Homework 7 Solution

Choose the best answer

1. The Nash equilibrium in a Bertrand game in which firms produce perfect substitutes and have equal marginal costs is

a. efficient because all mutually beneficial transactions will occur.
b. efficient because of the free entry assumption.
c. inefficient because some mutually beneficial transactions will be foregone.
d. inefficient because of the uncertainties inherent in the game.

2. All of the following are problems associated with maintaining a cartel except

a. cartels are illegal.
b. a large amount of information is needed to coordinate a cartel.
c. profits are not maximized by a cartel so it will evolve into a monopoly.
d. each member of the cartel has an incentive to "chisel" by expanding output

3. Which of the following factors might explain why the long-run equilibrium number of firms can in some instances exceed the socially optimal number?

a. The appropriability effect (the increase in consumers surplus following entry is not "appropriated" by entrants).
b. The feedback effect (an increase in the number of firms increases the competitiveness of the market).
c. The business-stealing effect (entry reduces rival firms' profits, a social loss that entrants do not account for).
d. The ratchet effect (the more profits the entrants earn, the more the stockholders expect them to earn in the future).

4. Suppose the more a firm invests in a new production technology, the lower its marginal costs. Which of the following scenarios involving this incumbent firm and a potential entrant makes the **least** economic sense?

a. The incumbent overinvests to deter entry when this investment is observable to the entrant.
b. The incumbent overinvests to deter entry when this investment is unobservable to the entrant.
c. The incumbent underinvests to accommodate entry when this investment is observable and they compete in prices.
d. The incumbent overinvests to accommodate entry when this investment is observable and they compete in quantities.

Analytical questions

1. Two firms $i = 1, 2$ engage in Cournot quantity competition. Each firm chooses its output level q_i. The inverse demand of the market is $p(q_i, q_j) = a - q_i - q_j$. The total cost of firm i depends both firm i and firm j's production: If the output is q_i, firm i's total cost is $(q_i - q_j)q_i$; firm j's total cost is $(q_j - q_i)q_j$.

a. What will the outcome be if the two firms choose their outputs simultaneously?

$$\max_{q_i} \pi_i(q_i, q_j) = (a - q_i - q_j)q_i - (q_i - q_j)q_i = (a - 2q_i)q_i$$

FOC yields

$$a - 4q_i = 0 \Rightarrow q_i^* = \frac{a}{4}.$$

NE: $q_1^* = q_2^* = a/4$. Each firm's output does not depend on each other, so this is a dominant strategy.

b. If instead firm 2 chooses its output after observing firm 1's output, what will be the outcome of the game?

Since each firm has a dominant strategy, the timing of the game does not affect the equilibrium and outcome of the game. The outcome is $q_1^* = a/4$, $q_2^* = a/4$.

2. A firm can offer the product at high quality (H) or low quality (L). The consumer needs to decide whether to buy or not without observing the quality of the good. The firm incurs a cost $c_H = 2$ and $c_L = 0$ in producing high-quality and low-quality good, respectively. Consumer obtain value $v_H = 5$ for high-quality good and value $v_L = 2$ for low-quality good. The price of the good is exogenously given at $p = 3$. If the consumer decides to buy, she pays the price p and the firm obtains p. If she chooses not to buy, she gets a payoff of zero.

		Consumer	
		Buy	Not
Firm	H	$p - c_H$, $v_H - p$	$-c_H$, 0
	L	$p - c_L$, $v_L - p$	$-c_L$, 0

a. Compute the payoff in each cell of the game. What is the Nash equilibrium of this game? What is the social optimal outcome (where the summation of payoffs is maximized)?

		Consumer	
		Buy	Not
Firm	H	,	, 0
	L	,	, 0

		Consumer	
		Buy	Not
Firm	H	1, $\underline{2}$	-2, 0
	L	$\underline{3}$, -1	$\underline{0}$, $\underline{0}$

The unique NE is (L, Not). The social optimal outcome is (H, Buy).

b. Continue with part (a). Suppose the regulator will punish the firm by a fine $f > 0$ if the firm sells a low-quality product to the consumer. Write down the normal form of the game with the fine. What is the least amount of fine that can induce the social optimal outcome in part (c)?

The game is changed to

		2	
		Buy	Not
1	H	1, $\underline{2}$	-2, 0
	L	$3-f$, -1	$\underline{0}$, $\underline{0}$

To induce the firm to supply at high quality, we need

$$\pi_1(H, Buy) > \pi_1(L, Buy) \Leftrightarrow 1 > 3 - f \Rightarrow f > 2$$

Therefore, the fine needs to be larger than 2.

*Answering $f \geq 2$ also receive full points.

c. Continue with part (a) (Ignore the fine in part (b)). Suppose that the firm and the consumer play this game infinitely number of times with a common discount factor $\delta \in (0, 1)$. The consumer decides to use a strategy that "If I experiences one time of the low-quality product, I will not buy from this firm any more." Translate this into a trigger strategy as contingent actions in period $t = 1, 2, 3, \ldots$.

The trigger strategy is

$$\begin{cases} t = 1 & \text{Buy,} \\ t = 2, 3, \ldots & \begin{cases} \text{Buy} & \text{if (H,Buy) at } t-1, \\ \text{Not} & \text{otherwise.} \end{cases} \end{cases}$$

d. Continue with part (c). Suppose that the consumer adopts the strategy in with part (d). What is the minimum discount factor δ that can induce the firm to choose H at all period of the game?

In the "cooperative" outcome, the firm receives $\pi^C = 1$. In the deviation outcome, the firm receives $\pi^D = 3$. In the NE outcome, $\pi^{NE} = 0$. So the firm to choose H when

$$\delta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}} = \frac{3-1}{3-0} \equiv \frac{2}{3} = \delta_{\min}.$$

3. Consider the following problem of Cournot quantity competition model with asymmetric costs. The market inverse demand is

$$p(q) = 16 - q,$$

where $q = q_1 + q_2$. q_1 denotes quantity produced firm 1. Firm 1 has a constant marginal cost at $c_1 = 1$ (per unit cost is 1). Firm 2 has a constant marginal cost at $c_2 = 2$.

a. Find the Nash (Cournot) equilibrium

$$\pi_1(q_1, q_2) = (16 - (q_1 + q_2)) * q_1 - 1 * q_1$$

$$\pi_2(q_1, q_2) = (16 - (q_1 + q_2)) * q_2 - 2 * q_2$$

$$\frac{\partial \pi_1}{\partial q_1} = 15 - 2q_1 - q_2 = 0$$

$$q_1 = BR_1(q_2) = \frac{1}{2}(15 - q_2)$$

$$\frac{\partial \pi_2}{\partial q_2} = 14 - 2q_2 - q_1 = 0$$

$$q_2 = BR_2(q_1) = \frac{1}{2}(14 - q_1)$$

Solve two unknown from two equations:

$$q_1^* = \frac{16}{3}, \quad q_2^* = \frac{13}{3}.$$

b. Firm 1 and firm 2 are considering whether to merge. They can merge and operate as one firm. However, the marginal cost of production will be 2 for every unit. Is it profitable for them to merge?

If two firm merges, the profit function becomes

$$\pi(q) = (16 - q)q - 2q$$

Maximize profit,

$$\pi'(q) = 16 - 2q - 2 = 0 \Rightarrow q^M = 7$$

$$\pi^M = \pi(q^M) = (16 - 7) * 7 - 2 * 7 = 63 - 14 = 49.$$

Compare the profit with the joint profit when they operate separately as in part (a)

$$q_1 + q_2 = \frac{29}{3}$$

$$\pi_1^{NE} = (16 - \frac{29}{3}) * \frac{16}{3} - \frac{16}{3} = \frac{256}{9}$$

$$\pi_2^{NE} = (16 - \frac{29}{3}) * 1\frac{13}{3} - 2 \times \frac{13}{3} = \frac{169}{9}$$

$$\pi_1^{NE} + \pi_2^{NE} \approx 47.22 < \pi^M = 49.$$

Because the joint profit of merging is greater than two firm operate separately, so it is profitable for them to merge. (As long as the sum is larger, they can divide the profit in a way so that both firms have incentive to be in the merge).

4. Consider two firms $i = 1, 2$ in a certain industry. They want to maximize their profit, they have no marginal cost of producing goods.

Consumers are uniformly distributed along the interval $x \in [0, 1]$

$t = 1$, two firms choose location a, b simultaneously, with restriction $a \in [0, \frac{1}{2}], b \in (\frac{1}{2}, 1]$.

$t = 2$, two firms choose their prices p_1, p_2 simultaneously, $p_i > 0$.

Consumer has quadratic transportation cost d^2 for traveling distance d. Each consumer will buy one product no matter how much it cost, and he will buy from the firm with cheaper cost.

a. Find demand functions q_1, q_2 as choice variables

Consider consumer x, his transportation cost

$$c = \begin{cases} (x-a)^2, & \text{buy from firm 1} \\ (x-b)^2, & \text{buy from firm 2} \end{cases}$$

Find indifference consume

$$p_1 + (x-a)^2 = p_2 + (x-b)^2 \Rightarrow \hat{x}$$

Demand from firm 1

$$q_1(p_1, p_2, a, b) = \hat{x} = \frac{p_2 - p_1}{2(b-a)} + \frac{a+b}{2}$$

Demand from firm 2

$$q_2(p_1, p_2, a, b) = 1 - \hat{x} = \frac{p_1 - p_2}{2(b-a)} + \frac{2-a-b}{2}$$

b. Taking a, b as given, find the optimal pricing scheme (best responses functions) of each firm and $t = 2$ Nash Equilibrium. Are prices strategic complement or substitute?

$$\max_{p_1} \pi_1(p_1, p_2, a, b) = q_1(p_1, p_2, a, b) \times p_1 = \left[\frac{p_2 - p_1}{2(b-a)} + \frac{a+b}{2}\right] \times p_1$$

$$\max_{p_1} \pi_2(p_1, p_2, a, b) = q_2(p_1, p_2, a, b) \times p_2 = \left[\frac{p_1 - p_2}{2(b-a)} + \frac{2-a-b}{2}\right] \times p_2$$

The profit function is concave in p_i, no need to consider corner solution.
FOC's

$$\begin{cases} \frac{\partial \pi_1}{\partial p_1} = \frac{p_2 - 2p_1}{2(b-a)} + \frac{a+b}{2} = 0 \\ \frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2}{2(b-a)} + \frac{2-a-b}{2} = 0 \end{cases}$$

Best responses

$$\begin{cases} BR_1^{t=2} = p_1(p_2, a, b) = \frac{p_2}{2} + \frac{(a+b)(b-a)}{2} \\ BR_2^{t=2} = p_2(p_1, a, b) = \frac{p_1}{2} + \frac{(2-a-b)(b-a)}{2} \end{cases}$$

$t = 2$ NE

$$\begin{cases} p_1^*(a,b) = \frac{(2+b+a)(b-a)}{3} = \frac{2}{3}(b-a) + \frac{1}{3}(b^2 - a^2) \\ p_2^*(a,b) = \frac{(4-a-b)(b-a)}{3} = \frac{4}{3}(b-a) - \frac{1}{3}(b^2 - a^2) \end{cases}$$

Best responses positively slope: firm 1 being aggressive by cutting price, firm 2 respond by also being aggressive and cutting price. Hence strategic complement.

c. Solve for $t = 1$ Nash equilibrium of location choice. Find the outcome of the entire game.

$$
\begin{aligned}
\max_a \pi_1(p_1^*(a,b), p_2^*(a,b), a, b) &= \left[\frac{\frac{(4-a-b)(b-a)}{3} - \frac{(2+b+a)(b-a)}{3}}{2(b-a)} + \frac{a+b}{2}\right] \times \frac{(2+b+a)(b-a)}{3} \\
&= \frac{2+a+b}{6} \times \frac{(2+b+a)(b-a)}{3} = \frac{(2+b+a)^2(b-a)}{18}
\end{aligned}
$$

$$
\begin{aligned}
\max_b \pi_2(p_1^*(a,b), p_2^*(a,b), a, b) &= \left[\frac{\frac{(2+b+a)(b-a)}{3} - \frac{(4-a-b)(b-a)}{3}}{2(b-a)} + \frac{2-a-b}{2}\right] \times \frac{(4-a-b)(b-a)}{3} \\
&= \frac{(4-a-b)}{6} \times \frac{(4-a-b)(b-a)}{3} = \frac{(4-a-b)^2(b-a)}{18}
\end{aligned}
$$

FOC's, because $a \in [0, \frac{1}{2}], b \in [\frac{1}{2}, 1]$, we don't have interior solution

$$
\begin{cases}
\frac{\partial \pi_1}{\partial a} = \frac{\overbrace{(2+b+a)}^{+}\overbrace{(b-3a-2)}^{-}}{18} < 0 \\
\frac{\partial \pi_2}{\partial b} = \frac{\overbrace{(4-a-b)}^{+}\overbrace{(4-3b+a)}^{+}}{18} > 0
\end{cases}
\Rightarrow
\begin{cases}
a^* = 0 \\
b^* = 1
\end{cases}
$$

So the $t = 1$ NE is $(a^* = 0, b^* = 1)$, where both firms located at the end of the interval (maximum differentiation).

Outcome: $\{p_1^* = 1, p_2^* = 1; a^* = 0, b^* = 1\}$

Note that the SPE is $\left\{p_1^*(a,b) = \frac{(2+b+a)(b-a)}{3}, p_2^*(a,b) = \frac{(4-a-b)(b-a)}{3}; a^* = 0, b^* = 1\right\}$

d. Consider a new game: firm 1 is leader and can commit on his location a first. Firm 2 is follower, it observes a and choose b. Then both firms choose their prices simultaneously. What's the outcome of location choice of firm 1 and 2?

Notice that, we cannot get firm 2's best response by setting $\frac{\partial \pi_2}{\partial b} = 0$. For any value of $a \in [0, \frac{1}{2}]$, $\frac{\partial \pi_2}{\partial b}$ is always negative and firm 2's dominant strategy of location choice is $b^* =$ [redacted]. So firm 1's choice of a cannot affect firm 2's location (Cannot push firm 2 any further).

At first stage

$$
\frac{\partial \pi_1}{\partial a} = \frac{\overbrace{(2+b+a)}^{+}\overbrace{(0-3a-2)}^{-}}{18} < 0
$$

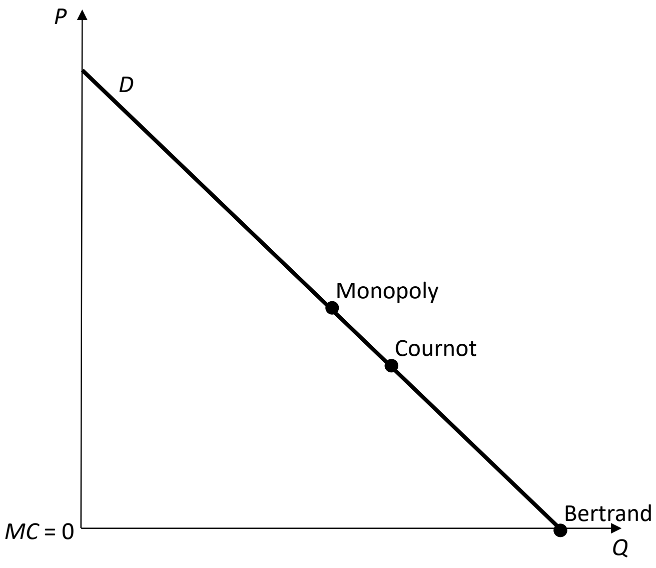
The location choice outcome is still $(a^* = 0, b^* = 1)$.

15.1 a. The monopolist maximizes profit $Q(150 - Q)$ yielding first-order condition $150 - 2Q = 0$ and monopoly outcome $P^m = Q^m = 75$ and $\Pi^m = 5,625.$

b. Cournot firm 1 maximizes profit $q_1(150 - q_1 - q_2)$ yielding first-order condition $150 - 2q_1 - q_2 = 0$ and best-response function $q_1 = 75 - q_2/2.$ Symmetrically, firm 2's best-response function is $q_2 = 75 - q_1/2.$ Solving simultaneously, $q_i^c = 50 = P^c$ and $\pi_i^c = 2,500.$

c. The Nash equilibrium of the Bertrand game is for both firms to charge marginal cost (here zero). Thus, $P^b = 0,$ $Q^b = 150,$ and $\pi_i^b = 0.$

d.



15.2 a. A monopolist maximizes $Q(a - bQ - c),$ yielding first-order condition $a - 2bQ - c = 0$ and the monopoly outcome

$$Q^m = \frac{a-c}{2b},$$

$$P^m = \frac{a+c}{2},$$

$$\Pi^m = \frac{(a-c)^2}{4b}.$$

b. Cournot firm 1 maximizes $q_1[a - b(q_1 + q_2) - c]$, yielding first-order condition $a - 2bq_1 - bq_2 - c = 0$ and best-response function

$$q_1 = \frac{a - bq_2 - c}{2b}.$$

Symmetrically for firm 2,

$$q_2 = \frac{a - bq_1 - c}{2b}.$$

The Nash equilibrium outcome is

$$q_i^c = \frac{a-c}{3b},$$

$$P^c = \frac{a}{3} + \frac{2c}{3},$$

$$\pi_i^c = \frac{(a-c)^2}{9b}.$$

c. The Nash equilibrium of the Bertrand game involves marginal-cost pricing:

$$P^b = c,$$

$$Q^b = \frac{a-c}{b},$$

$$\pi_i^b = 0.$$

d. Cournot firm i maximizes $q_i[a - b(Q_{-i} + q_i) - c]$, yielding first-order condition $a - bQ_{-i} - 2bq_i - c = 0$. Once the first-order condition has been taken, we can apply the fact that firms are symmetric and so the equilibrium will be symmetric. Substituting $Q_{-i}^c = (n-1)q_i^c$ into the first-order condition and solving for q_i^c yields

$$q_i^c = \frac{a-c}{b(n+1)}.$$

Therefore,

$$Q^c = \frac{n}{n+1} \cdot \frac{a-c}{b},$$

$$P^c = \frac{a+nc}{n+1},$$

$$\pi_i^c = \frac{(a-c)^2}{b(n+1)^2},$$

$$\Pi^c = \frac{n(a-c)^2}{b(n+1)^2}.$$

e. It is easy to verify the answers to parts (a)–(c) by making the indicated substitutions for n.

15.4 a. The most reasonable Nash equilibrium is for both firms to charge the high

marginal cost: $p_1^* = p_2^* = 10.$ (The are other Nash equilibria in which

both firms charge equal prices somewhere between 8 and 10, but these equilibria involve weakly dominated strategies for the high-cost firm. See Chapter 18 for further discussion of weakly dominated strategies. Charging a price of, for example, 9 is weakly dominated for firm 1. Charging a price of 10 weakly dominates charging lower prices: firm 1 earns 0 by charging 10 but can earn negative profit if it charges 9 and firm 2 charges a higher price.)

b. Firm 1 earns zero and produces zero. Firm 2 produces

$500-(20 \cdot 10)=300$ and earns $(10-8)(300)=600.$

c. No. The welfare-maximizing outcome is for firm 2 to charge its marginal cost (8). Social welfare in the Nash equilibrium from part (a) can be shown to be 2,850 (the 600 in profit plus 2,250 in consumer surplus). Social welfare in the welfare maximum is 2,890. Deadweight loss equals the difference, 40.