Problem Set 4[1]

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Question 1: Basic Concepts on Options (2/10)

(1) Explain the difference between European options and American options.

(2) Whether a European put option is just a short position of European call options? Explain your claim.

Question 2: Basic Concepts on Option Trading Strategies (Optional)

(1) Define what a "butterfly spread" is. If you believe the volatility of the underlying stock price is going to increase in the near future, will you find it profitable to take long positions in butterfly spreads?

(2) Define what a "straddle" is. If you believe the volatility of the underlying stock price is going to increase in the near future, will you find it profitable to take long positions in straddles?

Question 3: More on Butterfly Spreads (Optional) Suppose today is March 1st and the continuously compounded interest rate is $r = 10\%$. You decide to create a butterfly spread on GE stock. Specifically,

(1) you write one call on GE with strike \$30 and premium \$1.55,

(2) you write one put on GE with strike \$30 and premium \$0.85,

(3) you buy one put on GE with strike \$27.5 and premium \$0.30,

(4) buy one one call on GE with strike \$32.5 and premium \$0.45.

All four options are European and mature on April 1st.

(i) What is the net premium that you collect/pay on March 1st?

(ii) What are the cash flows (i.e. payoff) of this strategy on April 1st? Consider all possible stock price S_{April}.

(iii) Determine the profits of this strategy on April 1st and draw a profit diagram (NOT a payoff diagram).

[1]Note: optional questions are for your practice only. They are not counted toward your grades.

Question 4: One-Step Binomial Trees for Option Pricing (8/10) Shares in XYZ Corporation sell today for $20. The risk-free rate is 3% (continuously compounded). In the next six months, XYZ shares will either increase in price by 30%, or decrease in price by 40%. In the following six months, XYZ shares will again either increase in price by 30%, or decrease in price by 40%. XYZ pays no dividends.

- **One-Step Binomial Tree**

(1) What is the price of a European call with strike price $19 and expiration in 6 months? Calculate the price explicitly using a binomial tree and the dynamic hedging/replicating method.

(2) What is the price of a European put with strike price $19 and expiration in 6 months? Calculate the price explicitly using a binomial tree and the dynamic hedging/replicating method, and verify that put-call parity holds, together with your solution for (1).

(3) If the put of part (2) currently trades for $1, is there an arbitrage? If so, describe one. If not, why not?

- **Risk-Neutral Pricing**

(4) What are the risk-neutral probabilities.

(5) Write out the risk-neutral pricing formula for the 6-month call option and put option in (1) and (2). Verify that the result of the risk-neutral pricing method is the same as in (1) and (2).

Question 5 (Optional): Exotic Options Suppose Apple's stock is $139 today. There is an exotic option on Apple's stock which gives the investor a choice to buy a share of Apple stock using $156 whenever its price hits $160. Ignore all possible transaction costs and the settlement is all cash. What is the price of the exotic option today?