

Part 3

5. Two pharmaceutical companies, A and B, form a joint venture to collaborate on developing a vaccine for the latest pandemics. The outcome is uncertain, with the probability of success increasing in the research effort exerted by the companies. Specifically, if Company A exerts effort x_A and Company B exerts effort x_B , then the probability of successfully developing an effective vaccine is $p(x_A, x_B) = x_A + x_B + \frac{1}{5}x_Ax_B$. Exerting effort level x requires a cost of $12x^2$. A success brings a profit of 4 to **each company**, whereas a failure brings zero. Suppose the companies choose their research effort simultaneously.

- (a) (5 points) What is the Nash equilibrium effort profile?

Solution: The profit of Company A is

$$4p(x_A, x_B) - 12x_A^2 = 4\left(x_A + x_B + \frac{1}{5}x_Ax_B\right) - 12x_A^2.$$

The FOC is

$$\frac{\partial}{\partial x_A} \left(4\left(x_A + x_B + \frac{1}{5}x_Ax_B\right) - 12x_A^2 \right) = \frac{4}{5}x_B - 24x_A + 4 = 0 \Leftrightarrow x_A = \frac{1}{6} \left(\frac{1}{5}x_B + 1 \right) \equiv R_A(x_B).$$

By symmetry, Company B's best response is $R_B(x_A) = \frac{1}{6} \left(\frac{1}{5}x_A + 1 \right)$. The intersection of the best responses is $x_A = x_B = \frac{5}{29} \approx 0.17241$.

- (b) (1 point) Are the choices of research effort strategic complements or substitutes?

Solution: Complements

- (c) (3 points) Is effort over or under provided in equilibrium compared to the profit-maximizing level of the joint venture? What is the economic force behind this? *Explain without using any calculation.*

Solution: Effort creates positive externality which is not internalized, so it is so underprovided.

- (d) (6 points) Instead of choosing research effort simultaneously, suppose Company A takes the lead to commit to its x_A and makes it known to Company B before the latter chooses its x_B . Compared with the case of simultaneous move in part (b), would you expect the likelihood of project success to go up or down with sequential move? *Explain without using any calculation.*

Solution: Because of strategic complementarity, an increase in x_A induces a higher x_B by Company B, which in turn benefits Company A. Company A therefore has a strategic incentives to exert higher effort. As both exert higher effort, the likelihood of project success goes up.

(Total: 15 points)

6. Two sellers, A and B, can both supply a unit of a **homogeneous good at zero production cost**. There is a potential buyer for their good, who demands **at most one unit** and values it at $r > 0$. The buyer is initially uninformed about the existence of the sellers. To inform the buyer, each seller can independently send her a price quote. Receiving a price quote is free for the buyer, but sending a price quote requires a **quoting cost** $c \in (0, r)$ **to the firm**. If the buyer receives some price quotes, she can then decide whether and from which seller to buy. If she receives no price quote, then she does not know how and hence cannot buy from any seller.

Events unfold as follows. First, each seller simultaneously decides whether to send a price quote, and what price to ask for if she sends it. Next, depending on the price quotes received (if any), the buyer decides whether and from which seller to buy. Finally, all players collect their respective payoffs.

Clearly, the buyer will take the better deal as long as it is cheaper than her value r (suppose for simplicity that she randomizes equally if there is a tie in the price quotes). Therefore, we can focus attention on the first stage of price-quote competition between the sellers.

The payoffs facing a seller is as follows. If a seller does not send a price quote, her payoff is 0. If a seller sends a price quote, say p , that is accepted by the buyer, the seller's payoff is $p - c$. If a seller sends a price quote but is rejected, the seller's payoff is $-c$.

- (a) (3 points) Can it be an equilibrium to have both sellers sending a price quote of c (Bertrand paradox outcome)? Explain.

Solution: No. If the other seller does so, the best response of a seller is not to send the price quote, thus saving the quoting cost.

- (b) (3 points) Can it be an equilibrium to have both sellers sending a price quote of r (monopoly pricing outcome)? Explain.

Solution: No. If the other seller does so, the best response of a seller is to undercut slightly, thus winning the business for sure.

- (c) For the rest of this question, we focus on **symmetric mixed strategies**. Here, a mixed strategy consists of two components: a probability $\beta \in [0, 1]$ of sending the price quote, and a price distribution $F(p)$ in case of sending a price quote. Furthermore, we will focus on price distribution $F(p)$ that has no mass point and no "gap", so it has a density function $f(p)$ such that $f(p) > 0$ if and only if $p \in (\underline{p}, \bar{p})$.

- (i) (4 points) Explain why we cannot have $\beta = 0$ in any symmetric mixed-strategy equilibrium.

Solution: If the other seller does so, a seller can profitably deviate to submit a price quote of r , reaping a profit of $r - c > 0$.

- (ii) (4 points) Explain why $\bar{p} = r$ in any symmetric mixed-strategy equilibrium.

Solution: $\bar{p} > r$ is clearly suboptimal because the quoting cost is wasted. If $\bar{p} < r$, the payoff of \bar{p} is worse than r : the seller can make a sale only if the other seller does not submit a price quote, in which case the buyer will take r .

- (iii) (4 points) Explain why we cannot have $\beta = 1$ in any symmetric mixed-strategy equilibrium. Note that together with part (c.i), this implies that the seller's equilibrium payoff is zero (so that she is indifferent between sending and not sending a price quote).

Solution: If $\beta = 1$ and $\bar{p} = r$, sending a price quote $\bar{p} = r$ results in a loss for certain and hence negative payoff, a contradiction.

- (iv) (4 points) Explain why $\underline{p} = c$ in any symmetric mixed-strategy equilibrium.

Solution: $\underline{p} < c$ is clearly suboptimal because the implied payoff is surely negative. If $\underline{p} > c$, then sending a price quote \underline{p} implies a positive payoff, so we will have $\beta = 1$ contradicting part (iii) above.

- (v) (5 points) Use parts (c.i) to (c.iii) above to pin down the equilibrium value of β . (Note: your answer should depend on c and r .)

Solution: By part (iii), the equilibrium payoff of a sender is zero. As \bar{p} also gives the equilibrium payoff, we must therefore have

$$(1 - \beta)(r - c) + \beta(-c) = 0 \Leftrightarrow \beta = \frac{1}{r}(r - c).$$

- (vi) (8 points) Compute the equilibrium price distribution $F(p)$. (Note: your answer should depend on c and r .)

Solution: By quoting price p , a firm's payoff is

$$-c\beta F(p) + (p - c)(1 - \beta + \beta(1 - F(p))).$$

This has to be equal to zero in equilibrium, so

$$F(p) = \frac{p - c}{\beta p} = \frac{r p - c}{p r - c}.$$

(Total: 35 points)

End of Part 3