again, no additional calculations are required.)

Table 1: DJIA Futures

	DJ Industrial Average	DJIA Futures Price
Jan 27, 2009	8,174.73	8,136.00
Jan 28, 2009	8,375.45	8,336.50
Jan 29, 2009	8,149.01	8,112.00

Question 4: Futures Contracts (3/10)   The table above shows the daily closing values of the Dow Jones Industrial Average index (DJIA), along with the daily prices for the DJIA Futures contract with maturity date March 20 of the same year (Feb has 28 days). The futures contract size is 10 units of the index. value.

The initial margin required for the futures contract is $13,750/contract. The maintenance margin is $11,000/contract. The risk free rate is 1% (continuously compounded). Assume this is the rate you earn on your margin account.

 (i) Suppose you take a long position in 10 futures contracts on Jan 27. What is your profit/loss on Jan 28 and Jan 29? What is the balance in your margin account at the end of each of these days? Do you face any margin calls?

 (ii) Repeat part (i) for a short position of 5 futures contracts.

 (iii) Using the Jan 29 index value and futures price, what is the implied dividend yield on the DJIA? (Assume that the futures price is the same as the equivalent forward price.)

 (iv) For the three days shown, the index value is larger than the futures price. Why?

Question 5: Commodity Forward Contracts with Lease Rate (3/10)   The current price of crude oil is $32.00 per barrel. Forward prices for 3, 6, 9, and 12 months are $31.37, $30.75, $30.14, and $29.54. Assuming a 2% continuously-compounded annual risk-free rate.

 (i) Suppose there is an active leasing market for crude oil with continuously-compounded annual lease rate q. What is the lease rate q for each maturity?

 (ii) Suppose there is no leasing market or carry cost. What is the convenience yield c for each maturity?

(iii) Is this an example of contango or backwardation?

(Optional) Question 6: Commodity Forward Contracts with Convenience Yield

(This question is designed to help you understand why forward prices can be different across different investors. **This question is very difficult and will not be tested in exams.**)

Suppose you are a trader for an investment bank. An investor comes to you and want to enter a long forward position on one unit of soybeans. The forward price is $F_{0,T}$ which is determined by you. There is no lease market, carry cost, or transaction fee. The continuously-compounded interest rate is r.

(1) Suppose the investor has convenience yield c' by holding the soybean and can borrow from potential lenders who have convenience yield c'' by holding the soybean. Assume that $c' \leq c''$. Derive the interval of $F_{0,T}$ within which the investor cannot make arbitrage profits.

(2) If $c' = c''$ for the investor, what is the forward price you should set?

(3) Suppose here are many investors come to you with convenience yields $c_1 < \cdots < c_I$, and each investor $i \in \{1, \cdots, I\}$ has the same convenience yield c_i with her potential lenders. What is the minimum and maximum forward prices you would offer?

I write the notes below to provide some hints for Question 3

Let me first derive some general results for the non-arbitrage bound in the presence of bid-ask spread.

Table 2: Cash Flows of "Long Forward" Trading Strategy

Strategy Time	0	T
Long forward	0	$S_T - F_{0,T}^a$
Short stock	S_0^b	$-S_T$
Long T-bills	$-S_0^b$	$S_0^b e^{r^\ell \times T}$
Aggregate	0	$S_0^b e^{r^\ell \times T} - F_{0,T}^a$

Note on short selling: you borrow the stock from a lender at date zero and immediately sell it in the spot market to receive S_0^b. At T, you receive a share from your long forward counterparty and you return this share to the stock lender to close out your short stock position.

To ensure that this "Long Forward" strategy is not profitable, we need to have

$$F_{0,T}^a \geq S_0^b e^{r^\ell \times T} \equiv F_{low}^a \tag{1}$$

Similarly, we consider a "Short Forward" strategy.

Table 3: Cash Flows of "Short Forward" Trading Strategy

Strategy Time	0	T
Short forward	0	$F_{0,T}^b - S_T$
Long stock	$-S_0^a$	S_T
Short T-bills	S_0^a	$-S_0^a e^{r^b \times T}$
Aggregate	0	$F_{0,T}^b - S_0^a e^{r^b \times T}$

To ensure that this "Short Forward" strategy is not profitable, we need to have

$$F_{0,T}^b \leq S_0^a e^{r^b \times T} \equiv F_{high}^b \tag{2}$$

Note that because $S_0^a \geq S_0^b$ and $r^b \geq r^a$, we have

$$F_{high}^b \geq F_{low}^a \tag{3}$$

Note that because market makers make profits from bid-ask spread, we should have

$$F_{0,T}^b \leq F_{0,T}^a \tag{4}$$

Equations (1)-(4) give some general relations for the bid and ask forward prices. In general, we have the following cases all satisfying non-arbitrage conditions (by solving a standard combinatorial problem):

$$F_{0,T}^b \leq F_{low}^a \leq F_{high}^b \leq F_{0,T}^a \tag{5}$$

$$F_{0,T}^b \leq F_{low}^a \leq F_{0,T}^a \leq F_{high}^b \tag{6}$$

$$F_{low}^a \leq F_{0,T}^b \leq F_{0,T}^a \leq F_{high}^b \tag{7}$$

$$F_{low}^a \leq F_{0,T}^b \leq F_{high}^b \leq F_{0,T}^a \tag{8}$$