Homework 8

Due on Dec 10

Choose the best answer

1. Each of the following provides incentives to reduce a negative externality except
   a. merger with affected firms.
   b. subsidizing consumption of the good being produced.
   c. bargaining among firms.
   d. taxation of the externality.

2. Externalities between two firms can be "internalized" if:
   I. The two firms merge.
   II. The property right arrangement is clear and transaction costs are zero.
   III. The externalities affect each firm equally.
   IV. Marginal costs for both firms are constant.

   Which statement(s) correctly complete the sentence?
   a. Only II.
   b. All except III.
   c. I and II, but not III and IV.
   d. I and IV, but not II and III.

3. A non-exclusive good is a good which
   a. is sold in low-price markets.
   b. is impossible to keep people from enjoying the benefits the good provides.
   c. is produced by a perfectly competitive firm.
   d. is produced at the lowest possible cost.

4. A non-rival good is a good which
   a. is produced by a monopoly.
   b. is produced by a cartel.
   c. can provide benefits to additional users at a zero marginal cost.
   d. is sold in a single market.

Analytical questions

1. Consider the case described in Varian Ch 35: There is a steel firm (S) and fishery firm (F) located along the same river. Denote x as the amount of pollution. Firm S's cost of producing s unit of steel is $c_s(s,x) = s^2 x^{-\frac{1}{2}}$. Firm F's cost of fishing f unit of fish is $c_f(f,x) = f^2 x^{\frac{1}{2}}$. The maximum amount of pollution firm S can emit is $\bar{x} = 16$. The price of steel is $p_s = 4$ per unit and the price of fish is $p_f = 8$ per unit.

a. Write down the profit function of firm S and find out the level of steel production and pollution.

b. Given the profit maximizing behavior of firm S, how many units of fish will firm F produce?

c. Find the Pareto (social) optimal level of production and pollution (s^*, f^*, x^*).

2. Two roommates $i = 1, 2$ are considering to buy a TV and use it together. TV is a public good and the spending is contributed by two roommates, $G = g_1 + g_2$. Each roommate need to divide its income w_i into his private spending x_i and public spending g_i, that is, $x_i + g_i = w_i$. Roommate 1 has income $w_1 = 100$ and his utility function is

$$u_1(x_1, G) = x_1^{\frac{1}{2}} G^{\frac{1}{2}}.$$

Roommate 2 has income $w_2 = 60$ and his utility function is

$$u_2(x_2, G) = x_2^{\frac{1}{3}} G^{\frac{2}{3}}.$$

a. Assume that roommate 2 decides not to pay for the TV, $g_2 = 0$. Roommate 1 is selfish and maximizes his utility. How he will choose g_1 and x_1?

b. Solve for the non-cooperative (Nash equilibrium) allocation, $(x_1^{NE}, g_1^{NE}, x_2^{NE}, g_2^{NE})$.

c. Fix roommate 2's utility at $\bar{u}_2$. Find three equations characterize the Pareto efficient allocation $(x_1^{PE}, x_2^{PE}, G^{PE})$.

[You won't be able to solve for a specific allocation because the equations will contain $\bar{u}_2$.]

3. Consider a society with an electricity company and a representative consumer (all households). The electricity company uses labor l units and coal x units to produce electricity q units. The company's production function is

$$q = f(l, x) = 12 \times l^{\frac{1}{3}} x^{\frac{1}{2}}.$$

So its profit is

$$\pi(l, x) = 12 l^{\frac{1}{3}} x^{\frac{1}{2}} - wl - vx$$

The representative consumer's payoff is

$$u(l, x) = (w - 2)l - x,$$

which is measured in dollars. So the consumer is harmed by pollution from coal burning.

There is no market power: All prices are considered as fixed, and all electricity produced are being consumed. Each unit of labor costs $w = 4$, and each unit of coal costs $v = 3$. Electricity price is $p = 1$.

a. To maximize profit, how will the electricity company choose l and x. Compute the company's profit and the consumer's payoff.

b. Consider the company and the consumer together as a society. Determine the social efficient level of l and x.

[Hint: the social welfare is the summation of the company's profit and the representative consumer's payoff.]

c. Compute the social welfare in part (a) and part (b) and compare them.

d. Suppose the electricity company is selfish and only care its own profit. Using the concept of externality, how will you describe the electricity company's usage of coal? As a policy maker, how to fix the probem?

[Proposing one solution is enough.]

4. There are $n \geq 2$ people in a residential community indexed by $i = 1, 2, ..., n$. Each of them needs to contribute to a joint account for the maintenance of the facilities in the community. This joint account can be considered as a public good.

Each person i needs to determine to spend his income w_i between his private consumption y_i and joint account contribution x_i, i.e., $x_i + y_i = w_i$. Each person has the same utility function

$$U_i(X, y_i) = Xy^2,$$

where $X = \sum_{i=1}^{n} x_i$. Each person has an income $w_i = 120$.

a. Consider that n people voluntarily contributes to the joint account. Find the non-cooperative equilibrium $(x_1^{NE}, x_2^{NE}, ..., x_n^{NE}, y_1^{NE}, y_2^{NE}, ..., y_n^{NE})$. Note that the solution depends on n.

b. Assume that each person contribute the equal amount. What is the social optimal (Pareto efficient) allocation $(X^{SO}, y_1^{SO}, y_2^{SO}, ..., y_n^{SO})$? The solution also depends on n.

c. Compare the joint account amount under non-cooperative equilibrium (X^{NE}) and social optimal level (X^{SO}). When the number of people n increases, does the difference becomes larger or smaller? Does the free-rider problem become more severe when there are more people in the community?