ECON3113
Microeconomic Theory I

Online Assignment #5 Solution

Online assignment #5

Question 1 2 pts

Consider the lotteries below:

	$X_1 = 100$	$X_2 = 10$	$X_3 = -10$
L_1	0	0.9	0.1
L_2	0.2	0	0.8
L_3	0	0.675	0.325
L_4	0.15	0	0.85
L_5	0	0	1

Which of the following pairs of preferences violates the independence axiom?

A: $L_1 \succ L_2,\ L_3 \succ L_4$

B: $L_1 \succ L_5,\ L_2 \succ L_5$

C: $L_2 \succ L_1,\ L_4 \succ L_3$

D: $L_1 \succ L_2,\ L_4 \succ L_3$

- A
- B
- C
- D

Definition

Preference $\succsim$ over lotteries satisfies the **independence axiom** if for any three lotteries L, L', and L'', and any $\alpha \in [0,1]$,

$$L \succsim L' \Rightarrow \alpha L + (1-\alpha) L'' \succsim \alpha L' + (1-\alpha) L''.$$

- Note that L_3 is a compound lottery of L_1 and L_5 with $\alpha = 0.75$ and L_4 is a compound lottery of $L2_1$ and L_5 with $\alpha = 0.75$. Therefore, the independence axiom requires that if $L_1 \succ L_2$ then $L_3 \succ L_4$. Therefore, A is not a violation of the independence axiom, but D is.
- Regarding the other answers, there is no $\alpha \in [0,1]$ such that L_5 is a compound lottery of itself and L_2 is a compound lottery of L_1, so answer B cannot be a violation.
- Similarly, there is no $\alpha \in [0,1]$ such that L_5 is a compound lottery of itself and L_1 is a compound lottery of L_2, so answer C cannot be a violation.

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Question 2 **2 pts**

Consider the insurance model that we looked at in the lectures. Suppose that a person has total wealth $100,000 and owns a car worth $30,000. Also, suppose that there is a 20% chance that the car is stolen in any year, and that the person is risk averse. Assume that insurance companies are risk neutral and that the insurance market is perfectly competitive.

Under these conditions, the person will choose to buy insurance to cover [100% ▾] of the value of the car. The actuarially fair premium for the insurance is [$6,000 ▾]. In this way, the person will achieve a(n) [certain ▾] income per year of [$94,000 ▾]

- Correct answer as shown

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Question 3 2 pts

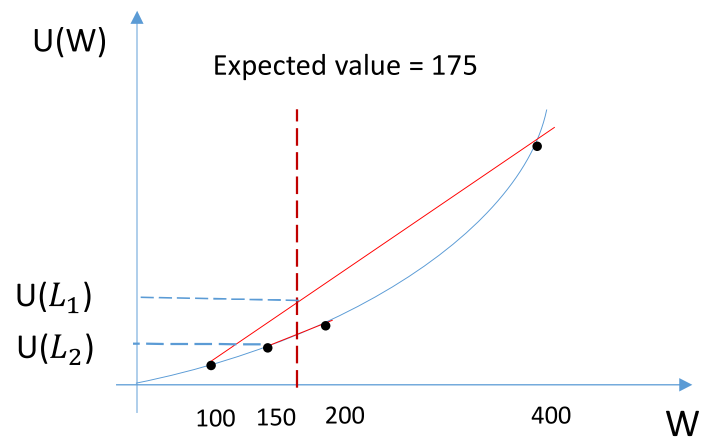
Assume that we can model an individual's preferences by a Von Neumann Morgernstern utility function. If the individual prefers lottery L_1 to lottery L_2 below, what can we conclude about the individual's preferences?

L_1	400	100
Probability	0.25	0.75

L_2	200	150
Probability	0.50	0.50

- Risk loving
- Risk neutral
- Risk averse

- See the diagram below:



- Since the expected values of the lotteries is the same and the individual prefers the riskier one, we may conclude that the individual is a risk lover

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Question 4 2 pts

Suppose that we are in the world of the asset investment model from the lectures. Assume that we have the following values for the model's variables and parameters:

starting wealth = $100,000

price of the asset per unit = $1

probability of good state = 0.4

pay-out by the asset in the good state (per unit owned) = $2.5

pay-out by the asset in the bad state = $0

According to the model, how much of this asset would you buy/short sell?

- Short sell 60,000 units
- Neither buy nor short sell any of the asset
- Buy 16,667 units of the asset
- Buy 33,333 of the asset

- A conclusion of the model is that when the price of an asset is actuarially fair, then its expected return is zero and none of it is bought
- The price of an asset is actuarially fair when its price is given by the probability of the good state x its return in the good state, that is when:
  - $\pi = (1 - p)R$
- In this case, we have $\pi = 1, p = 0.6$ and $R = 2.5$
- Therefore, $\pi = (1 - p)R$ and we conclude that the asset's price is actuarially fair and that none of it is bought

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Question 5 2 pts

Assume a Von Neumann Morgernstern utility function over income of $U(W) = W^{\frac{1}{3}}$. Suppose that during the next year, we could take a job as a trader in an investment bank. The amount that you can earn and probability of each are given below:

Income	\$1,728,000	\$64,000
Probability	0.10	0.90

Instead, we could take a job as an accountant that pays a certain income. How much would the accountant job have to pay to make us indifferent between being a trader or accountant?

- \$230,400
- \$110,592
- \$140,608
- \$46,656

- Expected utility from working as a trader is given by:
  - $E(U) = 0.10 \times 1{,}728{,}000^{1/3} + 0.90 \times 64{,}000^{1/3}$
    $= 12 + 36 = 48$
- Therefore, we need to find W such that $W^{1/3} = 48$
- This gives $W = 48^3 = 110{,}592$ which is the correct answer

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Question 6 2 pts

Consider the lotteries below:

	$x_1 = +10$	$x_2 = -5$
L_1	0.1	0.9
L_2	0.7	0.3
L_3	0.2	0.8
L_4	0.3	0.7

For which values of $a, b \in [0,1]$ are L_3 and L_4 compound lotteries of L_1 and L_2 such that $L_3 = aL_1 + (1-a)\,L_2$ and $L_4 = bL_1 + (1-b)\,L_2$?

- a=5/6, b = 7/8
- a=1/2, b = 2/3
- a=5/6, b = 2/3
- No such values are possible

- First, regarding L_3:
  - We want to find a such that $0.2 = a \times 0.1 + (1-a) \times 0.7 \Rightarrow a = 5/6$
- Next, regarding L_4:
  - We want to find b such that $0.3 = b \times 0.1 + (1-b) \times 0.7 \Rightarrow b = 2/3$