Midterm Test

ECON 4114 Industrial Organization and Competitive Strategies 2020

Part 1

Time allowed: 38 minutes **Total points: 50 points**

1. (10 marks) Consider a homogeneous-good Bertrand model with 2 firms; let's call them Firm A and Firm B. The market demand is $Q = 10 - P$. Firm A has a constant marginal cost of 3, whereas Firm B has a constant marginal cost of 4. Both firms do not face any capacity constraint.

   The two firms choose their respective prices p_A and p_B simultaneously. Whichever firm charges a lower price grabs the whole market demand. If there is a tie, all consumers buy from Firm A. What is/are the Nash equilibrium(s) of this pricing game? Explain.

   Solution:

   - *The only Nash equilibrium is* $p_A = p_B = 4$.
     - *At these prices, Firm A's profit is* $(10-4)\times(4-\overset{3}{\cancel{1}}) = \overset{6}{\cancel{18}}$, *whereas Firm B's profit is zero.*
     - *Firm B does not have any profitable deviation because any higher price results in no sale (and hence zero profit) whereas any lower price would mean selling below marginal cost (and hence negative profit).*
     - *Firm A does not have a profitable deviation neither because any higher price results in no sale (and hence zero profit) whereas any lower price would result in a lower profit*

       $$\frac{\partial}{\partial p_A}(10-p_A)(p_A-3) = 13 - 2p_A > 0 \text{ for all } p_A \in [3,4].$$

     - *There is no other equilibria. First, any equilibrium must have* $p_B = 4$. *To see this, take some* $p_B > 4$. *If* $p_A > p_B > 4$, *then Firm A has a profitable deviation to price slightly above 4 but below* p_B. *If* $p_B \geq p_A > 4$, *then Firm B has a profitable deviation to price slightly below* p_A. *If* $p_B > 4 \geq p_A$, *Firm A has a profitable deviation to price slightly above 4 but below* p_B. *Now, as* $p_B = 4$, *and the unique best response of Firm A to* $p_B = 4$ *is also pricing at 4, the unique equilibrium is* $p_A = p_B = 4$.

2. (40 marks) There are two and only two firms in the industry, selling differentiated products; let's call them Firm A and Firm B. The system of demands is given by

$$\begin{aligned} q_A &= 50 - 3p_A + 2p_B; \\ q_B &= 20 - 2p_B + p_A. \end{aligned}$$

Both firms have the same cost function $C(q) = q^2$.

In each firm, the owner delegates the pricing decision to a manager. To align the manager's incentive with that of the owner, the owner can offer a reward scheme based on the firm's profit. Suppose initially, both managers have their reward tied proportionally to the profit of the firm they are working in.

One day, Firm A's owner hires you as a consultant to work on reformulating its managerial compensation scheme. She tells you that she is contemplating which one of the following four reward schemes should be used.

- *Status quo*: reward proportional to Firm A's own profit.
- *Relative performance evaluation*: reward proportional to the amount by which Firm A's profit exceeds Firm B's profit.
- *Conquering market share:* reward proportional to the market share acquired.
- *Lerner index*: reward proportional to Firm A's Lerner index.

The choice of the reward scheme is publicly known to both managers before they choose their respective prices simultaneously.

For simplicity, assume the monetary cost of managerial compensation is negligible, so the firm owner cares only about the profit from the sales of goods. Also, Firm B's owner has already fixed and thus is **not** going to change the compensation scheme of her manager in any case.

Which reward scheme(s) above would you recommend to Firm A's owner? Explain **without using any calculation**. (Hint: In evaluating the reward schemes above, highlight the implications on the competitor's pricing. You may or may not use reaction-function diagrams to assist your presentation.)

Solution:

- *Relative performance evaluation is worse than the status quo of reward according to profit.*
  - *The direct effect (i.e., fixing competitor's responses) is negative, simply because p_A is no longer set to optimize profit.*
  - *The strategic effect is also negative. Compared with the status quo, relative performance evaluation induces more aggressive pricing by Firm A's manager: for each p_B, Firm A's best-responding price gets lower under relative performance evaluation (this can be represented by an inward shift of the reaction function). This is because a lower price hurts Firm B's profit, which benefits Firm A's manager. As Firm B's best response function is upward sloping (strategic complements), Firm B would respond by a lower price, which further hurts Firm A's profit.*

- *The total effect is thus unambiguously negative.*

- *The conquer market-share scheme is worse than the status quo of reward according to profit.*
  - *The direct effect (i.e., fixing competitor's responses) is negative, simply because p_A is no longer set to optimize profit.*
  - *The strategic effect is also negative. Compared with the status quo, the conquer market-share scheme induces more aggressive pricing by Firm A's manager: for each p_B, Firm A's best-responding price gets lower under the conquer market-share scheme (this can be represented by an inward shift of the reaction function). This is because a lower p_A is needed to expand the demand of Firm A and hurt the demand of Firm B. As Firm B's best response function is upward sloping (strategic complements), Firm B would respond by a lower price, which further hurts Firm A's profit.*
  - *The total effect is thus unambiguously negative.*

- The *Lerner index scheme may or may not better than the status quo of reward according to profit.*
  - *The direct effect (i.e., fixing competitor's responses) is negative, simply because p_A is no longer set to optimize profit.*
  - *The strategic effect is, however, positive. Compared with the status quo, the Lerner index scheme induces less aggressive pricing by Firm A's manager: for each p_B, Firm A's best-responding price gets higher under the Lerner index scheme (this can be represented by an outward shift of the reaction function). The reason is as follows. A higher price brings less sales and hence a lower marginal cost. A higher price thus certainly implies a higher profit margin and Lerner index. As Firm B's best response function is upward sloping (strategic complements), Firm B would respond by a higher price, which benefits Firm A's profit.*
  - *The total effect is thus ambiguously, depending on whether the direct effect or the strategic effect is stronger.*

- *Relative performance evaluation and the conquer market-share scheme should definitely not be used. There is no definite ranking between the status quo and the Lerner index scheme.*

End of Part 1

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Part 2

Time allowed: 38 minutes **Total points: 50 points**

3. The market demand is given by $P(Q) = 200 - 2Q$, where Q is the aggregate quantity. There are two firms in the market, and they compete by choosing quantities. Let's call them Firm 1 and Firm 2. Initially, both firms have a common cost function $C(q) = 5q^2$.

   (a) (10 marks) Compute the Cournot equilibrium. Calculate the equilibrium profit of each firm.

   Solution: The best-response of Firm 1 can be found by solving:

   $$\max_{q_1} (200 - 2(q_1 + q_2)) q_1 - 5q_1^2$$

   The FOC can be rewritten as

   $$q_1 = R_1(q_2) = \frac{100 - q_2}{7}.$$

   By symmetry, the best-response function of Firm 2 is given by

   $$q_2 = R_2(q_1) = \frac{100 - q_1}{7}.$$

   On solving, $q_1 = q_2 = \frac{25}{2} = 12.5$. *The profit of each firm in equilibrium is*

   $$\left(200 - 2\left(\frac{25}{2} + \frac{25}{2}\right)\right)\frac{25}{2} - 5\left(\frac{25}{2}\right)^2 = \frac{4375}{4} = 1093.75.$$

   (b) Prof Au has made a major breakthrough in his research in management strategy, which resulted in the invention of a new management system AMS. The adoption of AMS can lower the cost of production to $\tilde{C}(q) = 2q^2$.

   1. (8 marks) Suppose Firm 1 has adopted AMS (but Firm 2 has not). Explain, without any calculation, whether Firm 2 would increase or decrease its production.

      Solution:

      - *Firm 1 produces more since the marginal cost of production is now lower.*
      - *Since the reaction function of firm 2 is downward-sloping, this means firm 2 would cut its production.*

   2. (8 marks) What type of strategy does adopting AMS belong to (top dog, lean-and-hungry look, puppy dog, or fat cat)? Explain.

      Solution:

- *Since there is quantity competition, the reaction functions are downward sloping and there are strategic substitutes.*
- *Adopting AMS lowers the cost of production, which raises its own quantity of production, and in turn lowers the demand and hence profit of Firm 2. Therefore, it makes Firm 1 tough.*
- *Taken together, this is thus a top-dog strategy.*

(c) (24 marks) Instead of offering AMS exclusively to Firm 1, Prof Au decides to make AMS available for both firms, and he posts a licence fee L for the right to use it. After observing the fee L, the two firms simultaneously decide whether to pay for and adopt AMS. After making their own and observing the other firm's adoption decision, the two firms compete by choosing quantities simultaneously.

Find the subgame-perfect Nash equilibrium of the game described above. Note that Prof Au is also a player in the game, whose objective is to maximize the revenue from the collection of license fees. Also you may assume that firms adopt AMS when indifferent.

Solution: We apply generalized backward induction.

- *There are 4 subgames in the last stage corresponding to the following combination of adoption decisions.*

		Firm 2	
		Adopt	Not adopt
Firm 1	Adopt	$\square, \square$	$\square, \square$
	Not adopt	$\square, \square$	$\square, \square$

- *The subgame (Not adopt, Not adopt) has already been solved in part (a).*
- *Consider the subgame in which only Firm 1 adopts AMS. Firm 1's profit-maximizing problem is*

$$\max_{q_1} \left(200 - 2\left(q_1 + q_2\right)\right) q_1 - 2q_1^2.$$

The FOC is

$$\left(200 - 2\left(q_1 + q_2\right)\right) - 2q_1 - 4q_1 = 0 \Leftrightarrow q_1 = 25 - \frac{1}{4}q_2 \equiv R_1\left(q_2\right).$$

The best-response function of firm 2 remains at $R_2\left(q_1\right) = \frac{100-q_1}{7}$. *The intersection of best-response functions* $q_1 = R_1\left(q_2\right)$ *and* $q_2 = R_2\left(q_1\right)$ *gives*

$$q_1 = \frac{200}{9} = 22.222 \text{ and } q_2 = \frac{100}{9} = 11.111.$$

The profit of Firm 1 is thus $\left(200 - 2\left(\frac{200}{9} + \frac{100}{9}\right)\right)\frac{200}{9} - 2\left(\frac{200}{9}\right)^2 = \frac{160\,000}{81} \approx 1975.3.$
The profit of Firm 2 is thus $\left(200 - 2\left(\frac{200}{9} + \frac{100}{9}\right)\right)\frac{100}{9} - 5\left(\frac{100}{9}\right)^2 = \frac{70\,000}{81} \approx 864.20.$

- *Consider the subgame in which both Firm 1 and 2 adopt AMS. By the previous analysis, the best-response functions of Firm 1 and 2 are* $R_1(q_2) = 25 - \frac{1}{4}q_2$ *and* $R_2(q_1) = 25 - \frac{1}{4}q_1$ *respectively. The Nash equilibrium is given by* $q_1 = q_2 = 20$. *The profit for each firm is then* $(200 - 2(20+20))(20) - 2(20)^2 = 1600$.
- *At the stage of deciding whether to pay for AMS, the payoff table is given by*

		Firm 2	
		Adopt	Not adopt
Firm 1	Adopt	$1600 - L, 1600 - L$	$1975.3 - L, 864.20$
	Not adopt	$864.20, 1975.3 - L$	$1093.75, 1093.75$

  - *Both firms adopting is a Nash equilibrium if and only if* $1600 - L \geq 864.20 \Leftrightarrow L \leq 735.8$.
  - *Only one firm adopting is a Nash equilibrium if and only if* $1975.3 - L \geq 1093.75$ *and* $864.20 \geq 1600 - L$, *which is equivalent to* $735.8 \leq L \leq 881.55$.
  - *Neither firm adopting is a Nash equilibrium if and only if* $L \geq 881.55$.
- *Now going back to the very first decision node, Prof Au's revenue is*
  - $2L$ *if* $L \leq 735.8$,
  - L *if* $735.8 < L \leq 881.55$, *and*
  - 0 *if* $L > 881.55$.
- *The optimal fee is thus* 735.8.
- *A SPNE is as follows.*
  - *Prof Au charges* $L = 735.8$.
  - *Firm 1 adopts AMS if* $L \leq 881.55$ *and does not adopt otherwise. If both firms adopt AMS, then* $q_1 = 20$. *If only it adopts but not Firm 2, then produce* $q_1 = \frac{200}{9} = 22.222$. *If it does not adopt but Firm 2 adopts, then produce* $q_1 = \frac{100}{9} = 11.111$. *If neither firm adopts, then produce* $q_1 = \frac{25}{2} = 12.5$.
  - *Firm 2 adopts AMS if* $L \leq 735.8$ *and does not adopt otherwise. If both firms adopt AMS, then* $q_2 = 20$. *If only it adopts but not Firm 1, then produce* $q_2 = \frac{200}{9} = 22.222$. *If it does not adopt but Firm 1 adopts, then produce* $q_2 = \frac{100}{9} = 11.111$. *If neither firm adopts, then produce* $q_2 = \frac{25}{2} = 12.5$.
- *Swapping the indices for Firm 1 and Firm 2 give another SPNE.*

End of Part 2