Derivative Securities (FINA 3203)
Solutions to Problem Set 3

March 22, 2020

Question 1: Basic Concepts on Interest Rate Swaps (1/10)

Solution:

(1) First, they are used for risk management by financial and non-financial firms to hedge interest rate risk. Second, they are used by speculators because interest rate swaps require little capital upfront and thus they are often used to make levered bets on interest rate movements. Third, they are used by corporate managers to separate the choice of financing from the type of loans they have access to.

(2) A interest rate swap curve describes the relationship between the fixed swap interest rate r_{fix} and the swap term. The swap rate curve is smoother than the corresponding forward rate curve because a swap rate is a weighted average of the forward rates with different terms to maturity.

Question 2: Basic Concepts on Other Swaps (1/10)

Solution:

(1) A currency swap (or a "cross currency swap") is a foreign exchange OTC derivative/agreement in which two parties agree to exchange the principal and/or interest payments of two loans in one currency for equivalent amounts, in net present value terms, in another currency.

(2) A currency swap exchanges two loans in two different currencies, while a foreign exchange swap exchanges two currencies directly.

Question 3: Forward Rate Agreement (2/10)

Solution:

(i) You should long the FRA offered by bank XYZ and borrow \$1M from bank ABC in 3 months.

Table 1: Cash Flows of Trading Strategy

Strategy Time	0	3mth	15mth
Borrow \$1 M from bank ABC	0	\$1 M	$-\$1\text{M} \times (1+0.12) = -\$1,120,000$
Long FRA offered by bank XYZ	0	$\frac{(0.12-0.10)\times \$1M}{1+0.12} = \$17,857.14$	0
Invest the gain from the FRA	0	$-\$17,857.14$	$\$17,857.14 \times (1+0.12) = \$20,000$
Aggregate	0	\$1M	$-\$1,120,000 + \$20,000 = -\$1,100,000$

(ii) Suppose the one year interest in 3 months is 12%, i.e. $r_{\text{spot}} = 12\%$. The cash flows are: You owe bank ABC \$1, 120, 000 in 15 months. You will receive \$17, 857.14 from Bank XYZ in 3 months. Your actual cost of borrowing are \$100, 000 or 10%.

(iii) Suppose the one year interest in 3 months is 5%, i.e. $r_{\text{spot}} = 5\%$. The cash flows are:

Table 2: Cash Flows of Trading Strategy

Strategy Time	0	3mth	15mth
Borrow \$1 M from bank ABC	0	\$1 M	$-\$1\text{M} \times (1+0.05) = -\$1,050,000$
Long FRA offered by bank XYZ	0	$\frac{(0.05-0.10)\times \$1M}{1+0.05} = -\$47,619.05$	0
Borrow the loss from the FRA	0	$\$47,619.05$	$-\$47,619.05 \times (1+0.05) = -\$50,000$
Aggregate	0	\$1M	$-\$1,050,000 - \$50,000 = -\$1,100,000$

You owe bank ABC \$1, 050, 000 in 15 months. You have to pay bank XYZ \$47, 619.05 in 3 months. Your actual cost of borrowing are \$100, 000 or 10%.

Question 4: Currency Forward Contracts (3/10)

SOLUTION: The current spot price for 100 Euros is $St = (\$/_{€})130$. The continuously compounded US-Dollar interest rate is $r_{\$} = 1\%$ and the continuously compounded Euro interest rate is $r_{€} = 2\%$.

(i) The forward price for 100 Euros is $F_{t,T} = S_t e^{(r_{\$} - r_{€})(T-t)} = (\$/€)130 \times e^{(0.01-0.02)\times 9/12} = (\$/€)129$. Hence, the 9-month forward price for 100 Euros is 129 US-Dollars.

(ii) Note that the Euro is the asset and the 6-month forward price for 1 Euro is 1.2935 US-Dollars. Each contract entitles the long position to receive 125, 000 Euros for $€125,000 \times (\$/€)1.2935 = \$161,687.5$. You short 8 contracts and hence you will sell $8 \times €125,000 = €1,000,000$

for $8 \times €125,000 \times (\$/€)1.2935 = \$1,293,500$. In 6 months you will have no Euro and $3,293,500(= 2,000,000 + 1,293,500)$ US-Dollars.

Question 5: Foreign Exchange Swaps (3/10)

SOLUTION:

(i) The forward price is $F_{0,T} = S_0 e^{(r_\$ - r_¥)\times T}$. Hence,

$$\begin{aligned}
F_{0,1} &= (\$/¥)0.01 \times e^{(0.01-0.01)\times 1} = (\$/¥)0.01 \\
F_{0,2} &= (\$/¥)0.01 \times e^{(0.02-0.01)\times 2} = (\$/¥)0.010202 \\
F_{0,3} &= (\$/¥)0.01 \times e^{(0.03-0.01)\times 3} = (\$/¥)0.010618 \\
F_{0,4} &= (\$/¥)0.01 \times e^{(0.04-0.01)\times 4} = (\$/¥)0.011275 \\
F_{0,5} &= (\$/¥)0.01 \times e^{(0.05-0.01)\times 5} = (\$/¥)0.012214
\end{aligned}$$

(ii) The swap rate is the weighted average of forward rates:

$$\begin{aligned}
X &= \frac{F_{0,1}e^{-0.01} + F_{0,2}e^{0.02\times 2} + F_{0,3}e^{-0.03\times 3} + F_{0,4}e^{-0.04\times 4} + F_{0,5}e^{-0.05\times 5}}{e^{-0.01} + e^{0.02\times 2} + e^{-0.03\times 3} + e^{-0.04\times 4} + e^{-0.05\times 5}} \qquad (1) \\
&= (\$/¥)\frac{0.048527129}{4.49571503} = (\$/¥)0.010794085
\end{aligned}$$

You pay 1079.41 U.S. Dollars each year for 100, 000 Yen.