

ECON 3113 Microeconomic Theory I

Lecture 8: Social Choice

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- Can we talk about "a society's preference" in a meaningful way?
- Consider situations with the following features:
 - a public policy decision must be chosen among certain feasible alternatives; and
 - the decision would affect a group of individuals ("the society"); and
 - the individuals have (typically different) preferences towards the alternatives.
- Examples:
 - whether to build extra labs or student centers?
 - where to go in the next family vacation?
 - Donald Trump or Joe Biden?
 - allocation of the 5G spectrum right to telecoms
 - remuneration in an organization
 - placing high school students to universities...

- A social choice problem consists of
 - A set of n individuals (a typical individual is denoted by i);
 - A set of alternatives;
 - For each individual i , a strict preference ordering \succ_i that ranks the alternatives.
- We are interested in a social ranking \succ^* , which
 - forms the basis of deciding what alternative is "good for the society";
 - naturally should depend on the preference ordering of individuals in the society.

Social Welfare Function

- The mapping from individuals' preference orderings $(\succ_1, \succ_2, \dots, \succ_n)$ to a social ranking \succ^* is abstractly described by the **social welfare function**.

Definition

A **social welfare function** is a mathematical function which takes as an input the list of preferences $(\succ_1, \succ_2, \dots, \succ_n)$ and produces as an output a single preference ranking \succ^* .

Social Welfare Function: Examples

- Majority rule (2 alternatives; odd number of individuals)
- Plurality rule: count the "top choice votes" until all options are ranked
- Borda criterion ("point-system" voting)
- Dictatorship

Social Welfare Function: Some Requirements

- What is a reasonable social welfare function?
- Some minimal requirements:
 - 1 It should be able to **handle all possible lists** of preferences.
 - 2 It should respect **unanimity**: if all individuals prefer alternative A to B , then the output social ranking \succ^* should also rank A above B .
 - 3 how the output \succ^* ranks alternatives A versus B should depend **only** on the individuals' rankings of these two alternatives.

SWF Requirement 1: Universal Domain

Definition

A SWF satisfies **Universal Domain (UD)** if every possible preference list input results in a well-defined social ranking output.

- Pairwise majority is a SWF that violates UD. For example:
 - Individual 1: $A \succ B \succ C$
 - Individual 2: $B \succ C \succ A$
 - Individual 3: $C \succ A \succ B$
- Reason: 2 out of 3 prefer A to B , and 2 out of 3 prefer B to C , and 2 out of 3 prefer C to A . So we end up with $A \succ^* B \succ^* C \succ^* A$, which is not a well-defined social ranking.
- Condorcet paradox

SWF Requirement 2: Pareto

Definition

A SWF satisfies **Pareto** if it respects unanimity. Specifically, whenever it happens that for some pair of alternatives A and B , every individual i ranks $A \succ_i B$, Pareto requires that the social ranking output also ranks $A \succ^* B$.

- Pareto ensures that individuals' preferences are always respected, provided that there is unanimity.
- The SWF that ranks alternatives alphabetically regardless of individuals' preferences violates Pareto.

SWF Requirement 3: Independence of Irrelevant Alternatives

- Consider a setting with 2 individuals and 3 alternatives.
- Suppose the following Borda criterion is used.
 - The 1st choice gets 3 points; the 2nd choice gets 2 points; the last choice gets 1 point.
 - The total points are added up and used to rank alternatives in \succ^* . In case of a tie, rank alphabetically.
- Suppose individuals in the society have the following preferences:
 - Individual 1: $A \succ B \succ C$
 - Individual 2: $B \succ C \succ A$
- Compare alternatives A and B . Alternative A has 4 points, and B has 5 points. So, $B \succ^* A$.

SWF Requirement 3: Independence of Irrelevant Alternatives

- Suppose, instead, individuals in the society have the following preferences.
 - Individual 1: $A \succ C \succ B$
 - Individual 2: $B \succ A \succ C$
- Again, compare alternatives A and B . Now A has 5 points, and B has 4 points. So $A \succ^* B$.
- No individual has changed their ranking of A vs B ; yet the social ranking of A vs B has changed!
- This is problematic. Intuitively, if we are trying to figure out the social ranking of A vs B , what people think of C should not matter at all.
- IIA rules out this "defect".

SWF Requirement 3: Independence of Irrelevant Alternatives

Definition

A SWF satisfies **Independence of Irrelevant Alternatives (IIA)** if the social ranking of A versus B depends only on the individuals' ranking of those two alternatives.

In formal terms, if we are given two lists of preferences which are identical in terms of each individual's ranking of A and B , then the SWF should output the same social ranking of A versus B .

A Good SWF?

- A good SWF should satisfy UD, Pareto, and IIA; these are really minimal requirements.
- These requirements are silent on fairness, equity, justice, etc.
- A good SWF should satisfy these properties plus something more.
- A SWF that satisfies UD, Pareto, and IIA, but does not seem very good is dictatorship.

- A SWF is a dictatorship if for some individual i , the social ranking \succ^* is always exactly the same as individual i regardless of the preferences of individuals other than i . In this case, individual i is the dictator.
- A dictatorship satisfies
 - 1 UD because it always gives an answer ($\succ^* = \succ_i$);
 - 2 Pareto because if it is unanimous that A is better than B , then $A \succ_i B$ in particular, so $A \succ^* B$;
 - 3 IIA because the ranking of any alternative pair A and B depend only on the dictator's ranking of A and B .

Arrow's Impossibility Theorem

Theorem (Arrow's Impossibility Theorem)

If there are three or more alternatives, the only SWF that can satisfy UD, Pareto, and IIA is dictatorship.

Arrow's Impossibility Theorem

- Arrow's Impossibility Theorem leaves us 4 (and only 4) options in social choice problems:
 - drops UD (with something like pairwise majority voting); or
 - drops Pareto (with something that ignores individuals' preferences despite unanimity); or
 - drops IIA (with something like Borda criterion); or
 - adopts dictatorship.
- And we have not started worrying about how to figure out peoples' preferences:
 - People have incentives to lie about their preferences under some SWF.