ECON 3113 Microeconomic Theory I
Lecture 6: Demand

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Introduction

- What is the relationship between consumption and prices?
- What is the relationship between consumption and income?
- How to measure the strength of these relationship?
- How to measure the gains or losses of a consumer after a change in prices or income?

Demand Function

- Recall that the consumer's problem is

$$\max_{(x_1,x_2,...,x_n)\geq 0} u\left(x_1, x_2, ..., x_n\right)$$

subject to the budget constraint

$$p_1 x_1 + p_2 x_2 + ... + p_n x_n \leq I.$$

- The solution to the problem is dependent on prices p_1, p_2,..., p_n and income I.
- We will assume throughout that the preference is strictly convex, or equivalently DMRS is satisfied.
- An immediate implication is that the solution to the consumer's problem is unique.

Demand Function

- Denote this solution by

$$\begin{aligned} x_1 &= x_1\left(p_1, ..., p_n, I\right); \\ x_2 &= x_2\left(p_1, ..., p_n, I\right); \\ &.... \\ x_n &= x_n\left(p_1, ..., p_n, I\right). \end{aligned}$$

- These are the **demand functions** of the consumer.
- With only two goods, the demand functions are simply

$$x_1 = x_1\left(p_1, p_2, I\right) \text{ and } x_2 = x_2\left(p_1, p_2, I\right).$$

Demand Function: Homogeneity

- What happens to the consumer's demand if we double all the prices and incomes?
- The consumer's problem becomes

$$\max_{(x_1, x_2, ..., x_n) \geq 0} u\left(x_1, x_2, ..., x_n\right)$$

subject to the budget constraint

$$\lambda p_1 x_1 + \lambda p_2 x_2 + ... + \lambda p_n x_n \leq \lambda I.$$

- The problem is identical to the original one!
- So is the solution.

Theorem

The demand functions are **homogeneous of degree zero**. *That is,* $x_i\left(\lambda p_1, ..., \lambda p_n, \lambda I\right) = x_i\left(p_1, ..., p_n, I\right)$ *for all* $\lambda > 0$.

Demand Function: Walras' Law

- A bundle lying strictly within the budget set is clearly not optimal, if more goods is always preferred.

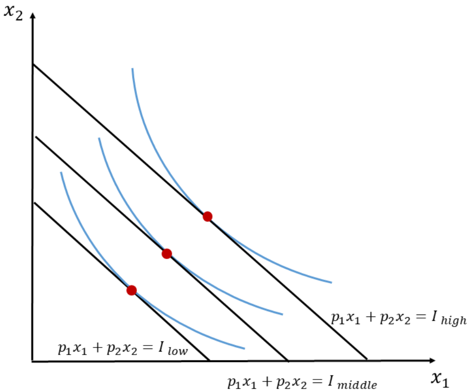
Theorem

If preference is monotone, the demand functions satisfy

$$p_1 x_1 (p_1, ..., p_n, I) + p_2 x_2 (p_1, ..., p_n, I) + ... + p_n x_n (p_1, ..., p_n, I) = I.$$

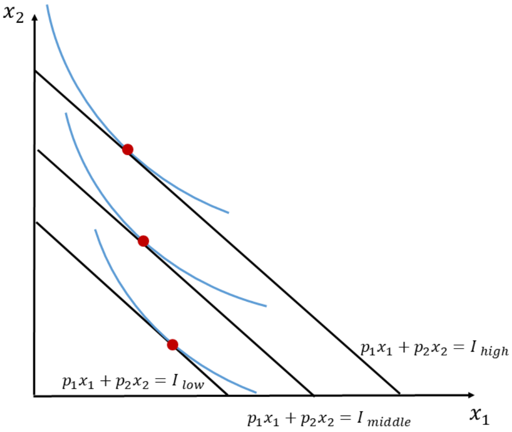
Changes in Income: Normal Goods

- Suppose the consumer's income increases, will she consume more of everything?
- An increase in income corresponds to a parallel outward shift of the budget line.
- These goods look normal.



Changes in Income: Inferior Goods

- Good 1 is called an inferior goods. E.g., instant noodles, budget airline flights, low-quality goods.



Normal Good vs Inferior Good

Definition

A good i is a **normal good** (in a range of income) if its consumption $x_i(p, I)$ increases with income, i.e., $\partial x_i / \partial I \geq 0$, in that range.
A good i is an **inferior good** (in a range of income) if its consumption $x_i(p, I)$ decreases with income, i.e., $\partial x_i / \partial I < 0$, in that range.

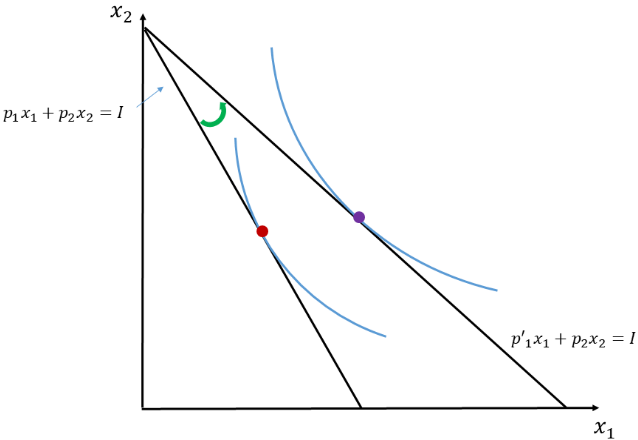
Changes in Prices

- Lower price $\Rightarrow$ more purchase?
  - aka Law of Demand
- A change in the price of a good has two effects:

1. it changes the "**exchange rate**" of the goods
2. it changes the consumer's **purchasing power**

- Suppose the price of goods 1 goes down.
  - She needs to give up less of goods 2 to acquire each extra unit of goods 1.
  - Her purchasing power goes up: she can afford every bundle previously affordable, plus something more.

- We can hypothetically decompose the change in budget line into two stages.
  - Rotation along the initial indifference curve
  - Parallel shift



Changes in Prices

- We can hypothetically decompose the change in budget line into two stages.
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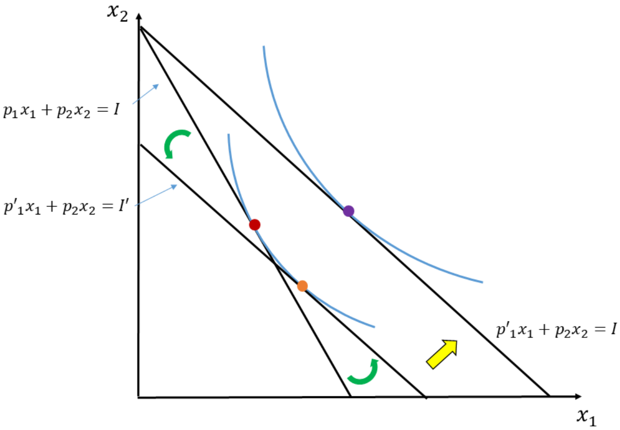
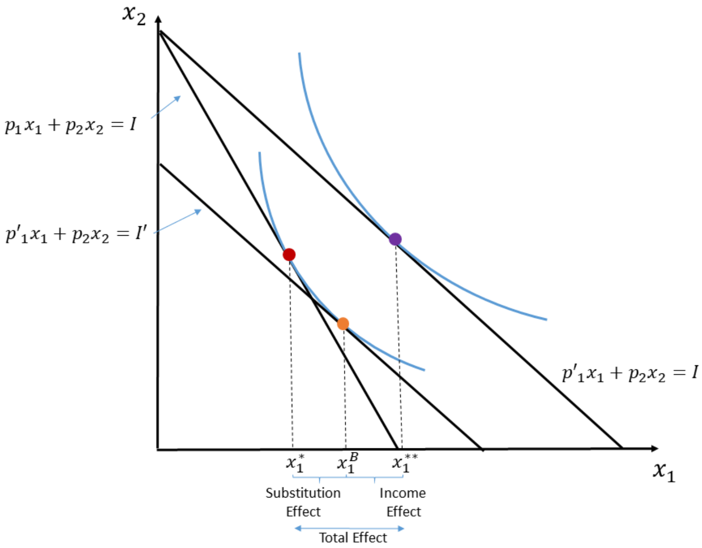
  - Parallel shift



Substitution and Income Effect

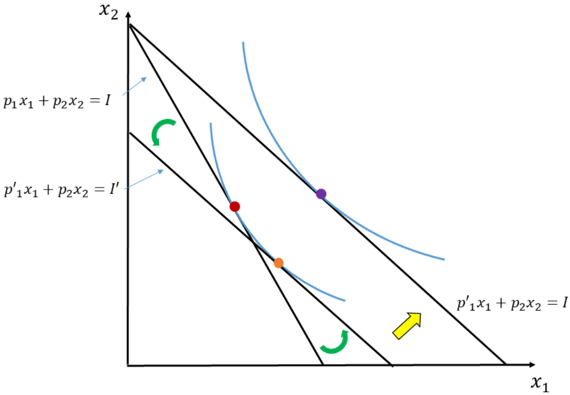
- The effect of a price change on demand can be (hypothetically) decomposed into the following two effects.
- **Substitution effect**: change in the demand due to a change in the price, holding utility constant.
- **Income effect**: change in demand due to a change in purchasing power, holding the relative prices constant.
- **Total effect**: sum of substitution effect and income effect.

Substitution and Income Effect



Substitution Effect

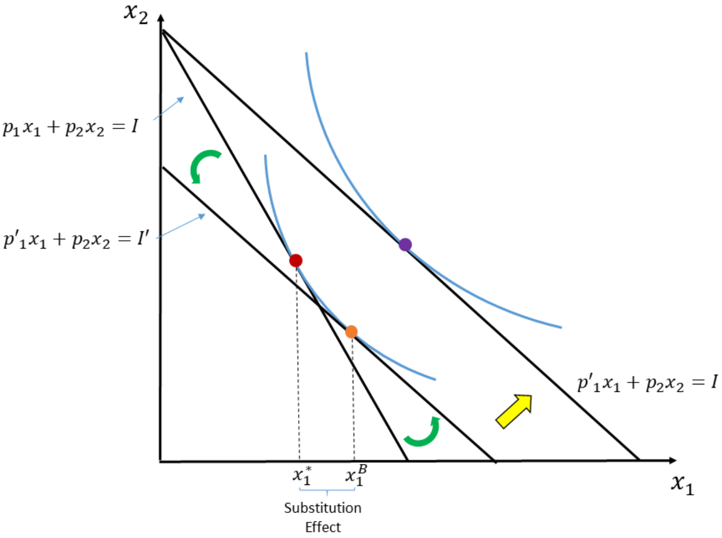
- Suppose the price of goods 1 decreases from p_1 to p_1'.
- The hypothetical budget line is $p_1' x_1 + p_2 x_2 = I'$, where **income is adjusted down** to some I' so that the consumer's attainable utility is preserved.



Substitution Effect

- Recall the optimal bundle (if interior) can be identified by equating MRS (slope of IC) with price ratio (p_1/p_2).
- As the consumer's preference satisfies DMRS, a reduction in p_1 always results in an increase in the optimal choice of x_1, holding utility fixed.
- As the substitution effect always drives price and quantity in opposite direction, **substitution effect is always negative**.

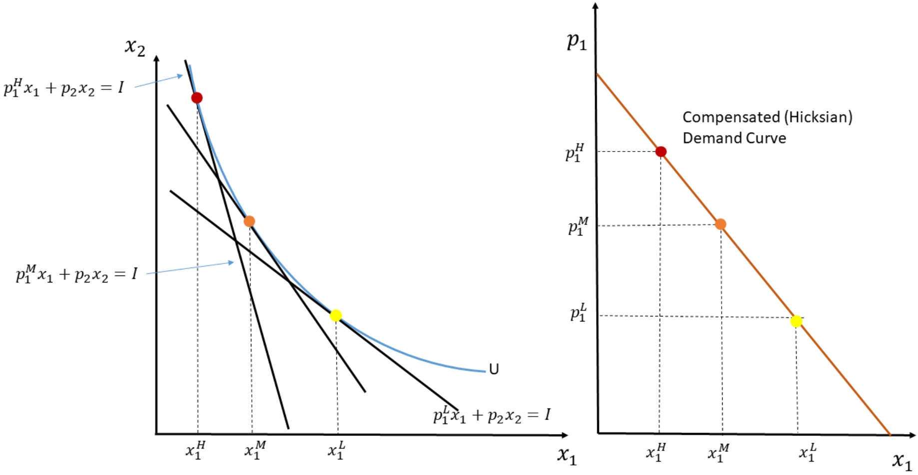
Substitution Effect



Compensated Demand Curve

- The **compensated (Hicksian) demand curve** of a consumer depicts the relationship between the price of a goods and her quantity of that goods, holding fixed the prices of other goods and the utility level.
- As the substitution effect is always negative, the compensated (Hicksian) demand curve is downward-sloping.

Compensated Demand Curve



Calculating the Substitution Effect

- Consider Cobb-Douglas utility $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$.
- Fix some utility level $\bar{u}$ and price of goods 2.
- What is the consumption bundle that satisfies the tangency condition while delivering a utility $\bar{u}$?

$$MRS = \frac{\alpha}{1-\alpha}\frac{x_2}{x_1} = \frac{p_1}{p_2} \text{ and } x_1^{\alpha} x_2^{1-\alpha} = \bar{u}.$$

- Substituting the second equation $\left(x_2 = \left(\bar{u} x_1^{-\alpha}\right)^{\frac{1}{1-\alpha}}\right)$ into the first and simplifying, we get

$$x_1 = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \bar{u}^{-1} \left(\frac{p_2}{p_1}\right)^{1-\alpha}.$$

- The compensated demand is thus

$$x_1^C(p_1, p_2, U) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} U^{-1} \left(\frac{p_2}{p_1}\right)^{1-\alpha}.$$

Calculating the Substitution Effect

- Consider quasi-linear utility $u(x_1, x_2) = v(x_1) + x_2$, with some strictly concave v.
- Fix some utility level $\bar{u}$ and price of goods 2 at $p_2 = 1$.
- What is the consumption bundle that satisfies the tangency condition while delivering a utility $\bar{u}$?

$$MRS = v'(x_1) = p_1 \text{ and } v(x_1) + x_2 = \bar{u}.$$

- For intermediate p_1 such that the solution is interior,

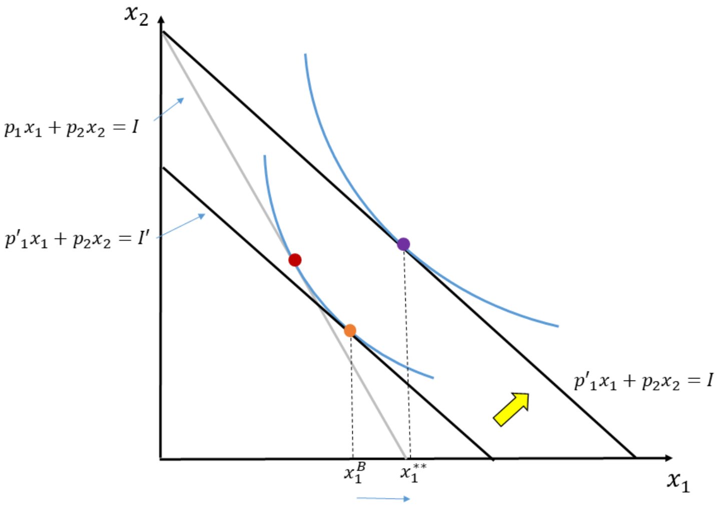
$$x_1^C(p_1, p_2, U) = (v')^{-1}(p_1).$$

- It coincides with the regular (uncompensated) demand!

Income Effect

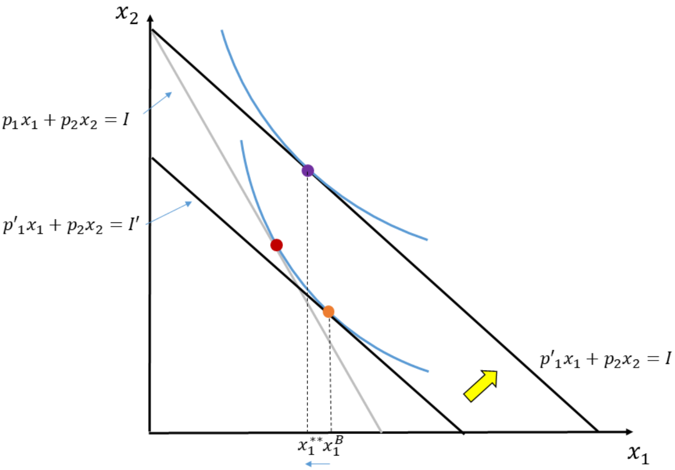
- Continue to consider a decrease in the price of goods 1 from p_1 to p_1'.
- Now consider the second stage of budget-line adjustment: parallel outward shift.
- If good 1 is a *normal goods*, this parallel outward shift of the budget line increases the consumption of goods 1.

Income Effect: Normal Goods



Income Effect: Inferior Goods

- If good 1 is an *inferior goods*, this parallel outward shift of the budget line decreases the consumption of goods 1.



Income Effect

- The direction of the income effect therefore depends on whether the goods is normal or inferior.
- Normal goods:
  lower price $\Rightarrow$ stronger purchasing power $\Rightarrow$ increase in consumption
- Inferior goods:
  lower price $\Rightarrow$ stronger purchasing power $\Rightarrow$ decrease in consumption

Calculating the Income Effect

- Consider again the Cobb-Douglas utility $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$.
- Fix the prices at some (p_1, p_2).
- How does the optimal bundle changes in response to income changes?
- Recall that when the consumer has income I, the optimal bundle has

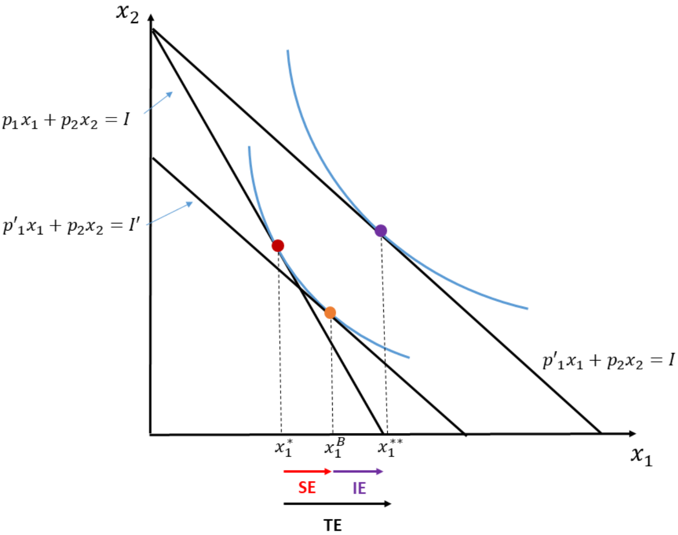
$$x_1(p_1, p_2, I) = \alpha \frac{I}{p_1}.$$

- Goods 1 is therefore a normal good, as its consumption always increases in I, i.e., the income effect is positive.

Total Effect

- Total effect is the sum of the substitution effect and the income effect.
- If the goods is a normal goods, both the substitution effect and the income effect operate in the same direction.
- Lower price $\Rightarrow$ higher quantity purchased: law of demand always holds for normal goods.

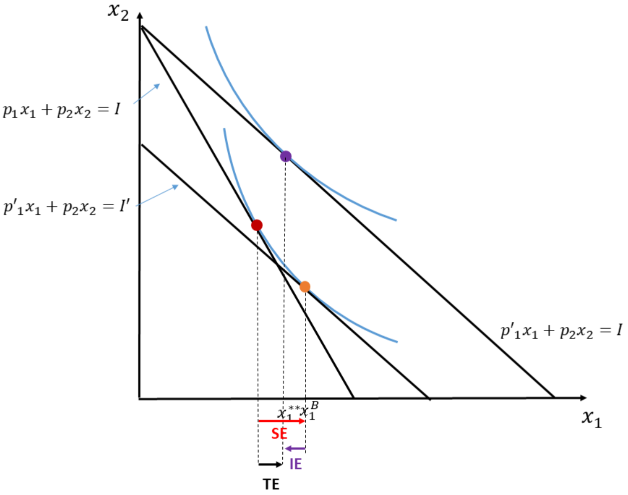
Total Effect: Normal Goods



Total Effect: Inferior Goods

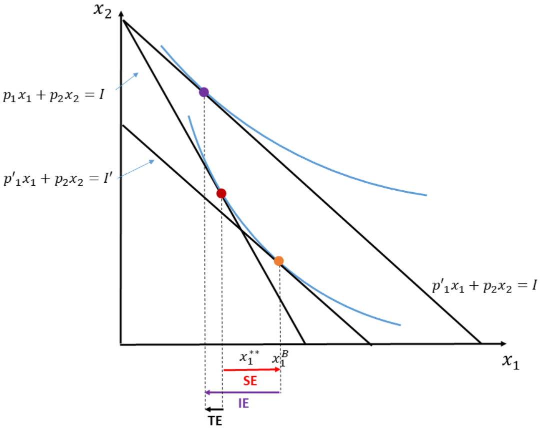
- If the goods is an inferior goods, the substitution effect and the income effect operate in opposite direction.
- Lower price may lead to higher or lower demand, depending on the relative strength of the two effects.

- If the substitution effect is stronger than the income effect, quantity demanded goes up following a price decrease.



Total Effect: Giffen Goods

- If the income effect is stronger than the substitution effect, quantity demanded goes down following a price decrease.
- In this case, we say the goods is a Giffen good.
  - No convincing evidence of its existence in the real world.

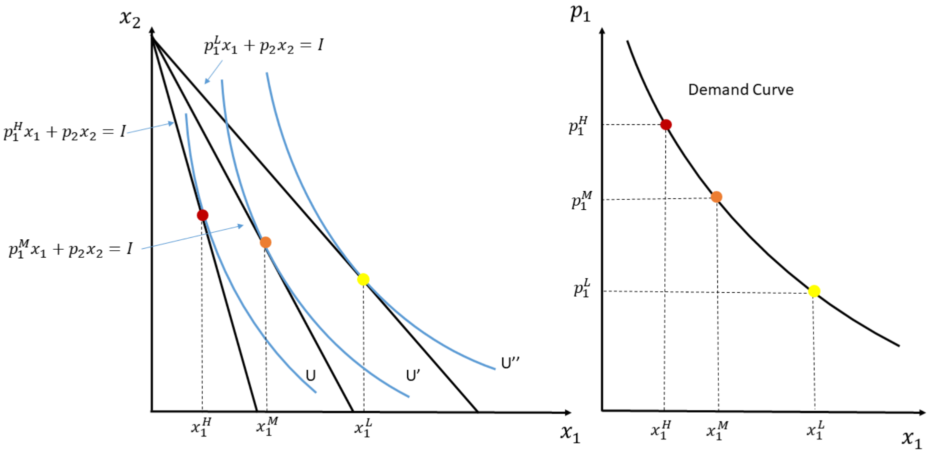


Interim Summary

- The effect of a price change on demand can be decomposed into the substitution effect and the income effect, by hypothetically decomposing the adjustment of the budget line into two steps.
    - Bear in mind this decomposition is a theoretical construct.
- Sign of the effects

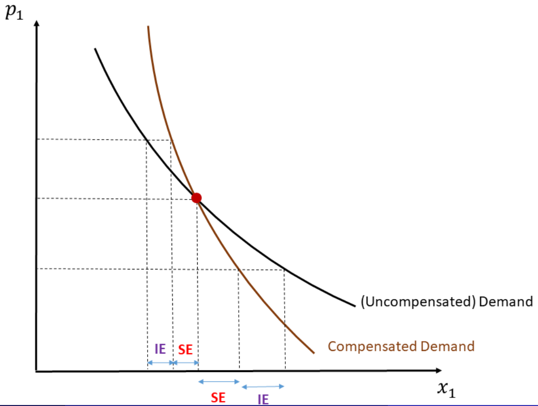
	Substitution Effect	Income Effect	Total Effect
Normal	-	-	-
Inferior, non-Giffen	-	+	-
Giffen	-	+	+

Tracing the Demand Curve Graphically



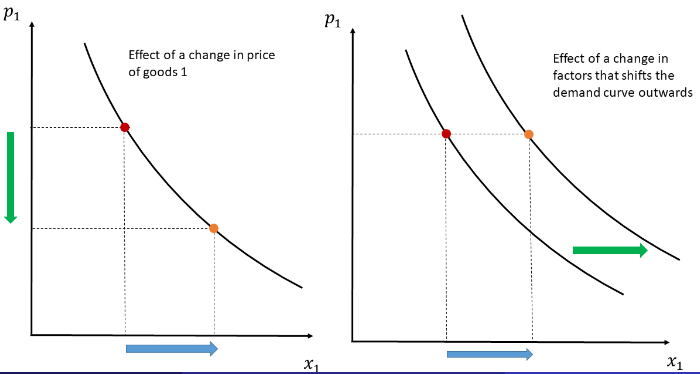
Demand Curve vs Compensated Demand Curve

- Compensated demand curve captures only the substitution effect.
- (Uncompensated) Demand curve captures both the substitution effect and the income effect.
- Therefore, for a normal goods, the demand curve is flatter than the compensated demand curve.



Movement along Demand Curve vs Shift of Whole Curve

- The demand curve plots how the good's demand varies with its own price, holding all other things constant (other goods' prices, income and preference).
- Any change in other goods' prices, the consumer's income or preference will shift the whole demand curve.

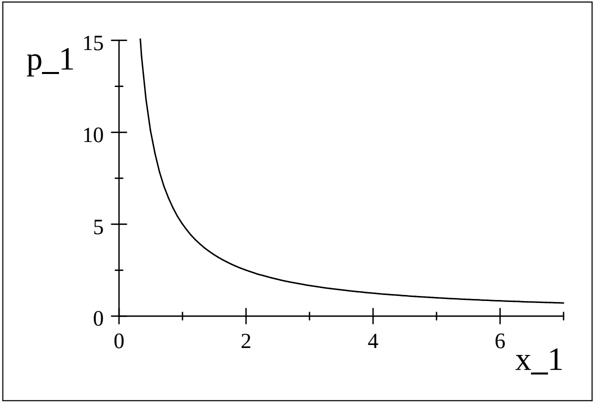


Movement along Demand Curve vs Shift of Whole Curve

- Recall the optimal consumption bundle given Cobb-Douglas utility is

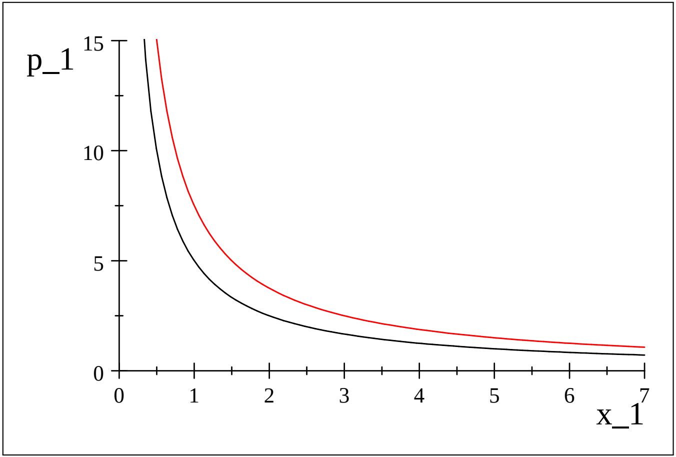
$$x_1(p_1, p_2, I) = \alpha \frac{I}{p_1} \text{ and } x_2(p_1, p_2, I) = (1-\alpha)\frac{I}{p_2}.$$

- With $\alpha = 0.5$ and $I = 10$, the demand curve of goods 1 looks



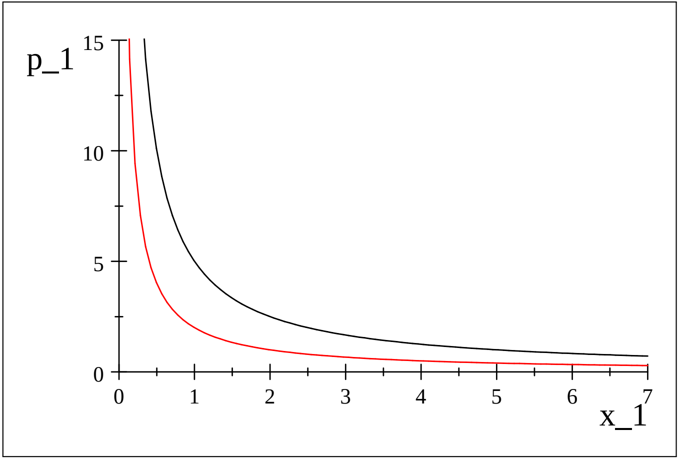
Movement along Demand Curve vs Shift of Whole Curve

- Now suppose the consumer's income increases to 15, the new demand curve looks



Movement along Demand Curve vs Shift of Whole Curve

- Next suppose α decreases to 0.2 so that the consumer becomes less interested in goods 1, the new demand curve looks



(Own-)Price Elasticity of Demand

- A measure of the sensitivity of the demand to price changes.

Definition

Price elasticity of demand of a goods is the percentage change in its quantity demanded in response to a unit percentage change in its price. In notation,

$$\varepsilon_{x_1,p_1} = \frac{\triangle x_1 / x_1}{\triangle p_1 / p_1} = \frac{\triangle x_1}{\triangle p_1}\frac{p_1}{x_1} = \frac{\partial x_1\left(p_1, p_2, I\right)}{\partial p_1}\frac{p_1}{x_1}.$$

(Own-)Price Elasticity of Demand

Example

With Cobb-Douglas utility $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$, the demand function of goods 1 is $x_1(p_1, p_2, I) = \alpha \frac{I}{p_1}$.

$$\varepsilon_{x_1, p_1} = \frac{\partial x_1(p_1, p_2, I)}{\partial p_1} \frac{p_1}{x_1} = \left(-\alpha \frac{I}{p_1^2}\right) \frac{p_1}{x_1} = -\frac{\alpha I}{p_1 x_1} = -1.$$

The demand is "unit-elastic."

Example

Suppose the demand function is linear: $x_1(p_1, p_2, I) = I - ap_1 + bp_2$.

$$\varepsilon_{x_1, p_1} = \frac{\partial x_1(p_1, p_2, I)}{\partial p_1} \frac{p_1}{x_1} = -a \frac{p_1}{x_1} = -a \frac{p_1}{I - ap_1 + bp_2},$$

so demand is more elastic at higher (own) prices.

Income Elasticity of Demand

- A measure of the sensitivity of the demand to income changes.

Definition

Income elasticity of demand of a goods is the percentage change in its quantity demanded in response to a unit percentage change in income. In notation,

$$\varepsilon_{x_1,I} = \frac{\triangle x_1 / x_1}{\triangle I / I} = \frac{\triangle x_1}{\triangle I}\frac{I}{x_1} = \frac{\partial x_1\left(p_1, p_2, I\right)}{\partial I}\frac{I}{x_1}.$$

Income Elasticity of Demand

Example

With Cobb-Douglas utility, the demand function of goods 1 is $x_1\left(p_1, p_2, I\right) = \alpha \frac{I}{p_1}$, so

$$\varepsilon_{x_1, I} = \frac{\partial x_1\left(p_1, p_2, I\right)}{\partial I} \frac{I}{x_1} = \left(\frac{\alpha}{p_1}\right) \frac{I}{x_1} = 1.$$

Example

With linear demand function, $x_1\left(p_1, p_2, I\right) = I - ap_1 + bp_2$, and

$$\varepsilon_{x_1, I} = \frac{\partial x_1\left(p_1, p_2, I\right)}{\partial I} \frac{I}{x_1} = \frac{I}{I - ap_1 + bp_2}.$$

Cross-price Elasticity of Demand

- A measure of the sensitivity of the demand to changes in prices of other goods.

Definition

Cross-price elasticity of demand of a goods is the percentage change in its quantity in response to a unit percentage change in the price of some other good. In notation,

$$\varepsilon_{x_1,p_2} = \frac{\triangle x_1 / x_1}{\triangle p_2 / p_2} = \frac{\triangle x_1}{\triangle p_2}\frac{p_2}{x_1} = \frac{\partial x_1\left(p_1, p_2, I\right)}{\partial p_2}\frac{p_2}{x_1}.$$

Cross-price Elasticity of Demand

Example

With Cobb-Douglas utility, the demand function of goods 1 is $x_1(p_1, p_2, I) = \alpha \frac{I}{p_1}$, so

$$\varepsilon_{x_1, p_2} = \frac{\partial x_1(p_1, p_2, I)}{\partial p_2} \frac{p_2}{x_1} = 0 \left(\frac{p_2}{x_1} \right) = 0.$$

Example

With linear demand function, $x_1(p_1, p_2, I) = I - ap_1 + bp_2$, and

$$\varepsilon_{x_1, p_2} = \frac{\partial x_1(p_1, p_2, I)}{\partial p_2} \frac{p_2}{x_1} = \frac{bp_2}{I - ap_1 + bp_2}.$$

Connection of Demand Elasticities: Homogeneity

- Recall demand function is homogeneous of degree zero, i.e., $x_1(\lambda p_1, \lambda p_2, \lambda I) = x_1(p_1, p_2, I)$ for all $\lambda > 0$.
- Differentiate this equation with respect to λ gives:

$$\frac{\partial x_1}{\partial p_1} \times p_1 + \frac{\partial x_1}{\partial p_2} \times p_2 + \frac{\partial x_1}{\partial I} \times I = 0.$$

- Dividing this equation by x_1, we get

$$\varepsilon_{x_1, p_1} + \varepsilon_{x_1, p_2} + \varepsilon_{x_1, I} = 0.$$

Connection of Demand Elasticities: Engel Aggregation

- Recall demand functions satisfy Walras' law, i.e., $p_1 x_1 (p_1, p_2, I) + p_2 x_2 (p_1, p_2, I) = I$.
- Differentiate this equation with respect to income gives:

$$p_1 \frac{\partial x_1 (p_1, p_2, I)}{\partial I} + p_2 \frac{\partial x_2 (p_1, p_2, I)}{\partial I} = 1.$$

- By definitions, $\varepsilon_{x_1, I} = \frac{\partial x_1}{\partial I} \times \frac{I}{x_1}$ and $\varepsilon_{x_2, I} = \frac{\partial x_2}{\partial I} \times \frac{I}{x_2}$, so

$$\frac{p_1 x_1}{I} \varepsilon_{x_1, I} + \frac{p_2 x_2}{I} \varepsilon_{x_2, I} = 1.$$

- Denote $s_i = p_i x_i / I$ as the share of income spent on goods i. The equation above can be simplified into

$$s_1 \varepsilon_{x_1, I} + s_2 \varepsilon_{x_2, I} = 1.$$

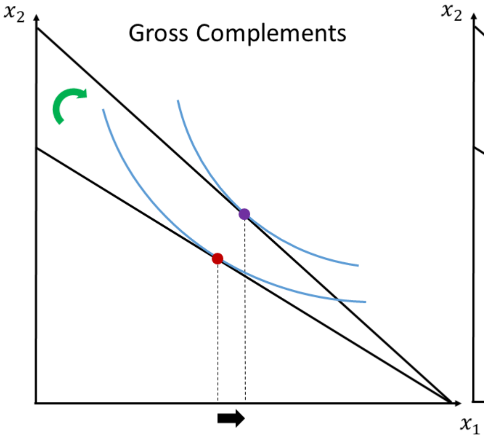
Substitutes and Complements

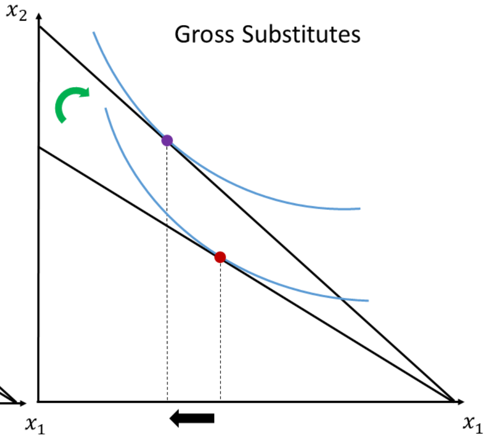
- It is natural to expect that if a consumer views two goods as substitutable, she consumes more of goods 1 if the price of goods 2 goes up.
- Conversely, if the consumer views two goods as complementary, she consumes less of goods 1 if the price of goods 2 goes up.

Definition

Good i is a **gross substitute** for goods j if an increase in the price of goods j increases the quantity of consumption of goods i.
Good i is a **gross complement** to goods j if an increase in the price of goods j decreases the quantity of consumption of goods i.

Substitutes and Complements





Substitutes and Complements

- These definitions turn out to be not always helpful because of asymmetry.
  - It is possible that goods i is a gross substitute of goods j but at the same time, goods j is a gross complement of goods i.
  - See textbook p.187 for an example.
- An alternative definition that overcomes the asymmetry problem above is to use compensated demands. The corresponding concepts are called net substitutes and net complements.

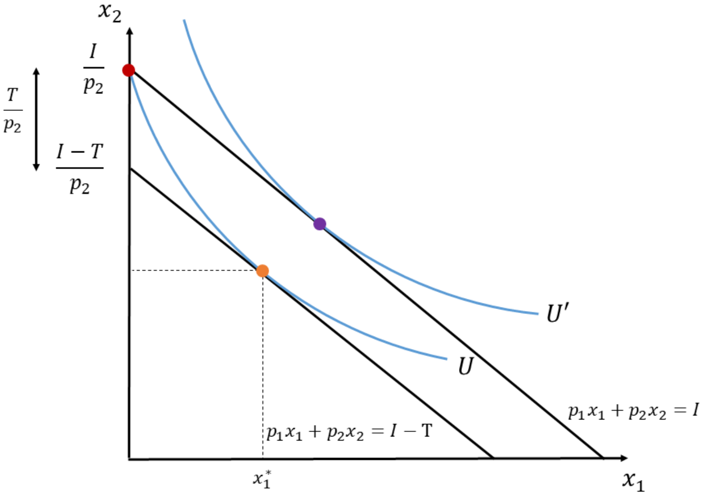
Welfare Analysis

- By how much does a consumer gain from having access to a market?
- Utility is not a cardinal concept, so it can't be used as a welfare measure.
- Let's reframe this question: "How much income is the consumer willing to give up in order to acquire access to a market?"

Welfare Analysis

- Initially the consumer does not have access to the market of goods 1 and so can consume only $(0, I/p_2)$.
  - Suppose this bundle gives her utility U.
- Suppose we offer her the deal "We can let you access market of goods 1 but you have to pay a lump sum T."
- What is the maximum amount of T that the consumer is willing to pay?
- Answer: T such that the optimal bundle given the budget set $\{x : p_1x_1 + p_2x_2 \leq I - T\}$ delivers exactly utility U.
- This is called the **consumer surplus**.

Welfare Analysis



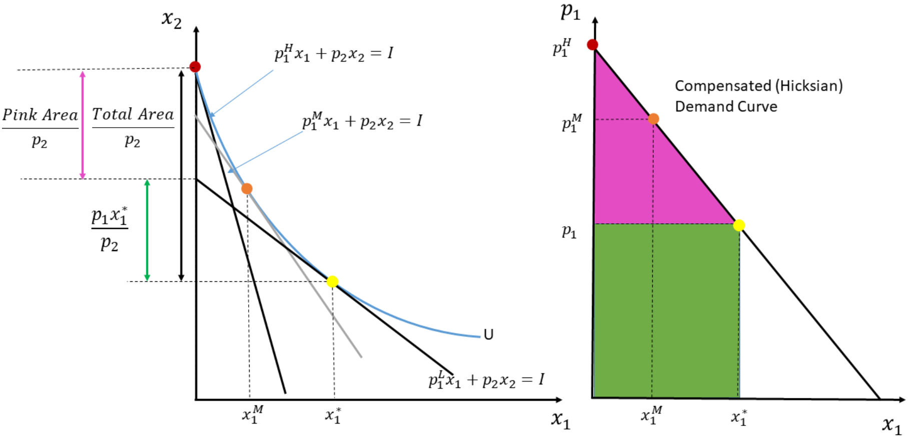
Welfare Analysis

- Another way to compute the consumer surplus is to use the compensated demand.
- Recall compensated demand fixes a utility level, say U, and ask how the quantity of goods 1 (say) changes in response to p_1.
    - obtained by solving

$$MRS\left(x_1, x_2\right) = \frac{p_1}{p_2} \text{ and } u\left(x_1, x_2\right) = U.$$

- Integrating the slope of a function returns the function itself.
- The area under the compensated demand therefore gives us the "height" of the indifference curve.
- The consumer surplus is equal to the pink area.

Welfare Analysis



Welfare Analysis: Remarks

- In some welfare analyses, the consumer surplus is evaluated using the uncompensated demand curve.
- This is completely legitimate if the utility function is quasi-linear in the goods under study.
  - With quasi-linear utility, the compensated demand coincides with regular (uncompensated) demand.
- Otherwise, this is only an approximation.

Summary

- The effect of a price change can be decomposed into the substitution effect (due to a change in exchange rate) and the income effect (due to an increase in purchasing power).
- If the income effect is positive or mildly negative, the law of demand holds.
- Elasticities of demand are measures of the responsiveness of demand to changes in own price, other goods' prices or income.
- Consumer surplus, given by the area under the compensated demand curve (less the "expenditure box"), is a welfare measure.
  - Though in practice, the area under the demand curve is used as an approximation (which is valid provided that the income effect is not strong).