Homework 3

Due on Oct 13

Choose the best answer

1. In order to maximize profits, a firm should produce at the output level for which
   a. average cost is minimized.
   b. marginal revenue equals marginal cost.
   c. marginal cost is minimized.
   d. price minus average cost is as large as possible.

2. For the cost function $C = q^{0.8}v^{0.4}w^{0.6}$, which of the following statements are true:
I. The function exhibits decreasing average cost.
II. The function is homogeneous of degree 1 in v and w.
III. The elasticity of marginal cost with respect to v exceeds the elasticity with respect to w.
   a. None is true.
   b. I, II, and III.
   c. I only.
   d. I and II.

3. A firm's demand for labor is known as a "derived demand" because
   a. the firm gains utility from hiring more labor.
   b. the amount of labor hired depends upon how much output the firm can sell.
   c. the wage rate paid to workers is derived from the market for labor.
   d. it is derived from the demand for capital.

4. If a firm is a price taker in both the input and output markets, its marginal revenue product of labor is given by
   a. the price of its output times labor's marginal physical productivity.
   b. the marginal value product of labor.
   c. the marginal revenue product of capital times the ratio of the wage rate to the rental rate on capital.
   d. all of the above.

Analytical questions

1. Consider the following production function

$$q = f(k, l) = \sqrt{k} + \sqrt{l}$$

   a. Does this production function has increasing, decreasing, or constant return to scale?

   b. Solve the profit max problem when the output price is p and factor prices are v and w respectively. Find the output supply function and input demand functions.

   c. Find the profit function and verify the Evnelop results: $\frac{\partial \pi}{\partial p} = q(p, v, w)$, $\frac{\partial \pi}{\partial v} = -k(p, v, w)$, $\frac{\partial \pi}{\partial w} = -l(p, v, w)$

d. Solve the cost minimization problem and find the conditional input demand functions and the cost function. Is the cost function concave or convex in output?

e. Take the cost function from part (d), solve the profit max problem with respect to output and confirm the profit function is the same as the profit function obtained in (c).

f. In the short-run, suppose k is fixed at k_1, show that the short-run cost function is no less than long-run cost function from (d), and the two costs are the same when conditional demand for k is equal to k_1.

2. Firm uses capital k, and labor l, to produce output, q. The firm is a price taker in the output market and in both input markets, denoted by (p, v, w) respectively. The firm's supply function is

$$q(p, v, w) = mp^a v^{-1} w^{-2},$$

its demand function for capital is

$$k(p, v, w) = 3p^4 v^b w^{-2},$$

and its demand function for labor is

$$l(p, v, w) = np^4 v^c w^{-3}.$$

What are the values of the constants a, b, c, m, and n? Explain your reasoning in each case. [Hint: these functions are homogeneous of degree zero in prices and satisfy the Hotelling lemma.]

3. A price-taking firm has production function

$$q = f(k, l) = k^{\frac{1}{3}} l^{\frac{1}{3}}.$$

The capital rental price is $v = 2$ and labor price is $w = 3$.

a. In the short run, capital is fixed at the level k_1. Set up the cost minimization problem and find the short-run cost function.

b. Find the short-run supply function given price p. What is the shut-down price?

c. Find the long-run supply function.

4. Textbook exercise 11.1

5. Textbook exercise 11.3

6. Textbook exercise 11.7