ECON3113 Microeconomic Theory I

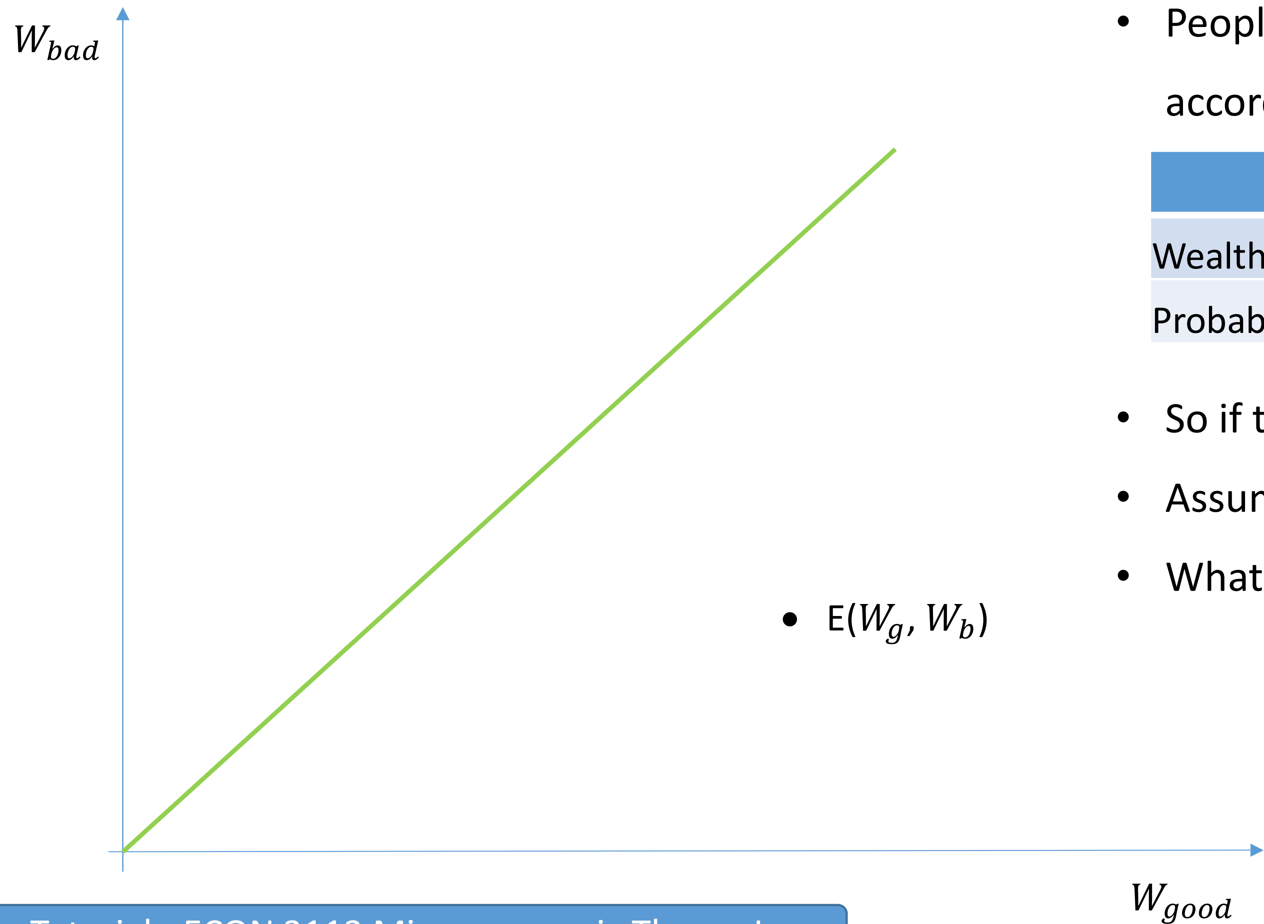
Tutorial #12

Insurance and Asset Investment

Today's tutorial

- The same model to understand:
  - Insurance
  - Asset Investment
- Assessment quiz #5

Insurance: the setting



- People have the same wealth that varies according to the state of the world

	Good state	Bad state
Wealth	$W_g = w$	$W_b = w - L$
Probability	1-p	p

- So if the bad state happens, wealth falls by L
- Assume people are all at point E
- What does the 45 degree line represent?

Insurance: expected utility

- Assume that insurance buyers maximise expected utility of wealth in each state
  - Expected utility $= p \times U(\text{wealth in bad state}) + (1-p) \times U(\text{wealth in good state})$
  - $= pU(W_b) + (1-p)U(W_g)$
  - Then for given expected utility $\bar{U}$, we have:
    - $\bar{U} = pU(W_b) + (1-p)U(W_g)$
- We can find the slope $\frac{dW_b}{dW_g} = -\frac{1-p}{p} . \frac{U\prime(W_g)}{U\prime(W_b)}$
- That is, the MRS
- Assume that insurance buyers are risk averse:
  - Concave expected utility function

Insurance: the proposition

- Insurance allows us to transfer wealth across states
- Suppose an insurance company offers us a deal:
  - Coverage q to repay q in the bad state, at a premium (paid in good and bad states) of πq
  - π is the price of insurance per unit of coverage
  - With insurance we then have:
    - $W_g = w - \pi q$
    - $W_b = w - \pi q - L + q$
- By choosing $q \in [0, L]$, we can achieve combinations of W_g, W_b such that the 'budget constraint' is satisfied:
  - $(1 - \pi)W_g + \pi W_b = w - \pi L$
- With boundary conditions $W_g \leq w$; $W_b \leq w - \pi L$

Insurance: solving the insurance buyer's optimization problem

- We can formulate the problem as follows:
  - $\max_{W_g \leq w; W_b \leq w - \pi L} (1-p)W_g + pW_b$ subject to the 'budget constraint' $(1-\pi)W_g + \pi W_b = w - \pi L$
- Assuming an interior solution, this has tangency condition:
  - MRS $=$ Price ratio
  - ie $-\frac{1-p}{p} . \frac{U'(W_g)}{U'(W_b)} = -\frac{1-\pi}{\pi}$

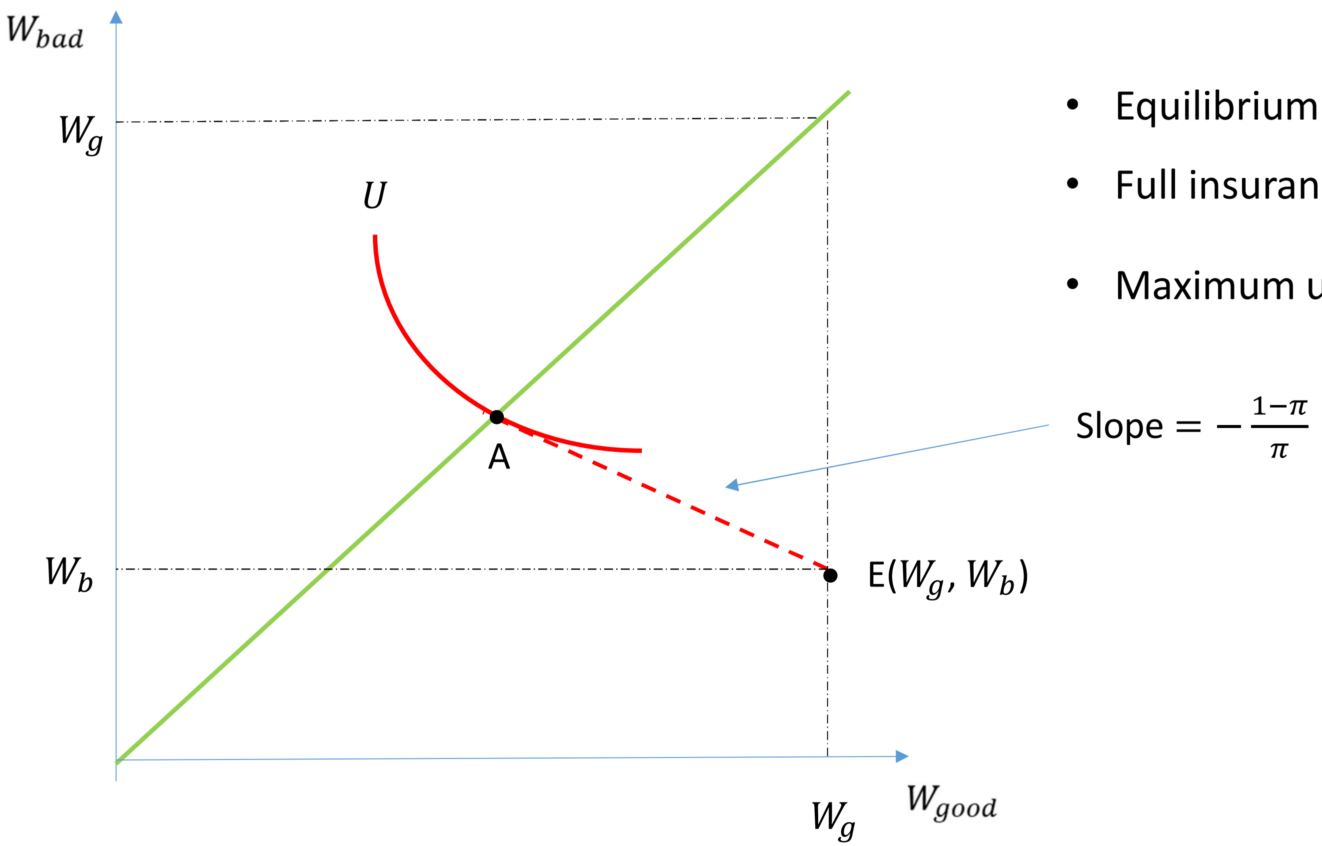
Insurance: solving the insurance buyer's optimization problem

- A key definition: Actuarially fair insurance
  - Insurance is actuarially fair when the premium is equal to the expected loss from the event being insured
  - In this case, expected loss $= pq$, and premium $= \pi q$
  - Therefore, actuarially fair insurance requires that $\pi = q$
- Note also that for an insurance company, for each insurance contract expected profit $= \pi q - pq$
- Therefore, actuarially fair insurance $\Rightarrow$ expected profits $= 0$
  - An outcome consistent with a perfectly competitive market for insurance

Insurance: solving the insurance buyer's optimization problem

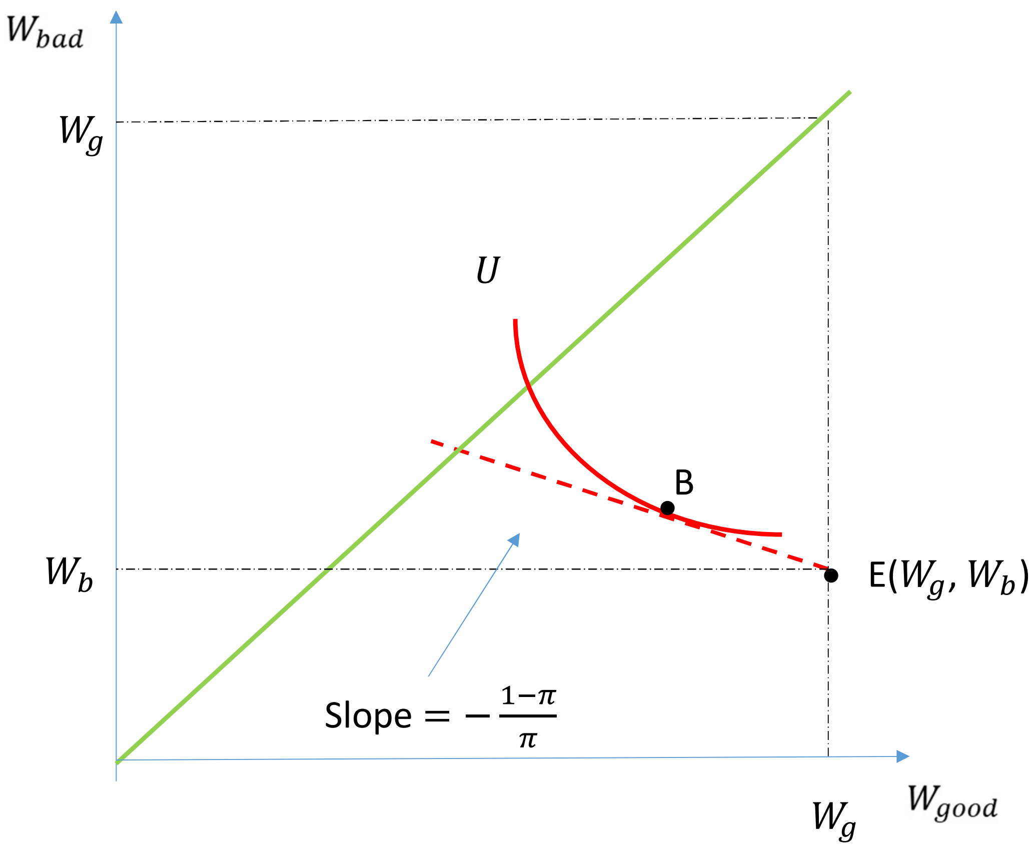
- If we assume a perfectly competitive insurance market, then we have expected profits $= 0$ and $\pi = p$
- We have tangency condition:
  - $-\frac{1-p}{p} . \frac{U'(W_g)}{U'(W_b)} = -\frac{1-\pi}{\pi}$
- With $\pi = p$, this becomes $U'\left(W_g\right) = U'(W_b)$
- Since we have assumed a concave expected utility function, it is the case that $W_g = W_b$ in this case
  - That is, the optimal choice in this case is full insurance (ie coverage is for the full amount of the expected loss)

Equilibrium with full information and risk averse buyers



- Equilibrium is at the point A
- Full insurance is chosen
- Maximum utility is achieved when MRS $= \frac{1-\pi}{\pi}$

The case with positive expected profits



- If the market for insurance is not perfectly competitive, then companies may make positive expected profits
  - $\pi q - pq > 0 \Rightarrow \pi > p$
- Now look at the tangency condition and the new 'price ratio':
  - $-\frac{1-p}{p} \cdot \frac{U'(W_g)}{U'(W_b)} = -\frac{1-\pi}{\pi}$
  - Since $\pi > p$, $\frac{1-\pi}{\pi} < \frac{1-p}{p} \Rightarrow U'(W_g) < U'(W_b)$
  - With U concave, this means $W_g > W_b$
  - That is, insurance buyers buy partial insurance

Investment: the setting

- Assume that we have starting wealth $\$w$ and we can invest it in as asset that costs $\$\pi$
- Each unit of the asset pays R in the good state of the world and zero in the bad state of the world
- The probability of the good state is $(1-p)$
- If we buy x units of the asset, then we have wealth:
  - Good state: $W_g = w - \pi x + Rx$
  - Bad state: $W_b = w - \pi x$
- And we can choose x to achieve combinations of W_g, W_b that satisfy:
  - $\pi\, W_g + (R-\pi)W_b = wR$

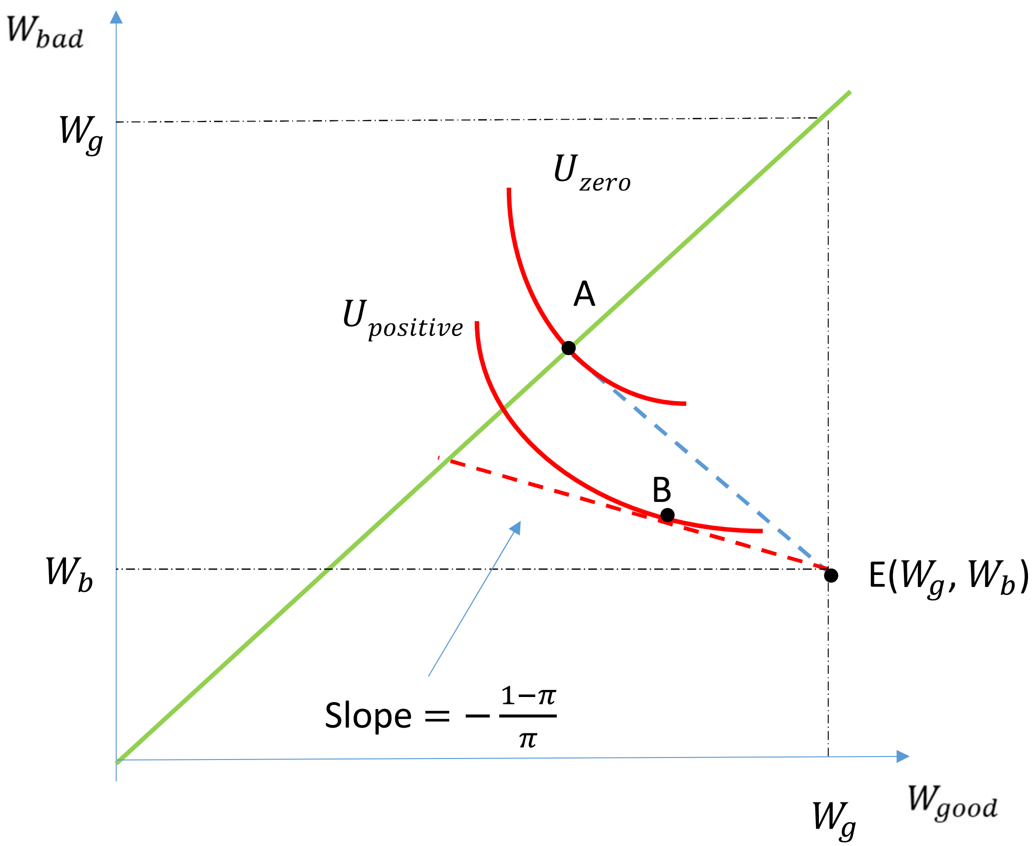
Investment: solving the investor's optimization problem

- We can formulate the problem as follows:
  - $\max_{W_g;W_b}(1-p)W_g + pW_b$ subject to the 'budget constraint' $\pi\, W_g + (R-\pi)W_b = wR$
- Assuming an interior solution, this has tangency condition:
  - MRS $=$ Price ratio
  - ie $\quad -\frac{1-p}{p}.\frac{U'(W_g)}{U'(W_b)} = -\frac{\pi}{R-\pi}$
- In this case, an actuarially fair price is defined as one at which price is given by expected pay-off
  - ie $\quad \pi = (1-\mathrm{p})\mathrm{R}$
- And then $\frac{1-p}{p} = \frac{\pi}{R-\pi}$, and so $U'\left(W_g\right) = U'(W_b)$
- If we assume that investors are risk averse, then $W_g = W_g$ and investors do not buy any of the asset

Investment: solving the investor's optimization problem

- If the asset has a positive expected value, then $\pi < (1 - \mathrm{p})\mathrm{R}$
- In this case $\frac{\pi}{R-\pi} < \frac{1-p}{p}$, and so $U'(W_g) < U'(W_b)$
- ie Given a concave expected utility function, $W_g > W_b$
- And so the investor buys a positive amount of the asset
- If the asset has a negative expected return, then the amount 'bought' is negative
  - The investor's optimal choice is to short sell the asset

Investment: solving the investor's optimization problem



- Summary:
  - With zero expected return, investors buy none of the asset
  - With positive expected return, investors buy a positive amount of the asset
  - With negative expected return, investors short sell the asset (not shown)