Econ 3143, Fall 2020, AS2 (Chapter 15) SOLUTION

2. Label each of the following statement true, false, or uncertain. Explain briefly.

a) Changes in consumption and investment typically move in the same direction and roughly of the same magnitude.

True/Uncertain.

Both consumption and investment are procyclical. If the consumers perceive a permanent increase in their incomes, they will increase their consumptions by an amount at most equal to the increase in their incomes. However, when firms expect a permanent increase in the sales, they will increase the investment spending by more than the increase in sales. Therefore, investment is more volatile than consumption. By looking at Figure 15-5, it is true that the investment is much more volatile than consumption.

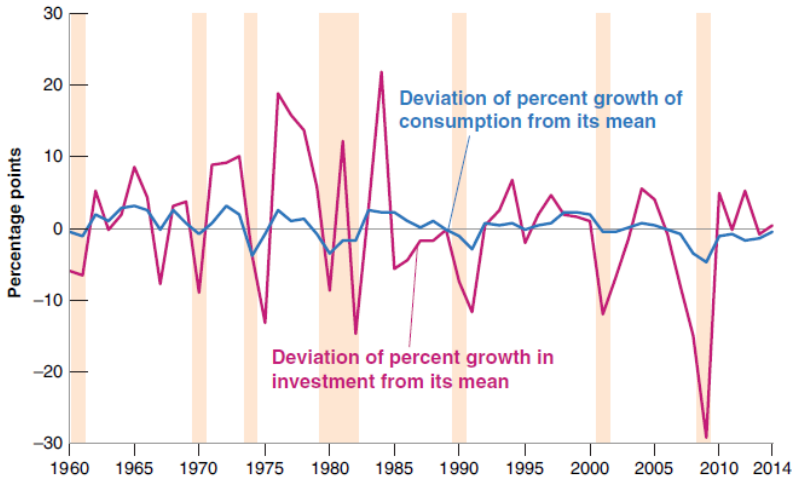

Figure 15-5

Rates of Change of Consumption and Investment in the United States since 1960

Relative movements in investment are much larger than relative movements in consumption.

Source: Series PCECC96, GPDI Federal Reserve Economic Data (FRED) http://research.stlouisfed.org/fred2/.

MyEconLab Real-time data

However, in US, since the investment only accounts for a small proportion of GDP relative to the consumption, changes in investment result in the same overall magnitude as changes in consumption in US. But in some countries, the investment is as high as 30%-40% of GDP. In those countries, changes in investment may not have the same magnitude as changes of consumption.

b) Investment depends on expected profits, but consumption only depends on current income.

False. Investment depends both on the present value of expected future profits and on the current level of profit. Consumption is an increasing function of total wealth and also an increasing function of current after-tax labor income. Total wealth is the sum of nonhuman wealth— financial wealth plus housing wealth — and human wealth — the present value of expected after-tax labor income.

c) A good indicator of current profit per unit of capital is the ratio of current output to capital.

True. According to Figure 15-3, there is a tight relation between changes in profit per unit of capital and changes in the ratio of output to capital. Intuition: Current profit depends on current sales, which usually depends on current output. Since we care about the profit per unit of capital, current output/capital ratio is a good indicator.

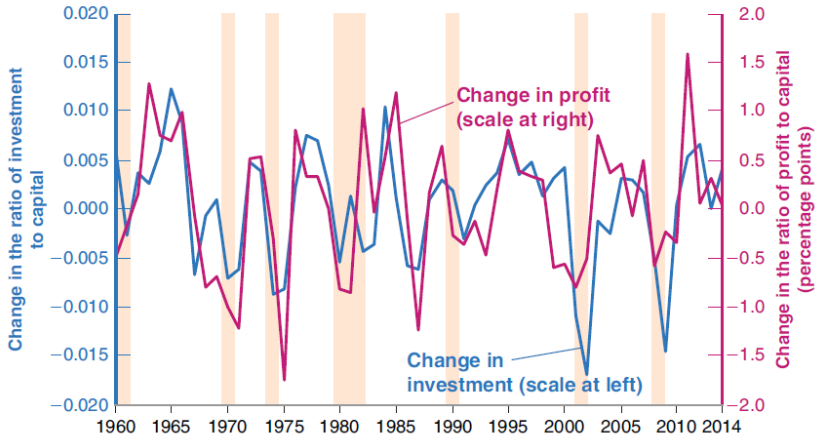

Figure 15-3

Changes in Investment and Changes in Profit in the United States since 1960

Investment and profit move very much together.

Source: Haver Analytics. Original source: Gross investment, Flow of funds variable FA105013005.A; Capital stock measured by Nonfinancial assets; Profit is constructed from Net operating surplus, taxes, and transfers, Bureau of Economic Analysis.

d) Unless current profits affected expected future profits, it should have no impact on investment.

False. The more transitory they expect a current increase in sales to be, the less they revise their assessment of the present value of profits, and thus the less likely they are to buy new machines or build new factories. Figure 15-3 shows that investment and current profit are strongly related.

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3. Consider a consumption-saving problem of an individual. This individual lives for two periods. In period one, the individual is young, and in period two the individual is old. The preferences are described by the utility function $U(C_1, C_2) = \sqrt{C_1} + \beta\sqrt{C_2}$, where $0 < \beta < 1$ is the discount factor. The individual get endowment $Y1$, $Y2$ from the Mother Nature in period 1 and 2 respectively. Assume that the discount factor $\beta = \frac{1}{1+r}$. Suppose this individual can freely borrow and lend on the loans market.

a) Derive the intertemporal optimality condition.

b) Show that $C1$, $C2$ as a function of $Y1$, $Y2$ and β . Explain the permanent income hypothesis.

Max $U(C_1, C_2) = C_1^{\frac{1}{2}} + \beta C_2^{\frac{1}{2}}$

s.t life-time b.c. $C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$

[derived from $C_1 + B = Y_1$, $C_2 = Y_2 + B(1+r)$]

Lagrangian $\mathcal{L} = C_1^{\frac{1}{2}} + \beta C_2^{\frac{1}{2}} + \lambda\left[Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r}\right]$

F.O.C

w.r.t C_1 $\quad \frac{1}{2} C_1^{-\frac{1}{2}} - \lambda = 0$

w.r.t C_2 $\quad \beta \frac{1}{2} C_2^{-\frac{1}{2}} - \lambda \frac{1}{1+r} = 0$

$\Rightarrow \beta \frac{1}{2} C_2^{-\frac{1}{2}} = \frac{1}{2} C_1^{-\frac{1}{2}} \cdot \frac{1}{1+r}$

$\Rightarrow \beta(1+r) C_2^{-\frac{1}{2}} = C_1^{-\frac{1}{2}}$ (1)

(a) The intertemporal optimality condition

LHS: The marginal benefit of saving, in terms of today's utility. When saving 1 unit of goods, get $(1+r)$ next period, which increase utility tomorrow by $(1+r) C_2^{-\frac{1}{2}}$, discounted by $\beta \Rightarrow \beta(1+r) C_2^{-\frac{1}{2}}$

RHS: The marginal cost of saving, in terms of today's utility. By saving ~~1 more unit~~, today's utility ↓ by $C_1^{-\frac{1}{2}}$

(b) From (1) $\beta(1+r) C_1^{\frac{1}{2}} = C_2^{\frac{1}{2}}$ (2)

$\Rightarrow C_2 = \beta^2 (1+r)^2 C_1 \Rightarrow C_2 = C_1$ since $\beta(1+r) = 1$

Substitute (2) into life-time b.c.

$$C_1 = C_2 = \left(Y_1 + \frac{Y_2}{1+r}\right) \cdot \frac{1}{1 + \frac{1}{1+r}}$$

$$= \left(Y_1 + \frac{Y_2}{1+r}\right) \frac{1+r}{2+r}$$

or $C_1 = C_2 = (Y_1 + \beta Y_2) \frac{1}{1+\beta}$ since $\beta = \frac{1}{1+r}$

This gives us PIH. Consumption is a function of present discounted value of life-time income

4. Using the method we described in lecture and the information I gave in the lecture notes, calculate your human wealth.

Suppose that you plan to start to work at the age y. Your human wealth will be:

$$\$180{,}000(1-0.16)[1+\frac{1.03}{1.02}+\frac{1.03^2}{1.02^2}+\cdots+(\frac{1.03}{1.02})^{65-y}]$$

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5. Suppose Sophie X will retire at the age of 60 this year with HKD $3 million in her pension account, to keep her living standard, he would like to have a constant real annuity in today's value from now to 87 years' old (life expectancy for women in Hong Kong in 2017). Suppose the annual nominal interest rate will be constant at 5% and the annual inflation rate is 3% per year. Can you calculate the real annuity (in today's value) she can get every year? (Note, we assume that she will start to get annuity at the age of 60).

Solution

$r \approx i - \pi$; r = 5%-3% = 2%

$V = Z + Z/(1+r) + Z/(1+r)^2 + \ldots + Z/(1+r)^{27}$

$$3000000 = Z * \frac{1-\left[\frac{1}{1.02}\right]^{28}}{1-\left[\frac{1}{1.02}\right]} \Rightarrow Z=137,248.$$

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6. Individual saving and aggregate capital accumulation. (Question 5 of Chapter 15 in 7th edition)

Suppose that every consumer is born with zero financial wealth and lives for three periods: youth, middle age, and old age. Consumers work in the first two periods and retire in the last one. Their income is $5 in the first period, $25 in the second, and $0 in the last one. Inflation and expected inflation are equal to zero, and so is the real interest rate.

a. What is the present discounted value of labor income at the beginning of the first period of life? What is the highest sustainable level of consumption such that consumption is equal in all three periods?

b. For each age group, what is the amount of saving that allows consumers to maintain the constant level of consumption you found in part (a)? (Hint: Saving can be a negative number if the consumer needs to borrow to maintain a certain level of consumption.)

c. Suppose there are n people born each period. What is total saving in the economy? (Hint: Add up the saving of each age group. Remember that some age groups may have negative saving.) Explain.

d. What is total financial wealth in the economy? (Hint: Compute the financial wealth of people at the beginning of the first period of life, the beginning of the second period, and the beginning of the third period. Add the three numbers. Remember that people can be in debt, so financial wealth can be negative.)

a. EPDV of future labor income = $30.

Consumption=$10 in all three periods.

b. youth: -5; middle age: 15; old age: -10

c. Total saving = n(-5+15-10) = 0

d. 0 – 5n + 10n = 5n
