Final Examination

ECON 4114 Industrial Organization and Competitive Strategies 2020

Part 2

Time allowed: 50 minutes **Total points: 50 points**

3. Consider a menu pricing problem but this time round, it is the buyer who designs the menu to induce sellers of different types to self-select. The buyer has a payoff function

$$u(q,T) = q - T,$$

where q is the quantity of consumption and T is the total payment made. There is a single firm that can supply the goods. Due to uncertainty in the supply chain, the cost of production fluctuates from time to time, and only the firm itself knows the actual cost. Specifically, the firm has a cost function of the form:

$$C(q,k) = \begin{cases} \frac{1}{10} + \frac{k}{2}q^2 & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases},$$

where $k = 1$ with probability $2/5 (= 0.4)$ and $k = 2$ with probability $3/5 (= 0.6)$ in a random period. The $1/10$ term in the cost function can be viewed as the (avoidable) fixed cost.

With **only** the knowledge about the distribution of k, the buyer designs a menu consisting of two quantity-payment pairs. The firm, which knows the actual value of k, then decides whether and which quantity-payment pair to pick. The contract is then implemented and the players collect their respective payoffs.

The question concerns the optimal menu design problem of the buyer. Call a firm with $k = 1$ an **efficient type (type-E)** and firm with $k = 2$ an **inefficient type (type-I)**. As usual, on the menu, there is an item (q_E, T_E) designed for the efficient type; and another item (q_I, T_I) designed for the inefficient type.

(a) (4 points) Write down the incentive compatibility constraints for both types of firms. Call them (IC-E) and (IC-I) for the efficient and inefficient types respectively.

Solution:

$$T_E - \frac{1}{2}q_E^2 \geq T_I - \frac{1}{2}q_I^2 \tag{IC-E}$$

$$T_I - q_I^2 \geq T_E - q_E^2 \tag{IC-I}$$

(b) (4 points) Write down the individual rationality constraints for both types of firms. Call them (IR-E) and (IR-I) for the efficient and inefficient types respectively.

Solution:

$$T_E - \frac{1}{2}q_E^2 \geq \frac{1}{10} \tag{IR-E}$$

$$T_I - q_I^2 \geq \frac{1}{10} \tag{IR-I}$$

(c) The expected payoff of the buyer is

$$\frac{2}{5}(q_E - T_E) + \frac{3}{5}(q_I - T_I).$$

The menu design problem is to choose (q_E, T_E) and (q_I, T_I) to maximize the buyer's expected payoff, subject to constraints (IC-E), (IC-I), (IR-E), and (IR-I).

(i) (10 points) Instead of solving the full problem above directly, solve the following relaxed problem.

$$\max_{q_E, T_E, q_I, T_I} \frac{2}{5}(q_E - T_E) + \frac{3}{5}(q_I - T_I) \text{ subject to (IC-E) and (IR-I).}$$

Solution: (IC-E) must hold with equality, for otherwise, we can lower T_E slightly. (IR-I) must hold with equality too, for otherwise, we can lower T_I slightly. The two equalities can be rewritten into the following payment formulas:

$$\begin{aligned} T_E &= \frac{1}{2}q_E^2 + \frac{1}{2}q_I^2 + \frac{1}{10} \\ T_I &= q_I^2 + \frac{1}{10} \end{aligned}$$

Substiuting these into the objective function gives:

$$\begin{aligned} & \frac{2}{5}\left(q_E - \left(\frac{1}{2}q_E^2 + \frac{1}{2}q_I^2 + \frac{1}{10}\right)\right) + \frac{3}{5}\left(q_I - \left(q_I^2 + \frac{1}{10}\right)\right) \\ = & \frac{2}{5}\left(q_E - \frac{1}{2}q_E^2\right) + \frac{3}{5}\left(q_I - q_I^2 - \frac{\frac{2}{5}}{\frac{3}{5}} \times \frac{1}{2}q_I^2\right) - \frac{1}{10} \\ = & \frac{2}{5}\left(q_E - \frac{1}{2}q_E^2\right) + \frac{3}{5}\left(q_I - \frac{4}{3}q_I^2\right) - \frac{1}{10} \end{aligned}$$

FOCs are

$$\begin{aligned} \frac{\partial}{\partial q_E}\left(q_E - \frac{1}{2}q_E^2\right) &= 1 - q_E = 0 \Leftrightarrow q_E = 1. \\ \frac{\partial}{\partial q_I}\left(q_I - \frac{4}{3}q_I^2\right) &= 1 - \frac{8}{3}q_I = 0 \Leftrightarrow q_I = \frac{3}{8}. \end{aligned}$$

Using the payment formulae:

$$\begin{aligned} T_E &= \frac{1}{2}(1)^2 + \frac{1}{2}\left(\frac{3}{8}\right)^2 + \frac{1}{10} = \frac{429}{640} \approx 0.670\,31; \\ T_I &= \left(\frac{3}{8}\right)^2 + \frac{1}{10} = \frac{77}{320} \approx 0.240\,63. \end{aligned}$$

(ii) (6 points) Show that the solution to the relaxed problem in part (c.i) indeed solves the full problem.

Solution: We have dropped (IC-I) and (IR-E). To check that (IC-I) is satisfied:

$$\frac{77}{320} - \left(\frac{3}{8}\right)^2 \geq \frac{429}{640} - 1 \Leftrightarrow \frac{1}{10} \geq -\frac{211}{640},$$

which is true.

To check (IR-E):

$$\frac{429}{640} - \frac{1}{2}(1)^2 \geq \frac{1}{10} \Leftrightarrow \frac{109}{640} \geq \frac{1}{10},$$

which is also true.

(Total: 24 points)

4. Consider a Hotelling city of unit length. Consumers are uniformly distributed on the unit interval, each of them is labelled by her location $\theta \in [0,1]$. Each consumer demands at most one unit of the goods, which she values at $V > 0$. There are two firms in the city: Firm A is located at 0 and Firm B is located at 1. Assume for simplicity that they have **zero marginal cost of production**.

The payoff of a consumer buying Firm $i \in \{A, B\}$ is

$$V - d - p_i,$$

where d is the distance she has to travel to the firm, and p_i is the price charged by Firm i. In this question, you may assume that V is so large that **the market is always fully covered**.

Suppose Firm A decides to shut down the brick-and-mortar store and go online. Consequently, consumers can no longer physically visit Firm A to purchase; instead, they must place their orders online and have them shipped to their home addresses. Firm A has partnered with Courier X to provide the shipping service. Specializing in logistics, Courier X has a lower transportation cost than consumers. Its shipping cost function is $\frac{1}{5} \times d$, where d is the distance of shipping (compared with d for a regular customer).

Courier X is a profit-maximizing firm, and adopts a shipping fee schedule that is linear in the distance of shipping. Denote by k its **shipping fee per unit distance**. For example, if a consumer living at location 0.3 buys from Firm A, she would have to pay a total of $p_A + k \times 0.3$ (recall Firm A is located at 0).

Firm B remains brick-and-mortar and its customers still have to travel to the store physically to buy.

(a) (4 points) Given all the prices and shipping fee posted, i.e., fixing p_A, p_B, and k, what is the demand of each firm? (*Hint: Your answers should depend on* p_A, p_B, *and* k. *The same applies to part (b).*)

Solution: The indifferent consumer

$$p_A + k\bar{\theta} = p_B + \left(1 - \bar{\theta}\right) \Leftrightarrow \bar{\theta} = \frac{1}{k+1}\left(1 - p_A + p_B\right).$$

The demand of Firm A and Courier X is thus

$$q_A\left(p_A, p_B, k\right) = \frac{1}{k+1}\left(1 - p_A + p_B\right),$$

and the demand of Firm B is

$$q_B\left(p_A, p_B, k\right) = 1 - \frac{1}{k+1}\left(1 - p_A + p_B\right).$$

(b) (4 points) Given all the prices and shipping fee posted, i.e., fixing p_A, p_B, and k, what is the total profit of Courier X? (*Hint: Courier X earns a profit of* $k\theta - \frac{1}{5}\theta$ *if the consumer located at* θ *buys from Firm A. Summing this over all Firm A's consumers gives the total profit of Courier X. A useful mathematical fact:* $\int_a^b x dx = \frac{1}{2}\left(b^2 - a^2\right)$.)

Solution: The profit derived from consumer $\theta \in \left[0, \bar{\theta}\right]$ is $k\theta - \frac{1}{5}\theta$. The total profit is thus

$$\int_0^{\bar{\theta}} \left(k\theta - \frac{1}{5}\theta\right) d\theta = \frac{1}{2}\left(k - \frac{1}{5}\right)\bar{\theta}^2 = \frac{1}{2}\left(k - \frac{1}{5}\right)\left(\frac{1}{k+1}\left(1 - p_A + p_B\right)\right)^2.$$

(c) (12 points) Suppose all firms, Firm A, Firm B and Courier X, make their pricing decision **simultaneously**. What are the Nash equilibrium prices and shipping fee?

Solution: The profit of Firm A is $p_A\left(\frac{1}{k+1}(1-p_A+p_B)\right)$, *with FOC*

$$\frac{\partial}{\partial p_A}\left(p_A\left(\frac{1}{k+1}(1-p_A+p_B)\right)\right)=\frac{1}{k+1}(p_B-2p_A+1)=0\Leftrightarrow p_A=\frac{1+p_B}{2}.$$

The profit of Firm B is $p_B\left(1-\frac{1}{k+1}(1-p_A+p_B)\right)$, *with FOC*

$$\frac{\partial}{\partial p_B}\left(p_B\left(1-\frac{1}{k+1}(1-p_A+p_B)\right)\right)=\frac{1}{k+1}(k+p_A-2p_B)=0\Leftrightarrow p_B=\frac{k+p_A}{2}.$$

The profit of Courier X is $\frac{1}{2}\left(k-\frac{1}{5}\right)\left(\frac{1}{k+1}(1-p_A+p_B)\right)^2$, *with FOC*

$$\frac{\partial}{\partial k}\left(\frac{1}{2}\left(k-\frac{1}{5}\right)\left(\frac{1}{k+1}(1-p_A+p_B)\right)^2\right)=\frac{1}{10}\frac{7-5k}{(k+1)^3}(p_B-p_A+1)^2=0\Leftrightarrow k=\frac{7}{5}.$$

Solving the system above gives the intersection of the best responses:

$$p_A=\frac{17}{15}\approx 1.133\,3 \text{ and } p_B=\frac{19}{15}\approx 1.266\,7.$$

(d) (6 points) In an attempt to conquer a larger market share, Firm A announces a free shipping policy. Specifically, Firm A enters into a contractual agreement with Courier X: after receiving a lump-sum fee from Firm A, Courier X will provide free shipping to all Firm A's customers so that $k=0$. What is the impact of announcing and committing to this free shipping policy on the (base) product prices p_A and p_B? *Explain without using any calculation.*

Solution: Both prices p_A *and* p_B *will go down because their products are less differentiated. The free-shipping policy is essentially a commitment to lower prices by Firm A. Strategic complementarity implies that Firm B will respond by more aggressive pricing.*

(Total: 26 points)

End of Part 2