# ECON3113
# Microeconomic Theory I

Tutorial #3:

## Today's tutorial

- Key characteristics of preference relations
  - Monotonicity
  - Continuity
  - (Convexity to come later)
- Implications for indifference curves
  - Proofs I, II and III
- Continuity of preferences

## Where we've got to

- We have shown the following:
- That if completeness, reflexivity and transitivity of a preference relation are met, then:

Utility function $\Leftrightarrow$ Choice function $\Leftrightarrow$ Preference relation

- That utility functions that are related via a strictly increasing function are equivalent representations of each other
- That in this theory, utility is an ordinal concept

## Where we go from here

- Now we consider a special case:
- $X$ consists of bundles of goods; each good in the bundle has a price; income is given
  - These conditions constrain the choice set to affordable $x_i \in X$
  - We can then do things like constrained optimisation, comparative statics (eg what happens the optimal bundle when prices, income change)
- We will add structure to the utility function and ask:
  - What can we say about choice behaviours?
  - What can we say about the utility function?

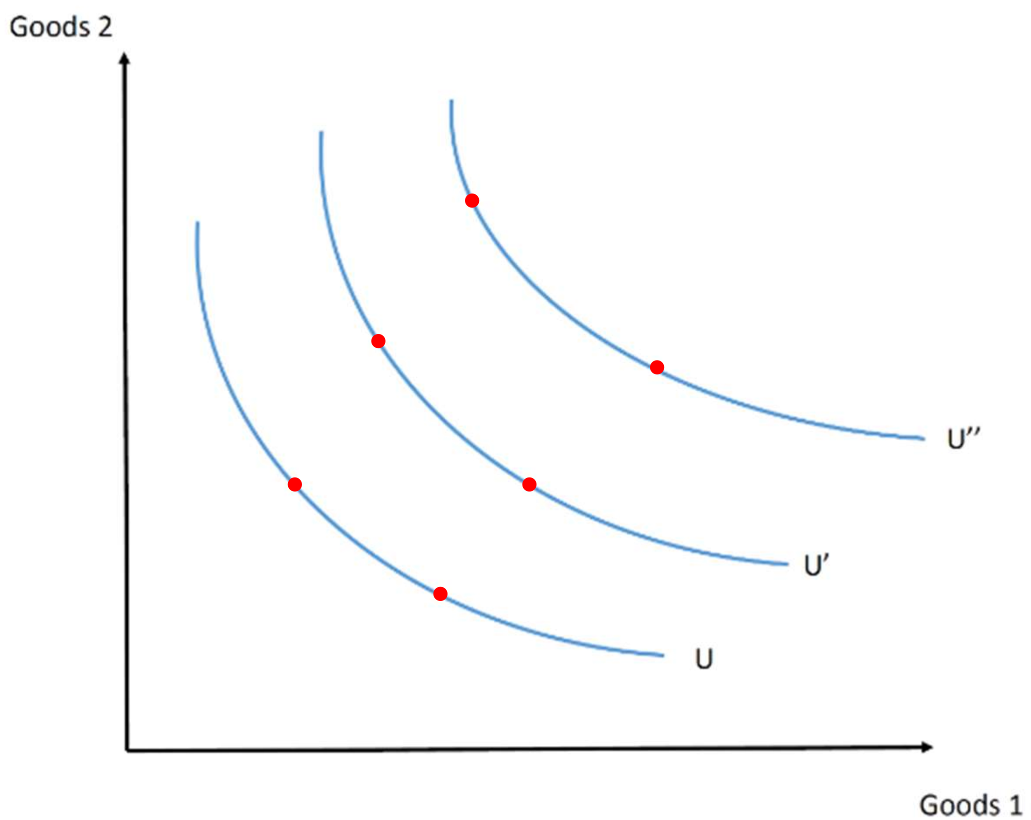
# The general setting

## Basics

- There are $n$ (infinitely divisible) goods available for consumption.
- The consumption set is $X = \mathbb{R}^n_+$, the set of all nonnegative $n$-dimensional lists/vectors.
- A generic consumption bundle is $x = (x_1, x_2, ..., x_n)$, where $x_i \geq 0$ represents the quantity of goods $i$ in the bundle.
- We write $x \geq y$ if $x_i \geq y_i$ for every goods $i$.
- We write $x \neq y$ if $x_i \neq y_i$ for at least one goods $i$.
- Preferences $\succsim$ and utility function $u$ are defined over $X$.
  - We will maintain the assumption that the consumer's preference is complete and transitive.

- Note the following:
- We're accustomed to the 2 good case; here we are in an $n$-good environment
- Quantities of goods in a bundle are always positive (ie $x_i \geq 0 \; \forall \; x_i \in x$)
- $x_i, y_i$ are quantities of goods

# Indifference curves in the two good setting




- Completeness of preferences implies every point lies on some indifference curve
- Every indifference curve represents a distinct utility level
- Indifference curves cannot cross

# Three (more) key properties of preferences

- Monotonicity:  more is better
- Continuity:  no jumps
- Convexity:  balanced consumption is better than extremes

# Monotonicity

**Definition**

Preference relation $\succsim$ is **monotone** if $x \succsim y$ for any two bundles $x$ and $y$ such that $x \geq y$.
It is **strictly monotone** if $x \succ y$ whenever $x \geq y$ and $x \neq y$.

- Monotonicity means preferring more to less

**Definition**

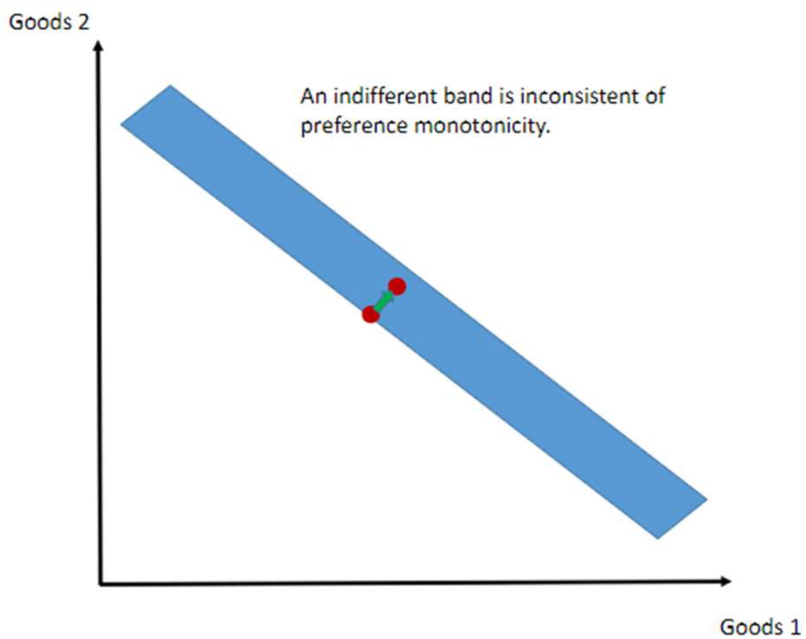
A utility function $u$ is **nondecreasing** if $u(x) \geq u(y)$ for any two bundles $x$ and $y$ such that $x \geq y$.
It is **strictly increasing** if $u(x) > u(y)$ whenever $x \geq y$ and $x \neq y$.

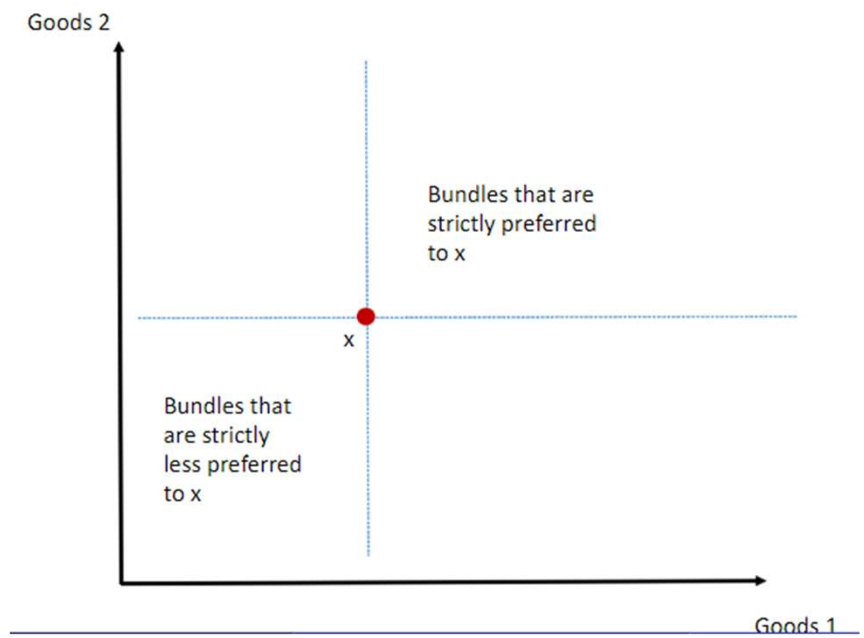
- More means greater utility

- If preference relation $\succsim$ can be represented by utility function $u$, then
  - $\succsim$ is monotone if and only if $u$ is nondecreasing;
  - $\succsim$ is strictly monotone if and only if $u$ is strictly increasing.

- Monotone preferences means $u$ non-decreasing
- Strictly monotone preferences means $u$ strictly increasing

# Two implications of monotonicity

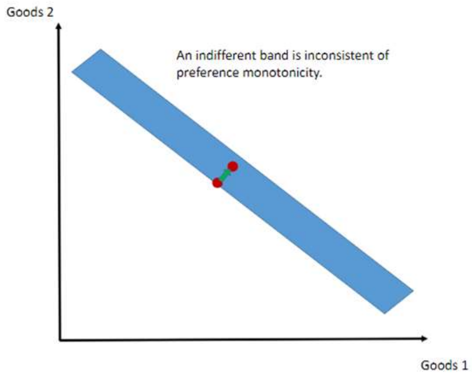



- If preferences are strictly monotone, indifference curves have no width
- If preferences are (strictly) monotone, indifference curves are (strictly) downward sloping

# Proof (I)

- Proof: If preferences are strictly monotone, indifference curves have no width

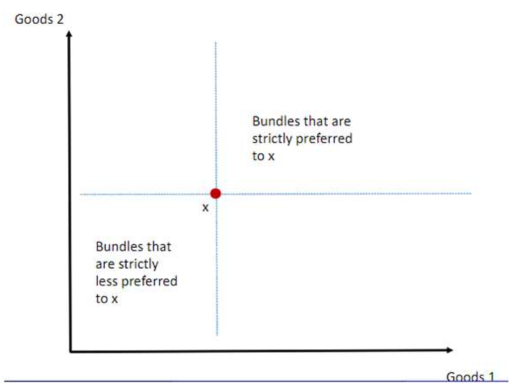



Step 1: What kind of proof?

Step 2: The proof

# Proof (II)

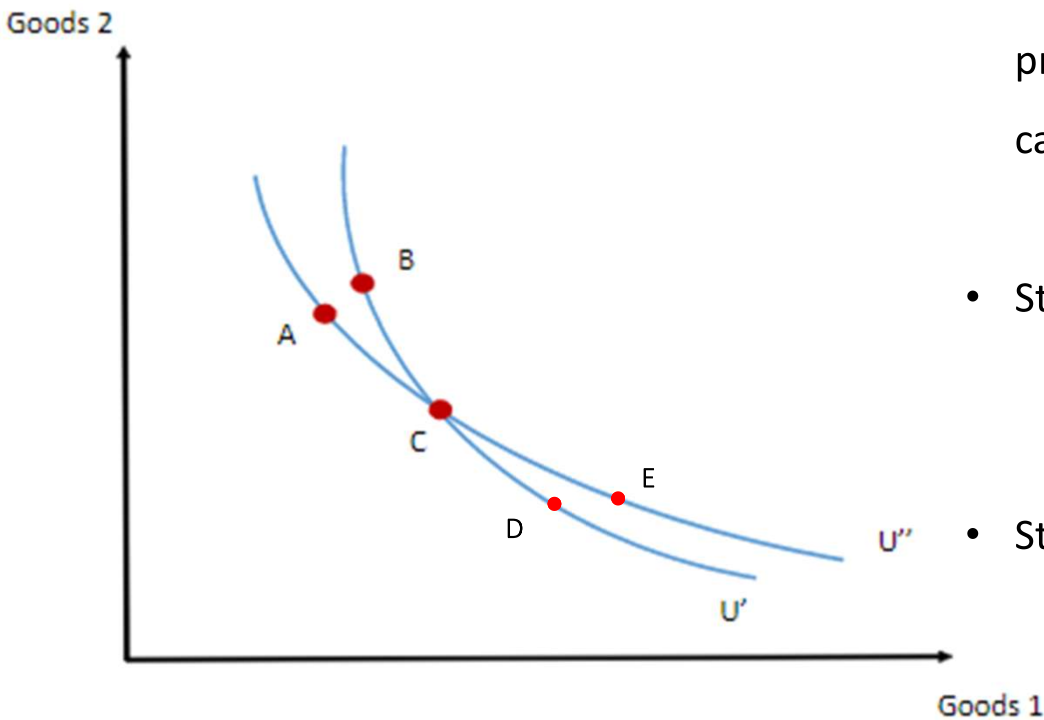
- Proof: If preferences are (strictly) monotone, indifference curves are (strictly) downward sloping
  - We'll do the strictly case only




Step 1: What kind of proof?

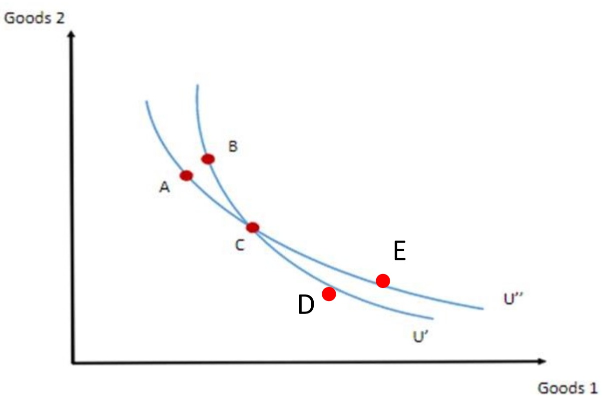
Step 2: The proof

## Proof (III)



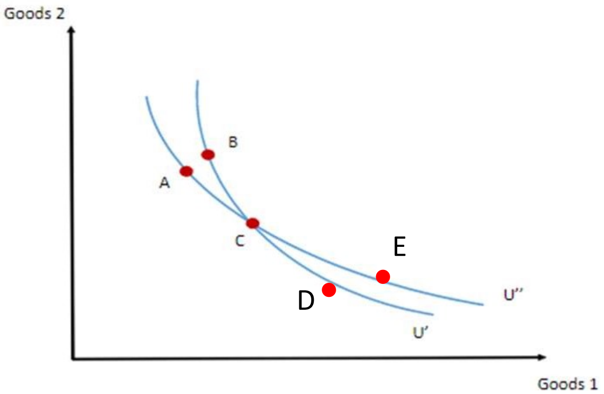

- Use the axioms of monotonicity and transitivity of preferences to prove that two indifference curves cannot intersect
- Step 1: What approach will we use?
- Step 2: The proof

# Indifference curves cannot cross: Proof



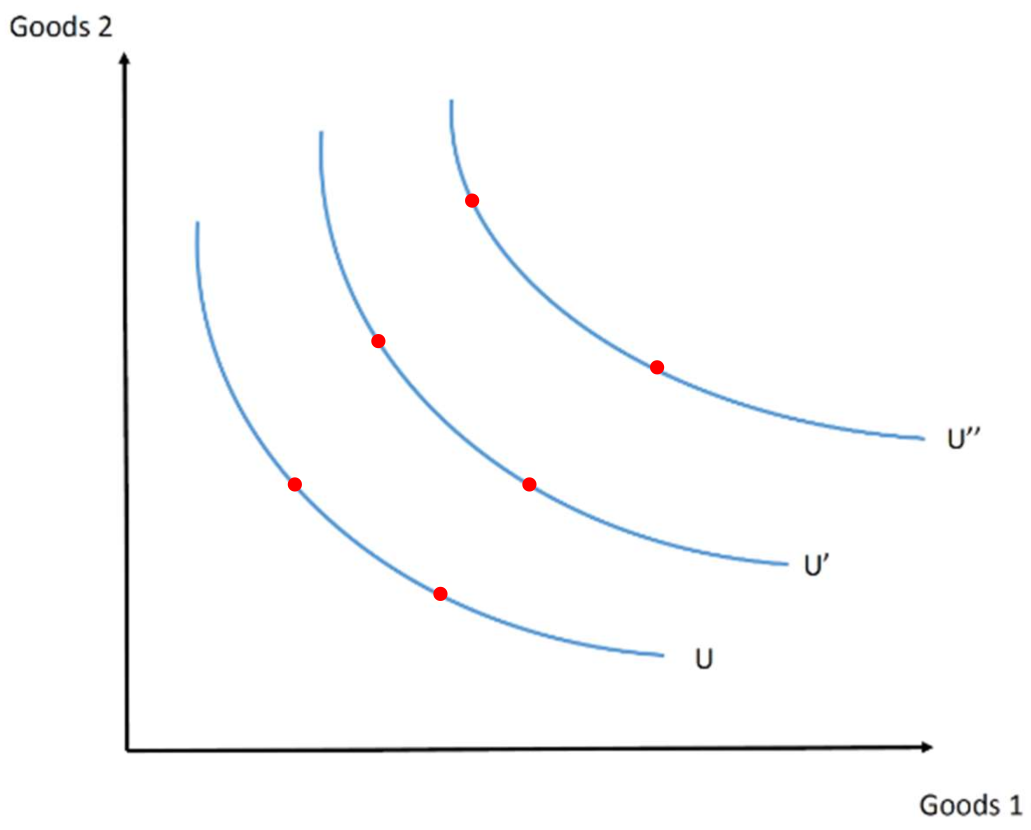

- Step 2: The proof

# Indifference curves cannot cross: Proof




- Step 2: The proof (cont.)

# Indifference curves in the two good setting




- Completeness of preferences implies every point lies on some indifference curve
- Every indifference curve represents a distinct utility level
- Indifference curves cannot cross

- Notice that these are familiar characteristics of indifference curves
- But we've derived them from our theory rather than from a cardinal utility function

## Making the utility function more useful: continuity

- At the moment, our utility function is only a mapping from a finite set to a set of numbers:
    - $\{x_1, x_2, \dots, x_n\} \rightarrow \{\mathrm{u}(x_1), \mathrm{u}(x_2), \dots, \mathrm{u}(x_n)\}$
    - {tea, espresso, latte} $\rightarrow \{30, 10, 30\}$
- Our utility function would be more useful if it were a continuous function
- Why?
    - We could use more mathematical tools on it
    - In particular, we could do calculus on it
- Note: a continuous utility function in this theory would still only give us an ordinal ranking
- How can we make the utility function continuous?

## Continuous preferences

- We make our preferences continuous and define continuous preferences, as follows:
- Given a set of alternatives, $X = \{x_1, x_2, \dots, x_n\}$ form an enumeration $d(X) = \{d(x_1), d(x_2), \dots, d(x_n)\}$ in which $d(x_i)$ is a number
- Then $d(x_i)$, $d(x_j)$, that are <u>close</u> to each other represent $x_i, x_j$ that are <u>similar</u> to each other for a given consumer
- For example, given a (finite) set of all the things a person can drink:
- $X = \{$…,green tea, English Breakfast tea, Darjeeling tea,….., caffe' macchiato, espresso, double espresso,…$\}$
- $d(X) =$ {…,100,101,102,….,450,451,452,…}
- We say that green tea, English Breakfast tea and Darjeeling tea are similar to each other (because their $\mathrm{d}(X)$ are close together) and that caffe' macchiato, espresso, and double espresso are similar (for the same reason)

## Continuous preferences

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## Continuous preferences (cont.)

- Now suppose that we have a complete, transitive preference relation on $X$ and that $x \succ y$ in $X$
- If we can find a positive number, ε > 0, such that:
  - Any $x$' that are less than distance ε from $x$ (ie $|\ d(x') -\ d(x)| <$ ε),
  - And
  - Any $y$' that are less than distance ε from $y$ (ie $|\ d(x') - d(x)| <$ ε)
  - Are such that $x$' > $y$'
- If this holds for all $x, y$ for which $x \succ y$, then we say that the <u>preference relation is continuous</u>

## Continuous preferences (cont.)

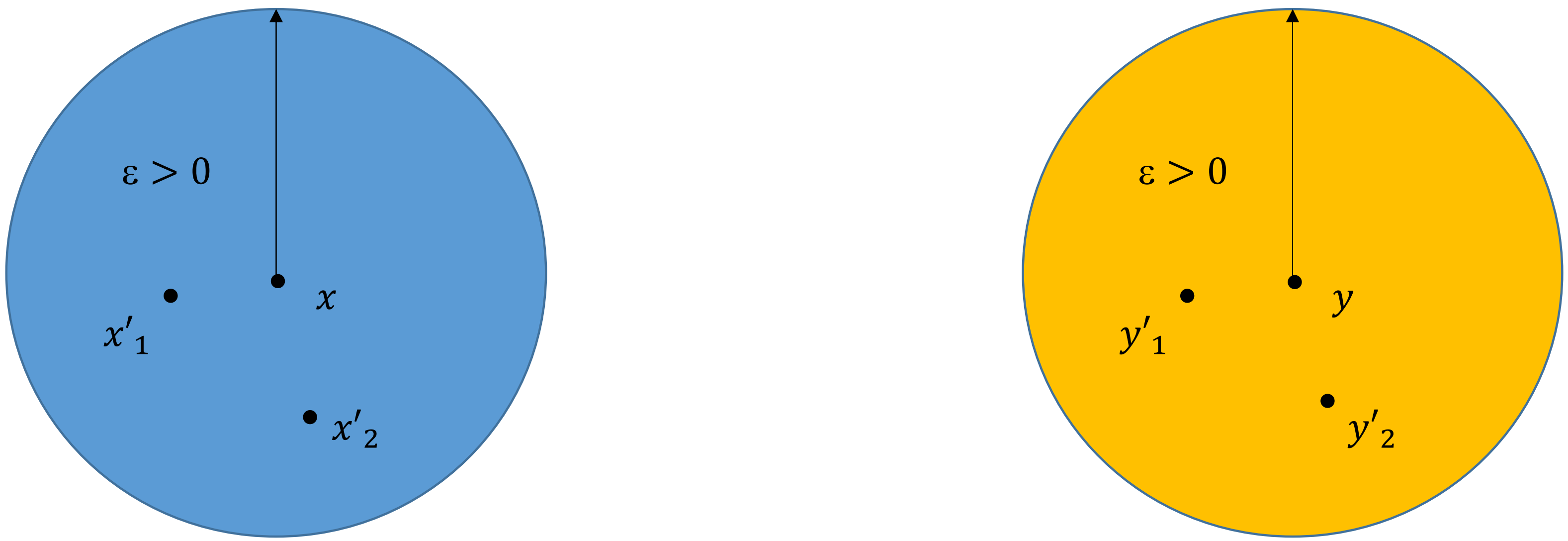
- In our example, suppose that we have English Breakfast tea > espresso, and that the preference relation also has the following:
- Green tea, English Breakfast tea, Darjeeling tea > caffe' macchiato, espresso, double espresso,
- Then with enumeration $d(X) = \{\ldots,100,101,102,\ldots.,450,451,452,\ldots\}$, for $\varepsilon = 2$ we have that all the $x'$ that are distance <u>less</u> than 2 away from $x$ are strictly preferred to all the $y'$ that are distance less than 2 away from $y$
- If this is true for all $x, y$ in $X$ for which $x > y$, then we say that the preference relation on $X$ is continuous

## Continuous preferences (cont.)

- Notice that so far our set $X$ has been finite
- The theory is usually constructed using an infinite set $X$
- In this case, the enumeration is itself an infinite set and is usually taken to be the positive real numbers
  - ie $d(X) = \mathbb{R}_+$
- Also, the members of $X$ are taken to be bundles of $k$ items. Then we say that $d(X) = \mathbb{R}_+^k$
- Because we are now in a $k$ dimensional world, we need to consider 'balls' around $x$

## Continuous preferences (cont.)

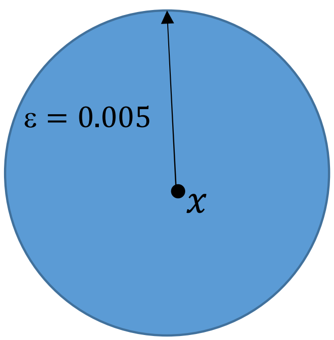
- We then have

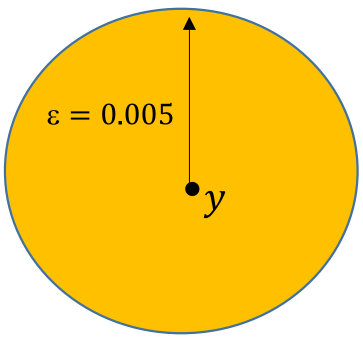



- The preference relation is continuous if for every $x, y$ such that $x > y$:
- We can find $\varepsilon > 0$ such that every $x', y'$ a distance $\varepsilon$ away from $x, y$ respectively has $x' > y'$
- Note that the balls are infinitely full of $x', y'$

# Continuous preferences (cont.)

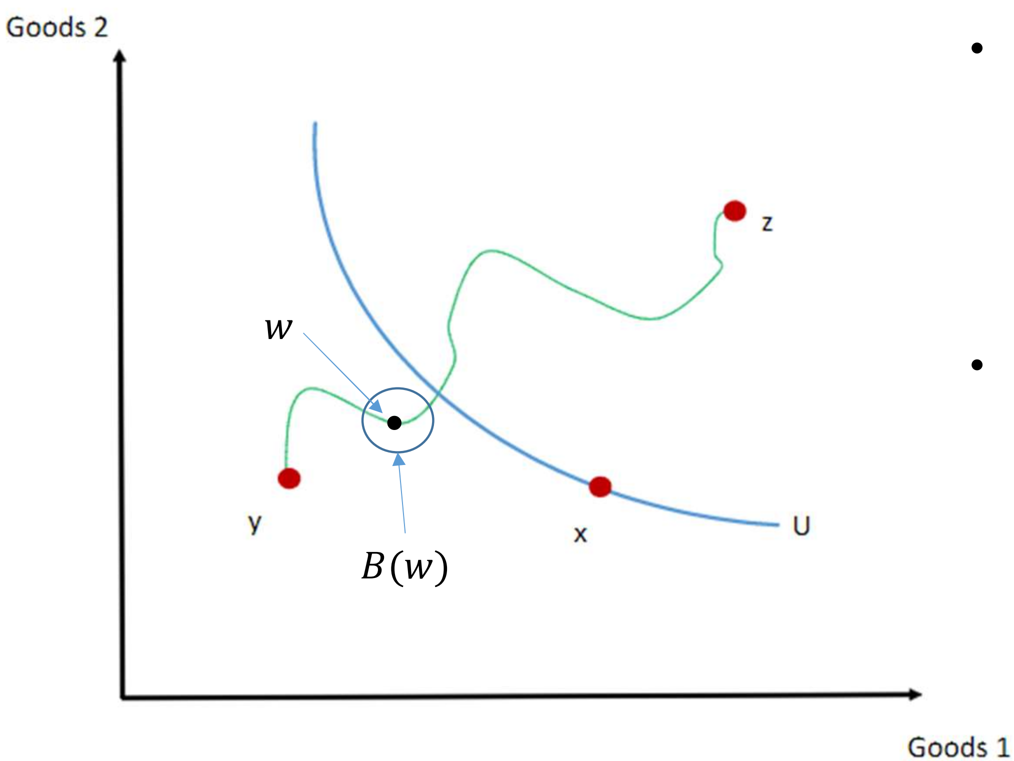
- Note that on a finite set $X$, a complete, transitive preference relation is always continuous
- On a finite set, the enumeration is always non-continuous
- Therefore, we can always find $\varepsilon > 0$ such that a ball contains at least $x$ and $y$ (and possibly only $x$ and $y$)
- For example, given enumeration $d(X) = \{\ldots, 0.01, 0.02, 0.03, \ldots, 0.55, 0.56, 0.57, \ldots\}$ on $X = \{\ldots, x_1, x_2, x_3 \ldots, y_1, y_2, y_3, \ldots\}$ and $x_2 > y_2$, then for $\varepsilon = 0.005$, the ball around $x_2$ contains only $x_2$, and the ball around $y_2$ contains only $y_2$, and $x_2 > y_2$
- For any $x_i, y_i$ we can always find an $\varepsilon > 0$ such that this is the case



# Continuity of preferences and indifference curves

- If $z \succ x \succ y$ then any continuous path from $z$ to $y$ must cross $x$




- If it's possible to get from from $z$ to $y$ without crossing the indifference curve of from $x$, then there must be a $w$ such that $w \prec x$ and for arbitrarily small ε, $\mathrm{w} + \varepsilon \succ x$
- But this contradicts continuity of preferences which says that if $w \prec x$, then there exists a ball around $w$ such that all $w' \in B(w)$ have $w' \prec x$

## From continuous preferences to a continuous utility function

- Why are continuous preferences useful?
- Because they are sufficient to guarantee a continuous utility function

- A utility function $u$ is continuous if for every $x \in X$, alternatives close to $x$ give utilities close to $u(x)$.

**Theorem (Debreu's Theorem)**

*If a preference relation is complete, transitive and continuous, then there exists a continuous utility function representing it. Conversely, if the utility function is continuous, then the implied preference relation is complete, transitive and continuous.*

# Where we've got to

- So we now have:
- For a complete, reflexive and transitive preference relation:

Utility function $\Leftrightarrow$ Choice function $\Leftrightarrow$ Preference relation

- Continuous preferences on a CRT preference relation imply the existence of a continuous utility function:

Continuous Preference relation $\Rightarrow$ Continuous utility function

So we have everything except Cardinal utility!