ECON3113
Microeconomic Theory I

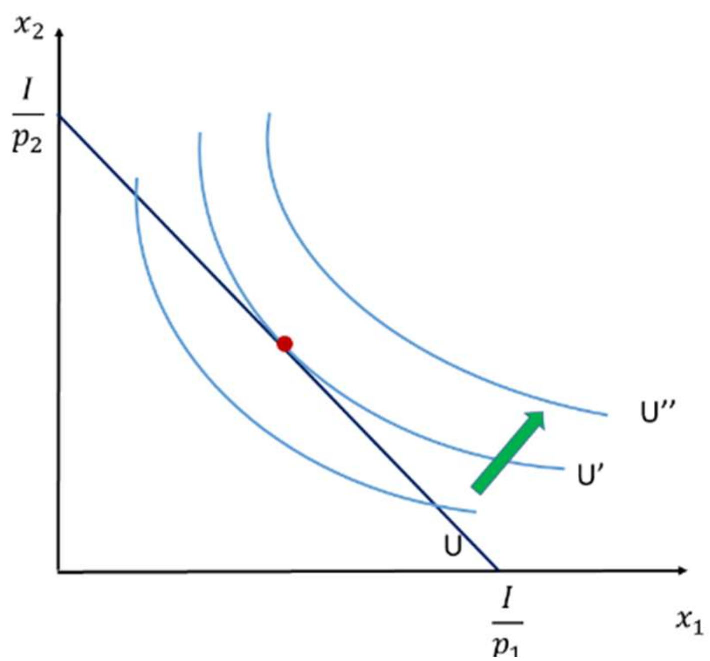
Tutorial #6: Demand Analysis

Today's tutorial

- Re-Cap on the theoretical framework
- Applications
  - Quasi-linear utility function
  - Stone-Geary utility function

Constrained utility maximisation: the framework

- Superposing the budget line with the indifference curves.
- Look for the bundle/point in the budget set that lies on the indifference curve with the highest utility.



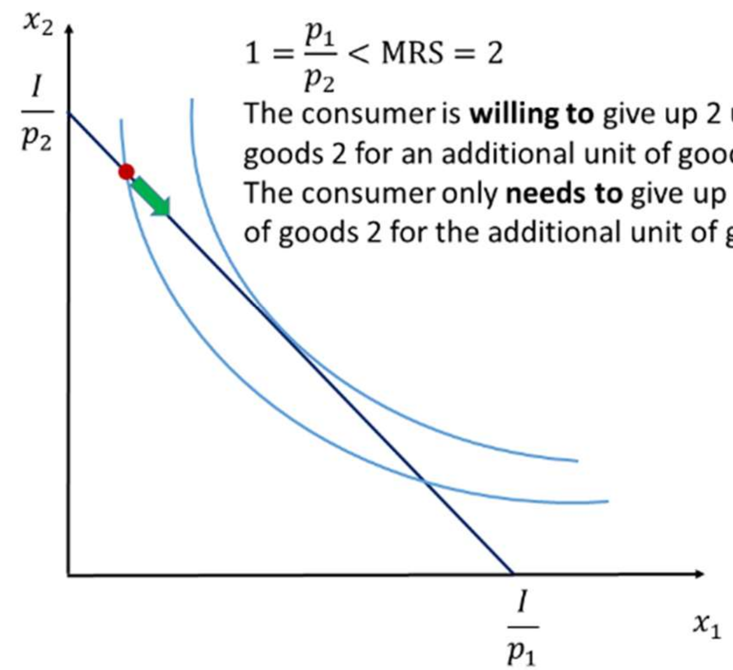
- We have:
  - $U(x, y)$
  - $I = P_x x + P_y y$
- Affordable bundles on or inside the budget constraint
- Tangency at: $MRS = \frac{P_x}{P_y}$

- Note: Limitations of this approach in lecture notes:
  - Corner solutions
  - Tangency not always optimal

Constrained utility maximisation: the framework

- Intuition of why the tangency condition works

- What bundle would make the consumer willing to stay put?
- Start with any bundle $(x_1, x_2) > (0, 0)$. If she wants to increase his consumption of goods 1 by one unit,
  - the amount of goods 2 she is *willing to* give up is MRS;
  - the amount of goods 2 she *has to* give up is $p_1 \times \frac{1}{p_2}$.
- She wants to consume more of goods 1 if $\frac{p_1}{p_2} < MRS$.



The Quasi-Linear utility function

- Suppose the consumer has a quasi-linear utility function:

$$u(x_1, x_2) = v(x_1) + x_2,$$

for some strictly increasing and strictly concave function v.

- The MRS is given by

$$MRS = \frac{\partial u / \partial x_1}{\partial u / \partial x_2} = v'(x_1),$$

so it is strictly decreasing in x_1 but independent of x_2.

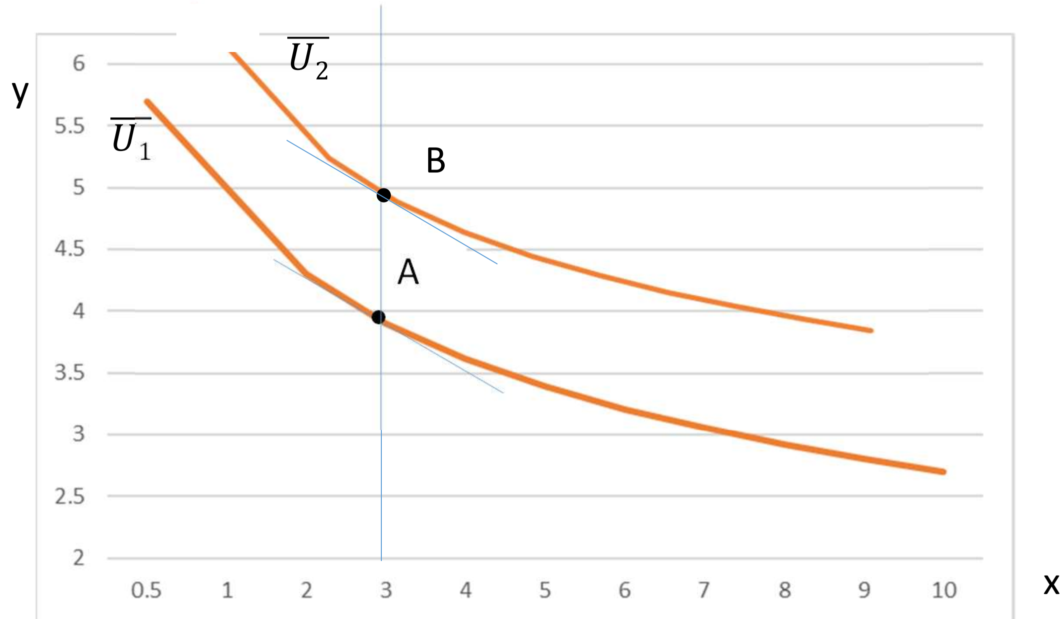
  - Strict concavity of v implies DMRS.

- A function f is strictly concave in x and y if for every $\alpha \in (0,1)$:

$$f((1-\alpha)x + \alpha y) > (1-\alpha)f(x) + \alpha f(y)$$

The Quasi-Linear Utility Function: Example

Consider the function $U(x, y) = y + \ln(x)$



- We have MRS $= \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$
- $= \frac{1}{x}$
- So we have DMRS
- How much of y a consumer requires to compensate for giving up 1 unit of x depends <u>only</u> on how much x the consumer already has
- MRS is constant for different indifference curves at equal points on the x axis (eg A and B)

The Quasi-Linear Utility Function: Example

- Consider the function $U(x, y) = y + \ln(x)$
- To find the demand curves for x and y:
- We have MRS $= \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{1}{x} = \frac{P_x}{P_y}$
- ie $x^* = \frac{P_y}{P_x}$
- Then $I = P_x x + P_y y = P_x \frac{P_y}{P_x} + P_y y$
- ie $y^* = \frac{I}{P_y} - 1$

Sensitivity of demand to $\boldsymbol{P_x}$, $\boldsymbol{P_y}$ and $\boldsymbol{I}$:

Good	P_x	P_y	I
x	Negative	Positive	Independent
y	Independent	Negative	Positive

- When $\boldsymbol{P_x}$ changes, all of the change in demand is accounted for by changes in demand for x
  - What is causing this?
- y is a normal good; x is not

The homogeneity of demand functions

Theorem

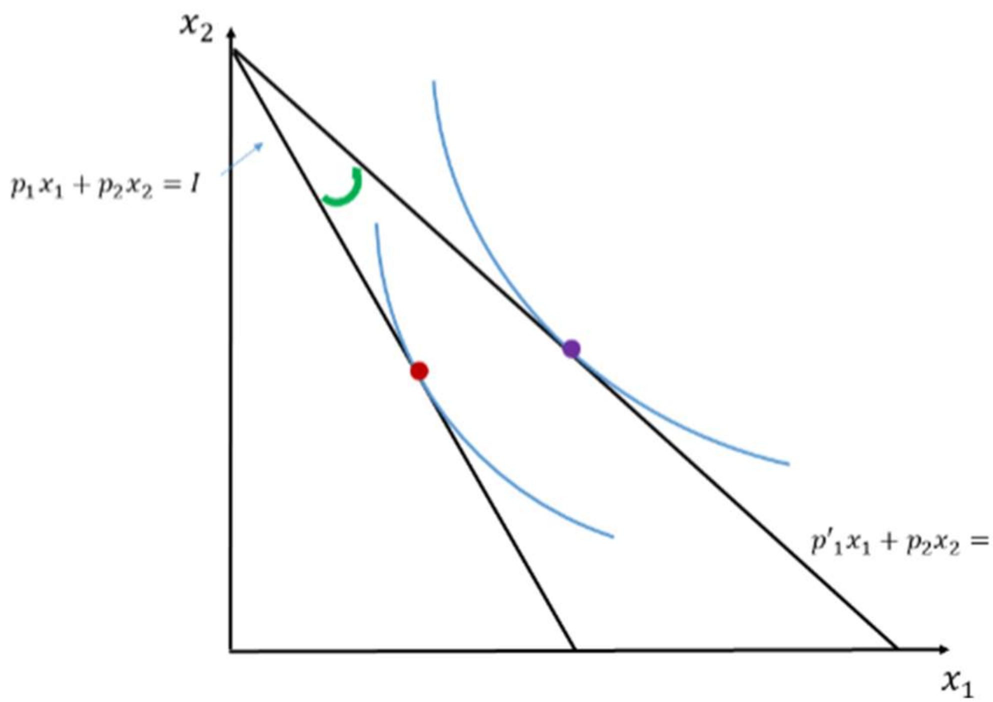
The demand functions are **homogeneous of degree zero**. *That is,* $x_i(\lambda p_1, \ldots, \lambda p_n, \lambda I) = x_i(p_1, \ldots, p_n, I)$ *for all* $\lambda > 0$.

<u>Examples:</u>

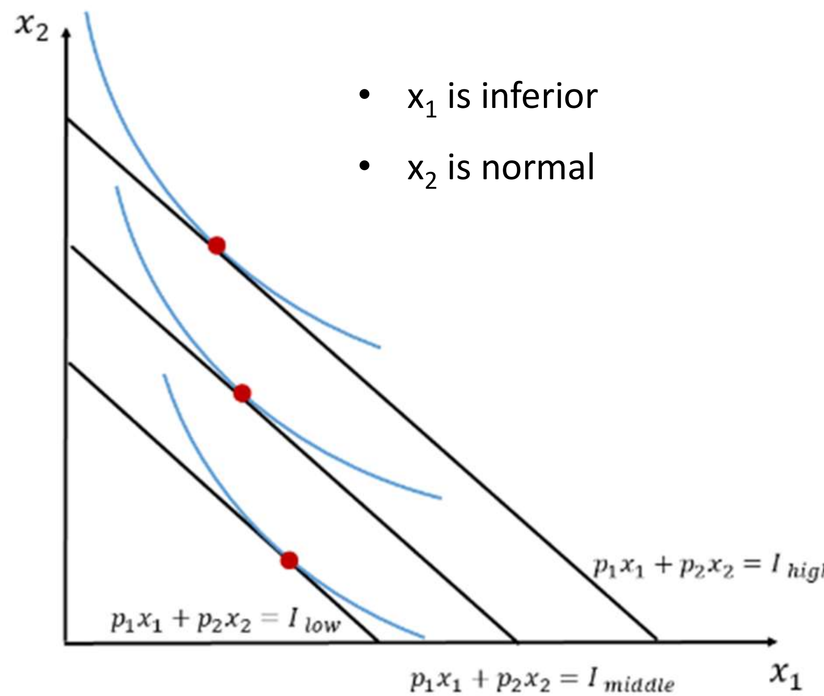
- For given I, P_x, P_y:
- Cobb Douglas $U(x,y) = x^{\alpha} y^{1-\alpha}$ $\quad x^* = \alpha \frac{I}{P_x} \quad y^* = (1-\alpha)\frac{I}{P_y}$
  - x^*, y^* invariant to scaling P_x, P_y, I by λ
- Quasi-Linear $U(x,y) = y + \ln(x)$ $\quad x^* = \frac{P_y}{P_x} \quad y^* = \frac{I}{P_y} - 1$
  - x^*, y^* invariant to scaling P_x, P_y, I by λ

Sensitivity of demand to changes in prices and income

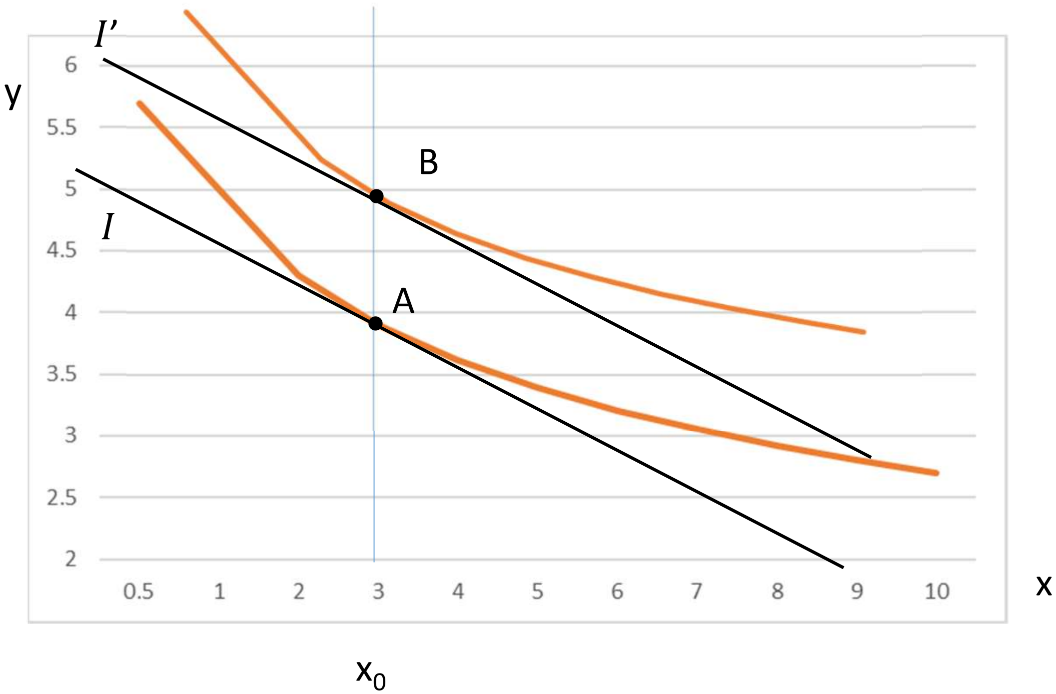
Decrease in price of x_1



Increase in income



Example: Quasi-linear utility function

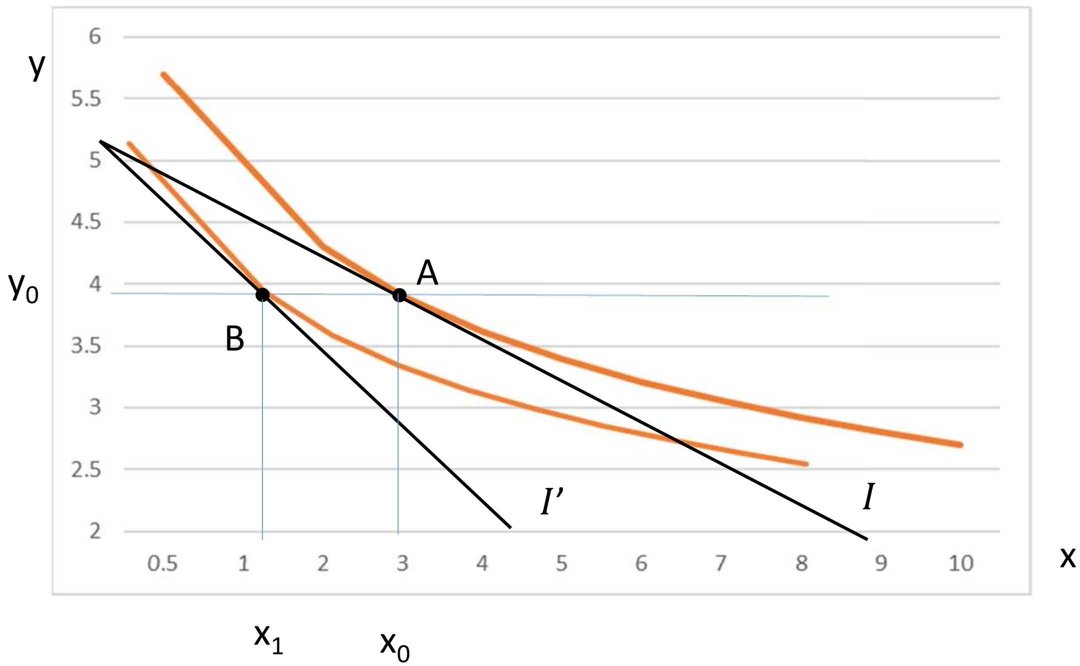


- Increase in income from I to I'
- We have:

$$x^* = \frac{P_y}{P_x} \qquad y^* = \frac{I}{P_y} - 1$$

- So demand for x doesn't change, demand for y increases by $\frac{1}{P_y}$

Example: Quasi-linear utility function

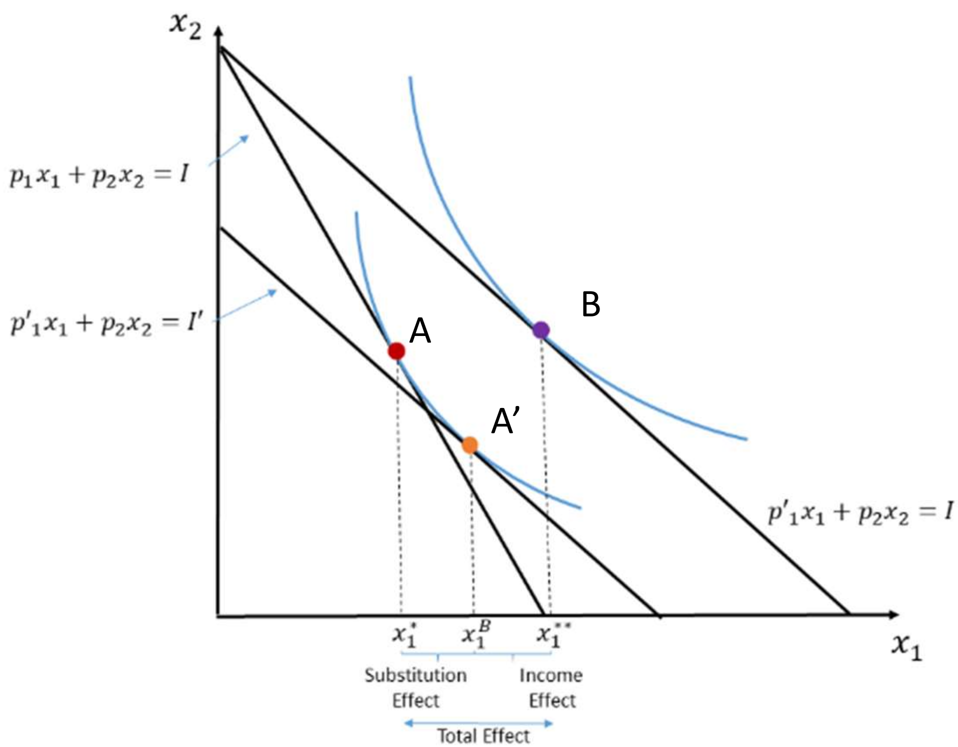


- Increase in the price of x
- We have:

$$x^* = \frac{P_y}{P_x} \qquad y^* = \frac{I}{P_y} - 1$$

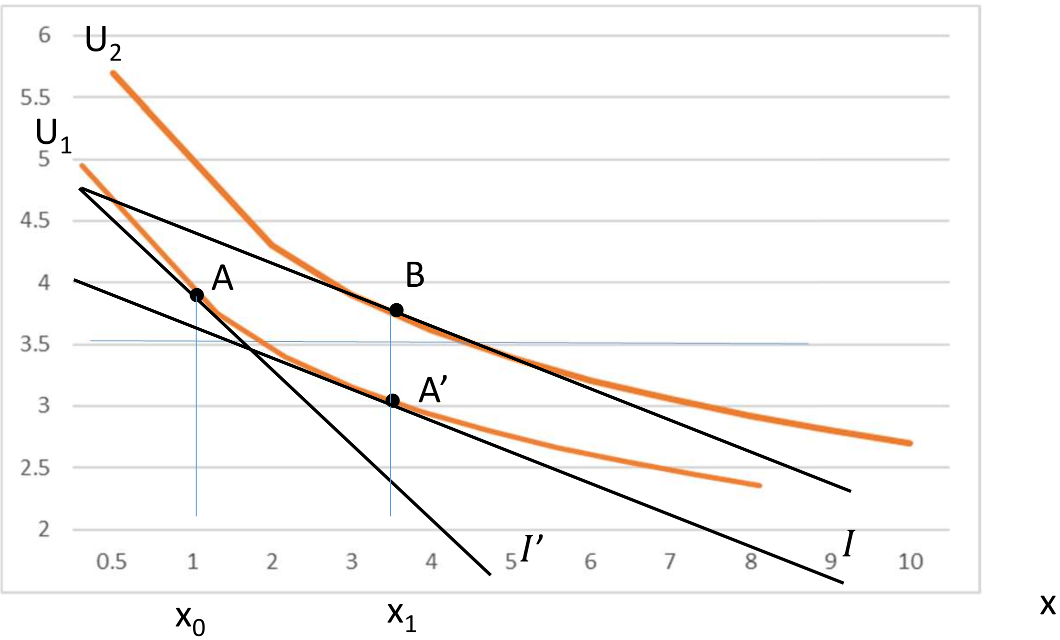
- So demand for y doesn't change, demand for x decreases by $\frac{P_y}{P_x^2}$

Decomposing a price change into income and substitution effect



- Given a fall in the price of x_1 from p_1 to p_1':
- Equilibrium moves from A to B
- We can de-compose the move into two parts:
  - Rotate the budget constraint around existing indifference curve
    - From A to A'
    - The substitution effect
    - With DMRS the substitution effect is always negative
  - Shift the budget constraint to the new budget constraint
    - from A' to B
    - The income effect

Example: Quasi-linear utility function



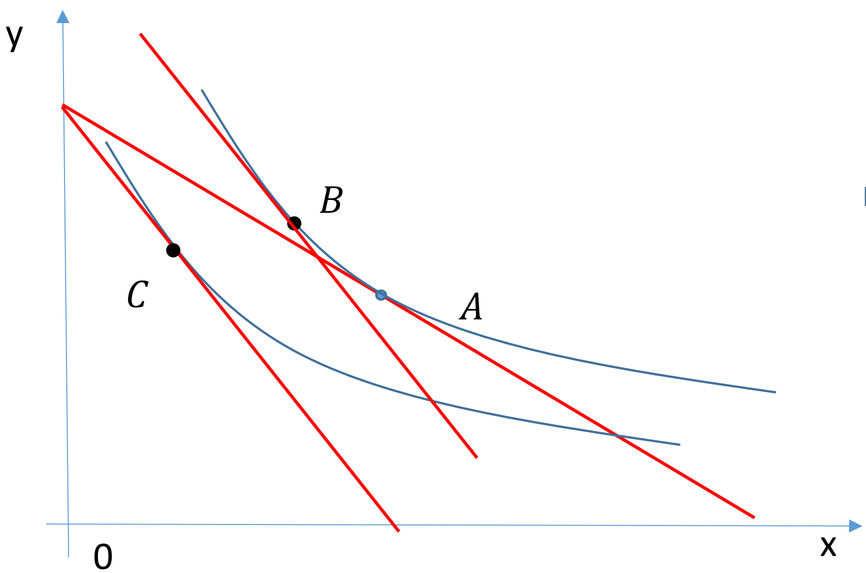
- Decrease in the price of x
- We have:

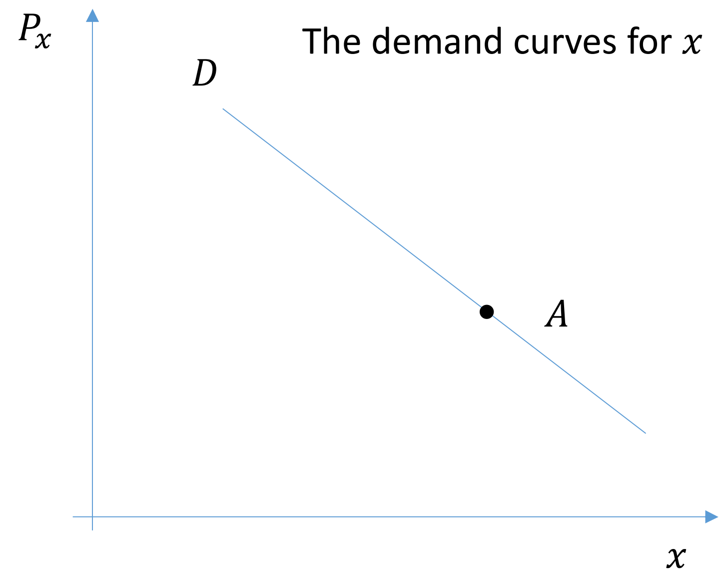
$$x^* = \frac{P_y}{P_x} \qquad y^* = \frac{I}{P_y} - 1$$

- Rotation around U_1 from A to A'
  - Demand for x increases from x_0 to x_1
  - Substitution effect
- Shift to the new budget constraint
  - Demand for x doesn't change!
  - There is no income effect for x with this utility function

Hicksian and Compensated demand curves

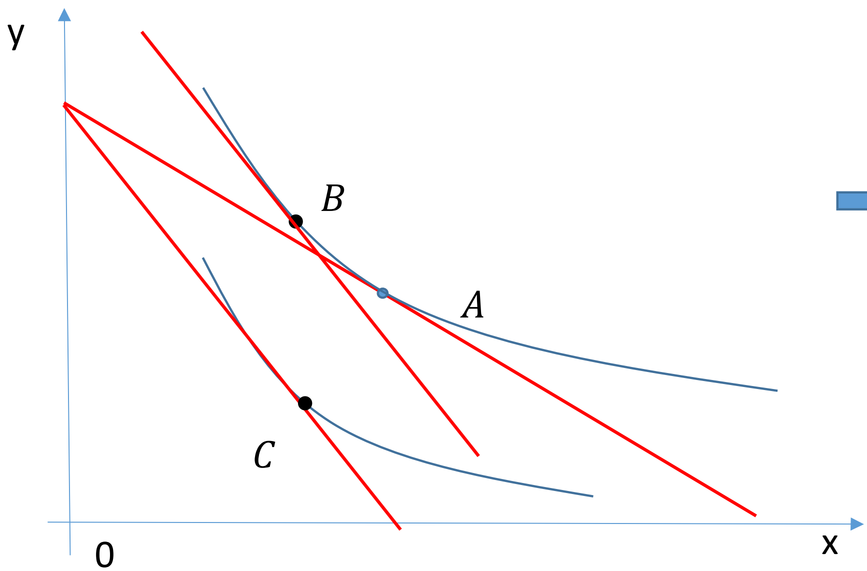
- Suppose that the price of good x doubles. What happens to equilibrium consumption?
- What can we say about goods x and y? Is x normal or inferior? Are they complements or substitutes?

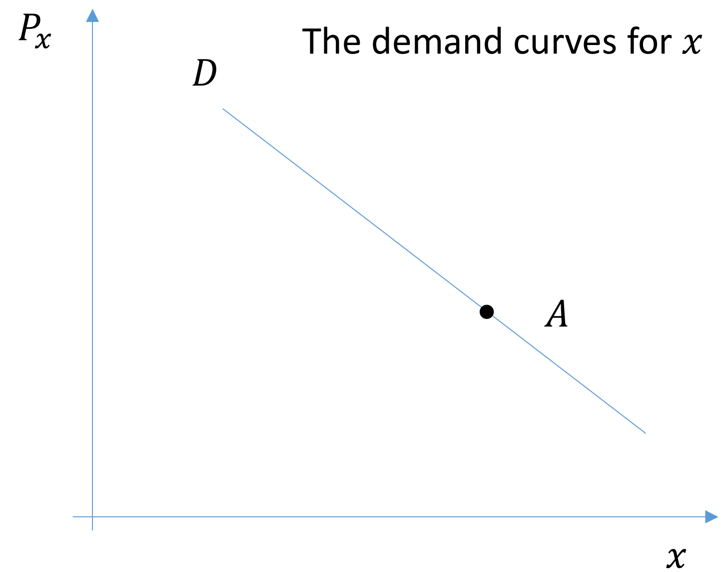




Hicksian and Compensated demand curves

- Now with x an inferior good





Hicksian and Compensated demand curves

- In the case of the quasi-linear utility function, we have:
- $x^c(P_x, P_y, \bar{U}) = x(P_x, P_y, I)$
- $y^c(P_x, P_y, \bar{U}) =$
- $y(P_x, P_y, I) =$

Case Study: the Stone Geary utility function

- Consider the utility function:

$$U(x, y) = (x - x_0)^{\alpha} y^{(1-\alpha)}$$

- In this function, x_0 represents the consumption of good x that a person needs to stay alive
- Note that when $x = x_0$, $U = 0$
- Let $z = x - x_0$
- Then $U(z, y) = z^{\alpha} y^{(1-\alpha)}$
- $I = P_x x + P_y y = P_x(\mathrm{z} + x_0) + P_y y$
- $I - P_x x_0 = P_x z + P_y y$
- $\frac{dy}{dz} = -\frac{P_x}{P_y}$

Case Study: the Stone Geary utility function

- We have:

$$U(z,y) = z^{\alpha} y^{(1-\alpha)}$$

- $\frac{\partial U}{\partial z} =$
- $\frac{\partial U}{\partial y} =$
- MRS $=$

- We have $\frac{\partial MRS}{\partial z} =$

- And so the utility function has DMRS with respect to z (and also with respect to x)

Case Study: the Stone Geary utility function

- To find the demand functions for x and y:
- MRS $=$
- $y =$
- Substitute into the budget constraint:
- $I - P_x x_0 = P_x z + P_y y$

Case Study: the Stone Geary utility function

To give $x^* = \alpha \frac{I}{P_x} + (1-\alpha)x_0$

- Note that x is a normal good (so we do have an income effect)

Case Study: the Stone Geary utility function

- To find the demand function for y:
- MRS $=$
- $y =$

Case Study: the Stone Geary utility function

- And so we have (uncompensated) demand functions for x and y
- $x^{*} = \alpha \frac{I}{P_x} + (1-\alpha)x_0\ y$
- $y^{*} = \frac{(1-\alpha)}{P_y}[I - P_x x_0]$
- Note that for $x_0 = 0$, these are Cobb-Douglas demand functions

Case Study: the Stone Geary utility function

- Exercise:
- Suppose we have:

$$U(x,y) = (x - x_0)^{\alpha} y^{(1-\alpha)}$$

- $\alpha = 0.25, x_0 = 2, I = 10, P_x = 1, P_y = 2$
- Questions:

1) Find the equilibrium consumption of x and y and the utility number generated by this consumption
2) Find the change in demand for x if its price rises to $P_x = 2$
3) Find the compensated demand curves for x and y
4) Check that in equilibrium, compensated and uncompensated demand for the two goods are equal
5) Decompose the total change in demand for x into a substitution effect and an income effect

Case Study: the Stone Geary utility function

1) Find the equilibrium consumption of x and y, and the utility number generated by this consumption

- $\alpha = 0.25, x_0 = 2, I = 10, P_x = 1, P_y = 2$
- $x^* = \alpha \frac{I}{P_x} + (1-\alpha)x_0 \ y$
- $y^* = \frac{(1-\alpha)}{P_y}[I - P_x x_0]$
- $U(x, y) = (x - x_0)^\alpha y^{(1-\alpha)}$

Case Study: the Stone Geary utility function

2) Find the change in demand for x if its price rises to $P_x = 2$

- $\alpha = 0.25, x_0 = 2, I = 10, P_x = 1, P_y = 2$
- $x^* = \alpha \frac{I}{P_x} + (1-\alpha)x_0\ y$
- $\frac{\partial x^*}{\partial P_x} =$
- Then change in demand for $x =$
- So demand for x falls by 1.25

Case Study: the Stone Geary utility function

3) Find the compensated demand curves for x and y

- MRS $= \frac{P_x}{P_y}$

- To give $y_c^* = \left(\frac{P_x}{P_y}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \overline{U}$

Case Study: the Stone Geary utility function

3) Find the compensated demand curves for x and y

- $\text{MRS} = \frac{P_x}{P_y}$

- To give $x_c^* = x_0 + \left(\frac{P_y}{P_x}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \overline{U}$

Case Study: the Stone Geary utility function

4) Check that in equilibrium, compensated and uncompensated demand for the two goods are equal

- We have:
- $\alpha = 0.25, x_0 = 2, I = 10, P_x = 1, P_y = 2, \overline{U} = 2.71, x^* = 4, y^* = 3$
- $x_c^* = x_0 + \left(\frac{P_y}{P_x}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \overline{U}$

Case Study: the Stone Geary utility function

4) Check that in equilibrium, compensated and uncompensated demand for the two goods are equal

- We have:
- $\alpha = 0.25, x_0 = 2, I = 10, P_x = 1, P_y = 2, \bar{U} = 2.71, x^* = 4, y^* = 3$
- $y_c^* = \left(\frac{P_x}{P_y}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \bar{U}$

Case Study: the Stone Geary utility function

5) Decompose the total change in demand for x into a substitution effect and an income effect

We have:

- $x_c^* = x_0 + \left(\frac{P_y}{P_x}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \bar{U}$
- $\frac{\partial x_c^*}{\partial P_x} = (-)(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{P_y}{P_x}\right)^{-\alpha} P_y P_x^{-2}\, \bar{U}$
- $= -\,(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} P_y^{1-\alpha}\, P_x^{\alpha-2}\, \bar{U}$
- Then change in x_c^* is:

Case Study: the Stone Geary utility function

5) Decompose the total change in demand for x into a substitution effect and an income effect

We have:

- $\alpha = 0.25, x_0 = 2, I = 10, P_x = 1, P_y = 2, \bar{U} = 2.71, x^* = 4, y^* = 3$

- Change in x_c^* is
- This is the substitution effect

Case Study: the Stone Geary utility function

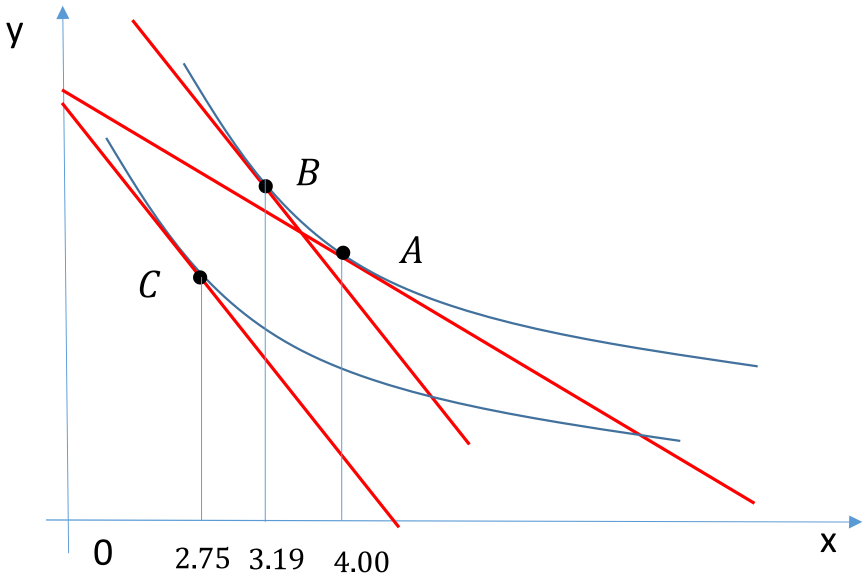
5) Decompose the total change in demand for x into a substitution effect and an income effect

We have:

- $Total\ effect\ of\ price\ change\ on\ demand\ for\ x = substitution\ effect + income\ effect$
- In our case:

- Therefore the income effect is

Case Study: the Stone Geary utility function



- Summary:
- $x^* = \alpha \frac{I}{P_x} + (1-\alpha)x_0$
- $y^* = \frac{(1-\alpha)}{P_y}[I - P_x x_0]$
- $x_c^* = x_0 + \left(\frac{P_y}{P_x}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \overline{U}$
- $y_c^* = \left(\frac{P_x}{P_y}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \overline{U}$

- Total change in demand for x given price change = -1.25
- Substitution effect = -0.81
- Income effect = -0.44
- x is a normal good
- When the price of x rises, demand for y falls
- Uncompensated demand for x does not depend on the price of y