ECON 3113 Microeconomic Theory I
Lecture 9: Modelling Risk and Information

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Introduction

- Decisions are often made without knowing the outcomes with certainty.
  - Will this topic be asked in the exam?
  - Will I enjoy this job/career?
  - Will the stock/real estate market do well in the coming year?
- In this lecture, we discuss how to **model risky situation**.
  - What could possibly happen?
  - What is the likelihood of each outcome?
  - What does it mean by having some knowledge or information about the case?
  - How to incorporate "new information" to existing "knowledge"?

Roadmap

- In the next lectures, we will see how this framework shed light on the following applications:
  - Search
  - Observational learning
  - Sample selection
- We will then move on to modelling peoples' risk attitude, and see applications in insurance and investment.

Formal Setup: State-space Approach

- A **state** is a complete description of outcome/happening of the environment relevant to us.
- In the end, one and only one state will occur/materialize.
- The point at which a state has occurred or materialized is often referred to **ex-post**.
- Before the occurrence or materialization of states, one can list out all the possibilities. This is often referred to the **ex-ante** stage.
- The set of **all possible states** is called the **state space**.
    - Notation: $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$ – we will content ourselves with finite state spaces for now.

An Interpretation of State-space Approach

- There is one and only one *true state*.
- We can see the true state **only** in the ex-post stage.
- We may have some partial knowledge/belief about the true state in the ex-ante stage.
  - Details will come.
- E.g.,
  - Stock price tomorrow
  - Result of United vs City
  - Exam questions
  - Marriage proposal

Example: Dice Roll

- Suppose we are interested in the outcome of a dice roll.
- A state is a number coming up top: 1, 2, 3, 4, 5, or 6.
- The state space is $\Omega = \{1, 2, 3, 4, 5, 6\}$.

Example: Stock Price

- Suppose we are interested in the closing price of Apple (AAPL) tomorrow.
- A state is a price: a positive number with two decimal places.
- The state space is $\Omega = \{0, 0.01, 0.02, 0.03, ...\}$.

Example: Match Result

- Suppose we are interested in the result of United vs City.
- A state is a pair of scores.
- The state space is $\Omega = \{0:0,\ 1:0,\ 0:1,\ 2:0,\ 1:1,\ 0:2, \ldots\}$.

Lottery

- Decision-making under risk can be modelled as picking a lottery among a set of available lotteries.
- A **lottery** consists of a description of

1. the set of all possible states;
2. the payoff of each state;
3. the probability of each state.

Probability

- Each state happens with some probability. Denote the probability that state ω happens by $\Pr(\omega)$.
- Being a probability, $\Pr(\omega)$ is between 0 and 1.
- As one (and only one) state will happen, the probability of all states in the state space add up to 1:

$$\sum_{\omega \in \Omega} \Pr(\omega) = \Pr(\omega_1) + \Pr(\omega_2) + ... + \Pr(\omega_n) = 1.$$

Expected Value

- The **expected value** of a lottery is the weighted average of the payoffs of the possible states, with the weights being the probabilities of outcomes.
- Consider a ticket that with the following payoffs:

State	H	M	L
Payoff	\$10	\$5	\$0
Probability	0.2	0.3	0.5

Expected Value

- If the ticket costs \$2, then **buying the ticket is a lottery** with

State	H	M	L
Payoff	\$8	\$3	$-$2
Probability	0.2	0.3	0.5

- The expected value of buying the ticket is

$$
\begin{aligned}
& EV \\
= \; & \underbrace{0.2}_{\Pr(H)} \times \underbrace{8}_{\text{payoff of state } H} + \underbrace{0.3}_{\Pr(M)} \times \underbrace{3}_{\text{payoff of state } M} + \underbrace{0.5}_{\Pr(L)} \times \underbrace{(-2)}_{\text{payoff of state } L} \\
= \; & 1.5.
\end{aligned}
$$

- Expected value is what you "expect" to get on average, if you purchase a large number of such tickets.
    - If you purchase 1000 tickets, then it is *very likely* that you get a total payoff *pretty close* to \$1500.
    - Mathematics jargon: Law of Large Numbers

Example: Miss More Flights!

- George Stigler (1982 Nobelist) once said, "If you never miss a plane, you're spending too much time in the airports."
- To see his point, suppose (for simplicity) there are only 3 options:
  1. arrive 2 hours before flight; miss flight with probability 2%;
  2. arrive 1.5 hours before flight, miss flight with probability 5%;
  3. arrive 1 hour before flight, miss flight with probability 15%.
- Assume that missing a flight brings you a disutility/discomfort equivalent to wasting 6 hours. The expected value (in hours) of each option is as follows:

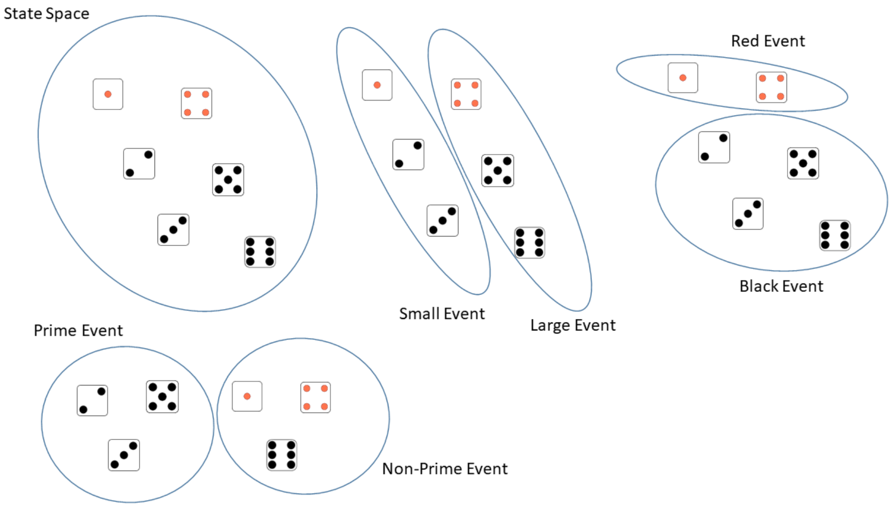
Option 1	$-2 + (0.02)(-6) = -2.12$
Option 2	$-1.5 + (0.05)(-6) = -1.8$
Option 3	$-1 + (0.15)(-6) = -1.9$

- Option 2 has the best expected value; so missing 5% of your flights is optimal!

Events

- An **event** is a subset of states.
- Examples of events in a dice-roll:
  - Null event: $\{ \}$
  - 1-event: $\{1\}$
  - Odd event: $\{1, 3, 5\}$
  - Even event: $\{2, 4, 6\}$
  - Small event: $\{1, 2, 3\}$
  - Large event: $\{4, 5, 6\}$
  - Prime event: $\{2, 3, 5\}$
  - Non-prime event: $\{1, 4, 6\}$
  - Red event: $\{1, 4\}$
  - Black event: $\{2, 3, 5, 6\}$
  - Whole-state-space event: $\{1, 2, 3, 4, 5, 6\}$

States and Events in Diagram



Events

- If the true state happens to be in set A, then we say event A occurs.
- If the outcome of the dice-roll is 5, then we say
  - the odd event occurs;
  - the large event occurs;
  - the prime event occurs;
  - the black event occurs.
- ...while
  - the even event does not occur;
  - the small event does not occur;
  - the non-prime event does not occur;
  - the red event does not occur.

Example: Stock Price

- Examples of events concerning the closing price of APPL tomorrow:
  - Stock price is higher than 250.
  - Stock price is between 230 and 280.
  - The last digit of the price is an even number.
- If the stock price (state) turns out to be 242.08, we say the last two events happen but not the first.

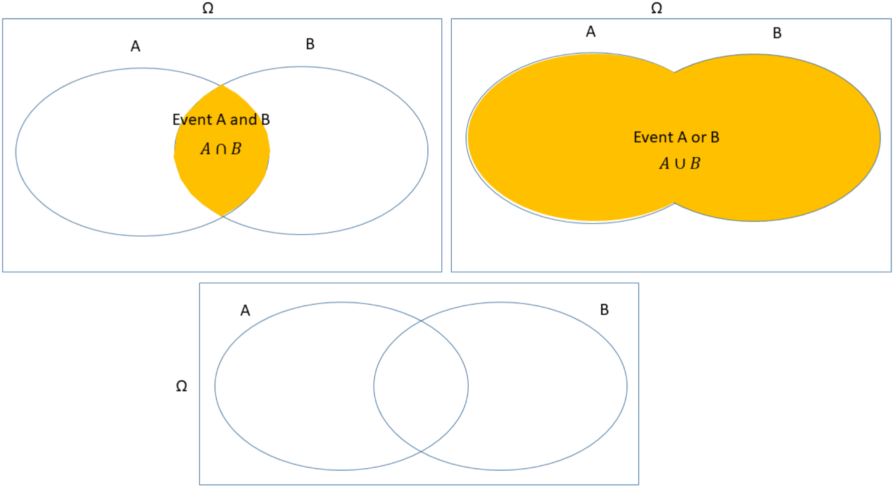
Example: Match Result

- Examples of events concerning the match result of United vs City.
  - United wins $\{1:0,\ 2:0,\ 2:1,\ 3:0,\ 3:1,\ 3:2,\ 4:0,\ 4:1,\ ....\}$
  - City wins $\{0:1,\ 0:2,\ 1:2,\ 0:3,\ 1:3,\ 2:3,\ 0:4,\ 1:4,\ ....\}$
  - They draw $\{0:0,\ 1:1,\ 2:2,\ 3:3,\ ....\}$
  - United beats City by more than 1.5 goals: $\{2:0,\ 3:1,\ 4:2, ...\}$
  - The total number of goals is no less than 3: $\{3:0,\ 2:1,\ 1:2,\ 0:3,\ 4:0,\ 3:1,\ 2:2,\ 1:3,\ 0:4,\ ....\}$
- If the match result (state) turns out to be $2:1$, then we say the event "United wins" and "The total number of goals is no less than 3" happen, but not the others listed above.

Events

- Event $A \cup B$ means either event A **or** event B or both.
- Event $A \cap B$ means both event A **and** event B.
- Two events, A and B, are disjoint if there is no overlapping states, i.e., $A \cap B = \varnothing$.

Events in Diagram



Probability of an Event

- The probability of an event is the sum of the probabilities of its constituent states.
- E.g. If event $A = \{\omega_1, \omega_4, \omega_n\}$, then

$$\Pr(A) = \sum_{\omega \in A} \Pr(\omega) = \Pr(\omega_1) + \Pr(\omega_4) + \Pr(\omega_n).$$

Probability of an Event: Examples

- In a dice-roll, $\Omega = \{1, 2, 3, 4, 5, 6\}$, and $\Pr(1) = \Pr(2) = ... = \Pr(6) = 1/6$.

$$\begin{aligned}
\Pr(\text{null}) &= 0. \\
\Pr(\text{odd}) &= \Pr(1) + \Pr(3) + \Pr(5) = \frac{1}{2}. \\
\Pr(\text{black}) &= \Pr(2) + \Pr(3) + \Pr(5) + \Pr(6) = \frac{4}{6}.
\end{aligned}$$

Law of Probability

- For any two events,

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

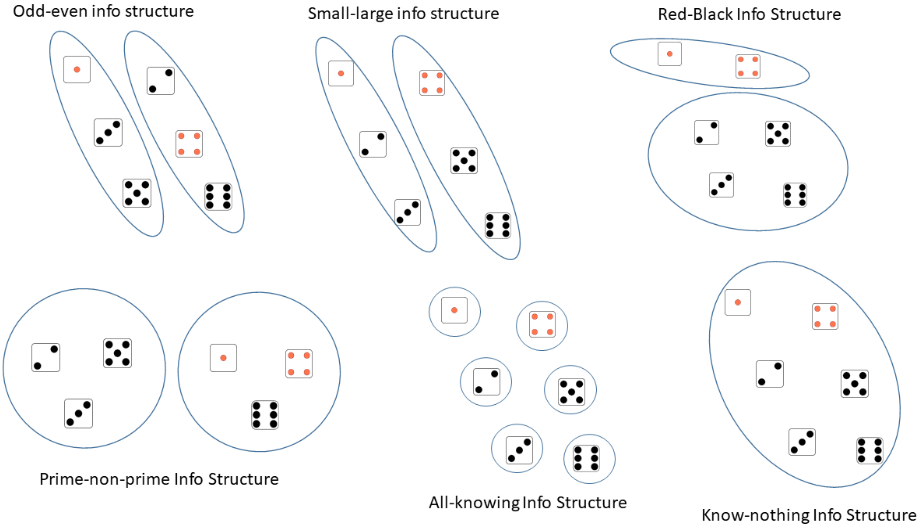
- If A and B are disjoint events (i.e., sharing no overlapping states), then

$$\Pr(A \cup B) = \Pr(A) + \Pr(B).$$

Information Structure

- Knowledge can be modelled as information structure.
- An **information structure** is a partition of the state space. For example:
  - "Odd-even" information structure: $\{\{1,3,5\},\{2,4,6\}\}$
  - "Small-large" information structure: $\{\{1,2,3\},\{4,5,6\}\}$
  - "Prime-non-prime" information structure: $\{\{1,4,6\},\{2,3,5\}\}$
  - "Red-black" information structure: $\{\{1,4\},\{2,3,5,6\}\}$
  - "Know-nothing" information structure: $\{\{1,2,3,4,5,6\}\}$
  - "All-knowing" information structure: $\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}\}$

Information Structure in Diagram



Comparing Information Structures

- An information structure is more **informative** than another if it is a **finer partition** (or segmentation) of the state space.
- "Know-everything" is more informative than "Small-large."
- $\{\{1,2\},\{3\},\{4,5\},\{6\}\}$ is a more informative structure than "Small-large."
- This criterion does not always give a ranking. E.g., "Odd-even" is neither more or less informative than "Small-large."

Signal

- Before the true state is revealed in the ex-post stage, a signal may be available.
- A signal structure is an informative structure that is potentially informative of the true state.
- A signal is a cell of the signal structure.
- The availability of a signal allows the decision-maker to refine his knowledge.

Signal Example

- Return to the dice-roll. We begin with no useful knowledge about its outcome; we have the "know-nothing" info structure.
- So our **prior belief** is

State	1	2	3	4	5	6
Prior	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- We meet Stephen Chow, who has supernatural power that can see through the cover and tell us whether the outcome is small or large.
  - Stephen is equipped with the "small-large" signal structure.

Conditional Probability

- Suppose he tells us that the outcome is *small*.
- Given the knowledge that the state is small, what is the probability that the state is 1?
- The answer is called the probability of state 1 **conditional on the event "small"**. In notation, $\Pr(1|\text{small})$.

Definition

Let A and B be two events such that $\Pr(B) > 0$. The **probability of** A **conditional on** B is

$$\Pr(A|B) = \frac{\Pr(A \text{ and } B)}{\Pr(B)}.$$

Signal Example

- How do we calculate $\Pr(1|\text{small})$?
- With the knowledge that the state lies in the "small" event, we can effectively revise the state space to $\{1, 2, 3\}$.
- And we know that the states are equally likely.
- We can **update** our belief into the following **posterior belief**:

State	1	2	3	4	5	6
Posterior	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0

Noisy Signal

- Often a signal structure is not perfectly precise; it comes with noise.
- Suppose Stephen's observation of the small-large signal is subject to noise. He sees it correctly with probability p, and he gets it wrong with probability $1 - p$.
- Suppose again, he tells us that he sees small.
- What is $\Pr\left(1 \middle| \text{noisy signal "small"}\right)$? What is $\Pr\left(6 \middle| \text{noisy signal "small"}\right)$?

Information Structure and Noisy Signal

- "Observation states": 1. correct observation; 2. wrong observation.
- Augment the "payoff state" with "observation state".

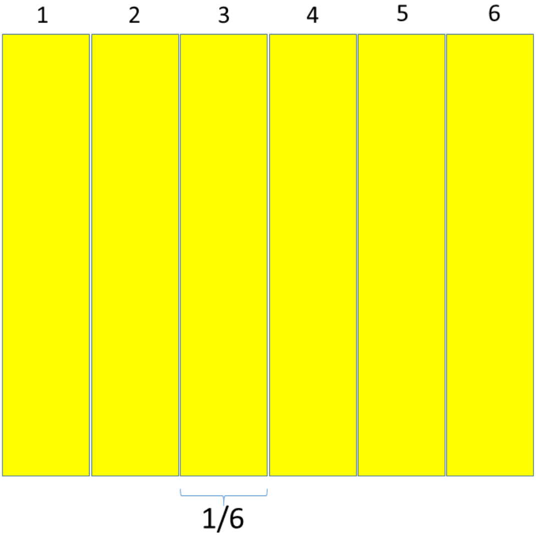
	1	2	3	4	5	6
Stephen sees it right	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$
Stephen sees it wrong	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$

Updating with Noisy Signal

- The event "noisy signal "small"" is equivalent to the event of ""*state is* 1, 2, *or* 3 *and Stephen sees it correctly*" or "*state is* 4, 5, *or* 6 *and Stephen sees it wrongly.*""

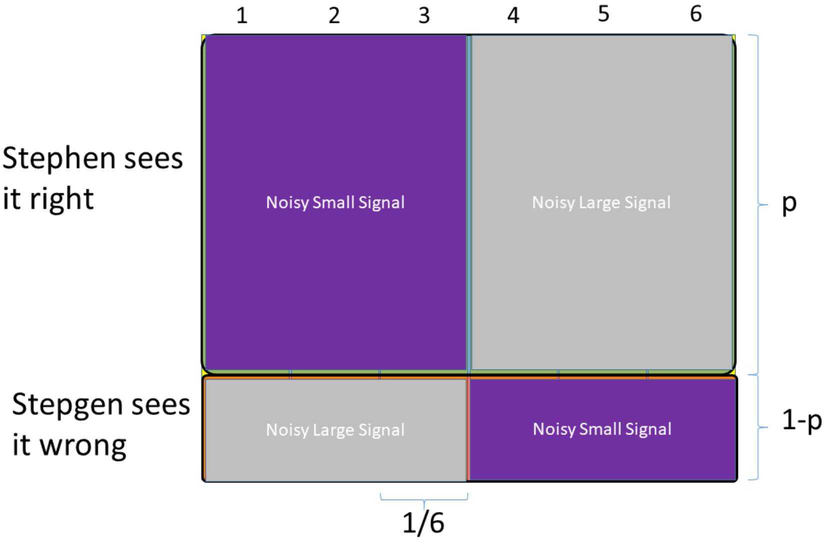
	1	2	3	4	5	6
Stephen sees it right	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$	$\frac{1}{6}\times p$
Stephen sees it wrong	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$	$\frac{1}{6}\times(1-p)$

Probability Square: Just the Payoff States



Probability Square: Augmenting the Observation States

Probability Square: Stephen's Information Structure

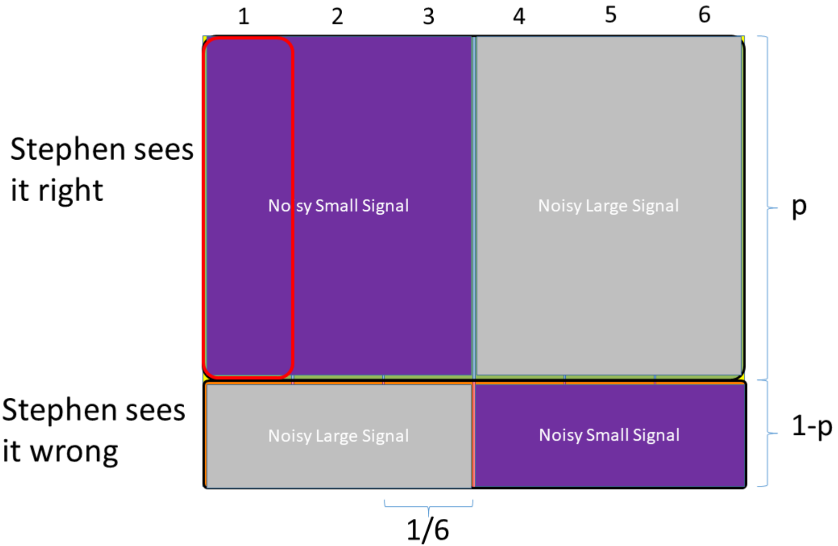


Updating with Noisy Signal

- $\Pr(1|\text{noisy signal "small"})$ is the **proportion** of the likelihood (area) of the event "state is 1 and Stephen sees it correctly" to the total relevant likelihood (areas):

$$
\begin{aligned}
& \Pr(1|\text{noisy signal "small"}) \\
= \; & \frac{\Pr(\text{state is 1 and Stephen sees it correctly})}{\Pr(\text{noisy signal "small"})} \\
= \; & \frac{\Pr(\text{state is 1 and Stephen sees it correctly})}{\left(\begin{array}{c} \Pr(\text{state is small and Stephen sees it correctly}) \\ +\Pr(\text{state is large and Stephen sees it wrongly}) \end{array}\right)} \\
= \; & \frac{\frac{1}{6}\times p}{\frac{1}{2}\times p+\frac{1}{2}\times(1-p)} = \frac{1}{3}p.
\end{aligned}
$$

Probability Square: Stephen's Information Structure

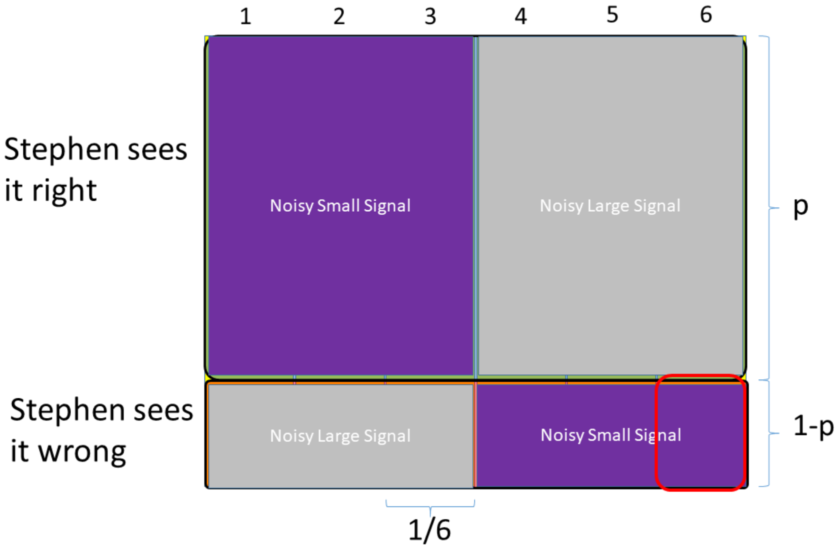


Updating with Noisy Signal

- $\Pr\left(6 \middle| \text{noisy signal "small"}\right)$ is the **proportion** of the likelihood (area) of the event "state is 6 and Stephen sees it wrongly" to the total relevant likelihood (areas):

$$
\begin{aligned}
& \Pr\left(6 \middle| \text{noisy signal "small"}\right) \\
= \; & \frac{\Pr\left(\text{state is 6 and Stephen sees it wrongly}\right)}{\Pr\left(\text{noisy signal "small"}\right)} \\
= \; & \frac{\Pr\left(\text{state is 6 and Stephen sees it wrongly}\right)}{\left(\begin{array}{c} \Pr\left(\text{state is small and Stephen sees it wrongly}\right) \\ +\Pr\left(\text{state is large and Stephen sees it correctly}\right) \end{array}\right)} \\
= \; & \frac{\frac{1}{6} \times (1-p)}{\frac{1}{2} \times (1-p) + \frac{1}{2} \times p} = \frac{1}{3}(1-p).
\end{aligned}
$$

Probability Square: Stephen's Information Structure



Updating with Noisy Signal

- The computation of conditional probability is thus essentially a *renormalization* exercise.
- The updated posterior belief after receiving the noisy small signal is

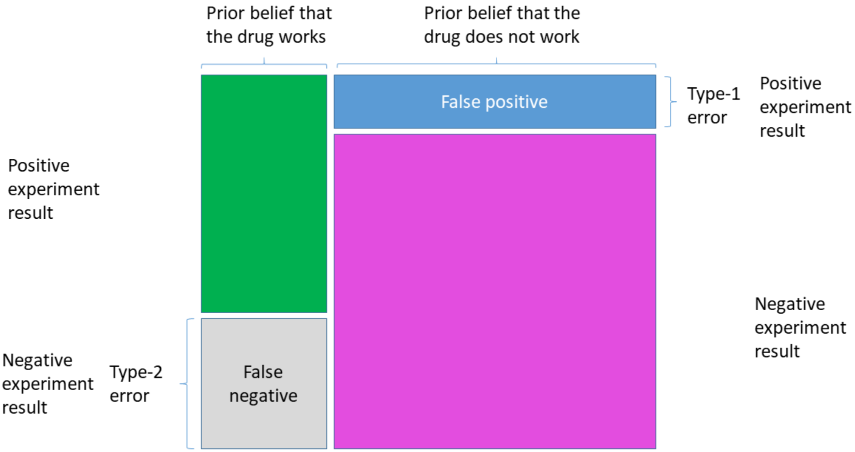
State	1	2	3	4	5	6
Posterior	$\frac{1}{3}p$	$\frac{1}{3}p$	$\frac{1}{3}p$	$\frac{1}{3}(1-p)$	$\frac{1}{3}(1-p)$	$\frac{1}{3}(1-p)$

- If $p = 1$, we are back to the previous case.
- If $p = \frac{1}{2}$, Stephen's signal is useless.
- If $p > \frac{1}{2}$, Stephen's signal is still informative.

Bayesian Inference

- This is how we can make sense of the world in the Bayesian framework.
- Start with some prior belief over the state space.
- Research, experiments, survey, etc...provides us with signal about the true state.
- Update our belief based on the evidence or signal (with math formula known as Bayes' rule)
- We end up with some posterior belief over the state space.
- If we need more precise knowledge about the state, keep seeking more evidence/signal.
- **Posterior belief depends on both the signal and the prior belief.**

Type-1 and Type-2 Errors



Type-1 and Type-2 Errors

- Conditional on a positive experiment result, the posterior probability that the drug works is

$$\frac{\text{green area}}{\text{green area } + \text{ blue area}}.$$

  This is higher than the prior belief that the drug works.

- Conditional on a negative experiment result, the posterior probability that the drug does not work is

$$\frac{\text{pink area}}{\text{pink area } + \text{ grey area}}.$$

  This is higher than the prior belief that the drug does not work.

- We will be getting closer to the truth with more and more evidence.

Prior Matters

- Consider an experiment with very low type-1 error, i.e., very unlikely to give false positive.
- And if we get a positive result, are we confident that the drug will work?
- Suppose a failing business hires a fengshui master as a consultant.
- And suppose indeed magically, the company is turned around.
- Type-1 error: 0.01
  - chance the company just got lucky for no particular reasons.
- Type-2 error: 0
  - if fengshui works, it always works.
- Fengshui master: "If fengshui doesn't work, you will have to attribute my success to the extremely unlikely event with 1% prob. My fengshui advice is most likely what contribute to the company's turnaround."

Prior Matters

- Prior belief that fengshui works: 0.000001.
- Posterior belief that fengshui works:

$$\frac{0.000001 \times 1}{0.000001 \times 1 + 0.9999 \times 0.01} = 0.0001.$$

- Still 99.99% it is not because of fengshui.
- To begin with, we have an extremely low prior belief that fengshui works.

Value of Information

- Recall this $2-ticket:

State	H	M	L
Payoff	$10	$5	$0
Probability	0.2	0.3	0.5

- Suppose all you care about are expected values.
- Suppose I have the "All-knowing" information structure and could tell you the signal I get.
- How much are you willing to pay for my tip?

Value of Information

- Knowing the state perfectly, you would buy the ticket if and only if it pays.
- Your expected value with my tip is

$$
\begin{aligned}
& 0.2 \times \underbrace{(10-2)}_{\text{Payoff of buying at state } H} + 0.3 \times \underbrace{(5-2)}_{\text{Payoff of buying at state } M} \\
& +0.5 \times \underbrace{0}_{\text{Payoff of \textbf{not} buying at state } L} \\
= \ & 2.5.
\end{aligned}
$$

- The expected value you get increases by \$1 — this is the value of my signal.
- **Informative signal is valuable as it improves decision-making.**

Value of Imperfect Information

- A signal structure can be valuable even if it is not perfectly informative.
- Continue with the same ticket but this time its price goes up to \$6.
- The expected value of buying it goes down to

$$\begin{aligned}
& \underbrace{0.2}_{\Pr(H)} \times \underbrace{4}_{\text{payoff of state } H} + \underbrace{0.3}_{\Pr(M)} \times \underbrace{(-1)}_{\text{payoff of state } M} \\
& + \underbrace{0.5}_{\Pr(L)} \times \underbrace{(-6)}_{\text{payoff of state } L} \\
= \quad & -2.5.
\end{aligned}$$

- So you are not buying it.

Value of Imperfect Information

- While I cannot tell the exact outcome, suppose I can still tell **whether the ticket will pay or not**.
- If I tell you that the ticket will pay, should you buy it?
- Formally, we need to calculate the expected value of buying the ticket, **conditional on the event that it pays**.

Updating Using New Information

- The event "pay" consists of two states "H" and "M".
- Conditional on the signal that the ticket pays, we can update the probabilities of each state as follows.

$$\Pr(H|\text{pay}) = \frac{\Pr(H \,\&\, \text{pay})}{\Pr(\text{pay})} = \frac{\Pr(H)}{\Pr(H)+\Pr(M)} = \frac{0.2}{0.2+0.3} = 0.4.$$

$$\Pr(M|\text{pay}) = \frac{\Pr(M \,\&\, \text{pay})}{\Pr(\text{pay})} = \frac{\Pr(M)}{\Pr(H)+\Pr(M)} = \frac{0.3}{0.2+0.3} = 0.6.$$

$$\Pr(L|\text{pay}) = \frac{\Pr(L \,\&\, \text{pay})}{\Pr(\text{pay})} = \frac{0}{\Pr(H)+\Pr(M)} = \frac{0}{0.2+0.3} = 0.$$

- The expected value of buying the ticket, conditional on the event that it pays, is thus

$$\begin{aligned}
& \Pr(H|\text{pay}) \times 4 + \Pr(M|\text{pay}) \times (-1) + \Pr(L|\text{pay}) \times (-6) \\
= \; & 0.4 \times 4 + 0.6 \times (-1) + 0 \times 0.6 \\
= \; & 1.
\end{aligned}$$

Value of Imperfect Information

- Without any information, you do not buy the ticket, so the expected value you get is 0.
- Equipped with my tip (i.e., information about whether the ticket pays or not), your expected value becomes

$$\begin{aligned} & \Pr(H) \times 4 + \Pr(M) \times (-1) + \Pr(L) \times 0 \\ = \; & 0.2 \times 4 + 0.3 \times (-1) + 0.5 \times 0 \\ = \; & 0.5. \end{aligned}$$

- The value of my tip is therefore \$0.5.

Value of Noisy Information

- Suppose I can tell you whether the ticket will pay or not, but there is a chance $1 - p < 0.5$ that my tip is wrong.
- If I tell you that the ticket will pay, should you buy it?
- Conditional on the noisy signal that the ticket pays, we can update the probabilities of outcomes as follows.

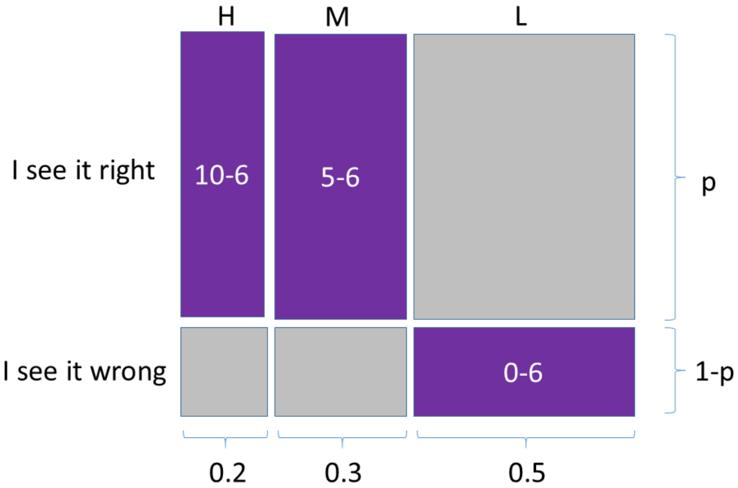
$$
\begin{aligned}
\Pr\left(H \middle| \text{noisy "pay"}\right) &= \frac{\Pr\left(H \text{ \& noisy "pay"}\right)}{\Pr\left(\text{noisy "pay"}\right)} \\
&= \frac{\Pr\left(H\right) \times p}{\left(\Pr\left(H\right) + \Pr\left(M\right)\right) \times p + \Pr\left(L\right) \times \left(1 - p\right)} \\
&= 0.4p.
\end{aligned}
$$

Value of Noisy Information

$$\begin{aligned}
\Pr(M|\text{noisy "pay"}) &= \frac{\Pr(M \,\&\, \text{noisy "pay"})}{\Pr(\text{noisy "pay"})} \\
&= \frac{\Pr(M) \times p}{(\Pr(H) + \Pr(M)) \times p + \Pr(L) \times (1-p)} \\
&= 0.6p.
\end{aligned}$$

$$\begin{aligned}
\Pr(L|\text{noisy "pay"}) &= \frac{\Pr(L \,\&\, \text{noisy "pay"})}{\Pr(\text{noisy "pay"})} \\
&= \frac{\Pr(L) \times (1-p)}{(\Pr(H) + \Pr(M)) \times p + \Pr(L) \times (1-p)} \\
&= 1-p.
\end{aligned}$$

Value of Noisy Information

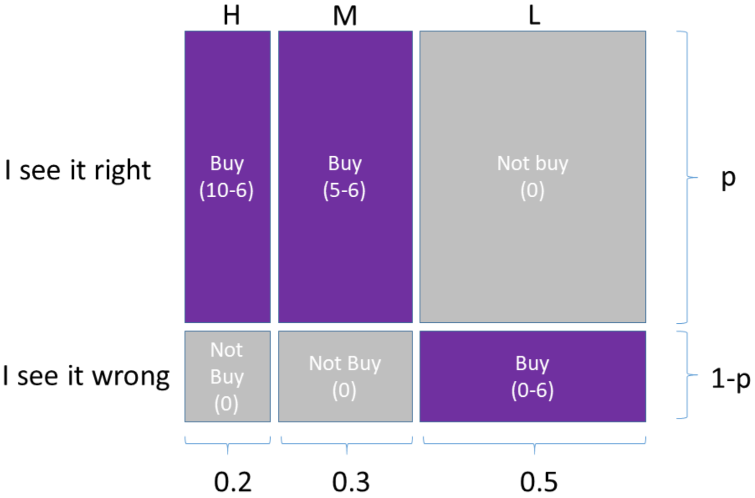


Value of Noisy Information

- The expected value of buying the ticket, conditional on the noisy "pay" signal, is thus

$$\begin{aligned} & \Pr\left(H|\text{noisy "pay"}\right) \times 4 + \Pr\left(M|\text{noisy "pay"}\right) \times (-1) \\ & + \Pr\left(L|\text{noisy "pay"}\right) \times (-6) \\ = \; & 0.4p \times 4 + 0.6p \times (-1) + (1-p) \times (-6) \\ = \; & 7p - 6. \end{aligned}$$

Value of Noisy Information



Value of Noisy Information

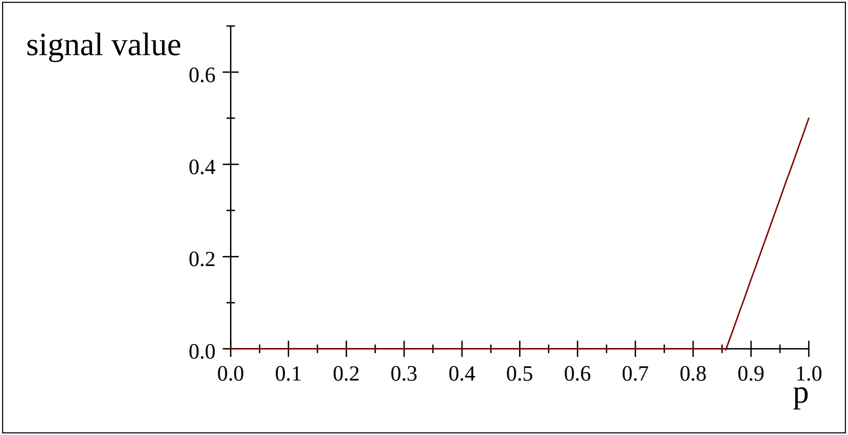
- If $p > 6/7$, you will buy the ticket if I tell you it will pay.
- You won't buy if I tell you it won't pay (Check!)
- In this case, your expected value equipped with my tip becomes

$$\begin{aligned} & \Pr(H) \times [p \times 4 + (1-p) \times 0] \\ & + \Pr(M) \times [p \times (-1) + (1-p) \times 0] \\ & + \Pr(L) \times [(1-p) \times (-6) + p \times 0] \\ = \; & 3.5p - 3. \end{aligned}$$

- Without any information, you do not buy the ticket, so the expected value you get is 0.
- The value of my signal is therefore $3.5p - 3$.

- On the other hand, if $p < 6/7$, the expected value of buying the ticket is negative regardless what my signal/tip is.
- You are not buying the ticket no matter what my tip is, so the expected value you get with my tip is 0 in any case.
- The value of my information is thus 0.
- Information is valuable only if it affects your decision.

Value of my (noisy) signal



Harmful Information

- Could information have a negative value?
    - Don't tell me the result!
    - Brother, sorry you know too much I can't let you exist.

Summary

- Decision-making under risk can be modelled as picking a lottery.
- One criterion for evaluating lotteries is the expected value/expected payoff.
- Signal structures can be modelled as partition of the state space.
- Informative signal improves our knowledge about the true state by allowing us to update our prior belief to form posterior belief.
- Information is valuable if it helps refines our decision-making.