SUMMARY

This chapter provided a structured way to think about strategic situations. We focused on the most important solution concept used in game theory, Nash equilibrium. We then progressed to several more refined solution concepts that are in standard use in game theory in more complicated settings (with sequential moves and incomplete information). Some of the principal results are as follows.

- All games have the same basic components: players, strategies, payoffs, and an information structure.
- Games can be written down in normal form (providing a payoff matrix or payoff functions) or extensive form (providing a game tree).
- Strategies can be simple actions, more complicated plans contingent on others' actions, or even probability distributions over simple actions (mixed strategies).
- A Nash equilibrium is a set of strategies, one for each player, that are mutual best responses. In other words, a player's strategy in a Nash equilibrium is optimal given that all others play their equilibrium strategies.
- A Nash equilibrium always exists in finite games (in mixed if not pure strategies).
- Subgame-perfect equilibrium is a refinement of Nash equilibrium that helps to rule out equilibria in sequential games involving noncredible threats.
- Repeating a stage game a large number of times introduces the possibility of using punishment strategies to attain higher payoffs than if the stage game is played once. If players are sufficiently patient in an infinitely repeated game, then a folk theorem holds implying that essentially any payoffs are possible in the repeated game.
- In games of private information, one player knows more about his or her "type" than another. Players maximize their expected payoffs given knowledge of their own type and beliefs about the others'.
- In a perfect Bayesian equilibrium of a signaling game, the second mover uses Bayes' rule to update his or her beliefs about the first mover's type after observing the first mover's action.
- The frontier of game-theory research combines theory with experiments to determine whether players who may not be hyper-rational come to play a Nash equilibrium, which particular equilibrium (if there are more than one), and what path leads to the equilibrium.

PROBLEMS

8.1

Consider the following game:

Player 1 \ Player 2	*D*	*E*	*F*
A	7, 6	5, 8	0, 0
B	5, 8	7, 6	1, 1
C	0, 0	1, 1	4, 4

a. Find the pure-strategy Nash equilibria (if any).
b. Find the mixed-strategy Nash equilibrium in which each player randomizes over just the first two actions.
c. Compute players' expected payoffs in the equilibria found in parts (a) and (b).
d. Draw the extensive form for this game.

8.2

The mixed-strategy Nash equilibrium in the Battle of the Sexes in Figure 8.3 may depend on the numerical values for the payoffs. To generalize this solution, assume that the payoff matrix for the game is given by

Player 1 (Wife) \ Player 2 (Husband)	Ballet	Boxing
Ballet	K, 1	0, 0
Boxing	0, 0	1, K

where $K \geq 1$. Show how the mixed-strategy Nash equilibrium depends on the value of K.

8.3

The game of Chicken is played by two macho teens who speed toward each other on a single-lane road. The first to veer off is branded the chicken, whereas the one who does not veer gains peer-group esteem. Of course, if neither veers, both die in the resulting crash. Payoffs to the Chicken game are provided in the following table.

Teen 1 \ Teen 2	Veer	Does not veer
Veer	2, 2	1, 3
Does not veer	3, 1	0, 0

a. Draw the extensive form.

b. Find the pure-strategy Nash equilibrium or equilibria.

c. Compute the mixed-strategy Nash equilibrium. As part of your answer, draw the best-response function diagram for the mixed strategies.

d. Suppose the game is played sequentially, with teen 1 moving first and committing to this action by throwing away the steering wheel. What are teen 2's contingent strategies? Write down the normal and extensive forms for the sequential version of the game.

e. Using the normal form for the sequential version of the game, solve for the Nash equilibria.

f. Identify the proper subgames in the extensive form for the sequential version of the game. Use backward induction to solve for the subgame-perfect equilibrium. Explain why the other Nash equilibria of the sequential game are "unreasonable."

8.4

Two neighboring homeowners, $i = 1, 2$, simultaneously choose how many hours l_i to spend maintaining a beautiful lawn. The average benefit per hour is

$$10 - l_i + \frac{l_j}{2},$$

and the (opportunity) cost per hour for each is 4. Homeowner i's average benefit is increasing in the hours neighbor j spends on his own lawn because the appearance of one's property depends in part on the beauty of the surrounding neighborhood.

a. Compute the Nash equilibrium.

b. Graph the best-response functions and indicate the Nash equilibrium on the graph.

c. On the graph, show how the equilibrium would change if the intercept of one of the neighbor's average benefit functions fell from 10 to some smaller number.

8.5

The Academy Award–winning movie *A Beautiful Mind* about the life of John Nash dramatizes Nash's scholarly contribution in a single scene: His equilibrium concept dawns on him while in a bar bantering with his fellow male graduate students. They notice several women, one blond and the rest brunette, and agree that the blond is more desirable than the brunettes. The Nash character views the situation as a game among the male graduate students, along the following lines. Suppose there are n males who simultaneously approach either the blond or one of the brunettes. If male i alone approaches the blond, then he is successful in getting a date with her and earns payoff a. If one or more other males approach the blond along with i, the competition causes them all to lose her, and i (as well as the others who approached her) earns a payoff of zero. On the other hand, male i earns a payoff of $b > 0$ from approaching a brunette because there are more brunettes than males; therefore, i is certain to get a date with a brunette. The desirability of the blond implies $a > b$.

a. Argue that this game does not have a symmetric pure-strategy Nash equilibrium.

b. Solve for the symmetric mixed-strategy equilibrium. That is, letting p be the probability that a male approaches the blond, find p^*.

c. Show that the more males there are, the less likely it is in the equilibrium from part (b) that the blond is approached by at least one of them. *Note:* This paradoxical result was noted by S. Anderson and M. Engers in "Participation Games: Market Entry, Coordination, and the Beautiful Blond," *Journal of Economic Behavior & Organization* 63 (2007): 120–37.

8.6

The following game is a version of the Prisoners' Dilemma, but the payoffs are slightly different than in Figure 8.1.

Suspect 1 \ Suspect 2	Fink	Silent
Fink	0, 0	3, −1
Silent	−1, 3	1, 1

a. Verify that the Nash equilibrium is the usual one for the Prisoners' Dilemma and that both players have dominant strategies.

b. Suppose the stage game is repeated infinitely many times. Compute the discount factor required for their suspects to be able to cooperate on silent each period. Outline the trigger strategies you are considering for them.

8.7

Return to the game with two neighbors in Problem 8.4. Continue to suppose that player *i*'s average benefit per hour of work on landscaping is

$$10 - l_i + \frac{l_j}{2}.$$

Continue to suppose that player 2's opportunity cost of an hour of landscaping work is 4. Suppose that player 1's opportunity cost is either 3 or 5 with equal probability and that this cost is player 1's private information.

a. Solve for the Bayesian–Nash equilibrium.

b. Indicate the Bayesian–Nash equilibrium on a best-response function diagram.

c. Which type of player 1 would like to send a truthful signal to player 2 if it could? Which type would like to hide his or her private information?

8.8

In Blind Texan Poker, player 2 draws a card from a standard deck and places it against her forehead without looking at it but so player 1 can see it. Player 1 moves first, deciding whether to stay or fold. If player 1 folds, he must pay player 2 \$50. If player 1 stays, the action goes to player 2. Player 2 can fold or call. If player 2 folds, she must pay player 1 \$50. If player 2 calls, the card is examined. If it is a low card (2–8), player 2 pays player 1 \$100. If it is a high card (9, 10, jack, queen, king, or ace), player 1 pays player 2 \$100.

a. Draw the extensive form for the game.

b. Solve for the hybrid equilibrium.

c. Compute the players' expected payoffs.

Analytical Problems

8.9 Alternatives to Grim Strategy

Suppose that the Prisoners' Dilemma stage game (see Figure 8.1) is repeated for infinitely many periods.

a. Can players support the cooperative outcome by using *tit-for-tat* strategies, punishing deviation in a past period by reverting to the stage-game Nash equilibrium for just one period and then returning to cooperation? Are two periods of punishment enough?

b. Suppose players use strategies that punish deviation from cooperation by reverting to the stage-game Nash equilibrium for 10 periods before returning to cooperation. Compute the threshold discount factor above which cooperation is possible on the outcome that maximizes the joint payoffs.

8.10 Refinements of perfect Bayesian equilibrium

Recall the job-market signaling game in Example 8.9.

a. Find the conditions under which there is a pooling equilibrium where both types of worker choose not to obtain an education (*NE*) and where the firm offers an uneducated worker a job. Be sure to specify beliefs as well as strategies.

b. Find the conditions under which there is a pooling equilibrium where both types of worker choose not to obtain an education (*NE*) and where the firm does not offer an uneducated worker a job. What is the lowest posterior belief that the worker is low-skilled conditional on obtaining an education consistent with this pooling equilibrium? Why is it more natural to think that a low-skilled worker would never deviate to *E* and thus an educated worker must be high-skilled? Cho and Kreps's *intuitive criterion* is one of a series of complicated refinements of perfect Bayesian equilibrium that rule out equilibria based on unreasonable posterior beliefs as identified in this part; see I. K. Cho and D. M. Kreps, "Signalling Games and Stable Equilibria," *Quarterly Journal of Economics* 102 (1987): 179–221.

Behavioral Problems

8.11 Fairness in the Ultimatum Game

Consider a simple version of the Ultimatum Game discussed in the text. The first mover proposes a division of \$1. Let r be the share received by the other player in this proposal (so the first mover keeps $1 - r$), where $0 \leq r \leq 1/2$. Then the other player moves, responding by accepting or rejecting the proposal. If the responder accepts the proposal, the players are paid their shares; if the responder rejects it, both players receive nothing. Assume that if the responder is indifferent between accepting or rejecting a proposal, he or she accepts it.

a. Suppose that players only care about monetary payoffs. Verify that the outcome mentioned in the text in fact occurs in the unique subgame-perfect equilibrium of the Ultimatum Game.

b. Compare the outcome in the Ultimatum Game with the outcome in the Dictator Game (also discussed in the text), in which the proposer's surplus division is implemented regardless of whether the second mover accepts or rejects (so it is not much of a strategic game!).

c. Now suppose that players care about fairness as well as money. Following the article by Fehr and Schmidt cited in the text, suppose these preferences are represented by the utility function

$$U_1(x_1, x_2) = x_1 - a|x_1 - x_2|,$$

where x_1 is player 1's payoff and x_2 is player 2's (a symmetric function holds for player 2). The first term reflects the usual desire for more money. The second term reflects the desire for fairness, that the players' payoffs not be too unequal. The parameter a measures how intense the preference for fairness is relative to the desire for more money. Assume $a < 1/2$.

1. Solve for the responder's equilibrium strategy in the Ultimatum Game.
2. Taking into account how the second mover will respond, solve for the proposer's equilibrium strategy r^* in the Ultimatum Game. (*Hint*: r^* will be a corner solution, which depends on the value of a.)
3. Continuing with the fairness preferences, compare the outcome in the Ultimatum Game with that in the Dictator Game. Find cases that match the experimental results described in the text, in particular in which the split of the pot of money is more even in the Ultimatum Game than in the Dictator Game. Is there a limit to how even the split can be in the Ultimatum Game?

8.12 Rotten Kid Theorem

In *A Treatise on the Family* (Cambridge, MA: Harvard University Press, 1981), Nobel laureate Gary Becker proposes his famous Rotten Kid Theorem as a sequential game between the potentially rotten child (player 1) and the child's parent (player 2). The child moves first, choosing an action r that affects both his own income $Y_1(r)$ and the income of his parent $Y_2(r)$, where $Y_1'(r) > 0$ and $Y_2'(r) < 0$. Later, the parent moves, leaving a monetary bequest L to the child. The child cares only for his own utility, $U_1(Y_1 + L)$, but the parent maximizes $U_2(Y_2 - L) + \alpha U_1$, where $\alpha > 0$ reflects the parent's altruism toward the child. Prove that, in a subgame-perfect equilibrium, the child will opt for the value of r that maximizes $Y_1 + Y_2$ even though he has no altruistic intentions. *Hint:* Apply backward induction to the parent's problem first, which will give a first-order condition that implicitly determines L^*; although an explicit solution for L^* cannot be found, the derivative of L^* with respect to r—required in the child's first-stage optimization problem—can be found using the implicit function rule.

Suggestions for Further Reading

Fudenberg, D., and J. Tirole. *Game Theory*. Cambridge, MA: MIT Press, 1991.

A comprehensive survey of game theory at the graduate-student level, although selected sections are accessible to advanced undergraduates.

Holt, C. A. *Markets, Games, & Strategic Behavior*. Boston: Pearson, 2007.

An undergraduate text with emphasis on experimental games.

Rasmusen, E. *Games and Information*, 4th ed. Malden, MA: Blackwell, 2007.

An advanced undergraduate text with many real-world applications.

Watson, Joel. *Strategy: An Introduction to Game Theory*. New York: Norton, 2002.

An undergraduate text that balances rigor with simple examples (often 2 × 2 games). Emphasis on bargaining and contracting examples.

A plausible approach to modeling improvements in labor and capital separately is to assume that the production function is

$$q = A(e^{\varphi t}k)^{\alpha}(e^{\varepsilon t}l)^{1-\alpha}, \tag{9.64}$$

where φ represents the annual rate of improvement in capital input and ε represents the annual rate of improvement in labor input. But because of the exponential nature of the Cobb–Douglas function, this would be indistinguishable from our original example:

$$q = Ae^{[\alpha\varphi+(1-\alpha)\varepsilon]t}k^{\alpha}l^{1-\alpha} = Ae^{\theta t}k^{\alpha}l^{1-\alpha}, \tag{9.65}$$

where $\theta = \alpha\varphi + (1-\alpha)\varepsilon$. Hence to study technical progress in individual inputs, it is necessary either to adopt a more complex way of measuring inputs that allows for improving quality or (what amounts to the same thing) to use a multi-input production function.

QUERY: Actual studies of production using the Cobb–Douglas tend to find $\alpha \approx 0.3$. Use this finding together with Equation 9.65 to discuss the relative importance of improving capital and labor quality to the overall rate of technical progress.

Summary

In this chapter we illustrated the ways in which economists conceptualize the production process of turning inputs into outputs. The fundamental tool is the production function, which—in its simplest form—assumes that output per period (q) is a simple function of capital and labor inputs during that period, $q = f(k, l)$. Using this starting point, we developed several basic results for the theory of production.

- If all but one of the inputs are held constant, a relationship between the single-variable input and output can be derived. From this relationship, one can derive the marginal physical productivity (*MP*) of the input as the change in output resulting from a one-unit increase in the use of the input. The marginal physical productivity of an input is assumed to decrease as use of the input increases.
- The entire production function can be illustrated by its isoquant map. The (negative of the) slope of an isoquant is termed the *marginal rate of technical substitution* (*RTS*) because it shows how one input can be substituted for another while holding output constant. The *RTS* is the ratio of the marginal physical productivities of the two inputs.
- Isoquants are usually assumed to be convex—they obey the assumption of a diminishing *RTS*. This assumption cannot be derived exclusively from the assumption of diminishing marginal physical productivities. One must also be concerned with the effect of changes in one input on the marginal productivity of other inputs.
- The returns to scale exhibited by a production function record how output responds to proportionate increases in all inputs. If output increases proportionately with input use, there are constant returns to scale. If there are greater than proportionate increases in output, there are increasing returns to scale, whereas if there are less than proportionate increases in output, there are decreasing returns to scale.
- The elasticity of substitution (σ) provides a measure of how easy it is to substitute one input for another in production. A high σ implies nearly linear isoquants, whereas a low σ implies that isoquants are nearly L-shaped.
- Technical progress shifts the entire production function and its related isoquant map. Technical improvements may arise from the use of improved, more productive inputs or from better methods of economic organization.

Problems

9.1

Power Goat Lawn Company uses two sizes of mowers to cut lawns. The smaller mowers have a 22-inch deck. The larger ones combine two of the 22-inch decks in a single mower. For each size of mower, Power Goat has a different production function, given by the rows of the following table.

	Output per Hour (square feet)	Capital Input (# of 22″ mowers)	Labor Input
Small mowers	5,000	1	1
Large mowers	8,000	2	1

a. Graph the $q = 40{,}000$ square feet isoquant for the first production function. How much k and l would be used if these factors were combined without waste?

b. Answer part (a) for the second function.

c. How much k and l would be used without waste if half of the 40,000-square-foot lawn were cut by the method of the first production function and half by the method of the second? How much k and l would be used if one fourth of the lawn were cut by the first method and three fourths by the second? What does it mean to speak of fractions of k and l?

d. Based on your observations in part (c), draw a $q = 40{,}000$ isoquant for the combined production functions.

9.2

Suppose the production function for widgets is given by

$$q = kl - 0.8k^2 - 0.2l^2,$$

where q represents the annual quantity of widgets produced, k represents annual capital input, and l represents annual labor input.

a. Suppose $k = 10$; graph the total and average productivity of labor curves. At what level of labor input does this average productivity reach a maximum? How many widgets are produced at that point?

b. Again assuming that $k = 10$, graph the MP_l curve. At what level of labor input does $MP_l = 0$?

c. Suppose capital inputs were increased to $k = 20$. How would your answers to parts (a) and (b) change?

d. Does the widget production function exhibit constant, increasing, or decreasing returns to scale?

9.3

Sam Malone is considering renovating the bar stools at Cheers. The production function for new bar stools is given by

$$q = 0.1k^{0.2}l^{0.8},$$

where q is the number of bar stools produced during the renovation week, k represents the number of hours of bar stool lathes used during the week, and l represents the number of worker hours employed during the period. Sam would like to provide 10 new bar stools, and he has allocated a budget of $10,000 for the project.

a. Sam reasons that because bar stool lathes and skilled bar stool workers both cost the same amount ($50 per hour), he might as well hire these two inputs in equal amounts. If Sam proceeds in this way, how much of each input will he hire and how much will the renovation project cost?

b. Norm (who knows something about bar stools) argues that once again Sam has forgotten his microeconomics. He asserts that Sam should choose quantities of inputs so that their marginal (not average) productivities are equal. If Sam opts for this plan instead, how much of each input will he hire and how much will the renovation project cost?

c. On hearing that Norm's plan will save money, Cliff argues that Sam should put the savings into more bar stools to provide seating for more of his USPS colleagues. How many more bar stools can Sam get for his budget if he follows Cliff's plan?

d. Carla worries that Cliff's suggestion will just mean more work for her in delivering food to bar patrons. How might she convince Sam to stick to his original 10-bar stool plan?

9.4

Suppose that the production of crayons (q) is conducted at two locations and uses only labor as an input. The production function in location 1 is given by $q_1 = 10l_1^{0.5}$ and in location 2 by $q_2 = 50l_2^{0.5}$.

a. If a single firm produces crayons in both locations, then it will obviously want to get as large an output as possible given the labor input it uses. How should it allocate labor between the locations to do so? Explain precisely the relationship between l_1 and l_2.

b. Assuming that the firm operates in the efficient manner described in part (a), how does total output (q) depend on the total amount of labor hired (l)?

9.5

As we have seen in many places, the general Cobb–Douglas production function for two inputs is given by

$$q = f(k, l) = Ak^{\alpha}l^{\beta},$$

where $0 < \alpha < 1$ and $0 < \beta < 1$. For this production function:

a. Show that $f_k > 0, f_l > 0, f_{kk} < 0, f_{ll} < 0$, and $f_{kl} = f_{lk} > 0$.

b. Show that $e_{q,k} = \alpha$ and $e_{q,l} = \beta$.

c. In footnote 5, we defined the scale elasticity as

$$e_{q,t} = \frac{\partial f(tk, tl)}{\partial t} \cdot \frac{t}{f(tk, tl)},$$

where the expression is to be evaluated at $t = 1$. Show that, for this Cobb–Douglas function, $e_{q,t} = \alpha + \beta$. Hence in this case the scale elasticity and the returns to scale of the production function agree (for more on this concept see Problem 9.9).

d. Show that this function is quasi-concave.

e. Show that the function is concave for $\alpha + \beta \leq 1$ but not concave for $\alpha + \beta > 1$.

9.6

Suppose we are given the constant returns-to-scale CES production function

$$q = (k^\rho + l^\rho)^{1/\rho}.$$

a. Show that $MP_k = (q/k)^{1-\rho}$ and $MP_l = (q/l)^{1-\rho}$.

b. Show that $RTS = (k/l)^{1-\rho}$; use this to show that $\sigma = 1/(1-\rho)$.

c. Determine the output elasticities for k and l; and show that their sum equals 1.

d. Prove that

$$\frac{q}{l} = \left(\frac{\partial q}{\partial l}\right)^\sigma$$

and hence that

$$\ln\left(\frac{q}{l}\right) = \sigma \ln\left(\frac{\partial q}{\partial l}\right).$$

Note: The latter equality is useful in empirical work because we may approximate $\partial q/\partial l$ by the competitively determined wage rate. Hence σ can be estimated from a regression of $\ln(q/l)$ on $\ln w$.

9.7

Consider a generalization of the production function in Example 9.3:

$$q = \beta_0 + \beta_1\sqrt{kl} + \beta_2 k + \beta_3 l,$$

where

$$0 \leq \beta_i \leq 1, \quad i = 0, \ldots, 3.$$

a. If this function is to exhibit constant returns to scale, what restrictions should be placed on the parameters $\beta_0, \ldots, \beta_3$?

b. Show that, in the constant returns-to-scale case, this function exhibits diminishing marginal productivities and that the marginal productivity functions are homogeneous of degree 0.

c. Calculate σ in this case. Although σ is not in general constant, for what values of the β's does $\sigma = 0$, 1, or ∞?

9.8

Show that Euler's theorem implies that, for a constant returns-to-scale production function $q = f(k, l)$,

$$q = f_k k + f_l l.$$

Use this result to show that, for such a production function, if $MP_l > AP_l$ then MP_k must be negative. What does this imply about where production must take place? Can a firm ever produce at a point where AP_l is increasing?

Analytical Problems

9.9 Local returns to scale

A local measure of the returns to scale incorporated in a production function is given by the scale elasticity $e_{q,t} = \partial f(tk, tl)/\partial t \cdot t/q$ evaluated at $t = 1$.

a. Show that if the production function exhibits constant returns to scale, then $e_{q,t} = 1$.

b. We can define the output elasticities of the inputs k and l as

$$e_{q,k} = \frac{\partial f(k, l)}{\partial k} \cdot \frac{k}{q},$$

$$e_{q,l} = \frac{\partial f(k, l)}{\partial l} \cdot \frac{l}{q}.$$

Show that $e_{q,t} = e_{q,k} + e_{q,l}$.

c. A function that exhibits variable scale elasticity is

$$q = (1 + k^{-1}l^{-1})^{-1}.$$

Show that, for this function, $e_{q,t} > 1$ for $q < 0.5$ and that $e_{q,t} < 1$ for $q > 0.5$.

d. Explain your results from part (c) intuitively. *Hint:* Does q have an upper bound for this production function?

9.10 Returns to scale and substitution

Although much of our discussion of measuring the elasticity of substitution for various production functions has assumed constant returns to scale, often that assumption is not necessary. This problem illustrates some of these cases.

a. In footnote 6 we pointed out that, in the constant returns-to-scale case, the elasticity of substitution for a two-input production function is given by

$$\sigma = \frac{f_k f_l}{f \cdot f_{kl}}.$$

Suppose now that we define the homothetic production function F as

$$F(k, l) = [f(k, l)]^{\gamma},$$

where $f(k, l)$ is a constant returns-to-scale production function and γ is a positive exponent. Show that the elasticity of substitution for this production function is the same as the elasticity of substitution for the function f.

b. Show how this result can be applied to both the Cobb–Douglas and CES production functions.

9.11 More on Euler's theorem

Suppose that a production function $f(x_1, x_2, \ldots, x_n)$ is homogeneous of degree k. Euler's theorem shows that $\sum_i x_i f_i = kf$, and this fact can be used to show that the partial derivatives of f are homogeneous of degree $k - 1$.

a. Prove that $\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j f_{ij} = k(k-1)f$.

b. In the case of $n = 2$ and $k = 1$, what kind of restrictions does the result of part (a) impose on the second-order partial derivative f_{12}? How do your conclusions change when $k > 1$ or $k < 1$?

c. How would the results of part (b) be generalized to a production function with any number of inputs?

d. What are the implications of this problem for the parameters of the multivariable Cobb–Douglas production function $f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} x_i^{\alpha_i}$ for $\alpha_i \geq 0$?

Suggestions for Further Reading

Clark, J. M. "Diminishing Returns." In *Encyclopaedia of the Social Sciences*, vol. 5. New York: Crowell-Collier and Macmillan, 1931, pp. 144–46.

Lucid discussion of the historical development of the diminishing returns concept.

Douglas, P. H. "Are There Laws of Production?" *American Economic Review* 38 (March 1948): 1–41.

A nice methodological analysis of the uses and misuses of production functions.

Ferguson, C. E. *The Neoclassical Theory of Production and Distribution*. New York: Cambridge University Press, 1969.

A thorough discussion of production function theory (as of 1970). Good use of three-dimensional graphs.

Fuss, M., and D. McFadden. *Production Economics: A Dual Approach to Theory and Application*. Amsterdam: North-Holland, 1980.

An approach with a heavy emphasis on the use of duality.

Mas-Collell, A., M. D. Whinston, and J. R. Green. *Microeconomic Theory*. New York: Oxford University Press, 1995.

Chapter 5 provides a sophisticated, if somewhat spare, review of production theory. The use of the profit function (see Chapter 11) is sophisticated and illuminating.

Shephard, R. W. *Theory of Cost and Production Functions*. Princeton, NJ: Princeton University Press, 1978.

Extended analysis of the dual relationship between production and cost functions.

Silberberg, E., and W. Suen. *The Structure of Economics: A Mathematical Analysis*, 3rd ed. Boston: Irwin/McGraw-Hill, 2001.

Thorough analysis of the duality between production functions and cost curves. Provides a proof that the elasticity of substitution can be derived as shown in footnote 6 of this chapter.

Stigler, G. J. "The Division of Labor Is Limited by the Extent of the Market." *Journal of Political Economy* 59 (June 1951): 185–93.

Careful tracing of the evolution of Smith's ideas about economies of scale.

- All cost curves are drawn on the assumption that the input prices are held constant. When input prices change, cost curves will shift to new positions. The extent of the shifts will be determined by the overall importance of the input whose price has changed and by the ease with which the firm may substitute one input for another. Technical progress will also shift cost curves.
- Input demand functions can be derived from the firm's total cost function through partial differentiation. These input demand functions will depend on the quantity of output that the firm chooses to produce and are therefore called "contingent" demand functions.
- In the short run, the firm may not be able to vary some inputs. It can then alter its level of production only by changing its employment of variable inputs. In so doing, it may have to use nonoptimal, higher-cost input combinations than it would choose if it were possible to vary all inputs.

Problems

10.1

Suppose that a firm produces two different outputs, the quantities of which are represented by q_1 and q_2. In general, the firm's total costs can be represented by $C(q_1, q_2)$. This function exhibits economies of scope if $C(q_1, 0) + C(0, q_2) > C(q_1, q_2)$ for all output levels of either good.

a. Explain in words why this mathematical formulation implies that costs will be lower in this multiproduct firm than in two single-product firms producing each good separately.

b. If the two outputs are actually the same good, we can define total output as $q = q_1 + q_2$. Suppose that in this case average cost $(= C/q)$ decreases as q increases. Show that this firm also enjoys economies of scope under the definition provided here.

10.2

Professor Smith and Professor Jones are going to produce a new introductory textbook. As true scientists, they have laid out the production function for the book as

$$q = S^{1/2}J^{1/2},$$

where q is the number of pages in the finished book, S is the number of working hours spent by Smith, and J is the number of hours spent working by Jones.

After having spent 900 hours preparing the first draft, time which he valued at \$3 per working hour, Smith has to move on to other things and cannot contribute any more to the book. Jones, whose labor is valued at \$12 per working hour, will revise Smith's draft to complete the book.

a. How many hours will Jones have to spend to produce a finished book of 150 pages? Of 300 pages? Of 450 pages?

b. What is the marginal cost of the 150th page of the finished book? Of the 300th page? Of the 450th page?

10.3

Suppose that a firm's fixed proportion production function is given by

$$q = \min(5k, 10l).$$

a. Calculate the firm's long-run total, average, and marginal cost functions.

b. Suppose that k is fixed at 10 in the short run. Calculate the firm's short-run total, average, and marginal cost functions.

c. Suppose $v = 1$ and $w = 3$. Calculate this firm's long-run and short-run average and marginal cost curves.

10.4

A firm producing hockey sticks has a production function given by

$$q = 2\sqrt{kl}.$$

In the short run, the firm's amount of capital equipment is fixed at $k = 100$. The rental rate for k is $v = \$1$, and the wage rate for l is $w = \$4$.

a. Calculate the firm's short-run total cost curve. Calculate the short-run average cost curve.

b. What is the firm's short-run marginal cost function? What are the SC, SAC, and SMC for the firm if it produces 25 hockey sticks? Fifty hockey sticks? One hundred hockey sticks? Two hundred hockey sticks?

c. Graph the SAC and the SMC curves for the firm. Indicate the points found in part (b).

d. Where does the SMC curve intersect the SAC curve? Explain why the SMC curve will always intersect the SAC curve at its lowest point.

Suppose now that capital used for producing hockey sticks is fixed at k_1 in the short run.

e. Calculate the firm's total costs as a function of q, w, v, and k_1.

f. Given q, w, and v, how should the capital stock be chosen to minimize total cost?

g. Use your results from part (f) to calculate the long-run total cost of hockey stick production.

h. For $w = \$4$, $v = \$1$, graph the long-run total cost curve for hockey stick production. Show that this is an envelope for the short-run curves computed in part (e) by examining values of k_1 of 100, 200, and 400.

10.5

An enterprising entrepreneur purchases two factories to produce widgets. Each factory produces identical products, and each has a production function given by

$$q_i = \sqrt{k_i l_i} \quad i = 1, 2.$$

The factories differ, however, in the amount of capital equipment each has. In particular, factory 1 has $k_1 = 25$, whereas factory 2 has $k_2 = 100$. Rental rates for k and l are given by $w = v = \$1$.

a. If the entrepreneur wishes to minimize short-run total costs of widget production, how should output be allocated between the two factories?

b. Given that output is optimally allocated between the two factories, calculate the short-run total, average, and marginal cost curves. What is the marginal cost of the 100th widget? The 125th widget? The 200th widget?

c. How should the entrepreneur allocate widget production between the two factories in the long run? Calculate the long-run total, average, and marginal cost curves for widget production.

d. How would your answer to part (c) change if both factories exhibited diminishing returns to scale?

10.6

Suppose the total-cost function for a firm is given by

$$C = qw^{2/3}v^{1/3}.$$

a. Use Shephard's lemma to compute the (constant output) demand functions for inputs l and k.

b. Use your results from part (a) to calculate the underlying production function for q.

10.7

Suppose the total-cost function for a firm is given by

$$C = q(v + 2\sqrt{vw} + w).$$

a. Use Shephard's lemma to compute the (constant output) demand function for each input, k and l.

b. Use the results from part (a) to compute the underlying production function for q.

c. You can check the result by using results from Example 10.2 to show that the CES cost function with $\sigma = 0.5, \rho = -1$ generates this total-cost function.

10.8

In a famous article [J. Viner, "Cost Curves and Supply Curves," *Zeitschrift fur Nationalokonomie* 3 (September 1931): 23–46], Viner criticized his draftsman who could not draw a family of *SAC* curves whose points of tangency with the U-shaped *AC* curve were also the minimum points on each *SAC* curve. The draftsman protested that such a drawing was impossible to construct. Whom would you support in this debate?

Analytical Problems

10.9 Generalizing the CES cost function

The CES production function can be generalized to permit weighting of the inputs. In the two-input case, this function is

$$q = f(k, l) = [(\alpha k)^\rho + (\beta l)^\rho]^{\gamma/\rho}.$$

a. What is the total-cost function for a firm with this production function? *Hint:* You can, of course, work this out from scratch; easier perhaps is to use the results from Example 10.2 and reason that the price for a unit of capital input in this production function is v/α and for a unit of labor input is w/β.

b. If $\gamma = 1$ and $\alpha + \beta = 1$, it can be shown that this production function converges to the Cobb—Douglas form $q = k^\alpha l^\beta$ as $\rho \to 0$. What is the total cost function for this particular version of the CES function?

c. The relative labor cost share for a two-input production function is given by wl/vk. Show that this share is constant for the Cobb–Douglas function in part (b). How is the relative labor share affected by the parameters α and b?

d. Calculate the relative labor cost share for the general CES function introduced above. How is that share affected by changes in w/v? How is the direction of this effect determined by the elasticity of substitution, σ? How is it affected by the sizes of the parameters α and β?

10.10 Input demand elasticities

The own-price elasticities of contingent input demand for labor and capital are defined as

$$e_{l^c,w} = \frac{\partial l^c}{\partial w} \cdot \frac{w}{l^c},$$

$$e_{k^c,v} = \frac{\partial k^c}{\partial v} \cdot \frac{v}{k^c}.$$

a. Calculate $e_{l^c,w}$ and $e_{k^c,v}$ for each of the cost functions shown in Example 10.2.

b. Show that, in general, $e_{l^c,w} + e_{k^c,v} = 0$.

c. Show that the cross-price derivatives of contingent demand functions are equal—that is, show that $\partial l^c/\partial v = \partial k^c/\partial w$. Use this fact to show that $s_l e_{l^c,v} = s_k e_{k^c,w}$ where s_l, s_k are, respectively, the share of labor in total cost (wl/C) and of capital in total cost (vk/C).

d. Use the results from parts (b) and (c) to show that $s_l e_{l^c,w} + s_k e_{k^c,w} = 0$.

e. Interpret these various elasticity relationships in words and discuss their overall relevance to a general theory of input demand.

10.11 The elasticity of substitution and input demand elasticities

The definition of the (Morishima) elasticity of substitution s_{ij} in Equation 10.54 can be recast in terms of input demand elasticities. This illustrates the basic asymmetry in the definition.

a. Show that if only w_j changes, $s_{ij} = e_{x_i^c, w_j} - e_{x_j^c, w_j}$.

b. Show that if only w_i changes, $s_{ji} = e_{x_j^c, w_i} - e_{x_i^c, w_i}$.

c. Show that if the production function takes the general CES form $q = \left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\gamma/\rho}$ for $\rho \neq 0$, then all of the Morishima elasticities are the same: $s_{ij} = 1/(1-\rho) = \sigma$. This is the only case in which the Morishima definition is symmetric.

10.12 The Allen elasticity of substitution

Many empirical studies of costs report an alternative definition of the elasticity of substitution between inputs. This alternative definition was first proposed by R. G. D. Allen in the 1930s and further clarified by H. Uzawa in the 1960s. This definition builds directly on the production function-based elasticity of substitution defined in footnote 6 of Chapter 9: $A_{ij} = C_{ij}C/C_iC_j$, where the subscripts indicate partial differentiation with respect to various input prices. Clearly, the Allen definition is symmetric.

a. Show that $A_{ij} = e_{x_i^c, w_j}/s_j$, where s_j is the share of input j in total cost.

b. Show that the elasticity of s_i with respect to the price of input j is related to the Allen elasticity by $e_{s_i, p_j} = s_j(A_{ij} - 1)$.

c. Show that, with only two inputs, $A_{kl} = 1$ for the Cobb–Douglas case and $A_{kl} = \sigma$ for the CES case.

d. Read Blackorby and Russell (1989: "Will the Real Elasticity of Substitution Please Stand Up?") to see why the Morishima definition is preferred for most purposes.

Suggestions for Further Reading

Allen, R. G. D. *Mathematical Analysis for Economists.* New York: St. Martin's Press, 1938, various pages—see index.

Complete (though dated) mathematical analysis of substitution possibilities and cost functions. Notation somewhat difficult.

Blackorby, C., and R. R. Russell. "Will the Real Elasticity of Substitution Please Stand Up? (A Comparison of the Allen/Uzawa and Morishima Elasticities)." *American Economic Review* (September 1989): 882–88.

A nice clarification of the proper way to measure substitutability among many inputs in production. Argues that the Allen/Uzawa definition is largely useless and that the Morishima definition is by far the best.

Ferguson, C. E. *The Neoclassical Theory of Production and Distribution.* Cambridge: Cambridge University Press, 1969, Chap. 6.

Nice development of cost curves; especially strong on graphic analysis.

Fuss, M., and D. McFadden. *Production Economics: A Dual Approach to Theory and Applications*. Amsterdam: North-Holland, 1978.

Difficult and quite complete treatment of the dual relationship between production and cost functions. Some discussion of empirical issues.

Knight, H. H. "Cost of Production and Price over Long and Short Periods." *Journal of Political Economics* 29 (April 1921): 304–35.

Classic treatment of the short-run, long-run distinction.

Silberberg, E., and W. Suen. *The Structure of Economics: A Mathematical Analysis,* 3rd ed. Boston: Irwin/McGraw-Hill, 2001.

Chapters 7–9 have a great deal of material on cost functions. Especially recommended are the authors' discussions of "reciprocity effects" and their treatment of the short-run long, run distinction as an application of the Le Chatelier principle from physics.

Sydsaeter, K., A. Strom, and P. Berck. *Economists' Mathematical Manual*, 3rd ed. Berlin: Springer-Verlag, 2000.

Chapter 25 provides a succinct summary of the mathematical concepts in this chapter. A nice summary of many input cost functions, but beware of typos.

We can decompose the fall in labor hiring from $l = 50$ to $l = 14.8$ into substitution and output effects by using the contingent demand function. If the firm had continued to produce $q = 40$ even though the wage rose, Equation 11.56 shows that it would have used $l = 33.33$. Capital input would have increased to $k = 300$. Because we are holding output constant at its initial level of $q = 40$, these changes represent the firm's substitution effects in response to the higher wage.

The decline in output needed to restore profit maximization causes the firm to cut back on its output. In doing so it substantially reduces its use of both inputs. Notice in particular that, in this example, the rise in the wage not only caused labor usage to decline sharply but also caused capital usage to fall because of the large output effect.

QUERY: How would the calculations in this problem be affected if all firms had experienced the rise in wages? Would the decline in labor (and capital) demand be greater or smaller than found here?

Summary

In this chapter we studied the supply decision of a profit-maximizing firm. Our general goal was to show how such a firm responds to price signals from the marketplace. In addressing that question, we developed a number of analytical results.

- To maximize profits, the firm should choose to produce that output level for which marginal revenue (the revenue from selling one more unit) is equal to marginal cost (the cost of producing one more unit).
- If a firm is a price-taker, then its output decisions do not affect the price of its output; thus, marginal revenue is given by this price. If the firm faces a downward-sloping demand for its output, however, then it can sell more only at a lower price. In this case marginal revenue will be less than price and may even be negative.
- Marginal revenue and the price elasticity of demand are related by the formula

$$MR = P\left(1 + \frac{1}{e_{q,p}}\right),$$

  where P is the market price of the firm's output and $e_{q,p}$ is the price elasticity of demand for its product.
- The supply curve for a price-taking, profit-maximizing firm is given by the positively sloped portion of its marginal cost curve above the point of minimum average variable cost (AVC). If price falls below minimum AVC, the firm's profit-maximizing choice is to shut down and produce nothing.
- The firm's reactions to changes in the various prices it faces can be studied through use of its profit function, $\Pi(P, v, w)$. That function shows the maximum profits that the firm can achieve given the price for its output, the prices of its input, and its production technology. The profit function yields particularly useful envelope results. Differentiation with respect to market price yields the supply function, whereas differentiation with respect to any input price yields (the negative of) the demand function for that input.
- Short-run changes in market price result in changes to the firm's short-run profitability. These can be measured graphically by changes in the size of producer surplus. The profit function can also be used to calculate changes in producer surplus.
- Profit maximization provides a theory of the firm's derived demand for inputs. The firm will hire any input up to the point at which its marginal revenue product is just equal to its per-unit market price. Increases in the price of an input will induce substitution and output effects that cause the firm to reduce hiring of that input.

Problems

11.1

John's Lawn Mowing Service is a small business that acts as a price-taker (i.e., $MR = P$). The prevailing market price of lawn mowing is \$20 per acre. John's costs are given by

$$\text{total cost} = 0.1q^2 + 10q + 50,$$

where $q =$ the number of acres John chooses to cut a day.

a. How many acres should John choose to cut to maximize profit?

b. Calculate John's maximum daily profit.

c. Graph these results, and label John's supply curve.

11.2

Universal Widget produces high-quality widgets at its plant in Gulch, Nevada, for sale throughout the world. The cost function for total widget production (q) is given by

$$\text{total cost} = 0.25q^2.$$

Widgets are demanded only in Australia (where the demand curve is given by $q_A = 100 - 2P_A$) and Lapland (where the demand curve is given by $q_L = 100 - 4P_L$); thus, total demand equals $q = q_A + q_L$. If Universal Widget can control the quantities supplied to each market, how many should it sell in each location to maximize total profits? What price will be charged in each location?

11.3

The production function for a firm in the business of calculator assembly is given by

$$q = 2\sqrt{l},$$

where q denotes finished calculator output and l denotes hours of labor input. The firm is a price-taker both for calculators (which sell for P) and for workers (which can be hired at a wage rate of w per hour).

a. What is the total cost function for this firm?

b. What is the profit function for this firm?

c. What is the supply function for assembled calculators [$q(P, w)$]?

d. What is this firm's demand for labor function [$l(P, w)$]?

e. Describe intuitively why these functions have the form they do.

11.4

The market for high-quality caviar is dependent on the weather. If the weather is good, there are many fancy parties and caviar sells for \$30 per pound. In bad weather it sells for only \$20 per pound. Caviar produced one week will not keep until the next week. A small caviar producer has a cost function given by

$$C = 0.5q^2 + 5q + 100,$$

where q is the weekly caviar production. Production decisions must be made before the weather (and the price of caviar) is known, but it is known that good weather and bad weather each occur with a probability of 0.5.

a. How much caviar should this firm produce if it wishes to maximize the expected value of its profits?

b. Suppose the owner of this firm has a utility function of the form

$$\text{utility} = \sqrt{\pi},$$

where π is weekly profits. What is the expected utility associated with the output strategy defined in part (a)?

c. Can this firm owner obtain a higher utility of profits by producing some output other than that specified in parts (a) and (b)? Explain.

d. Suppose this firm could predict next week's price but could not influence that price. What strategy would maximize expected profits in this case? What would expected profits be?

11.5

The Acme Heavy Equipment School teaches students how to drive construction machinery. The number of students that the school can educate per week is given by $q = 10\min(k, l)^r$, where k is the number of backhoes the firm rents per week, l is the number of instructors hired each week, and γ is a parameter indicating the returns to scale in this production function.

a. Explain why development of a profit-maximizing model here requires $0 < \gamma < 1$.

b. Supposing $\gamma = 0.5$, calculate the firm's total cost function and profit function.

c. If $v = 1000$, $w = 500$, and $P = 600$, how many students will Acme serve and what are its profits?

d. If the price students are willing to pay rises to $P = 900$, how much will profits change?

e. Graph Acme's supply curve for student slots, and show that the increase in profits calculated in part (d) can be plotted on that graph.

11.6

Would a lump-sum profits tax affect the profit-maximizing quantity of output? How about a proportional tax on profits? How about a tax assessed on each unit of output? How about a tax on labor input?

11.7

This problem concerns the relationship between demand and marginal revenue curves for a few functional forms.

a. Show that, for a linear demand curve, the marginal revenue curve bisects the distance between the vertical axis and the demand curve for any price.

b. Show that, for any linear demand curve, the vertical distance between the demand and marginal revenue curves is $-1/b \cdot q$, where $b(<0)$ is the slope of the demand curve.

c. Show that, for a constant elasticity demand curve of the form $q = aP^b$, the vertical distance between the demand and marginal revenue curves is a constant ratio of the height of the demand curve, with this constant depending on the price elasticity of demand.

d. Show that, for any downward-sloping demand curve, the vertical distance between the demand and marginal revenue curves at any point can be found by using a linear approximation to the demand curve at that point and applying the procedure described in part (b).

e. Graph the results of parts (a)–(d) of this problem.

11.8

How would you expect an increase in output price, P, to affect the demand for capital and labor inputs?

a. Explain graphically why, if neither input is inferior, it seems clear that a rise in P must not reduce the demand for either factor.

b. Show that the graphical presumption from part (a) is demonstrated by the input demand functions that can be derived in the Cobb–Douglas case.

c. Use the profit function to show how the presence of inferior inputs would lead to ambiguity in the effect of P on input demand.

Analytical Problems

11.9 A CES profit function

With a CES production function of the form $q = (k^\rho + l^\rho)^{\gamma/\rho}$ a whole lot of algebra is needed to compute the profit function as $\Pi(P, v, w) = KP^{1/(1-\gamma)}(v^{1-\sigma} + w^{1-\sigma})^{\gamma/(1-\sigma)(\gamma-1)}$, where $\sigma = 1/(1-\rho)$ and K is a constant.

a. If you are a glutton for punishment (or if your instructor is), prove that the profit function takes this form. Perhaps the easiest way to do so is to start from the CES cost function in Example 10.2.

b. Explain why this profit function provides a reasonable representation of a firm's behavior only for $0 < \gamma < 1$.

c. Explain the role of the elasticity of substitution (σ) in this profit function.

d. What is the supply function in this case? How does σ determine the extent to which that function shifts when input prices change?

e. Derive the input demand functions in this case. How are these functions affected by the size of σ?

11.10 Some envelope results

Young's theorem can be used in combination with the envelope results in this chapter to derive some useful results.

a. Show that $\partial l(P, v, w)/\partial v = \partial k(P, v, w)/\partial w$. Interpret this result using substitution and output effects.

b. Use the result from part (a) to show how a unit tax on labor would be expected to affect capital input.

c. Show that $\partial q/\partial w = -\partial l/\partial P$. Interpret this result.

d. Use the result from part (c) to discuss how a unit tax on labor input would affect quantity supplied.

11.11 Le Châtelier's Principle

Because firms have greater flexibility in the long run, their reactions to price changes may be greater in the long run than in the short run. Paul Samuelson was perhaps the first economist to recognize that such reactions were analogous to a principle from physical chemistry termed the *Le Châtelier's Principle*. The basic idea of the principle is that any disturbance to an equilibrium (such as that caused by a price change) will not only have a direct effect but may also set off feedback effects that enhance the response. In this problem we look at a few examples. Consider a price-taking firm that chooses its inputs to maximize a profit function of the form $\Pi(P, v, w) = Pf(k, l) - wl - vk$. This maximization process will yield optimal solutions of the general form $q^*(P, v, w)$, $l^*(P, v, w)$, and $k^*(P, v, w)$. If we constrain capital input to be fixed at $\bar{k}$ in the short run, this firm's short-run responses can be represented by $q^s(P, w, \bar{k})$ and $l^s(P, w, \bar{k})$.

a. Using the definitional relation $q^*(P, v, w) = q^s(P, w, k^*(P, v, w))$, show that

$$\frac{\partial q^*}{\partial P} = \frac{\partial q^s}{\partial P} + \frac{-\left(\dfrac{\partial k^*}{\partial P}\right)^2}{\dfrac{\partial k^*}{\partial v}}.$$

Do this in three steps. First, differentiate the definitional relation with respect to P using the chain rule. Next, differentiate the definitional relation with respect to v (again using the chain rule), and use the result to substitute for $\partial q^s/\partial k$ in the initial derivative. Finally, substitute a result analogous to part (c) of Problem 11.10 to give the displayed equation.

b. Use the result from part (a) to argue that $\partial q^*/\partial P \geq \partial q^s/\partial P$. This establishes Le Châtelier's Principle for supply: Long-run supply responses are larger than (constrained) short-run supply responses.

c. Using similar methods as in parts (a) and (b), prove that Le Châtelier's Principle applies to the effect of the wage on labor demand. That is, starting from the definitional relation $l^*(P, v, w) = l^s(P, w, k^*(P, v, w))$, show that $\partial l^*/\partial w \leq \partial l^s/\partial w$, implying that long-run labor demand falls more when wage goes up than short-run labor demand (note that both of these derivatives are negative).

d. Develop your own analysis of the difference between the short- and long-run responses of the firm's cost function $[C(v, w, q)]$ to a change in the wage (w).

11.12 More on the derived demand with two inputs

The demand for any input depends ultimately on the demand for the goods that input produces. This can be shown most explicitly by deriving an entire industry's demand for inputs. To do so, we assume that an industry produces a homogeneous good, Q, under constant returns to scale using only capital and labor. The demand function for Q is given by $Q = D(P)$, where P is the market price of the good being produced. Because of the constant returns-to-scale assumption, $P = MC = AC$. Throughout this problem let $C(v, w, 1)$ be the firm's unit cost function.

a. Explain why the total industry demands for capital and labor are given by $k = QC_v$ and $l = QC_w$.

b. Show that

$$\frac{\partial k}{\partial v} = QC_{vv} + D'C_v^2 \quad \text{and} \quad \frac{\partial l}{\partial w} = QC_{ww} + D'C_w^2.$$

c. Prove that

$$C_{vv} = \frac{-w}{v}C_{vw} \quad \text{and} \quad C_{ww} = \frac{-v}{w}C_{vw}.$$

d. Use the results from parts (b) and (c) together with the elasticity of substitution defined $\sigma = CC_{vw}/C_vC_w$ to show that

$$\frac{\partial k}{\partial v} = \frac{wl}{Q}\cdot\frac{\sigma k}{vC} + \frac{D'k^2}{Q^2} \quad \text{and} \quad \frac{\partial l}{\partial w} = \frac{vk}{Q}\cdot\frac{\sigma l}{wC} + \frac{D'l^2}{Q^2}.$$

e. Convert the derivatives in part (d) into elasticities to show that

$$e_{k,v} = -s_l\sigma + s_k e_{Q,P} \quad \text{and} \quad e_{l,w} = -s_k\sigma + s_l e_{Q,P},$$

where $e_{Q,P}$ is the price elasticity of demand for the product being produced.

f. Discuss the importance of the results in part (e) using the notions of substitution and output effects from Chapter 11.

Note: The notion that the elasticity of the derived demand for an input depends on the price elasticity of demand for the output being produced was first suggested by Alfred Marshall. The proof given here follows that in D. Hamermesh, *Labor Demand* (Princeton, NJ: Princeton University Press, 1993).

11.13 Cross-price effects in input demand

With two inputs, cross-price effects on input demand can be easily calculated using the procedure outlined in Problem 11.12.

a. Use steps (b), (d), and (e) from Problem 11.12 to show that

$$e_{k,w} = s_l(\sigma + e_{Q,P}) \quad \text{and} \quad e_{l,v} = s_k(\sigma + e_{Q,P}).$$

b. Describe intuitively why input shares appear somewhat differently in the demand elasticities in part (e) of Problem 11.12 than they do in part (a) of this problem.

c. The expression computed in part (a) can be easily generalized to the many-input case as $e_{x_i,w_j} = s_j(A_{ij} + e_{Q,P})$, where A_{ij} is the Allen elasticity of substitution defined in Problem 10.12. For reasons described in Problems 10.11 and 10.12, this approach to input demand in the multi-input case is generally inferior to using Morishima elasticities. One oddity might be mentioned, however. For the case $i = j$ this expression seems to say that $e_{l,w} = s_l(A_{ll} + e_{Q,P})$, and if we jumped to the conclusion that $A_{ll} = \sigma$ in the two-input case, then this would contradict the result from Problem 11.12. You can resolve this paradox by using the definitions from Problem 10.12 to show that, with two inputs, $A_{ll} = (-s_k/s_l)\cdot A_{kl} = (-s_k/s_l)\cdot\sigma$ and so there is no disagreement.

11.14 Profit functions and technical change

Suppose that a firm's production function exhibits technical improvements over time and that the form of the function is $q = f(k, l, t)$. In this case, we can measure the proportional rate of technical change as

$$\frac{\partial \ln q}{\partial t} = \frac{f_t}{f}$$

(compare this with the treatment in Chapter 9). Show that this rate of change can also be measured using the profit function as

$$\frac{\partial \ln q}{\partial t} = \frac{\Pi(P, v, w, t)}{Pq}\cdot\frac{\partial \ln \Pi}{\partial t}.$$

That is, rather than using the production function directly, technical change can be measured by knowing the share of profits in total revenue and the proportionate change in profits over time (holding all prices constant). This approach to measuring technical change may be preferable when data on actual input levels do not exist.

11.15 Property rights theory of the firm

This problem has you work through some of the calculations associated with the numerical example in the Extensions. Refer to the Extensions for a discussion of the theory in the case of Fisher Body and General Motors (GM), who we imagine are deciding between remaining as separate firms or having GM acquire Fisher Body and thus become one (larger) firm. Let the total surplus that the units generate together be $S(x_F, x_G) = x_F^{1/2} + ax_G^{1/2}$, where x_F and x_G are the investments undertaken by the managers of the two units before negotiating, and where a unit of investment costs \$1. The parameter a measures the importance of GM's manager's investment. Show that, according to the property rights model worked out in the Extensions, it is efficient for GM to acquire Fisher Body if and only if GM's manager's investment is important enough, in particular, if $a > \sqrt{3}$.

Suggestions for Further Reading

Hart, O. *Firms, Contracts, and Financial Structure*. Oxford, UK: Oxford University Press, 1995.

Discusses the philosophical issues addressed by alternative theories of the firm. Derives further results for the property rights theory discussed in the Extensions.

Hicks, J. R. *Value and Capital*, 2nd ed. Oxford, UK: Oxford University Press, 1947.

The Appendix looks in detail at the notion of factor complementarity.

Mas-Colell, A., M. D. Whinston, and J. R. Green. *Microeconomic Theory*. New York: Oxford University Press, 1995.

Provides an elegant introduction to the theory of production using vector and matrix notation. This allows for an arbitrary number of inputs and outputs.

Samuelson, P. A. *Foundations of Economic Analysis*. Cambridge, MA: Harvard University Press, 1947.

Early development of the profit function idea together with a nice discussion of the consequences of constant returns to scale for market equilibrium. Pages 36–46 have extensive applications of Le Châtelier's Principle (see Problem 11.11).

Sydsaeter, K., A. Strom, and P. Berck. Economists' *Mathematical Manual*, 3rd ed. Berlin: Springer-Verlag, 2000.

Chapter 25 offers formulas for a number of profit and factor demand functions.

Varian, H. R. *Microeconomic Analysis*, 3rd ed. New York: W. W. Norton, 1992.

Includes an entire chapter on the profit function. Varian offers a novel approach for comparing short- and long-run responses using Le Châtelier's Principle.

Summary

In this chapter we developed a detailed model of how the equilibrium price is determined in a single competitive market. This model is basically the one first fully articulated by Alfred Marshall in the latter part of the nineteenth century. It remains the single most important component of all of microeconomics. Some of the properties of this model we examined may be listed as follows.

- Short-run equilibrium prices are determined by the interaction of what demanders are willing to pay (demand) and what existing firms are willing to produce (supply). Both demanders and suppliers act as price-takers in making their respective decisions.
- In the long run, the number of firms may vary in response to profit opportunities. If free entry is assumed, then firms will earn zero economic profits over the long run. Therefore, because firms also maximize profits, the long-run equilibrium condition is $P = MC = AC$.
- The shape of the long-run supply curve depends on how the entry of new firms affects input prices. If entry has no impact on input prices, the long-run supply curve will be horizontal (infinitely elastic). If entry increases input prices, the long-run supply curve will have a positive slope.
- If shifts in long-run equilibrium affect input prices, this will also affect the welfare of input suppliers. Such welfare changes can be measured by changes in long-run producer surplus.
- The twin concepts of consumer and producer surplus provide useful ways of measuring the welfare impact on market participants of various economic changes. Changes in consumer surplus represent the monetary value of changes in consumer utility. Changes in producer surplus represent changes in the monetary returns that inputs receive.
- The competitive model can be used to study the impact of various economic policies. For example, it can be used to illustrate the transfers and welfare losses associated with price controls.
- The competitive model can also be applied to study taxation. The model illustrates both tax incidence (i.e., who bears the actual burden of a tax) and the welfare losses associated with taxation (the excess burden). Similar conclusions can be derived by using the competitive model to study transaction costs.

Problems

12.1

Suppose there are 100 identical firms in a perfectly competitive industry. Each firm has a short-run total cost function of the form

$$C(q) = \frac{1}{300}q^3 + 0.2q^2 + 4q + 10.$$

a. Calculate the firm's short-run supply curve with q as a function of market price (P).

b. On the assumption that firms' output decisions do not affect their costs, calculate the short-run industry supply curve.

c. Suppose market demand is given by $Q = -200P + 8{,}000$. What will be the short-run equilibrium price–quantity combination?

12.2

Suppose there are 1,000 identical firms producing diamonds. Let the total cost function for each firm be given by

$$C(q, w) = q^2 + wq,$$

where q is the firm's output level and w is the wage rate of diamond cutters.

a. If $w = 10$, what will be the firm's (short-run) supply curve? What is the industry's supply curve? How many diamonds will be produced at a price of 20 each? How many more diamonds would be produced at a price of 21?

b. Suppose the wages of diamond cutters depend on the total quantity of diamonds produced, and suppose the form of this relationship is given by

$$w = 0.002Q;$$

here Q represents total industry output, which is 1,000 times the output of the typical firm.

In this situation, show that the firm's marginal cost (and short-run supply) curve depends on Q. What is the industry supply curve? How much will be produced at a price of 20? How much more will be produced at a price of 21? What do you conclude about the shape of the short-run supply curve?

12.3

Suppose that the demand function for a good has the linear form $Q = D(P, I) = a + bP + cI$ and the supply function is also of the linear form $Q = S(P) = d + gP$.

a. Calculate equilibrium price and quantity for this market as a function of the parameters a, b, c, d, and g and of I (income), the exogenous shift term for the demand function.

b. Use your results from part (a) to calculate the comparative statics derivative dP^*/dI.

c. Now calculate the same derivative using the comparative statics analysis of supply and demand presented in this chapter. You should be able to show that you get the same results in each case.

d. Specify some assumed values for the various parameters of this problem and describe why the derivative dP^*/dI takes the form it does here.

12.4

A perfectly competitive industry has a large number of potential entrants. Each firm has an identical cost structure such that long-run average cost is minimized at an output of 20 units $(q_i = 20)$. The minimum average cost is \$10 per unit. Total market demand is given by

$$Q = D(P) = 1{,}500 - 50P.$$

a. What is the industry's long-run supply schedule?

b. What is the long-run equilibrium price (P^*)? The total industry output (Q^*)? The output of each firm (q^*)? The number of firms? The profits of each firm?

c. The short-run total cost function associated with each firm's long-run equilibrium output is given by

$$C(q) = 0.5q^2 - 10q + 200.$$

Calculate the short-run average and marginal cost function. At what output level does short-run average cost reach a minimum?

d. Calculate the short-run supply function for each firm and the industry short-run supply function.

e. Suppose now that the market demand function shifts upward to $Q = D(P) = 2{,}000 - 50P$. Using this new demand curve, answer part (b) for the very short run when firms cannot change their outputs.

f. In the short run, use the industry short-run supply function to recalculate the answers to (b).

g. What is the new long-run equilibrium for the industry?

12.5

Suppose that the demand for stilts is given by

$$Q = D(P) = 1{,}500 - 50P$$

and that the long-run total operating costs of each stilt-making firm in a competitive industry are given by

$$C(q) = 0.5q^2 - 10q.$$

Entrepreneurial talent for stilt making is scarce. The supply curve for entrepreneurs is given by

$$Q_S = 0.25w,$$

where w is the annual wage paid.

Suppose also that each stilt-making firm requires one (and only one) entrepreneur (hence the quantity of entrepreneurs hired is equal to the number of firms). Long-run total costs for each firm are then given by

$$C(q, w) = 0.5q^2 - 10q + w.$$

a. What is the long-run equilibrium quantity of stilts produced? How many stilts are produced by each firm? What is the long-run equilibrium price of stilts? How many firms will there be? How many entrepreneurs will be hired, and what is their wage?

b. Suppose that the demand for stilts shifts outward to

$$Q = D(P) = 2{,}428 - 50P.$$

How would you now answer the questions posed in part (a)?

c. Because stilt-making entrepreneurs are the cause of the upward-sloping long-run supply curve in this problem, they will receive all rents generated as industry output expands. Calculate the increase in rents between parts (a) and (b). Show that this value is identical to the change in long-run producer surplus as measured along the stilt supply curve.

12.6

The handmade snuffbox industry is composed of 100 identical firms, each having short-run total costs given by

$$STC = 0.5q^2 + 10q + 5$$

and short-run marginal costs given by

$$SMC = q + 10,$$

where q is the output of snuffboxes per day.

a. What is the short-run supply curve for each snuffbox maker? What is the short-run supply curve for the market as a whole?

b. Suppose the demand for total snuffbox production is given by

$$Q = D(P) = 1{,}100 - 50P.$$

What will be the equilibrium in this marketplace? What will each firm's total short-run profits be?

c. Graph the market equilibrium and compute total short-run producer surplus in this case.

d. Show that the total producer surplus you calculated in part (c) is equal to total industry profits plus industry short-run fixed costs.

e. Suppose the government imposed a \$3 tax on snuffboxes. How would this tax change the market equilibrium?

f. How would the burden of this tax be shared between snuffbox buyers and sellers?

g. Calculate the total loss of producer surplus as a result of the taxation of snuffboxes. Show that this loss equals the change in total short-run profits in the snuffbox industry. Why do fixed costs not enter into this computation of the change in short-run producer surplus?

12.7

The perfectly competitive videotape-copying industry is composed of many firms that can copy five tapes per day at an average cost of \$10 per tape. Each firm must also pay a royalty to film studios, and the per-film royalty rate (r) is an increasing function of total industry output (Q):

$$r = 0.002Q.$$

Demand is given by

$$Q = D(P) = 1{,}050 - 50P.$$

a. Assuming the industry is in long-run equilibrium, what will be the equilibrium price and quantity of copied tapes? How many tape firms will there be? What will the per-film royalty rate be?

b. Suppose that demand for copied tapes increases to

$$Q = D(P) = 1{,}600 - 50P.$$

In this case, what is the long-run equilibrium price and quantity for copied tapes? How many tape firms are there? What is the per-film royalty rate?

c. Graph these long-run equilibria in the tape market, and calculate the increase in producer surplus between the situations described in parts (a) and (b).

d. Show that the increase in producer surplus is precisely equal to the increase in royalties paid as Q expands incrementally from its level in part (b) to its level in part (c).

e. Suppose that the government institutes a \$5.50 per-film tax on the film-copying industry. Assuming that the demand for copied films is that given in part (a), how will this tax affect the market equilibrium?

f. How will the burden of this tax be allocated between consumers and producers? What will be the loss of consumer and producer surplus?

g. Show that the loss of producer surplus as a result of this tax is borne completely by the film studios. Explain your result intuitively.

12.8

The domestic demand for portable radios is given by

$$Q = D(P) = 5{,}000 - 100P,$$

where price (P) is measured in dollars and quantity (Q) is measured in thousands of radios per year. The domestic supply curve for radios is given by

$$Q = S(P) = 150P.$$

a. What is the domestic equilibrium in the portable radio market?

b. Suppose portable radios can be imported at a world price of \$10 per radio. If trade were unencumbered, what would the new market equilibrium be? How many portable radios would be imported?

c. If domestic portable radio producers succeeded in having a \$5 tariff implemented, how would this change the market equilibrium? How much would be collected in tariff revenues? How much consumer surplus would be transferred to domestic producers? What would the deadweight loss from the tariff be?

d. How would your results from part (c) be changed if the government reached an agreement with foreign suppliers to “voluntarily” limit the portable radios they export to 1,250,000 per year? Explain how this differs from the case of a tariff.

12.9

Suppose that the market demand for a product is given by $Q_D = D(P) = A - BP$. Suppose also that the typical firm’s cost function is given by $C(q) = k + aq + bq^2$.

a. Compute the long-run equilibrium output and price for the typical firm in this market.

b. Calculate the equilibrium number of firms in this market as a function of all the parameters in this problem.

c. Describe how changes in the demand parameters A and B affect the equilibrium number of firms in this market. Explain your results intuitively.

d. Describe how the parameters of the typical firm’s cost function affect the long-run equilibrium number of firms in this example. Explain your results intuitively.

Analytical Problems

12.10 Ad valorem taxes

Throughout this chapter our analysis of taxes has assumed that they are imposed on a per-unit basis. Many taxes (such as sales taxes) are proportional, based on the price of the item. In this problem you are asked to show that, assuming the tax rate is reasonably small, the market consequences of such a tax are quite similar to those already analyzed. To do so, we now assume that the price received by suppliers is given by P and the price paid by demanders is $P(1 + t)$, where t is the ad valorem tax rate (i.e., with a tax rate of 5 percent, $t = 0.05$, the price paid by demanders is $1.05P$). In this problem then the supply function is given by $Q = S(P)$ and the demand function by $Q = D[(1 + t)P]$.

a. Show that for such a tax
$$\frac{d\ln P}{dt} = \frac{e_{D,P}}{e_{S,P} - e_{D,P}}.$$
(*Hint*: Remember that $d\ln P/dt = \frac{1}{P} \cdot \frac{dP}{dt}$ and that here we are assuming $t \approx 0$.)

b. Show that the excess burden of such a small ad valorem tax is given by:
$$DW = -0.5\frac{e_{D,P}e_{S,P}}{e_{S,P} - e_{D,P}}t^2P^*Q^*.$$

c. Compare these results to those derived in this chapter for a per-unit tax. Can you make any statements about which tax would be superior in various circumstances?

12.11 The Ramsey formula for optimal taxation

The development of optimal tax policy has been a major topic in public finance for centuries.[17] Probably the most famous result in the theory of optimal taxation is due to the English economist Frank Ramsey, who conceptualized the problem as how to structure a tax system that would collect a given amount of revenues with the minimal deadweight loss.[18] Specifically, suppose there are n goods (x_i with prices p_i) to be taxed with a sequence of ad valorem taxes (see Problem 12.10) whose rates are given by t_i $(i = 1, n)$. Therefore, total tax revenue is given by $T = \sum_{i=1}^{n} t_i p_i x_i$. Ramsey's problem is for a fixed T to choose tax rates that will minimize total deadweight loss $DW = \sum_{i=1}^{n} DW(t_i)$.

a. Use the Lagrange multiplier method to show that the solution to Ramsey's problem requires $t_i = \lambda(1/e_S - 1/e_D)$, where λ is the Lagrange multiplier for the tax constraint.

b. Interpret the Ramsey result intuitively.

c. Describe some shortcomings of the Ramsey approach to optimal taxation.

12.12 Cobweb models

One way to generate disequilibrium prices in a simple model of supply and demand is to incorporate a lag into producer's supply response. To examine this possibility, assume that quantity demanded in period t depends on price in that period $(Q_t^D = a - bP_t)$ but that quantity supplied depends on the previous period's price—perhaps because farmers refer to that price in planting a crop $(Q_t^S = c + dP_{t-1})$.

a. What is the equilibrium price in this model $(P^* = P_t = P_{t-1})$ for all periods, t.

b. If P_0 represents an initial price for this good to which suppliers respond, what will the value of P_1 be?

c. By repeated substitution, develop a formula for any arbitrary P_t as a function of P_0 and t.

d. Use your results from part (a) to restate the value of P_t as a function of P_0, P^*, and t.

e. Under what conditions will P_t converge to P^* as $t \to \infty$?

f. Graph your results for the case $a = 4, b = 2, c = 1, d = 1$, and $P_0 = 0$. Use your graph to discuss the origin of the term *cobweb model*.

12.13 More on the comparative statics of supply and demand

The supply and demand model presented earlier in this chapter can be used to look at many other comparative statics questions. In this problem you are asked to explore three of them. In all of these, quantity demanded is given by $D(P, \alpha)$ and quantity supplied by $S(P, \beta)$.

a. **Shifts in supply:** In Chapter 12 we analyzed the case of a shift in demand by looking at a comparative statics analysis of how changes in α affect equilibrium price and quantity. For this problem you are to make a similar set of computations for a shift in a parameter of the supply function, β. That is, calculate $dP^*/d\beta$ and $dQ^*/d\beta$. Be sure to calculate your results in both derivative and elasticity terms. Also describe with some simple graphs why the results here differ from those shown in the body of Chapter 12.

b. **A quantity "wedge":** In our analysis of the imposition of a unit tax we showed how such a tax wedge can affect equilibrium price and quantity. A similar analysis can be done for a quantity "wedge" for which, in equilibrium, the quantity supplied may exceed the quantity demanded. Such a situation might arise, for example, if some portion of production were lost through spoilage or if some portion of production were demanded by the government as a payment for the right to do business. Formally, let $\overline{Q}$ be the amount of the good lost. In this case equilibrium requires $D(P) = Q$ and $S(P) = Q + \overline{Q}$. Use the comparative statics methods developed in this chapter to calculate $dP^*/d\overline{Q}$ and $dQ^*/d\overline{Q}$. [In many cases it might be more reasonable to assume $\overline{Q} = \delta Q$ (where δ is a small decimal value). Without making any explicit calculations, how do you think this case would differ from the one you explicitly analyzed?]

c. **The identification problem:** An important issue in the empirical study of competitive markets is to decide whether observed price–quantity data points represent

[17]The seventeenth-century French finance minister Jean-Baptiste Colbert captured the essence of the problem with his memorable statement that "the art of taxation consists in so plucking the goose as to obtain the largest possible amount of feathers with the smallest amount of hissing."

[18]See F. Ramsey, "A Contribution to the Theory of Taxation," *Economic Journal* (March 1927): 47–61.

demand curves, supply curves, or some combination of the two. Explain the following conclusions using the comparative statics results we have obtained:

i. If only the demand parameter α takes on changing values, data on changing equilibrium values of price and quantity can be used to estimate the price elasticity of supply.

ii. If only the supply parameter β takes on changing values, data on changing equilibrium values of price and quantity can be used to evaluate the price elasticity of supply (to answer this, you must have done part (a) of this problem).

iii. If demand and supply curves are both only shifted by the same parameter [i.e., the demand and supply functions are $D(P, \alpha)$ and $S(P, \alpha)$], neither of the price elasticities can be evaluated.

12.14 The Le Chatelier principle

Our analysis of supply response in this chapter focused on the fact that firms have greater flexibility in the long run both in their hiring of inputs and in their entry decisions. For this reason, price increases resulting from an increase in demand may be large in the short run, but price will tend to return toward its initial equilibrium value over the longer term. Paul Samuelson noted that this tendency resembled a similar principle in chemistry in which an initial disturbance to an equilibrium tends to be moderated over the longer term. He therefore introduced the term used in chemistry (the Le Chatelier principle) to economics. To examine this principle, we now write the supply function as $S(P, t)$, where t represents time and our discussion in this chapter shows why $S_{P,t} > 0$—that is, the effect of a price increase on quantity supplied becomes greater over time.

a. Using this new supply function, differentiate Equations 12.24 with respect to t. This results in two equations in the two second-order cross-derivatives $\frac{d^2P^*}{d\alpha dt}$ and $\frac{d^2Q^*}{d\alpha dt}$. These derivatives show how equilibrium price and quantity react to a given shift in demand over time.

b. Solve these two equations for the second-order cross-partial derivatives identified in the previous part. Show that $\frac{d^2P^*}{d\alpha dt}$ has the opposite sign from $\frac{dP^*}{d\alpha}$. This is the Le Chatelier result—the initial change in equilibrium price is moderated over time.

c. Show that $\frac{d^2Q^*}{d\alpha dt}$ has the same sign as $\frac{dP^*}{d\alpha}$. This is a situation therefore in which the Le Chatelier "moderating" result is not reflected in all of the equilibrium values of all outcomes.

d. Describe how your mathematical results mirror the graphical analysis presented in this chapter.

SUGGESTIONS FOR FURTHER READING

Arnott, R. "Time for Revision on Rent Control?" *Journal of Economic Perspectives* (Winter 1995): 99–120.

Provides an assessment of actual "soft" rent-control policies and provides a rationale for them.

Knight, F. H. *Risk, Uncertainty and Profit*. Boston: Houghton Mifflin, 1921, chaps. 5 and 6.

Classic treatment of the role of economic events in motivating industry behavior in the long run.

Marshall, A. *Principles of Economics*, 8th ed. New York: Crowell-Collier and Macmillan, 1920, book 5, chaps. 1, 2, and 3.

Classic development of the supply–demand mechanism.

Mas-Colell, A., M. D. Whinston, and J. R. Green. *Microeconomic Theory*. New York: Oxford University Press, 1995, chap. 10.

Provides a compact analysis at a high level of theoretical precision. There is a good discussion of situations where competitive markets may not reach an equilibrium.

Reynolds, L. G. "Cut-Throat Competition." *American Economic Review* 30 (December 1940): 736–47.

Critique of the notion that there can be "too much" competition in an industry.

Robinson, J. "What Is Perfect Competition?" *Quarterly Journal of Economics* 49 (1934): 104–20.

Critical discussion of the perfectly competitive assumptions.

Salanie, B. *The Economics of Taxation*. Cambridge, MA: MIT Press, 2003.

This provides a compact study of many issues in taxation. Describes a few simple models of incidence and develops some general equilibrium models of taxation.

Stigler, G. J. "Perfect Competition, Historically Contemplated." *Journal of Political Economy* 65 (1957): 1–17.

Fascinating discussion of the historical development of the competitive model.

Varian, H. R. *Microeconomic Analysis*, 3rd ed. New York: W. W. Norton, 1992, chap. 13.

Terse but instructive coverage of many of the topics in this chapter. The importance of entry is stressed, although the precise nature of the long-run supply curve is a bit obscure.

Summary

In this chapter we have examined models of markets in which there is only a single monopoly supplier. Unlike the competitive case investigated in Part 4, monopoly firms do not exhibit price-taking behavior. Instead, the monopolist can choose the price–quantity combination on the market demand curve that is most profitable. A number of consequences then follow from this market power.

- The most profitable level of output for the monopolist is the one for which marginal revenue is equal to marginal cost. At this output level, price will exceed marginal cost. The profitability of the monopolist will depend on the relationship between price and average cost.
- Relative to perfect competition, monopoly involves a loss of consumer surplus for demanders. Some of this is transferred into monopoly profits, whereas some of the loss in consumer supply represents a deadweight loss of overall economic welfare.
- Monopolists may opt for higher or lower levels of quality than would perfectly competitive firms, depending on the circumstances.
- A monopoly may be able to increase its profits further through price discrimination—that is, charging different prices to different buyers based in part on their valuations. Various strategies can be used, including segmenting markets based on identifiable characteristics or letting buyers sort themselves on a non-uniform price schedule. The ability of the monopoly to practice price discrimination depends on its ability to prevent arbitrage among buyers.
- Governments often choose to regulate natural monopolies (firms with diminishing average costs over a broad range of output levels). The type of regulatory mechanisms adopted can affect the behavior of the regulated firm.
- The deadweight loss from high monopoly prices can be dwarfed in the long run by dynamic gains if monopolies can be shown to be more innovative than competitive firms, still an open empirical question.

Problems

14.1

A monopolist can produce at constant average and marginal costs of $AC = MC = 5$. The firm faces a market demand curve given by $Q = 53 - P$.

a. Calculate the profit-maximizing price–quantity combination for the monopolist. Also calculate the monopolist's profits.

b. What output level would be produced by this industry under perfect competition (where price = marginal cost)?

c. Calculate the consumer surplus obtained by consumers in case (b). Show that this exceeds the sum of the monopolist's profits and the consumer surplus received in case (a). What is the value of the "deadweight loss" from monopolization?

14.2

A monopolist faces a market demand curve given by

$$Q = 70 - p.$$

a. If the monopolist can produce at constant average and marginal costs of $AC = MC = 6$, what output level will the monopolist choose to maximize profits? What is the price at this output level? What are the monopolist's profits?

b. Assume instead that the monopolist has a cost structure where total costs are described by

$$C(Q) = 0.25Q^2 - 5Q + 300.$$

With the monopolist facing the same market demand and marginal revenue, what price–quantity combination will be chosen now to maximize profits? What will profits be?

c. Assume now that a third cost structure explains the monopolist's position, with total costs given by

$$C(Q) = 0.0133Q^3 - 5Q + 250.$$

Again, calculate the monopolist's price–quantity combination that maximizes profits. What will profit be? *Hint:* Set $MC = MR$ as usual and use the quadratic formula to solve the second-order equation for Q.

d. Graph the market demand curve, the MR curve, and the three marginal cost curves from parts (a), (b), and (c). Notice that the monopolist's profit-making ability is constrained by (1) the market demand curve (along with its associated MR curve) and (2) the cost structure underlying production.

14.3

A single firm monopolizes the entire market for widgets and can produce at constant average and marginal costs of

$$AC = MC = 10.$$

Originally, the firm faces a market demand curve given by

$$Q = 60 - P.$$

a. Calculate the profit-maximizing price–quantity combination for the firm. What are the firm's profits?

b. Now assume that the market demand curve shifts outward (becoming steeper) and is given by

$$Q = 45 - 0.5P.$$

What is the firm's profit-maximizing price-quantity combination now? What are the firm's profits?

c. Instead of the assumptions of part (b), assume that the market demand curve shifts outward (becoming flatter) and is given by

$$Q = 100 - 2P.$$

What is the firm's profit-maximizing price–quantity combination now? What are the firm's profits?

d. Graph the three different situations of parts (a), (b), and (c). Using your results, explain why there is no real supply curve for a monopoly.

14.4

Suppose the market for Hula Hoops is monopolized by a single firm.

a. Draw the initial equilibrium for such a market.

b. Now suppose the demand for Hula Hoops shifts outward slightly. Show that, in general (contrary to the competitive case), it will not be possible to predict the effect of this shift in demand on the market price of Hula Hoops.

c. Consider three possible ways in which the price elasticity of demand might change as the demand curve shifts: It might increase, it might decrease, or it might stay the same. Consider also that marginal costs for the monopolist might be increasing, decreasing, or constant in the range where $MR = MC$. Consequently, there are nine different combinations of types of demand shifts and marginal cost slope configurations. Analyze each of these to determine for which it is possible to make a definite prediction about the effect of the shift in demand on the price of Hula Hoops.

14.5

Suppose a monopoly market has a demand function in which quantity demanded depends not only on market price (P) but also on the amount of advertising the firm does (A, measured in dollars). The specific form of this function is

$$Q = (20 - P)(1 + 0.1A - 0.01A^2).$$

The monopolistic firm's cost function is given by

$$C = 10Q + 15 + A.$$

a. Suppose there is no advertising $(A = 0)$. What output will the profit-maximizing firm choose? What market price will this yield? What will be the monopoly's profits?

b. Now let the firm also choose its optimal level of advertising expenditure. In this situation, what output level will be chosen? What price will this yield? What will the level of advertising be? What are the firm's profits in this case? *Hint:* This can be worked out most easily by assuming the monopoly chooses the profit-maximizing price rather than quantity.

14.6

Suppose a monopoly can produce any level of output it wishes at a constant marginal (and average) cost of \$5 per unit. Assume the monopoly sells its goods in two different markets separated by some distance. The demand curve in the first market is given by

$$Q_1 = 55 - P_1,$$

and the demand curve in the second market is given by

$$Q_2 = 70 - 2P_2.$$

a. If the monopolist can maintain the separation between the two markets, what level of output should be produced in each market, and what price will prevail in each market? What are total profits in this situation?

b. How would your answer change if it costs demanders only \$4 to transport goods between the two markets? What would be the monopolist's new profit level in this situation?

c. How would your answer change if transportation costs were zero and then the firm was forced to follow a single-price policy?

d. Now assume the two different markets 1 and 2 are just two individual consumers. Suppose the firm could adopt a linear two-part tariff under which marginal prices charged to the two consumers must be equal but their lump-sum entry fees might vary. What pricing policy should the firm follow?

14.7

Suppose a perfectly competitive industry can produce widgets at a constant marginal cost of \$10 per unit. Monopolized marginal costs increase to \$12 per unit because \$2 per unit must be paid to lobbyists to retain the widget producers' favored position. Suppose the market demand for widgets is given by

$$Q_D = 1{,}000 - 50P.$$

a. Calculate the perfectly competitive and monopoly outputs and prices.
b. Calculate the total loss of consumer surplus from monopolization of widget production.
c. Graph your results and explain how they differ from the usual analysis.

14.8

Suppose the government wishes to combat the undesirable allocational effects of a monopoly through the use of a subsidy.

a. Why would a lump-sum subsidy not achieve the government's goal?
b. Use a graphical proof to show how a per-unit-of-output subsidy might achieve the government's goal.
c. Suppose the government wants its subsidy to maximize the difference between the total value of the good to consumers and the good's total cost. Show that, to achieve this goal, the government should set

$$\frac{t}{P} = -\frac{1}{e_{D,P}},$$

where t is the per-unit subsidy and P is the competitive price. Explain your result intuitively.

14.9

Suppose a monopolist produces alkaline batteries that may have various useful lifetimes (X). Suppose also that consumers' (inverse) demand depends on batteries' lifetimes and quantity (Q) purchased according to the function

$$P(Q, X) = g(X \cdot Q),$$

where $g' < 0$. That is, consumers care only about the product of quantity times lifetime: They are willing to pay equally for many short-lived batteries or few long-lived ones. Assume also that battery costs are given by

$$C(Q, X) = C(X)Q,$$

where $C'(X) > 0$. Show that, in this case, the monopoly will opt for the same level of X as does a competitive industry even though levels of output and prices may differ. Explain your result. *Hint:* Treat XQ as a composite commodity.

Analytical Problems

14.10 Taxation of a monopoly good

The taxation of monopoly can sometimes produce results different from those that arise in the competitive case. This problem looks at some of those cases. Most of these can be analyzed by using the inverse elasticity rule (Equation 14.1).

a. Consider first an ad valorem tax on the price of a monopoly's good. This tax reduces the net price received by the monopoly from P to $P(1 - t)$, where t is the proportional tax rate. Show that, with a linear demand curve and constant marginal cost, the imposition of such a tax causes price to increase by less than the full extent of the tax.
b. Suppose that the demand curve in part (a) were a constant elasticity curve. Show that the price would now increase by precisely the full extent of the tax. Explain the difference between these two cases.
c. Describe a case where the imposition of an ad valorem tax on a monopoly would cause the price to increase by more than the tax.
d. A specific tax is a fixed amount per unit of output. If the tax rate is τ per unit, total tax collections are τQ. Show that the imposition of a specific tax on a monopoly will reduce output more (and increase price more) than will the imposition of an ad valorem tax that collects the same tax revenue.

14.11 Flexible functional forms

In an important recent working paper, M. Fabinger and E. G. Weyl characterize tractable monopoly problems.[18] A "tractable" problem satisfies three conditions. First, it must be possible to move back and forth between explicit expressions for inverse and direct demand (invertibility). Second, inverse demand—which can also be interpreted as average revenue—must have the same functional form as marginal revenue, and average cost must have the same functional form as marginal cost (form preservation). Third, the monopolist's first-order condition must be a linear equation (linearity), if not immediately after differentiation, then at least after suitable substitution. The authors show that the broadest possible class of tractable problems has the following functional form for inverse demand and average cost:

$$P(Q) = a_0 + a_1 Q^{-s}$$
$$AC(Q) = c_0 + c_1 Q^{-s},$$

where a_0, a_1, c_0, c_1, and s are non-negative constants.

a. Solve for the monopoly equilibrium quantity and price given these functional forms. What substitution $x = f(Q)$ do you need to make the first-order condition linear in x?
b. Derive the solution in the special case with constant average and marginal cost.

[18] M. Fabinger and E. G. Weyl, "A Tractable Approach to Pass-Through Patterns," (March 2015) SSRN working paper no. 2194855.

c. If one is willing to relax tractability a bit to allow the monopoly's first-order condition to be a quadratic equation (at least after suitable substitution), the authors show that the broadest class of tractable problems then involves the following functional forms:

$$P(Q) = a_0 + a_1 Q^{-s} + a_2 Q^s$$
$$AC(Q) = c_0 + c_1 Q^{-s} + c_2 Q^s.$$

Solve for the monopoly equilibrium quantity and price. What substitution $x = f(Q)$ is needed to make the first-order condition quadratic in x?

d. While slightly complicated, the functional forms in part (c) have the advantage of being flexible enough to allow for U-shaped average cost curves such as drawn in Figure 14.2 in addition to constant, increasing, and decreasing. Demonstrate this by graphing this average cost curve for well-chosen values of c_0, c_1, c_2 to illustrate the various cases.

The flexible functional forms in part (c) also allow for realistic demand shapes, for example, one that closely fits the U.S. income distribution (which implicitly takes income to proxy for consumers' willingness to pay). These realistic demand shapes can be used in calibrations to address important policy questions. For example, the text mentioned that, in theory, the welfare effects of monopoly price discrimination can go either way, either being higher or lower than under uniform pricing. Calibrations involving the demand curves from part (c) invariably show that welfare is higher under price discrimination.

14.12 Welfare possibilities with different market segmentations

The article by D. Bergemann, B. Brooks, and S. Morris discussed in the text highlights the fundamental ambiguity of the welfare effects of price discrimination. This question guides you through the construction of market segmentations that can achieve extreme welfare gains and losses relative to a single-price policy. Here we focus on the simple case of a market containing two consumer types, but the results hold generally for any number of types and indeed for arbitrary continuous distributions of types.

Consider a market served by a monopolist in which $\overline{q}$ consumers have value (maximum willingness to pay) $\overline{v}$ for the good and $\underline{q}$ consumers have value $\underline{v}$, where $\overline{v} > \underline{v} > 0$. Production is costless.

a. For comparison, first solve for the socially efficient output and welfare associated with the perfectly competitive outcome.

b. Find a way to segment consumers into just two markets that allows the monopolist to recover the profit from perfect price discrimination. Compute the associated profit, consumer surplus, and social welfare.

c. The analysis in the rest of the problem is divided into two exhaustive cases. First suppose $\overline{q}\,\overline{v} > (\overline{q} + \underline{q})\underline{v}$.
   i. Find the monopoly price, quantity, profit, consumer surplus, and welfare when the monopolist charges a single price in the initial market before any segmentation.
   ii. Divide the single market into two segments by moving all of the low-value consumers and a fraction b of the high-value ones into segment B, leaving the remaining consumers in the initial market to constitute segment A, and assume the monopolist engages in price discrimination across the segments. Show that there exists b^* in the interval $(0, 1)$ such that the monopolist is indifferent between charging a high and low price in segment B. Consider the equilibrium in which the monopolist charges the low price on a segment when indifferent. Solve for the monopolist's discriminatory prices across the segments. Solve for profit, consumer surplus, and welfare in total across the two segments. Compare this outcome to a single-price monopoly, showing that consumer surplus and welfare is created. How do surpluses compare to those under perfect competition?
   iii. Plot the outcomes from parts (i) and (ii) on a graph with consumer surplus on the horizontal axis and monopoly profit on the vertical. Also plot the perfect price discrimination from part (b). Connect the points as vertices of a triangle. For a challenge, think of ways to further segment the market to achieve the surplus divisions along the sides and in the interior of the triangle.

d. Now suppose $\overline{q}\,\overline{v} < (\overline{q} + \underline{q})\underline{v}$.
   i. Find the monopoly price, quantity, profit, consumer surplus, and welfare when the monopolist charges a single price before segmentation.
   ii. Divide the single market into two segments by moving all of the high-value consumers and a fraction a of the low-value consumers into segment A, leaving the remaining consumers in the initial market to constitute segment B, and assume the monopolist engages in price discrimination across the segments. Show that there exists a^* in the interval $(0, 1)$ such that the monopolist is indifferent between charging a high and low price in segment A. Consider the equilibrium in which the monopolist charges the high price on a segment when indifferent. Solve for the monopolist's discriminatory prices across the segments. Solve for profit, consumer surplus, and welfare in total across the two segments. Compare this outcome to a single-price monopoly, showing that consumer surplus and welfare is destroyed.

iii. Show that plotting the consumer surplus and monopoly profit from the various pricing strategies yields a similar triangle as in part (c).

The analysis is similar when production involves a positive marginal cost c rather than being costless. We just need to reinterpret consumer values above as being net of c.

Behavioral Problem

14.13 Shrouded prices

Some firms employ the marketing strategy of posting a low price for the good, but then tack on hidden fees or high prices for add-ons that can add up to an "all-in" price that is exorbitant compared to the posted price. A television ad may blare that a perpetually sharp knife sells for $20, leaving the additional $10 handling charge—or worse, that the $20 is just one of three installments—for the small print. A laser printer printing photo-quality color prints may seem like a bargain at $300 if one doesn't consider that the five toner cartridges must be replaced each year at $100 each. If consumers understand and account for these additional expenses, we are firmly in a neoclassical model, which can be analyzed using standard methods. Behavioral economists worry about the possibility that unsophisticated consumers may underestimate or even ignore these shrouded prices and firms do their best to keep it that way. This question introduces a model of shrouded prices and analyzes their efficiency consequences.

a. Consumers' demand for a good whose price they perceive to be P is given by $Q = 10 - P$. A monopolist produces the good at constant average and marginal cost equal to $6. Compute the monopoly price, quantity, profit, consumer surplus, and welfare (the sum of consumer surplus and profit) assuming the perceived is the same as the actual price, so there is no shrouding.

b. Now assume that while the perceived price is still P, the actual price charged by the monopolist is $P + s$, where s is the shrouded part, which goes unrecognized by consumers. Compute the monopoly price, quantity, and profit assuming the same demand and cost as in part (a). What amount of shrouding does the firm prefer?

c. Compute the consumer surplus (CS) associated with the outcome in (b). This requires some care because consumers spend more than they expect to. Letting P_s and Q_s be the equilibrium price and quantity charged by the monopoly with shrouded prices,

$$CS = \int_0^{Q_s} P(Q)dQ - P_sQ_s.$$

This equals gross consumer surplus (the area under inverse demand up to the quantity sold) less actual rather than perceived expenditures.

d. Compute welfare. Find the welfare-maximizing level of shrouding. Explain why this is positive rather than zero.

e. Return to the case of no shrouding in part (a) but now assume the government offers a subsidy s. Show that the welfare-maximizing subsidy equals welfare-maximizing level of shrouding found in part (d). Are the distributional consequences (surplus going to consumers, firm, and government) the same in the two cases? Use the connection between shrouding and a subsidy to argue informally that any amount of shrouding will be inefficient in a perfectly competitive market.

Suggestions for Further Reading

Posner, R. A. "The Social Costs of Monopoly and Regulation." *Journal of Political Economy* 83 (1975): 807–27.

An analysis of the probability that monopolies will spend resources on the creation of barriers to entry and thus have higher costs than perfectly competitive firms.

Schumpeter, J. A. *Capitalism, Socialism and Democracy*, 3rd ed. New York: Harper & Row, 1950.

Classic defense of the role of the entrepreneur and economic profits in the economic growth process.

Spence, M. "Monopoly, Quality, and Regulation." *Bell Journal of Economics* (April 1975): 417–29.

Develops the approach to product quality used in this text and provides a detailed analysis of the effects of monopoly.

Stigler, G. J. "The Theory of Economic Regulation." *Bell Journal of Economics and Management Science* 2 (Spring 1971): 3.

Early development of the "capture" hypothesis of regulatory behavior—that the industry captures the agency supposed to regulate it and uses that agency to enforce entry barriers and further enhance profits.

Tirole, J. *The Theory of Industrial Organization*. Cambridge, MA: MIT Press, 1989, chaps. 1–3.

A complete analysis of the theory of monopoly pricing and product choice.

Varian, H. R. *Microeconomic Analysis*, 3rd ed. New York: W. W. Norton, 1992, chap. 14.

Provides a succinct analysis of the role of incentive compatibility constraints in second-degree price discrimination.

engage in a race to see who can produce a viable vaccine from this initial idea. Continue to assume that firm 2 already has a malaria vaccine on the market and that this new vaccine would be a perfect substitute for it.

The difference between the models in this and the previous section is that if firm 2 does not win the race to develop the idea, then the idea does not simply fall by the wayside but rather is developed by the competitor, firm 1. Firm 2 has an incentive to win the innovation competition to prevent firm 1 from becoming a competitor. Formally, if firm 1 wins the innovation competition, then it enters the market and is a competitor with firm 2, earning duopoly profit π_D. As we have repeatedly seen, this is the maximum that firm 1 would pay for the innovation. Firm 2's profit is Π_M if it wins the competition for the innovation but π_D if it loses and firm 1 wins. Firm 2 would pay up to the difference, $\Pi_M - \pi_D$, for the innovation. If $\Pi_M > 2\pi_D$—that is, if industry profit under a monopoly is greater than under a duopoly, which it is when some of the monopoly profit is dissipated by duopoly competition—then $\Pi_M - \pi_D > \pi_D$, and firm 2 will have more incentive to innovate than firm 1.

This model explains the puzzling phenomenon of dominant firms filing for "sleeping patents": patents that are never implemented. Dominant firms have a substantial incentive—as we have seen, possibly greater than entrants'—to file for patents to prevent entry and preserve their dominant position. Whereas the replacement effect may lead to turnover in the market and innovation by new firms, the dissipation effect may help preserve the position of dominant firms and retard the pace of innovation.

Summary

Many markets fall between the polar extremes of perfect competition and monopoly. In such imperfectly competitive markets, determining market price and quantity is complicated because equilibrium involves strategic interaction among the firms. In this chapter, we used the tools of game theory developed in Chapter 8 to study strategic interaction in oligopoly markets. We first analyzed oligopoly firms' short-run choices such as prices and quantities and then went on to analyze firms' longer-run decisions such as product location, innovation, entry, and the deterrence of entry. We found that seemingly small changes in modeling assumptions may lead to big changes in equilibrium outcomes. Therefore, predicting behavior in oligopoly markets may be difficult based on theory alone and may require knowledge of particular industries and careful empirical analysis. Still, some general principles did emerge from our theoretical analysis that aid in understanding oligopoly markets.

- One of the most basic oligopoly models, the Bertrand model involves two identical firms that set prices simultaneously. The equilibrium resulted in the Bertrand paradox: Even though the oligopoly is the most concentrated possible, firms behave as perfect competitors, pricing at marginal cost and earning zero profit.
- The Bertrand paradox is not the inevitable outcome in an oligopoly but can be escaped by changing assumptions underlying the Bertrand model—for example, allowing for quantity competition, differentiated products, search costs, capacity constraints, or repeated play leading to collusion.
- As in the Prisoners' Dilemma, firms could profit by coordinating on a less competitive outcome, but this outcome will be unstable unless firms can explicitly collude by forming a legal cartel or tacitly collude in a repeated game.
- For tacit collusion to sustain supercompetitive profits, firms must be patient enough that the loss from a price war in future periods to punish undercutting exceeds the benefit from undercutting in the current period.
- Whereas a nonstrategic monopolist prefers flexibility to respond to changing market conditions, a strategic oligopolist may prefer to commit to a single choice. The firm can commit to the choice if it involves a sunk cost that cannot be recovered if the choice is later reversed.
- A first mover can gain an advantage by committing to a different action from what it would choose in the Nash equilibrium of the simultaneous game. To deter entry, the first mover should commit to reducing the entrant's profits using an aggressive "top dog" strategy (high output or low price). If it does not deter entry, the first mover should commit to a strategy leading its rival to compete less aggressively. This is sometimes a "top dog" and sometimes a "puppy dog" strategy, depending on the slope of firms' best responses.

- Holding the number of firms in an oligopoly constant in the short run, the introduction of a factor that softens competition (e.g., product differentiation, search costs, collusion) will increase firms' profit, but an offsetting effect in the long run is that entry—which tends to reduce oligopoly profit—will be more attractive.
- Innovation may be even more important than low prices for total welfare in the long run. Determining which oligopoly structure is the most innovative is difficult because offsetting effects (dissipation and replacement) are involved.

Problems

15.1

Assume for simplicity that a monopolist has no costs of production and faces a demand curve given by $Q = 150 - P$.

a. Calculate the profit-maximizing price–quantity combination for this monopolist. Also calculate the monopolist's profit.

b. Suppose instead that there are two firms in the market facing the demand and cost conditions just described for their identical products. Firms choose quantities simultaneously as in the Cournot model. Compute the outputs in the Nash equilibrium. Also compute market output, price, and firm profits.

c. Suppose the two firms choose prices simultaneously as in the Bertrand model. Compute the prices in the Nash equilibrium. Also compute firm output and profit as well as market output.

d. Graph the demand curve and indicate where the market price–quantity combinations from parts (a)–(c) appear on the curve.

15.2

Suppose that firms' marginal and average costs are constant and equal to c and that inverse market demand is given by $P = a - bQ$, where $a, b > 0$.

a. Calculate the profit-maximizing price–quantity combination for a monopolist. Also calculate the monopolist's profit.

b. Calculate the Nash equilibrium quantities for Cournot duopolists, which choose quantities for their identical products simultaneously. Also compute market output, market price, and firm and industry profits.

c. Calculate the Nash equilibrium prices for Bertrand duopolists, which choose prices for their identical products simultaneously. Also compute firm and market output as well as firm and industry profits.

d. Suppose now that there are n identical firms in a Cournot model. Compute the Nash equilibrium quantities as functions of n. Also compute market output, market price, and firm and industry profits.

e. Show that the monopoly outcome from part (a) can be reproduced in part (d) by setting $n = 1$, that the Cournot duopoly outcome from part (b) can be reproduced in part (d) by setting $n = 2$ in part (d), and that letting n approach infinity yields the same market price, output, and industry profit as in part (c).

15.3

Let c_i be the constant marginal and average cost for firm i (so that firms may have different marginal costs). Suppose demand is given by $P = 1 - Q$.

a. Calculate the Nash equilibrium quantities assuming there are two firms in a Cournot market. Also compute market output, market price, firm profits, industry profits, consumer surplus, and total welfare.

b. Represent the Nash equilibrium on a best-response function diagram. Show how a reduction in firm 1's cost would change the equilibrium. Draw a representative isoprofit for firm 1.

15.4

Suppose that firms 1 and 2 operate under conditions of constant average and marginal cost but that firm 1's marginal cost is $c_1 = 10$ and firm 2's is $c_2 = 8$. Market demand is $Q = 500 - 20P$.

a. Suppose firms practice Bertrand competition, that is, setting prices for their identical products simultaneously. Compute the Nash equilibrium prices. (To avoid technical problems in this question, assume that if firms charge equal prices, then the low-cost firm makes all the sales.)

b. Compute firm output, firm profit, and market output.

c. Is total welfare maximized in the Nash equilibrium? If not, suggest an outcome that would maximize total welfare, and compute the deadweight loss in the Nash equilibrium compared with your outcome.

15.5

Consider the following Bertrand game involving two firms producing differentiated products. Firms have no costs of production. Firm 1's demand is

$$q_1 = 1 - p_1 + bp_2,$$

where $b > 0$. A symmetric equation holds for firm 2's demand.

a. Solve for the Nash equilibrium of the simultaneous price-choice game.
b. Compute the firms' outputs and profits.
c. Represent the equilibrium on a best-response function diagram. Show how an increase in b would change the equilibrium. Draw a representative isoprofit curve for firm 1.

15.6

Recall Example 15.6, which covers tacit collusion. Suppose (as in the example) that a medical device is produced at constant average and marginal cost of \$10 and that the demand for the device is given by

$$Q = 5{,}000 - 100P.$$

The market meets each period for an infinite number of periods. The discount factor is δ.

a. Suppose that n firms engage in Bertrand competition each period. Suppose it takes two periods to discover a deviation because it takes two periods to observe rivals' prices. Compute the discount factor needed to sustain collusion in a subgame-perfect equilibrium using grim strategies.
b. Now restore the assumption that, as in Example 15.7, deviations are detected after just one period. Next, assume that n is not given but rather is determined by the number of firms that choose to enter the market in an initial stage in which entrants must sink a one-time cost K to participate in the market. Find an upper bound on n. *Hint:* Two conditions are involved.

15.7

Assume as in Problem 15.1 that two firms with no production costs, facing demand $Q = 150 - P$, choose quantities q_1 and q_2.

a. Compute the subgame-perfect equilibrium of the Stackelberg version of the game in which firm 1 chooses q_1 first and then firm 2 chooses q_2.
b. Now add an entry stage after firm 1 chooses q_1. In this stage, firm 2 decides whether to enter. If it enters, then it must sink cost K_2, after which it is allowed to choose q_2. Compute the threshold value of K_2 above which firm 1 prefers to deter firm 2's entry.
c. Represent the Cournot, Stackelberg, and entry-deterrence outcomes on a best-response function diagram.

15.8

Recall the Hotelling model of competition on a linear beach from Example 15.5. Suppose for simplicity that ice cream stands can locate only at the two ends of the line segment (zoning prohibits commercial development in the middle of the beach). This question asks you to analyze an entry-deterring strategy involving product proliferation.

a. Consider the subgame in which firm A has two ice cream stands, one at each end of the beach, and B locates along with A at the right endpoint. What is the Nash equilibrium of this subgame? *Hint:* Bertrand competition ensues at the right endpoint.
b. If B must sink an entry cost K_B, would it choose to enter given that firm A is in both ends of the market and remains there after entry?
c. Is A's product proliferation strategy credible? Or would A exit the right end of the market after B enters? To answer these questions, compare A's profits for the case in which it has a stand on the left side and both it and B have stands on the right to the case in which A has one stand on the left end and B has one stand on the right end (so B's entry has driven A out of the right side of the market).

Analytical Problems

15.9 Herfindahl index of market concentration

One way of measuring market concentration is through the use of the Herfindahl index, which is defined as

$$H = \sum_{i=1}^{n} s_i^2,$$

where $s_t = q_i/Q$ is firm i's market share. The higher is H, the more concentrated the industry is said to be. Intuitively, more concentrated markets are thought to be less competitive because dominant firms in concentrated markets face little competitive pressure. We will assess the validity of this intuition using several models.

a. If you have not already done so, answer Problem 15.2d by computing the Nash equilibrium of this n-firm Cournot game. Also compute market output, market price, consumer surplus, industry profit, and total welfare. Compute the Herfindahl index for this equilibrium.
b. Suppose two of the n firms merge, leaving the market with $n - 1$ firms. Recalculate the Nash equilibrium and the rest of the items requested in part (a). How does the merger affect price, output, profit, consumer surplus, total welfare, and the Herfindahl index?
c. Put the model used in parts (a) and (b) aside and turn to a different setup: that of Problem 15.3, where Cournot duopolists face different marginal costs. Use your answer to Problem 15.3a to compute equilibrium firm outputs, market output, price, consumer surplus, industry profit, and total welfare, substituting the particular cost parameters $c_1 = c_2 = 1/4$. Also compute the Herfindahl index.
d. Repeat your calculations in part (c) while assuming that firm 1's marginal cost c_1 falls to 0 but c_2 stays at 1/4. How does the cost change affect price, output, profit, consumer surplus, total welfare, and the Herfindahl index?
e. Given your results from parts (a)–(d), can we draw any general conclusions about the relationship between

market concentration on the one hand and price, profit, or total welfare on the other?

15.10 Inverse elasticity rule

Use the first-order condition (Equation 15.2) for a Cournot firm to show that the usual inverse elasticity rule from Chapter 11 holds under Cournot competition (where the elasticity is associated with an individual firm's residual demand, the demand left after all rivals sell their output on the market). Manipulate Equation 15.2 in a different way to obtain an equivalent version of the inverse elasticity rule:

$$\frac{P - MC}{P} = -\frac{s_i}{e_{Q,P}},$$

where $s_i = q_i/Q$ is firm i's market share and $e_{Q,P}$ is the elasticity of market demand. Compare this version of the inverse elasticity rule with that for a monopolist from the previous chapter.

15.11 Competition on a circle

Hotelling's model of competition on a linear beach is used widely in many applications, but one application that is difficult to study in the model is free entry. Free entry is easiest to study in a model with symmetric firms, but more than two firms on a line cannot be symmetric because those located nearest the endpoints will have only one neighboring rival, whereas those located nearer the middle will have two.

To avoid this problem, Steven Salop introduced competition on a circle.[18] As in the Hotelling model, demanders are located at each point, and each demands one unit of the good. A consumer's surplus equals v (the value of consuming the good) minus the price paid for the good as well as the cost of having to travel to buy from the firm. Let this travel cost be td, where t is a parameter measuring how burdensome travel is and d is the distance traveled (note that we are here assuming a linear rather than a quadratic travel-cost function, in contrast to Example 15.5).

Initially, we take as given that there are n firms in the market and that each has the same cost function $C_i = K + cq_i$, where K is the sunk cost required to enter the market [this will come into play in part (e) of the question, where we consider free entry] and c is the constant marginal cost of production. For simplicity, assume that the circumference of the circle equals 1 and that the n firms are located evenly around the circle at intervals of $1/n$. The n firms choose prices p_i simultaneously.

a. Each firm i is free to choose its own price (p_i) but is constrained by the price charged by its nearest neighbor to either side. Let p^* be the price these firms set in a symmetric equilibrium. Explain why the extent of any firm's market on either side (x) is given by the equation

$$p + tx = p^* + t[(1/n) - x].$$

b. Given the pricing decision analyzed in part (a), firm i sells $q_i = 2x$ because it has a market on both sides. Calculate the profit-maximizing price for this firm as a function of p^*, c, t, and n.

c. Noting that in a symmetric equilibrium all firms' prices will be equal to p^*, show that $p_i = p^* = c + t/n$. Explain this result intuitively.

d. Show that a firm's profits are $t/n^2 - K$ in equilibrium.

e. What will the number of firms n^* be in long-run equilibrium in which firms can freely choose to enter?

f. Calculate the socially optimal level of differentiation in this model, defined as the number of firms (and products) that minimizes the sum of production costs plus demander travel costs. Show that this number is precisely half the number calculated in part (e). Hence this model illustrates the possibility of overdifferentiation.

15.12 Signaling with entry accommodation

This question will explore signaling when entry deterrence is impossible; thus, the signaling firm accommodates its rival's entry. Assume deterrence is impossible because the two firms do not pay a sunk cost to enter or remain in the market. The setup of the model will follow Example 15.4, so the calculations there will aid the solution of this problem. In particular, firm i's demand is given by

$$q_i = a_i - p_i + \frac{p_j}{2},$$

where a_i is product i's attribute (say, quality). Production is costless. Firm 1's attribute can be one of two values: either $a_1 = 1$, in which case we say firm 1 is the low type, or $a_1 = 2$, in which case we say it is the high type. Assume there is no discounting across periods for simplicity.

a. Compute the Nash equilibrium of the game of complete information in which firm 1 is the high type and firm 2 knows that firm 1 is the high type.

b. Compute the Nash equilibrium of the game in which firm 1 is the low type and firm 2 knows that firm 1 is the low type.

c. Solve for the Bayesian–Nash equilibrium of the game of incomplete information in which firm 1 can be either type with equal probability. Firm 1 knows its type, but firm 2 only knows the probabilities. Because we did not spend time in this chapter on Bayesian games, you may want to consult Chapter 8 (especially Example 8.6).

d. Which of firm 1's types gains from incomplete information? Which type would prefer complete information (and thus would have an incentive to signal its type if possible)? Does firm 2 earn more profit on average under complete information or under incomplete information?

[18] See S. Salop, "Monopolistic Competition with Outside Goods," *Bell Journal of Economics* (Spring 1979): 141–56.

e. Consider a signaling variant of the model chat has two periods. Firms 1 and 2 choose prices in the first period, when firm 2 has incomplete information about firm 1's type. Firm 2 observes firm 1's price in this period and uses the information to update its beliefs about firm 1's type. Then firms engage in another period of price competition. Show that there is a separating equilibrium in which each type of firm 1 charges the same prices as computed in part (d). You may assume that, if firm 1 chooses an out-of-equilibrium price in the first period, then firm 2 believes that firm 1 is the low type with probability 1. *Hint:* To prove the existence of a separating equilibrium, show that the loss to the low type from trying to pool in the first period exceeds the second-period gain from having convinced firm 2 that it is the high type. Use your answers from parts (a)–(d) where possible to aid in your solution.

Behavioral Problem

15.13 Can competition unshroud prices?

In this problem, we return to the question of shrouded product attributes and prices, introduced in Problem 6.14 and further analyzed in Problem 14.13. Here we will pursue the question of whether market forces can be counted on to attenuate consumer behavioral biases. In particular, whether competition and advertising can serve to unshroud previously shrouded prices.

We will study a model inspired by Xavier Gabaix and David Laibson's influential article.[19] A population of consumers (normalize their mass to 1) have gross surplus v for a homogeneous good produced by duopoly firms at constant marginal and average cost c. Firms $i = 1, 2$ simultaneously post prices p_i. In addition to these posted prices, each firm i can add a shrouded fee s_i, which are anticipated by some consumers but not others. For example, the fees could be for checked baggage associated with plane travel or for not making a minimum monthly payment on a credit card. A fraction α are sophisticated consumers, who understand the equilibrium and anticipate equilibrium shrouded fees. At a small inconvenience cost e, they are able to avoid the shrouded fee (packing only carry-ons in the airline example or being sure to make the minimum monthly payment in the credit-card example). The remaining fraction $1 - \alpha$ of consumers are myopic. They do not anticipate shrouded fees, only considering posted prices in deciding from which firm to buy. Their only way of avoiding the fee is void the entire transaction (saving the total expenditure $p_i + s_i$ but forgoing surplus v). Suppose firms choose posted prices simultaneously as in the Bertrand model.

a. Argue that in equilibrium, $p_i^* + s_i^* = v$ (at least as long as e is sufficiently small that firms do not try to induce sophisticated consumers not to avoid the shrouded fee). Compute the Nash equilibrium posted prices p_i^*. (*Hint:* As in the standard Bertrand game, an undercutting argument suggests that a zero-profit condition is crucial in determining p_i^* here, too.) How do the posted prices compare to cost? Are they guaranteed to be positive? How is surplus allocated across consumers?

b. Can you give examples of real-world products that seem to be priced as in part (a)?

c. Suppose that one of the firms, say firm 2, can deviate to an advertising strategy. Advertising has several effects. First, it converts myopic consumers into sophisticated ones (who rationally forecast shrouded fees and who can avoid them at cost e). Second, it allows firm 2 to make both p_2 and s_2 transparent to all types of consumers. Show that this deviation is unprofitable if

$$e < \left(\frac{1 - \alpha}{\alpha}\right)(v - c).$$

d. Hence we have shown that even costless advertising need not result in unshrouding. Explain the forces leading advertising to be an unprofitable deviation.

e. Return to the case in part (a) with no advertising, but now suppose firms cannot post negative prices. (One reason is that sophisticated consumers could exact huge losses by purchasing an untold number of units to earn the negative price, which are simply disposed of.) Compute the Nash equilibrium. How does it compare to part (a)? Can firms earn positive profits?

Suggestions for Further Reading

Carlton, D. W., and J. M. Perloff. *Modern Industrial Organization*, 4th ed. Boston: Addison-Wesley, 2005.

Classic undergraduate text on industrial organization that covers theoretical and empirical issues.

Kwoka, J. E., Jr., and L. J. White. *The Antitrust Revolution*, 4th ed. New York: Oxford University Press, 2004.

Summarizes economic arguments on both sides of a score of important recent antitrust cases. Demonstrates the policy relevance of the theory developed in this chapter.

Pepall, L., D. J. Richards, and G. Norman. *Industrial Organization: Contemporary Theory and Practice*, 2nd ed. Cincinnati, OH: Thomson South-Western, 2002.

[19] X. Gabaix and D. Laibson, "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *Quarterly Journal of Economics* (May 2006): 461–504.

An undergraduate textbook providing a simple but thorough treatment of oligopoly theory. Uses the Hotelling model in a variety of additional applications including advertising.

Sutton, J. *Sunk Costs and Market Structure*. Cambridge, MA: MIT Press, 1991.

Argues that the robust predictions of oligopoly theory regard the size and nature of sunk costs. Second half provides detailed case studies of competition in various manufacturing industries.

Tirole, J. *The Theory of Industrial Organization*. Cambridge, MA: MIT Press, 1988.

A comprehensive survey of the topics discussed in this chapter and a host of others. Standard text used in graduate courses, but selected sections are accessible to advanced undergraduates.

19.10.2 The Clarke mechanism

A similar mechanism was proposed by E. Clarke, also in the early 1970s.[18] This mechanism also envisions asking individuals about their net valuations for some public project, but it focuses mainly on "pivotal voters"—those whose reported valuations can change the overall evaluation from negative to positive or vice versa. For all other voters, there are no special transfers, on the presumption that reporting a nonpivotal valuation will not change either the decision or the (zero) payment, so he or she might as well report truthfully. For voters reporting pivotal valuations, however, the Clarke mechanism incorporates a Pigovian-like tax (or transfer) to encourage truth telling. To see how this works, suppose that the net valuations reported by all other voters are negative $(\sum_{j \neq i} \tilde{v}_j < 0)$, but that a truthful statement of the valuation by person i would make the project acceptable $(v_i + \sum_{j \neq i} \tilde{v}_j > 0)$. Here, as for the Groves mechanism, a transfer of $t_i + \sum_{j \neq i} \tilde{v}_j$ (which in this case would be negative—i.e., a tax) would encourage this pivotal voter to report $\tilde{v}_i = v_i$. Similarly, if all other individuals reported valuations favorable to a project $(\sum_{j \neq i} \tilde{v}_j > 0)$ but inclusion of person i's evaluation of the project would make it unfavorable, then a transfer of $t_i = \sum_{j \neq i} \tilde{v}_j$ (which in this case is positive) would encourage this pivotal voter to choose $\tilde{v}_i = v_i$ also. Overall, then, the Clarke mechanism is also truth revealing. Notice that in this case the transfers play much the same role that Pigovian taxes did in our examination of externalities. If other voters view a project as unfavorable, then voter i must compensate them for accepting it. On the other hand, if other voters find the project acceptable, then voter i must be sufficiently against the project that he or she cannot be "bribed" by other voters into accepting it.

19.10.3 Generalizations

The voter mechanisms we have been describing are sometimes called *VCG mechanisms* after the three pioneering economists in this area of research (Vickrey, Clarke, and Groves). These mechanisms can be generalized to include multiple governmental projects, alternative concepts of voter equilibrium, or an infinite number of voters. One assumption behind the mechanisms that does not seem amenable to generalization is the quasi-linear utility functions that we have been using throughout. Whether this assumption provides a good approximation for modeling political decision making remains an open question, however.

Summary

In this chapter we have examined market failures that arise from externality (or spillover) effects involved in the consumption or production of certain types of goods. In some cases it may be possible to design mechanisms to cope with these externalities in a market setting, but important limits are involved in such solutions. Some specific issues we examined were as follows.

- Externalities may cause a misallocation of resources because of a divergence between private and social marginal cost. Traditional solutions to this divergence include mergers among the affected parties and adoption of suitable (Pigovian) taxes or subsidies.
- If transaction costs are small, then private bargaining among the parties affected by an externality may bring social and private costs into line. The proof that resources will be efficiently allocated under such circumstances is sometimes called the *Coase theorem*.
- Public goods provide benefits to individuals on a nonexclusive basis—no one can be prevented from consuming such goods. Such goods are also usually nonrival in that the marginal cost of serving another user is zero.
- Private markets will tend to underallocate resources to public goods because no single buyer can appropriate all of the benefits that such goods provide.

[18]E. Clarke, "Multipart Pricing for Public Goods," *Public Choice* (Fall 1971): 19–33.

- A Lindahl optimal tax-sharing scheme can result in an efficient allocation of resources to the production of public goods. However, computing these tax shares requires substantial information that individuals have incentives to hide.
- Majority rule voting does not necessarily lead to an efficient allocation of resources to public goods. The median voter theorem provides a useful way of modeling the actual outcomes from majority rule in certain situations.
- Several truth-revealing voting mechanisms have been developed. Whether these are robust to the special assumptions made or capable of practical application remain unresolved questions.

Problems

19.1

A firm in a perfectly competitive industry has patented a new process for making widgets. The new process lowers the firm's average cost, meaning that this firm alone (although still a price taker) can earn real economic profits in the long run.

a. If the market price is \$20 per widget and the firm's marginal cost is given by $MC = 0.4q$, where q is the daily widget production for the firm, how many widgets will the firm produce?

b. Suppose a government study has found that the firm's new process is polluting the air and estimates the social marginal cost of widget production by this firm to be $SMC = 0.5q$. If the market price is still \$20, what is the socially optimal level of production for the firm? What should be the rate of a government-imposed excise tax to bring about this optimal level of production?

c. Graph your results.

19.2

On the island of Pago Pago there are 2 lakes and 20 anglers. Each angler can fish on either lake and keep the average catch on his particular lake. On Lake x, the total number of fish caught is given by

$$F^x = 10l_x - \frac{1}{2}l_x^2,$$

where l_x is the number of people fishing on the lake. For Lake y, the relationship is

$$F^y = 5l_y.$$

a. Under this organization of society, what will be the total number of fish caught?

b. The chief of Pago Pago, having once read an economics book, believes it is possible to increase the total number of fish caught by restricting the number of people allowed to fish on Lake x. What number should be allowed to fish on Lake x in order to maximize the total catch of fish? What is the number of fish caught in this situation?

c. Being opposed to coercion, the chief decides to require a fishing license for Lake x. If the licensing procedure is to bring about the optimal allocation of labor, what should the cost of a license be (in terms of fish)?

d. Explain how this example sheds light on the connection between property rights and externalities.

19.3

Suppose the oil industry in Utopia is perfectly competitive and that all firms draw oil from a single (and practically inexhaustible) pool. Assume that each competitor believes that it can sell all the oil it can produce at a stable world price of \$10 per barrel and that the cost of operating a well for 1 year is \$1,000.

Total output per year (Q) of the oil field is a function of the number of wells (n) operating in the field. In particular,

$$Q = 500n - n^2,$$

and the amount of oil produced by each well (q) is given by

$$q = \frac{Q}{n} = 500 - n. \qquad \textbf{(19.72)}$$

a. Describe the equilibrium output and the equilibrium number of wells in this perfectly competitive case. Is there a divergence between private and social marginal cost in the industry?

b. Suppose now that the government nationalizes the oil field. How many oil wells should it operate? What will total output be? What will the output per well be?

c. As an alternative to nationalization, the Utopian government is considering an annual license fee per well to discourage overdrilling. How large should this license fee be if it is to prompt the industry to drill the optimal number of wells?

19.4

There is considerable legal controversy about product safety. Two extreme positions might be termed *caveat emptor* (let the buyer beware) and *caveat vendor* (let the seller beware).

Under the former scheme producers would have no responsibility for the safety of their products: Buyers would absorb all losses. Under the latter scheme this liability assignment would be reversed: Firms would be completely responsible under law for losses incurred from unsafe products. Using simple supply and demand analysis, discuss how the assignment of such liability might affect the allocation of resources. Would safer products be produced if firms were strictly liable under law? How do possible information asymmetries affect your results?

19.5

Suppose a monopoly produces a harmful externality. Use the concept of consumer surplus in a partial equilibrium diagram to analyze whether an optimal tax on the polluter would necessarily be a welfare improvement.

19.6

Suppose there are only two individuals in society. Person A's demand curve for mosquito control is given by

$$q_a = 100 - p;$$

for person B, the demand curve for mosquito control is given by

$$q_b = 200 - p.$$

a. Suppose mosquito control is a pure public good; that is, once it is produced, everyone benefits from it. What would be the optimal level of this activity if it could be produced at a constant marginal cost of \$120 per unit?

b. If mosquito control were left to the private market, how much might be produced? Does your answer depend on what each person assumes the other will do?

c. If the government were to produce the optimal amount of mosquito control, how much will this cost? How should the tax bill for this amount be allocated between the individuals if they are to share it in proportion to benefits received from mosquito control?

19.7

Suppose the production possibility frontier for an economy that produces one public good (x) and one private good (y) is given by

$$100x^2 + y^2 = 5{,}000.$$

This economy is populated by 100 identical individuals, each with a utility function of the form

$$\text{utility} = \sqrt{xy_i},$$

where y_i is the individual's share of private good production ($= y/100$). Notice that the public good is nonexclusive and that everyone benefits equally from its level of production.

a. If the market for x and y were perfectly competitive, what levels of those goods would be produced? What would the typical individual's utility be in this situation?

b. What are the optimal production levels for x and y? What would the typical individual's utility level be? How should consumption of good y be taxed to achieve this result? *Hint:* The numbers in this problem do not come out evenly, and some approximations should suffice.

Analytical Problems

19.8 More on Lindahl equilibrium

The analysis of public goods in this chapter exclusively used a model with only two individuals. The results are readily generalized to n persons—a generalization pursued in this problem.

a. With n persons in an economy, what is the condition for efficient production of a public good? Explain how the characteristics of the public good are reflected in these conditions.

b. What is the Nash equilibrium in the provision of this public good to n persons? Explain why this equilibrium is inefficient. Also explain why the underprovision of this public good is more severe than in the two-person cases studied in the chapter.

c. How is the Lindahl solution generalized to n persons? Is the existence of a Lindahl equilibrium guaranteed in this more complex model?

19.9 Taxing pollution

Suppose that there are n firms each producing the same good but with differing production functions. Output for each of these firms depends only on labor input, so the functions take the form $q_i = f_i(l_i)$. In its production activities each firm also produces some pollution, the amount of which is determined by a firm-specific function of labor input of the form $g_i(l_i)$.

a. Suppose that the government wishes to place a cap of amount K on total pollution. What is the efficient allocation of labor among firms?

b. Will a uniform Pigovian tax on the output of each firm achieve the efficient allocation described in part (a)?

c. Suppose that, instead of taxing output, the Pigovian tax is applied to each unit of pollution. How should this tax be set? Will the tax yield the efficient allocation described in part (a)?

d. What are the implications of the problem for adopting pollution control strategies? (For more on this topic see the Extensions to this chapter.)

19.10 Vote trading

Suppose there are three individuals in society trying to rank three social states (A, B, and C). For each of the methods of social choice indicated, develop an example to show how the resulting social ranking of A, B, and C will be intransitive (as in the paradox of voting) or indeterminate.

a. Majority rule without vote trading.

b. Majority rule with vote trading.

c. Point voting where each voter can give 1, 2, or 3 points to each alternative and the alternative with the highest point total is selected.

19.11 Public choice of unemployment benefits

Suppose individuals face a probability of u that they will be unemployed next year. If they are unemployed they will receive unemployment benefits of b, whereas if they are employed they receive $w(1 - t)$, where t is the tax used to finance unemployment benefits. Unemployment benefits are constrained by the government budget constraint $ub = tw(1-u)$.

a. Suppose the individual's utility function is given by

$$U = (y_i)^\delta/\delta,$$

where $1 - \delta$ is the degree of constant relative risk aversion. What would be the utility-maximizing choices for b and t?

b. How would the utility-maximizing choices for b and t respond to changes in the probability of unemployment, u?

c. How would b and t change in response to changes in the risk aversion parameter δ?

19.12 Probabilistic voting

Probabilistic voting is a way of modeling the voting process that introduces continuity into individuals' voting decisions. In this way, calculus-type derivations become possible. To take an especially simple form of this approach, suppose there are n voters and two candidates (labeled A and B) for elective office. Each candidate proposes a platform that promises a net gain or loss to each voter. These platforms are denoted by θ_i^A and θ_i^B, where $i = 1, \ldots, n$. The probability that a given voter will vote for candidate A is given by $\pi_i^A = f(U_i(\theta_i^A) - U_i(\theta_i^B))$, where $f' > 0 > f''$. The probability that the voter will vote for candidate B is $\pi_i^B = 1 - \pi_i^A$.

a. How should each candidate choose his or her platform so as to maximize the probability of winning the election subject to the constraint $\sum_i \theta_i^A = \sum_i \theta_i^B = 0$? (Do these constraints seem to apply to actual political candidates?)

b. Will there exist a Nash equilibrium in platform strategies for the two candidates?

c. Will the platform adopted by the candidates be socially optimal in the sense of maximizing a utilitarian social welfare? [Social welfare is given by $SW = \sum_i U_i(\theta_i)$.]

Suggestions for Further Reading

Alchian, A., and H. Demsetz. "Production, Information Costs, and Economic Organization." *American Economic Review* 62 (December 1972): 777–95.

Uses externality arguments to develop a theory of economic organizations.

Barzel, Y. *Economic Analysis of Property Rights.* Cambridge: Cambridge University Press, 1989.

Provides a graphical analysis of several economic questions that are illuminated through use of the property rights paradigm.

Black, D. "On the Rationale of Group Decision Making." *Journal of Political Economy* (February 1948): 23–34. Reprinted in K. J. Arrow and T. Scitovsky, Eds., *Readings in Welfare Economics.* Homewood, IL: Richard D. Irwin, 1969.

Early development of the median voter theorem.

Buchanan, J. M., and G. Tullock. *The Calculus of Consent.* Ann Arbor: University of Michigan Press, 1962.

Classic analysis of the properties of various voting schemes.

Cheung, S. N. S. "The Fable of the Bees: An Economic Investigation." *Journal of Law and Economics* 16 (April 1973): 11–33.

Empirical study of how the famous bee-orchard owner externality is handled by private markets in the state of Washington.

Coase, R. H. "The Market for Goods and the Market for Ideas." *American Economic Review* 64 (May 1974): 384–91.

Speculative article about notions of externalities and regulation in the "marketplace of ideas."

———. "The Problem of Social Cost." *Journal of Law and Economics* 3 (October 1960): 1–44.

Classic article on externalities. Many fascinating historical legal cases.

Cornes, R., and T. Sandler. *The Theory of Externalities, Public Goods, and Club Goods.* Cambridge: Cambridge University Press, 1986.

Good theoretical analysis of many of the issues raised in this chapter. Good discussions of the connections between returns to scale, excludability, and club goods.

Demsetz, H. "Toward a Theory of Property Rights." *American Economic Review, Papers and Proceedings* 57 (May 1967): 347–59.

Brief development of a plausible theory of how societies come to define property rights.

Mas-Colell, A., M. D. Whinston, and J. R. Green. *Microeconomic Theory.* New York: Oxford University Press, 1995.

Chapter 11 covers much of the same ground as this chapter does, though at a somewhat more abstract level.

Olson, M. *The Logic of Collective Action.* Cambridge, MA: Harvard University Press, 1965.

Analyzes the effects of individual incentives on the willingness to undertake collective action. Many fascinating examples.

Persson, T., and G. Tabellini. *Political Economics: Explaining Economic Policy.* Cambridge, MA: MIT Press, 2000.

A complete summary of recent models of political choices. Covers voting models and issues of institutional frameworks.

Posner, R. A. *Economic Analysis of Law*, 5th ed. Boston: Little, Brown, 1998.

In many respects the "bible" of the law and economics movement. Posner's arguments are not always economically correct but are unfailingly interesting and provocative.

Samuelson, P. A. "The Pure Theory of Public Expenditures." *Review of Economics and Statistics* 36 (November 1954): 387–89. Classic statement of the efficiency conditions for public goods production.