



Strategic Interaction and Oligopoly

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Announcements

■ Assigned reading:

- Textbook Chapter 15 except Antitrust Law (p394 to 397)
- Textbook Chapter 17 – externalities
- ~~Textbook Chapter 16 – public goods~~ (other than materials on my slides)

■ Problem set 8

- Ch 15: 1, 4, 5, 7, 12, 18
- Ch 17: 1-9
- Due date will be announced on CANVAS.

STRATEGIC INTERACTION

Strategic Interaction

- Suppose, two fancy restaurants are located across the street from one another.
- The owner of each restaurant will be concerned about the “pricing strategy” of the other,... ..
- And the other’s business strategy in general.
- In fact, each owner will base her own business strategy on her beliefs about the strategy of the other! (What is the other doing???)
- **This is an example of “strategic interaction”.**

Strategic Interaction

- In perfectly competitive market, firms do not compete with other firms on individual basis.
- You sell Coca-Cola, but you would not care what the others are doing – because too many of them are around.
- What you care: Equilibrium market price. Take the price as given (price takers), you decide how many units to sell (business strategy).
- You DO NOT “*interact strategically*” with your competitors.
- Same for Monopoly, no strategic interaction because only one seller in the market.

Strategic Interaction

- “**Strategic**” interaction is very important when two or a small number of people or firms engage in **bargaining**, **conflict** or **competition**.
- Duopoly (two competing firms)
- Oligopoly (several competing firms)
- To study strategic interaction, we need a tool: **“Game Theory”**.

Example - Coordination in Business

- We can see sometimes firms can increase profits by coordinating their strategies.
- **EXAMPLE:** If a men's clothing shop and a women's clothing shop locate in the same mall, both may attract more customers.
- **EXAMPLE:** If two hardware stores locate in different areas, rather than in the same area, both may attract more customers.

Game Theory

- Game Theory: A set of mathematical-oriented tools used to analyze strategic interaction.
 - Often applied in economics, political science, and military science
- In a game theory:
 - Players (decision makers)
 - Strategies (Players' complete plans of action)
 - Payoffs (rewards or punishments – outcome)
- At least two players in a game, but games with any number of players can be analyzed.

More on “Strategies”

- **Strategy:** A plan of action that describes what a player will do “**in every circumstance**” that we can observe.
- Strategy can be as simple as involving only one choice: Ask/Not ask her/him to be my gf / bf.
- Some strategies are complex that involve many contingencies (If xxx, then aaa; if yyy, then bbb; if zzz, then ccc;).

Battle of the Sexes – Cooperation?

- The Battle of the Sexes is a game-theory model of coordination in business.
- To keep the game simple, only two players are modeled.
- Husband wants to go to a football game (F) and wife wants to go to an Opera (R).
- If they both do F, then husband gets utility 2 and wife gets 1; If they both do R, then wife gets utility 2 and husband gets 1.
- If they do different things, both get 0.
- Both must choose their strategies **simultaneously** without knowing what the other will choose.

Battle of the Sexes

- Both must choose their strategies simultaneously, without knowing what the other has done.
- Husband and wife are “**players**”.
- F and R are “**strategies**”.
- 2,1,0 are “**payoffs**”.
- Each box corresponds to a “**strategy profile**” (one strategy for each player): $\langle \text{R}, \text{F} \rangle$ (wife chooses R and husband chooses F).
- **0** for wife and **0** for husband are the corresponding payoffs of a strategy profile.

		Husband	
		F	R
Wife	F	1, 2	0, 0
	R	0, 0	2, 1

Battle of the Sexes

- This game is modeled as a “normal-form game”.
- Each row represents a strategy for one player (wife).
- Each column represents a strategy for the other (husband).

		Husband	
		F	R
Wife	F	1, 2	0, 0
	R	0, 0	2, 1

Applying Game Theory

- Can we use game theory to study the outcomes of strategic interaction?
- What strategies should we expect the husband and wife to adopt in their “battle of the sexes”?
- The most commonly-used solution concept is the “**Nash Equilibrium**”, named after John Nash (Nobel Prize Winner of 1994).
- Movie: The Beautiful Mind

Nash Equilibrium

- A Nash Equilibrium is a strategy profile in which each player has chosen the strategy that is her “**best response**” to the strategies of the other players.
- Equivalently, in a Nash Equilibrium, when all players found out what the others were going to do, no player would want to “deviate” from her chosen strategy.
- Does the “equilibrium” make sense for this “game”? Why?

Nash Equilibrium – Battle of the Sexes

■ If both husband and wife decide to go to football match,

- Is it an equilibrium?

■ Let's first look at the wife:

- If husband chooses F, will the wife deviate from F?

- No!

- F is her best response (higher payoff value) given husband chooses F.

■ Then look at the husband:

- If wife chooses F, will the husband deviate from F?

- Result: **<F,F> IS an Equilibrium.**

- Any other equilibrium?

		Tony	
		F	R
Janet	F	1, 2	0, 0
	R	0, 0	2, 1

Nash Equilibrium – Battle of the Sexes

■ Yes, $\langle R, R \rangle$ is also an Equilibrium.

● Why?

● Try please

		Tony	
		F	R
Janet	F	1, 2	0, 0
	R	0, 0	2, 1

Nash Equilibrium – Battle of the Sexes

■ Suppose wife goes R and husband goes F.

● Is $\langle R, F \rangle$ an equilibrium?

■ Let's first look at the wife:

● If husband chooses F, will the wife deviate from R?

■ Then look at the husband:

● If wife chooses R, will the husband deviate from F?

● **Result: $\langle R, F \rangle$ IS NOT an Equilibrium (and neither is $\langle F, R \rangle$).**

		Tony	
		F	R
Janet	F	1, 2	0, 0
	R	0, 0	2, 1

Nash Equilibrium – Battle of the Sexes

- In the “Battle of the Sexes” game, “coordination failure” is not an **equilibrium**!
- Players want to deviate!
- Both of these two (cooperative) equilibria are called “pure-strategy” equilibria, because neither player chooses his strategy randomly.
- There is a “mixed-strategy” equilibrium also:
 - Husband goes to F with probability $2/3$ and to R with probability $1/3$. Wife does the opposite.
 - Mixed-strategy is NOT required in the exam.

Friend or Foe Game Show

Example: OPEC – Cooperation?

- Sometimes cooperation is more profitable or productive for firms than competition.
- **BUT** cooperation can be hard to maintain:
 - If all other firms (or players) are cooperating, it may be profitable for an individual firm to “defect” or “cheat”.
- **EXAMPLE:** OPEC

Example: OPEC – Cooperation?

- **OPEC:** Organization of the Petroleum Exporting Countries
- OPEC sets production quotas for each member country.
- But some countries cheat and produce more than OPEC allows them to.
- Some analysts believe that OPEC is completely ineffective ...
- And the price of petroleum ends up at the competitive price.

Prisoners' Dilemma

- Michael and Albert have been caught by the police.
- Police have evidence to put them behind bars for 5 years each.
- But with a confession, the police could get 20-year sentence for a more severe crime.
- So the police offers them the following:
 - If you confess but the other does not, you will get only 2 years, but the other gets 20 years.
 - If both confess, each gets 15 years.

Prisoners' Dilemma

- Brother, what should I do?
 - Silence (S): Good brother! Wow!
 - Confession (C): It is business! Don't blame me.
- Each has to make his choice without knowing what the other will do.
 - Of course, police will separate them in different rooms to prevent potential communication.
- Suppose, both Michael and Albert decide not to confess (S).
 - Is that an equilibrium?

Nash Equilibrium – Prisoners' Dilemma

- First, draw the game in normal-form.
- Given Michael chooses S, what happen if Albert deviates from S?
- Albert will get -2 instead of -5!
- Albert will deviate to C.
- $\langle S, S \rangle$ IS NOT an equilibrium!
- Is $\langle C, S \rangle$ an equilibrium?
- Michael chooses S, should Albert deviates from C? No!
- However, if Albert chooses C, Michael wants to deviate to C!!!
- $\langle C, S \rangle$ IS NOT an equilibrium!

		Michael	
		S	C
Albert	S	-5, -5	-20, -2
	C	-2, -20	-15, -15

Nash Equilibrium – Prisoners' Dilemma

- Is $\langle C, C \rangle$ an equilibrium?
- Michael chooses C, should Albert deviate from C?
- No! -20 instead of -15
- Only $\langle C, C \rangle$ is an equilibrium, even though both would be **better off** if they could commit to $\langle S, S \rangle$.
- For each player, C is a **strictly dominant strategy** – i.e. it is better to play C, **no matter what** the other may do.

		Michael	
		S	C
Albert	S	-5, -5	-20, -2
	C	-2, -20	-15, -15

Cooperation and the Prisoners' Dilemma

- The Prisoners' Dilemma illustrates how difficult it is for competing firms to cooperate with each other.
- Whatever they have agreed to, each player can do better by cheating (following self-interest).
- That is why OPEC countries cheat and overproduce.
- If possible, binding commitment between players would be good for them all.

OLIGOPOLY

Oligopoly

■ **Oligopoly:** A market with a small number of firms, linked by strategic interaction.

- Decision on “P” or “Q” can affect other firms and cause them to react accordingly.
- The firm will consider others’ reactions when making its own decision.

■ Here, we use game theory to model “**duopoly**”, a market with only two firms:

- **Cournot Duopoly:** Firms compete by setting output quantities.
- ~~**Bertrand Duopoly:** Firms compete by setting prices.~~

Example: Mobile Phone Duopoly

P	Q
\$0	140
5	130
10	120
15	110
20	100
25	90
30	80
35	70
40	60
45	50

- Smalltown has 140 residents.
- The “good”: Mobile phone services
- Smalltown’s demand schedule is given.
- Two firms: T-Mobile and Verizon (**duopoly**: an oligopoly with two firms)
- Each firm’s costs: $FC = \$0$, $MC = \$10$

Example: Mobile Phone Duopoly

<i>P</i>	<i>Q</i>	Revenue	Cost	Profit
\$0	140	\$0	\$1,400	−1,400
5	130	650	1,300	−650
10	120	1,200	1,200	0
15	110	1,650	1,100	550
20	100	2,000	1,000	1,000
25	90	2,250	900	1,350
30	80	2,400	800	1,600
35	70	2,450	700	1,750
40	60	2,400	600	1,800
45	50	2,250	500	1,750

Competitive
outcome:

$$P = MC = \$10$$

$$Q = 120$$

$$\text{Profit} = \$0$$

Monopoly
outcome:

$$P = \$40$$

$$Q = 60$$

$$\text{Profit} = \$1,800$$

Example: Mobile Phone Duopoly

- One possible duopoly outcome: **Collusion**
- **Collusion:** An agreement among firms in a market about quantities to produce or prices to charge
- T-Mobile and Verizon could agree to each produce half of the monopoly output:
 - For each firm: $Q = 30$, $P = \$40$, profits = \$900
 - “Bake the biggest cake”, then divide it !!!
- **Cartel:** A group of firms acting in “unison”

Collusion vs. Self-Interest

<i>P</i>	<i>Q</i>
\$0	140
5	130
10	120
15	110
20	100
25	90
30	80
35	70
40	60
45	50

- Duopoly outcome with collusion:
Each firm agrees to produce $Q = 30$,
earns profit = \$900.
- If T-Mobile reneges on the agreement
and produces $Q = 40$, what happens to
the market price? T-Mobile's profits?
- Is it in T-Mobile's interest to renege on
the agreement?
- If both firms renege and produce $Q = 40$,
determine each firm's profits.

Collusion vs. Self-Interest

<i>P</i>	<i>Q</i>
\$0	140
5	130
10	120
15	110
20	100
25	90
30	80
35	70
40	60
45	50

- If both firms stick to agreement, each firm's profit = \$900.
- If T-Mobile reneges on agreement and produces $Q = 40$:
 Market quantity = 70, $P = \$35$
 T-Mobile's profit = $40 \times (\$35 - 10) = \text{\textcolor{blue}{\$1000}}$
T-Mobile's profits are higher if it reneges.
- Verizon will conclude the same, so both firms renege, each produces $Q = 40$:
 Market quantity = 80, $P = \$30$
 Each firm's profit = $40 \times (\$30 - 10) = \text{\textcolor{red}{\$800}}$

Collusion vs. Self-Interest

- Both firms would be better off if both stick to the cartel agreement.
- But each firm has incentive to renege on the agreement (**cheat**).
- **Lesson:** It is difficult for oligopoly firms to form cartels and honor their agreements.

Equilibrium of another kind: Cell Phone Duopoly

P	Q
\$0	140
5	130
10	120
15	110
20	100
25	90
30	80
35	70
40	60
45	50

- If each firm produces $Q = 40$,
market quantity = 80
 $P = \$30$
each firm's profit = \$800
- Is it in T-Mobile's interest to increase its output further, to $Q = 50$?
- Is it in Verizon's interest to increase its output to $Q = 50$?

Equilibrium of another kind: Cell Phone Duopoly

<i>P</i>	<i>Q</i>
\$0	140
5	130
10	120
15	110
20	100
25	90
30	80
35	70
40	60
45	50

- If each firm produces $Q = 40$,
then each firm's profit = \$800

- If T-Mobile increases output to $Q = 50$:

Market quantity = 90, $P = \$25$

T-Mobile's profit = $50 \times (\$25 - 10) = \750

T-Mobile's profits are higher at $Q = 40$ than at $Q = 50$.

The same is true for Verizon.

The Equilibrium for an Oligopoly

- **Nash Equilibrium:** Our duopoly example has a Nash Equilibrium, in which each firm produces $Q = 40$.
- Given that Verizon produces $Q = 40$, T-Mobile's best response is to produce $Q = 40$.
- Given that T-Mobile produces $Q = 40$, Verizon's best response is to produce $Q = 40$.

Oligopolies as a Prisoners' Dilemma

- When oligopolies form a cartel in the hopes of reaching the monopoly outcome, they become players in a prisoners' dilemma.
- T-Mobile and Verizon Example:
 - T-Mobile and Verizon are duopolists in Smalltown.
 - The cartel outcome maximizes profits: Each firm agrees to serve $Q = 30$ customers.
 - Game and payoff matrix?

T-Mobile & Verizon in the Prisoners' Dilemma

Each firm's dominant strategy: renege on agreement, produce $Q = 40$.

		T-Mobile	
		$Q = 30$	$Q = 40$
Verizon	$Q = 30$	<div>T-Mobile's profit = \$900</div> <div>Verizon's profit = \$900</div>	<div>T-Mobile's profit = \$1000</div> <div>Verizon's profit = \$750</div>
	$Q = 40$	<div>T-Mobile's profit = \$750</div> <div>Verizon's profit = \$1000</div>	<div>T-Mobile's profit = \$800</div> <div>Verizon's profit = \$800</div>

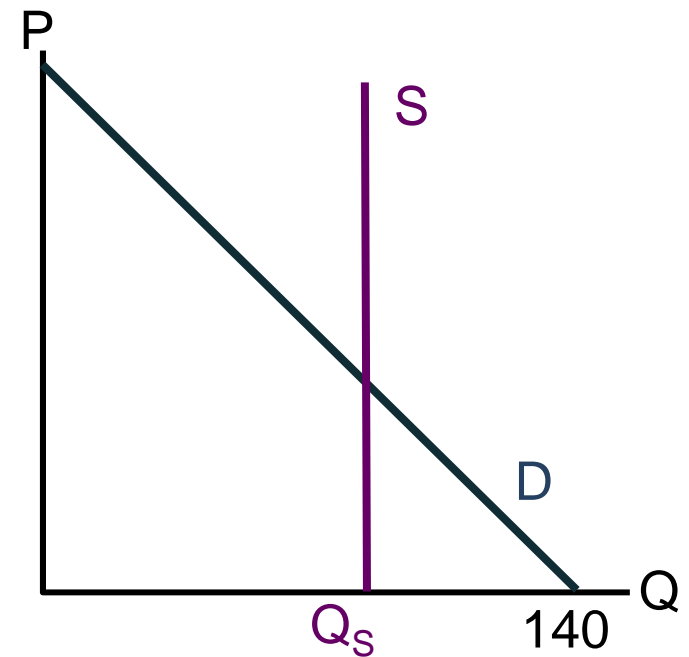
Prisoners' Dilemma and Society's Welfare

- The non-cooperative oligopoly equilibrium:
 - Bad for oligopoly firms: Prevents them from achieving monopoly profits.
 - Good for the society: Q is closer to the socially efficient output; P is closer to MC .

A COURNOT DUOPOLY – IN EQUATION

The Cournot Game

- The market demand curve for cell phone service: $Q_D = 140 - 2P$.
- Each firm sets its own production
- **T** selects q_T (T's strategy).
- **V** selects q_V (V's strategy).
- Market supply is therefore:
- $Q_S = q_T + q_V$
- We find the marketing-clearing price
- $Q_D = Q_S$
- $140 - 2P = q_T + q_V$
- $P = 70 - 1/2(q_T + q_V)$
- **MC = 10**, what is q_T^* & q_V^* ?

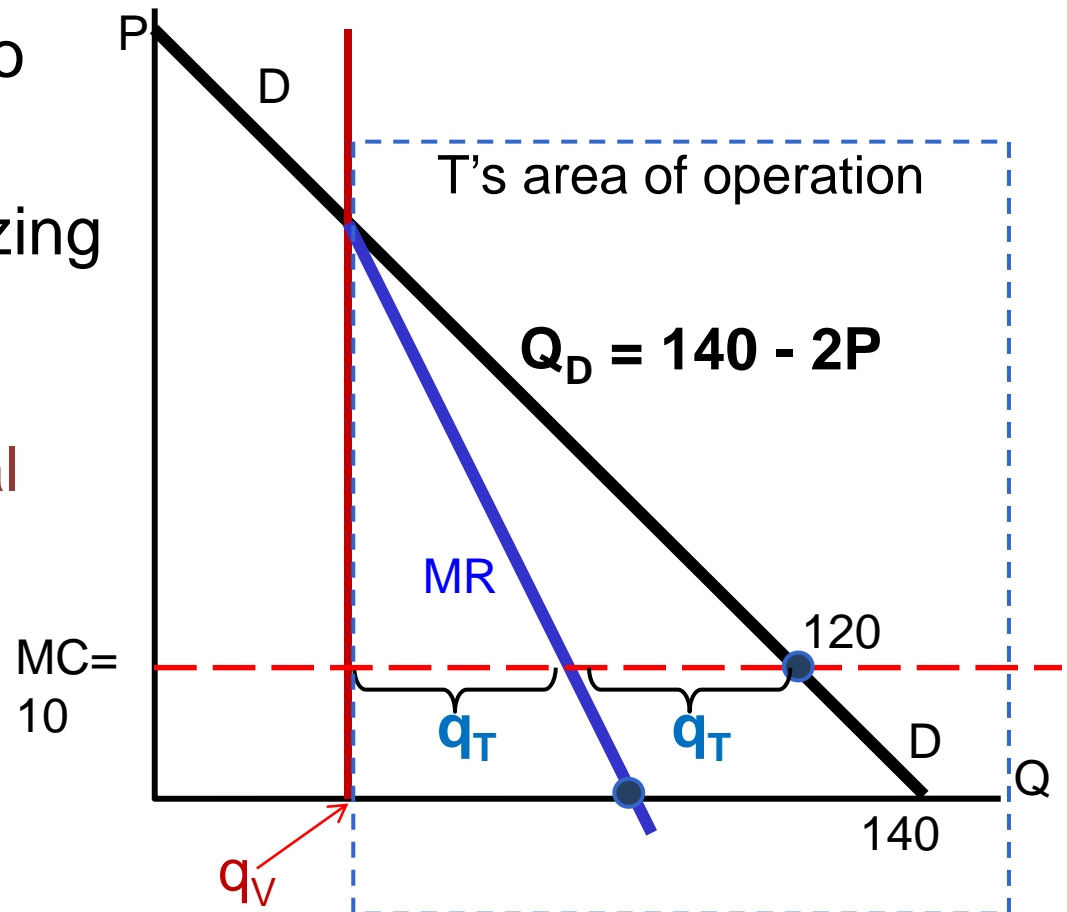


The Cournot Game

- Suppose V's strategy is to produce q_V .
- What is T's profit maximizing (best) response?
- T cannot control q_V , so his demand curve and marginal revenue curve begin at q_V .
- MR crosses MC halfway between q_V and 140, ...
- So T's best response is

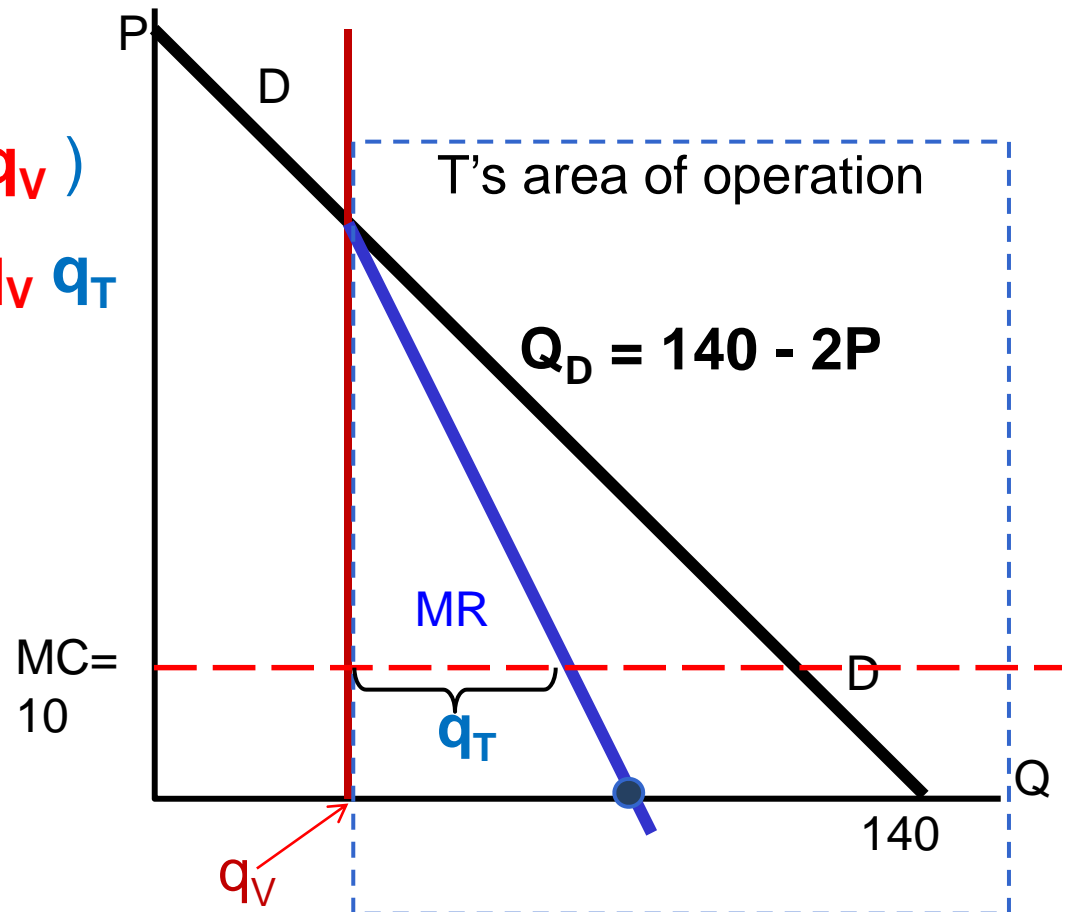
$$q_T^* = 60 - \frac{1}{2} q_V$$
- V's best response is

$$q_V^* = 60 - \frac{1}{2} q_T$$



A formal way in finding the best response function

- $TR_T = q_T \times P$
- $TR_T = q_T \times (70 - \frac{1}{2} q_T - \frac{1}{2} q_V)$
- $TR_T = 70 q_T - \frac{1}{2} q_T^2 - \frac{1}{2} q_V q_T$
- $MR_T = 70 - q_T - \frac{1}{2} q_V$
- At equilibrium: $MR_T = MC_T$
- $= 70 - q_T - \frac{1}{2} q_V = 10$
- So T's best response is
 $q_T^* = 60 - \frac{1}{2} q_V$
- V's best response is
 $q_V^* = 60 - \frac{1}{2} q_T$



Equilibrium of the Cournot Game

- How can we find the equilibrium?
- Solution concept: If $\langle q_T^*, q_V^* \rangle$ is an equilibrium, then q_T^* must be the best response to q_V^* and vice versa, which means
 - For T : given q_V^* , T chooses q_T^* to maximize its profit;
 - For V : given q_T^* , V chooses q_V^* to maximize its profit;
 - Both T and V have no intention to change, when they know what choice the other picks.

Equilibrium of the Cournot Game

- The best-response equations must be satisfied:

$$q_T^* = 60 - \frac{1}{2} q_V^*$$

$$q_V^* = 60 - \frac{1}{2} q_T^*$$

- Two equations with two unknown; by substitution:

$$q_T^* = 40 ; \quad q_V^* = 40 .$$

Is the Cournot Equilibrium efficient?

- The efficient level of output (Q^*) is 120.
- We know that total supply under Cournot Equilibrium is:

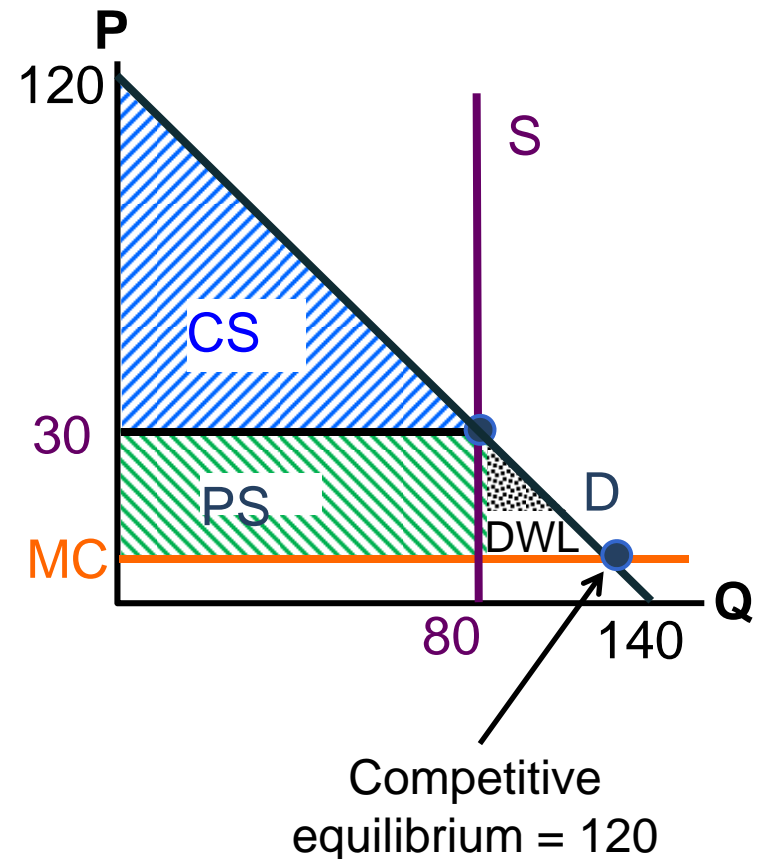
$$Q_s = q_T + q_V = 40 + 40 = 80$$

$$\text{So, } P = \frac{1}{2}(140 - Q_s) = 30$$

- We have

● PS, CS, and DW Loss

- **Cournot Equilibrium is NOT efficient!**



Efficiency with many Cournot Competitors

- If the market demand curve is a downward-sloping straight line, and MC is constant, then
 - A monopoly would produce $\frac{1}{2}$ of the efficient (competitive) level of output.
 - 2 Cournot competitors would produce a total of $\frac{2}{3}$ of the efficient level of output.
 - 3 Cournot competitors would produce a total of $\frac{3}{4}$ of the efficient level of output.
 - 99 Cournot competitors would produce a total of $\frac{99}{100}$ of the efficient level of output.
- **Conclusion:** A very large number of Cournot competitors behave like perfect competitors and are almost efficient.

WHY PEOPLE SOMETIMES COOPERATE

Why People sometimes cooperate

- In a prisoner dilemma game, players always have the incentive to cheat.
- “Non-cooperative” equilibrium is the result, which is not the “most beneficial” outcome.
- However, in the real world, we can observe that “cooperation” is possible, at least under some occasions, for example:
- **Repeated Game**: A game repeated itself many times.

Why People sometimes cooperate

- If you cheat in one period, you (may) gain
 - As in mobile service example, your profit increases from \$900 to \$1000, given the other player does not cheat.
- However, if you cheat this period, very likely the other player will “revenge” by “cheating”, which means a future loss due to “non-cooperative” equilibrium.
 - Cooperative equilibrium, profit = **\$900**; VS.
 - Non-cooperative equilibrium, profit = **\$800**

Why People sometimes cooperate

- You compare the gains and losses.
 - If the gain by cheating $<$ the lose in the future, you will decide not to cheat in the 1st period.
- In fact, a strategy “**Tit-for-tat**”
 - Play “cooperate” strategy first;
 - Once other parties cheat (cooperate), you will cheat (cooperate) next round.
- Tit-for-tat strategy encourages the cooperative equilibrium to appear, by playing cooperative (at first), AND “a tooth for a tooth” – Punishment !!!

Repeated Game

- **Tit-for-tat strategy:** One player cooperates this period if the other player cooperated in the previous period but cheats in the current period if the other player cheated in the previous period.
- A more severe punishment strategy is a **Trigger strategy:** One in which a player cooperates if the other player cooperates but plays the Nash equilibrium strategy forever thereafter if the other player cheats.

Repeated Game

TABLE 15.4 Cheating with Punishment

Period of play	Collude		Cheat with tit-for-tat	
	Trick's profit (millions of dollars)	Gear's profit (millions of dollars)	Trick's profit (millions of dollars)	Gear's profit (millions of dollars)
1	2	2	4.5	-1.0
2	2	2	-1.0	4.5
3	2	2	2.0	2.0
4	•	•	•	•

- There is gain from cheating.
- Punishment is big enough to out-weigh the gain from cheating, then incentive for cheating disappears.

SEQUENTIAL GAME

Sequential Game

- Previously, players make their choices simultaneously.
- Game can be “**sequential**”, a player will make her decision first and the other player then makes her decision.
- What would be the Nash Equilibrium, if the “Battle of the Sexes” game is in a “sequential” context?

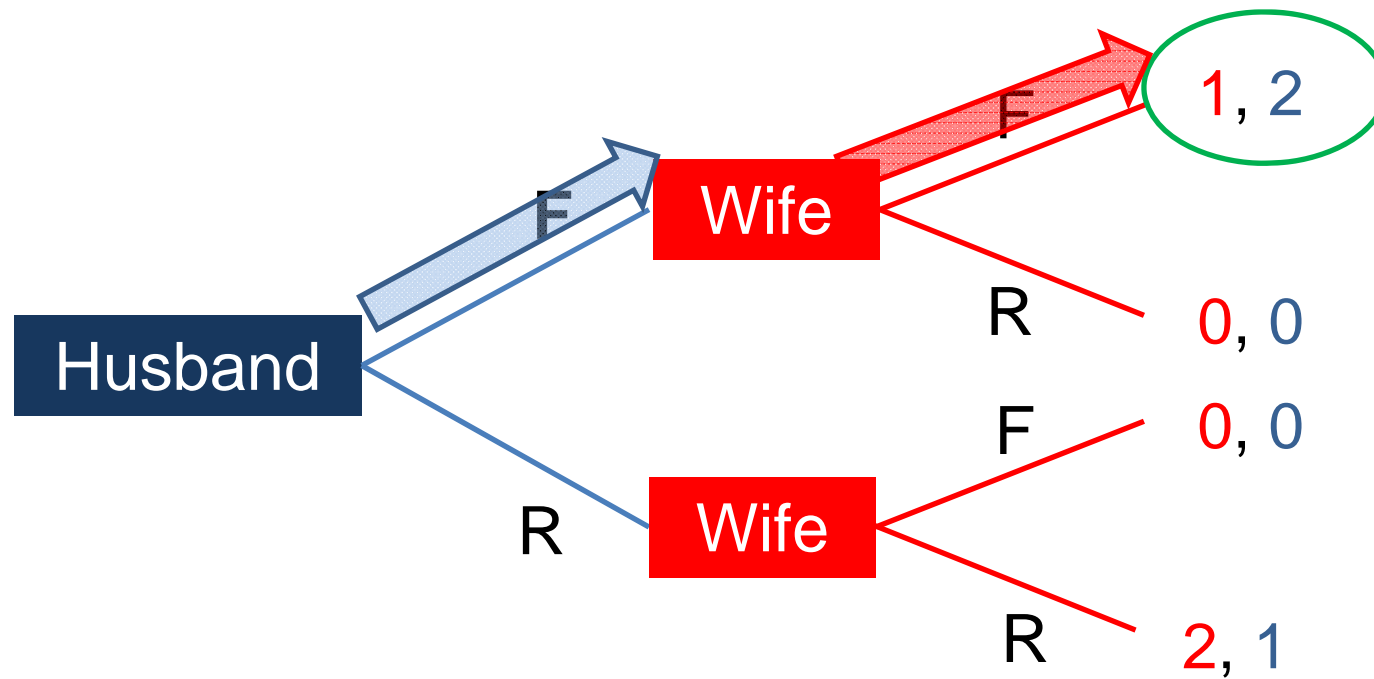
Nash Equilibrium – Battle of the Sexes

- If the husband makes his choice first,
- The wife observes husband's choice and then makes hers:
<F,F> would be the equilibrium, agree?
- “**First-Mover Advantage**”!

		Tony	
		F	R
Janet	F	1, 2	0, 0
	R	0, 0	2, 1

Sequential Game – Time Consistency

■ Game in “Extensive Form”



Sequential Game – Time Consistency

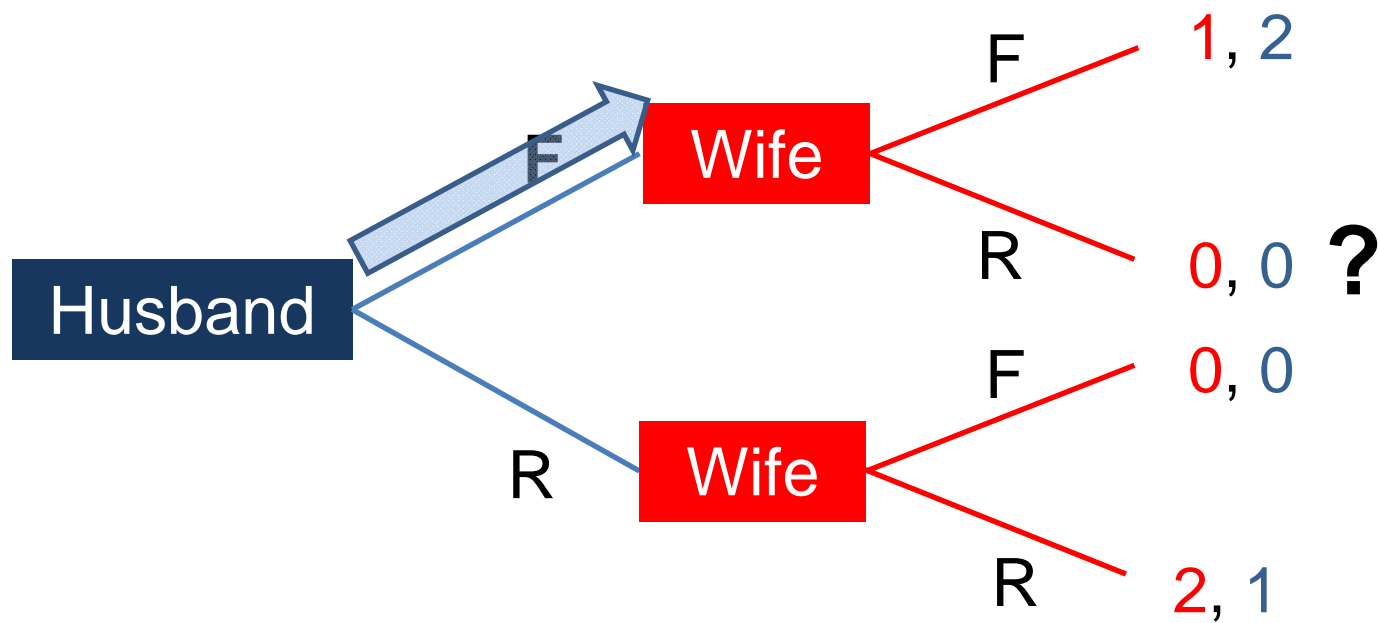
- However, wife may do the following:
- Telling Husband: **I will choose R no matter what you choose!!!**
- Purpose: To “force” husband into choosing **R** by announcing “Always R Strategy”.
- Will the wife succeed?

Sequential Game – Time Consistency

- Recall: A **strategy** is a plan of action that specifies what a player will do in **every circumstance** that a player can observe.
- Think of a strategy as a plan made at the beginning of the game.
- The strategy is “**time-consistent**” if the player is willing to follow her plan as the game progress.
- What is wife's strategy / Game Plan: Always R?

Sequential Game – Time Consistency

■ Game in “Extensive Form”



Sequential Game – Time Consistency

- Not likely: Because wife's "threat" is not **"time-consistent"**.
- If the husband chooses F, wife would not want to follow her "threat" (always R), and should switch to F.
- **EXAMPLE:** A existing firm threatens "I will reduce my price to a very low level (using price war as a response), if any competitor enters into the market" !!!
- A common strategy (announced) by existing firms in the market – credible???

Another Example

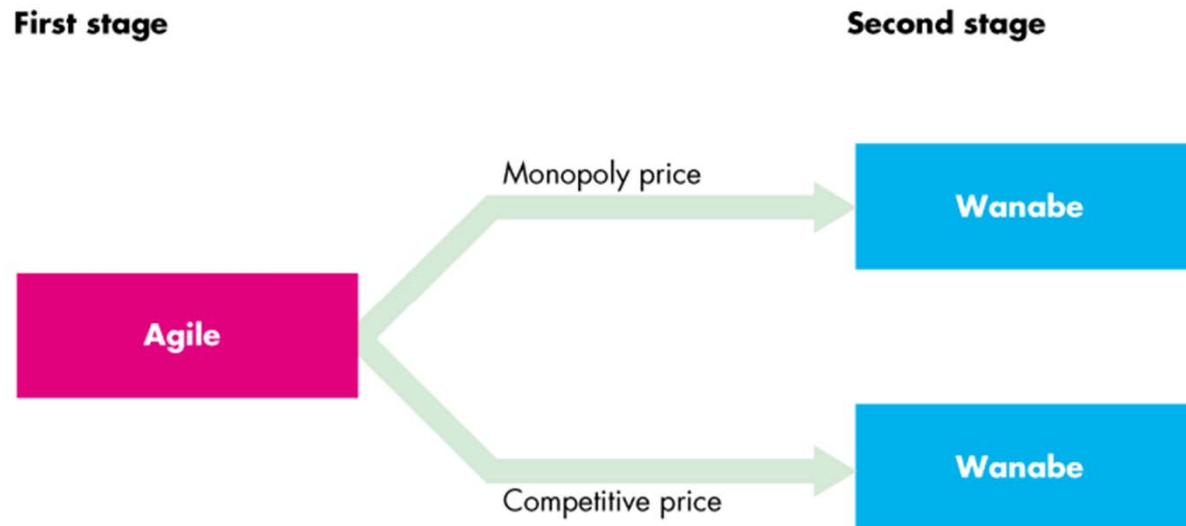
- North Korea's nuclear test in 2006
 - North Korea had to choose between “test” or “no test”.
 - US had to choose between “war/sanction” or “negotiate”.
- North Korea moved first and “test”ed.
- Too late for sanction, and US decided to negotiate.
- “Always war/sanction” may not be time-consistent (credible).
- First-mover advantage of North Korea

Sequential Game – Contestable Market

- A Sequential Entry Game in a Contestable Market
- In a contestable market—a market in which firms can enter and leave so easily that firms in the market **face competition from potential entrants**—firms play a sequential entry game.

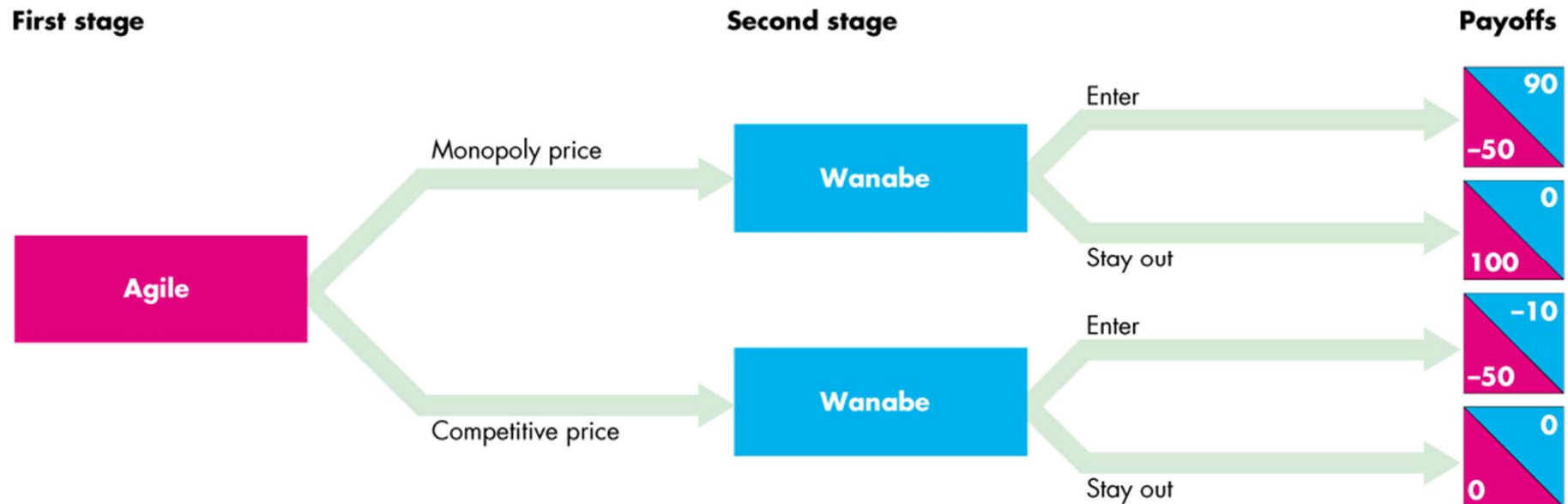
Sequential Game – Contestable Market

- In the first stage, Agile decides whether to set the monopoly price or the competitive price.



Sequential Game – Contestable Market

- In the second stage, Wanabe decides whether to enter or stay out.



Sequential Game – Contestable Market

- In the equilibrium of this entry game:
- Agile sets a competitive price and makes zero economic profit to keep Wanabe out.

Summary – Oligopoly

- Benefits from acting like “monopoly”,
- But, driven by self-interest, cooperation is not likely achievable and sustainable.
- Game Theory is a useful tool to analyze the behavior of oligopoly.
- Firms cooperate, cheat, threat, negotiate, all to maximize their own profit.
- Cheating is good from society’s perspective.
- Number of firms increases in oligopoly, closer to perfect competitive results (p and q).

Revisit: Nash-Equilibrium Concept

- In equilibrium, after finding out what the other players have done, each player is happy with the strategy that she chose.
- If there is regret, then the strategy profile is not an equilibrium.
- We can think about a Nash equilibrium like:
 - Each player chooses a best response to what she believes will be the strategies of the other players.
 - And her beliefs about the strategies of other players turn out to be correct.

Using Nash-Equilibrium to Predict

- A problem with the Nash-equilibrium concept is that the formation of beliefs about the strategies of other players is **NOT** explained.
- The best way to show that players will end up in a Nash equilibrium,...
- is to argue that players will adopt correct beliefs about the strategies of others.
- Unfortunately, it isn't always clear why those beliefs ought to be correct.

End for today 😊
Thank you very much
See you next time !