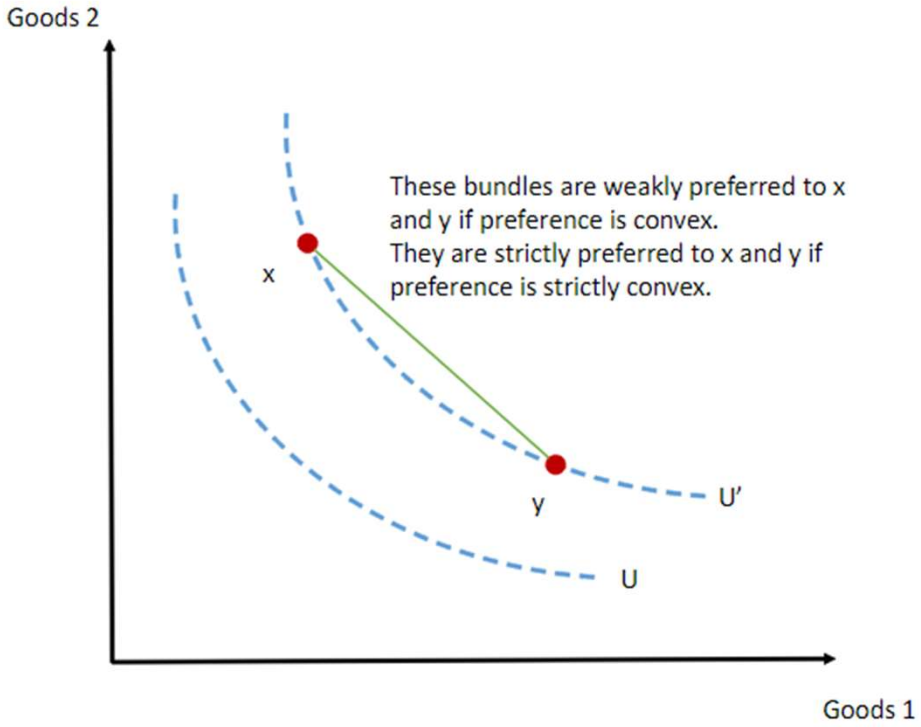
ECON3113
Microeconomic Theory I

Tutorial #4:

Today's tutorial

- Key characteristics of preference relations
  - Convexity
- Characteristics of indifference curves
  - Marginal rate of substitution and marginal utility
  - Examples:
    - Perfect substitutes
    - Perfect complements
    - Quasi-Linear utility functions
- Online Assessed Quiz #1

Convexity of preferences

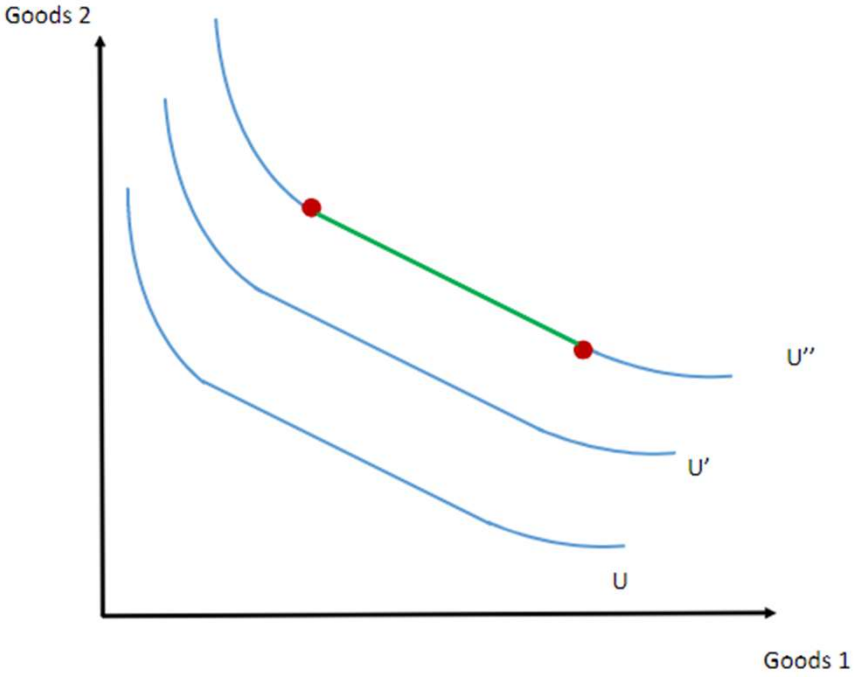


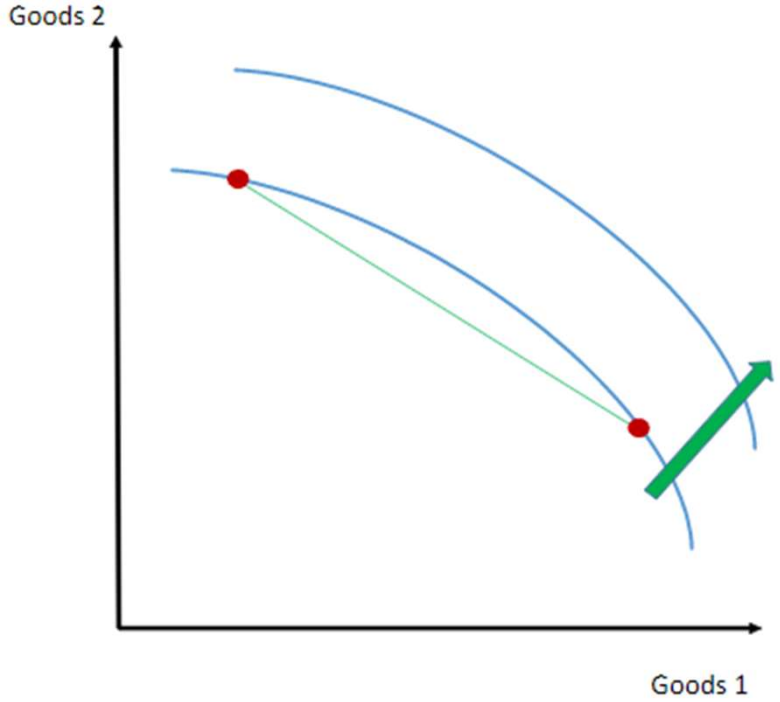
- Take a pair of bundles x and y that are on the same higher indifference curve.
- Consider moving along a line segment from y to x.
- Convexity of preference means that we are never worse off, at any point along the line segment, than the two end points.
- Strict convexity means that we are strictly better off, at any point along the line segment, than the two end points.

Example:

- Given $x = (5,15)$,) and $y = (15,5)$, convex preferences mean we are never worse off than at x or y at any point on $y = 30 - 3x$

Convexity of preferences: non-strict convexity and other possibilities

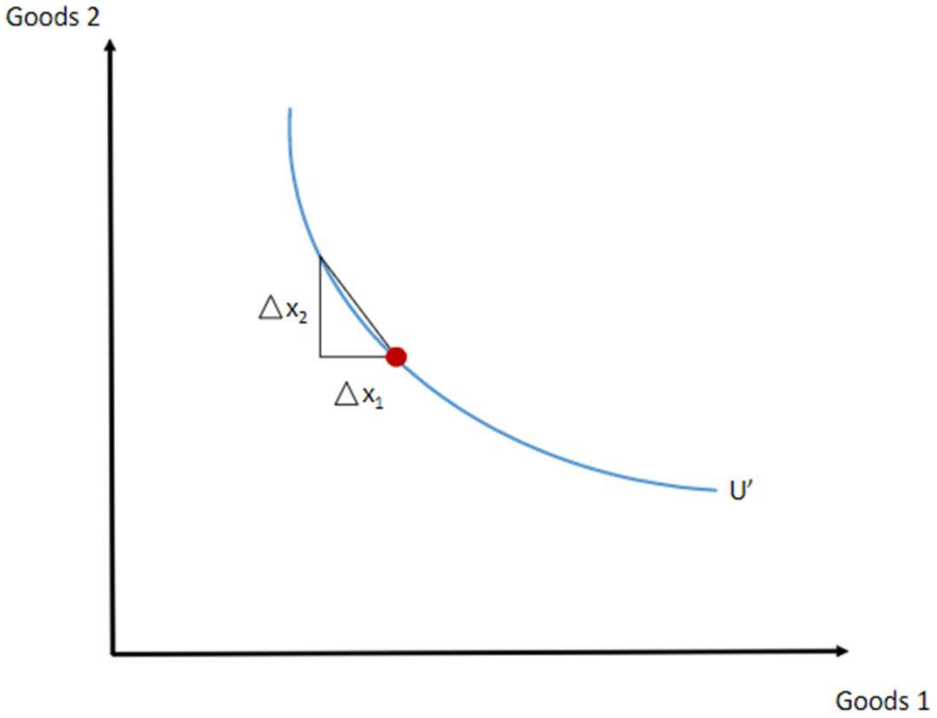




- Non-strictly convex preferences – points on the line segment can have utility equal to utility at the end points

- Strictly concave preferences – points on the line segment may have utility lower than at the end points

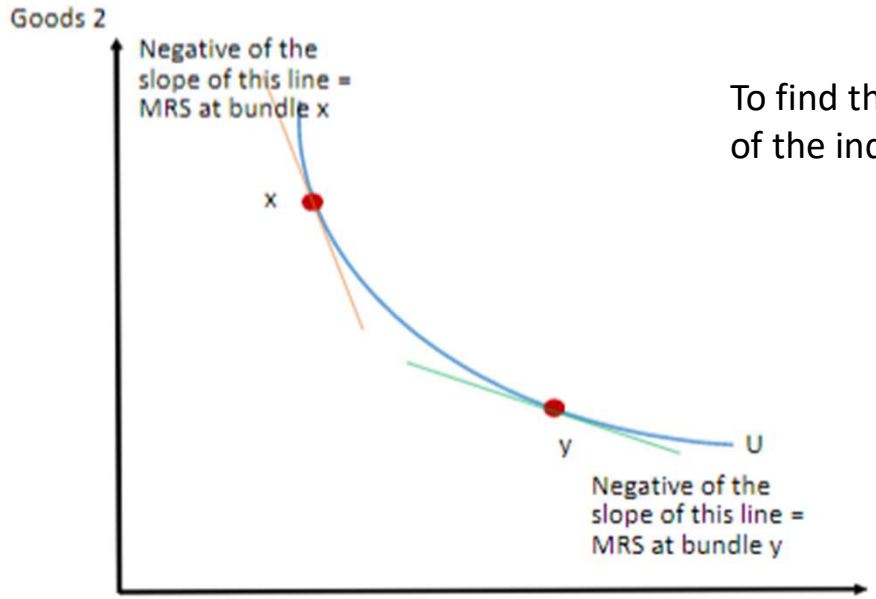
Marginal rate of substitution



- Start with some initial consumption bundle, say (x_1, x_2).
- Now, let's take away $\triangle_1$ units of good 1 from the bundle, how many units of goods 2 do we need to compensate the consumer to keep him indifferent?
- That is, what is the value of $\triangle_2$ so that $(x_1, x_2) \sim (x_1 - \triangle_1, x_2 + \triangle_2)$?
- This ratio $\triangle_2/\triangle_1$ is the rate at which the consumer is willing to **substitute goods** 2 **for goods** 1**.**
- If we take $\triangle_1$ to be extremely small, the ratio $\triangle_2/\triangle_1$ is called the **marginal rate of substitution (MRS)**.
- The value of MRS, in general, depends on the bundle we evaluate it.

Marginal rate of substitution

- Graphically, MRS at a bundle (x_1, x_2) can be read off by the slope of the indifference curve passing through (x_1, x_2).
  - Note the difference in sign.



To find the MRS of x with respect to y, find the slope of the indifference curve at a particular point

Marginal utility and the marginal rate of substitution

- Given utility function $U = U(x, y)$, marginal utility of x (or y) is the partial derivative of U with respect to x (or y):
  - Marginal utility of $x = \frac{\partial U(x,y)}{\partial x}$, marginal utility of $y = \frac{\partial U(x,y)}{\partial y}$
- Marginal rate of substitution of x with respect to y is the ratio of marginal utilities:
- MRS $= \frac{\partial U(x,y)}{\partial x} / \frac{\partial U(x,y)}{\partial y}$
- Example: General Cobb-Douglas utility function $U = x^{\alpha} y^{1-\alpha}$
- MRS $=$

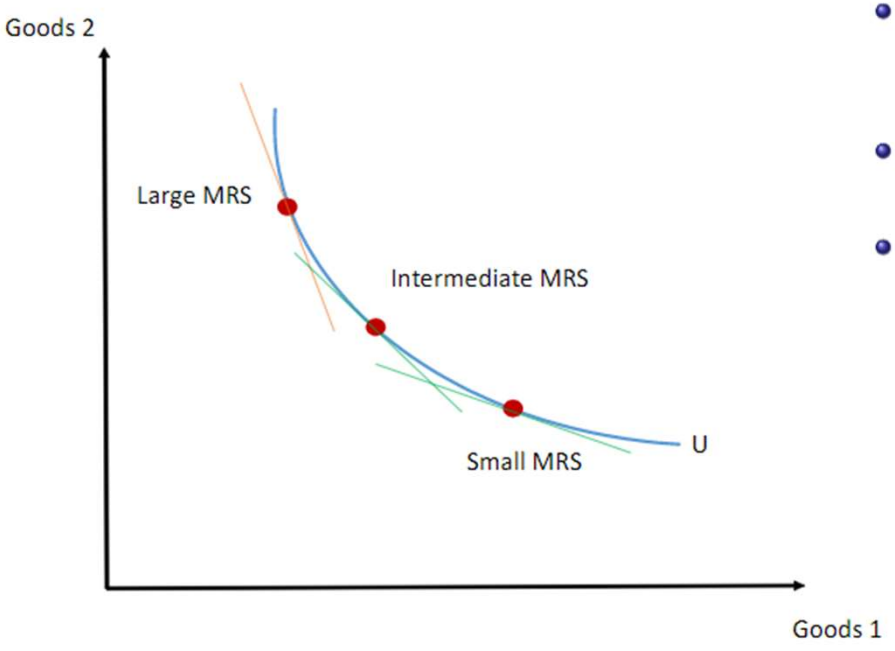
Marginal utility and the marginal rate of substitution

- Note the invariance of MRS to a monotone transformation of the utility function
- That is, if we have utility function $U = U(x, y)$ and strictly increasing function f, then:
  - MRS $_U$ = MRS $_{f(U)}$
- Example: Consider the utility functions $U_1 = x^{1/2}y^{1/2}$ and $U_2 = e^{(1+x^{\frac{1}{2}}y^{\frac{1}{2}})^2}$
- In this case, $f(x) = e^{(1+x)^2}$
- MRS =

Marginal utility and the marginal rate of substitution

- MRS =

Diminishing marginal rate of substitution



- A consumer's preference satisfies **diminishing marginal rate of substitution (DMRS)** if MRS is decreasing in the quantity of good 1 in the bundle (holding utility constant).
- This is intuitive: when you have little goods 1, you really want them and are willing to give up a lot of other things (goods 2) to get them.
- When you have lots of goods 1, you don't really want them anymore and are willing to give up little other things (goods 2) to get them.

- DMRS means that the negative of the slope of every indifference curve is decreasing as x_1 increases.
- Mathematic fact: a function with increasing derivative is strictly convex.
- DMRS implies indifference curves are strictly convex, which in turn implies that the preference is strictly convex.

Convexity and diminishing marginal rate of substitution

- To determine convexity property of preference, we ask whether *MRS* decreases in x_1 along the indifference curves.
- Fix an indifference curve with utility $\bar{u}$:

$$u(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2} = \bar{u}.$$

- We can rewrite this indifference curve into

$$x_2 = \left(\bar{u} x_1^{-\alpha_1}\right)^{\frac{1}{\alpha_2}}.$$

- The MRS of a bundle on the indifference curve as a function of x_1:

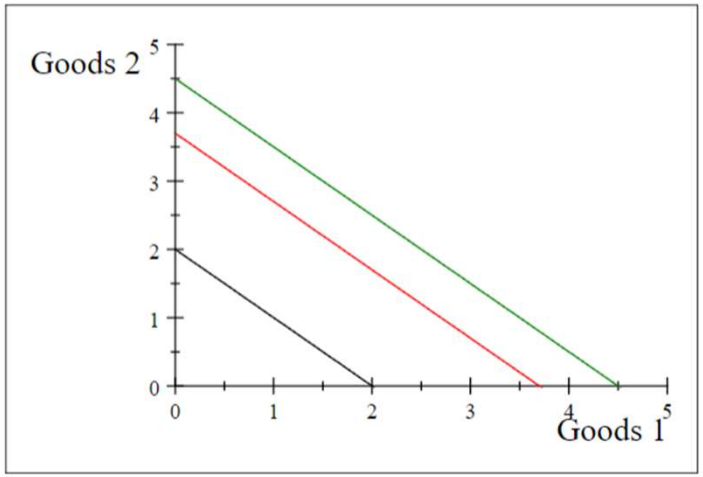
$$MRS = \frac{\alpha_1}{\alpha_2}\frac{x_2}{x_1} = \frac{\alpha_1}{\alpha_2}\frac{\left(\bar{u} x_1^{-\alpha_1}\right)^{\frac{1}{\alpha_2}}}{x_1} = \frac{\alpha_1}{\alpha_2}\bar{u}^{\frac{1}{\alpha_2}} x_1^{-\left(\frac{\alpha_1}{\alpha_2}+1\right)},$$

which is clearly decreasing in x_1, so Cobb-Douglas preference is strictly convex.

Examples of utility functions

Perfect substitutes

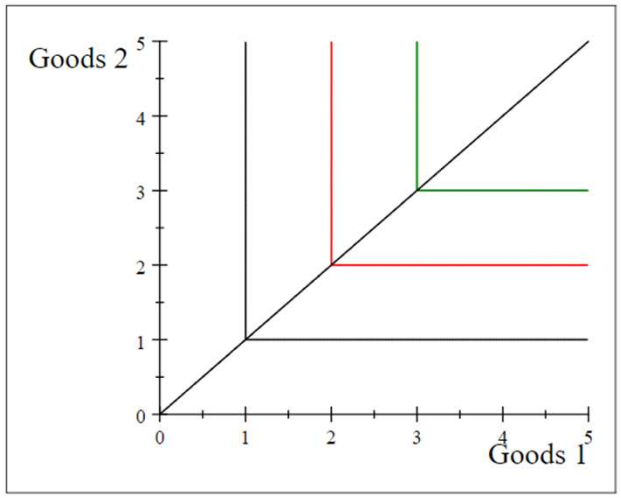
$$u(x_1, x_2) = x_1 + x_2.$$



- As the utility is strictly increasing, the preference is strictly monotone.
- The MRS is constant at 1; the preference is convex but not strictly convex.

Perfect complements

$$u(x_1, x_2) = \min\{x_1, x_2\}.$$



- The preference is convex but not strictly convex.

Takeaways

1. Monotonicity of preference (more is better) implies
   - ICs have no width;
   - ICs are downward sloping.

Strictly monotone

2. Adding continuity (no jumps), we get
   - ICs are continuous;
   - utility function is continuous.
3. Adding further convexity (balanced consumption is better than extremes), we get
   - ICs are convex;
   - Diminishing Marginal Rate of Substitution (with strict convexity).

- Online Assessed Quiz #1

- Online Assessed Quiz #1 is available in Canvas/Quizzes
- You have 15 minutes to complete the quiz
- 6 multiple choice questions on the lecture material up to the end of Lecture Notes #3
- There is no countdown clock available; the Canvas clock will count upwards and will stop at 15.00 minutes
- Any problems during the quiz, let me know
- Good luck!