4.1. (a) The predicted average test score is

$$\widehat{TestScore} = 640.3 - 4.93 \times 25 = 517.05$$

(b) The predicted change in the classroom average test score is

The change is

$$\Delta\widehat{TestScore} = (-4.93 \times 24) - (-4.93 \times 21) = -14.79$$

that is, the regression predicts that test scores will fall by 14.79 points.

(c) Using the formula for $\hat{\beta}_0$ in Equation (4.8), we know the sample average of the test scores across the 50 classrooms is

$$\overline{TestScore} = \widehat{\beta_0} + \widehat{\beta_1} \times \overline{CS} = 640.3 - 4.93 \times 22.8 = 527.9$$

(d) Use the formula for the standard error of the regression (SER) in Equation (4.19) to get the sum of squared residuals:

$$SSR = (n-2)SER^2 = (50-2) \times 8.7^2 = 3633.12$$

Use the formula for R^2 in Equation (4.16) to get the total sum of squares:

$$TSS = \frac{SSR}{1-R^2} = \frac{3633.12}{1-0.11} = 4082.16$$

The sample variance is $s_Y^2 = \frac{TSS}{n-1} = \frac{4082.16}{49} = 83.31$. Thus, standard deviation is $s_Y = \sqrt{s_Y^2} = 9.13.$

4.5. (a) u_i represents factors other than time that influence the student's performance on the exam including amount of time studying, aptitude for the material, and so forth. Some students will have studied more than average, other less; some students will have higher than average aptitude for the subject, others lower, and so forth.

(b) Because of random assignment u_i is independent of X_i. Since u_i represents deviations from average $E(u_i) = 0$. Because u and X are independent $E(u_i|X_i) = E(u_i) = 0$. The estimated coefficients are unbiased.

(c) (2) is satisfied if this year's class is typical of other classes, that is, students in this year's class can be viewed as random draws from the population of students that enroll in the class. (3) is satisfied because $0 \le Y_i \le 100$ and X_i can take on only two values (90 and 120).

(d) (i) 55 + 0.17*60 = 65.2; 55 + 0.17*75 = 67.75; 55 + 0.17*90 = 70.3

(ii) 0.17*5 = 0.85.

4.9. (a) With $\hat{\beta}_1 = 0$, $\hat{\beta}_0 = \bar{Y}$, and $\hat{Y}_i = \hat{\beta}_0 = \bar{Y}$. Thus ESS = 0 and $\boldsymbol{R^2} = 0$.

(b) If $\boldsymbol{R^2} = 0$, then ESS = 0, so that $\hat{\boldsymbol{Y}}_{\boldsymbol{i}} = \bar{\boldsymbol{Y}}$ for all i. But, using the formula for $\hat{\boldsymbol{\beta}}_{\boldsymbol{0}}$,

$\widehat{Y_\iota} = \widehat{\beta_0} + \widehat{\beta_1} X_i = \bar{Y} - \widehat{\beta_1}(\bar{X} - X_i)$, Thus $\hat{Y}_i = \bar{Y}$ for all i implies that $\hat{\beta}_1 = 0$, or that X_i is constant for all i. If X_i is constant for all i, then $\sum_{i-1}^{n}(X_i - \bar{X})^2 = 0$ and $\hat{\beta}_1$ is undefined (see equation (4.7)).

4.14. Because $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$, $\bar{Y} = \hat{\beta}_0 + \beta_1 \bar{X}$. The sample regression line is $\boldsymbol{y} = \hat{\boldsymbol{\beta}}_{\boldsymbol{0}} + \hat{\boldsymbol{\beta}}_{\boldsymbol{1}} \boldsymbol{x}$, so that the sample regression line passes through ($\bar{X}, \bar{Y}$).