Practice Problem Set
ECON 3113 Microeconomics Theory I 2020

Questions from the textbook

Please note that this list is **only suggestive**. Questions outside of this list are not necessarily irrelevant. Conversely, being able to solve all these questions does not automatically guarantee good exam results.
Also, please **do not** expect either Dominic or I will provide a detailed solution to individual questions listed below (we do not have the answers provided by the publisher neither). Having said that, if there are questions that really puzzle you, please feel free to consult either Dominic or me and we will try to suggest how you may approach them.

Chapter 3

3.3, 3.4, 3.7, 3.12 (check page 105 for definition of a homothetic preference)

Chapter 4

4.1, 4.3, 4.4, 4.5 (a, b), 4.11 (a, b, c)

Chapter 5

5.1, 5.2, 5.5a, 5.6

Chapter 6

6.3, 6.10

Chapter 7

7.1, 7.3, 7.4, 7.5, 7.8, 7.9a

Chapter 18

18.5, 18.7

Extra Questions

These problems are provided to help you prepare for the final exam with extra practice opportunities. Solutions to these questions will be posted early next week, though you are encouraged to work on the questions before consulting the solutions.

1. Irma has an initial wealth of $\$50,000$ and a car that is valued at $\$100,000$. She faces a probability of 5% of the car being stolen. She can purchase insurance to cover her potential loss. The insurance term is as follows. If she wants to get a coverage of $\$x$ (i.e., compensation in case her car is being stolen), she has to pay a (upfront) premium of $0.1x$. Irma has a von-Neumann-Morgenstern function $u(w) = 4\ln w$.

   (a) How much insurance coverage x would Irma purchase?

   (b) Suppose Irma can install a burglar alarm, which costs $5000 and lowers the risk of car theft to 1%. Suppose the premium rate stays fixed at 0.1. How much insurance coverage would Irma purchase after installing the burglar alarm?

   (c) Would Irma install the burglar alarm? (Hint: Compare the expected utilities under the optimal insurance purchase obtained in part (a) and (b) respectively.)

Solution: (a) By purchasing a coverage of $\$x$*, Irma has wealth* $150000 - 0.1x$ *with probability* 95%*, and a payoff of* $50000 + x - 0.1x$ *with probability* 5%*. Her expected utility is thus*

$$\begin{aligned} E[u(\cdot)] &= 0.95 \times (4\ln(150000 - 0.1x)) + 0.05 \times (4\ln(50000 + x - 0.1x)) \\ &= 3.8\ln(150000 - 0.1x) + 0.2\ln(50000 + 0.9x). \end{aligned}$$

The FOC of expected utility maximization gives

$$\begin{aligned} \frac{\partial E[u(\cdot)]}{\partial x} &= 0 \Leftrightarrow 3.8(-0.1)(150000 - 0.1x)^{-1} + 0.2(50000 + 0.9x)^{-1}(0.9) = 0 \\ &\Rightarrow x = 22222. \end{aligned}$$

(b) By installing the burglar alarm and purchasing a coverage of $\$x$*, Irma has wealth* $150000 - 5000 - 0.1x$ *with probability* 99%*, and a payoff of* $50000 - 5000 + x - 0.1x$ *with probability* 1%*. Her expected utility is thus*

$$\begin{aligned} E[u(\cdot)] &= 0.99 \times (4\ln(150000 - 5000 - 0.1x)) + 0.01 \times (4\ln(50000 - 5000 + x - 0.1x)) \\ &= 3.96 \times \ln(145000 - 0.1x) + 0.04 \times \ln(45000 + 0.9x) \end{aligned}$$

The derivative of $E[u(\cdot)]$ *with respect to* x *is*

$$\begin{aligned} \frac{\partial E[u(\cdot)]}{\partial x} &= 3.96 \times (-0.1) \times (145000 - 0.1x)^{-1} + 0.04 \times 0.9 \times (45000 + 0.9x)^{-1} \\ &= \frac{-0.36x - 12600}{(145000 - 0.1x)(45000 + 0.9x)} < 0. \end{aligned}$$

Therefore, the optimal insurance purchase after the installation of the burglar alarm is zero.

(c) Under the optimal insurance purchase, the expected utility in part (a) is

$$3.8\ln(150000 - 0.1(22222)) + 0.2\ln(50000 + 0.9(22222)) \approx 47.464.$$

Under the optimal insurance purchase, the expected utility in part (b) is

$$3.96 \times \ln(145000) + 0.04 \times \ln(45000) \approx 47.491.$$

Therefore, Irma would install the alarm and go for no insurance coverage.

2. Irma is considering investing in the stock of a company APH. She is a long-term investor, so she is going to hold any stock purchased for a year. She believes that by the end of the year, the stock price will increase by 10 dollars with probability 0.6, and will decrease by 10 dollars with probability 0.4. Irma's money utility function is $u(w) = \sqrt{w}$, and her initial wealth is 100. Irma has to decide how many units of the stock to buy, with the objective of maximizing her year-end expected utility.

   (a) How much unit of stock would Irma buy?

   (b) Before she decides her investment, she can buy a financial analyst's report about the stock. The report issues the stock either a good rating or a bad rating. Historically, stocks that were rated "good" showed a price increase with probability 0.8, and suffered a price-drop only with probability 0.2. On the other hand, stocks that were rated "bad" showed a price increase with only probability 0.2, and suffered a price-drop with probability 0.8. The probability that the stock receives a good rating is $\frac{2}{3}$, and the probability that the stock receives a bad rating is $\frac{1}{3}$. The cost of buying the analyst's report is 10. Does Irma find it a good idea to buy the analyst's report? (Hint: We can allow Irma to buy a "negative amount of stock". In reality, this can be achieved by short-selling.)

   Solution: (a) If Irma buys x units of the stock, her wealth would be $100 + 10x$ with probability 0.6, and it would be $100 - 10x$ with probability 0.4. By buying x units, her expected utility would be

$$(0.6)\sqrt{100 + 10x} + (0.4)\sqrt{100 - 10x}.$$

   The FOC is

$$(0.6)\frac{10}{2\sqrt{100 + 10x}} + (0.4)\frac{-10}{2\sqrt{100 - 10x}} = 0.$$

   Upon solving, $x = \frac{50}{13}$. The expected utility under the optimal stock purchase is

$$(0.6)\sqrt{100 + 10\left(\frac{50}{13}\right)} + (0.4)\sqrt{100 - 10\left(\frac{50}{13}\right)} \approx 10.198.$$

   (b) Suppose Irma has purchased the report and the report issues a good rating. Then she chooses x to maximize

$$(0.8)\sqrt{100 - 10 + 10x} + (0.2)\sqrt{100 - 10 - 10x},$$

   with FOC

$$(0.8)\frac{1}{2}\frac{10}{\sqrt{90 + 10x}} + (0.2)\frac{1}{2}\frac{-10}{\sqrt{90 - 10x}} = 0.$$

Upon solving, $x = \frac{135}{17} \approx 7.941\,2$. *The expected utility is*

$$\begin{aligned} & (0.8)\sqrt{90+10\left(\frac{135}{17}\right)} + (0.2)\sqrt{90-10\left(\frac{135}{17}\right)} \\ = \; & (0.8)\sqrt{\frac{2880}{17}} + (0.2)\sqrt{\frac{180}{17}} \approx 11.063. \end{aligned}$$

Next suppose the report issues a bad rating. Then she chooses x *to maximize*

$$(0.2)\sqrt{100-10+10x} + (0.8)\sqrt{100-10-10x}.$$

The problem is "symmetric" to the one above: the two problems are identical by replacing x *with* $-x$. *Thus, the solution is* $x = -\frac{135}{17} \approx -7.941\,2$. *The expected utility is equal to* $(0.8)\sqrt{\frac{2880}{17}} + (0.2)\sqrt{\frac{180}{17}} \approx 11.063$.

Thus, the expected utility of buying the report is $\frac{2}{3}(11.063) + \frac{1}{3}(11.063) = 11.063$, *exceeding the payoff of not buying the report, which is* 10.2. *As a result, Irma finds it a good idea to buy the analyst's report.*

3. Consider the Akerlof's model of lemons. Assume the sellers have all the bargaining power: they can charge the buyers their expected value of buying the car. Suppose there are 3 possible car qualities: high, medium and low. Suppose also that the proportion of each type of car is 1/3. The values of buyers and sellers for different car qualities are tabulated below.

	Sellers' value	Buyers' value
Low-quality car	5	8
Medium-quality car	10	13
High-quality car	15	18

   (a) Is there an equilibrium in which all types of qualities are traded?

   (b) Is there an equilibrium in which only low and medium qualities are traded?

   (c) Is there an equilibrium in which only low qualities are traded?

   (d) Instead of equal proportion of cars, suppose the proportion of high-, medium-, and low-quality cars are θ_H, θ_M, and θ_L respectively (clearly, all of θs' are between zero and one, and they add up to one). Derive conditions on θ_H, θ_M, and θ_L under which there is an equilibrium in which all cars are traded.

   Solution: (a) Suppose there is an equilibrium in which all types of qualities are traded. Since all three types are equal in numbers, the expected valuation of a car to the buyers is

$$\frac{1}{3}(8) + \frac{1}{3}(13) + \frac{1}{3}(18) = 13.$$

   Thus, the market price is 13. *Observe that* $13 < 15 = v_S^H$. *Thus, sellers of high-quality cars are not willing to put up the car for sale, contradicting the assumption that all three qualities are traded. Thus, there is no equilibrium in which all types of qualities are traded.*

(b) Suppose there is an equilibrium in which only low and medium qualities are traded. Since low and medium qualities are in equal numbers, the expected value of a car to the buyers is

$$\frac{1}{2}(8) + \frac{1}{2}(13) = 10.5.$$

Thus, the market price is 10.5*. Observe that*

$$v_S^H > 10.5 > v_S^M > v_S^L.$$

Thus, sellers of medium and low qualities are willing to put the cars for sale, while sellers of high qualities are not willing to. This is consistent with the initial assumption. Thus, there is an equilibrium in which market price is 10.5*, and both low and medium qualities are traded.*

(c) Suppose there is an equilibrium in which only low quality is traded. The expected value of a car to the buyers is 8*. The market price is thus* 8*. Sellers of low quality is willing to put the car for sale, while sellers of medium and high qualities are not willing to do so, since* $8 < v_S^M < v_S^H$*. This is consistent with the initial assumption. Thus there is an equilibrium in which the market price is* 8*, and only low quality cars are traded.*

(d) If all three qualities are traded in the market, the expected value of a car to a buyer is $18\theta_H + 13\theta_M + 8\theta_L$*. As sellers have all the bargaining power, this will be the equilibrium price. Sellers of all three qualities are willing to sell provided that* $18\theta_H + 13\theta_M + 8\theta_L \geq 15$*. This requires* θ_H *to be sufficiently large.*

4. Recall the Akerlof's model of lemons discussed in the lecture. There are many more buyers than sellers, so sellers have full bargaining power. The value that each agent places on each type of car:

	Peach	Lemon
Seller	7000	3000
Buyer	8000	4000

Suppose each seller (of either a peach or lemon) can offer a warranty for her car. Suppose also that the cost of offering a warranty to the seller is 0 and C for a peach and a lemon respectively. For simplicity, assume the buyer does not derive additional (direct) value from the warranty, so his value for each type of car is still given by the table above.

A strategy for the seller is a function from her car's quality to whether offering warranty or not. A strategy for the buyer is a function from warranty offer to a price offer.

A separating equilibrium is a pair of strategies that satisfy the following conditions:

- Different types of sellers make different warranty offer.
- Given the price-offer function of the buyers, the seller's strategy is optimal.
- Given the strategy of the sellers, the buyers' price-offer function is perfectly competitive, i.e., each buyer earns a zero payoff.

If $C = 2000$, is there a separating equilibrium in which warranty is used as a signal for good-quality? What if $C = 5000$?

Solution: Consider the following "candidate" separating equilibrium:

- *Seller's strategy:*
  - *If he owns a peach, offer a warranty;*
  - *If he owns a lemon, offer no warranty.*
- *Buyer's strategy:*
  - *Offer* 8000 *for a car with a warranty;*
  - *Offer* 4000 *for a car without a warranty.*

Does each type of seller offer different warranty? Yes.

Are sellers' strategy optimal given buyers' strategy?

- *If the seller has a peach, the expected payoff of offering a warranty is* 8000*; the expected payoff of offering no warranty is* $\max\{4000, 7000\} = 7000$. *Thus, it is optimal to offer warranty.*
- *If the seller has a lemon, the expected payoff of offering a warranty is* $\max\{8000 - C, 3000\}$*; the expected payoff of offering no warranty is* 4000. *Thus, it is optimal to offer no warranty if and only if* $\max\{8000 - C, 3000\} \leq 4000$.
- *The last inequality is satisfied if* $C = 5000$*, but not satisfied if* $C = 2000$.

Are buyers' strategy perfectly competitive given sellers' strategy?

- *If the car has a warranty, buyers infers that it is a peach, as it is the only type of cars that would be sold with a warranty. The expected value of the car is thus* 8000*, and the price offer of* 8000 *is perfectly competitive.*
- *If the car has no warranty, buyers infers that it is a lemon, as it is the only type of cars that would be sold without a warranty. The expected value of the car is thus* 4000*, and the price offer of* 4000 *is perfectly competitive.*
- *Therefore, buyers' strategy is perfectly competitive given sellers' strategy.*

Conclusion: the strategy pair is indeed a separating equilibrium if $C = 5000$*, but not if* $C = 2000$.