Derivative Securities (FINA 3203)
Solutions to Problem Set 1

February 24, 2020

Question 1 Basic Concepts (1/10)

1. Define what a zero-coupon bond is. (1 sentence)

   Answer: A zero-coupon bond is a bond that is issued at a deep discount to its face value but pays no interest.

2. Define what it means to "short-sell a stock". (at most 2 sentences)

   Answer: The short-seller borrows the stock from a stockholder and agrees to repay the stock at a prespecified date. The shortseller sells the stock right after borrowing, betting on a price drop to buy cheaply when she is obliged to repay.

3. Describe four major differences between exchanges and over-the-counter (OTC) markets for derivatives.

   Answer:

   - Exchanges are the central clearing house of trade, while OTC markets do not have a central clearing house.
   - Exchanges eliminate counterparty risk faced by traders.
   - Exchange traded contracts are standardized, while OTC traded contracts are customized.
   - Information is transparent for exchange traded contracts, while OTC contract information is opaque.

4. Suppose a risk-free bond which pays \$100 in three years is trading at \$90 now. Compute its annual interest rate in both the "simple interest rate" convention and the "continuously-compounded interest rate" convention.

   Answer: Simple interest rate is calculated by $90 \times (1 + 3r_s) = 100$, so $r_s = 3.7\%$.

   Continuously compounded interest rate is calculated by $90 \times e^{3r_c} = 100$, so $r_c = 3.51\%$.

Question 2: Forwards on Stocks without Dividends (1/10)

If a share in XYZ currently trades for \$100 and the risk free interest rate is 5% (annual, continuously compounded), what is the 6-month forward price?

Answer: The 6-month forward price is $\$100 \times e^{5\% \times \frac{6}{12}} = \$102.53$.

Question 3: Forwards on Stocks with Dividends (2/10)

Intel stock is trading at $100 per share. The risk-free interest rate (annualized, continuously compounded) is 5.00%. The market assumes that Intel will not pay any dividend within the next 3 months.

1. What must be the forward price to purchase one share of Intel stock in 3 months?

   Answer: The forward price must be $\$100 \times e^{5\% \times \frac{3}{12}} = \$101.26$.

2. Suppose that Intel suddenly announces a dividend of $1 per share in exactly 2 months, and assume that the Intel stock price does not change upon the announcement. What must be the new 3-month forward price for the Intel stock?

   Answer: The new forward price must be $(\$100 - PV(D)) \times e^{5\% \times \frac{3}{12}} = (\$100 - \$1 \times e^{-5\% \times \frac{2}{12}}) \times e^{5\% \times \frac{3}{12}} = \$100.25$.

3. If after the dividend announcement, the 3-month forward price still stays the same, how would you make arbitrage profit from the market mis-pricing?

   Answer: If the forward price remains the same after the dividend announcement, then I will implement the following strategy to arbitrage: I borrow $100 at t=0 to buy one share of Intel stock, and short a forward contract. After the dividend is paid, I use the dividend to repay after two months. After three months, I sell the share at the forward price, and use the proceeds to repay the rest of my borrowing. The payoffs are shown in the following table (S_T is the spot price after 3 months):

Portfolio Holdings	$t = 0$ Payoff	$t = 2M$ Payoff	$t = 3M$ Payoff
Borrow $100	$100	0	0
One share of stock	-$100	$1	S_T
Repay $1 after 1M	0	-$1	0
Repay the rest after 3M	0	0	-$100.25
Short the forward	0	0	$\$101.26 - S_T$
Total Payoff	0	0	$1.01

4. Suppose that Intel suddenly announces two dividend payments of $1 per share in exactly 1 month and 2 months, and assume that the Intel stock price does not change upon the announcement. What must be the new 3-month forward price for the Intel stock?

   Answer: The new forward price shoule be $(\$100 - PV(D)) \times e^{5\% \times \frac{3}{12}} = (\$100 - \$1 \times e^{-5\% \times \frac{2}{12}} - \$1 \times e^{-5\% \times \frac{1}{12}}) \times e^{5\% \times \frac{3}{12}} = \$99.25$.

Question 4: Arbitrage Opportunities in Forward Markets (2/10)

Suppose the S&P 500 index spot price is 1100, the risk-free rate is 5% (annual, continuously compounded), and the dividend yield on the index is 0.

1. Suppose you observe a 6-month forward price of 1135. What arbitrage would you undertake?

   Answer: The theoretical price of the 6M forward should be $\$1100 \times e^{5\% \times \frac{6}{12}} = \$1127.85$. If the forward price is $1135, I will implement the following strategy to arbitrage: Borrow

\$1100 to buy one share of S&P 500 index, and short a 6M forward contract. After 6 months, I sell the share at the forward price and use the proceeds to repay my borrowing. Payoffs are shown in the following table (S_T is the spot price after 6 months):

Portfolio Strategy	$t = 0$ Payoff	$t = 6M$ Payoff
Borrow \$1100	\$1100	0
One share	-\$1100	S_T
Repay after 6M	0	-\$1127.85
Short the forward	0	$\$1135 - S_T$
Total Payoff	0	\$7.15

2. Suppose you observe a 6-month forward price of 1115. What arbitrage would you undertake?

   Answer: If the price of a 6-month forward contract is \$1115, I will implement the following stragety: I short-sell one share of the S&P 500 index at t=0, and deposit into the risk free account. Meanwhile I enter a long position of the 6-month forward contract. After 6 months, I purchase the share at the forward price and repay it back to the shareholder I short sell with. The payoffs are shown in the following table (S_T is the spot price after 6 months):

Portfolio Strategy	$t = 0$ Payoff	$t = 6M$ Payoff
Short sell one share	\$1100	$-S_T$
Risk free account	-\$1100	\$1127.85
Long the forward	0	$S_T - \$1115$
Total Payoff	0	\$12.85

Question 5 S&P 500 Futures Contracts (2/10)

Suppose the S&P 500 index is currently 950 and the initial margin is 10%. You wish to enter into a long position for 10 S&P 500 futures contracts.

1. What is the contract size for S&P 500 Futures?

   Answer: 250 units of the index

2. What is the notional value of your position?

   Answer: $\$950 \times 10 \times 250 = \$2,375,000$.

3. What is the initial margin in dollars?

   Answer: $\$2,375,000 \times 10\% = \$237,500$.

4. Suppose you earn a continuously-compounded interest rate of 6% on your margin balance, your position is marked to market weekly, and the maintenance margin is 80% of the initial margin. For simplicity, we assume that today's futures price is the same as the spot price, 950. What is the greatest S&P 500 index futures price 1 week from today at which will you receive a margin call?

   Answer: The margin account has $\$237,500 \times e^{6\% \times \frac{1}{52}} = \$237,774.20$ after a week if there is no price fluctuations. The margin call occurs when the price of future contract drops, suppose to P. The required margin is $\$950 \times 8\% \times 250 \times 10$, must be greater than or equal to $\$237,774.20 - (\$950 - P) \times 250 \times 10$, which is the amount in the margin account after marking the loss to the market. Therefore, the greatest future price for margin call satisfies $95 \times 8\% \times 250 \times 10 = \$237,774.20 - (\$950 - P) \times 250 \times 10$. We solve for $P = \$930.89$.

Question 6: The Value of A Forward Contract (2/10)

This question asks you to think about how the value of a forward contract on a non-dividend paying stock changes over time.

1. On February 20 you enter into forward contract to buy ABC shares on December 20. ABC shares currently trade at \$100. What is the forward price? (The continuously compounded interest rate is 10% and assumed to be constant for the whole calendar year.)

   Answer: The forward price is $\$100 \times e^{10\% \times \frac{10}{12}} = \$108.69$ on February 20.

2. On May 20, the price of one ABC share is \$150. What is the forward price of a forward contract with delivery date December 20 (this is a different contract)?

   Answer: The forward price is $\$150 \times e^{10\% \times \frac{7}{12}} = \$159.01$ on May 20.

3. Forward contracts are not traded on an exchange, they do not have a market price. However, a reasonable way to define the value of a forward contract is as the amount of money someone would have to pay you today to give up your forward contract. Using this definition, what is the value of the original forward contract (that you entered into in February) on May 20?

   Answer: The value of the original contract is $(\$159.01 - \$108.69) \times e^{-10\% \times (\frac{10}{12} - \frac{3}{12})} = \$47.47$.

4. Sometimes it is the case that you would be prepared to actually pay someone else for the right to walk away from a forward contract. In this case, the value of the forward contract is negative. Re-do part (iii) under the assumption that on May 20, the price of one ABC share is \$50.

   Answer: If the price of ABC share on May 20 is \$50, the forward price on May 20 will be $\$50 \times e^{10\% \times \frac{7}{12}} = \$53.00$. Then the value of the forward contract is $(\$53.00 - \$108.69) \times e^{-10\% \times (\frac{10}{12} - \frac{3}{12})} = -\$52.53$.

5. (Optional) What is the May 20 value of a short position in the original forward contract if the ABC share price on May 20 is \$150?

   Answer: The value of short position is -\$47.47 if the ABC share price is \$150 on May 20.

6. (Optional) What is the May 20 value of a short position in the original forward contract if the ABC share price on May 20 is \$50?

   Answer: The value of short position is \$52.53 if the ABC share price is \$50 on May 20.

7. (Optional) What is the value of a long position in the original forward contract on February 20 (the entry date)?

   Answer:The value of a long position in the original forward contract on the entry date is 0.