ECON 3113 Microeconomic Theory I
Lecture 10: Applications of Risk and Information Models

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Roadmap

- We will look at three applications involving information processing in economics

1. Search for information
2. Sample selection
3. Information cascade due to observational learning

Search

- "*Life was like a box of chocolates. You never know what you're gonna get.*" Forrest Gump.
- Imagine you are given a number of boxes, and hidden within each box is a prize.
- You can grab one and only one of these prizes.
- Each box is like an independent lottery ticket – you know its state space, payoff of each state (prize), and probability of each state.
- Every time you open one box, you have to first pay an unboxing cost.

Search

- You can open these boxes sequentially, one and a time, to inspect the prize contained within.
- At any point in your search, you are free to grab any one of the prizes discovered so far, or you may continue to open more boxes until you find a satisfactory prize.
- Examples: job/career, romantic partner/spouse, shopping for ideal costumes, ...
- Weitzman (1979) "Optimal search for the best alternative," Econometrica.

Search

- Suppose you have only two boxes.
- Box A offers prize 20 with probability 0.2, and 0 with probability 0.8. Cost of opening is 2.4.
- Box B offers prize 9 with probability 0.9 and 0 with probability 0.1. Cost of opening is 1.
- Which box should you open first?
- The expected value of box A's lottery ticket is $0.2 \times 20 - 2.4 = 1.6$.
    - The variance of box A's ticket is $0.2 \times (20 - 2.4 - 1.6)^2 + 0.8 \times (0 - 2.4 - 1.6)^2 = 64.0$.
- The expected value of box B's lottery ticket is $0.9 \times 9 - 1 = 7.1$.
    - The variance of box B's ticket is $0.9 \times (9 - 1 - 7.1)^2 + 0.1 \times (0 - 1 - 7.1)^2 = 7.29$.

Search

- Strategy 1: open both boxes in any case.
  - Expected value

$$0.2 \times 20 + 0.8 \times 0.9 \times 9 - 2.4 - 1 = 7.08.$$

- Strategy 2: open box B first, then open box A only if prize is zero in box B.
  - Expected value

$$0.9 \times 9 - 1 + 0.1 \times (0.2 \times 20 - 2.4) = 7.26.$$

- Strategy 3: open box A first, then open box B only if prize is zero in box A.
  - Expected value

$$0.2 \times 20 - 2.4 + 0.8 \times (0.9 \times 9 - 1) = 7.28.$$

Optimal Search Strategy (if you really want to know)

Theorem (Weitzman 1979)

Each box is assigned a score that depends on the characteristics of its lottery tickets (cost, possible prizes and their probabilities).
The optimal strategy is as follows. Open the boxes in descending order of the scores. Once the highest prize discovered so far exceeds the scores of all the unopened boxes, stop and take that prize.

Moral

- Options are valuable.
- Risky box gives you a higher option value.
- Once you have seen the box's prize, you have the option to stop or continue.

Sample Selection: Winning a prize turns a novel bad

- Kovács and Sharkey (2014) "The paradox of publicity: How awards can negatively affect the evaluation of quality." Administrative Science Quarterly
- Studied a group of English-language novels that won major literary awards.
- After winning the prize, their ratings on Goodreads dropped from an average of ~ 4 to ~ 3.75.
- A group of comparably rated novels that were short-listed for prizes, but didn't win, showed no such drop.

Sample Selection: Handsome guys are jerks!

- Think about the gentlemen in your social group/ dating pool.
- The handsome ones tend not to be nice, and the nice ones tend not to be handsome.
- Is it that a good look makes people nasty? Or that being nice makes people ugly?

Sample Selection

- A guy enters into your fields of attention if and only if

$$\underbrace{\text{Handsomeness score}}_{H} + \underbrace{\text{Niceness score}}_{N} \geq \underbrace{\text{Cut}}_{c}$$

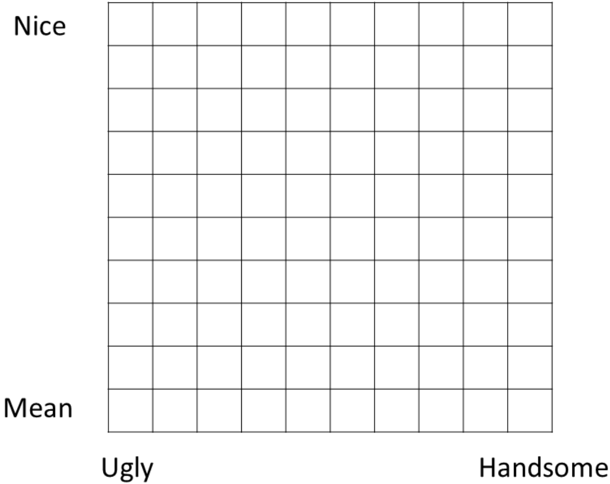
- Let's say $H \in \{1, 2, ..., 10\}$ and $N \in \{1, 2, ..., 10\}$ and they are independent/uncorrelated.
  - Knowing how handsome a guy is doesn't tell you anything informative about how nice he is. For any n and h,

$$\Pr(N = n | H = h) = \Pr(N = n).$$

  - An implication is that

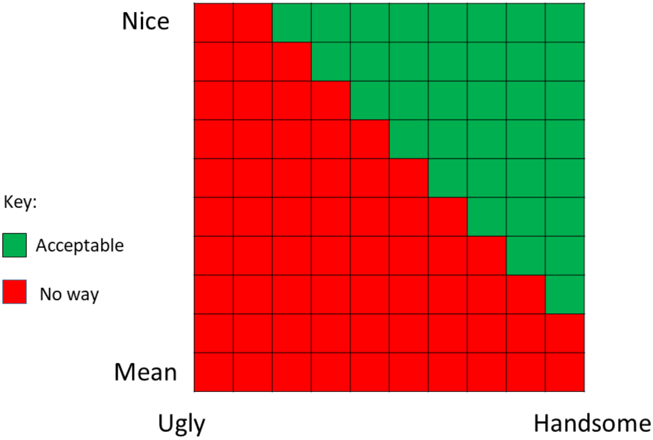
$$\frac{\Pr(N = n \text{ and } H = h)}{\Pr(H = h)} = \Pr(N = n)$$
$$\Leftrightarrow \quad \Pr(N = n \text{ and } H = h) = \Pr(H = h) \times \Pr(N = n).$$

The Great Square of Guys



The Great Square of Guys

- Let's say the cut is $c = 13$.



Handsome guys are jerks

- What is the probability of a certain niceness n, conditional on being noticeable and being super handsome ($H = 10$)?
  - If $n < 13 - 10$, the cond. prob. is 0. So take a number $n \geq 3$.

$$
\begin{aligned}
& \Pr\left(N = n | H = 10 \text{ and } H + N \geq c\right) \\
= \; & \frac{\Pr\left(N = n \text{ and } H = 10 \text{ and } H + N \geq c\right)}{\Pr\left(H = 10 \text{ and } H + N \geq c\right)} \\
= \; & \frac{\Pr\left(N = n \text{ and } H = 10 \text{ and } n \geq 3\right)}{\Pr\left(H = 10 \text{ and } N \geq 3\right)} \\
= \; & \frac{\Pr\left(N = n \text{ and } H = 10\right)}{\Pr\left(H = 10 \text{ and } N = 3\right) + ... + \Pr\left(H = 10 \text{ and } N = 10\right)} \\
= \; & \frac{\Pr\left(N = n\right) \times \Pr\left(H = 10\right)}{\Pr\left(H = 10\right) \times \Pr\left(N = 3\right) + ... + \Pr\left(H = 10\right) \times \Pr\left(N = 10\right)} \\
= \; & \frac{\Pr\left(N = n\right)}{\Pr\left(N = 3\right) + ... + \Pr\left(N = 10\right)}.
\end{aligned}
$$

Handsome guys are jerks

- For simplicity, suppose both qualities are evenly distributed.
- The table of prior probabilities of niceness

Niceness	1	2	3	4	5	6	7	8	9	10
Prob.	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

- Probabilities of niceness conditional on being noticeable and being super handsome:

Niceness	1	2	3	4	5	6	7	8	9	10
Cond. Prob.	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

- The conditional expected value of niceness is 6.5.

Ugly guys are nice

- What is the probability of a certain niceness n, conditional on being noticeable and looking ok-ish ($H = 5$)?
- If $n < 13 - 5$, the conditional probability is zero. So take a number $n \geq 8$. Same calculation as above gives:

$$\begin{aligned} & \Pr\left(N = n \middle| H = 5 \text{ and } H + N \geq c\right) \\ = \quad & \frac{\Pr\left(N = n\right)}{\Pr\left(N = 8\right) + \Pr\left(N = 9\right) + \Pr\left(N = 10\right)}. \end{aligned}$$

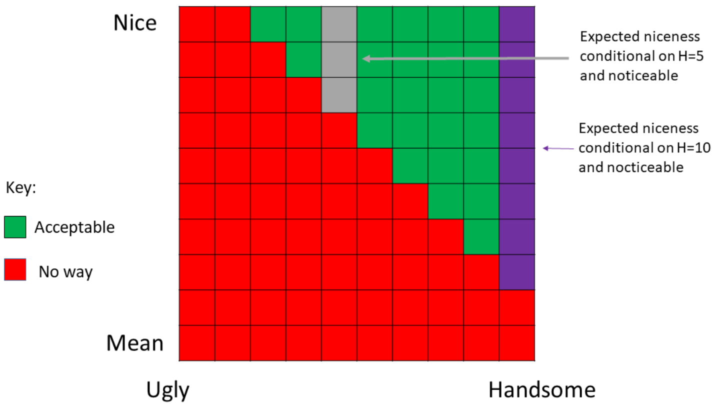
- The probabilities of niceness conditional on being noticeable and looking ok-ish:

Niceness	1	2	3	4	5	6	7	8	9	10
Cond. Prob.	0	0	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

- The conditional expected value of niceness is 9.

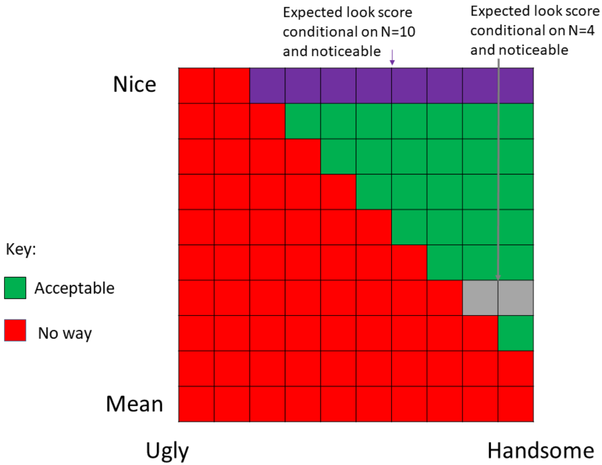
Handsome guys are jerks

- Conclusion: within the pool of noticeable guys, handsome guys tend to be less nice.



Nice guys are ugly

- A symmetric conclusion can be reached by going the other direction: within the pool of noticeable guys, nice guys tend to be less handsome.



Berkson's paradox

- **Berkson's paradox**: being sampled/being noticeable can be an informative signal about the state.
  - If some researchers study the relation between the likelihoods of different diseases (say heart disease and kidney problem)...
  - ...by collecting data from hospitalized patients....
  - ...chances are that they will find having heart disease lowers the likelihood of having kidney problem.

The Curse of Prize-Winning

- Going back to the study on novel popularity....
- A novel grabs your attention either because it is well-known or it looks like it's your cup of tea.
    - Within the pool of novels you read, those famous ones tend not to be matching with your taste....
    - ...whereas those less well-known pieces tend to be matching with your taste.
    - When a novel gets a major award, the audience expands, and many of them aren't predisposed to like it.

Introduction: Information Cascades

- Why do people tend to 'herd' on similar actions?
  - Can be people happen to have the same belief and preference;
  - Can be well-functioning convention, like driving on the left;
  - Can be preference for fitting-in, like wearing similar clothes in fashion
- These explanations do not cover cases in which mass behaviours are error-prone, idiosyncratic, and often fragile.
  - Investors/fund managers rush to buy (sell) in stock-market boom (crashes)
  - IPO frenzy
  - Adoption of products by consumers and medicine by doctors
  - Academic research
  - Toilet papers frenzy?

Introduction: Information Cascades

- Bikhchandani, Hirshleifer, and Welch (1992) "A theory of fads, fashion, custom, and cultural change as informational cascades." Journal of Political Economy.
  - A theory based on rational information processing of individuals by observing others' actions.
- Imagine you arrive at MK around noon on a weekday.
  - After some searching, you finally decide to choose between two side-by-side restaurants (both of which you have never tried before).
  - Judging from cuisine, price, decorations, etc, you are indifferent between the two restaurants.
  - However, one restaurant is packed with people (well but there are still seats), while the other is almost empty.
  - Which one would you choose?

Model

- A large number of people decide *in sequence* whether to adopt or reject a proposal.
- The order in which each person decides is pre-determined and known to all. We call the i-th person in the line Individual i.
- The payoff of rejecting the proposal is 0.
- The payoff of adopting the proposal is common to everyone. It is equal to 1 with probability $1/2$ and equal -1 with probability $1/2$.
    - We will write $V = 1$ ($V = -1$) to stand for the state that the payoff of adoption is positive (negative).

Model

- Before deciding, Individual i privately observes a noisy signal about the payoff of adoption. The noisy signal is correct with probability $p > 1/2$.
    - We will write $s_i = G$ ($s_i = B$) to stand for the event that Individual i sees a favorable signal about the proposal.
- The signals s_1, s_2, s_3, ... are all results of independent investigations.

Model

- Before deciding, Individual i **can observe the actions chosen by all preceding individuals** $1, 2, ..., i-1$ (but not their signals or payoffs).
- After collecting all info (own private signals and observations of others' actions) , an individual form a posterior belief about V, and would find it profitable to adopt if her posterior belief that $V = 1$ exceeds $1/2$.
  - If an individual is indifferent between adopting and rejecting, she flips a coin (say adopt if head and reject if tail).

Individual 1's Problem

- After observing $s_1 = G$, Individual 1 believes that

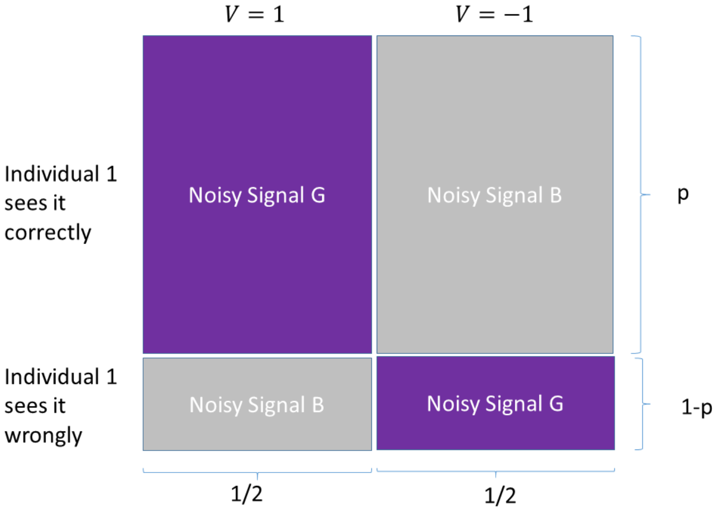
$$
\begin{aligned}
& \Pr\left(V = 1 | s_1 = G\right) \\
= \quad & \frac{\Pr\left(V = 1 \text{ and } s_1 = G\right)}{\Pr\left(s_1 = G\right)} \\
= \quad & \frac{\Pr\left(V = 1 \text{ and } s_1 = G\right)}{\Pr\left(V = 1 \text{ and } s_1 = G\right) + \Pr\left(V = -1 \text{ and } s_1 = G\right)} \\
= \quad & \frac{\frac{1}{2} \times p}{\frac{1}{2} \times p + \frac{1}{2} \times (1 - p)} = p > \frac{1}{2}.
\end{aligned}
$$

- The expected payoff of adopting is

$$p \times 1 + (1 - p) \times (-1) = 2p - 1 > 0.$$

- Therefore, person 1 finds it optimal to follow her own signal:
  - $s_1 = G$: adopt
  - $s_2 = B$: reject

Individual 1's Problem



If only everyone follows their signal....

- If there are a lot of people and everyone follows their own signal, the true state (i.e., whether the proposal is profitable or not) is revealed (almost) perfectly by observing peoples' choices.
  - If $V = 1$, a fraction p of all the people will see $s_i = G$ and adopt, whereas a fraction $1 - p$ will see $s_i = B$ and reject.
  - If $V = 0$, a fraction p of all the people will see $s_i = B$ and reject, whereas a fraction $1 - p$ will see $s_i = G$ and accept.
- Therefore, if the observed fraction of adoption is close to p, we can infer very confidently that the state is $V = 1$.
  - And, if the observed fraction of adoption is close to $1 - p$, we can infer very confidently that the state is $V = 0$.
  - Law of large numbers

Individual 2's Problem

- Individual 2 has two pieces of information: (i) her own private signal s_2; and (ii) Individual 1's action.
- Individual 1's action perfectly reveals her information
    - If she has adopted, she must have seen $s_1 = G$;
    - If she has rejected, she must have seen $s_1 = B$.
- Four possible combo of "signals"

1. $s_1 = G$ (inferred from Individual 1 adopting) ; $s_2 = G$;
2. $s_1 = G$ (inferred from Individual 1 adopting); $s_2 = B$;
3. $s_1 = B$ (inferred from Individual 1 rejecting); $s_2 = G$;
4. $s_1 = B$ (inferred from Individual 1 rejecting); $s_2 = B$.

Individual 2's Problem: Cases (i) and (iv)

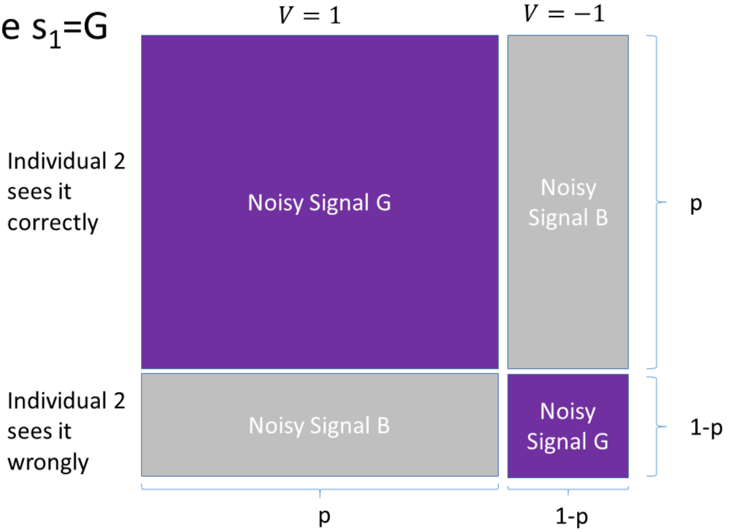
- Case (i) $s_1 = s_2 = G$
- We already know that $\Pr(V = 1 | s_1 = G) = p$.
- This forms the "prior" for the next step of updating. We write $\Pr_G(V = 1) = p$ to denote this "new prior".

$$
\begin{aligned}
& \Pr_G(V = 1 | s_2 = G) \\
= \; & \frac{\Pr_G(V = 1 \text{ and } s_2 = G)}{\Pr_G(s_2 = G)} \\
= \; & \frac{\Pr_G(V = 1 \text{ and } s_2 = G)}{\Pr_G(V = 1 \text{ and } s_2 = G) + \Pr_G(V = -1 \text{ and } s_2 = G)} \\
= \; & \frac{p \times p}{p \times p + (1-p) \times (1-p)} = \frac{1}{2 - \frac{2p-1}{p^2}} > \frac{1}{2}.
\end{aligned}
$$

- Thus, Individual 2's optimal action is to adopt.
- Case (iv) is symmetric and Individual 2's optimal action is to reject.

Individual 2's Problem: Cases (i) and (iv)

Case s_1=G



Individual 2's Problem: Cases (ii) and (iii)

- In cases (ii) and (iii), Individual 2 infers that there is one H signal and one L signal.
- In these cases, the two signals cancel each other out, and Individual 2 is indifferent between adopting and rejecting. E.g., in case (ii):

$$
\begin{aligned}
& \Pr_G\left(V=1 \middle| s_2=B\right) \\
= & \frac{\Pr_G\left(V=1 \text{ and } s_2=B\right)}{\Pr_G\left(s_2=B\right)} \\
= & \frac{\Pr_G\left(V=1 \text{ and } s_2=B\right)}{\Pr_G\left(V=1 \text{ and } s_2=B\right)+\Pr_G\left(V=-1 \text{ and } s_2=B\right)} \\
= & \frac{p\times(1-p)}{p\times(1-p)+(1-p)\times p}=\frac{1}{2}.
\end{aligned}
$$

- Case (iii) is symmetric.

Individual 2's Problem: Summary

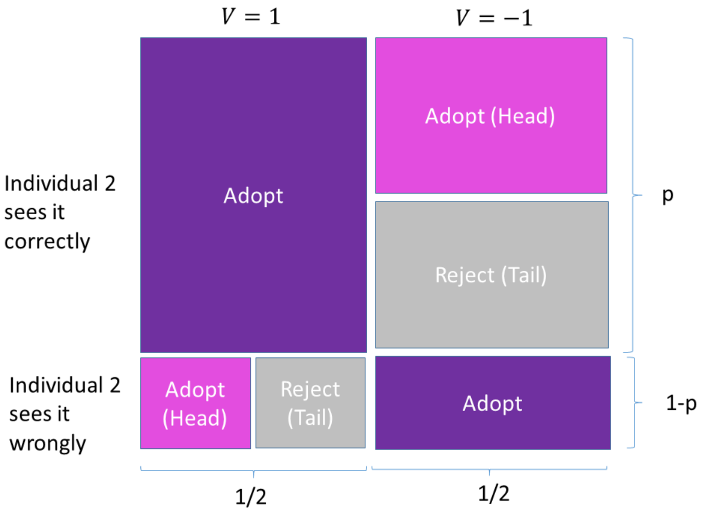
1. $s_1 = G$; $s_2 = G$: adopts
2. $s_1 = G$; $s_2 = B$: flips a coin
3. $s_1 = B$; $s_2 = G$: flips a coin
4. $s_1 = B$; $s_2 = B$: rejects

Individual 3's Problem

- Individual 3 has three pieces of information: (i) her own private signal s_3; (ii) Individual 1's action; and (iii) Individual 2's action.
- Suppose both Individual 1 and Individual 2 adopt, and $s_3 = B$.
- Individual 3 infers that $s_1 = G$ and this cancels out with his $s_3 = B$. The only remaining piece of info is that Individual 2 adopts.
- Her decision is therefore based on $\Pr(V = 1 | \text{Ind 2 adopts})$.
- What did Individual 2 see?
  - She may have seen $s_2 = G$ or
  - she may have seen $s_2 = B$ but the coin flip leads her to adopt.
  - Therefore,

$$\begin{aligned}\Pr(\text{Ind 2 adopts}) &= \Pr(s_2 = G) + \Pr(s_2 = B \text{ and head}) \\ &= \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}.\end{aligned}$$

Individual 3's Problem



Individual 3's Problem

$$
\begin{aligned}
& \Pr\left(V=1 \middle| \text{Ind 2 adopts}\right) \\
= \;& \Pr\left(V=1 \middle| \text{Ind 2 adopts}\right) \\
= \;& \frac{\Pr\left(V=1 \text{ and Ind 2 adopts}\right)}{\Pr\left(\text{Ind 2 adopts}\right)} \\
= \;& \frac{\Pr\left(V=1 \text{ and } s_2=G\right)+\Pr\left(V=1 \text{ and } s_2=B \text{ and head}\right)}{\Pr\left(\text{Ind 2 adopts}\right)} \\
= \;& \frac{\frac{1}{2}\times p+\frac{1}{2}\times(1-p)\times\frac{1}{2}}{\frac{3}{4}}=\frac{1+p}{3}>\frac{1}{2}.
\end{aligned}
$$

- The optimal action of Individual 3 is adoption, despite his private signal being $s_3 = B$.

Information Cascade

- If the first two individuals adopt, then Individual 3 ignores her own private signal: she adopts even if her private signal is B.
- In this case, Individual 3's action contains **no information content** about her signal.
- All subsequent individuals then face exactly the same problem as Individual 3, and they all choose to adopt regardless of what signals they see!
- The adoption of the first two individuals "causes" everyone in line to follow suit: an **information cascade** arises.
- Similarly, the rejection of the first two individuals "causes" an information cascade of rejection.

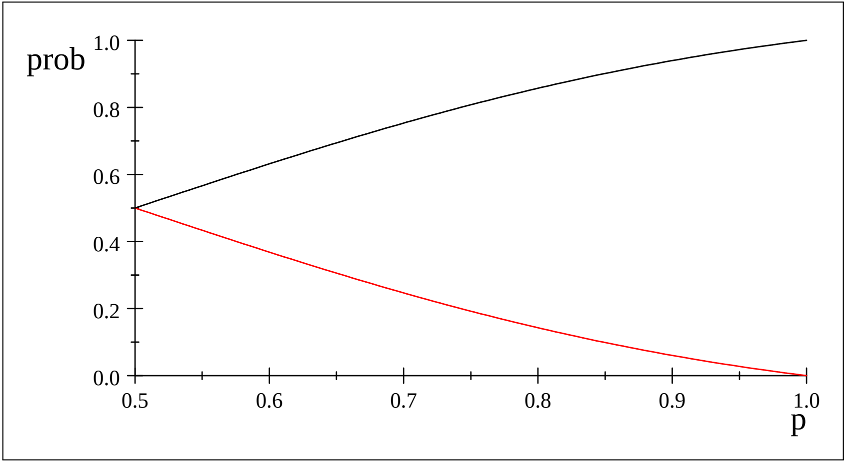
Information Cascade: General Case

- If the actions of Individual 1 and 2 are different, they must receive different signals. Thus, Individual 3 learns nothing useful from observing their actions.
- Individual 3's problem is then similar to Individual 1's, and Individual 4's problem is similar to Individual 2's.
- The optimal strategy for each individual is as follows:

Let d be the difference between the number of previous individuals who adopted and the number who rejected.
When $d \geq 2$, adopt regardless of private signal.
When $d = 1$, adopt if signal is G, and flip a coin if signal is B.
When $d = 0$, follow own signal.
Rules for $d = -1$, and $d \leq -2$ are symmetric.

Discussion

- With a long line of people, an information cascade *almost surely* arises.
- There is a reasonable chance that the "wrong" cascade forms.



Black: correct cascade; Red: wrong cascade.

Moral

- We have looked at a model with the features:
  1. Individuals take action in sequence.
  2. They observe the actions, but not the signals and payoffs, of all previous individuals.
- When the actions by previous individuals become more informative than private signals of subsequent individuals, the latter completely ignore their private information and simply follow what the majority did — an information cascade arises.
- Even when private signals are quite accurate, there is a reasonable chance that a wrong cascade arises.