ECON3133
Microeconomic Theory II

Tutorial #8: Introduction to Game Theory

Today's tutorial:

- Introduction to Game Theory
- Pure strategies in static games
  - The Normal Form of a game
  - Dominant and secure strategies
  - Nash equilibrium
  - The socially optimal choice
  - Co-ordination and equilibrium choice
  - The power of a dominant strategy
  - Sequentially eliminating strictly dominated strategies
  - Games with three players
  - Voting games

Game theory: the setting

- The structure of a game
  - Players
  - Strategies and choice of strategy
  - Pay-offs
  - Timing of making choices
    - Simultaneous or sequential
  - Frequency
    - How often is the game played? · Once only.

Example: a Normal Form game

		Player B	
		L	H
Player A	L	$(20,20)$	$(0,30)$
	H	$(30,0)$	$(10,10)$

- 2 players, A and B
- 2 strategies for each player, S_1 and S_2
- One pay-off per player per strategy $\left(P_i^A, P_i^B\right)$
  - Note: Player B is always in the second position in (,)
- Eg if Player A chooses L and player B chooses H then the pay-off is $(0,30)$, 0 to player A and 30 to player B

(H, L): (30, 0)

A plays H

B plays L

Example: a Normal Form game

- Strategy types:
  - A dominant strategy
    - A strategy that a player should choose regardless of the other player's choice
  - A secure strategy
    - A strategy that guarantees a sure pay-off of a given amount

at least

risk preferences of players

expected

1. Players max pay-off: mixed strategy game.
2. Players max pay-off.: pure strategy game.
3. minimizing risk of loss
4. guarantees a certain minimum.

Example: a Normal Form game

		Player B	
		L	H
Player A	L	(20,20)	(0,30)
	H	(30,0)	(10,10)

- Does either Player have a dominant strategy in this example?
- Player A:

Player B chooses	Player A's Best Response	Pay-off to A
L	H	30
H	H	10

- We may write:
  - $u_A(H,L) = 30 > u_A(L,L) = 20$
  - $s_A^* = H = BR_A(s_B = L)$
  - $u_A(H,H) = 10 > u_A(L,H) = 0$
  - $s_A^* = H = BR_A(s_B = H)$
  - $\Rightarrow$ $s_A^* = H$ is a dominant strategy for player A

Example: a Normal Form game

		Player B	
		L	H
Player A	L	(20,20)	(0,30)
	H	(30,0)	(10,10)

- Does either Player have a dominant strategy in this example?
- Player B:

Player A chooses	Player B's Best Response	Pay-off to B
L	H	30
H	H	10

- We may write:
  - $u_B(L,H) = 30 > u_B(L,L) = 20$
  - $s_B^* = H = BR_B(s_A = L)$
  - $u_B(H,H) = 10 > u_B(H,L) = 0$
  - $s_B^* = H = BR_B(s_A = H)$
  - $\Rightarrow\ s_B^* = H$ is a dominant strategy for player B

Example: a Normal Form game

		Player B	
		L	H
Player A	L	(20,20)	(0,30)
	H	$(30,\cancel{0})$ $\boxed{15}$	(10,10)

- What secure strategies are available for either player?
- Player A:

· Always playing H earns at least 10

· H is a secure strategy · guarantees a minimum.

- Player B:

· Always playing H earns at least 10.

· if $\pi(H,L) = (30,15)$ then L is a secure strategy for B guaranteeing 15

Key idea: A Nash equilibrium

- A Nash equilibrium:
  - A NE is an equilibrium in which each player's strategy is the Best Response to the other players' Best Responses:
  - An equilibrium in which no player can unilaterally improve their outcome by changing their own strategy given all the other players' strategies

    alone
  - That is, an equilibrium in which one strategies are decided, no player has an incentive to change their strategy . no incentive to cheat. eg OPEC
  - Note: any equilibrium is a combination of strategies and not pay-offs!

. (H, L) not eg (30, 10)

Example: a Normal Form game

		Player B	
		L	H
Player A	L	(20,20)	(0,30)
	H	(30,0)	(10,10)

- Is there a Nash equilibrium in this example?

Player B chooses	Player A's Best Response	Pay-off
L	H	30
H	H	10

Player A chooses	Player B's Best Response	Pay-off
L	H	30
H	H	10

- $s_A^* = H = BR_A(s_B^* = H)$
- $s_B^* = H = BR_B(s_A^* = H)$

Mutual Best Response of A and B

(H, H)

Example: a Normal Form game

		Player B	
		L	H
Player A	L	(20,20)	(0,30)
	H	(30,0)	(10,10)

- What is the efficient outcome in this game?
- Pareto efficient:
  - (L, L)
  - (H, L)
  - (L, H)
  - (H, H)
- Maximise total pay-off

(L, L) (20,20) sum = 40

- The efficient outcome is different to the Nash equilibrium
  - The socially optimal outcome is to co-operate

(H, H) (10,10)

Pareto efficient: one player then not Pareto efficient

: Both players can change, then can

improve by going (20,20)

Example: a Normal Form game with multiple NE

		Player B	
		L	H
Player A	L	(10,10)	(20,30)
	H	(30,20)	(10,10)

- Is there a Nash equilibrium in this example?

Player B chooses	Player A's Best Response	Pay-off to Player A
L	H	30
H	L	20

Player A chooses	Player B's Best Response	Pay-off to Player B
L	H	30
H	L	20.

- Therefore in this case we have two NE:
  - (L, H) and (H, L)

The Nash Equilibrium: some issues

- The Nash Equilibrium (NE) is such that once it has emerged, no player has an incentive to deviate from it
  - But nothing in the game tells us how we get to a Nash Equilibrium
- A Nash Equilibrium is not unique
  - A game may have more than one NE (eg any symmetric two-player game will have an even number of NE), or none at all (eg Rock, Paper, Scissors)
  - A game may have too many NE to be useful
- Over time, information, expectations and pay-offs may change, so the NE in a game may change

Example: Co-ordination and equilibrium selection

		Player B	
		L	H
Player A	L	(10,10)	(20,20)
	H	(30,30)	(10,10)

- What are the Nash equilibria in this example?

Player B chooses	Player A's Best Response	Pay-off to Player A
L	H	30
H	L	20

Player A chooses	Player B's Best Response	Pay-off to Player B
L	H	20.
H	H	30.

- Therefore in this case we have two NE:
  - (L, H) and (H, L) (H, L).
  - A rational player will realise that $\cancel{(L, H)}$ is better than $\cancel{(H, L)}$ and so (L, H) is considered superior

(L, H)

Example: Co-ordination and equilibrium selection

B

A

	L	H
L	(10,10)	(20,20)
H	(30,30)	(10,10)

- Two NE: (L, H) and (H, L) (H,L) (L,H)
- A rational player will realise that $\cancel{(L, H)}$ is better than $\cancel{(H, L)}$ and so $\cancel{(L, H)}$ (H,L) is considered superior

B

A

	L	H
L	(10,10)	(20,30)
H	(30,20)	(10,10)

- Two NE: (L, H) and (H, L)
- No clear reason to expect one NE over the other

Example: Co-ordination and equilibrium selection

B

A

	L	H
L	(30,20)	(10,10)
H	(10,30)	(20,30)

- The NE are: (L, L) , (H, H)
  (30,20) , (20,30).
- Can we say anything about which NE is more likely?
  - Consider Player B's secure strategy:

    L: min. 20
    H: min: 10 } always play L
  - Then consider Player A's response to Player B's secure strategy:

    . Player A plays L too.
  - Therefore (L,L) is the more likely NE

. Based on risk preferences of player B.

Example: Co-ordination and equilibrium selection

B

	L	H
L	(20,20)	(30,0)
H	(0,30)	(50,50)

A

- The NE are: (L,L) (H,H)
- Is NE $(50{,}50)$ more likely than NE $(20{,}20)$?
  - What if Player B plays randomly?

· A would like to be at (H,H) through co-operation, but if B plays randomly then playing H might earn 0 for A. Playing L is more secure

  - Player A's best strategy may be the secure strategy L to earn a minimum of 20

· assuming A prefers a secure strategy.

- It's not enough to know that you are rational
- You also need to know that the other person is rational
- And the other person needs to know that you are rational

Example: The power of a dominant strategy

	L	H
L	(20,20)	(0,30)
H	(30,0)	(10,10)

- However, if a Player has a dominant strategy, then it doesn't matter whether the other Player is rational or not
- The dominant strategy is more likely to be played
- Suppose that Player B is playing irrationally and Player A is playing rationally
  - Player A's best response is always the dominant strategy, which is to play H

Example: Two players and four strategies

			B		
		C1	C2	C3	C4
	R1	(6,2)	(8,0)	(4,4)	(8,2)
A	R2	(6,0)	(2,12)	(0,4)	(16,2)
	R3	(8,8)	(6,2)	(2,8)	(8,4)
	R4	(4,2)	(4,4)	(0,2)	(12,0)

- The NE are:

(R3, C1), (R1, C3).

Example: Two players and four strategies

			B		
		C1	C2	C3	C4
	R1	(4,0)	(1,4)	(2,1)	(6,2)
A	R2	(8,2)	(2,2)	(6,3)	(8,4)
	R3	(3,1)	(6,1)	(8,2)	(1,2)
	R4	(6,3)	(8,4)	(4,6)	(6,6)

- We can sometimes reduce the possible equilibria by eliminating those strategies that are strictly dominated by other strategies
    - Sequentially eliminating dominated strategies

(R3, C3) and (R2, C4)

1. For Player B:
2. For Player A:
3. For Player B:
4. For Player A:

Games with three players

		C		
	1		**2**	
	B		**B**	
	L	R	L	R
A U	(5,5,10)	(-10,-5,5)	(-4,-4,0)	(-10,-5,5)
D	(-10,-5,5)	(10,10,-5)	(-10,-5,5)	(-2,-4,6)

(a, b, c)

- We now have 3 players, A, B and C
- How to show a 3 dimensional game?
  - Players A and B choose U or D, L or R respectively
  - Player C chooses version 1 or 2 of the game

Games with three players

1

		B: L	B: R
A	U	(5,5,10)	(-10,-5,5)
	D	(-10,-5,5)	(10,10,-5)

2

		B: L	B: R
A	U	(-4,-4,0)	(-10,-5,5)
	D	(-10,-5,5)	(-2,-4,6)

- Approach:

1. Fix Player C plays #1
   - 1.1 Player B plays L => Player A plays U
   - 1.2 Player B plays R => Player A plays D
   - 1.3 Player A plays U => Player B plays L
   - 1.4 Player A plays D => Player B plays R
2. Fix Player C plays #2
   - 1.1 Player B plays L => Player A plays U
   - 1.2 Player B plays R => Player A plays D
   - 1.3 Player A plays U => Player B plays L
   - 1.4 Player A plays D => Player B plays R

Games with three players

1

		B	
		L	R
A	U	(5,5,10)	(-10,-5,5)
	D	(-10,-5,5)	(10,10,-5)

2

		B	
		L	R
A	U	(-4,-4,0)	(-10,-5,5)
	D	(-10,-5,5)	(-2,-4,6)

3. Player C's choices

3.1 Player A plays U, Player B plays L=> Player C chooses #1

3.2 Player A plays U, Player B plays R => Player C indifferent between #1 and #2

3.3 Player A plays D, Player B plays L => Player C indifferent between #1 and #2

3.4 Player A plays D, Player B plays R => Player C chooses #2

To give 2 NE at:

- $(U, L, 1)$
- $(D, R, 2)$

Voting games: The intransitivity of collective preferences

- Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$
$$\pi_2(B) > \pi_2(C) > \pi_2(A)$$
$$\pi_3(C) > \pi_3(A) > \pi_3(B)$$

- What is the outcome if candidate C is disqualified?
  - Player 1: A
  - Player 2: B
  - Player 3: A
  - The winner: A

- Assume 3 players (1,2,3) and 3 strategies (A, B, C eg candidates, preferred election choices)
- A majority voting rule (ie all votes count equal, a simple majority wins)
- Pay-offs to the players π_1, π_2, π_3

$A > B$

$B > C$

$\Rightarrow A > C$

Voting games: The intransitivity of collective preferences

- Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$

$$\pi_2(B) > \pi_2(C) > \pi_2(A)$$

$$\pi_3(C) > \pi_3(A) > \pi_3(B)$$

- What is the outcome if B and C are the candidates?
  - Player 1:
  - Player 2:
  - Player 3:
  - The winner: B
- What is the outcome if A and C are the candidates?
  - Player 1:
  - Player 2:
  - Player 3:
  - The winner: C

Voting games: The intransitivity of collective preferences

- Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$
$$\pi_2(B) > \pi_2(C) > \pi_2(A)$$
$$\pi_3(C) > \pi_3(A) > \pi_3(B)$$

- Summary:
- Between A and B, A is preferred
- Between B and C, B is preferred
- Between A and C, C is preferred

- In terms of preference relations we have:
- $A \succ B$
- $B \succ C$
- Transitivity requires that therefore $A \succ C$, but in this case we have $C \succ A$
- Therefore the collective preference under simple majority voting is not transitive
- Note that individuals may have transitive preferences, but the collective preference need not be transitive

Voting games: the status quo

- Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$
$$\pi_2(B) > \pi_2(C) > \pi_2(A)$$
$$\pi_3(C) > \pi_3(A) > \pi_3(B)$$

- Suppose that if the voting result is 1-1-1 then nothing changes
  - The "status quo" is maintained
- Suppose that A is the incumbent/status quo/nothing changes candidate or policy

- What might Player #2 do in this situation?
  - In the simple majority, the result is 1-1-1
  - Then Player #2's least preferred candidate/policy wins
  - Knowing this beforehand, Player #2 might vote for their second preferred candidate/policy, C, and then C wins
  - But then Player #1 might know this beforehand, and so vote for B
  - And so on…