Exercise 3:

1. (b): $f(1,2) = -1$, $f(-1,1) = 0$, $f(2,3) = 1$

   $f_x = 2x$, $f_y = -1$, thus $\nabla f(x,y) = (2x, -1)$

   $\nabla f(1,2) = (2,-1)$, $\nabla f(-1,1) = (-2,-1)$, $\nabla f(2,3) = (4,-1)$
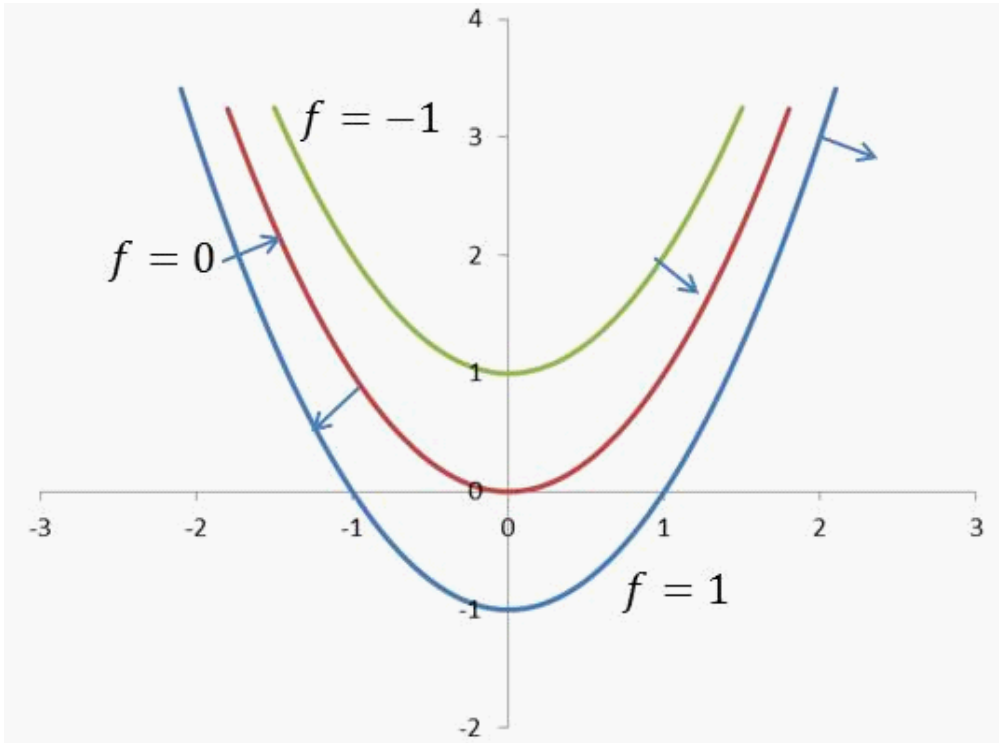
   (c): Hessian matrix$= \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$

2. (a): $\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) = (3yz + 2xy - z^3, 3xz + x^2, 3xy - 3xz^2)$

   $$f''(x,y,z) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix} = \begin{pmatrix} 2y & 3z + 2x & 3y - 3z^2 \\ 3z + 2z & 0 & 3x \\ 3y - 3z^2 & 3x & -6xz \end{pmatrix}$$

   (b): $\nabla f(x,y,z) = (-2x, -2y, -2z)$

   $$f''(x,y,z) = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} = -2I_3$$

   (c): $\nabla f(x,y,z) = (2x + y - 2z, 6y + x + 4z, -2z - 2x + 4y)$

   $$f''(x,y,z) = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 6 & 4 \\ -2 & 4 & -2 \end{pmatrix}$$

3.
   (a) $(3, -2, 1)$
   (b) $x_1 = t$, $x_2 = 0.5 - 2t$, $x_3 = t$ where $t \in R$ (real number)
   (c) no solution

4.

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 0 & 1/3 & 1/3 \\ 0 & 1/3 & -2/3 \\ -1 & 2/3 & -1/3 \end{pmatrix}$$