Homework 4 Solution

Choose the best answer

1. The market demand curve for any good is
   a. independent of individuals' demand curves for the good.
   b. the vertical summation of individuals' demand curves.
   c. the horizontal summation of individuals' demand curves.
   d. derived from the firm's marginal cost of production.

2. In the short run
   a. new firms may enter an industry.
   b. existing firms may change the quantity they are supplying.
   c. firm can adjust levels of all inputs.
   d. quantity supplied is absolutely fixed.

3. If the market for bottled spring water is characterized by a very elastic supply curve and a very inelastic demand curve, an outward shift in the supply curve would be reflected primarily in the form of
   a. higher prices.
   b. higher output.
   c. lower prices.
   d. lower output.

4. In the short run, a sales tax is
   a. wholly absorbed by the producer.
   b. shared between the consumer and the producer.
   c. deferred until the market is able to re-establish an equilibrium price.
   d. wholly absorbed by the consumer.

Analytical questions

1. Consider a generic firm in a perfectly competitive industry with price p. Its average cost is

$$AC(q) = \frac{16}{q} + q.$$

   a. What is the total cost and marginal cost of this firm?

$$C(q) = AC(q) \times q = 16 + q^2$$

$$MC(q) = C'(q) = 2q$$

b. Find the long run supply curve of this firm. Be careful of the zero profit point.

The seller will produce at where $p = MC(q) = 2q$, therefore $S(p) = \frac{p}{2}$. However, in the long run, the firm will exit the industry if it has negative profit. We can find the zero profit point by finding the minimum of average cost.

$$AC'(q) = -\frac{16}{q^2} + 1 = 0, \quad \Rightarrow AC_{min} = AC(4) = 8.$$

Therefore, the supply function shall be

$$S(p) = \begin{cases} 0 & \text{if } p < 8 \\ \frac{p}{2} & \text{if } p \geq 8. \end{cases}$$

c. Suppose there are 30 firms with this same technology, what is the market supply?

We shall add supply from all these firms horizontally. The market supply is

$$Q_S(p) = S_1(p) + S_2(p) + \cdots + S_{30}(p) = 30 \times \frac{p}{2} = 15p, \quad p \geq 8$$

(Supply is zero when $p < 8$).

d. Suppose there is free entry to the industry and all firms have the same technology. The market demand of this industry is

$$Q_D(p) = 160 - 10p.$$

What will be the long run equilibrium price and the number of firms of the industry?

The long run equilibrium price will be $p = 8$, because any price higher than 8 results in positive profit and attract more entrants. Plug in $p = 8$ to the demand $Q_D(8) = 80$ and to firm's supply $S(8) = 4$. $80 \div 4 = 20$, there will be 20 firms in the industry.

2. For the car industry of nation A, the market demand is

$$Q_D(P, I) = 2000 \times P^{-2} I^{0.5},$$

where I is income.

a. Compute the price elasticity $e_{D,P}$ and income elasticity $e_{D,I}$ of the car demand

$$e_{D,P} = \frac{dQ_D}{dP}\frac{P}{Q_D} = 2000 \times (-2)P^{-3}I^{0.5} \times \frac{P}{2000 \times P^{-2}I^{0.5}} = -2$$

$$e_{D,I} = \frac{dQ_D}{dI}\frac{I}{Q_D} = 2000 \times (0.5)P^{-2}I^{-0.5} \times \frac{I}{2000 \times P^{-2}I^{0.5}} = 0.5.$$

b. Suppose there is a 10% increase of income I. We do not know the supply function $Q_S(P)$, but knows that the supply elasticity is $e_{S,P} = 3$ from empirical data. Predict how much the equilibrium price will increase (in percentage).

The goal is to compute $\frac{dP}{dI}$, We know that in the equilibrium $\frac{dQ_D(P,I)}{dI} = \frac{dQ_S(P)}{dI}$

$$\frac{dQ_D(P,I)}{dI} = \frac{\partial Q_D}{\partial P}\frac{dP}{dI} + \frac{\partial Q_D}{\partial I}, \qquad \frac{dQ_S(P)}{dI} = \frac{\partial Q_S}{\partial P}\frac{dP}{dI}$$

$$\frac{\partial Q_D}{\partial P}\frac{dP}{dI} + \frac{\partial Q_D}{\partial I} = \frac{\partial Q_S}{\partial P}\frac{dP}{dI} \Rightarrow \frac{dP}{dI} = \frac{\frac{\partial Q_D}{\partial I}}{\frac{\partial Q_S}{\partial P} - \frac{\partial Q_D}{\partial P}}$$

$$\frac{dP}{dI}\frac{I}{P} = \frac{\frac{\partial Q_D}{\partial I}\frac{I}{Q}}{\frac{\partial Q_S}{\partial P}\frac{P}{Q} - \frac{\partial Q_D}{\partial P}\frac{P}{Q}} \Rightarrow e_{P,I} = \frac{e_{Q,I}}{e_{S,P} - e_{D,P}} = \frac{0.5}{3-(-2)} = 0.1$$

Therefore, by the defintion of elasticity $e_{P,I}$, the 10% increase will result in 1% increase in equilibrium car price (approximately).

3. Consider the car industry in nation C. The domestic demand for car can be represented by inverse demand

$$p^D(q) = 40 - \frac{1}{2}q.$$

The domestic car inverse supply is

$$p^S(q) = \frac{1}{2}q.$$

a. If nation C probibit international trade, calculate equilibrium price, quantity, consumer surplus, (domestic) producer surplus.

$$q^* = 40, \quad p^* = 20$$
$$CS = \frac{1}{2} \times 40 \times 20 = 400$$
$$PS = \frac{1}{2} \times 40 \times 20 = 400.$$

b. If nation C allow for free trade. The world price of car is 10. How many cars will nation C import? Use a diagram to illustrate the effect. Calculate consumer surplus, (domestic) producer surplus.

At $p_w = 10$, the quantity demand is

$$10 = 40 - \frac{1}{2}q \Rightarrow q^D = 60.$$

At $p_w = 10$, the domestic car quantity supply is

$$10 = \frac{1}{2}q \Rightarrow q^S = 20.$$

So 40 units of car are imported.

$$CS = \frac{1}{2} \times (40 - p_w) \times q^D = 900$$

$$PS = \frac{1}{2} \times 10 \times q^S = 100.$$

Consumer gets better off, but domestic producer are worse off.

c. The domestic car producers successfully convinces the government to product "infant industry". The government decides to impose a tarriff of 5 on each car imported. Illustrate the new equilibrium on the graph you draw for (b), how many cars will nation C import? How much tariff revenue will the government collected?

The price of foreign car rise to 15. So $q^D = 50$, $q^S = 30$, so import 20 cars. The government can collect $20 \times 5 = 100$ tariff revenue.

d. If the government replace the policy of tariff by a import quota of 20. Compare to the tariff case in (c), which party is worse-off? Which party is better-off?

The government is worse-off because there is not tariff revenue. The foreign car producers are better-off.

4. The cost function of a firm is given by

$$C(q) = q^2 + 4q + 4.$$

a. Suppose that the firm is operating in a perfectly competitive market and the current price is p. What is the firm's supply function? Draw a demand-supply diagram. Clearly indicate which part denote the supply curve (be careful on the shutdown point).

Firm uses $MC(q) = p$ rule to determine production

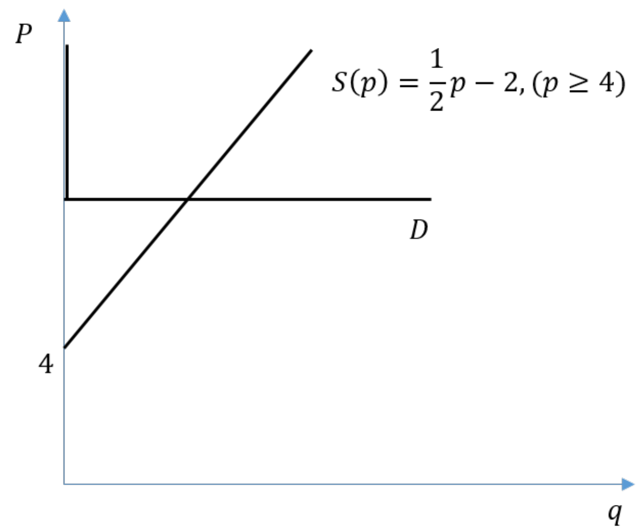
$$MC(q) = 2q + 4 = p,$$

therefore the supply function is

$$S(p) = \begin{cases} \frac{1}{2}p - 2 & , \quad \text{for } p \geq 4, \\ 0 & , \quad \text{for } p < 4. \end{cases}$$

The shutdown point is easy to obtained from a diagram. Here, average variable cost is $AVC(q) = q + 4$, therefore, it intersects $MC(q)$ at $q = 0$.

Because the market is perfectly competitive, the firm is a price-taking and faces a horizontal demand curve.



b. Compute the change of producer surplus when the price rises from $p = 5$ to $p = 6$.

$$\Delta PS = \int_5^6 (\frac{1}{2}p - 2)dp = \left[\frac{1}{4}p^2 - 2p\right]_5^6$$
$$= \left[\frac{1}{4} \times 36 - 2 \times 6\right] - \left[\frac{1}{4} \times 25 - 2 \times 5\right] = \frac{3}{4}$$

c. Suppose that the firm is operating in a market with downward-sloping market demand curve $Q_D(p) = 20 - p$ and there is no entry barrier. Potential entrants of this market all have the same cost function. Compute the LONG RUN equilibrium of this market with price, quantity, and number of firms.

The long run equilibrium price is determined by the minimum point of average cost.

$$AC(q) = q + 4 + \frac{4}{q}$$
$$AC'(q) = 1 - \frac{4}{q^2} = 0 \Rightarrow q_{min} = 2,\ AC_{min} = AC(2) = 8.$$

Therefore, the long run equilibrium price is $p_{LR} = 8$, and each firm produce 2 units. LR equilibrium quantity $Q_{LR} = Q_D(8) = 20 - 8 = 12$. $n_{LR} = Q_{LR}/q_{LR} = 6$. There will be 6 firms, each produce 2 units, and sell at price 8.

Textbook exercise 12.3, part (c)

The simplest formula to use is (see lecture note 4)

$$\frac{\partial P}{\partial \alpha} = \frac{\frac{\partial Q_D}{\partial \alpha}}{\frac{\partial Q_S}{\partial P} - \frac{\partial Q_D}{\partial P}} \Rightarrow \frac{\partial P^*}{\partial I} = \frac{\frac{\partial D}{\partial I}}{\frac{\partial S}{\partial P} - \frac{\partial D}{\partial P}} = \frac{c}{g - b}.$$

c. Suppose that the firm is operating in a market with downward-sloping market demand curve $Q_D(p) = 20 - p$ and there is no entry barrier. Potential entrants of this market all have the same cost function. Compute the LONG RUN equilibrium of this market with price, quantity, and number of firms.

The long run equilibrium price is determined by the minimum point of average cost.

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Therefore, the long run equilibrium price is $p_{LR} = 8$, and each firm produce 2 units. LR equilibrium quantity $Q_{LR} = Q_D(8) = 20 - 8 = 12$. $n_{LR} = Q_{LR}/q_{LR} = 6$. There will be 6 firms, each produce 2 units, and sell at price 8.

Textbook Questions

12.2 Given the cost function

$$C = q^2 + wq.$$

a. Differentiating total cost gives marginal cost:

$$MC = 2q + w.$$

Substituting $w = 10$,

$$C = q^2 + 10q.$$

$$MC = 2q + 10.$$

Set $MC = P$ and solve for the firm's (short-run) supply curve:

$$2q + 10 = P$$
$$\Rightarrow q = 0.5P - 5.$$

The industry's supply curve is

$$Q = \sum_{1}^{1000} q = 500P - 5{,}000.$$

At $P = 20$, $Q = 5{,}000$. At $P = 21$, $Q = 5{,}500$.

b. Here, $MC = 2q + 0.002Q$. For profit maximum, set $MC = P$:

$$q = 0.5P - 0.001Q.$$

The industry supply curve is

$$Q = \sum_{1}^{1000} q = 500P - Q.$$

$$Q = 250P.$$

If $P = 20,\ Q = 5{,}000.$ If $P = 21,\ Q = 5{,}250.$ Supply is more steeply sloped in the case where expanded output bids up wages.

12.3

Demand: $Q = a + bP + cI$ or $a + bP + cI - Q = 0\ (b < 0),$
Supply: $Q = d + gP$ or $d + gP - Q = 0 \quad (g > 0).$

a. Equating quantity demanded to quantity supplied yields:

$$a + bP + cI = d + gP \text{ or } P^* = \frac{a-d}{g-b} + \frac{c}{g-b} I,$$
$$Q^* = d + gP^* = d + \frac{g(a-d)}{g-b} + \frac{cg}{g-b} I.$$

b. $\dfrac{dP^*}{dI} = \dfrac{c}{g-b} > 0, \quad \dfrac{dQ^*}{dI} = \dfrac{cg}{g-b} > 0.$

c. Differentiation of the demand and supply equations yields:

$$b\frac{dP}{dI} + c - \frac{dQ}{dI} = 0,$$
$$g\frac{dP}{dI} - \frac{dQ}{dI} = 0.$$

Putting this into matrix notation:

$\begin{bmatrix} b & -1 \\ g & -1 \end{bmatrix} \cdot \begin{bmatrix} dP/dI \\ dQ/dI \end{bmatrix} = \begin{bmatrix} -c \\ 0 \end{bmatrix}$ and applying Cramer's rule gives

$$\frac{dP^*}{dI} = \frac{\begin{vmatrix} -c & -1 \\ 0 & -1 \end{vmatrix}}{\begin{vmatrix} b & -1 \\ g & -1 \end{vmatrix}} = \frac{c}{g-b} \qquad \frac{dQ^*}{dI} = \frac{\begin{vmatrix} b & -c \\ g & 0 \end{vmatrix}}{\begin{vmatrix} b & -1 \\ g & -1 \end{vmatrix}} = \frac{cg}{g-b}.$$

d. Suppose
$a = 10, b = -1, c = 0.1, d = -10, g = 1, I = 100: \quad P^* = 10 + 0.05 \cdot 100 = 15, \quad Q^* = 5.$

an increase of income of 10 would increase quantity demanded by 1 if price were held constant. This would create an excess demand of 1 that must be closed by a price rise. Because the demand and supply relations have price coefficients that are equal and opposite in sign, a price rise of 0.5 will reduce quantity demanded by 0.5 and increase quantity supplied by the same amount. Equilibrium quantity would also increase by 0.5. Hence, the new equilibrium is $P^* = 15.5, Q^* = 5.5$ as would have been predicted by the multipliers from parts (b) and (c).

12.6 a. Short-run supply is $q = P - 10.$ Market supply is $100q = 100P - 1{,}000.$

b. Equilibrium where $100P - 1{,}000 = 1{,}100 - 50P$. So, $P^* = 14$, $Q^* = 400$.

c. Since $Q_S = 0$ when $P = 10$, $\text{producer surplus} = 0.5(14 - 10)(400) = 800$.

d. Total industry fixed cost $= 500$. For a single firm,

$$\pi = 14 \cdot 4 - \left[0.5(4)^2 + 40 + 5\right] = 56 - 5 = 3.$$

Total industry profits $= 300$. Finally, we have

$$\text{Short-run profits} + \text{fixed cost} = 800 = \text{producer surplus}.$$

e. With tax, $P_D = P_S + 3$. Equating supply and demand,

$$\begin{aligned} &1{,}100 - 50P_D = 100P_S - 1{,}000 \\ &\Rightarrow 1{,}100 - 50P_D = 100(P_D - 3) - 1{,}000 \\ &\Rightarrow 150P_D = 2{,}400. \end{aligned}$$

Thus,

$$P_D^* = 16$$

$$P_S^* = 16 - 3 = 13$$

$$Q^* = 1{,}100 - (50 \cdot 16) = 300.$$

Total tax $= 900$.

f. Consumers pay $300(16 - 14) = 600$. Producers pay $300(14 - 13) = 300$.

g. $PS = 0.5(300)(13 - 10) = 450$, a loss of 350 from Problem 11.2, part (d). Short-run profits equal $13(300) - 100C$. But

$$C = 0.5(3)^2 + 30 + 5 = 39.5.$$

Hence,

$$\pi = 3{,}900 - 3{,}950 = -50.$$

Since total profits were 300, this is a reduction of 350 in short-run profits.