HKUST **FINA3203 Fall 2018**
School of Business **Yan Ji**

Final Mock Exam (solutions)

Thursday, 20 Dec, 2018

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1 (20 points) Short Questions

(Please briefly explain your answers.)

(i) (5 points) How to use binary options to replicate a standard European call option with strike price K?

Answer: We long an asset-or-nothing call with strike K and short K shares of cash-or-nothing call with strike K. The binary options we use should have the same maturity as the standard European call.

(ii) (5 points) The Delta of a call option will increase as the volatility of the stock increases.

Answer: False. $\Delta = N(d_1)$, where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

This formula shows that d_1 is not monotonic in σ. Particularly, when the option is deeply in the money, higher σ reduces the option's Δ.

(iii) (5 points) All else equal, when the default correlation rises, the value of the equity tranche of a CDO increases while the values of the senior tranches typically decrease.

Answer: True. The equity tranche is long default correlation while the senior tranches are short default correlation.

(iv) (5 points) Compare the following two positions. There are three strike prices, $K1 = 10$, $K2 = 20$ and $K3 = 30$. Position 1 holds a butterfly position (using all three options) and borrows the present value of 10 dollars (borrow $10 \times B(t;T)$ dollars today, repay 10 dollars tomorrow). Position 2 holds a short straddle (sell the 20 call, sell the 20 put). Draw the payoff diagrams for the two positions. In each case, what are you betting on? Which position is cheaper today and why?

Answer: The payoff diagrams of both positions resemble tents. The maximum payoff of both positions is 0 (when the strike is 20). (If you depicted a regular butterfly with positive payoff, you forgot to include the borrowing.) The difference is that the butterfly position has wings that do not go below -10, while the short straddle has wings that extend much further. From the payoff diagrams, both positions are bets that the stock price will not move far from 20. In the case that the stock price does not move far from 20, the proceeds from each position today will outweigh the non-positive payoff at maturity. From the payoff diagrams, position 2 will be cheaper today because it does not bound the negative payoffs at -10.

2 (20 points) Long Question 1

European puts and calls with one year until expiration are available on ABC common stock at the following prices:

Strike price	Call price	Put price
30	-	5
40	10	-
50	-	10

The dashes indicate that the corresponding options are not available. The stock pays no dividends, and its current price is \$38 per share. The current price of a zero coupon bond paying one dollar one year from now is \$0.90.

(i) (10 points) Calculate the three missing prices in the table.

Answer: Each of the missing options could be created synthetically. For example, the call with a strike price of \$30 could be created by buying the stock, buying the put with a strike price \$30, and borrowing the present value of \$30. The net investment required would be \$[5 + 38 - 30(.90)] = \$16. In the same way, the net investment required to create the call with a strike price of \$50 is \$[10 + 38- 50(.90)] = \$3. Finally, the put with a strike price of \$40 could be created synthetically by shorting the stock, buying the call with a strike price of \$40, and lending the present value of \$40. The net investment required here would be \$[10 + 38 - 40(.90)] = \$8.

Here are the prices for all of the options, both actual and synthetic:

Strike price	Call price	Put price
30	16	5
40	10	8
50	3	10

(ii) (10 points) Explain how you could make an arbitrage profit trading at the quoted prices.

Answer: Clearly, the arbitrage profit cannot be made by trading puts and calls with the same strike price. This is because we used the put-call parity to derive the missing prices, which ensures that the no arbitrage condition holds. Thus, we have to think whether there exist arbitrage opportunities by trading calls/puts with different strike prices.

Both the call prices and the put prices violate the condition that the option value must be a convex function of the striking price. In each case, the middle strike option is too expensive relative to the two end strike options. One way to take advantage of this would be to buy synthetic calls with strike prices of \$30 and \$50 and sell two calls with a strike price of \$40. The overall portfolio would thus be long one put with a strike price of \$30, long one put with a strike price of \$50, short two calls with a strike price of \$40, long two shares of stock, and borrowing of \$(30+50)(.90) = \$72. This strategy would produce an immediate cash inflow of \$(-5-10 + 20 - 76 + 72) = \$1 . The following table shows the outcomes for each possible final stock price S:

	$S < 30$	$30 \leq S < 40$	$40 \leq S < 50$	$S < 50$
Long 1 put with $K = 30$	$30 - S$	0	0	0
Long 1 put with $K = 50$	$50 - S$	$50 - S$	$50 - S$	0
Short 2 calls with $K = 40$	0	0	$-2(S - 40)$	$-2(S - 40)$
Long 2 shares	$2S$	$2S$	$2S$	$2S$
Borrow 72	-80	-80	-80	-80
Total payoff	0	$S - 30$	$50 - S$	0

In no circumstances would you have a loss later, and if the final stock price is between \$30 and \$50 you would have an additional profit. Alternatively, we could think of taking advantage of the opportunity by shorting two of the synthetic middle strike puts and buying the end strike puts. However, this will lead to exactly the same overall position that we just considered, as it must because of the relationship between the actual and synthetic options.

3 (30 points) Long Question 2

Alex, a manager at Fast Food, Inc.(FFI) received 1,000 shares of company stock as part of his compensation package. The stock currently sells at \$40 at share. Alex would like to defer selling the stock until the next tax year. In January, however, he will need to sell all his holdings to provide for a down payment on his new house. Alex is worried about the price risk involved in keeping his shares. At current prices, he would receive \$40,000 for the stock. If the value of his stock holdings falls below \$35,000, his ability to come up with the necessary down payment would be jeopardized. On the other hand, if the stock value rises to \$45,000, he would be able to maintain a small cash reserve even after making the down payment. Alex considers three investment strategies:

1. Strategy A is to write January call options on the FFI shares with strike price \$45. These calls are currently selling for \$3 each.

2. Strategy B is to buy January put options on FFI with strike price \$35. These options also sell for \$3 each.

3. Strategy C is to establish a zero-cost collar by writing the January calls with strike price \$45 and buying the January puts with strike price \$35.

Evaluate each of these strategies with respect to Alex's investment goals. What are the advantages and disadvantages of each? Which would you recommend?

Answer: Let us examine the payoffs of each of the strategies:

1. By writing call options (these are called covered calls given the long position in the stock), Alex takes in premium income of \$3,000. If the price of the stock in January is less than or equal to \$45, he will have his stock plus the premium income. But the most he can have is \$45,000 + \$3,000 because the stock will be called away from him if its price exceeds \$45. (We are ignoring interest earned on the premium income from writing the options in this very short period of time.) The payoff as a function of the stock price in January, S_T, is

Stock price	Portfolio value
$S_T \leq \$45$	$(1,000)S_T + 3,000$
$S_T > \$45$	$45,000 + 3,000 = \$48,000$

   This strategy offers some extra premium income (by selling the upside) but leaves substantial downside risk. At an extreme, if the stock price fell to zero, Jones would be left with only \$3,000. The strategy also puts a cap on the final value at \$48,000, but this is more than sufficient to purchase the house.

2. By buying put options with a \$35 strike price, Alex will be paying \$3,000 in premiums to insure a minimum level for the final value of his portfolio. That minimium value is $(35)(1,000) - 3,000 = \$32,000$. This strategy allows for upside gain, but exposes Jones to the possibility of a moderate loss equal to the cost of the puts. The payoff structure is

Stock price	Portfolio value
$S_T \leq \$35$	$35,000 - 3,000 = \$32,000$
$S_T > \$35$	$(1,000)S_T - 3,000$

3. The cost of the collar is zero. The value of the portfolio in January will be as follows:

Stock price	Portfolio value
$S_T \leq \$35$	$\$35,000$
$\$35 < S_T < \$45$	$(1,000)S_T$
$S_T \geq \$45$	$45,000

If the stock price is less than or equal to $35, the collar preserves the $35,000 in principal. If the stock price exceeds $45, the value of the portfolio can rise to a cap of $45,000. In between, the proceeds equal 1,000 times the stock price

Given the objective of Alex, the best strategy would be (c) since it satisfies the two requirements of preserving the $35,000 in principal while offering a chance of getting $45,000. Strategy (a) should be ruled out since it leaves Alex exposed to the risk of substantial loss of principal.

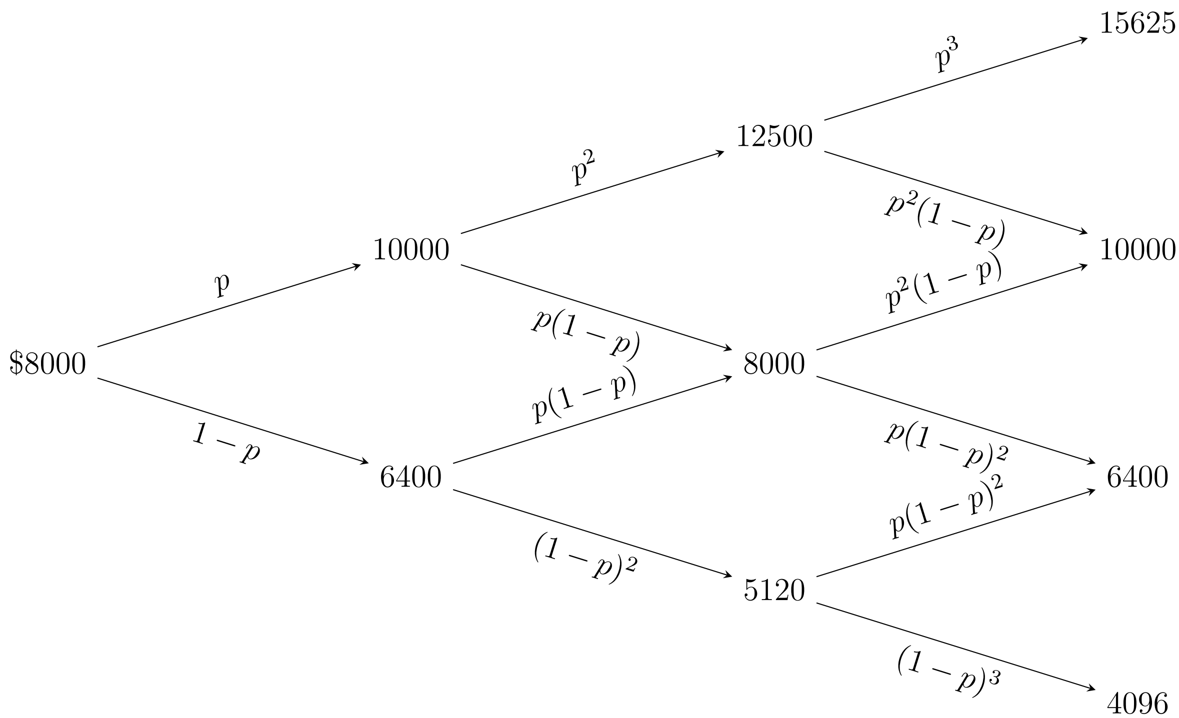
4 (30 points) Long Question 3

A chooser option gives its owner the right to specify at the time of exercise whether the option is a put or a call. Your firm is considering a three-period American chooser option with a time-dependent strike price.

Each option covers 100 shares of ABC stock. The current price of the stock is $80 per share. Over each period, the stock price will either increase by 25% or decrease by 20%. The strike price of the option grows at 5% per period. The initial strike price is $8,000. The stock does not pay dividends, and the interest rate is 2.5% per period.

(i) (15 points) Given the information above, what is the proper value of the chooser option?

Answer: For this problem the stock price tree over the next three periods will be



The risk-neutral probability is $p = (1.025 - 0.8)/(1.25 - 0.8) = 0.5$. After one period, the strike price will be 8000 (1.05) = 8400. After two periods, it will be 8820, and after three periods it will be 9261. At the time of exercise, the owner of the option will choose for it to be a call if the stock price at that time is greater than the prevailing striking price. Similarly, the owner will chose for it to be a put if the stock price at that time is less than the prevailing striking price. The problem is solved by successive application of the one-period risk-neutral valuation equation, working backwards from the expiration date. At each point one must check for the possibility of optimal early exercise.

Let $V(S, n)$ be the value of the chooser option when the stock price is S and there are n periods until expiration, we have

$$\begin{aligned} V(15625, 0) &= 6364 \\ V(10000, 0) &= 739 \\ V(6400, 0) &= 2861 \\ V(4096, 0) &= 5165 \end{aligned}$$

The option will be designated as a call if the final value of the stock is \$15625 or \$10000 and as a put if the final value of the stock is \$6400 or \$4096.

Applying the risk-neutral valuation equation gives the values in the previous periods as

$$
\begin{aligned}
V(12500,1) &= \max\{12500 - 8820, (0.5 \times 6364 + 0.5 \times 739)/1.025\} \\
&= \max\{3680, 3464.88\} = \underline{3680} \\
V(8000,1) &= \max\{8820 - 8000, (0.5 \times 739 + 0.5 \times 2861)/1.025\} \\
&= \max\{820, 1756.10\} = 1756.10 \\
V(5120,1) &= \max\{8820 - 5120, (0.5 \times 2861 + 0.5 \times 5165)/1.025\} \\
&= \max\{3700, 3915.12\} = 3915.12 \\
V(10000,2) &= \max\{10000 - 8400, (0.5 \times 3680 + 0.5 \times 1756.10)/1.025\} \\
&= \max\{1600, 2651.76\} = 2651.76 \\
V(6400,2) &= \max\{8400 - 6400, (0.5 \times 1756.10 + 0.5 \times 3915.12)/1.025\} \\
&= \max\{2000, 2766.45\} = 2766.45 \\
V(8000,3) &= \max\{8000 = 8000, (0.5 \times 2651.76 + 0.5 \times 2766.45)/1.025\} \\
&= \max\{0, 2643.03\} = 2643.03
\end{aligned}
$$

The current value of the chooser option is \$2,643.03.

(ii) (5 points) Under what circumstances should the option be exercised before the expiration date?

Answer: The underlined value above indicates that immediate exercise should occur. This option should be exercised immediately if the value of the stock reaches \$12,500 with one period remaining.

(iii) (10 points) If the seller of the option wishes to hedge its position using stock shares, how many shares should it hold initially?

Answer: The current delta of the option is

$$(V(10000,2) - V(6400,2))/(10000 - 6400) = -0.031858 \tag{1}$$

To hedge the sale of the option, the issuing firm should short stock worth $\$8,000 \times 0.031858$, which are 3.1858 shares.