Derivative Securities (FINA 3203)
Solutions to Problem Set 2

Question 1: Basic Concepts on Forward Curves (2/10)

SOLUTION:

(1) A forward curve consists of a series of forward prices plotted together plotted against their corresponding terms to maturity, reflecting a range of today's tradable values for specified dates in the future; it represents a term structure of forward prices.

(2) If the forward curve is ascending (upward-sloping), i.e. forward prices increase with term to maturity, it is called contango.

(3) If the forward curve is descending (downward-sloping), i.e. forward prices decrease with term to maturity, is is called backwardation.

(4) Commodity holdings provide an extra nonpecuniary gain which is called convenience yield.

Question 2: Basic Concepts on Currency Forward Contracts (2/10)

SOLUTION:

(1) The forward exchange rate is the pre-determined exchange rate at which a party agrees to exchange one currency for another at a pre-determined future date when it enters into a foreign exchange forward contract with the other party.

(2) The currency carry trade strategy in the foreign exchange market is a trading strategy which borrows money at a lower interest rate currency and lend money at a higher interest rate currency.

(3) The covered interest rate parity (CIP) refers to the theoretical non-arbitrage relationship between interest rate differentials, forward rate, and spot rate. It leads to the pricing formula of forward exchange rate.

(4) The uncovered interest rate parity (UIP) refers to a hypothesis that the difference in interest rates between two countries is equal to the expected change in exchange rates between the two countries' currencies. If UIP holds, it means that the carry trade or reverse carry trade has zero profits on average.

(Optional) Question 3: Non-Arbitrage and Transaction Costs

SOLUTION:

(i) The forward ask price violates the no arbitrage bound and hence there is an arbitrage opportunity. The following strategy represent an arbitrage: (a) long the forward, (b) short the stock, (c) invest the proceeds of the stock sale at the T-Bill rate.

Table 1: Cash Flows of Trading Strategy

Strategy Time	0	6 Month
Long forward	0	$S_{6mth} - F^{a}_{0,6mth} = S_{6mth} - \$20.45$
Short stock	$S^{b}_{0} = \$20$	$-S_{6mth}$
Long T-bills	$-S^{b}_{0} = -\$20$	$S^{b}_{0}e^{r^{\ell}\times\frac{6}{12}} = \$20.51$
Aggregate	0	$\$20.51 - \$20.45 = \$0.06$

Note on short selling: you borrow the stock from a lender at date zero and immediately sell it in the spot market to receive S^{b}_{0}. In 6 months you receive a share from your long forward counterparty and you return this share to the stock lender to close out your short stock position.

(ii) There may be additional costs associated with short selling NDV stock. Specifically, you will earn the rebate rate on the proceeds which is usually lower than T-Bill rate. Depending on how much less than the T-Bill rate you earn, the above strategy may no longer represent an arbitrage.

(iii) The forward bid price violates the no arbitrage bound and hence there is an arbitrage opportunity. The following strategy represent an arbitrage: (a) short the forward, (b) buy the stock, (c) borrow at LIBOR to finance the stock purchase.

Table 2: Cash Flows of Trading Strategy

Strategy Time	0	6 Month
Short forward	0	$F^{b}_{0,6mth} - S_{6mth} = \$20.81 - S_{6mth}$
Long stock	$-S^{a}_{0} = -\$20.10$	S_{6mth}
Short T-bills	$S^{a}_{0} = \$20.10$	$-S^{a}_{0}e^{r^{b}\times\frac{6}{12}} = -\$20.71$
Aggregate	0	$\$20.81 - \$20.71 = \$0.10$

(iv) Your answer does not change - the above strategy does not entail short selling.

Question 4: Futures Contracts (3/10)

SOLUTION: Note that the continuously-compounded interest rate is always annual rate and the day count convention is actual/365.

(i) Your Jan 27 margin balance is $10 \times \$13,750 = \$137,500$. On Jan 28 your profit is

$$10 \times 10 \times (8,336.5 - 8,136) = \$20,050.$$

Your margin account pays the risk-free interest of 1%. Your Jan 28 margin balance is

$$\$137,500 \times e^{0.01 \times 1/365} + \$20,050 = \$157,554.$$

On Jan 29 your profit is

$$10 \times 10 \times (8,112 - 8,336.50) = -\$22,450,$$

and so your Jan 30 margin balance is

$$\$157,554 \times e^{0.01 \times 1/365} - \$22,450 = \$135,108.$$

The maintenance margin for 10 contracts is $10 \times \$11,000 = \$110,000$. The value of your margin account is above the maintenance margin on all dates and hence you do not face any margin calls.

(ii) Your Jan 27 margin balance is $5 \times \$13,750 = \$68,750$. On Jan 28 your profit is

$$-5 \times 10 \times (8,336.5 - 8,136) = -\$10,025.$$

Your margin account pays the risk-free interest of 1%. Your Jan 28 margin balance is

$$\$68,750 \times e^{0.01 \times 1/365} - \$10,025 = \$58,727$$

On Jan 29 your profit is

$$-5 \times 10 \times (8,112 - 8,336.50) = \$11,225$$

and so your Jan 30 margin balance is

$$\$58,727 e^{0.01 \times 1/365} + \$11,225 = \$69,954$$

The maintenance margin for 5 contracts is $5 \times \$11,000 = \$55,000$. The value of your margin account is above the maintenance margin on all dates and hence you do not face any margin calls.

(iii) From class we know that $F_{t,T} = S_t e^{(r-\delta)(T-t)}$. Thus,

$$\ln\left(\frac{F_{t,T}}{S_t}\right) = (r-\delta)(T-t)$$

and also

$$\delta = r - \frac{1}{T-t} \ln\left(\frac{F_{t,T}}{S_t}\right).$$

Here

$$T - t = (2 + 28 + 20)/365 = 50/365.$$

Thus, we have

$$0.01 - \frac{1}{50/365} \ln\left(\frac{8112}{8149.01}\right) = 0.0432.$$

Therefore, the dividend yield implied by the prices is 4.32%.

(iv) The index value is larger than the futures price because the interest rate is lower than the dividend yield.

Question 5: Commodity Forward Contracts with Lease Rate (3/10)

SOLUTION:

(i) When there is an active leasing market, then the non-arbitrage condition leads to the pricing formula:

$$F_{0,T} = S_0 e^{(r - q_{0,T})T}, \quad \text{with } T = 0.25, 0.5, 0.75, 1.$$

where $q_{0,T}$ is the leasing rate with time to maturity T. Thus,

$$q_{0,T} = r - \frac{1}{T} \ln\left(\frac{F_{0,T}}{S_0}\right) = 0.02 - \frac{1}{T} \ln\left(\frac{F_{0,T}}{\$32}\right).$$

Therefore, for $T = 0.25$, $q_{0,0.25} = 0.02 - \frac{1}{0.25} \ln\left(\frac{31.37}{32}\right) = 9.95\%$; for $T = 0.5$, $q_{0,0.5} = 0.02 - \frac{1}{0.5} \ln\left(\frac{30.75}{32}\right) = 9.97\%$; for $T = 0.75$, $q_{0,0.75} = 0.02 - \frac{1}{0.75} \ln\left(\frac{30.14}{32}\right) = 9.98\%$; for $T = 1$, $q_{0,0.75} = 0.02 - \frac{1}{1} \ln\left(\frac{29.54}{32}\right) = 10\%$.

(ii) If there is no leasing market or carry cost, the pricing formula becomes

$$F_{0,T} = S_0 e^{(r - c_{0,T})T}.$$

Similarly, we can see the implied convenience yields are

$$c_{0,0.25} = 9.95\%; \quad c_{0,0.5} = 9.97\%; \quad c_{0,0.75} = 9.98\%; \quad \text{and } c_{0,1} = 10\%.$$

(iii) The forward curve is downward-sloping, then this is an example of backwardation.

(Optional) Question 6: Commodity Forward Contracts with Convenience Yield

SOLUTION:

(1) First, consider the cash-and-carry strategy. The following is its payoff compared with an actual bond position. Note that c' is the investor's holding convenience yield. The non-arbitrage

Table 3: Payoffs and Present Values

	Holdings	PV at 0	Payoffs at T
Portfolio #1	Long bond	$S_0e^{-c'T}$	$S_0e^{(r-c')T}$
	Total: actual bond	$S_0e^{-c'T}$	$S_0e^{(r-c')T}$
Portfolio #2	Long commodity	$S_0e^{-c'T}$	S_T
	Short forward	0	$F_{0,T} - S_T$
	Total: synthetic bond	$S_0e^{-c'T}$	$F_{0,T}$

condition implies that

$$\underbrace{S_0e^{(r-c')T}}_{\text{payoff \#1}} \geq \underbrace{F_{0,T}}_{\text{payoff \#2}}$$

Second, consider the reverse cash-and-carry strategy. The following is its payoff compared with an actual bond position. Note that $c^{''}$ is the convenience yield that the investor is required to compensate the lender.

Table 4: Payoffs and Present Values

	Holdings	PV at 0	Payoffs at T
Portfolio #1	Short bond	$-S_0e^{-c^{''}T}$	$-S_0e^{(r-c^{''})T}$
	Total: actual bond	$-S_0e^{-c^{''}T}$	$-S_0e^{(r-c^{''})T}$
Portfolio #2	Short commodity	$-S_0e^{-c^{''}T}$	$-S_T$
	Long forward	0	$S_T - F_{0,T}$
	Total: synthetic bond	$-S_0e^{-c^{''}T}$	$-F_{0,T}$

The non-arbitrage condition implies that

$$\underbrace{-S_0e^{(r-c^{''})T}}_{\text{payoff \#1}} \geq \underbrace{-F_{0,T}}_{\text{payoff \#2}}$$

Therefore, the non-arbitrage region for forward price is

$$S_0e^{(r-c')T} \geq F_{0,T} \geq S_0e^{(r-c'')T}.$$

(2) When $c = c' = c''$, the non-arbitrage forward price is

$$F_{0,t} = S_0e^{(r-c)T}.$$

(3) Suppose, instead of just one investor, there are many investors $i = 1, \cdots, I$ such that $c_i = c_i' = c_i''$ with each $i = 1, \cdots, I$. Then, the non-arbitrage (fair) forward price for each investor i is

$$F_{0,T}^i = S_0 e^{(r-c_i)T}.$$

Because $c_1 < \cdots < c_I$, the largest forward price offered by the dealer is $F_{0,T}^1$ and the smallest forward price offered by the dealer is $F_{0,T}^I$.