4. Partial Equilibrium Competitive Model

Market demand

Example

$$x_1 = 10 - 2p_x + 0.1I_1 + 0.5p_y$$
$$x_2 = 17 - p_x + 0.05I_2 + 0.5p_y$$

Market demand is

$$X(p_x, p_y, I_1, I_2) = 27 - 3p_x + 0.1I_1 + 0.05I_2 + p_y$$

(Allocation of demand matters. If $I = I_1 + I_2$ increases, the effect is different for different I_1 and I_2).

Let us set $I_1 = 40$, $I_2 = 20$, $p_y = 4$

The market demand function becomes

$$\begin{aligned} X(p_x) &= 27 - 3p_x + 4 + 1 + 4 \\ &= 36 - 3p_x \end{aligned}$$

If p_y increases to 6, then it causes a shift of demand curve

$$X(p_x) = 38 - 3p_x.$$

Partial equilibrium

Example

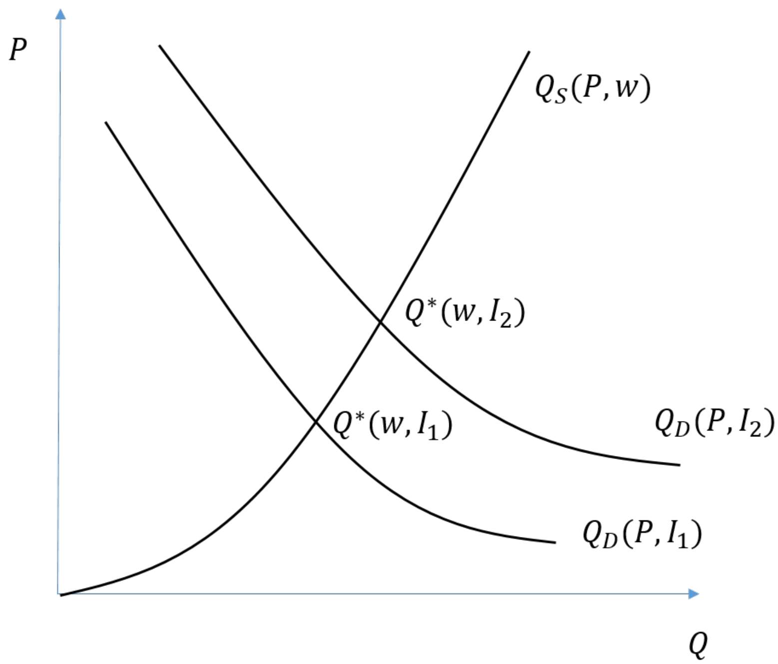
$$\begin{cases} Q_D(P,I) = 0.1 \times P^{-1.2} I^3 \\ Q_S(P,w) = 6400 \times P w^{-0.5} \end{cases}$$

$$e_{D,P} = 0.1 \times (-1.2) P^{-2.2} I^3 \frac{P}{0.1 \times P^{-1.2} I^3} = -1.2$$

$$e_{D,I} = 3$$

$$e_{S,P} = 1$$

$$e_{S,w} = -0.5$$



This functional form of demand and supply has some good properties. They can be connected to empirical study.

Take log on both side $Q_D(P,I) = 0.1 \times P^{-1.2} I^3$

$$\ln Q_D = \ln 0.1 - 1.2 \ln P + 3 \ln I$$

$$Q_D(P,P',I) = AP^{\beta_1} P'^{\beta_2} I^{\beta_3}$$

$$\ln Q_D = \ln A + \beta_1 \ln P + \beta_2 \ln P' + \beta_3 \ln I$$

Coefficients $\beta_1, \beta_2, \beta_3$ are elasticities.

$$\begin{cases} Q_D(P,I) = 0.1 \times P^{-1.2} I^3 \\ Q_S(P,w) = 6400 \times P w^{-0.5} \end{cases}$$

Solve for equilibrium price and quantity

$$0.1 \times P^{-1.2} I^3 = 6400 \times P w^{-0.5}$$

$$P^{2.2} = \frac{0.1 \times I^3}{6400 \times w^{-0.5}} = \frac{I^3 w^{0.5}}{64000}$$

Here, P^* and Q^* are endogenously determined by the partial equilibrium of the market.

Becasue I and w are exogenous variable, so the solution will have the form of

$$\begin{cases} P^*(w,I) = \left(\frac{I^3 w^{0.5}}{64000}\right)^{\frac{5}{11}} \\ Q^*(w,I) = 6400 \times P^* w^{-0.5} \end{cases}$$

Comparative statics prediction

Goal: predict how equilibrium price/quantity change if there is demand/supply shock.

$$\frac{dP^*}{d\alpha},\ \frac{dP^*}{d\beta},\ e_{P,\alpha} = \frac{dP^*}{d\alpha}\frac{\alpha}{P},\ e_{P,\beta} = \frac{dP^*}{d\beta}\frac{\beta}{P}$$

Demand is $Q_D(P,\alpha)$; supply is $Q_S(P,\beta)$
Equilibrium is characterized by

$$Q_D(P^*(\alpha,\beta),\alpha) = Q_S(P^*(\alpha,\beta),\beta)$$

Differentiate both sides with respect to α

$$\frac{\partial Q_D}{\partial P}\frac{\partial P}{\partial \alpha} + \frac{\partial Q_D}{\partial \alpha} = \frac{\partial Q_S}{\partial P}\frac{\partial P}{\partial \alpha}$$

$$\frac{\partial Q_D}{\partial P}\frac{\partial P}{\partial \alpha} - \frac{\partial Q_S}{\partial P}\frac{\partial P}{\partial \alpha} = -\frac{\partial Q_D}{\partial \alpha}$$

$$\frac{\partial P}{\partial \alpha}\left[\frac{\partial Q_D}{\partial P} - \frac{\partial Q_S}{\partial P}\right] = -\frac{\partial Q_D}{\partial \alpha}$$

$$\frac{\partial P}{\partial \alpha} = \frac{\frac{\partial Q_D}{\partial \alpha}}{\frac{\partial Q_S}{\partial P} - \frac{\partial Q_D}{\partial P}}$$

Rewrite the fomula with elasticity

$$\frac{\partial P}{\partial \alpha}\frac{\alpha}{P} = \frac{\frac{\partial Q_D}{\partial \alpha}\alpha}{(\frac{\partial Q_S}{\partial P} - \frac{\partial Q_D}{\partial P})P}$$

$$e_{P,\alpha} = \frac{\partial P}{\partial \alpha}\frac{\alpha}{P} = \frac{\frac{\partial Q_D}{\partial \alpha}\frac{\alpha}{Q}}{(\frac{\partial Q_S}{\partial P} - \frac{\partial Q_D}{\partial P})\frac{P}{Q}} = \frac{e_{D,\alpha}}{e_{S,P} - e_{D,P}}$$

If income increases by 10%, how will equilibrium price P^* change?

$$e_{P,\alpha} = \frac{e_{D,I}}{e_{S,P} - e_{D,P}} = \frac{3}{1-(-1.2)} = \frac{3}{2.2} = \frac{15}{11}$$

$$e_{P,\alpha} = \frac{\text{\% change of } P^*}{\text{\% change of } \alpha} = \frac{\text{\% change of } P^*}{10} = \frac{15}{11}$$

$$\text{\% change of } P^* = \frac{150}{11} = 13.63$$

So the price will rise by 13.63%.
How much will equilibrium quantity Q^* changes?

$$e_{Q,\alpha} = e_{S,P} \times e_{P,\alpha} = 1 \times \frac{15}{11} = 13.63$$

Equilibrium quantity will also rise by 13.63%.

Long run equilibrium

Cost function is

$$C(q) = q^3 - 20q^2 + 100q + 8000$$

The market demand is

$$Q_D = 2500 - 3P$$

(1) Zero-profit condition determines P_{LR}
Zero profit condition is the minimum of average cost

$$AC(q) = q^2 - 20q + 100 + \frac{8000}{q}$$

$$AC'(q) = 2q - 20 - \frac{8000}{q^2} = 0$$

$$q^3 - 10q^2 - 4000 = 0$$

$$q^2(q-10) = 4000$$

$$q_{min} = 20$$

q_{min} is the minimum efficient scale (where average cost is minimized).
The minimum of average cost is

$$\min AC = AC(20) = 400 - 400 + 100 + 8000/20 = 500$$

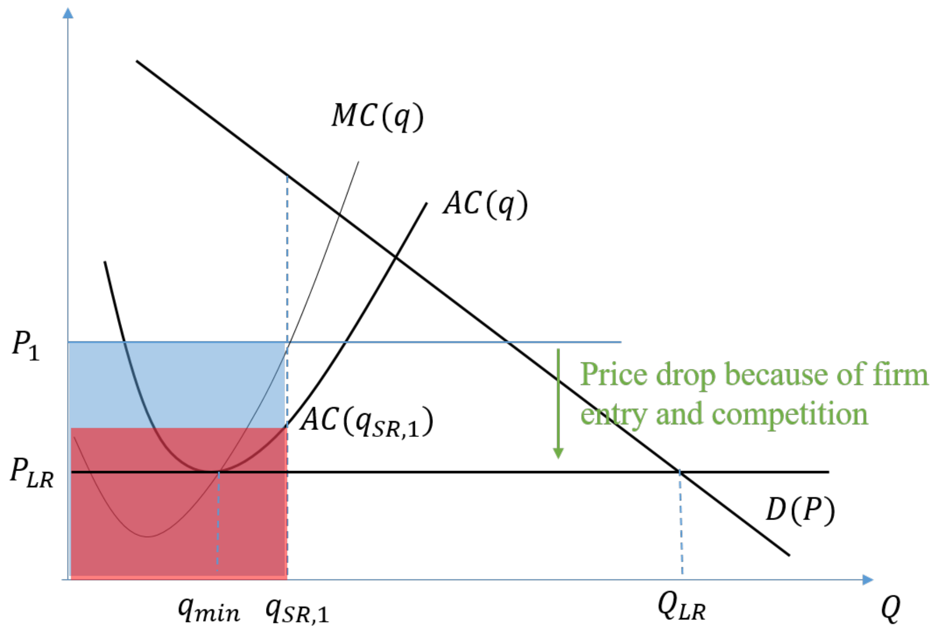
$$P_{LR} = \min AC = 500$$

(2) Demand determines Q_{LR}

$$\begin{aligned} Q_{LR} &= 2500 - 3P \\ &= 2500 - 3P_{LR} = 1000 \end{aligned}$$

(3) Q_{LR} determines n_{LR}

$$n_{LR} = \frac{Q_{LR}}{q_{min}} = \frac{1000}{20} = 50$$



Welfare analysis

Triangluar approximation for deadweight loss caused by distortionary regulation.

$$Q_S(P),\ Q_D(P)$$

With the regulation (quota, rationing, price floor, price ceiling), the equilibrium quantity (Q_1) will be smaller than the equilibrium without regulation (Q^*).

With Q_1, we can find the supply side price (willingness to pay), and the demand side price.

Let the inverse demand be $P_D(Q)$, the demand side price is $P_D(Q_1)$

Let the inverse supply be $P_S(Q)$, the supply side price is $P_S(Q_1)$

The deadweight loss is

$$DWL = Area(\Delta EFG) = \frac{1}{2}|Q^* - Q_1| \times |P_D - P_S|$$

To perform this approximation, we need the functional form of demand/supply.

Example, $\bar{Q} = 3$

$$D(P) = 10 - P$$
$$S(P) = P - 2$$

$$Q^* = 10 - P = P - 2$$
$$P^* = 6,\ Q^* = 4$$

If there is a quota being imposed

$$\bar{Q} = 3 = D(P) = 10 - P \Rightarrow P_D = 7$$
$$\bar{Q} = 3 = S(P) = P - 2 \Rightarrow P_S = 5$$
$$DWL = \frac{1}{2}|Q^* - \bar{Q}| \times |P_D - P_S| = \frac{1}{2} \times 1 \times 2 = 1$$

Example

$$D(P) = 200P^{-1.2}$$

$$S(P) = 1.3P$$

(i) Find equilibirum P and Q

$$D(P) = 200P^{-1.2} = S(P) = 1.3P$$
$$P^{2.2} = \frac{200}{1.3}$$
$$P_1 = (\frac{200}{1.3})^{\frac{5}{11}} = 9.87$$
$$Q_1 = 1.3 \times 9.87 = 12.83$$

(ii) Elasticities

$$e_{S,P} = 1, \quad e_{D,P} = -1.2$$

(iii) Use the formula

$$DWL = -0.5t^2 \frac{e_{S,P}e_{D,P}}{e_{S,P} - e_{D,P}} \frac{Q_1}{P_1}$$
$$= 0.5t^2 \frac{1.2}{2.2} \frac{12.83}{9.87}$$

Note that, we do not need the entire demand or supply curve.

We only need to know the elasticity of demand and supply at the neighborhood of the equilibrium price and quantity.

(iv) Find the tax level to reduce equilibrum quantity to $\bar{Q}$

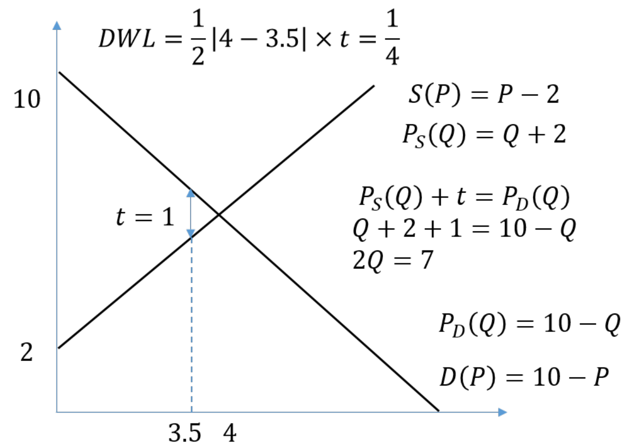
$$P_S(\bar{Q}) + t = P_D(\bar{Q})$$

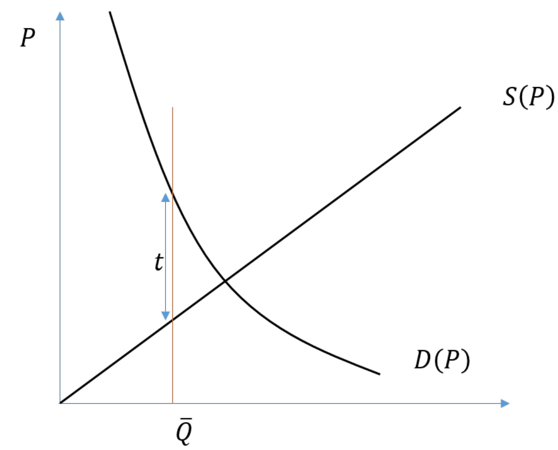
$$D(P) = 200P^{-1.2} \Rightarrow P_D(Q) = (\frac{1}{200})^{-\frac{5}{6}} Q^{-\frac{5}{6}}$$

$$S(P) = 1.3P \Rightarrow P_S(Q) = \frac{10}{13} Q$$
$$\frac{10}{13}\bar{Q} + t = (\frac{1}{200})^{-\frac{5}{6}} \bar{Q}^{-\frac{5}{6}}$$

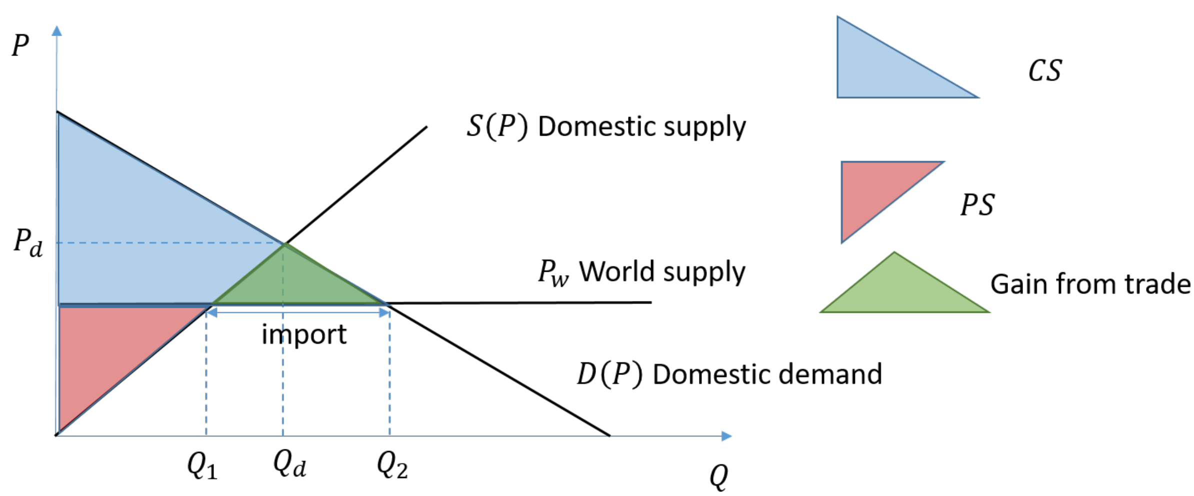
If $\bar{Q} = 11$

$$\frac{10}{13} \times 11 + t = (\frac{1}{200})^{-\frac{5}{6}}(11)^{-\frac{5}{6}}$$

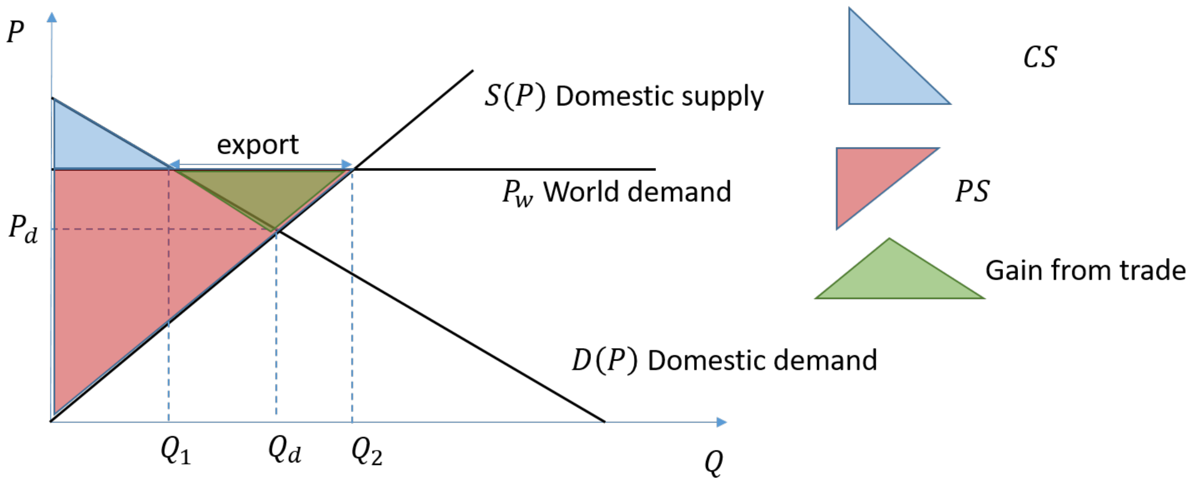




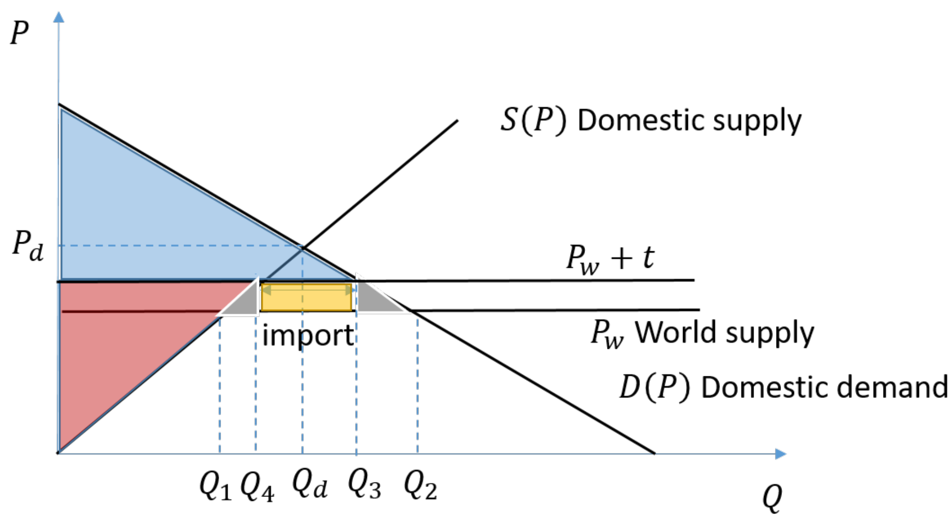
International trade



Q_1 produced by domestic suppliers
Q_2 consumed by domestic consumers
$Q_2 - Q_1$ is the import amount

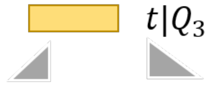


Q_1 sell to domestic consumers
Q_2 produced by domestic supplier
$Q_2 - Q_1$ is the export amount



$Q_3 - Q_4$ is the import amount after tariff t

 $t|Q_3 - Q_4|$ is the tariff revenue

 Loss from trade protection