7. Imperfect Competition

Bertrand model with differential products

Demand for firm 1 is

$$q_1(p_1, p_2) = a - p_1 + \frac{1}{2}p_2.$$

The positive coefficient $\frac{1}{2}$ indicates that product of firm 2 is a substitute to product of firm 1.

Demand for firm 2 is

$$q_2(p_1, p_2) = a - p_2 + \frac{1}{2}p_1.$$

$$u(q_1, q_2) = q_1^{\frac{1}{3}} q_2^{\frac{2}{3}}, \text{ s.t. } p_1 q_1 + p_2 q_2 = m$$

$$q_1^*(\overbrace{p_1}^{-}, \overbrace{p_2}^{+}, \overbrace{m}^{+})$$

Firm 1 chooses price p_1 by solving the following profit max problem

$$\max_{p_1} \pi_1(p_1, p_2) = p_1 q_1(p_1, p_2) - c(q_1(p_1, p_2))$$

For simplicity, set cost to zero.

$$\max_{p_1} \pi_1(p_1, p_2) = p_1 q_1(p_1, p_2) = p_1(a - p_1 + \frac{1}{2}p_2)$$

$$= ap_1 - p_1^2 + \frac{1}{2}p_1 p_2$$

Treating p_2 as given, find the best response of firm 1

$$\frac{\partial \pi_1}{\partial p_1} = a - 2p_1 + \frac{1}{2}p_2 = 0.$$

The optimal price p_1^* is determined by this FOC

$$a - 2p_1^* + \frac{1}{2}p_2 = 0$$

$$2p_1^* = a + \frac{1}{2}p_2$$

$$p_1^* = \frac{1}{2}a + \frac{1}{4}p_2 = BR_1(p_2)$$

$$p_2^* = \frac{1}{2}a + \frac{1}{4}p_1 = BR_2(p_1)$$

NE is determined by mutual best responses

$$\begin{cases} p_1^* = BR_1(p_2^*) \\ p_2^* = BR_2(p_1^*) \end{cases}$$

$$\begin{aligned} p_1^* &= \frac{1}{2}a + \frac{1}{4}(\frac{1}{2}a + \frac{1}{4}p_1^*) \\ &= \frac{1}{2}a + \frac{1}{8}a + \frac{1}{16}p_1^* \end{aligned}$$

$$\frac{15}{16}p_1^* = \frac{5}{8}a$$

$$p_1^* = \frac{2}{3}a, \quad p_2^* = \frac{2}{3}a$$

At this equilibrium, the profit of firm 1 is

$$\pi_1(p_1^*, p_2^*) = ap_1^* - (p_1^*)^2 + \frac{1}{2}p_1^* p_2^*$$

If there is a constant marginal cost c,

$$\begin{aligned} \max_{p_1} \pi_1(p_1, p_2) &= p_1 q_1(p_1, p_2) - c \times q_1(p_1, p_2) \\ &= (p_1 - c)q_1(p_1, p_2) \end{aligned}$$

Hotelling model

At $t = 1$

Firm 1/A chooses location a; firm 2/B chooses location b.

At $t = 2$, a and b are fixed.

Firm 1 chooses p_1, firm 2 chooses p_2.

Suppose each consumer derive a utility v from purchasing one unit. Each consumer only purchase one unit.

Consider consumer x, where x is his location on the (taste) space $[0,1]$.

The payoff from purchasing firm 1's product is

$$u_1 = v - p_1 - \text{transportation cost} = \begin{cases} v - p_1 - \tau|x-a| \\ v - p_1 - \tau(x-a)^2 \end{cases}$$

The payoff from purchasing firm 2's product is

$$u_2 = \begin{cases} v - p_2 - \tau|x-b| \\ v - p_2 - \tau(x-b)^2 \end{cases}$$

Consumer will purchase the one with higher utility. If $u_1 > u_2$, then purchase one unit of good 1.

The indifference consumer, $\hat{x}$, is determined by $u_1 = u_2$.

$$u_1 = v - p_1 - \tau(\hat{x} - a)^2 = u_2 = v - p_2 - \tau(\hat{x} - b)^2$$

$$\tau(\hat{x} - b)^2 - \tau(\hat{x} - a)^2 = p_1 - p_2$$

$$\hat{x}^2 - 2b\hat{x} + b^2 - \hat{x}^2 + 2a\hat{x} - a^2 = \frac{p_1 - p_2}{\tau}$$

$$(2a - 2b)\hat{x} = \frac{p_1 - p_2}{\tau} + a^2 - b^2$$

$$\hat{x} = \frac{p_1 - p_2}{2\tau(a-b)} + \frac{a+b}{2} = \frac{a+b}{2} + \frac{1}{2\tau(a-b)}p_1 - \frac{1}{2\tau(a-b)}p_2$$

Assume that consumers are uniformly distributed. Consumers with location $x < \hat{x}$ will purchase product a; consumers with location $x > \hat{x}$ will purchase product b.

$$q_1(a, b, p_1, p_2) = \hat{x} = \frac{p_1 - p_2}{2\tau(a-b)} + \frac{a+b}{2}$$

$$\begin{aligned} q_2(a, b, p_1, p_2) &= 1 - \hat{x} = 1 - \frac{p_1 - p_2}{2\tau(a-b)} - \frac{a+b}{2} \\ &= \frac{p_2 - p_1}{2\tau(a-b)} + 1 - \frac{a+b}{2} \end{aligned}$$

With demand, we can write firm's profits (assume zero cost)

$$\pi_1(a, b, p_1, p_2) = p_1 \times q_1(a, b, p_1, p_2)$$

$$\pi_2(a, b, p_1, p_2) = p_2 \times q_2(a, b, p_1, p_2)$$

When $a = b$,two products are exactly the same. Homogenous product price competition.
When a and b are closer, then price competition is more intense.
When a and b are further apart, the price competition is weaker.

Solve this game by backward induction
(i) Given a, b, at stage 2, firm 1 solves

$$\max_{p_1} \pi_1(p_1 | a, b, p_2) \Rightarrow p_1 = BR_1^{II}(p_2 | a, b)$$

Firm 2 solves

$$\max_{p_2} \pi_2(p_2 | a, b, p_1) \Rightarrow p_2 = BR_2^{II}(p_1 | a, b)$$

Note that we take $a \leq b$.

$$\max_{p_1} \pi_1(a, b, p_1, p_2) = p_1 \times \left[\frac{p_1 - p_2}{2\tau(a-b)} + \frac{a+b}{2} \right]$$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1 = BR_1^{II}(p_2|a,b)$$

$$\max_{p_2} \pi_2(a,b,p_1,p_2) = p_2 \times \left[\frac{p_2 - p_1}{2\tau(a-b)} + 1 - \frac{a+b}{2}\right]$$

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Rightarrow p_2 = BR_2^{II}(p_1|a,b)$$

Both functions are concave in the choice variable, so there is a unique maximum.

(ii) Stage 2 price competition equilibrium given (a,b)

$$\begin{cases} p_1 = BR_1^{II}(p_2|a,b) \\ p_2 = BR_2^{II}(p_1|a,b) \end{cases} \Rightarrow \begin{cases} p_1^*(a,b) \\ p_2^*(a,b) \end{cases}$$

(iii) At stage 1, knowing how location combinations will result in stage-2 NE $\{p_1^*(a,b), p_2^*(a,b)\}$, firm 1 solves

$$\max_a \pi_1(a,b,p_1^*(a,b),p_2^*(a,b))$$
$$\Rightarrow a = BR_1^I(b)$$

Firm 2 solves

$$\max_b \pi_2(a,b,p_1^*(a,b),p_2^*(a,b))$$
$$\Rightarrow b = BR_2^I(a)$$

Stage 1 NE is

$$\begin{cases} a = BR_1^I(b) \\ b = BR_2^I(a) \end{cases} \Rightarrow (a^*, b^*)$$

(iv) The outcome of this game

$$\{a^*, b^*, p_1^*, p_2^*\} = \{a^*, b^*, p_1^*(a^*,b^*), p_2^*(a^*,b^*)\}\}$$

SPE

$$\{a^*, b^*, p_1^*(a,b), p_2^*(a,b)\}\}$$

Cournot model

Firms choose quantity simultaneously. Price is determined by a market inverse demand $p(Q)$.

(i) In the case of $n = 2$

$Q = q_1 + q_2$. $p(Q) = a - Q = a - q_1 - q_2$, $c(q) = c \times q$

Firm 1's profit is

$$\begin{aligned} \pi_1(q_1,q_2) &= [p(Q) - c] \times q_1 \\ &= [a - q_1 - q_2 - c] \times q_1 \\ &= (a - c - q_2)q_1 - q_1^2 \end{aligned}$$

Firm 2's profit is

$$\pi_2(q_1,q_2) = (a-c-q_1)q_2 - q_2^2$$

To find the NE (Cournot equilibrium)

$$\max_{q_1} \pi_1(q_1,q_2) = (a-c-q_2)q_1 - q_1^2$$

$$\frac{\partial \pi_1}{\partial q_1} = a-c-q_2-2q_1 = 0$$

$$q_1 = BR_1(q_2) = \frac{a-c}{2} - \frac{1}{2}q_2$$

$$\max_{q_2} \pi_2(q_1,q_2) = (a-c-q_1)q_2 - q_2^2$$

$$\frac{\partial \pi_2}{\partial q_2} = a-c-q_1-2q_2 = 0$$

$$q_2 = BR_2(q_1) = \frac{a-c}{2} - \frac{1}{2}q_1$$

Mutual best responses

$$\begin{cases} q_1 = BR_1(q_2) \\ q_2 = BR_2(q_1) \end{cases}$$

$$\begin{aligned} q_1 &= \frac{a-c}{2} - \frac{1}{2}\left(\frac{a-c}{2} - \frac{1}{2}q_1\right) \\ &= \frac{a-c}{2} - \frac{a-c}{4} + \frac{1}{4}q_1 \end{aligned}$$

$$\frac{3}{4}q_1 = \frac{a-c}{4}$$

$$q_1^* = \frac{a-c}{3}, \quad q_2^* = \frac{a-c}{3}.$$

$$Q^{NE} = \frac{2}{3}(a-c)$$

Compute prices and profit

$$p^{NE} = p(Q^{NE}) = p(q_1^*+q_2^*) = a - \frac{2}{3}(a-c) = \frac{3a-2a+2c}{3} = \frac{a+2c}{3}$$

$$\pi_1^{NE} = [p^{NE}-c]q_1^{NE} = \left(\frac{a+2c}{3} - c\right)\frac{a-c}{3} = \frac{1}{9}(a-c)^2$$

$$\pi_2^{NE} = \frac{1}{9}(a-c)^2$$

$$\Pi^{NE} = \pi_1^{NE} + \pi_2^{NE} = \frac{2}{9}(a-c)^2$$

(ii) $n = 1$

If firm 1 is a monopoly ($q_2 = 0$)

$$\pi_1(q_1) = [p(q_1) - c]q_1 = (a - q_1 - c)q_1$$

$$\frac{\partial \pi}{\partial q_1} = a - c - 2q_1 = 0 \Rightarrow q_1^M = \frac{a-c}{2} < Q^{NE}$$

Compute the monopoly profit

$$p^M = p(q_1^M) = a - q_1^M = a - \frac{a-c}{2} = \frac{a+c}{2} > p^{NE}$$

$$\pi_1^M = [p(q_1^M) - c]q_1^M = \left[\frac{a+c}{2} - c\right]\frac{a-c}{2} = \frac{1}{4}(a-c)^2 > \Pi^{NE}$$

Compared the case with Cournot ($n = 2$), monopoly firm produce less output, set higher price, and earn higher profit.

NE is not efficient (not maximizing joint profit of these two firms). (Note that consumer prefer a larger equilibrium quantity level).

The two firms in duopoly have an incentive to coordinate their behavior and even merge.

If the firms can collude, they will each produce half of the monopoly output

$$q_1^C = \frac{a-c}{4}, \;\; q_2^C = \frac{a-c}{4}.$$

(iii) The case of n firms

Market quantity is

$$Q = q_1 + q_2 + \cdots + q_n$$

The price is determined by inverse demand $p(Q) = a - q_1 - q_2 - \cdots - q_n$.

Let q_i be the quantity of firm i, $i = 1, 2, ..., n$.

Let Q_{-i} be $Q_{-i} = \sum_{j \neq i} q_i$.

$$\begin{aligned}\max_{q_1} \pi_1(q_1, q_2, \cdots, q_n) &= (p(Q) - c)q_1 \\ &= (a - q_1 - q_2 - \cdots - q_n - c)q_1\end{aligned}$$

For convenience, we express the problem of firm i as

$$\begin{aligned}\max_{q_i} \pi_i(q_1, q_2, \cdots, q_n) &= (a - q_1 - q_2 - \cdots - q_n - c)q_i \\ &= (a - q_i - Q_{-i} - c)q_i, \;\; i = 1, 2, ..., n\end{aligned}$$

FOC

$$\frac{\partial \pi_i}{\partial q_i} = a - c - Q_{-i} - 2q_i = 0$$

Best response of firm i to all other firms

$$q_i = BR_i(Q_{-i}) = \frac{a-c}{2} - \frac{1}{2}Q_{-i}, \quad i = 1, 2, ..., n$$

Cournot NE is determined by listing all n best response equations together.

Because this problem is symmetric, in the equilibrium,

$$q_1^{NE} = q_2^{NE} = \cdots = q_n^{NE} = q^{NE}.$$

They need to satisfy all best response equations. Plug in firm i's BR equation

$$q^{NE} = \frac{a-c}{2} - \frac{1}{2}\sum_{j\neq i} q^{NE} = \frac{a-c}{2} - \frac{1}{2}(n-1)q^{NE}$$

$$\begin{aligned} q^{NE} + \frac{1}{2}(n-1)q^{NE} &= \frac{a-c}{2} \\ 2q^{NE} + (n-1)q^{NE} &= a-c \\ (n+1)q^{NE} &= a-c \\ q^{NE}(n) &= \frac{a-c}{n+1} \end{aligned}$$

Note that, if $n=1$, $q^{NE} = \frac{a-c}{2} = Q^M$.
If $n=2$, $q^{NE} = \frac{a-c}{3}$, which is the duopoly Cournot NE level.
The market quantity is

$$Q^{NE}(n) = \frac{n(a-c)}{n+1}$$

$$P^{NE}(n) = a - Q^{NE}(n) = a - \frac{n(a-c)}{n+1} = \frac{(n+1)a - na + nc}{n+1} = \frac{a+nc}{n+1}$$

If $n \to \infty$, $P^{NE}(n) \to c$, which is the perfect competition price.

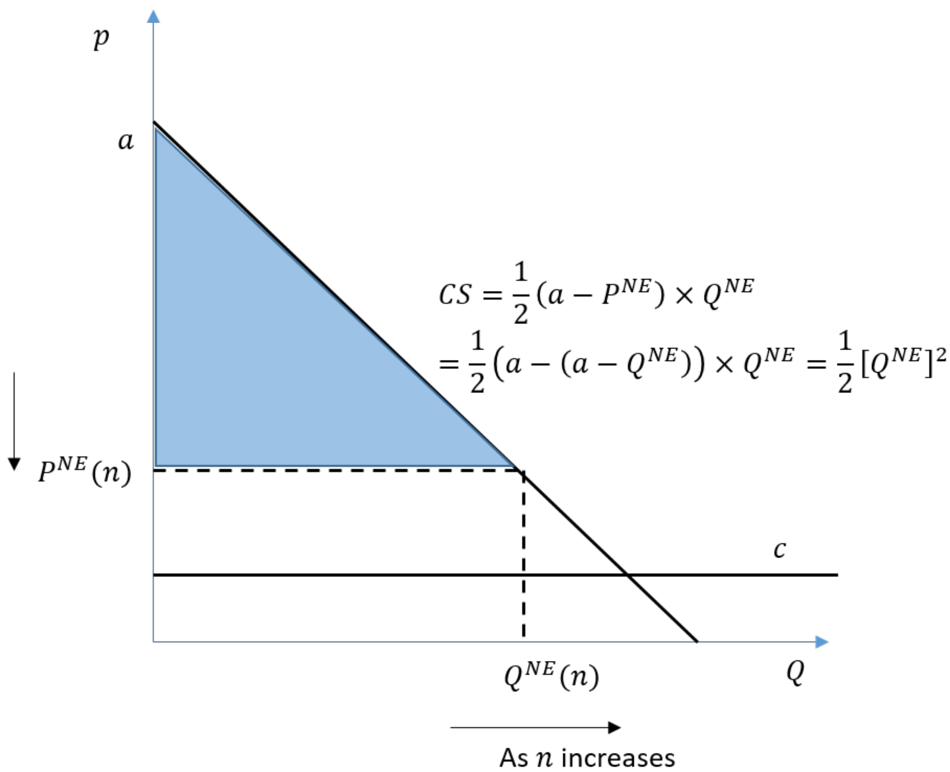
$$\pi^{NE}(n) = [P^{NE}(n) - c]q^{NE}(n) = [\frac{a+nc}{n+1} - c]\frac{a-c}{n+1} = (\frac{a-c}{n+1})^2$$

If $n \to \infty$, $\pi^{NE}(n) \to 0$.

$$\Pi^{NE}(n) = n(\frac{a-c}{n+1})^2$$

$$CS^{NE}(n) = \frac{1}{2}\left[Q^{NE}(n)\right]^2$$

which increases in n.



Tacit Collusion

(1) Homogenous product Bertrand market

$n = 2$, two firms

NE outcome, $p_1 = p_2 = c$, $\pi_1^{NE} = \pi_2^{NE} = 0$.

Cooperative/Collusion outcome,

The most profitable way to collude is setting the price at the monopoly level, $p_1 = p_2 = p^M$.

$$\begin{aligned}
\max_p \Pi(p) &= p \times Q(p) - c \times Q(p) \\
&= p \times (5000 - 100p) - 10(5000 - 100p) \\
&= 5000p - 100p^2 - 50000 + 1000p \\
&= 6000p - 100p^2 - 50000
\end{aligned}$$

$$\Pi'(p) = 6000 - 200p = 0$$
$$p^M = 30$$

$$\begin{aligned}
\Pi^M &= \Pi(30) = 30 \times (5000 - 3000) - 10(5000 - 3000) \\
&= 30 \times 2000 - 10 \times 2000 = 40000
\end{aligned}$$

The two firm divide the profit evenly

$$\pi_1^C = \pi_2^C = \Pi^M/2 = 20000.$$

What is the profit of deviating from the collusion? π^D.

What is the most profitable way to deviate? It is by setting the price slightly below the monopoly level.

$$p^D = p^M - \varepsilon$$

For simplicity, assume that ε is very small. By undercutting the other firm by a little, it can obtain the entire market.

$$\pi^D = \Pi(p = p^M - \varepsilon) = \Pi^M = 40000$$

(If $\varepsilon = 1$, $p^D = 29$, $\pi^D = \Pi(29)$.)

With π^{NE}, π^C and π^D, we can compute the threshold discount factor. The trigger strategy is

$$p_i = \begin{cases} p^M \text{ (collusion)} & \text{if there is no deviation in the past} \\ c \text{ (NE level)} & \text{if there is deviation in the last period.} \end{cases}$$

$\delta \geq \delta_{\min}$ is the condition that the above trigger strategy is the equilibrium strategy. Each firm does not have incentive to deviate from this tacit collusion agreement.

$$\delta_{\min} = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}} = \frac{40000 - 20000}{40000 - 0} = \frac{1}{2}.$$

What if there are more than 2 firms, say there is n firms? Is it more difficulty to maintain tacit collusion?

Player $i = 1, 2, 3, ..., n$ use trigger strategy. Under collusion, firms divide the profit evenly,

$$\pi_i^C = \Pi^M/n = 40000/n.$$

$$\delta_{\min} = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}} = \frac{40000 - 40000/n}{40000 - 0} = \frac{1 - \frac{1}{n}}{1} = 1 - \frac{1}{n}.$$

Therefore, $\delta_{\min}$ increases in n.

Tacit collusion is harder to maintain when there are more firms.

(2) Tacit collusion in Cournot market

$n = 2$

If two firm collude, they choose the monopoly price/quantity.

$$p^M = 30, \;\; Q^M = 5000 - 100p^M = 2000$$

$$q_1^C = q_2^C = 1000$$

$$\Pi^M = 40000$$

Suppose two firm divide the market share evenly in the collusion

$$\pi_1^M = \pi_2^M = 20000$$

NE outcome
Inverse demand

$$P(Q) = 50 - \frac{1}{100}Q$$

$$P(Q) = a - bQ, \;\; MC = c$$

$$q_1^{NE} = q_2^{NE} = \frac{a-c}{3b} = \frac{50-10}{3 \times \frac{1}{100}} = \frac{40 \times 100}{3} = \frac{4000}{3}$$

$$Q^{NE} = q_1^{NE} + q_2^{NE} = \frac{8000}{3} > Q^M$$

$$\begin{aligned}
\pi_1^{NE} = \pi_2^{NE} &= [50 - \frac{1}{100}Q^{NE} - c]q_1^{NE} \\
&= [50 - \frac{1}{100}\frac{8000}{3} - 10]\frac{4000}{3} \\
&= [40 - \frac{80}{3}]\frac{4000}{3} \\
&= [\frac{40}{3}]\frac{4000}{3} \\
&= \frac{160000}{9}
\end{aligned}$$

Lastly, π^D. What is the most profitable way to deviate?

$$\begin{aligned}
\max_{q_1} \pi_1(q_1, q_2^C) &= [50 - \frac{1}{100}(q_1 + q_2^C) - 10]q_1 \\
&= [40 - \frac{1}{100}q_1 - \frac{1}{100}q_2^C]q_1
\end{aligned}$$

$$\frac{\partial \pi_1}{\partial q_1} = 40 - \frac{1}{50}q_1 - \frac{1}{100}q_2^C = 0$$

$$2000 - q_1 - \frac{1}{2}q_2^C = 0$$

$$\begin{aligned}
q_1^D &= BR_1(q_2^C) = 2000 - \frac{1}{2}q_2^C \\
&= 2000 - \frac{1}{2}q_2^C = 2000 - \frac{1}{2} \times 1000 \\
&= 1500
\end{aligned}$$

The most profitable way to deviate is to produce $q_1^D = 1500$ instead of $q_1^C = 1000$.

$$Q^D = q_1^D + q_2^C = 1500 + 1000 = 2500$$

$$\begin{aligned}
\pi_1^D &= [p(Q^D) - c]q_1^D \\
&= [40 - \frac{1}{100}Q^D] \times 1500 \\
&= 15 \times 1500 = 22500 > \pi_1^{NE} \approx 17778
\end{aligned}$$

What is the δ_{min} to maintain trigger strategy as equilibrium for tacit collusion?

$$\begin{aligned}
\delta_{min} &= \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}} = \frac{22500 - 20000}{22500 - 17778} \\
&= 0.53
\end{aligned}$$

Sequential move

(1) Cournot

$$\max_{q_1} \pi_1(q_1, BR_2(q_1))$$

$$\frac{d\pi_1}{dq_1} = \underbrace{\frac{\partial \pi_1}{\partial q_1}}_{\text{direct effect}} + \underbrace{\overbrace{\frac{\partial \pi_1}{\partial q_2}}^{-} \times \overbrace{\frac{dBR_2}{dq_1}}^{-}}_{\text{indirect effect (+)}}$$

Compare to the case that q_2 is fixed

$$\max_{q_1} \pi_1(q_1, q_2)$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow q_1^{NE}$$

Expanding the output q_1 has an extra benefit from the indirect effect (firm 2 will produce less). Therefore, we should have $q_1^L > q_1^{NE}$.

The first-mover advantage comes from best response being downward sloping.

(2) Differentiated-product Bertrand

Firm 1 (leader)'s problem is

$$\max_{p_1} \pi_1(p_1, BR_2(p_1))$$

$$\frac{d\pi_1}{dp_1} = \frac{\partial \pi_1}{\partial p_1} + \overbrace{\frac{\partial \pi_1}{\partial p_2}}^{+} \times \overbrace{\frac{dBR_2}{\partial p_1}}^{+}$$

Compare to the case of NE

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1^{NE}$$

There is an extra benefit from raising price because it also pushes up firm 2's price.

The result shows

$$p_1^L > p_1^{NE}, \quad p_2^F > p_2^{NE}.$$

After computing profit, we find that

$$\pi_1^L > \pi_1^{NE}$$

$$\pi_2^F > \pi_2^{NE}$$

There is a second-mover advantage because it earns more than the leader:

$$\pi_2^F > \pi_1^L$$

Entry deterence

(1) Stackelberg

Demand $p(Q) = a - Q$, marginal cost is c.

By backward induction, at $t = 1$, firm 2 chooses q_2 after observing q_1.

$$\max_{q_2} \pi_2(q_1, q_2) = (a - q_1 - q_2 - c)q_2$$

FOC for best response

$$a - q_1 - 2q_2 - c = 0$$

$$q_2 = BR_2(q_1) = \frac{a - c - q_1}{2}.$$

Given that firm 2 will response optimally in the way above, firm 1 solve his profit maximization problem at $t = 1$.

$$\begin{aligned}\max_{q_1} \pi_1(q_1, BR_2(q_1)) &= [a - q_1 - BR_2(q_2) - c]\, q_1 \\ &= \left[a - c - q_1 - \frac{a - c - q_1}{2}\right] q_1 \\ &= \frac{a - c - q_1}{2} q_1 \\ &= \frac{a - c}{2} q_1 - \frac{1}{2} q_1^2\end{aligned}$$

FOC

$$\frac{d\pi_1}{dq_1} = \frac{a - c}{2} - q_1 = 0$$

$$q_1^L = \frac{a-c}{2}$$

$$q_2^F = BR_2(q_1^L) = \frac{a-c-\frac{a-c}{2}}{2} = \frac{a-c}{4}$$

The Stacklerberg leader-follower equilibrium (outcome of sequential game) is

$$q_1^L = \frac{a-c}{2},\ q_2^F = \frac{a-c}{4}.$$

Note that, SPE is $\{q_1^L = \frac{a-c}{2}, q_2^F = \frac{a-c-q_1}{2}\}$.

(2) Introduce fixed cost K

Suppose that firm 2 has a fixed cost $K > 0$. The total cost of firm 2

$$C_2(q_2) = \begin{cases} 0 & \text{if } q_2 = 0 \text{ (do not enter the market)} \\ K & \text{if } q_2 > 0 \text{ (enter the market)} \end{cases}$$

$t = 1$, firm 1 chooses q_1.

$t = 2$, firm 2 (entrant), observing q_1, decides whether to enter the market (by paying the fixed cost). If enters, firm 2 chooses q_2.

This framework can be used to analyze entre deterence.

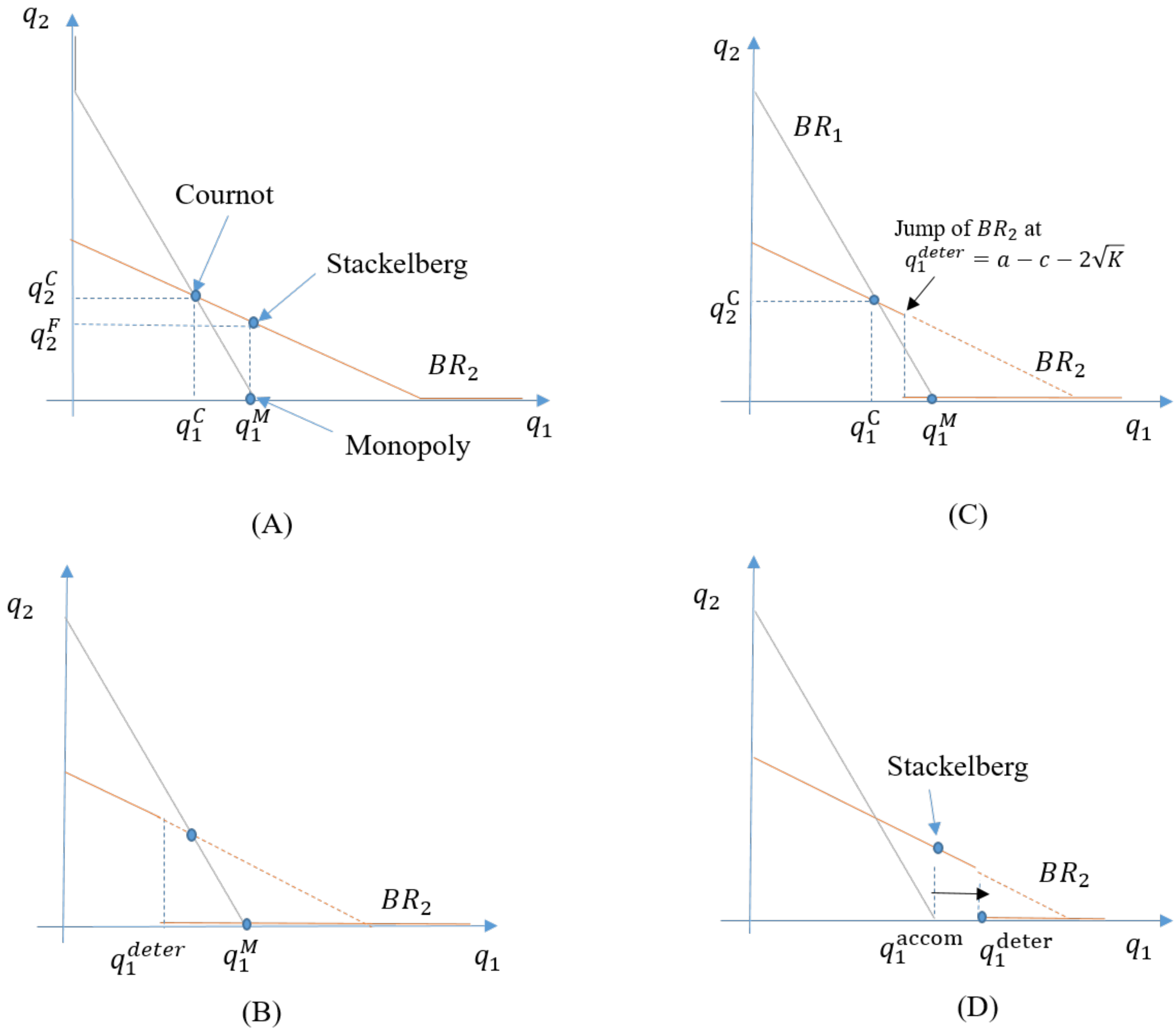
Firm 2 will not enter the market ($q_2 = 0$) if $\pi_2 \leq 0$.

$$\max_{q_2} \pi_2(q_1, q_2) = (a - q_1 - q_2 - c)q_2 - K$$

In equilibirum, with firm 2 optimally respond to firm 1, its profit is

$$\begin{aligned} \pi_2^F &= (a - q_1^L - q_2^F - c)q_2^F - K \\ &= \left[a - c - \frac{a-c}{2} - \frac{a-c}{4}\right]\frac{a-c}{4} - K \\ &= (\frac{a-c}{4})^2 - K \geq 0 \end{aligned}$$

$$K \leq (\frac{a-c}{4})^2$$



(3) Entry deterrence
Find q_1^{deter} such that when firm 1 produce $q_1 \geq q_1^{deter}$, firm 2 chooses not to enter $(q_2 = 0)$.
q_1^{deter} is where firm 2's best responses have a jump.

$$q_2 = BR_2(q_1) = \begin{cases} \frac{a-c-q_1}{2} & \text{if } q_1 < q_1^{deter} \\ 0 & \text{if } q_1 \geq q_1^{deter} \end{cases}$$

$$\max_{q_2} \pi_2(q_1, q_2) = (a - q_1 - q_2 - c)q_2 - K$$

The solution of optimization problem is the best reponse, so the value function is

$$\begin{aligned}
\pi_2(q_1, BR_2(q_1)) &= (a - q_1 - \frac{a-c-q_1}{2} - c)\frac{a-c-q_1}{2} - K \\
&= (a - q_1 - c - \frac{a-c-q_1}{2})\frac{a-c-q_1}{2} - K \\
&= (\frac{a-c-q_1}{2})^2 - K \leq 0 \\
(\frac{a-c-q_1}{2})^2 &\leq K \\
\frac{a-c-q_1}{2} &\leq \sqrt{K} \\
a - c - q_1 &\leq 2\sqrt{K} \\
q_1 &\geq a - c - 2\sqrt{K} = q_1^{deter}
\end{aligned}$$

If firm 1 produce $q_1 \geq q_1^{deter}$, firm 2, doing its best, will earn a negative profit. Therefore, firm 2 will not enter the market.

(4) Cases of entry deterrence

$$q_1^{deter} = a - c - 2\sqrt{K}$$

The deterence quantity level decreases in K.

(i) When K is sufficiently large, q_1^{deter} is small.

The case of blockade happens when firm 1 produces the monopoly level q_1^M, and firm 2 do not find it profitable to enter.

Firm 1, being passively behaving as a monopoly, firm 2 cannot enter (entry is blocked).

$$\pi_2(q_1^M, BR_2(q_1^M)) \leq 0$$

(ii) When K is small, firm 1 needs to produce more than the monopoly level $q_1^{deter} > q_1^M$ in order to deter entry.

Firm 1 needs to decide whether doing so is worthy.

If firm 1 keeps producing at the Stackelberg level $q_1 = q_1^L$,firm 2 can enter and produce $q_2 = q_2^F$. (Accommodate)

$$\pi_1^{accom} = \pi_1(q_1^L, BR_2(q_1^L)) = \pi_1(q_1^L, q_2^F)$$

If firm 1 produces q_1^{deter}, firm 2 will not enter $q_2 = 0$. (Strategic entry deterrence)

$$\pi_1^{deter} = \pi_1(q_1^{deter}, BR_2(q_1^{deter})) = \pi_1(q_1^{deter}, 0)$$

Firm 1 will compare π_1^{accom} and π_1^{deter} and decide whether to strategically deter entry.

$$\begin{aligned}
\pi_1^{accom} = \pi_1^L &= (a - c - q_1^L - q_2^F)q_1^L - K \\
&= (a - c - \frac{a-c}{2} - \frac{a-c}{4})\frac{a-c}{2} - K \\
&= \frac{a-c}{4}\frac{a-c}{2} - K = \frac{(a-c)^2}{8} - K
\end{aligned}$$

$$\begin{aligned}\pi_1^{deter} &= \pi_1(q_1^{deter}, 0) = (a - c - q_1^{deter} - 0)q_1^{deter} - K \\ &= (a - c - (a - c - 2\sqrt{K}))(a - c - 2\sqrt{K}) - K \\ &= 2\sqrt{K}(a - c - 2\sqrt{K}) - K\end{aligned}$$

Firm 1 will choose to deter entry if

$$\pi_1^{deter} > \pi_1^{accom}$$

$$2\sqrt{K}(a - c - 2\sqrt{K}) > \frac{(a-c)^2}{8}$$