

PROBLEM SET 5¹

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Question 1: European and American Puts (4/10) You wish to price an European put on a stock which currently trades for \$100. The put expires in nine months, and has a strike of \$100. The nine-month interest rate (annualized continuously compounded) is 5%. The estimated volatility of the stock is 25%. The stock pays no dividends.

Important hint: To solve this question, you could use the excel worksheet on CANVAS titled “LN19_BinomialTreeAmerican.xls”. Doing your own calculations is also fine, but the algebra is very messy.

- (i) What is the Black-Scholes-Merton price of the European put?
- (ii) What is the price of the European put according to a standard 3-step binomial tree?
- (iii) Suppose the standard 3-step binomial tree is the true description of stock price movements in the real world. If the European put is trading for \$6, is there an arbitrage? If not, explain why not. If so, explain in detail what your strategy is.
- (iv) What is the price of the American put according to a 3-step binomial tree?
- (v) Under what circumstances, if any, do you exercise the put before maturity?

Question 2: Binomial Option Pricing (3/10) A share in the company no dividends (ND) currently trades at \$80. The volatility of the stock price is 25% and the expected rate of return is 12%; i.e. $E[S_1] = 80e^{0.12}$. The continuously compounded risk-free rate is 3%. Assume that the volatility, the expected rate of return, and the risk-free rate are constant.

- (i) What is the price of an at the money European call and put option that matures in one year?

Use the Excel macro on CANVAS to determine the price of the at money call for different $h = T/N$. Specifically, consider five different cases for N : $N = 5$, $N = 10$, $N = 50$, and $N = 100$. Use put-call parity to determine the price of the put.

¹Note: optional questions are for your practice only. They are not counted toward your grades.

- (ii) What is the price of an American at-the-money put that matures in one year?
 - (ii.a) Use the Excel macro on CANVAS to determine the price of the American put. Choose $N = 100$.
 - (ii.b) Compare the price of the American put to the price of the European put. Explain intuitively why somebody would like to exercise an American put early.
- (iii) Show that it is never optimal to exercise a call on a non-dividend paying stock early without making any assumptions about the movements of the stock.

Question 3: Implied Volatility and Put-Call Parity (3/10)

Suppose $S = 100$ and there are both a 9-month European call and a 9-month European put with $K = 100$. The continuously compounded risk-free rate is 5%, and there are no payouts.

- (i) The call currently trades at a price of 14.087. What is the Black-Scholes implied volatility?
- (ii) The put trades at an implied volatility of 36.85%. Is there an arbitrage opportunity here? If so, how would you take advantage of it and what are the cash flows?

Question 4: Greeks for Black-Scholes-Merton Model (Optional) Consider the Black-Scholes-Merton Model,

- (i) What are the Delta (Δ) and the Gamma (Γ) of an European call option? You need to show how to derive the formula.
- (ii) What is the Vega of an European call option? You need to show how to derive the formula.