ECON3133
Microeconomic Theory II

Tutorial #10: Game Theory (cont.)

Today's tutorial:

- Repeated games: emphasis on how to support co-operation by dynamic incentive
- Finitely repeated games
  - One NE
  - Multiple NE: allow dynamic incentive to support co-operation
  - History dependent strategy with punishment and reward

- Infinitely repeated games.
- Case study: Oil market last year.

Finitely repeated games

		2		
		L_2	M_2	H_2
1	L_1	1,1	5,0	½,0
	M_1	0,5	4,4	0,0
	H_1	0, ½	0,0	0,0

- In this static game played once, what are the NE?

(L_1, L_2)

- What is the social optimum outcome?

(M_1, M_2)

- Question: If the game were played twice, is there any way that cooperation at stage 1 could be enforced by behaviour at stage 2?
  - threats (credible)
  - retaliations
  - punishment
  - reward.

Finitely repeated games

		2		
		L_2	M_2	H_2
1	L_1	1,1	5,0	½,0
	M_1	0,5	4,4	0,0
	H_1	0, ½	0,0	0,0

- Question: If the game were played twice, is there any way that cooperation at stage 1 (M_1, M_2) could be enforced by behaviour at stage 2?
- "If you don't co-operate at T then I won't co-operate at $T+1$"
- But in stage 2, there is no $T+1$
- Therefore, in stage 2 given M_i $(\mathrm{i} = 1,2)$, both players have an incentive to play L_i to earn a pay-off of 5
- This will be known by both players, and so they will play the NE at (L_1, L_2) in stage 2

P1 played M_1 in stage 2

P2 can play L_2 in stage 2

Finitely repeated games

		2		
		L_2	M_2	H_2
1	L_1	1,1	5,0	½,0
	M_1	0,5	4,4	0,0
	H_1	0, ½	0,0	0,0

- P_1 plays M_2 in stage 1
- P_2 has incentive to play L_2 in stage 1

- Therefore, both players play the NE at (L_1, L_2) in stage 2
- Key point (1): because stage 2 is the final stage, there is no strategy at stage 1 that can enforce co-operation at stage 2
  - There's no punishment for uncooperative behaviour at stage 2 that can be played in stage 3 because there is no stage 3
- Key point (2): because behaviour at stage 2 is independent of behaviour at stage 1, there is no punishment available at stage 2 for uncooperative behaviour at stage 1
  - In stage 1, both players have an incentive to play L_i to earn a pay-off of 5
  - This will be known by both players, and so they will play the NE at (L_1, L_2) in stage 1
- So we get the static game NE (L_1, L_2) in stages 1 and 2

- Never get to social optimum (M_1, M_2).

Finitely repeated games

		2		
		L_2	M_2	H_2
1	L_1	1,1	5,0	½,0
	M_1	0,5	4,4	0,0
	H_1	0, ½	0,0	0,0

- We can generalise to a finite game of T stages:
  - At t $= T$ players will play the NE
  - Therefore at t $= T - 1$ players will play the NE
  - ….Therefore at $t = 1$ players will play the NE
- Conclusion: In finitely repeated games with a single NE, increasing the number of times that the game is played does not increase the possibility for co-operation

- As long as T is finite,

Finitely repeated games

		2		
		L_2	M_2	H_2
1	L_1	1,1	5,0	½,0
	M_1	0,5	4,4	0,0
	H_1	0, ½	0,0	3,3

0, 0.

- In this static game played once, what are the NE?

(L_1, L_2) ; (H_1, H_2)

- What is the social optimum outcome?

(M_1, M_2)

- Question: In this game, is there any way that cooperation at $T-1$ could be enforced by behaviour at T?

1. At T, players play a NE; not play social optimum because you're vulnerable to be free ridden of cheating. 2. Punishment strategy: L_i

Finitely repeated games

- Game lasts T rounds.

		2		
		L_2	M_2	H_2
1	L_1	1,1	5,0	½,0
	M_1	0,5	4,4	0,0
	H_1	0, ½	0,0	3,3

- Question: In this game, is there any way that cooperation at $T-1$ could be enforced by behaviour at T?
- Suppose player i plays M_i at $T-1$
- If player j plays M_j at $T-1$, then player i 'rewards' this by playing H_i at T
  - Player i does not play M_i at T because that leaves player j free to play L_j at T
  - Best response behaviour gives 'good' NE at (H_i, H_j)
- If player j plays anything apart from M_j at $T-1$, then player i 'punishes' this by playing L_i at T
  - Best response behaviour gives 'bad' NE at (L_i, L_j)
- Conclusion: The existence of multiple NE makes enforcement of co-operation possible

(1)
- P_1 plays M_1 at $T-1$
- If P_2 plays M_2 at $T-1$, player 1 plays H_1 at T
- P_1 plays H_1
- P_2 plays H_2
- Good NE at H_1, H_2.

(2)
- Pl plays M_1 at $T-1$ but P_2 doesn't play M_2 at $T-1$.
- Pl plays L_1 at T.

T: (5,5).

Stay at (M_1, M_2) until someone cheats: Then go to punishment strategy.

P_2 also plays L2 at T.

"bad NE": (L_1, L_2).

Infinitely repeated games

- If a game is played infinitely, then there is always a credible $t+1$ threat to punish non co-operative behaviour at t.
- Therefore, a game does not require more than one NE for co-operative behaviour to be enforced
- And because the game is played forever, then any stage may be considered the first stage of the game (ie the end of the game is always an equal – and infinite – time away)
- Therefore we may consider behaviour at the first stage as representative of behaviour at any stage

Example: the oil market in 2020

- In mid-2020, the oil price collapsed, and oil price futures went negative for a short time (why?)
- Since then the market has stabilised somewhat
- Can we analyse the behaviour of the oil price as an infinitely played game by two players?

Source: Trading Economics

The world oil market

- The leading oil producers in 2019

Rank	Country	% share
1	US	18%
2	S Arabia	12%
3	Russia	11%
4	Canada	5%
4	China.	5%

- The leading oil importers in 2019

Rank	Country	% share (of world oil imports)
1	china	23%
2	US	13%
3	India	10%
4	Japan	7%
5	Skorea.	4%

- 13 oil producing countries are members of OPEC, which agrees production amongst its members (leader is S Arabia)
- Since the early 1970s, OPEC has determined the direction of world oil prices
- But of the top 5 oil producers, only Saudi Arabia is an OPEC member
- The balance of power in the world oil market is changing

Source: wikipedia

Example: the oil market in 2020

		Russia: Deviate (D) (high production)	Russia: Cooperate (C) (agreed production)
S Arabia	D	100,100	1000,0
	C	0,1000	600,600

#3 producer

- What are the NE in the stage game?

(D,D)

- Are there any dominant strategies?

· S Arabia : D

· Russia : D.

· Social Optimum (C,C).

· Cooperate: limit production / support price.

· Deviate: increasing production / price falls.

Example: the oil market in 2020

		Russia	
		Deviate (D) (high production)	Cooperate (C) (agreed production)
S Arabia	D	100,100	1000,0
	C	0,1000	600,600

- What happened in 2020?
- Mutual co-operation gave the outcome (C, C) with average oil price \$50-60/brl and pay-offs $(600,600)$
- Then Russia decided to deviate and play D
  - Massive increase in production
  - Increased pay-off of 1000 and move to (C, D)
  - Oil price fell sharply from \$60 to \$25-\$30/brl
- Then Saudi Arabia retaliated and also deviated, playing D for the NE $(100,100)$
  - Oil price completely collapsed

Example: the oil market in 2020

		Russia	
		Deviate (D) (high production)	Cooperate (C) (agreed production)
S Arabia	D	100,100	1000,0
	C	0,1000	600,600

- Consider the trigger strategy:
  - If player j deviates, then player i won't cooperate any more
  - If Russia deviates, then Saudi Arabia won't co-operate any more

Trigger strategy

$$\begin{cases} \text{for } i = 1,2: & t = 1, & \text{play } C \\ & t \geq 2, & \begin{cases} \text{play } C \text{ if only } (C,C) \text{ in the past} \\ \text{play } D \text{ otherwise} \end{cases} \end{cases}$$

1: C

2: C → (C, C)

N → (D, D).

Example: the oil market in 2020

Co-operative outcome: pay-off to Russia

Russia	C	C	C	C	C
S Arabia	C	C	C	C	C

- $\Pi(C) = \pi_1(C,C) + \delta\pi_2(C,C) + \delta^2\pi_3(C,C) + \cdots = \frac{600}{1-\delta}$

Russia deviates outcome: pay-off to Russia

Russia	D	D	D	D	D
S Arabia	C	D	D	D	D

- $\Pi(D) = \pi_1(C,D) + \delta\pi_2(D,D) + \delta^2\pi_3(D,D) + \cdots = 1000 + \delta 100 + \delta^2 100 + \cdots = 1000 + \frac{\delta}{1-\delta}100$

Example: the oil market in 2020

- Russia will not deviate when:
  - $\Pi(C) > \Pi(D)$
- ie when:
  - $\frac{600}{1-\delta} > 1000 + \frac{\delta}{1-\delta}100$

- We may also use the general formula to find δ_{min}:
- $\delta_{min} = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$

$= 0.44$

$= \frac{4}{9}$.

- $\delta_{min} = \frac{4}{9}$

$= 0.44$

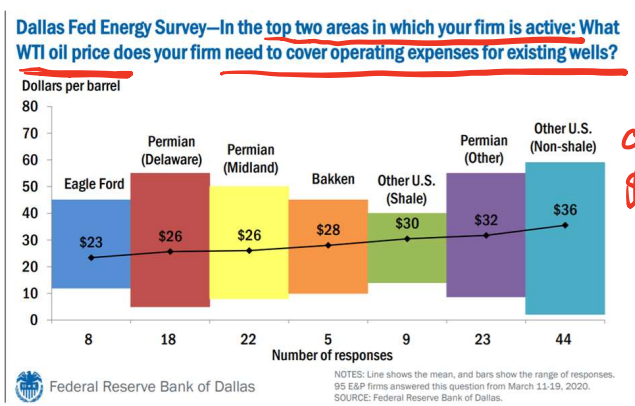
- $\delta > \delta_{min}$ and Russia does better by co-operating.

Example: the oil market in 2020

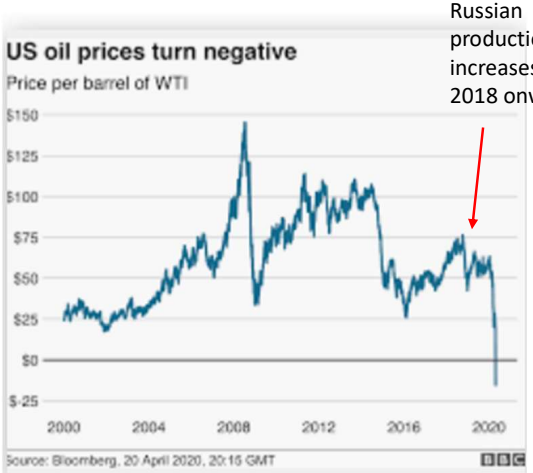
- What is a good estimate for the discount factor in the Russian oil industry at present?
- Assume that it's between 10 and 20%, so a one year discount factor of between $\frac{1}{1.20}$ to $\frac{1}{1.10}$
- That is a one year discount factor in the range 0.83-0.91
- This is much higher than $\delta_{\min}$
- So why would Russia deviate in the way it did?
  - The pay-offs are illustrative but it is still likely that $\delta_{\min}$ is a lot lower than actual discount factors
- So why did Russia deviate?

Example: the oil market in 2020

- In fact, we have seen that the US is the world's largest oil producer
- About 40% of this production comes from shale oil
- If you can push the oil price below USD30 per barrel, you can destroy the US shale oil industry and remove a global competitor
- And withstand the losses to your own oil industry in the meantime



about $30/bl.



Russian production increases from 2018 onwards

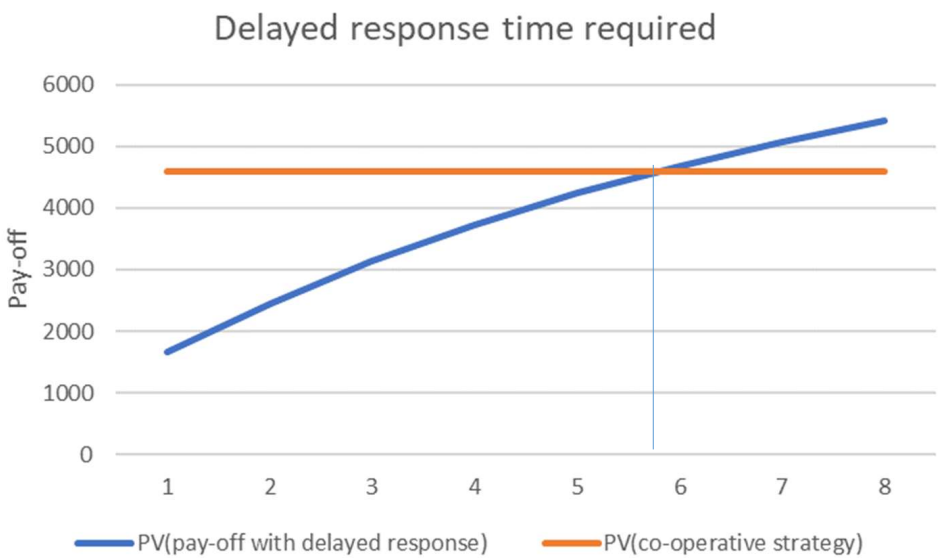
Example: the oil market in 2020

- Suppose that the weighted average cost of capital in the Russian oil industry is 15%
  - This gives a 1 year discount factor of $\frac{1}{1.15} = 0.87$ $\delta_{est} = 0.87.$
- Assume that Russia believed that Saudi Arabia would not play the 'punishment' strategy immediately
- That is, that Saudi Arabia would delay in deviating
- This would give Russia time to drive the US producers out of business
- How much delay by Saudi Arabia would be needed to make Russia indifferent between co-operating and deviating?

Example: the oil market in 2020

- If Russia continues to cooperate:
  - $\Pi(C) = \pi_1(C,C) + \delta\pi_2(C,C) + \delta^2\pi_3(C,C) + \cdots = \frac{600}{1-\delta}$
  - $\delta = 0.87$
  - $\Rightarrow \Pi(C) = 4600$
- If Russia deviates but Saudi Arabia only retaliates after P years:
  - $$\begin{aligned}\Pi(D) &= \pi_1(C,D) + \delta\pi_2(C,D) + \ldots + \delta^{P-1}\pi_{P-1}(C,D) + \ldots + \delta^P\pi_P(D,D) + \ldots + \delta^{P+1}\pi_{P+1}(D,D) + \cdots \\ &= 1000 + \delta 1000 + \cdots + \delta^{P-1}1000 + \delta^P 100 + \delta^{P+1}100 + \cdots \\ &= 1000 + \delta 1000 + \cdots + \delta^{P-1}1000 + \delta^P 100[1 + \delta + \delta^2 + \cdots] \\ &= 1000 + \delta 1000 + \cdots + \delta^{P-1}1000 + \delta^P 100\left[\frac{1}{1-\delta}\right]\end{aligned}$$
- Solve for P such that $\Pi(C) = \Pi(D)$

Example: the oil market in 2020



- For Russia to be as well-off by deviating as in the co-operative case, Saudi Arabia would have to delay retaliation for 5.9 years
- This seems too long a time to expect no action from Saudi Arabia
- It is more likely that Russia would lose profits compared to the strategy of cooperation

Example: the oil market in 2020

- Is there another strategy that Russia could play?
- What if Russia expects that Saudi Arabia will not punish Russia forever (punishment hurts Saudi Arabia too)
- Suppose Russia believes that Saudi Arabia will return to a cooperative strategy (ie (C, C) after a certain number of years
- In the meantime, the US shale oil industry might be driven out of business
- Suppose that Saudi Arabia retaliates immediately, but then negotiations bring about a cooperative settlement at (C, C)
- How much time does Russia have to drive the US shale oil industry out of business before its deviation strategy causes it to lose relative to the cooperative strategy?

Example: the oil market in 2020

- If Russia continues to cooperate:
  - $\Pi(C) = 4600$

- If Russia deviates and Saudi Arabia retaliates immediately, how much (in PV terms) does Russia lose until cooperation returns?
  - $\Pi(D) = \pi_1(C,D) + \delta\pi_2(D,D) + \ldots + \delta^{P-1}\pi_{P-1}(D,D) + \ldots + \delta^{P}\pi_P(C,C) + \ldots + \delta^{P+1}\pi_{P+1}(C,C) + \cdots$

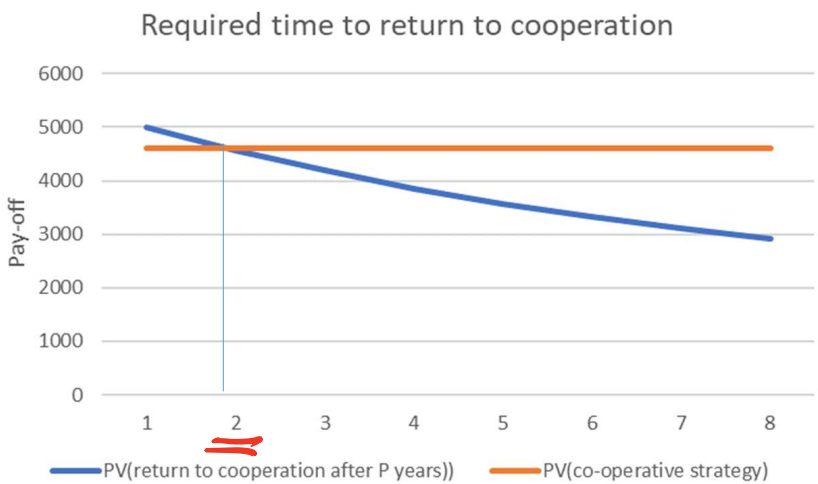
$$= 1000 + \delta 100 + \cdots + \delta^{P-1}100 + \delta^{P}600 + \delta^{P+1}600 + \cdots$$

$$= 1000 + \delta 100 + \cdots + \delta^{P-1}100 + \delta^{P}600[1 + \delta + \delta^2 + \cdots]$$

$$= 1000 + \delta 100 + \cdots + \delta^{P-1}100 + \delta^{P}600\left[\frac{1}{1-\delta}\right]$$

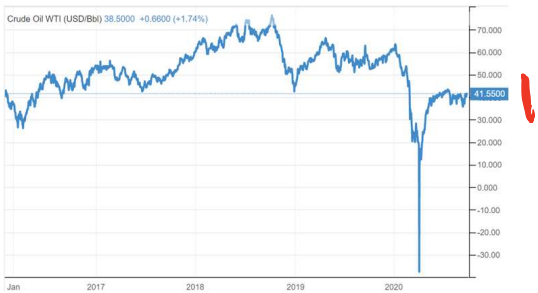
- Solve for P such that $\Pi(C) = \Pi(D)$

Example: the oil market in 2020



- Russia has about 2 years to return to a cooperative agreement before its Deviate strategy causes it losses compared to the cooperative strategy (C, C)
- Therefore, Russia has 2 years to drive the US shale oil industry out of business before its strategy will cause it to lose relative to the cooperative strategy

Example: the oil market in 2020



Saudi Arabia and Russia urge compliance on oil cuts

Producers push for others to observe Opec agreement made during price collapse in April

Shale Oil & Gas + Add to myFT

US shale producers bleed cash despite slashed spending

North American oil and gas groups suffer dismal second quarter

- Where are we now?
- Since the oil price collapse, Covid has occurred, reducing global oil demand significantly
- Possibly in response to Covid, Saudi Arabia and Russia have agreed to limit production
- This is a partial return to (C, C)
- The oil price has recovered, but the market has yet to return fully to cooperative equilibrium
- The US shale oil industry is still chronically weak, but has survived so far

Source: Financial Times October 2020, Trading Economics