5. Monopoly

Monopoly behavior

Example

$$Q = 2000 - 20P, \quad C(Q) = \frac{1}{20}Q^2 + 10000$$

Profit objective function

$$\max_Q R(Q) - C(Q)$$

FOC

$$MR(Q) = MC(Q)$$

Inverse demand

$$P(Q) = 100 - \frac{1}{20}Q$$

$$R(Q) = Q \times P(Q) = 100Q - \frac{1}{20}Q^2$$

$$MR(Q) = 100 - \frac{1}{10}Q = MC(Q) = \frac{1}{10}Q$$

$$\frac{1}{5}Q = 100 \Rightarrow Q_m = 500$$

$$P_m = 100 - \frac{1}{20}Q_m = 100 - 25 = 75$$

Monopoly profit

$$\begin{aligned} &R(Q_m) - C(Q_m) \\ =&P_m Q_m - AC(Q_m)Q_m = [P_m - AC(Q_m)] \times Q_m \end{aligned}$$

Example

$$P = a - bQ, \ MC = c$$

For linear demand curve, marginal revenue is twice as steep as the inverse demand

$$R(Q) = Q \times (a - bQ) = aQ - bQ^2$$

$$MR(Q) = a - 2bQ$$

Solve the monopoly problem

$$MR(Q) = a - 2bQ = c$$

$$Q_m = \frac{a-c}{2b}, \quad P_m = a - bQ_m = a - \frac{a-c}{2} = \frac{a+c}{2}$$

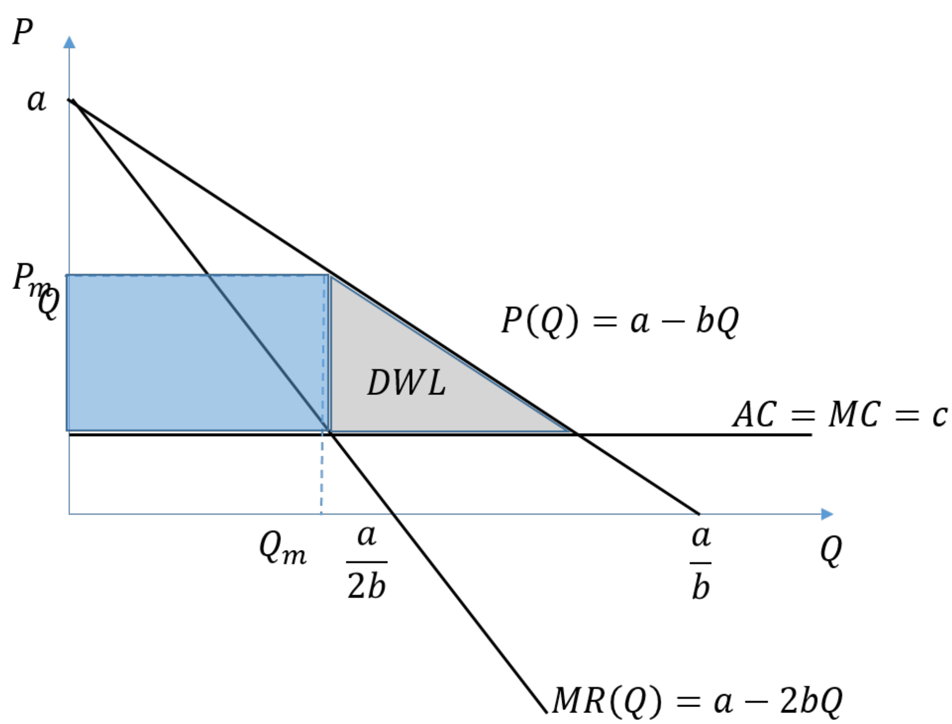
$$\pi_m = (P_m - c)Q_m = [\frac{a+c}{2} - c] \times \frac{a-c}{2b} = \frac{(a-c)^2}{4b}$$

Under competitive equilibrium, the market quantity is determined by

$$P = a - bQ = c$$

$$Q_c = \frac{a-c}{b}$$

$$\begin{aligned} DWL &= \frac{1}{2}|Q_c - Q_m| \times |P_m - P_c| \\ &= \frac{1}{2}(\frac{a-c}{b} - \frac{a-c}{2b})(\frac{a+c}{2} - c) \\ &= \frac{1}{2}\frac{a-c}{2b} \times \frac{a-c}{2} = \frac{(a-c)^2}{8b} \end{aligned}$$



$$Q = aP^e, (e < -1) \;\; MC = c$$

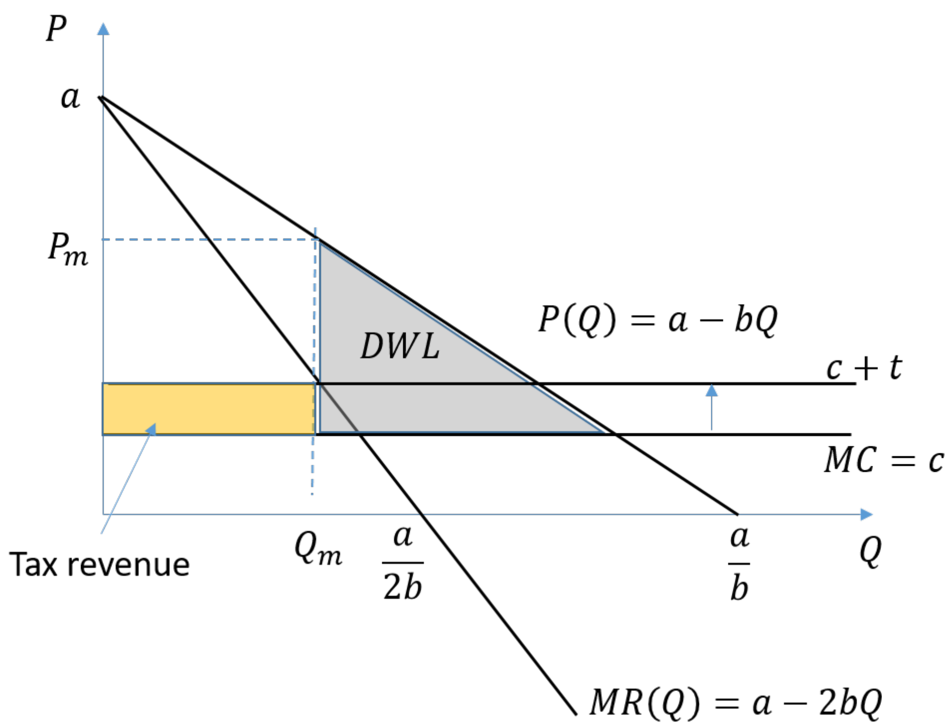
$$\frac{P-c}{P} = -\frac{1}{e}$$
$$1-\frac{c}{P} = -\frac{1}{e}$$
$$\frac{c}{P} = 1+\frac{1}{e}$$
$$P = \frac{c}{1+\frac{1}{e}} = \frac{c}{1-\frac{1}{|e|}}$$

Taxing monopoly

Quantity tax t

For linear demand

$$MR(Q) = a-2bQ = MC+t = c+t$$
$$Q_m' = \frac{a-c-t}{2b}, \quad P_m' = \frac{a+c+t}{2} = \frac{a+c}{2}+\frac{t}{2}$$
$$\pi_m' = \frac{(a-c-t)^2}{4b}$$



For constant elastic demand

$$P = \frac{c+t}{1-\frac{1}{|e|}} = \frac{c}{1-\frac{1}{|e|}} + \frac{t}{1-\frac{1}{|e|}}$$

Note that $0 < 1 - \frac{1}{|e|} < 1$ (for $|e| > 1$, elastic demand), so the increase of price

$$\frac{t}{1 - \frac{1}{|e|}} > t$$

Monopolistic competition (long run)

$$C(q) = q + 4, \ \ P = a - q$$

Note that here a is endogenous. It varies with entry/exit of firms in the long run.
Two conitions that characterize the equilibrium of a monopolistic competition market

$$\begin{cases} \text{zero profit} & AC(q) = P \\ \text{profit max} & MR(q) = MC(q) \end{cases}$$

$$AC(q) = 1 + \frac{4}{q}, \ \ MC(q) = 1$$

$$MR(q) = a - 2q = MC = 1$$

$$AC(q) = 1 + \frac{4}{q} = P(q) = a - q$$

Two equations, two unknown, q_m and a.

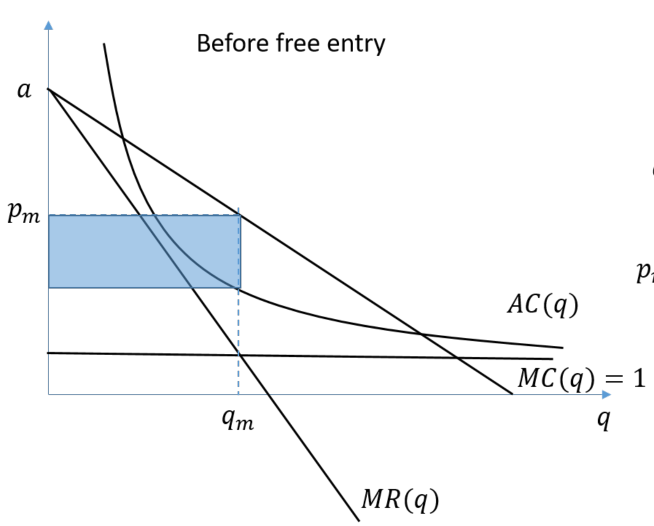
$$\begin{cases} a - 2q = 1 & \quad a = 1 + 2q \\ 1 + \frac{4}{q} = a - q & \end{cases}$$

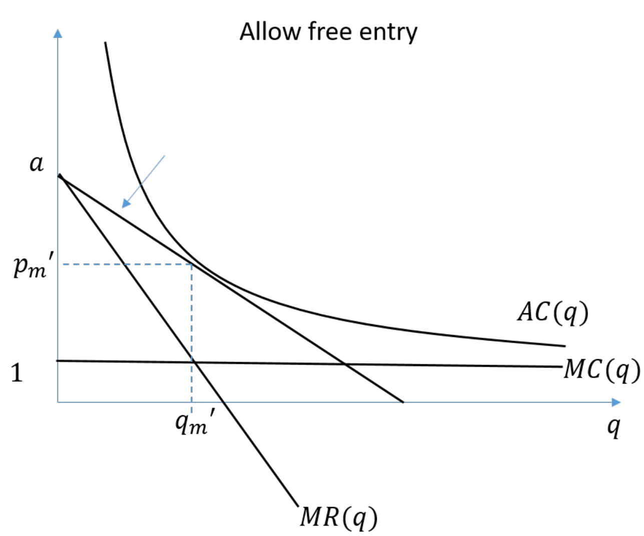
$$1 + \frac{4}{q} = 1 + 2q - q = 1 + q$$

$$q + 4 = q + q^2$$

$$q_m = 2$$

$$a = 5$$





Natural monopoly and regulation

Average cost

$$AC(q) = 1 + \frac{4}{q}, \quad MC(q) = 1$$

Demand is

$$P(q) = 10 - q$$

When the government does not regulate price, the monopoly will set price by

$$MR(q) = MC(q).$$

This implies a higher price, low quantity, and low social welfare.

The quantity that maximize social welfare is determined by MC pricing

$$MC(q) = 1 = P(q) = 10 - q$$

$$q_2 = 9, \; p_2 = 1$$

In this case, consumer surplus is

$$CS = \frac{1}{2} \times (10 - 1) \times 9 = 40.5$$

producer surplus $PS = 0$. Total social welfare is 22.5.

Therefore, producer make a loss (AFEp_2)

$$\pi = (p_2 - AC(q_2)) \times q_2 = (1 - 1 - \frac{4}{9}) \times 9 = -4$$

(This amount is the fixed cost.)

If the government regulate price at $p_2 = 1$, quantity level is efficient, but the monopoly makes a loss in the long run and need compensation.

If the government does not want to pay compensation, it can allow the monopoly to charge a high price to some consumers who has high willingness to pay.

For example, if the first two consumers who has the highest valuation pay a high price.

$$P(q) = 10 - q$$

The first consumer is willing to pay 9.

The second consumer is willing to pay 8.

If the monopoly charge $p_3 = 8$ for the first two units. Then it earns $p_3 \times 2 = 16$ revenue from them.

There is an extra 14 revenue paid by these two consumers. This extra revenue can be use to cover the long-run loss.

First degree price discrimination

Example

$$P = 100 - \frac{Q}{20}$$

$$MC = 10$$

Under first-degree price discrimination

The first consumer (who has the highest willingness to pay) is willing to pay

$$P = 100 - \frac{1}{20} = \frac{1999}{20}$$

The second consumer is willing to pay

$$P = 100 - \frac{2}{20} = \frac{1998}{20}$$

The monopoly can offer price the same as this willingness to pay.

What will be its profit?

The monopoly can supply to where

$$MC = 10 = P = 100 - \frac{Q}{20}$$

$$200 = 2000 - Q$$
$$Q^* = 1800$$

For every unit, the monopoly charges the price exactly the same as the willingness to pay. The entire consumer surplus is captured by the monopoly.

In this case, the profit is

$$\pi^{1st} = \frac{1}{2}(100 - 10) \times 1800$$

If there is a uniform price policy, the monopoly cannot sell its good to different consumers at different prices. Then it will supply by

$$MC = 10 = MR = 100 - \frac{1}{10}Q$$

$$100 = 1000 - Q$$

$$Q^u = 900,\ p^u = 100 - \frac{Q^u}{20} = 55.$$
$$\pi^u = (p^u - MC)Q^u = (55 - 10) \times 900 < \pi^{1st}$$

Two part tarriff

The monopoly can use two-part tariff to achieve the profit from first degree price discrimination. The total payment/revenue is

$$T(q) = A + pq$$

Set $p = MC = 10$.

Consumers will earn a consumer surplus CS.

Then, the monopoly will charge a fixed fee/membership fee exactly equal to the CS and extract it.

Quantity discount

Suppose a consumer has the following decreasing marginal willingness to pay

$$\begin{aligned} v_1 &= 10 \\ v_2 &= 9 \\ v_3 &= 8 \\ &\vdots \\ v_{10} &= 1 \end{aligned}$$

The monopoly can set a price schedule appear to be quantity discount.

If the consumer only buy one unit, then

$$p_1 = 12.$$

If the conusmer buy three units, then for each unit, the price is

$$p_3 = 9.$$

The consumer has a total surplus of $V_3 = v_1 + v_2 + v_3 = 27$.

The total payment from three units is $p_3 \times 3 = 27$.

This way, all consumer surplus is extracted by the monopoly.

Third degree price discrmination

$$P_1 = 24 - Q_1, \;\; P_2 = 12 - 0.5Q_2, \;\; MC = 6$$

(1) When price discrimination is allowed.

Monopoly can different prices for two markets.

For market 1,

$$MR_1 = 24 - 2Q_1 = MC = 6$$

$$2Q_1 = 18 \Rightarrow Q_1^* = 9$$

$$P_1^* = 24 - Q_1^* = 15$$

$$\pi_1^* = (P_1^* - MC) \times Q_1^* = (15 - 6) \times 9 = 81$$

For market 2,

$$MR_2 = 12 - Q_2 = MC = 6$$
$$Q_2^* = 6 \Rightarrow P_2^* = 12 - 0.5 \times Q_2^* = 9$$
$$\pi_2^* = (P_2^* - MC) \times Q_2^* = (9-6) \times 6 = 18$$

The total profit by exercising 3rd degree price discrimination

$$\Pi^* = \pi_1^* + \pi_2^* = 99.$$

$$\max_{Q_1, Q_2} P_1(Q_1)Q_1 + P_2(Q_2)Q_2 - 6 \times (Q_1 + Q_2)$$

$$\max_{P_1, P_2} P_1 \times Q_1^D(P_1) + P_2 \times Q_2^D(P_2) - 6 \times (Q_1^D(P_1) + Q_2^D(P_2))$$

(2) If price discrimination is not allowed. Regulator impose uniform price rule.

$$\max_{P_1, P_2} P_1 \times Q_1^D(P_1) + P_2 \times Q_2^D(P_2) - 6 \times (Q_1^D(P_1) + Q_2^D(P_2)) \quad \text{s.t. } P_1 = P_2$$

$$\max_{P} P \times Q_1^D(P) + P \times Q_2^D(P) - 6 \times (Q_1^D(P) + Q_2^D(P))$$

Because there is an extra constraint, so the monopoly must earn less (or equal) profit than the previous case.

Find market demand (aggregate the quantity demanded by group 1 and group 2)

$$P_1 = 24 - Q_1, \quad P_2 = 12 - 0.5Q_2$$
$$Q_1^D(P_1) = 24 - P_1, \quad Q_2^D(P_2) = 24 - 2P_2$$

Under uniform price policy, $P_1 = P_2 = P$. The market demand is

$$Q^D(P) = \begin{cases} Q_1^D(P) + Q_2^D(P) = 48 - 3P & , \text{for } P \leq 12 \\ Q_1^D(P) + 0 = 24 - P & , \text{for } P > 12 \end{cases}$$

Given the market demand, find monopoly supply

$$Q^D(P) = 48 - 3P$$

$$\Rightarrow P(Q) = 16 - \frac{1}{3}Q$$

$$MR(Q) = 16 - \frac{2}{3}Q = MC = 6$$

$$\frac{2}{3}Q = 10, \quad Q^u = 15$$

$$P^u = 16 - \frac{1}{3}Q^u = 11$$

Note that $11 < 12$, so the supply point is on the first part of demand curve.

$$\Pi^u = (P^u - MC) \times Q^u = (11 - 6) \times 15 = 75.$$

There is a tricky thing: $\Pi^u = 75 < \pi_1^* = 81$. Setting price at P^u and serve two groups of consumers ends with a smaller profit than serving only group 1 consumers.

In this case, the monopoly will shut down the group 2 consumers. It will choose a high price $P^u = P_1^* = 15$ and only sell to group 1 consumers.

Therefore, the welfare effect of uniform price policy is negative.

The equilibrium quantity drops from $Q^* = 9 + 6 = 15$ to $Q^u = 9$.

This is an exmple of unintended consequence of policy/regulation.