Homework 5 Solution

Choose the best answer

1. All monopolies exist because of
   a. firms' desire to maximize profits.
   b. failure of antitrust laws.
   c. barriers to entry.
   d. natural selection.

2. For the practice of price discrimination to be successful, the monopoly must
   a. be able to prevent resale of its product.
   b. face similar demand curves for various markets.
   c. have similar costs among markets.
   d. have a downward sloping marginal cost curve.

3. The "deadweight loss" from a monopoly refers to
   a. the portion of a monopolist's profits that are above the competitive profit level.
   b. the increase in price due to the monopolization of a market.
   c. the inefficient use of factors of production by a monopoly.
   d. the loss of welfare due to the monopolization of a market that is not transferred to another economic actor.

4. Relative to uniform price policy, third degree price discrimination
   a. always reduces welfare.
   b. always increases welfare.
   c. may increase welfare if total output falls.
   d. may increase welfare if total output rises.

Analytical questions

1. Rock Oil Company is the only gasoline seller on an island. It has a constant marginal cost of production as $MC(q) = 2$. The demand for gas of all residents on the island is

$$Q_D(p) = 80 - 4p.$$

You don't need to consider fixed cost in this question.

a. Rock Oil Company maximizes its profit as a monopoly. How many oil will it supplies to the island? What price will it charge?

We first need to calculate marginal revenue. The inverse demand is $p_D(q) = 20 - \frac{1}{4}q$. Linear demand has marginal revenue curve with twice steeper slope:

$$MR(q) = 20 - \frac{1}{2}q$$

Use the MC equals MR rule, we have

$$20 - \frac{1}{2}q = 2 \Rightarrow q^M = 36$$

$$p^M = 20 - \frac{1}{4}q^M = 11.$$

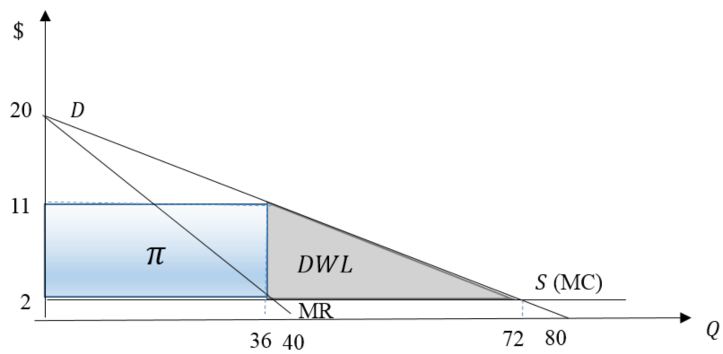
b. Calculate how much profit Rock Oil company earned in (a). What is the deadweight loss? Illustrate them on a graph.

For each unit, Rock oil company cost 2 to produce and sell 11 on the market, so

$$\pi = (p^M - MC) \times q^M = (11 - 2) \times 36 = 324.$$

The efficient price is $P = 2$, $q = 72$,

$$DWL = \frac{1}{2}(72 - 36) \times (11 - 2) = 162.$$



c. If the government imposes a per-unit tax of $t = 2$. Compute the new equilibrium price and deadweight loss.

Imposing the tax will raise the marginal cost of the company to $MC(q) = 4$.

$$MR(q) = 20 - \frac{1}{2}q = 32$$

$$q^M = 32, \quad p^M = 20 - \frac{1}{4}q^M = 12$$

So the price rises by 1.

The deadweight loss is

$$DWL = \frac{1}{2}(72 - 32) \times (12 - 2) = 200.$$

d. Now consider that the government directly regulates price, what price should the government set to maximzie the welfare? What will be consumer surplus and the producer surplus of Rock Oil Company?

The government can set price at $P = 2 = MC$. Then the welfare of gasoline market is maximized.

The consumer surplus is

$$CS = \frac{1}{2} \times 72 \times (20 - 2) = 648.$$

The producer surplus of the company is

$$PS = 0.$$

e. Due to the change of regulation, there is another seller, Stone oil company, enters into the gasoline market of the island. It also has a marginal cost of $MC = 2$. Suppose Rock oil and Stone oil sell exactly same product, and they do not collude. What do you think is the equilibrium price?

The equilibrium price shall be very closed to 2 (or equal to 2). Because the product is exactly the same, the company set price higher will lose all the consumers, and the one undercut price can gain all the consumers. They will be likely to fall into price war and ends up at $P = MC = 2$.

f. Consider the same situation in (e), but now Rock Oil and Stone Oil decide to collude. What will the equilibrium price be?

Collusive oligopoly results in the same situation of monopoly, so $p = 11$ as we calculated in (c). They maximize their joint profit, then divide it.

2. A theme park is designing the pricing of its ticket. Denote q as the number of facilities a representative consumer used in the theme park. The demand (marginal utility) of the consumer can be approximate by

$$p_D(q) = 105 - 5q.$$

The theme park has a constant marginal cost of \$5 of operating each facility usage. (No need to consider fixed cost in this question)

a. If the theme park does not charge an admission fee, but charge a uniform price p_u for each use of facility. Compute the optimal monopoly price and the theme park's profit.

Use monopoly output decision rule

$$MR(q) = 105 - 10q = MC = 5$$

$$p_u = 10, \quad p_u = p_D(10) = 55$$
$$\pi^M = p_u q^M - MC \times p_u = 500.$$

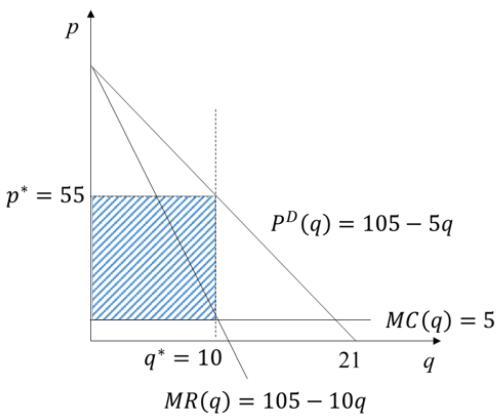
b. Suppose now the theme park learns the demand curve of the consumer, so it can exercise first-degree price discrimination. Describe the theme park's pricing strategy and compute (approximately) the profit.

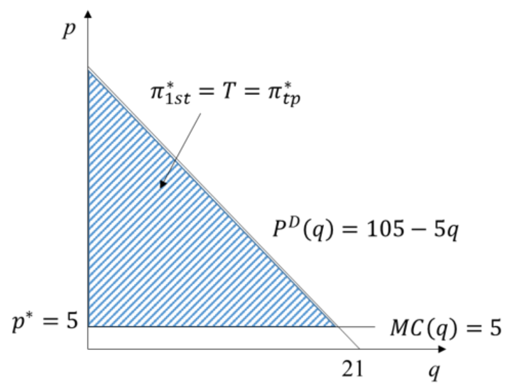
The theme park will charge a price as the demand curve (marginal utility) and capture all the consumer surplus, that is $p(q) = p_D(q)$. Specifically, for the first unit, it charges

$p_D(1) = 105 - 5 = 100$; for the second unit, it charges $p_D(2) = 95$; ...; for the 20th unit, it charges $p_D(20) = 5$, which is equal to its marginal cost.

If we ignore the discreteness, its profit will be the entire consumer surplus (above marginal cost)

$$\pi^{1st} = \frac{1}{2} \times 100 \times 20 = 1000$$





c. Because charging each unit for different price is hard to implement, the theme park decides to use "two-part tariff". It sets up an admission fee T and then charges each use of facility a price p. Show that by setting T and p properly, the theme park can reach the same profit as in part (b).

The theme park will charge $p = MC = 5$, so consumers will derive the most consumer surplus from using 20 facilities. We have computed it in part (b) that this consumer surplus is of amount 1000. Then the park will ask for an admission fee $T = 1000$ to capture the entire consumer surplus as profit, $\pi^{2nd} = 1000$.

3. HK Airline company is operating a flight from city A to city B. The consumer can be categorized into two groups: business travelers and leisure travelers. Business traveler's demand is

$$D^B(p) = 1000 - p,$$

and the leisure traveler's demand is

$$D^L(p) = 500 - p.$$

The marginal cost of handling each passenger is constant at \$50.

a. If the Airline company cannot distinguish between two travelers, therefore cannot charge different price. (No price discrimination) What price shall it charge to maximize profit?

[Hint: you need to compute the market demand first. Use a diagram to help you]

The market demand shall be computed by horizontal aggregation.

$$Q_D(p) = D^B(p) + D^L(p) = \begin{cases} 1000 - p & \text{if } p > 500 \\ 1500 - 2p & \text{if } p \leq 500. \end{cases}$$

Because it serves both groups of consumers, therefore we only need to consider the part with $p \leq 500$. The profit is therefore

$$\pi(p) = (p - 50) \times (1500 - 2p)$$

By taking first order derivative and making it to zero, we get $p^* = 400$. Plug back to the profit function, the profit is

$$\pi(400) = (400 - 50)(1000 - 2 \times 400) = 245,000.$$

Alternatively, we can solve quantity first by $MC = MR$ rule. It requires inverse demand first. From

$$q = 1500 - 2q, \quad P_D(q) = 750 - \frac{1}{2}q.$$

MR for linear demand is twice steeper:

$$MR(q) = 750 - q = MC = 50 \Rightarrow q^* = 700.$$

The computation above is assuming that HK Airline serve both type of consumer. It has another option of serving only business travel. The inverse demand of business travelers is

$$p^B(q) = 1000 - q$$

$$MR^B(q) = 1000 - 2q = MC = 50$$

So $q' = 475$, and $p' = 1000 - 475 = 525$. By doing so, no leisure travel will purchase any ticket. The profit is

$$\pi(525) = (525 - 50) \times 475 = 225,625 < 245,000.$$

Therefore, HK Airline would rather serve both markets at a lower price.

b. The Airline Company hires a group of data scientists and now can identify these two group of travelers by big data techniques. So it can charge p_B to business travelers and p_L to leisure travelers. Solve for p_B and p_L that maximizes profit. How much more profit it earns compare to part (a).

Solve profit maximization problem separately for two groups of consumers

$$\pi^B(p) = (p - 50) \times (1000 - p)$$

$$\pi^{B'}(p) = (1000 + 50) - 2p = 0$$

$$p^B = 525$$

From business travelers, it earns

$$\pi^B(525) = (525 - 50)(1000 - 525) = 225,625$$

Similar for leisure traveler

$$\pi^L(p) = (p - 50) * (500 - p)$$

$$\pi^{L'}(p) = (500 + 50) - 2p = 0$$

$$p_L = 275$$

From leisure travelers, it earns

$$\pi^L(275) = (275 - 50)(500 - 275) = 50,625.$$

So the total profit now is

$$\pi_{3rd} = 225,625 + 50,625 = 276,250$$

Compare to part (a), $276,250 - 245,000 = 31,250$. This is the profit increment.

c. Compare consumer surpluses of part (a) and part (b), for both groups of consumers.

Consumer surplus can be computed as the area of triangle in linear demand for each group. In part (a), when price is at 400, business traveler purchase 600 tickets, while leisure traveler purchase 100 tickets.

$$CS_a = \frac{1}{2}(1000 - 400)600 + \frac{1}{2}(500 - 400)100 = 185,000$$

In part (b),

$$CS_b = \frac{1}{2}(1000 - 525)475 + \frac{1}{2}(500 - 275)225 = 138,125$$

Therefore consumer surplus shrinks by $46,875 (= 185,500 - 138,125)$ when HK Airline start price discriminating.

4. Textbook exercise 14.1

5. Textbook exercise 14.3

6. Textbook exercise 14.6

Similar for leisure traveler

$$\pi^L(p) = (p - 50) * (500 - p)$$

$$\pi^{L'}(p) = (500 + 50) - 2p = 0$$
$$p_L = 275$$

From leisure travelers, it earns

$$\pi^L(275) = (275 - 50)(500 - 275) = 50,625.$$

So the total profit now is

$$\pi_{3rd} = 225,625 + 50,625 = 276,250$$

Compare to part (a), $276,250 - 245,000 = 31,250$. This is the profit increment.

c. Compare consumer surpluses of part (a) and part (b), for both groups of consumers.

Consumer surplus can be computed as the area of triangle in linear demand for each group. In part (a), when price is at 400, business traveler purchase 600 tickets, while leisure traveler purchase 100 tickets.

$$CS_a = \frac{1}{2}(1000 - 400)600 + \frac{1}{2}(500 - 400)100 = 185,000$$

In part (b),

$$CS_b = \frac{1}{2}(1000 - 525)475 + \frac{1}{2}(500 - 275)225 = 138,125$$

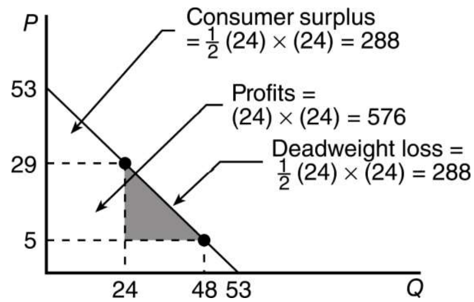
Therefore consumer surplus shrinks by $46,875 (= 185,500 - 138,125)$ when HK Airline start price discriminating.

Textbook questions

14.1 a. Given $P = 53 - Q$. Then $TR = PQ = 53Q - Q^2$, implying $MR = 53 - 2Q$. Profit maximization yields $MR = 53 - 2q = MC = 5$, implying $Q_m = 24$, $P_m = 29$, and $\pi_m = (P - AC)Q = 576.$

b. $MC = P = 5$ implies $P_c = 5$ and $Q_c = 48.$

c. Consumer surplus under competition is $2(48)^2 = 1,152.$ See the graph for monopoly.



14.3 a. Given $AC = MC = 10$ and $Q = 60 - P$, implying $MR = 60 - 2Q$. For profit maximum, $MC = MR \Rightarrow 10 = 60 - 2Q \Rightarrow Q_m = 25$. Solving for the other equilibrium variables, $P_m = 35$ and $\pi_m = TR - TC = 25 \cdot 35 - 25 \cdot 10 = 625$.

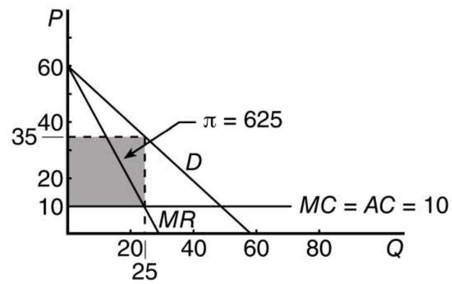
b. Given $AC = MC = 10$ and $Q = 100 - 2P$, implying $MR = 90 - 4Q$. For profit maximum, $MC = MR \Rightarrow 10 = 90 - 4Q \Rightarrow Q_m = 20$. Solving for the other equilibrium variables, $P_m = 50$ and $\pi_m = 40 \cdot 30 - 40 \cdot 10 = 800$.

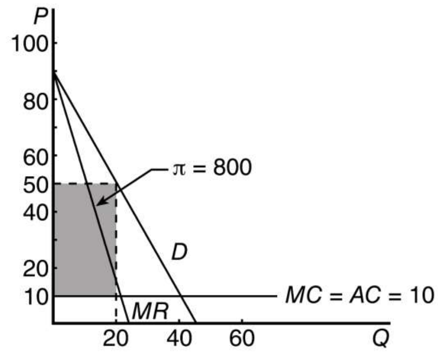
c. Given $AC = MC = 10$ and $Q = 100 - 2P$, implying $MR = 50 - Q$. For profit maximum, $MC = MR \Rightarrow 10 = 50 - Q \Rightarrow Q_m = 40$. Solving for the other equilibrium variables, $P_m = 30$ and $\pi_m = 40 \cdot 30 - 40 \cdot 10 = 800$. π = (40)(30) – (40)(10) = 800.

Note: Here the inverse elasticity rule is clearly illustrated:

Problem part	$e_{Q,P} = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q}$	$\frac{-1}{e_{Q,P}} = \frac{P - MC}{P}$
(a)	$-1(35/25) = -1.4$	$0.71 = (35 - 10)/35$
(b)	$-0.5(50/20) = -1.25$	$0.80 = (50 - 10)/50$
(c)	$-2(30/40) = -1.5$	$0.67 = (30 - 10)/30$

d.





The supply curve for a monopoly is a single point, namely, that quantity–price combination that corresponds to the quantity for which $MC = MR$. Any attempt to connect equilibrium points (price–quantity points) on the market demand curves has little meaning and brings about a strange shape. One reason for this is that as the demand curve shifts, its elasticity (and its *MR* curve) usually changes bringing about widely varying price and quantity changes.

14.6

a. In the first market, $Q_1 = 55 - P_1 \quad \Rightarrow \quad R_1 = (55 - Q_1)Q_1 = 55Q_1 - Q_1^2$
$\Rightarrow \quad MR_1 = 55 - 2Q_1$. Setting $MR_1 = MC = 5$ yields $Q_1^* = 25$ and $P_1^* = 30$. In the second market, $Q_2 = 70 - 2P_2 \quad \Rightarrow \quad R_2 = [(70 - Q_2)/2]Q_2 = (70Q_2 - Q_2^2)/2$
$\Rightarrow \quad MR_2 = 35 - Q_2$. Setting $MR_2 = MC = 5$ yields $Q_2^* = 30$ and $P_2^* = 20$. Profits across both markets are $\pi = (30-5)\cdot 25 + (20-5)\cdot 30 = 1,075$.

b. If the producer ignores the problem of arbitrage among consumers, the price differential between the two markets found to be optimal in the previous part ($10) induces arbitrage. The producer does better by preventing arbitrage by keeping the price differential to $4, that is, $P_1 = P_2 + 4$. We can solve this as a constrained maximization problem. Setting up the associated Lagrangian,

$$\mathrm{L} = (P_1 - 5)(55 - P_1) + (P_2 - 5)(70 - 2P_2) + \lambda(4 - P_1 + P_2).$$

Taking the first-order conditions,

$$\mathrm{L}_{P_1} = 60 - 2P_1 - \lambda = 0,$$
$$\mathrm{L}_{P_2} = 80 - 4P_1 + \lambda = 0,$$
$$\mathrm{L}_{\lambda} = 4 - P_1 + P_2 = 0.$$

This yields two equations in two unknowns $60-2P_1=4P_2-80$ and $P_1=P_2+4$. Solving, or $P_2^*=22$. Further, $P_1^*=26$ and $\pi^*=1{,}051$. (The same answer can be obtained by substituting $P_1=P_2+4$ into profits from the two markets and solving as a single-variable, unconstrained maximization problem.)

c. Now $P_1=P_2=P$. So $\pi=140P-3P^2-625$. Taking the first-order condition, $d\pi/dP=140-6P=0$, implying $P^*=140/6=23.33$, $Q_1^*=31.67$, $Q_2^*=23.33$, and $\pi^*=1{,}008.33$.

d. If the firm adopts a linear tariff of the form $T(Q_i)=\alpha_i+mQ_i$, it can maximize profit by setting $m=5$,

$$\alpha_1=0.5(55-5)(50)=1{,}250,$$
$$\alpha_2=0.5(35-5)(60)=900,$$

earning $\pi^*=2{,}150$. Notice that in this problem neither market can be uniquely identified as the "least willing" buyer, so a solution similar to Example 14.5 is not possible. If the entry fee were constrained to be equal in the two markets, the firm could set $m=0$ and charge a fee of 1,225 (the most buyers in market 2 would pay). This would yield profits of $2{,}450-125\cdot 5=1{,}825$, which is inferior to profits obtained with $T(Q_i)$.