

## Externality and Public Goods

# Externality

- Definition of externality: one agent's action (choice variable) goes into the other agent's payoff function.
  - ▶ between consumers
  - ▶ between firms
  - ▶ between a consumer and a firm
- Each economic agent solve its own optimization problem without considering its externality imposed on other agents.
- Non-cooperative/Nash equilibrium (NE) or competitive allocation is different from the social optimal allocation.
- Find the social optimal allocation
  - ▶ by solving Pareto efficient allocation
  - ▶ by solving the decision after merger
  - ▶ Coase theorem: what matters is property right.

# Externality

- Example: smoking roommate

$$\begin{cases} u_1(x_1, y_1), & x_1 + y_1 = w_1 \\ u_2(x_2, y_2), & x_2 + y_2 = w_2 \end{cases}$$
$$\Rightarrow \begin{cases} u_1(x_1, y_1, x_2), & x_1 + y_1 = w_1 \\ u_2(x_2, y_2), & x_2 + y_2 = w_2 \end{cases}$$

Assume  $p_x = p_y = 1$ , MRS conditions

$$MRS_1 = \frac{\frac{\partial u_1}{\partial x_1}}{\frac{\partial u_1}{\partial y_1}} = \frac{p_x}{p_y} = 1, \quad MRS_2 = \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}} = 1.$$

# Externality

- Solve for Pareto efficient allocation

$$\max_{x_1, x_2, y_1, y_2} u_1(x_1, y_1, x_2), \quad \text{s.t.} \quad \begin{cases} u_2(x_2, y_2) \geq \bar{u}_2 \\ x_1 + y_1 + x_2 + y_2 = w_1 + w_2 \end{cases}$$

$$\mathcal{L} = u_1(x_1, y_1, x_2) + \lambda(u_2(x_2, y_2) - \bar{u}_2) + \mu(w_1 + w_2 - x_1 - y_1 - x_2 - y_2).$$

$$\frac{\frac{\partial u_1}{\partial x_2}}{\frac{\partial u_1}{\partial y_1}} + \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}} = \frac{\frac{\partial u_1}{\partial x_1}}{\frac{\partial u_1}{\partial y_1}} = MRS_1 = 1$$

$$MRS_2 = \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}} = 1 - \frac{\frac{\partial u_1}{\partial x_2}}{\frac{\partial u_1}{\partial y_1}} > 1,$$

Suppose that  $\frac{\partial u_1}{\partial x_2} < 0$  (negative externality),  $\frac{\partial u_2}{\partial x_2}$  is decreasing in  $x_2$ ,  $x_2$  shall be lower than competitive equilibrium level.

# Externality

- Coase theorem

- ▶ If trading of the externality source is possible and there are sufficiently low transaction costs, bargaining will lead to a Pareto optimal outcome regardless of the initial allocation of property.
- ▶ The level of pollution is independent of who initially owns the rights.
- ▶ Smoking roommate example. Transfer between agents close the gap between private marginal cost and social marginal cost.

# Externality

- Fishery and steel firm example in Varian Ch 34.
  - ▶ Steel firm profit-maximization problem ( $x$  indicates pollution)

$$\max_{s,x} \pi_s(s, x) = p_s s - c_s(s, x)$$

$$\frac{\partial c_s}{\partial s} > 0, \quad \frac{\partial c_s}{\partial x} < 0$$

- ▶ Fishery's profit maximization problem

$$\max_f \pi_f(f, x) = p_f f - c_f(f, x)$$

$$\frac{\partial c_f}{\partial s} > 0, \quad \frac{\partial c_f}{\partial x} > 0$$

- ▶ First-order conditions

$$p_s = \frac{\partial c_s}{\partial s}, \quad p_f = \frac{\partial c_f}{\partial f}, \quad 0 = \frac{\partial c_s}{\partial x}$$

# Externality

- Solution
  - ▶ Pigouvian tax
  - ▶ assign property right
  - ▶ internalize (merger)
- Efficiency allocation

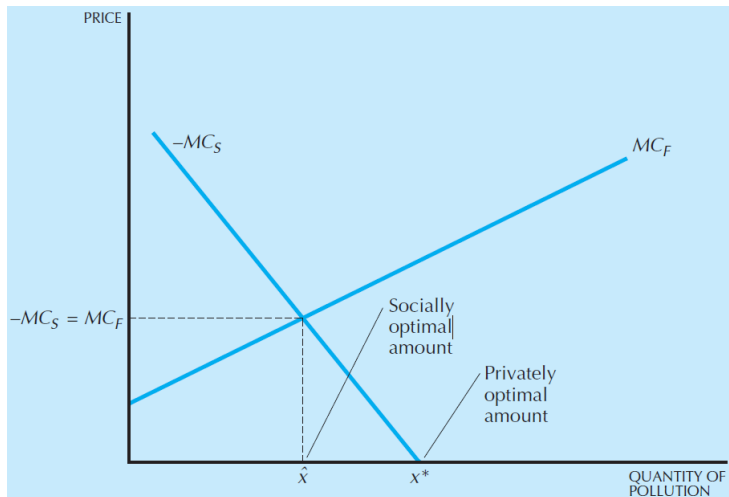
$$\begin{aligned}\max_{s,x,f} \Pi(s, f, x) &= \pi_s(s, x) + \pi_f(f, x) \\ &= p_s s - c_s(s, x) + p_f f - c_f(f, x)\end{aligned}$$

$$p_s = \frac{\partial c_s}{\partial s}, \quad p_f = \frac{\partial c_f}{\partial f}, \quad 0 = \frac{\partial c_s}{\partial x} + \frac{\partial c_f}{\partial x}$$

- The social cost of pollution is not taken into account at the competitive allocation.

# Externality

- Competitive allocation and social optimal allocation





# Public Goods

- Public goods

- ▶ Non-exclusive: a good that must be provided in the same amount to all consumers.
- ▶ Non-rival: one consumer's consumption does not prevent others from enjoying it.

		Exclusive/excludable	
		Yes	No
Rival	Yes	food, automobile houses, energy	clean air, radio spectrum international ocean
	No	bridge, digital content intellectual property	national defense, public health judicial system, team output

# Public Goods

- Free rider problem in public good provision
  - ▶ Two roommates are considering to buy a TV.
  - ▶ Each willing to pay  $r_1 = r_2 = 100$  for the TV. The price of TV is  $c = 150$ .
  - ▶ They are considering whether each pays  $g_1 = g_2 = 75$  to buy the TV. Each get a payoff of 25.

		<b>2</b>	
		Buy	Not
<b>1</b>	Buy	25, 25	-50, 100
	Not	100, -50	0, 0

# Public Goods

- When to provide a public good?
  - ▶ necessary condition (Pareto improvement):  $r_1 + r_2 > c = g_1 + g_2$
  - ▶ sufficient condition:  $r_1 > g_1$  and  $r_2 > g_2$
- Private provision of public good
  - ▶ If both tell the truth about  $r_1$  and  $r_2$ . Any payment arrangement with  $r_1 > g_1$  and  $r_2 > g_2$  (and  $g_1 + g_2 = c$ ) will be accepted by both roommate.
  - ▶ Lindahl pricing rule: share cost proportionally to based on reported valuation.
  - ▶ However, each individual has incentive to under-report.

# Public Goods

- One important role of government: provide public goods and goods with positive externality
  - ▶ education
  - ▶ public health
  - ▶ basic research
  - ▶ infra-structure
  - ▶ platform
  - ▶ institution and property right arrangement
- Inter-government coordination
  - ▶ global warming
  - ▶ over-fishing in international ocean
  - ▶ reducing nuclear weapon

# Public Goods

## ● Example 19.3

Two roommates contribute to buy a TV with value  $X = x_1 + x_2$ .  $y_i$  is roommate  $i$ 's private consumption ( $p_y = 1$ ).

$$\begin{cases} \max_{x_1, y_1} u_1(X, y_1), & \text{s.t. } p_X x_1 + y_1 = w_1 \\ \max_{x_2, y_2} u_2(X, y_2), & \text{s.t. } p_X x_2 + y_2 = w_2 \end{cases}$$

$$U_i(x_1 + x_2, y_i) = (x_1 + x_2)^{\frac{1}{2}} y_i^{\frac{1}{2}}, \quad i = 1, 2$$

$$p_x = 10, w_1 = w_2 = 300.$$

$$\begin{cases} MRS_1 = \frac{\frac{\partial u_1}{\partial X}}{\frac{\partial u_1}{\partial y_1}} = \frac{p_X}{p_y} \\ MRS_2 = \frac{\frac{\partial u_2}{\partial X}}{\frac{\partial u_2}{\partial y_2}} = \frac{p_X}{p_y} \end{cases} \Rightarrow \begin{cases} x_1 = BR_1(x_2) \\ x_2 = BR_2(x_1) \end{cases}$$

Solution to private provision of public good is a NE  $(x_1^{NE}, x_2^{NE})$ .

# Public Goods

- Pareto efficient level of public good provision

$$\max_{y_1, y_2, X} u_1(X, y_1), \quad \text{s.t.} \quad \begin{cases} u_2(X, y_2) \geq \bar{u}_2 \\ p_X X + y_1 + y_2 = w_1 + w_2 \end{cases}$$

Set up Lagrangian

$$\mathcal{L} = u_1(X, y_1) + \lambda(u_2(X, y_2) - \bar{u}_2) + \mu(w_1 + w_2 - p_X X - y_1 - y_2).$$

$$MRS_1 + MRS_2 = \frac{\frac{\partial u_1}{\partial X}}{\frac{\partial u_1}{\partial y_1}} + \frac{\frac{\partial u_2}{\partial X}}{\frac{\partial u_2}{\partial y_2}} = \frac{p_X}{p_y}.$$

This condition characterizes the Pareto optimal allocation.

- Free rider problem: because marginal utility is decreasing,  $MRS_1 = 1$ ,  $MRS_2 = 1$  imply a lower public good level than  $MRS_1 + MRS_2 = 1$ .

# Public Goods

- Competitive (NE) allocation

$$MRS_1 = MRS_2 = \frac{p_X}{p_Y}.$$

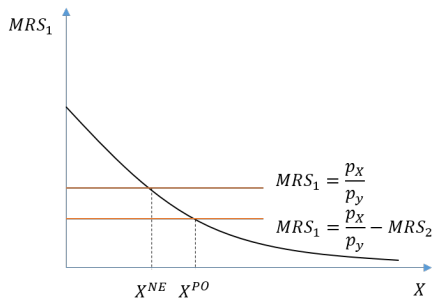
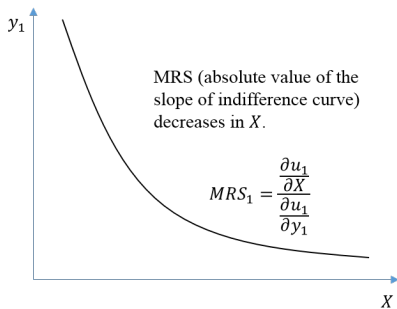
- Efficient allocation

$$MRS_1 + MRS_2 = \frac{p_X}{p_Y}.$$

- Free-rider problem: because marginal utility is decreasing,  $MRS_1 = \frac{p_X}{p_Y}$  implies a lower public good level than  $MRS_1 + MRS_2 = \frac{p_X}{p_Y}$ .

# Public Goods

- In general, we have  $X^{NE} < X^{PO}$





# Public Goods

- Public good production

Suppose now  $G = f(x_1 + x_2)$ ,  $y_1 = w_1 - x_1$ ,  $y_2 = w_2 - x_2$

Competitive equilibrium

$$\begin{cases} \max_{x_1} u_1(G, w_1 - x_1) = u_1(f(x_1 + x_2), w_1 - x_1) \\ \max_{x_2} u_2(G, w_2 - x_2) = u_2(f(x_1 + x_2), w_2 - x_2) \end{cases}$$

First-order condition yields

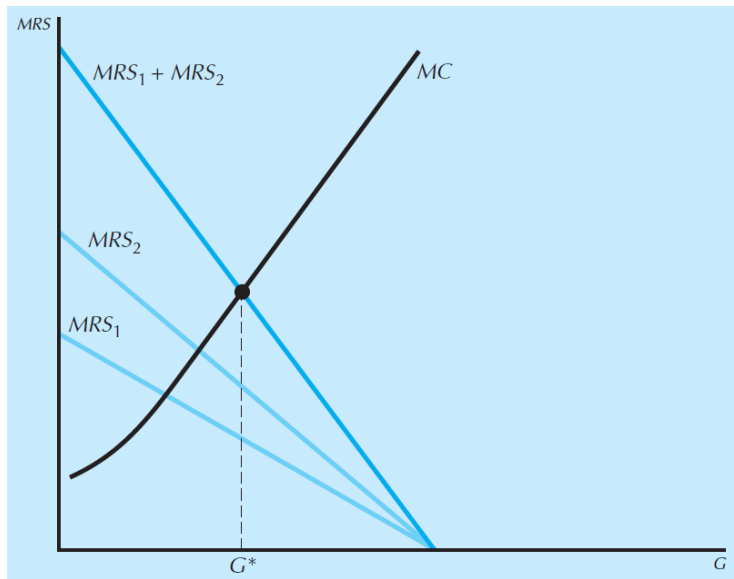
$$\frac{\partial u_1}{\partial G} f'(x_1 + x_2) + \frac{\partial u_1}{\partial y_1} (-1) = 0$$

$$\frac{\partial u_2}{\partial G} f'(x_1 + x_2) + \frac{\partial u_2}{\partial y_2} (-1) = 0$$

$$MRS_1 = \frac{\frac{\partial u_1}{\partial G}}{\frac{\partial u_1}{\partial y_1}} = \frac{1}{f'}, \quad MRS_2 = \frac{\frac{\partial u_2}{\partial G}}{\frac{\partial u_2}{\partial y_2}} = \frac{1}{f'}.$$

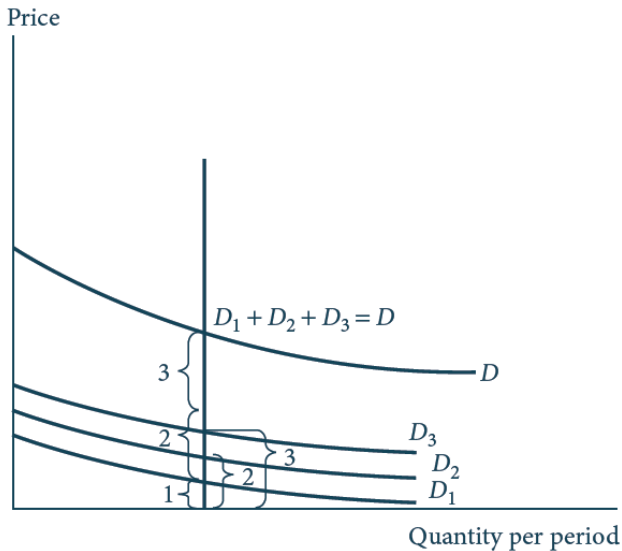
# Public Goods

- Determining the efficient amount of a public good



# Public Goods

- Demand for public good



# Public Goods

- Private provision of public good
  - ▶ Solve the solution by Nash equilibrium.
  - ▶ Free rider problem
  - ▶ In general, there will be under provision of public good.
  - ▶ Decentralized decision does not generate an efficient outcome.
- Public provision of public good
  - ▶ Tax and public choice
  - ▶ Mechanism / market design
  - ▶ Pareto-efficient allocation is not unique: utility possibility frontier

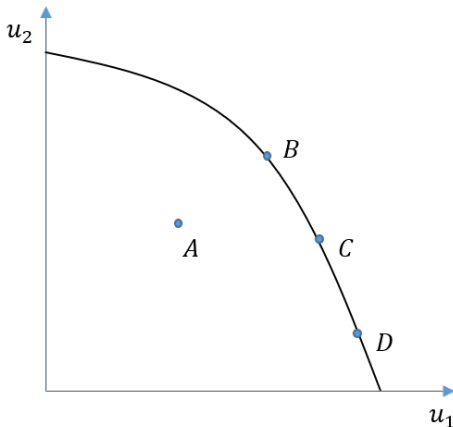
# Public Goods

- Yi Wen: “The ‘free’ market is not free. It is a fundamental **public good** that is extremely costly to create. The ongoing industrial revolution in China has been driven not by technology adoption, per se, but instead by continuous market creation led by a capable mercantilist government.”



# Public Choice

- Pareto-efficient allocation is genenerally not unique.
  - ▶ A: inefficient allocation
  - ▶ B, C, D: efficient allocation
  - ▶ Inefficient but equal allocation vs. efficient but unequal allocaiton (A vs. D)



# Public Choice

- Objective function of a society
  - ▶ In order to make an optimal choice for the public, we need a objective function.
- Social welfare function
  - ▶ classical utilitarian (Benthamite)

$$W(u_1, \dots, u_n) = \sum_i u_i$$
$$\text{or} = \sum_i \alpha_i u_i$$

- ▶ minimax (Rawlsian)

$$W(u_1, \dots, u_n) = \min \{u_1, \dots, u_n\}$$

- ▶ other theory

# Public Choice

- Market or government?
  - ▶ Competitive / decentralized / non-cooperative (NE) equilibrium
  - ▶ Efficient / centralized / cooperative equilibrium (allocation)
- Market failure: when competitive equilibrium is not efficient.
  - ▶ There is a room for Pareto improvement.
  - ▶ Government (agreement, association, law, regulation, authorities...) may be able to achieve it.
- Government failure: intrinsic principal-agent problem
  - ▶ corruptible
  - ▶ costly
  - ▶ asymmetric information



# Public Choice

- Aggregation of preference

- ▶ social preference by aggregating from individual preferences
- ▶ incomplete

Person A	Person B
x	y
y	x

- ▶ intransitive

Person A	Person B	Person C
x	y	z
y	z	x
z	x	y

- Way to aggregate preference

- ▶ by vote (each individual has the same weight)
- ▶ by money, such as lobbying (each individual's option has different weight)

# Public Choice

- Arrow's impossibility theorem
  - ▶ (1) social preference is complete, reflexive, and transitive
  - ▶ (2) If everybody prefers  $x$  to  $y$ , then social preference shall rank  $x$  ahead of  $y$
  - ▶ (3) independent of irrelevant alternative
  - ▶ If a social decision mechanism satisfies properties (1) (2) (3), then it must be a dictatorship.
- There is no desirable way to aggregate heterogeneous preferences of many individuals.

# Discussion

- Problems of forming and maximizing social welfare function
  - ▶ What objectives? efficiency, equity, justice ...
  - ▶ How to aggregate?
  - ▶ heterogeneous preferences  
e.g., high-speed railway, reclamation (filling the sea)
  - ▶ moral puzzle  
e.g., statistical value of life; automatic driving vehicles; one life vs. five lives
  - ▶ envy (reference-dependent utility)
  - ▶ feasibility of political process  
e.g., labor union, location of hazardous facility, change city zoning ...
  - ▶ voluntary (mutually benefit) but immoral activity  
e.g., child workers, selling blood, transplant tourism, prostitution, gamble, drug.

# Discussion

- Statistical value of life  
“Value analysis of auto fuel fed fire related fatalities” (General Motors, 1973)

Analyzing these figures indicates that fatalities related to accidents with fuel fed fires are costing General Motors \$2.40 per automobile in current operation.

$$\frac{500 \text{ fatalities} \times \$200,000 / \text{fatality}}{41,000,000 \text{ automobiles}} = \$2.40 / \text{automobile}$$

This cost will be with us until a way of preventing all crash related fuel fed fires is developed.

-----  
If we assume that all crash related fuel fed fires can be prevented commencing with a specific model year another type analysis can be made.

Along with the assumptions numbered above the following assumptions are necessary:

1. G.M. builds approximately 5,000,000 automobiles per year.
2. Approximately 11% of the automobiles on the road are of the current model year at the end of that model year.



# Discussion

- Evidence based public policy
  - ▶ intuition (rule of thumb) vs. data
- Data scientists cannot work on
  - ▶ value judgment (what objective function?)  
e.g., can we eat dogs?  
e.g., rich but unequal society vs. poor but equal society
  - ▶ evaluate subjective experiences and things that cannot be quantified  
e.g., joy from having country parks and white dolphin vs. suffer from small living space
  - ▶ situations with few data  
e.g., scarce cases in health database

# Discussion

- Figure from "Regressive sin taxes, with an application to the optimal soda tax."

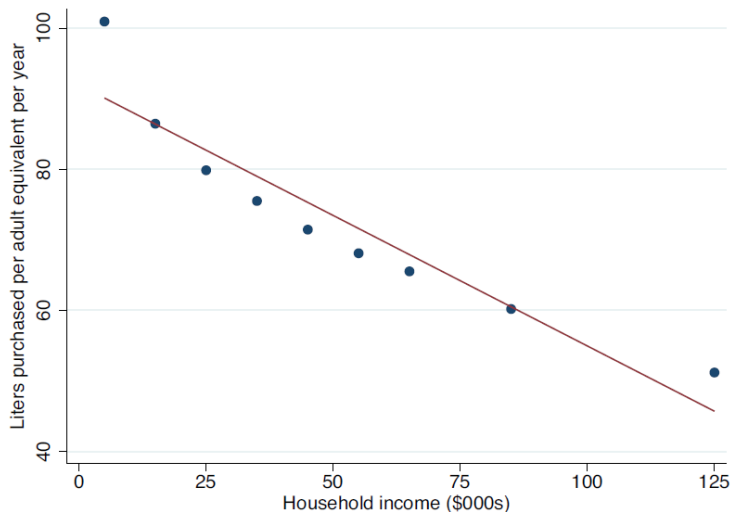


FIGURE 1

# Discussion

- Figure from "Regressive sin taxes, with an application to the optimal soda tax."

“The only way to protect all of us, including the poor, from further harm is through a sugary drink tax.”

—[Huehnergath \(2016\)](#)

“A tax on soda and juice drinks would disproportionately increase taxes on low-income families in Philadelphia.”

—U.S. Senator Bernie [Sanders \(2016\)](#)

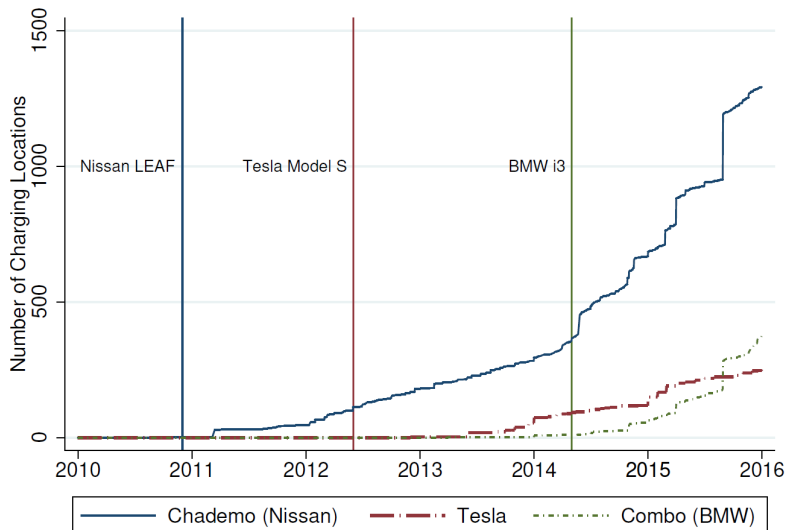
“They’ve [big soda] made their money off the backs of poor people, but this money [soda tax revenue] will stay in poor neighborhoods.”

—Philadelphia Mayor Jim Kenney  
(quoted in [Blumgart 2016](#))



# Discussion

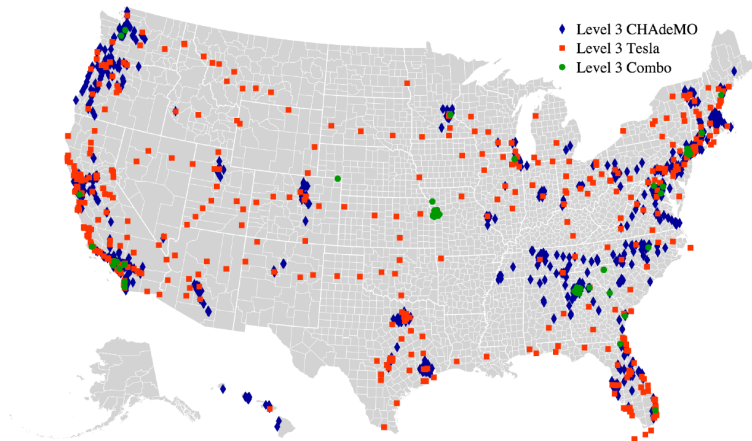
- Figure from "Compatibility and investment in the US electric vehicle market."



Data Source: Alternative Fuel Data Center

# Discussion

- Figure from "Compatibility and investment in the US electric vehicle market."






*Notes:* This figure shows the Level 3 charging locations for each standard as of September 2015, using data from the Alternative Fuels Data Center of the Department of Energy.

# Discussion

- Discussion on "Compatibility and investment in the US electric vehicle market."
  - ▶ if charging system is exclusive
  - ▶ if charging system is compatible

Figure 1: Types of Level 3 (DC, Fast) Charging Standards

Level 3 (DC, Fast) Charging Standards		
		
SAE J1772 Combo	Chademo	Tesla
BMW: i3 GM: Bolt, Spark EV Volkswagen: e-Golf Ford Chrysler Daimler	Nissan: LEAF Mitsubishi: i-MiEV Kia: Soul EV Toyota Peugeot Citroën	Tesla: Model S, X

# Course Conclusion

- To regulate or not?
- When first welfare theorem holds
  - ▶ Decentralized decisions from agents generate efficient allocation.
  - ▶ Policy brings in distortion.
  - ▶ Use policy for equity or allocation purpose.
- When first welfare theorem fails (market failure)
  - ▶ Decentralized decisions from agents is not efficient
  - ▶ Policy may improve efficiency.

# Course Conclusion

- Regulate action vs. regulate incentive
  - ▶ Regulating action is intuitive, but may cause unintended consequences.
- Because economic agents will respond to incentive, there is trade-off in policy
  - ▶ tax
  - ▶ monopoly
  - ▶ innovation
  - ▶ oligopoly
  - ▶ public goods

# Course Conclusion

- Need to exercise our ability of reasoning to consider consequences of intervention.
  - ▶ How economic agent will response in choosing  $y$  in response to intervention  $x$ ?
- Three-level of reasoning in *The Book of Why* by computer scientists
  - ① observation / descriptive  
e.g. People who buy tooth paste also tend to buy floss.
  - ② intervention / comparative statics  
e.g. If I cut the price of tooth paste, how the sales of floss will change?
  - ③ imagination / counterfactual  
e.g. What if I sell tooth paste and floss as a bundle?

# Course Conclusion

- Evidence based policy making
  - ① causal inference ( $x$  affects  $y$ ) and quantitative prediction (by how much?)  
Note that correlation is not causal relationship.
  - ② marginal intervention (what if  $x$  change to  $x'$ ?)
  - ③ fundamental intervention (what if the environment changes?)
- Think carefully about strategic responses and unintended consequences.

# Course Conclusion

- Example: Go Green movement with the goal of reducing plastic cutlery and plastic bag usage
  - ▶ plastic bag fee ( $x$ ); plastic bag usage ( $y$ )
  - ▶ How plastic bag usage related to the fee? (descriptive)
  - ▶ How much it affect the merchant's business? (comparative statics)
  - ▶ Unintended consequence: may increased the usage of plastic bag because the thought of "I have already paid". (counterfactual)
  - ▶ Unintended consequence: Substitutes may damage the environment even more. (counterfactual)
  - ▶ Nudging?



# Course Conclusion



# Course Conclusion

- Example: financial fraud and P2P collapse
  - ▶ Increase legal punishment and raise qualification ( $x$ ), cases of financial fraud ( $y$ ). (descriptive)
  - ▶ How many financial fraud cases can be avoid? (comparative statics)
  - ▶ Unintended consequence: information disclosure hurt good firms; reduce competition; rule out some financial products; hard for small firms to borrow. (counterfactual)

Table 1: Overview of Collapsed Cases of Chinese P2P Lending Platforms

Year	N.platform collapsed	N.lender involved (thousand)	Unpaid loan (CNY million)
2013	94	16	1,620
2014	301	47	5,200
2015	1,291	209	9,930
2016	1,721	182	9,790
2017	723	122	6,650
2018	1,279	1,578	143,990
2019	729	191	39,235
Total	6,138	2,345	216,415

Sources: [www.wdzj.com](http://www.wdzj.com).

# Course Conclusion

- Example: Mitigate traffic congestion
  - ▶ Building more and wider roads
  - ▶ Build public transportation
  - ▶ Impose new vehicle quote
  - ▶ Promote share car service (DiDi, Uber, Car2Go)

# Course Conclusion

- Computer cannot do the second and third kind of reasoning (at this stage)
- Human are good at thinking about intervention and imagination
  - ▶ learn Economics (work with model)
  - ▶ learn Econometrics
    - ★ Most machine learning tools are mostly for the first level of reasoning.
    - ★ In Econometrics, we are more interested in the second two levels of reasoning.
    - ★ causal inference to predict comparative statics
    - ★ structural model to predict counterfactual

# Course Conclusion

**Bon voyage!**

Please finish take some time to finish SFQ at [canvas.ust.hk](https://canvas.ust.hk).

Thank you!