Topic 1

Economic models and Equilibrium analysis

Outline

1. Sets
2. Variables, constants, and parameters
3. Equilibrium analysis
4. Functions of one variable
5. Functions of two or more independent variables

1. Sets

- A set is a collection of objects. Examples:
  - $A = \{1,2\}$ is a finite set.
  - The set of integers Z = $\{n: n = 0, \pm 1, \pm 2, \ldots$ are integers$\}$ is a countable infinite set
  - R={all real numbers} is an uncountable infinite set

- If A is a set, $x \in A$ denotes that x is an element of A. $x \notin A$ denotes that x is not an element of A. Examples:
  - If $A = \{1,2\}$, then $1 \in A, -1 \notin A$

Sets: examples

- for $a < b$,
  - $[a, b] = \{x \in R, a \leq x \leq b\}$
  - $(a, b] = \{x \in R, a < x \leq b\}$
  - $[a, b) = \{x \in R, a \leq x < b\}$
  - $(a, b) = \{x \in R, a < x < b\}$
- Empty set $\emptyset$ is the set with no elements.
  - $\emptyset = \{x: x \neq x\} = \{x \in R: x^2 < 0\}$
- $R_{++} = \{x \in R: x > 0\}$; $R_{+} = \{x \in R: x \geq 0\}$
- $Z_{++} = \{1,2,3, \dots\}$={all positive integers}; $Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$={all integers}
- $A = \{(x, y):\ x \in R, y \in R, y = x\}$
- $A = \{(x, y):\ x \in R, y \in R, y \geq x^2\}$
- $A = \{(x, y):\ x \in R, y \in R, x^2 + y^2 \leq 1\}$

If A and B are two sets

- A is a subset of B, denoted $A \subset B$ or $B \supset A$, if $x \in A \Rightarrow x \in B$; example: $[1,2] \subset R$
- Two sets equal $A = B$ if $A \subset B$ and $B \subset A$
- Complement: $A^c = \{x\colon\ x \notin A\}$; example:

  $A = \{x: x \leq 1\}$, then $A^c = \{x: x > 1\}$
- Intersection: $A \cap B = \{x: x \in A\ and\ x \in B\}$;
- Union: $A \cup B = \{x: x \in A\ or\ x \in B\}$;
- example:

  $A = \{x \in R\colon\ x^2 - 2x - 3 \geq 0\} = (-\infty, -1] \cup [3, \infty)$,

  $B = \{x \in R\colon\ x < 0\ \} = (-\infty, 0)$, then

  $A \cap B = (-\infty, -1]$; $A \cup B = (-\infty, 0) \cup [3, \infty)$
- Difference: $A \backslash \mathrm{B}$(or $A - B$)

  $A \backslash \mathrm{B} = \{x: x \in A$ and $x \notin B\} = A \cap B^c$

- A and B are disjoint if $A \cap B = \emptyset$. example: $A = (0,1), B = (2,3)$

- Cartesian product:

$$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$$
$$R^2 = R \times R = \{(x, y) : x \in R, y \in R\}$$
$$R^n = R \times \cdots \times R = \{(x_1, x_2, \ldots, x_n) : x_i \in R \text{ for } i = 1, 2, \ldots n\}$$
$$R^n_{++} = R_{++} \times \cdots \times R_{++} = \{(x_1, x_2, \ldots, x_n) : x_i \in R,\ x_i > 0 \text{ for } i = 1, 2, \ldots n\}$$

- A set is closed if it contains all its limit points.

- A set $S \subset R^k$ is open if $x \in S \Rightarrow$ its small neighborhood $\subset S$.

- **Examples**:
  $A = \{(x, y): 1 \leq x^2 + y^2 \leq 2\}$ and $B = [1,2]$ are closed sets
  $C = \{(x, y): 1 < x^2 + y^2 < 2\}$ and $C = (1,2)$ are open sets

2. Variables, constants, and parameters

- A variable is something that can take on different values.
- Some economic variables are determined by our models, while others are usually assumed to be determined by factors outside of our models. We call the former endogenous variables and the latter exogenous variables.
- A constant is a magnitude which does not change.
- Example: $f(x) = ax$ is the function for a line with x a variable and a a given constant.
  - Since no specific number is assigned to a, it can take any value. i.e, a is a constant that is a variable. a is called a parameter
  - Although different values can be assigned to a, it is regarded as a constant in the model

3. Equilibrium analysis

- **Single-good equilibrium model**:

  Demand function: $Q^d = a - bP$

  Supply function: $Q^s = -c + dP$

  Equilibrium condition: Q^d=Q^s

  where a, b, c, and d are positive constants, P is the price of the good, and Q is the output.

- To solve the model, set

  $$a - bP = -c + dP$$

- The equilibrium solution to the model is (P^*, Q^*), where

  $$P^* = \frac{a+c}{b+d}, \quad Q^* = \frac{ad-bc}{b+d}$$

  – Note that in this model, a, b, c and d are parameters, and P and Q are endogenous variables

- **Two-good equilibrium model:**

$$\begin{aligned}
&\text{Demand function 1:} && Q_1^{\,d} = a_0 + a_1 P_1 + a_2 P_2 \\
&\text{Demand function 2:} && Q_2^{\,d} = \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2 \\
&\text{Supply function 1:} && Q_1^{\,s} = b_0 + b_1 P_1 + b_2 P_2 \\
&\text{Supply function 2:} && Q_2^{\,s} = \beta_0 + \beta_1 P_1 + \beta_2 P_2 \\
&\text{Equilibrium conditions:} && Q_1^{\,d} = Q_1^{\,s}, \quad Q_2^{\,d} = Q_2^{\,s}
\end{aligned}$$

  - where $a_i, b_i, \alpha_i, \beta_i$ $(i = 0,1,2)$ are parameters.
  - The model can be solved through

$$\begin{cases} (a_0 - b_0) + (a_1 - b_1)P_1 + (a_2 - b_2)P_2 = 0 \\ (\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)P_1 + (\alpha_2 - \beta_2)P_2 = 0 \end{cases} \tag{1}$$

  - Denote $c_i = a_i - b_i, \gamma_i = \alpha_i - \beta_i, \;\; i = 0,1,2,$ then (1) becomes

$$\begin{cases} c_0 + c_1 P_1 + c_2 P_2 = 0 \\ \gamma_0 + \gamma_1 P_1 + \gamma_2 P_2 = 0 \end{cases}$$

- **The Keynesian national income model**:

$$\begin{cases} Y = C + I_0 + G_0 \\ C = a + bY \end{cases} \qquad (a > 0, \quad 0 < b < 1)$$

- Where

Endogenous variables:	Y = national income, C = consumption
Exogenous variables:	I_0 = investment, G_0= government expenditure
Parameters:	a, b

- These two equations determine Y and C

$$Y^* = \frac{a + I_0 + G_0}{1 - b}, \ C^* = \frac{a + b(I_0 + G_0)}{1 - b}$$

4. Functions of one variable

- A function of a real variable x with domain D is a rule that assigns a unique real number to each number x in D.

- As x varies over the whole domain, the set of all possible resulting values $f(x)$ is called the range of f.

- Note: A function is only completely specified if besides the rule its domain and range are fixed.

- **Example**: Find the domain and range of $f(x) = \sqrt{x-1}$
  - Domain: $x \geq 1$ since negative numbers have no roots
  - Range: $[0,\infty)$

- **Example**: Find the domain and range of $\mathrm{y} = f(x) = \frac{1}{x+1}$
  - Domain: $x \neq -1$ since $(1/0)$ is meaningless
  - Range: $y \neq 0$ or $(-\infty,0) \cup (0,\infty)$

- **Graph** of a function $y = f(x)$ is the curve consisting of all points $(x, y) = (x, f(x))$ drawn in coordinate system with x on the horizontal and y on the vertical axis where x varies over the domain of the function.

- Which of the following two curves is the graph of a function?

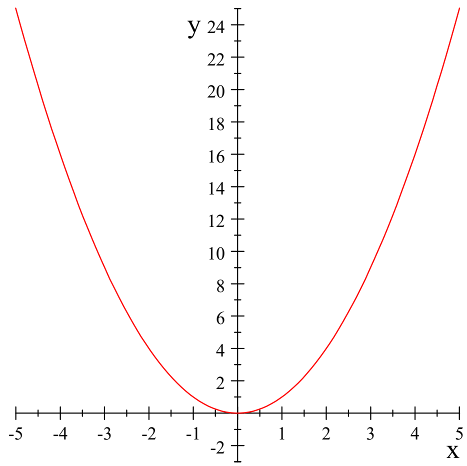


Figure 1

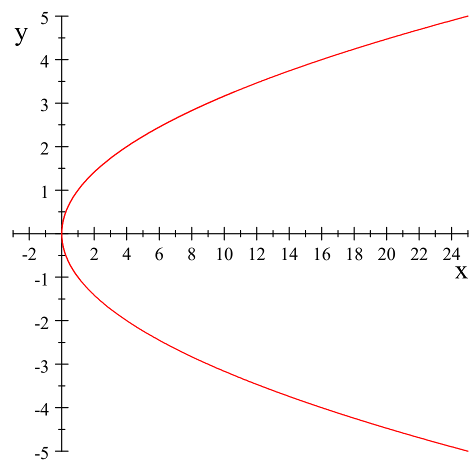


Figure 2

- A vertical line test: A curve is the graph of a function if and only if no vertical line intersects the curve more than once.

Inverse functions

- Recall that a function $y = f(x)$ is a rule that assigns a unique real number y to each number x in its domain

- If the function represents a one-to-one mapping, i.e., if the function is such that each value of y is associated with a unique value of x, the function is said to have an inverse function $x = f^{-1}(y)$.

- the symbol f^{-1} is a function symbol, it does not mean the reciprocal of the function $f(x)$

Monotone functions

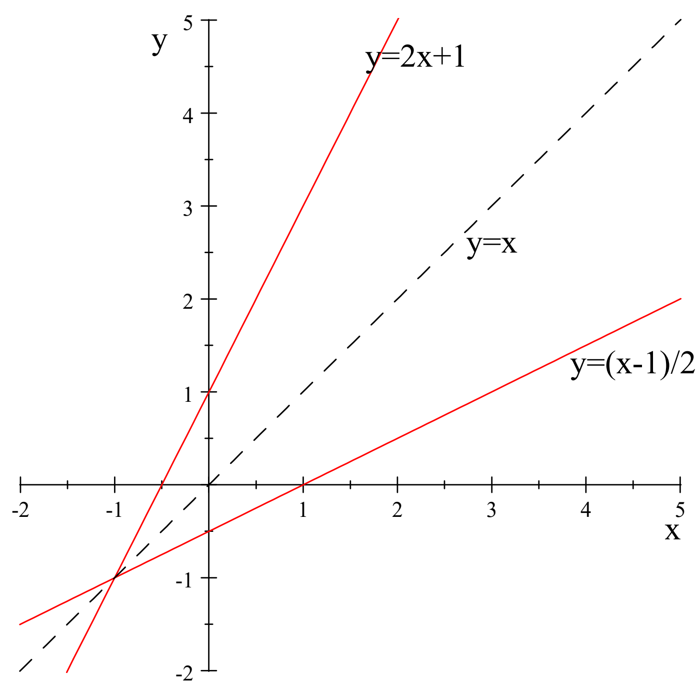
- Increasing function: $f(x_2) \geq f(x_1)$ whenever $x_2 > x_1$
- Strictly increasing function: $f(x_2) > f(x_1)$ whenever $x_2 > x_1$
- Decreasing function: $f(x_2) \leq f(x_1)$ whenever $x_2 > x_1$
- Strictly decreasing function: $f(x_2) < f(x_1)$ whenever $x_2 > x_1$
- f is monotone if it is increasing or decreasing -
- f is strictly monotone if it is strictly increasing or strictly decreasing
- Note: f is invertible (its inverse function exists) when it is strictly monotone

Example: $y = f(x) = 2x + 1$ for $x \in R$

- Each y is associated with a unique x, in fact, you can solve for

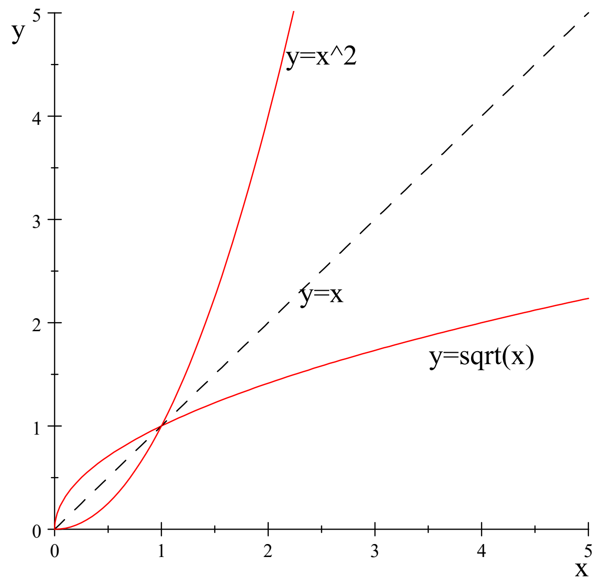
  $$x = f^{-1}(y) = \frac{y-1}{2}$$

- Now if we adopt the convention that x is the independent and y is the dependent variable, then, the inverse function is $y = f^{-1}(x) = \frac{x-1}{2}$



Example: $y = x^2$ for $x \in R$

- Each y is associated with two x's, ($x = \pm\sqrt{y}$)
- Inverse function does not exist
- However, if the domain is restricted to $x \in R_+$ (positive real number), then $x = \sqrt{y}$, i.e., the inverse function is $y = f^{-1}(x) = \sqrt{x}$ if we use x as the independent and y as the dependent variable



5. Functions of two or more independent variables

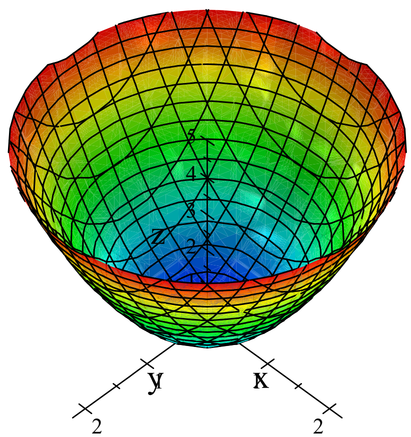
- A realistic description of many economic phenomena requires considering a large number of variables.

- A functions f of two variables x and y with domain D is a rule that assigns a specified number $z = f(x, y)$ to each point (x, y) in D

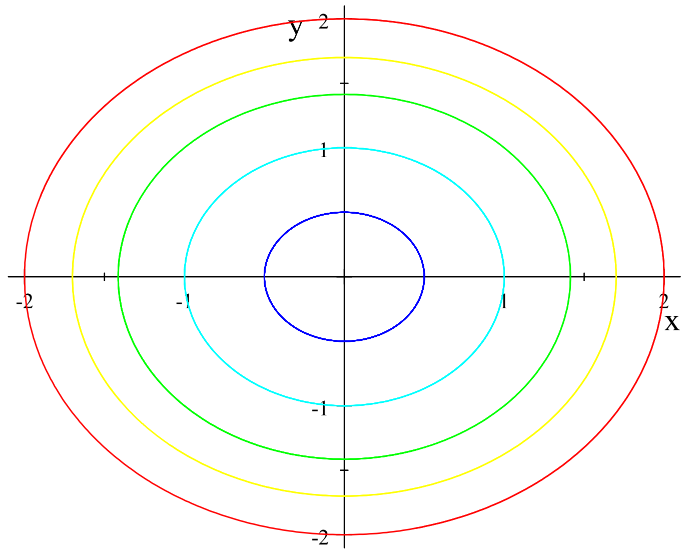
- The graph of f is the surface in 3-dimensional space considering of all points (x, y, z) with (x, y) in D

- The level curve of the function $z = f(x, y)$ at level c is the solution set to the equation $f(x, y) = c$

- Example: function $z = f(x, y) = x^2 + y^2$

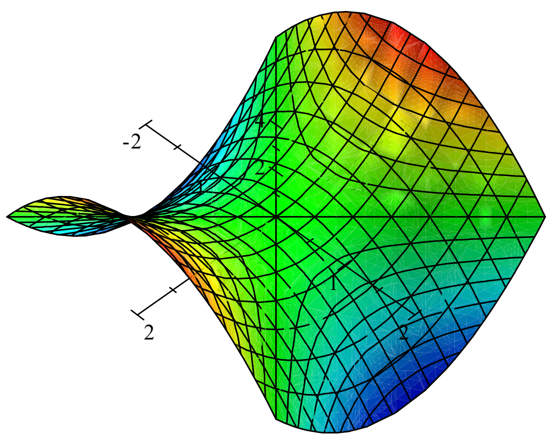


3D graph

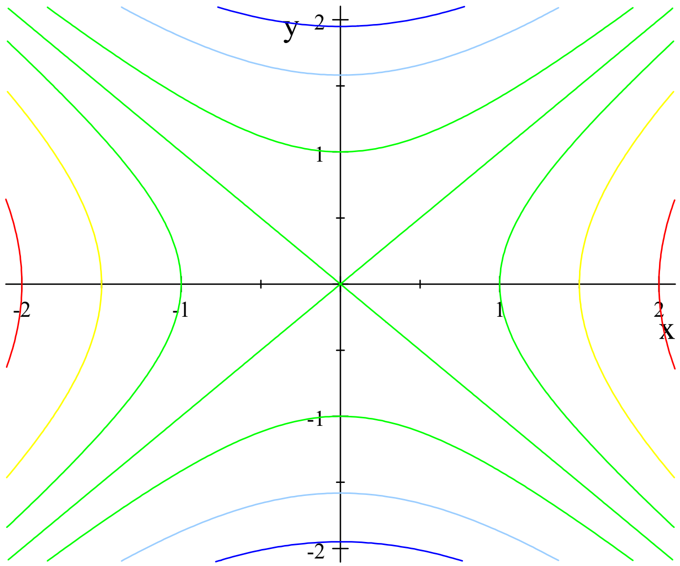


Level curves

- Example: function: $z = x^2 - y^2$



3D graph



Level curves