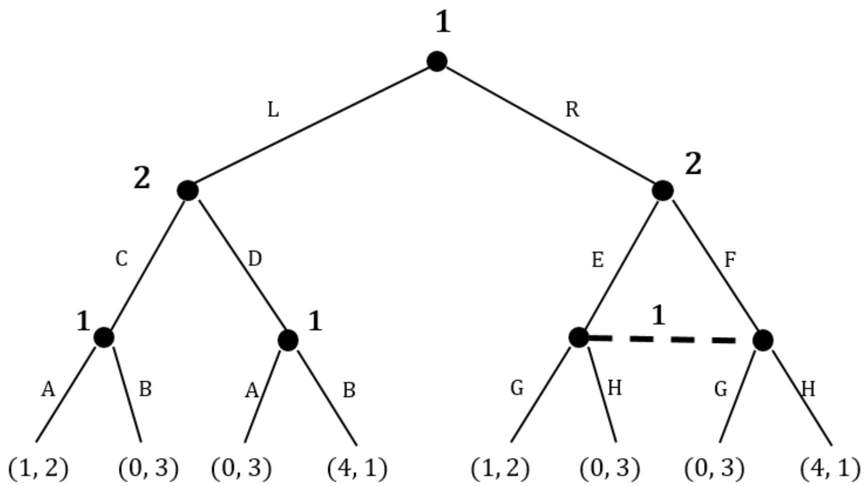
ECON3133 Final Exam Solution

Fall 2020, 120 minutes, 150 points

There are 7 questions.

1. (15 points) Consider the following game.



For each payoff pair, the left-hand side number is the payoff of player 1; the right-hand side number is the payoff of player 2. Predict the outcome of this game.

2. (15 points) A group project is assigned to a team of $n > 2$ group members. Each teammate i choose an effort level e_i to spend on the project. The quality of the project, $q(e_1, e_2, ..., e_n) = e_1 \times e_2 \times \cdots \times e_n$, is determined by the joint effort. Teammate i's payoff is

$$u_i(e_1, e_2, ..., e_n) = q(e_1, e_2, ..., e_n) - c(e_i),$$

where $c(e_i) = e_i^2$ is his effort cost.

a. All n teammates choose their effort simultaneously. Find the NE.

b. Continue with part (a). Does the quality of the project increase or decrease with the number of team members n?

3. (20 points) Consider the following game

		2	
		A	B
1	A	x, x	5, 1
	B	1, 5	3, 3

a. What is the range of x that the game has only one NE?

b. What is the range of x that the game has three NEs?

c. Given that x is in the range of part (b). Compute the mixed strategy NE. Let player 1 choose A with probability p and player 2 choose B with probability q.

d. Continue with part (c). Does the player's payoff in the equilibrium increase or decrease in x?

4. (25 points) There are two restaurants, $i = 1, 2$, operating on the University campus. Their demands are, respectively,

$$q_1(p_1, p_2) = 2 - p_1 + \frac{1}{3}p_2,$$
$$q_2(p_1, p_2) = 2 - p_2 + \frac{1}{3}p_1.$$

Assume their costs are zero. Suppose the maximum of price they can charge is 2 due to University regulation.

[Note: the price cap is to restrict the restaurants from setting infinite prices.]

a. Two firms choose their prices simultaneously. Compute the Nash equilibrium prices, (p_1^{NE}, p_2^{NE}) and profits.

b. Consider the leader-follower case. Suppose that firm 1 is an incumbent of the market and chooses its price p_1 first. Firm 2 is an entrant. Firm 2 chooses p_2 after observing p_1. Compute the equilibrium prices, (p_1^L, p_2^F). Does the leader or follow earn a higher profit?

c. What happens if the two restaurants merge? What will be the prices? What is the total profit after the merger?

d. Continue with part (a). Consider an infinitely repeated game of the simultaneous price setting game. These two restaurant interact with each other for infinitely many periods. The discount factor is $\delta \in [0, 1)$. The University does not allow them to merge. What is the condition of δ that they can tacitly collude on pricing using trigger strategy?

5. (25 points) The firms in the face mask industry engage in Cournot competition. Let n denote the number of firms. Firm i's quantity is denoted as q_i. All firms are identical and have a constant marginal cost $c = 2$. The market demand is $P(Q) = 32 - Q$, where Q is the summation of supply by all firms, $Q = \sum_{i=1}^{n} q_i$.

a. Suppose that there is only one profit-maximizing monopoly firm in the market. Compute the market supply $Q(n = 1)$ and consumer surplus $CS(n = 1)$.

b. Suppose that there are two firms. Compute the market supply $Q(n = 2)$ and consumer surplus $CS(n = 2)$.

c. Suppose there are $n \geq 1$ firms. Derive general formula for the market supply $Q(n)$, consumer surplus $CS(n)$, and the joint profit of all firms $\Pi(n)$.

d. Setting up each face mask firm requires a fixed cost of $K = 5$. Write down a formula for the total social welfare $W(n)$, which is the summation of consumer surplus and the joint profit netting off the total fixed costs. Compare two industrial policy that induces the number of firms to be $n_1 = 5$ and $n_2 = 10$, respectively. Which one maximizes the total surplus?

6. (25 points) Two fishermen, $i = 1, 2$, are suffering from the pollution poured by a steel firm. They decided to go to the congress and lobby for their right to fishing in clean water. Going for lobbying is costly. Suppose the time cost of lobbying spent by fisherman 1 is g_1, the cost spent by fisherman 2 is g_2. The expected benefit of lobbying is $G = g_1 + g_2$, which will be enjoyed by both fishermen as a public good.

Instead of spending the time lobbying, each fisherman will spend their time on fishing, denoted by x_i. Both of them are considering allocating their 30 days of time to fishing and lobbying. So each of them is facing the constraint $x_i + g_i = 30$. Fisherman 1's utility is

$$u_1(x_1, G) = \ln x_1 + \ln G.$$

Fisherman 2's utility is

$$u_2(x_2, G) = \ln x_2 + 2 \ln G.$$

Each of them has a utility function

a. Find the non-cooperative (Nash) equilibrium allocation. What is the level of G^{NE}?

b. Fix fisherman 1's utility at $\bar{u}_1$. List three equations that determines the Pareto optimal allocation $\{x_1^{PO}, x_2^{PO}, G^{PO}\}$.

c. Is the Pareto optimal allocation always make both fishermen better off compare to the non-cooperative equilibrium? Explain your argument.

7. (25 points) Consider the crude oil market in two countries. Country A has firm 1 producing oil at a constant marginal cost $c_1 = 2$. Country B has firm 2 producing oil at a constant marginal cost $c_2 = 2$.

If international trade is not allowed, the oil market in each country has a monopoly market structure. Firm 1 supplies country A with output level $Q_A = q_{1A}$. Firm 2 supplies country B with output level $Q_B = q_{2B}$.

If international trade is allowed, the oil market in each country is duopoly under Cournot competition. Firm 1 supplies both country A and B with quantities q_{1A} and q_{1B}, respectively. Firm 2 supplies both country A and B with quantities q_{2A} and q_{2B}, respectively.

The inverse demand for oil of country A is

$$p_A(Q_A) = 38 - Q_A, \quad \text{where } Q_A = q_{1A} + q_{2A}.$$

The inverse demand for oil of country B is

$$p_B(Q_B) = 20 - Q_B, \quad \text{where } Q_B = q_{1B} + q_{2B}.$$

a. Consider the case of no international trade. Compute the output level in country A (q_{1A}^M) and country B (q_{2B}^M). Then, compute the corresponding firm profits (π_1^M,π_2^M) and computer surplus (CS_A^M,CS_B^M).

b. Consider the case of free trade. Compute the Cournot equilibrium output levels q_{1A}^{NE}, q_{1B}^{NE}, q_{2A}^{NE}, and q_{2B}^{NE}. Then, compute the corresponding firm profits (π_1^{NE},π_2^{NE}) and computer surplus (CS_A^{NE},CS_B^{NE}). Note that each firm earns profit from two markets, that is $\pi_1^{NE} = \pi_{1A}^{NE} + \pi_{1B}^{NE}$ and $\pi_2^{NE} = \pi_{2A}^{NE} + \pi_{2B}^{NE}$.

c. Compare the results in part (a) and (b), which party is the winner and which party is the loser after opening up for international trade? (Party here refers to consumers of a country or a firm.)

For the rest of this question, consider the following game about trade policy.

At stage 1, country A and country B choose tariff level t_A and t_B, respectively.

At stage 2, after observing t_A and t_B, firm 1 and firm 2 engage in Cournot competition in two markets as described above.

Country A and B both try to maximize the total surplus of its side. Country A's total welfare includes the consumer surplus of country A, firm 1's profit, and tariff revenue collected from firm 2. Country B's total welfare includes the consumer surplus of country B, firm 2's profit, and tariff revenue collected from firm 1. That is

$$W_A(t_A, t_B) = CS_A(t_A) + \pi_1(t_A, t_B) + q_{2A}t_A,$$

$$W_B(t_A, t_B) = CS_B(t_B) + \pi_2(t_A, t_B) + q_{2B}t_B.$$

d. With the tariff, it is as if the marginal cost of the firm increases when it supplies to a foreign country. More specifically, in country A, firm 1 has a marginal cost $c_1 = 2$, while firm 2 has a marginal cost $c_2 = 2 + t_A$. Solve for the Cournot equilibrium quantities in country A and country B.

e. Intuitively, can you guess which country tends to send a higher tariff?

f. Continue with part (d), find the equilibrium tariff levels, t_A^{NE} and t_B^{NE}.