ECON 3123: Macroeconomic Theory 1

Problem Set #4

Due Date: May 19, 2020

Instructions:

- Please upload your answers on Gradescope by 10:00 pm.
- Late submissions will not be accepted.
- The following clip on how to submit your homework may be useful. (LINK)
- Please put your name and student ID at the upper right corner of the first page.

1. The Solow growth model with a Cobb-Douglas production function

In this question, you will study the Solow growth model with a Cobb-Douglas production function. Consider the following model of economic growth:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{1}$$

$$Y_t = \mathcal{A}_t K_t^{\alpha} N_t^{1-\alpha} \tag{2}$$

$$S_t = sY_t \tag{3}$$

$$I_t = S_t \tag{4}$$

$$\mathcal{A}_t = \mathcal{A} \quad \text{for all } t \tag{5}$$

$$N_t = N \quad \text{for all } t \tag{6}$$

(a) Is the production function (2) characterized by constant returns to scale? Explain.

(b) Transform the production function (2) into a relation between output per worker, y_t, and capital per worker, k_t.

(c) Show that $k_{t+1} = (1-\delta)k_t + s\mathcal{A}k_t^{\alpha}$.

(d) Show that the steady-state level of capital per worker, k^*, is given by $\left(\frac{s\mathcal{A}}{\delta}\right)^{1/(1-\alpha)}$.

(e) Give an expression for output per worker in the steady state, y^*, and consumption per worker in the steady state, c^*. For c^*, assume that $T = 0$.

(f) Explain what happens to the steady state output, Y^*, and output per worker y^*, when N doubles.

2. Numerical analysis of the Solow model

Consider the model in question 1. Here, we will draw some figures using Excel or your favorite software to understand the working of the Solow model.

(a) Let $s = 0.4$, $\delta = 0.4$, $\mathcal{A} = 1, \alpha = 1/3$, and $N = 1$. Using the results in 1(d), compute the value of k^* and y^*.

(b) Draw a graph similar to Figure 11-2 (i.e., plot y_t, sy_t, and δy_t against k_t). You may want to begin with constructing a grid for the value of k_t between 0 and 2 spaced by 0.05 (HINT: look at my spreadsheet below).

	A	B	C	D	E	F	G
1	capital per worker	output per worker	investment per work	depreciation per wor		Parameters	
2	0	=$G$4*A2^$G$5	=$G$2*B2	=$G$3*A2		s	0.4
3	=A2+0.05	=$G$4*A3^$G$5	=$G$2*B3	=$G$3*A3		delta	0.4
4	=A3+0.05	=$G$4*A4^$G$5	=$G$2*B4	=$G$3*A4		A	=1
5	=A4+0.05	=$G$4*A5^$G$5	=$G$2*B5	=$G$3*A5		alpha	=1/3
6	=A5+0.05	=$G$4*A6^$G$5	=$G$2*B6	=$G$3*A6		N	=1

(c) Suppose that this economy in year 0 is very poor. Specifically, $k_0 = 0.01$. Show how this economy grows between year 0 and year 50 using figures similar to Figure 11-7. You need to plot y_t and the growth rate of y_t against t (HINT: look at my spreadsheet below).

	A	B	C	D	E	F	G
1	Years	Capital per worker	Output per worker	Growth rate of outpu		Parameters	
2	0	0.01	=$G$4*B2^$G$5			s	0.4
3	=A2+1	=(1-$G$3)*B2+$G$2*C2	=$G$4*B3^$G$5	=(C3-C2)/C2*100		delta	0.4
4	=A3+1	=(1-$G$3)*B3+$G$2*C3	=$G$4*B4^$G$5	=(C4-C3)/C3*100		A	=1
5	=A4+1	=(1-$G$3)*B4+$G$2*C4	=$G$4*B5^$G$5	=(C5-C4)/C4*100		alpha	=1/3
6	=A5+1	=(1-$G$3)*B5+$G$2*C5	=$G$4*B6^$G$5	=(C6-C5)/C5*100		N	=1
7	=A6+1	=(1-$G$3)*B6+$G$2*C6	=$G$4*B7^$G$5	=(C7-C6)/C6*100			
8	=A7+1	=(1-$G$3)*B7+$G$2*C7	=$G$4*B8^$G$5	=(C8-C7)/C7*100			

(d) How long does it take for this economy to grow half-way close to its steady state? That is, after how many years does k_t becomes greater than $k^*/2$?

(e) Suppose that the economy stays in a steady state associated with $s = 0.4$ for $t \leq 5$. Then, from year 6, the saving rate increases from 0.4 to 0.5. Draw a graph showing how y_t evolves between year 0 and year 20, which should be similar to Figure 11-4 (HINT: look at my spreadsheet below).

	A	B	C	D	E	F
1	Years	Capital per worker	Output per worker		Parameters	
2	0	=1	=$F$4*B2^$F$5		s0	0.4
3	=A2+1	=(1-$F$3)*B2+$F$2*C2	=$F$4*B3^$F$5		delta	0.4
4	=A3+1	=(1-$F$3)*B3+$F$2*C3	=$F$4*B4^$F$5		A	=1
5	=A4+1	=(1-$F$3)*B4+$F$2*C4	=$F$4*B5^$F$5		alpha	=1/3
6	=A5+1	=(1-$F$3)*B5+$F$2*C5	=$F$4*B6^$F$5		N	=1
7	=A6+1	=(1-$F$3)*B6+$F$2*C6	=$F$4*B7^$F$5			
8	=A7+1	=(1-$F$3)*B7+$F$8*C7	=$F$4*B8^$F$5		s1	0.5
9	=A8+1	=(1-$F$3)*B8+$F$8*C8	=$F$4*B9^$F$5			
10	=A9+1	=(1-$F$3)*B9+$F$8*C9	=$F$4*B10^$F$5			

(f) Let's find the golden rule saving rate, which maximizes the steady-state consumption per worker in this economy. Rather than analytically maximizing c^* with respect to $s \in [0,1]$, we will use a numerical optimization toolbox in Excel. We first need to add "Solver Add-in." Click File, Options, and Add-ins. Select Excel Add-ins, click Go, select "Solver Add-in," and click Ok. Then, write a spreadsheet which calculates the value of c^* as a function of s (HINT: look at my spreadsheet below).

	A	B	C	D	E
1	Parameters			Steady-state values	
2	s	0.4		k	=(B2*B4/B3)^(1/(1-B5))
3	delta	0.4		y	=B4*E2^B5
4	A	=1		c	=(1-B2)*E3
5	alpha	=1/3			
6	N	=1			

Then, open "Solver" under Data tab. Note that we want to maximize c^* by changing s subject to the conditions that $s \geq 0$ and $s \leq 1$. What is the golden rule saving rate s_G (HINT: look at my spreadsheet below)?