ECON 3113 Microeconomic Theory I
Lecture 5: Applications of Utility Maximization

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March 2020

We will look at two applications of the utility-maximization model

- Lump-sum principle
- Intertemporal consumption

Lump-sum Principle

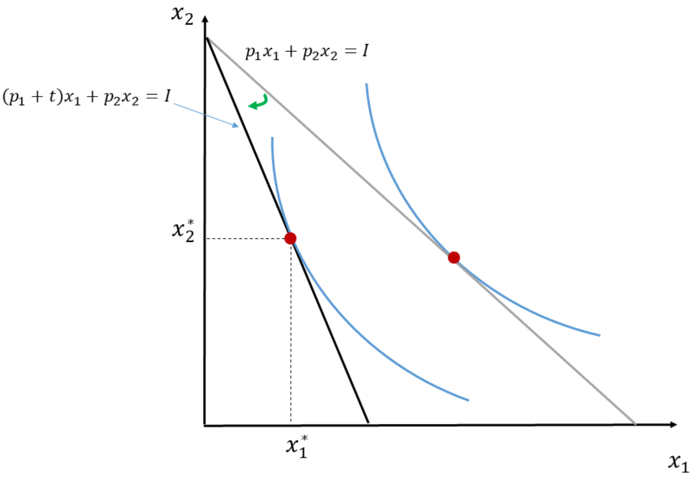
- "*[I]n this world nothing can be said to be certain, except death and taxes.*" Benjamin Franklin (1789)
- Which is the lesser of the two evils, sales tax or income tax?
- Sales tax (aka per-unit tax, quantity tax): a tax rate is charged for each unit of the goods purchased.
- Income tax: a lump-sum amount of income is taxed regardless of consumption choice.

Lump-sum Principle

- Initial condition: prices are p_1 and p_2 and income I.
  - Initial budget line: $p_1 x_1 + p_2 x_2 = I$.
- With an introduction of sales tax on goods 1 at a rate of t, the budget line becomes $(p_1 + t)\, x_1 + p_2 x_2 = I$.
- With an introduction of income tax of T, the budget line becomes $p_1 x_1 + p_2 x_2 = I - T$.
- For a fair comparison, we consider the case of equal total tax payment under the two tax schemes.
- Let (x_1^*, x_2^*) be the optimal bundle under the sales tax scheme. The total tax payment is thus tx_1^*.
- For equal tax payment, the income tax is set at $T = tx_1^*$.

Sales Tax

- The sales tax of t on goods 1 rotates the budget line inwards:

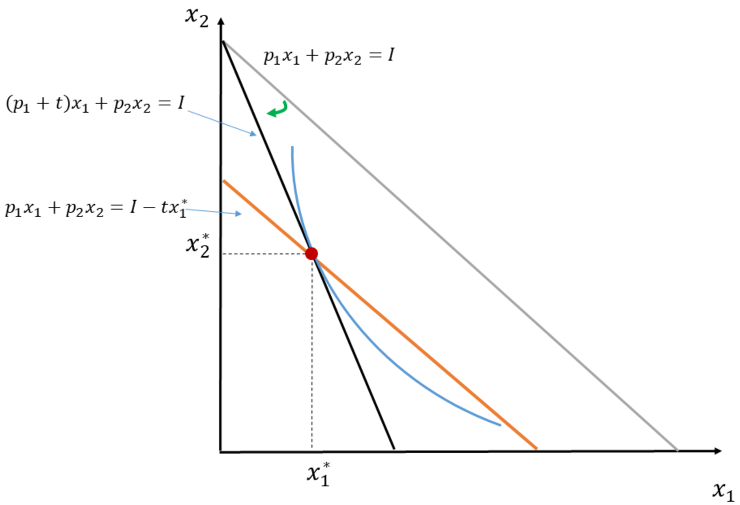


Income Tax

- An income tax of $T = tx_1^*$ will parallelly shift the budget line inwards.
- The budget line under both schemes pass through the bundle (x_1^*, x_2^*), but with different slopes:

$$(p_1 + t)\, x_1^* + p_2 x_2^* = I \Leftrightarrow p_1 x_1^* + p_2 x_2^* = I - \underbrace{tx_1^*}_{T}.$$

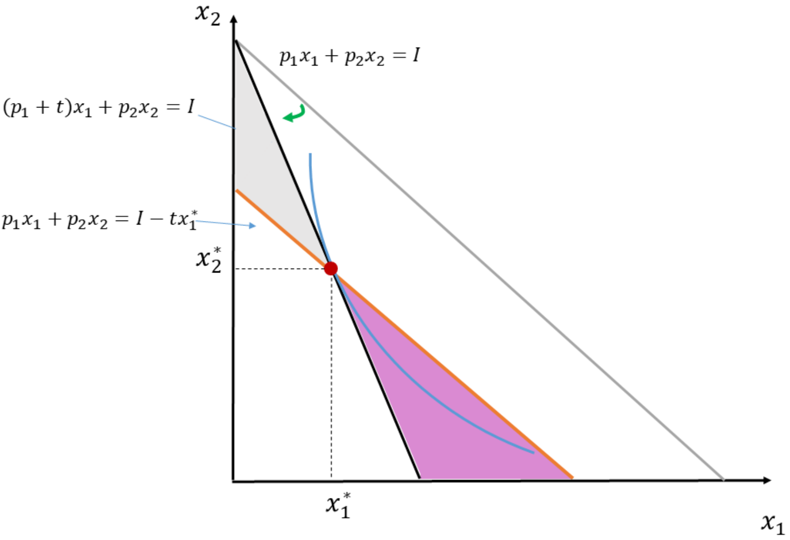
Two Tax Schemes in One Picture



Lump-sum Principle

- Lump-sum principle: the consumer prefers an income tax over a sales tax of equal amount.
- A revealed-preference argument
  - the income tax scheme offers the bundle (x_1^*, x_2^*) (which is the most preferred bundle under the sales tax scheme), plus something else.

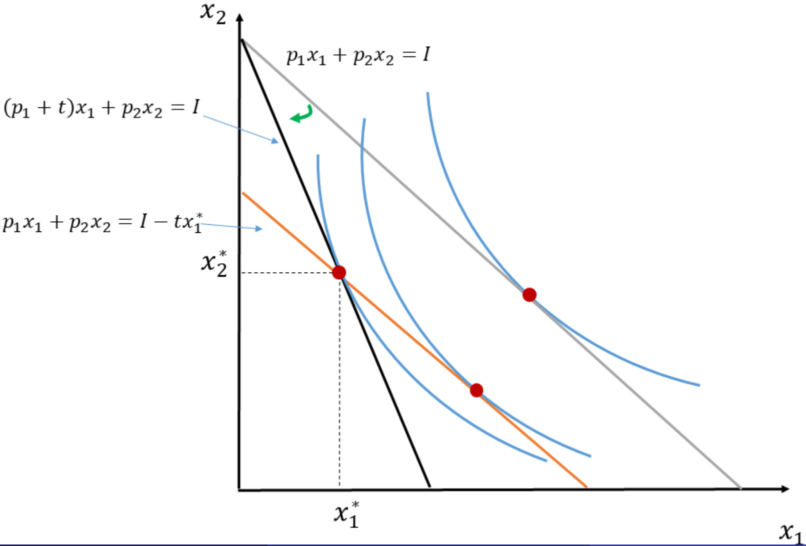
Lump-sum Principle



Lump-sum Principle

- Lump-sum principle: the consumer prefers an income tax over a sales tax of equal amount.
- An MRS argument
  - The MRS at the bundle (x_1^*, x_2^*) is $(p_1 + t) / p_2$, which exceeds (the magnitude of) the slope of the budget line p_1 / p_2 under the income tax scheme.
  - At bundle (x_1^*, x_2^*), she is willing to give up more of goods 2 than is required by the market to acquire an additional unit of goods 1.
  - She will therefore buy more of goods 1 beyond x_1^*, which makes her strictly better off than staying at (x_1^*, x_2^*).

Lump-sum Principle



Lump-sum Principle: Moral and Caveats

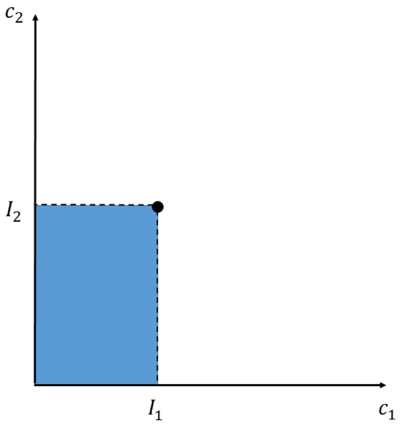
- The use of sales tax is less desirable than that of an income tax because it changes not only the consumer's purchasing power, but also **distorts the relative prices** between the goods.
- A similar principle applies to subsidy: a general income grant improves the consumer's well-being more than a per-unit subsidy on specific goods (after controlling for the total amount of money received).
- Caveats:
  - Consumers may be heterogeneous: I wouldn't care whether there is a sales tax on Lamborghini.
  - Income tax/subsidy may distort work incentives.
  - Sales tax/subsidy may affect supplier's incentives.

Intertemporal Consumption

- Consume now or save for later?
- There are two time periods, period 1 and period 2.
- Consumptions in period 1 and 2 are denoted by c_1 and c_2 respectively.
- The consumer has income I_1 in period 1 and I_2 in period 2.
- Her utility of consumption is denoted by $U(c_1, c_2)$, satisfying the usual assumptions.
  - monotonicity, convexity and continuity
  - In a lot of applications, time-separability is also assumed.

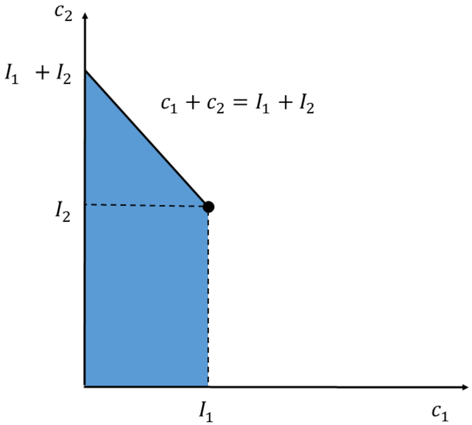
Intertemporal Consumption: Budget Set

- If the consumer can neither save nor borrow, the budget constraints are $c_1 \leq I_1$ and $c_2 \leq I_2$.
- The "budget line" consists of a single point (I_1, I_2).



Intertemporal Consumption: Budget Set

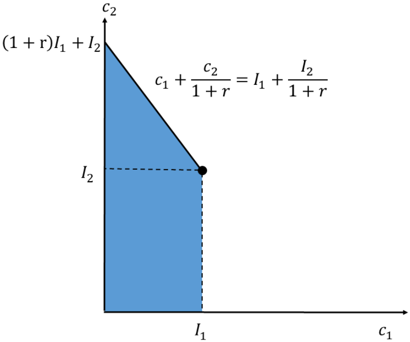
- If the consumer cannot borrow but can hoard money under her bed, the budget constraints are $c_1 \leq I_1$ and $c_2 \leq I_2 + (I_1 - c_1)$.
- The budget line is a line segment with slope -1.



Intertemporal Consumption: Budget Set

- Now if the consumer can save and borrow at some interest rate r, what is/are the budget constraint(s)?
- Suppose she decides to be a saver, so that $c_1 \leq I_1$. In period 2, she has $I_2 + (1+r)(I_1 - c_1)$ to spend in period 2, i.e.,

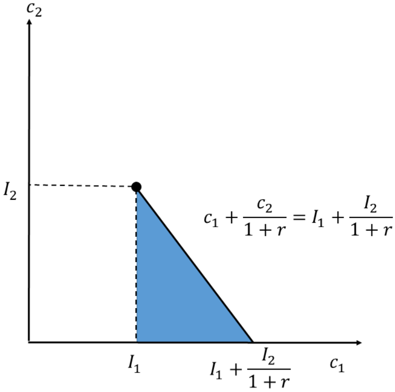
$$c_2 \leq I_2 + (1+r)(I_1 - c_1) \Leftrightarrow c_1 + \frac{c_2}{1+r} \leq I_1 + \frac{I_2}{1+r}.$$



Intertemporal Consumption: Budget Set

- Conversely, suppose she decides to be a borrower, so that $c_1 \geq I_1$. In period 2, she has to pay back a total debt of $(1+r)(c_1 - I_1)$, so her period-2 consumption is at most

$$c_2 \leq I_2 - (1+r)(c_1 - I_1) \Leftrightarrow c_1 + \frac{c_2}{1+r} \leq I_1 + \frac{I_2}{1+r}.$$



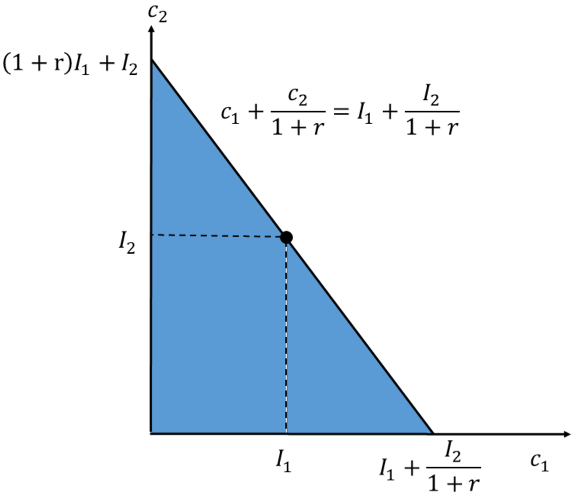
Intertemporal Consumption: Budget Set

- In sum, regardless of whether the consumer is a borrower or a saver, the budget constraint is described by the same relation.

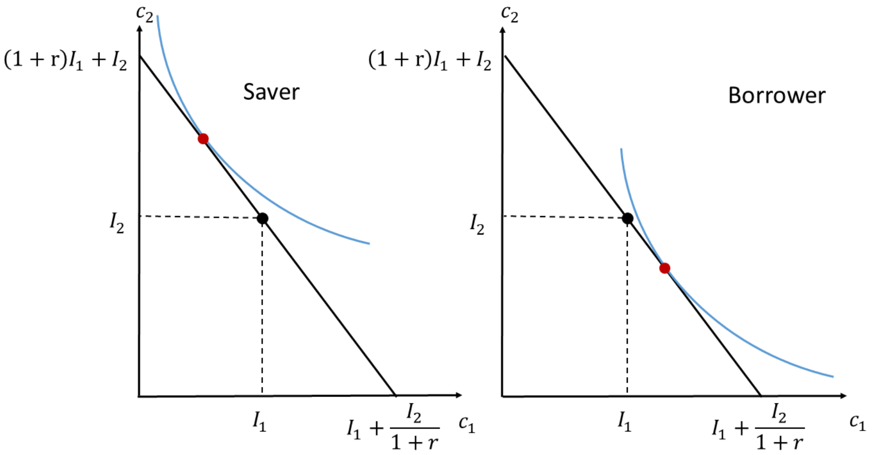
$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\text{present value of consumption}} \leq \underbrace{I_1 + \frac{I_2}{1+r}}_{\text{present value of income}} .$$

- If the consumer wants to increase her current consumption by one unit, she would have to give up $1+r$ units of period-2 consumption. The "relative price" of period-1 consumption is therefore $1+r$.
- The horizontal intercept of the budget line is the present value of income $I_1 + \frac{I_2}{1+r}$.

Intertemporal Consumption: Budget Set

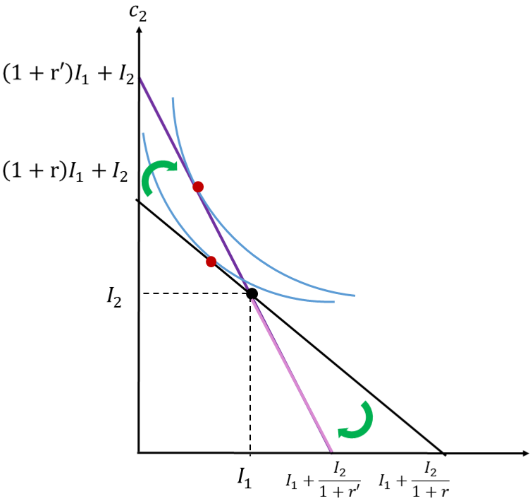


Borrower or Saver?



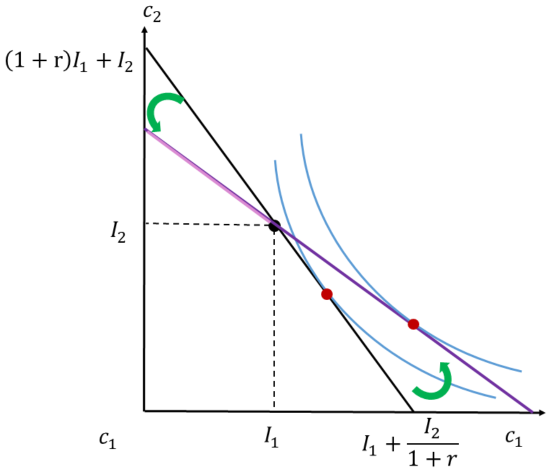
Effect of Interest Rate Increase

- Suppose the consumer is initially a lender and the interest rate goes up.



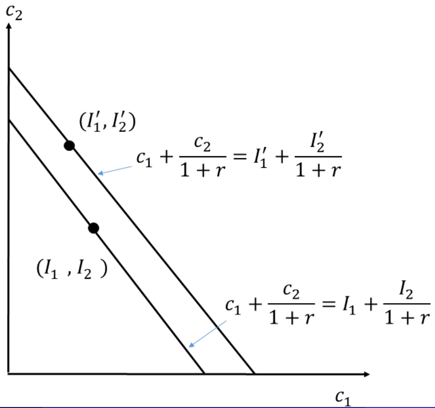
Effect of Interest Rate Increase

- Suppose the consumer is initially a borrower and the interest rate goes down.



Present Value

- Given two income streams, say (I_1, I_2) and (I_1', I_2') where $I_1 > I_1'$ and $I_2 < I_2'$, which one should the consumer pick?
- Assuming that the consumer has access to a frictionless capital market, he should simply go for the income stream with the **highest present value**.



Separable Time Preference

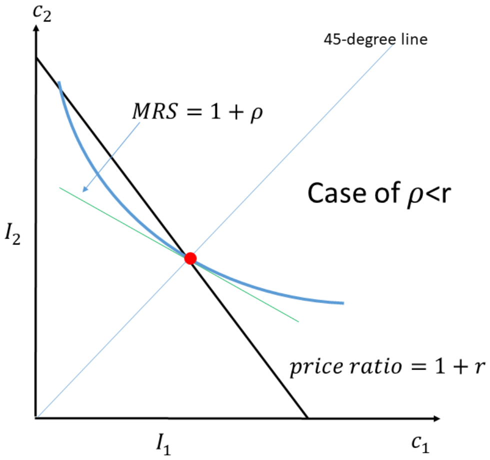
- Suppose the consumer has a utility function $U(c_1, c_2) = u(c_1) + \frac{1}{1+\rho} u(c_2)$.
  - $u(\cdot)$ is a strictly increasing and concave function.
  - $\rho \geq 0$ is the consumer's subjective discount rate.
- Suppose also that the consumer has stable income: $I_1 = I_2 = I$.
- The consumer's problem is to choose c_1 and c_2 to maximize $u(c_1) + \frac{1}{1+\rho} u(c_2)$ subject to $c_1 + \frac{c_2}{1+r} = I + \frac{I}{1+r}$.
- Assume interior solution. The slope condition is given by

$$(1+\rho) \frac{u'(c_1)}{u'(c_2)} = 1 + r \Leftrightarrow \frac{u'(c_1)}{u'(c_2)} = \frac{1+r}{1+\rho}.$$

- Whether the consumer is a saver or a borrower depends on the comparison between r and ρ.

Saver or Borrower?

- If $r > \rho$, then $c_1 < c_2$ at the optimal choice and the consumer is a saver.



Saver or Borrower?

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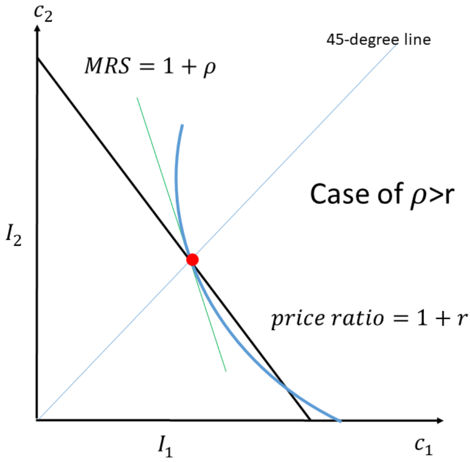
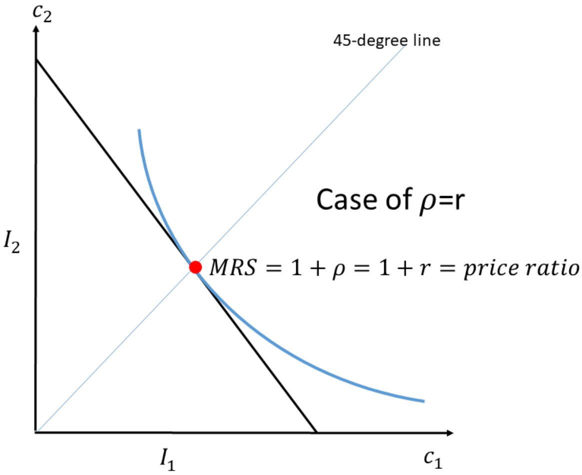


Saver or Borrower?

- If $r = \rho$, then $c_1 = c_2$ and the consumer neither save nor borrow.



Caveats

- In practice, borrowing usually entails a higher interest rate than saving.
- There is a cap on how much you can borrow.
- Interest rate schedule depends on the level of borrowing/saving.