Profit Maximization

Profit Maximization

- Firm chooses output (q) to maximize profit
  - Input choice has been solved in cost minimization problem.
  - economic profit= $\pi(q) = R(q) - C(q)$
  - FOC, marginal revenue (MR) equals marginal cost (MC)

$$MR(q^*) = MC(q^*)$$

  - SOC

$$\left.\frac{d^2\pi}{dq^2}\right|_{q=q^*} < 0$$

Profit Maximization

- Case 1: price-taking firm

$$R(q) = q \times p$$

- Case 2: firm facing downward-sloping (residual) demand

$$R(q) = q \times p(q)$$

  - The firm has market power to determine price.
  - In other words, the firm can raise price without losing too many consumers.
  - sources of market power

Price-taking Firm

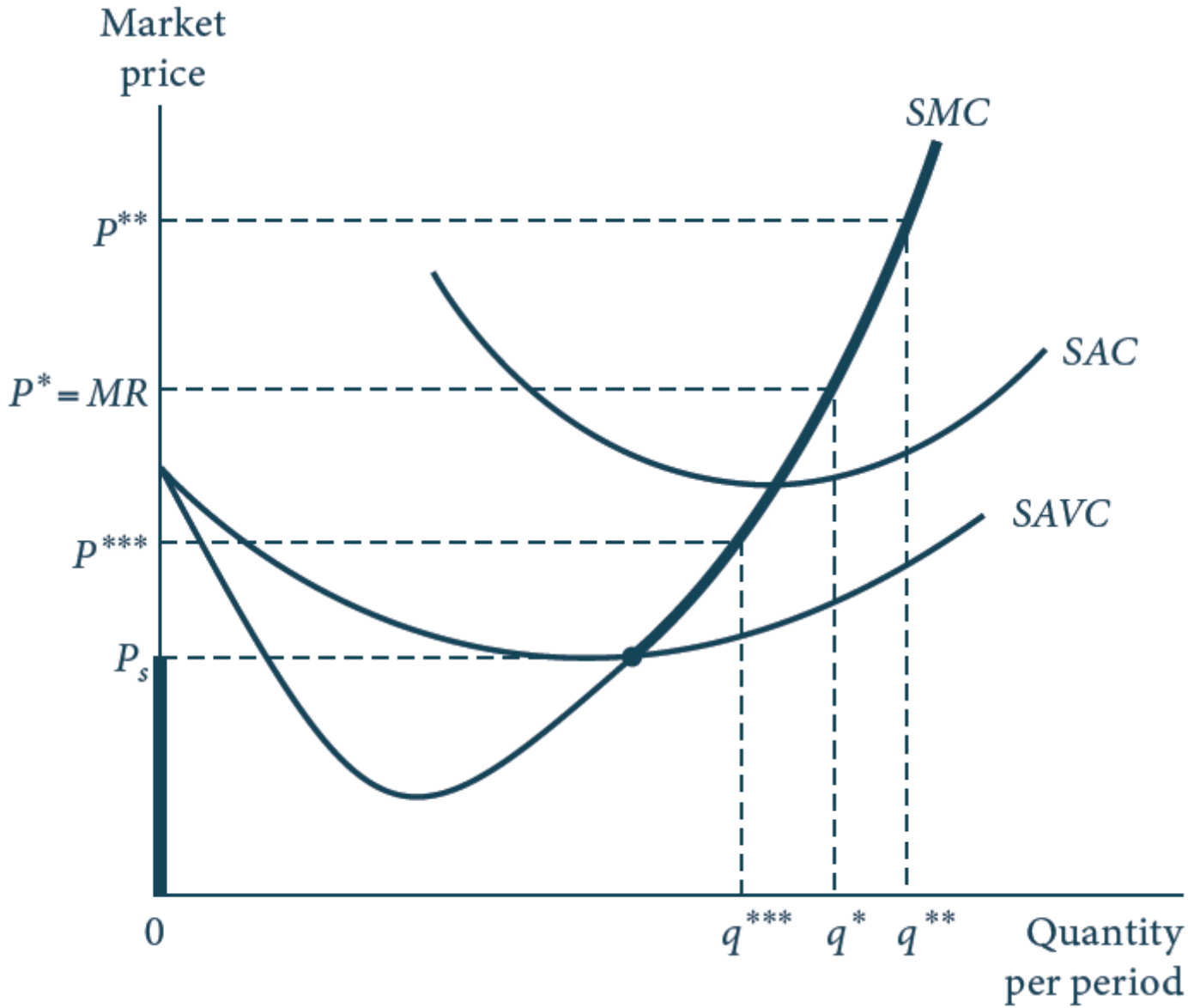
- Price-taking firm
  - The firm faces a horizontal demand curve (has no market power).
  - $MR(q) = p$ for all q
  - Long-run entry-exit decision (whether pays the fixed cost)
- Supply by a price-taking firm
  - Supply $S(p)$ coincides with marginal cost

$$S(p) = \begin{cases} MC^{-1}(q), & \text{for } q \geq \underline{q} \\ 0, & \text{for } q < \underline{q} \end{cases}$$

  - $\underline{q}$ is the shut-down point in the short run
  - $\underline{q}$ is the zero-profit / breakeven point in the long run
  - graphical analysis

Price-taking Firm

- Short-run firm supply



Price-taking Firm

- Example 11.3, find the supply function

$$SC(v, w, q, k_1) = vk_1 + wq^{\frac{1}{\beta}} k_1^{-\frac{\alpha}{\beta}}$$

  $(w = 12,\ v = 3,\ k_1 = 80,\ \alpha = \beta = 0.5)$

- Find zero-profit point and shut-down point with the help of a diagram

$$C(q) = q^3 - 4q^2 + 6q + 18$$

- Fixed cost in the long run?
  - Opportunity cost of the rent/profit from an alternative industry

Price-taking Firm

- Choosing inputs $(k,\, l)$ to maximize profit

$$\Pi(p, v, w) = \max_{q}\{pq - C(q)\} = \max_{k,l}\{pf(k, l) - vk - wl\}$$

  - FOC
  - SOC (positive/negative definite of a matrix)
  - The value function is called *profit function*.

- Input demand functions

$$k(p, v, w), \quad l(p, v, w)$$

- Example $f(k, l) = k^{\alpha} l^{\beta}$.

Price-taking Firm

- Properties of profit function
  - homogeneity of degree 1 in all prices
  - nondecreasing in p
  - nonincreasing in v and w
  - convex in p (graph)

- Results from envelop theorem (Hotelling's lemma)

$$\frac{\partial \Pi}{\partial p} = q(p, v, w), \quad \frac{\partial \Pi}{\partial v} = -k(p, v, w), \quad \frac{\partial \Pi}{\partial w} = -l(p, v, w)$$

Price-taking Firm

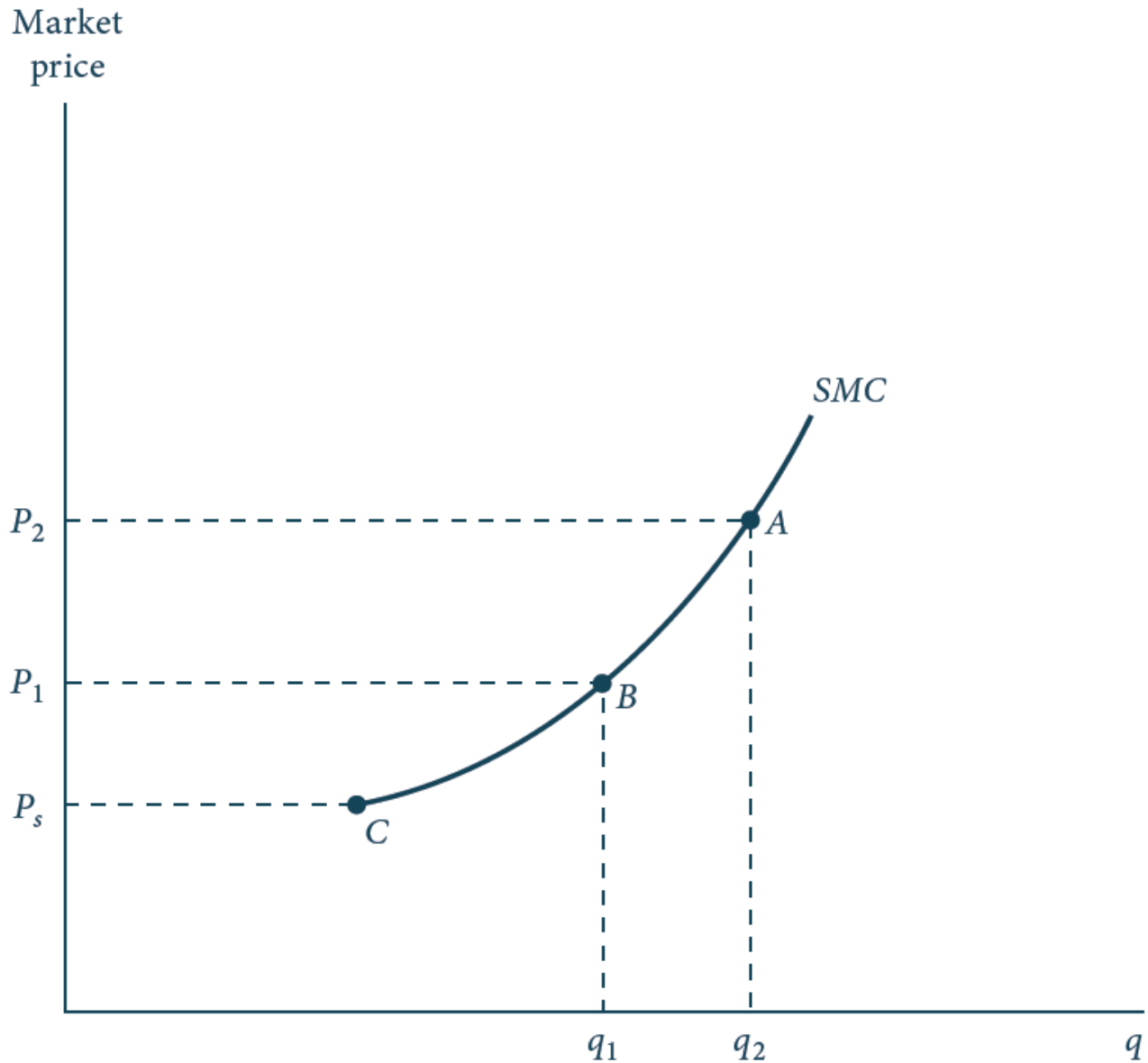
- Producer surplus
  - definition $PS = \pi(p_2) - \pi(p_1) = \int_{p_1}^{p_2} q(p)dp$
  - note on integration
  - difference between PS and profit: PS does not account for fixed cost.
- Example 11.4 ($w = 12$, $v = 3$, $k_1 = 80$, $\alpha = \beta = 0.5$)

$$\pi = pk_1^{\alpha} l^{\beta} - vk_1 - wl \Rightarrow q = S(p) = \frac{10}{3}p$$

Producer surplus at $p = 12$.

Price-taking Firm

- Producer surplus



Firm with Market Power

- Now consider firm facing downward-sloping demand

$$R(q) = q \times p(q)$$

$$MR(q) = p(q) + q\frac{dp}{dq} = p \times \left(1 + \frac{1}{e_{q,p}}\right) = p(q) \times \left(1 - \frac{1}{|e_{q,p}|}\right)$$

  - MR is steeper than demand curve: to sell one more unit, firm must reduce the good's price **of all units** (unless the firm exercises price discrimination).

- MR of linear demand curve, Example 11.1

$$q(p) = a - bp$$

Marginal Revenue

- MR and elasticity

$$\begin{cases} e_{q,p} < -1 \text{ (elastic)} & MR > 0 \\ e_{q,p} = -1 & MR = 0 \\ e_{q,p} > -1 \text{ (inelastic)} & MR < 0 \end{cases}$$

  - elastic demand

$$p \downarrow\downarrow \Rightarrow q \uparrow\uparrow\uparrow \Rightarrow R \uparrow \ (MR > 0)$$
$$p \uparrow\uparrow \Rightarrow q \downarrow\downarrow\downarrow \Rightarrow R \downarrow$$

  - inelastic demand

$$p \downarrow\downarrow \Rightarrow q \uparrow \Rightarrow R \downarrow \ (MR < 0)$$
$$p \uparrow\uparrow \Rightarrow q \uparrow \Rightarrow R \uparrow$$

- Firm will choose to supply at the elastic portion of the demand curve.
  - If firm faces an inelastic demand, it can increase profit by raising price.

Marginal Revenue

- Markup pricing rule

$$\frac{p - MC}{p} = \frac{1}{|e_{q,p}|}$$

- Example 11.2, MR of constant elasticity demand

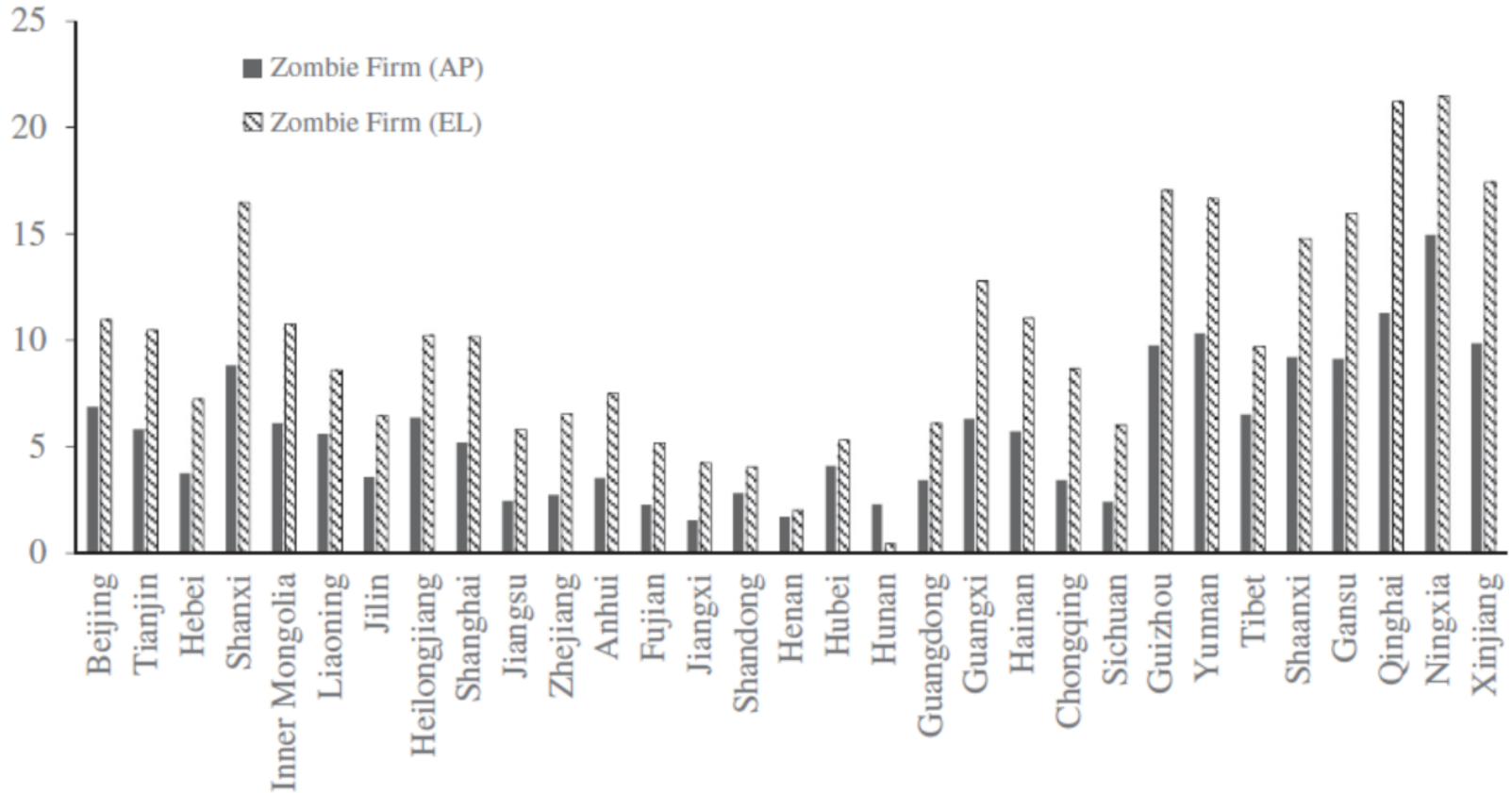
$$q(p) = ap^b$$

Discussion

- Micro-level productivity variation and persistence
  - Each firm is like a resource container. Resources do not flow instantaenously.
  - Resources are kept in less productive firm: misallocation issue.
  - Cross-firm productivity difference can be very large.
- Darwinian view of firm selection
  - External drivers: infrastructure, critical mass, tax/subsidy, employment policy, financial/capital market, trade, regulation ...
  - Avoid misallocation: Let inefficient firm whittle vs. "too-big-to-fall".
  - New challenge in the age of information technology: strong network effect.

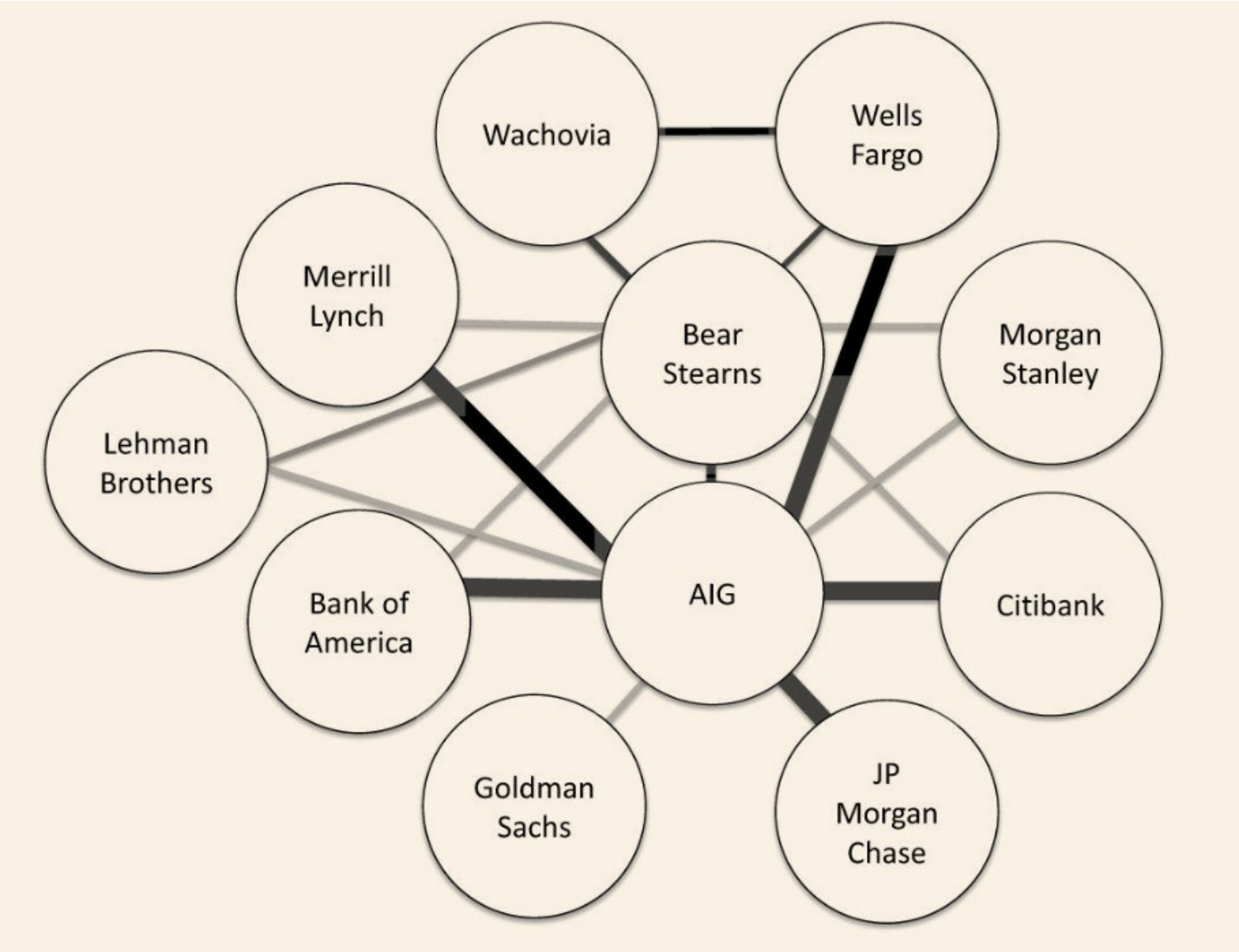
Discussion

- Figure from "Zombie firms and over-capacity in Chinese manufacturing"



Discussion

- US government saved AIG in 2008 financial crisis.



Recap of Producer Theory

- Production function
- Cost minimization
- **Cost function**
- Profit maximization
- **Supply**

Cost function and supply function are important building blocks for analysis of output market.