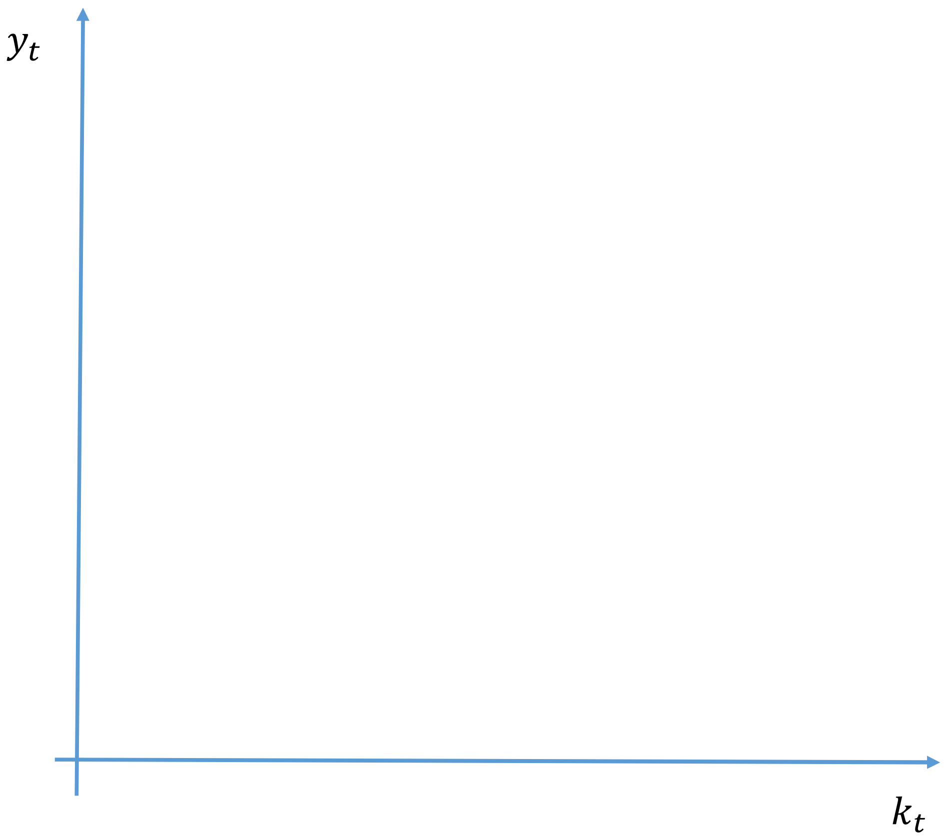
ECON3123
Macroeconomic Theory I

Tutorial #12: The Solow Growth Model

Today's tutorial

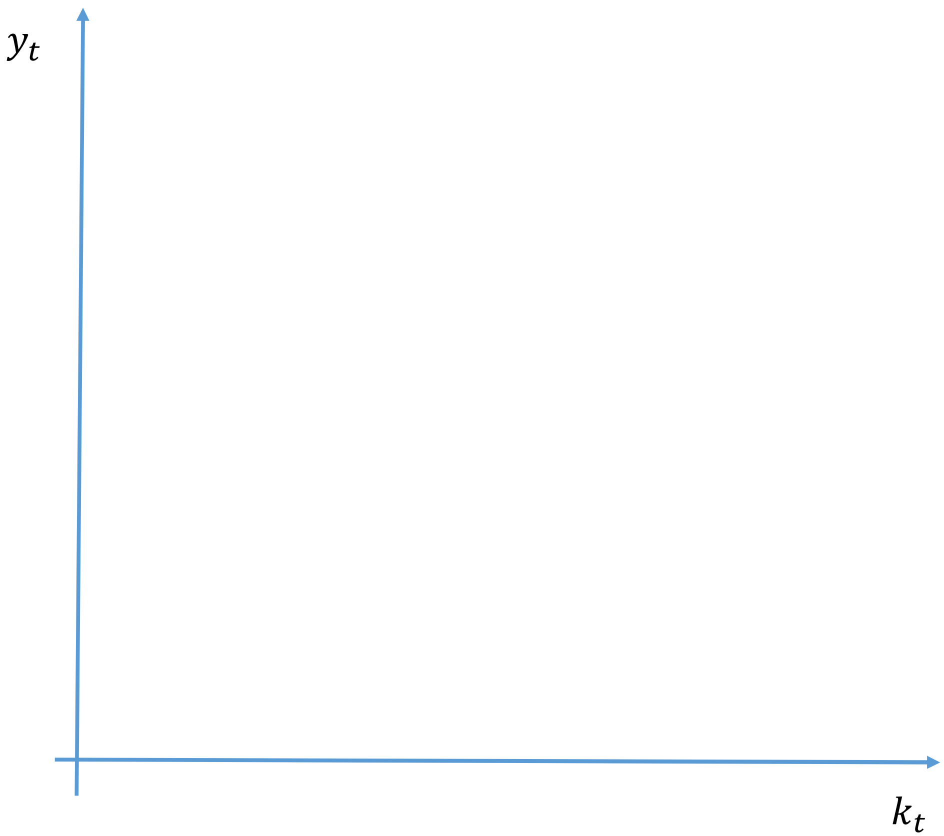
- Resuming from last week:
  - Re-cap on equilibrium in the model
  - Example: a decrease in the savings rate
  - The Golden Rule: savings rate and consumption
- Examples and exercises

The model: principal equations



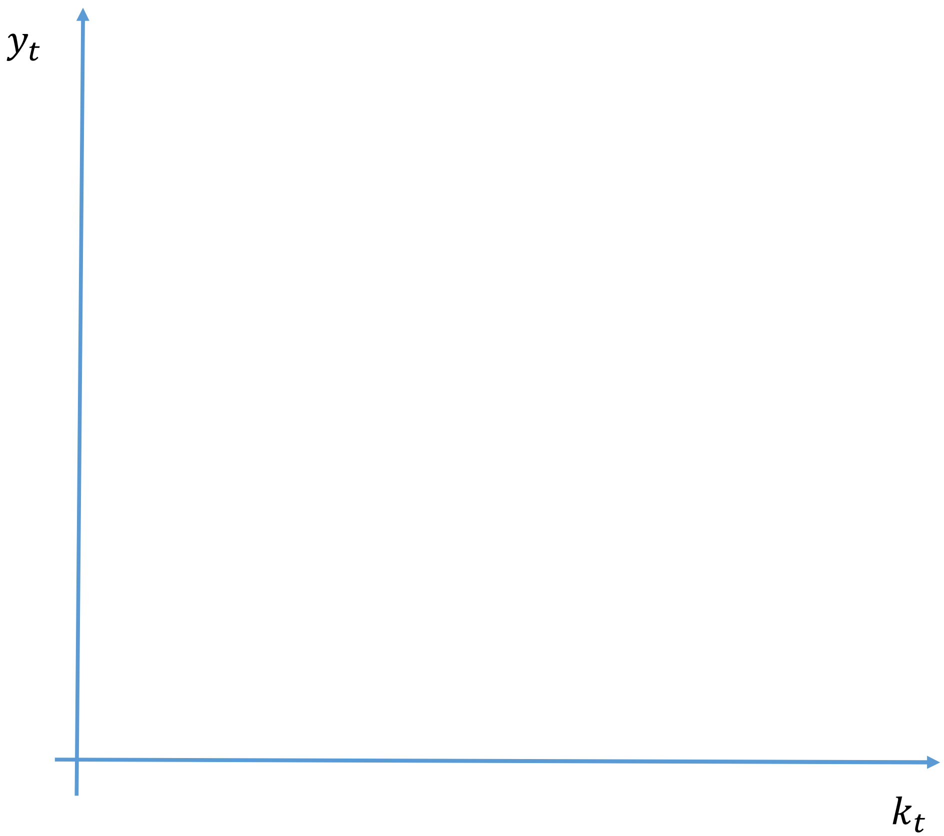
- Production function:
  - $y_t = f(k_t)$
  - eg $y_t = Ak_t^{\alpha}$
- Investment and savings
  - $S_t/N = I_t/N = sy_t$
- Depreciation
  - $\text{Depreciation}_t = \delta k_t$

The model: equilibrium and the steady state



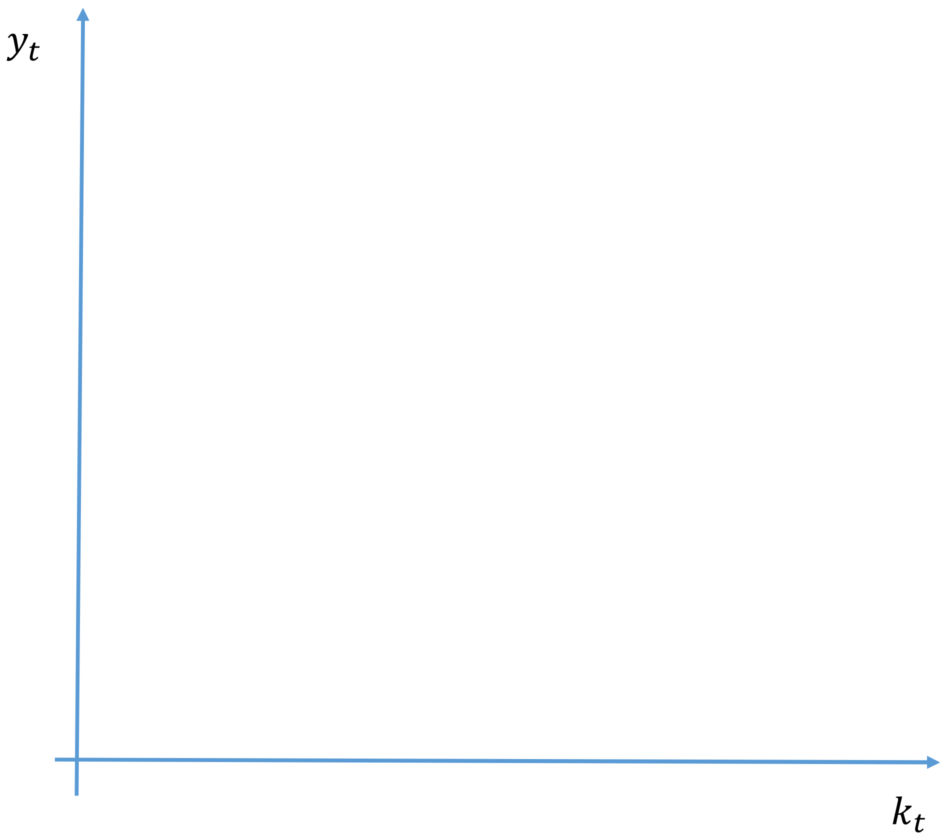
- Capital accumulation
  - $k_{t+1} = (1-\delta)k_t + sy_t$
  - $\Delta k_{t+1} = k_{t+1} - k_t = sy_t - \delta k_t$
- Steady-state equilibrium
  - y_t^*, k_t^* such that $\Delta k_{t+1} = 0$:
  - $sy_t^* = \delta k_t^*$
- This happens where the δk_t line crosses the sy_t line
- At this steady-state point, investment is replacing depleted capital

Model dynamics (1)



1) $k_t < k_t^*$:
   - $\Delta k_{t+1} => 0 \Rightarrow k_{t+1} > k_t$ and $sy_t > \delta k_t$
   - Therefore, investment at time t is adding to the capital stock at $t+1$
   - This process continues until $k_t = k_{t+1}$

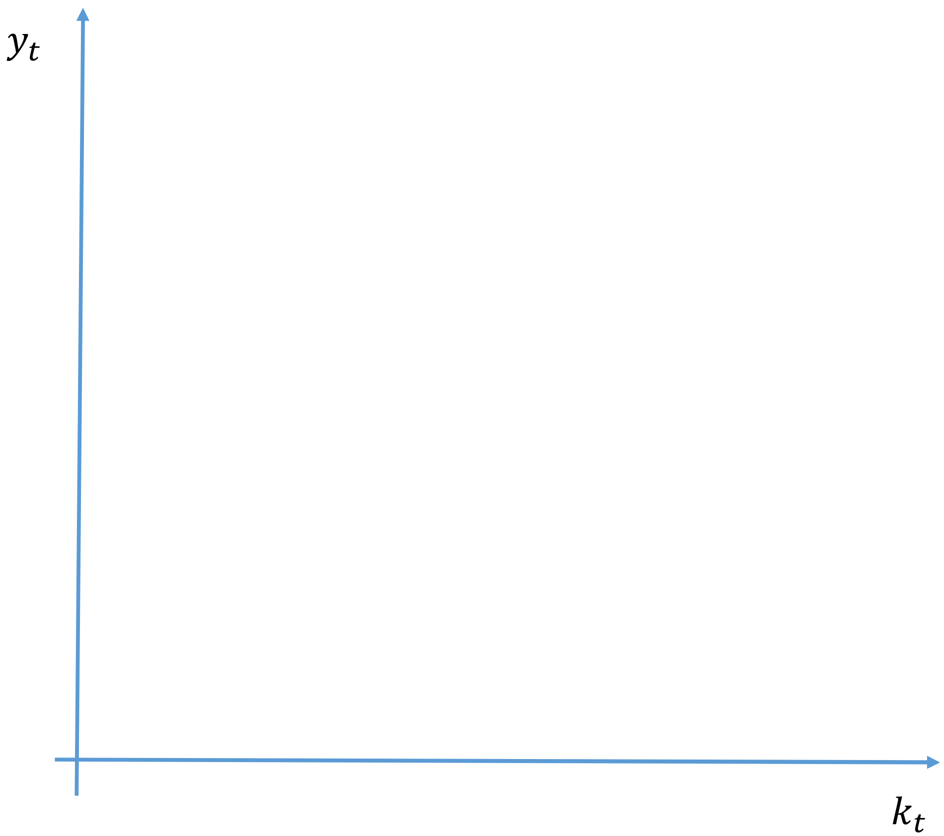
Model dynamics (2)



2) $k_t > k_t^*$:

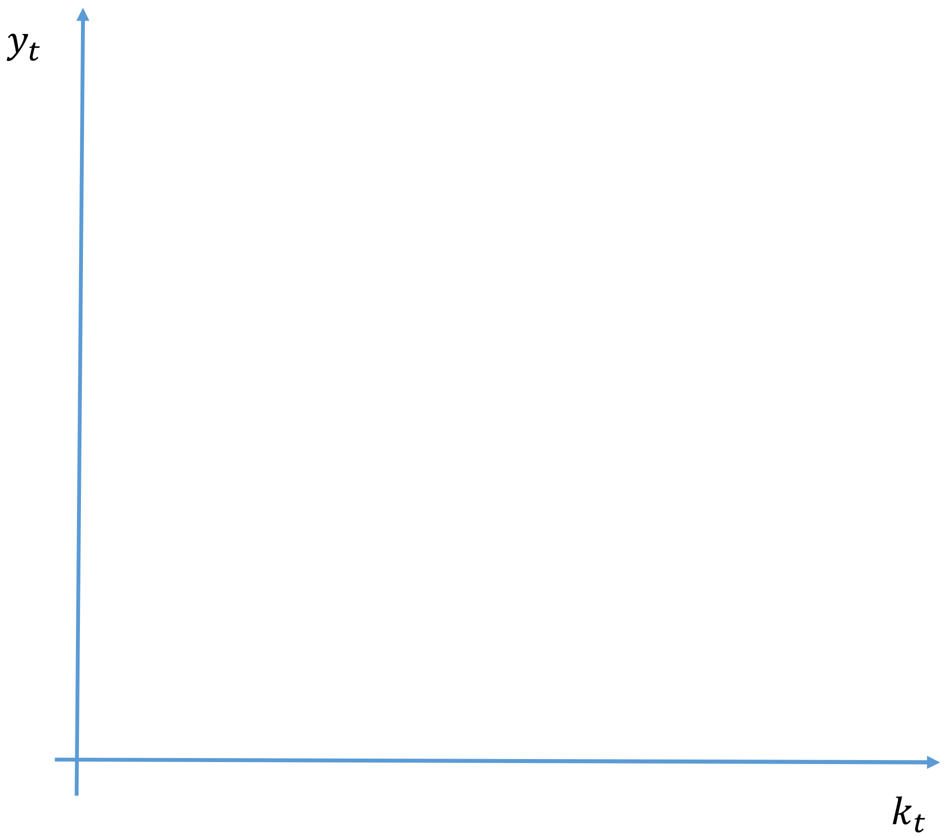
- $\Delta k_{t+1} < 0 \Rightarrow k_{t+1} < k_t \Rightarrow sy_t < \delta k_t$
- Therefore, investment at time t is less than depreciation, and the capital stock is falling
- This process continues until $k_t = k_t^*$

The steady state and the role of the savings rate



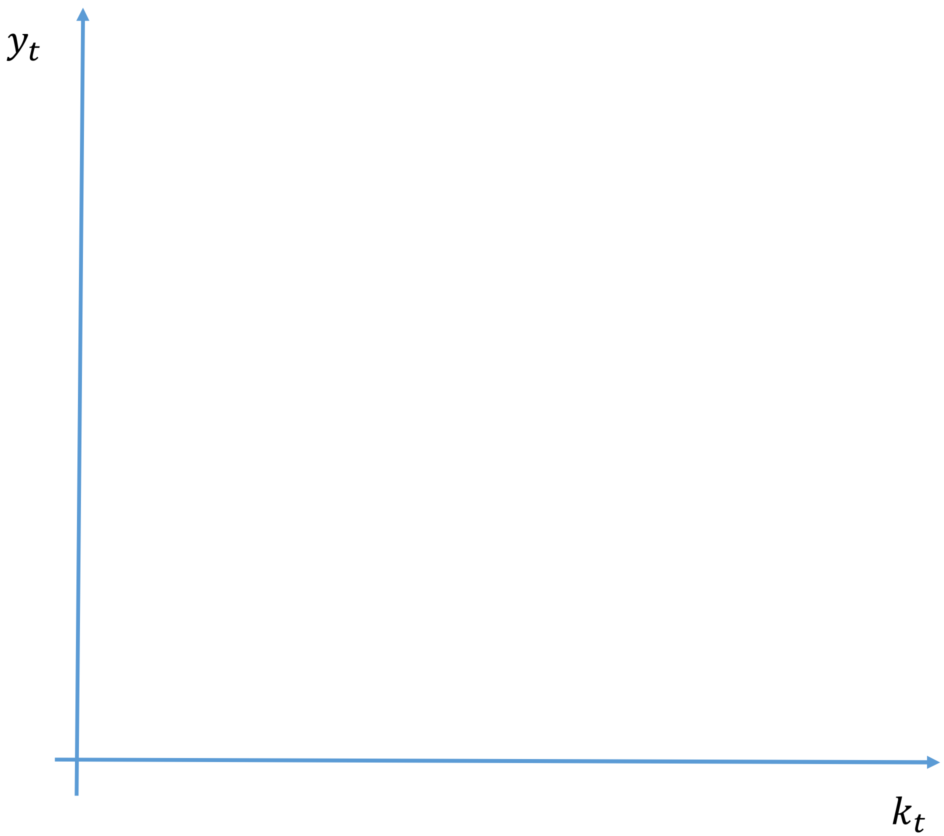
- Now we focus on the steady state
- For given values of A, N and δ, the savings rate, s, determines the steady state values:
  - k_t^*
  - y_t^*
- Different values of s give different values for k_t^*, y_t^*

Consumption in the model and in the steady state



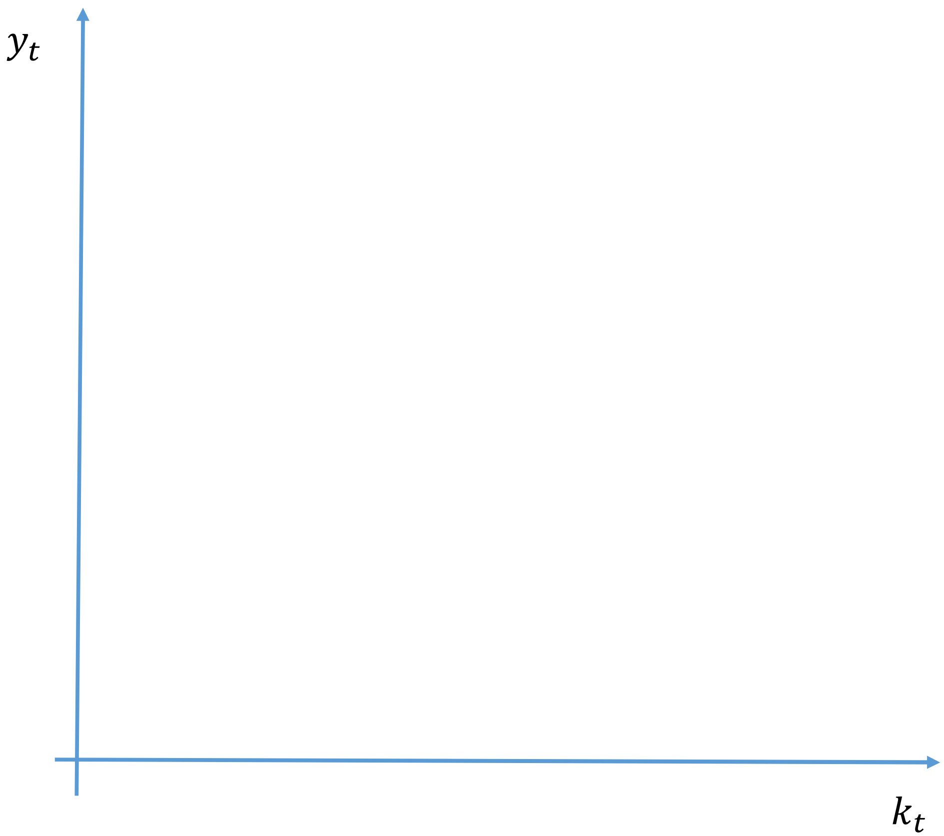
- In the model, consumption per worker is always the difference between output per worker and savings per worker (and therefore investment per worker)
- $c_t = y_t - sy_t$
- This is the difference between the **black** line and the **blue** line

Consumption in the model and in the steady state



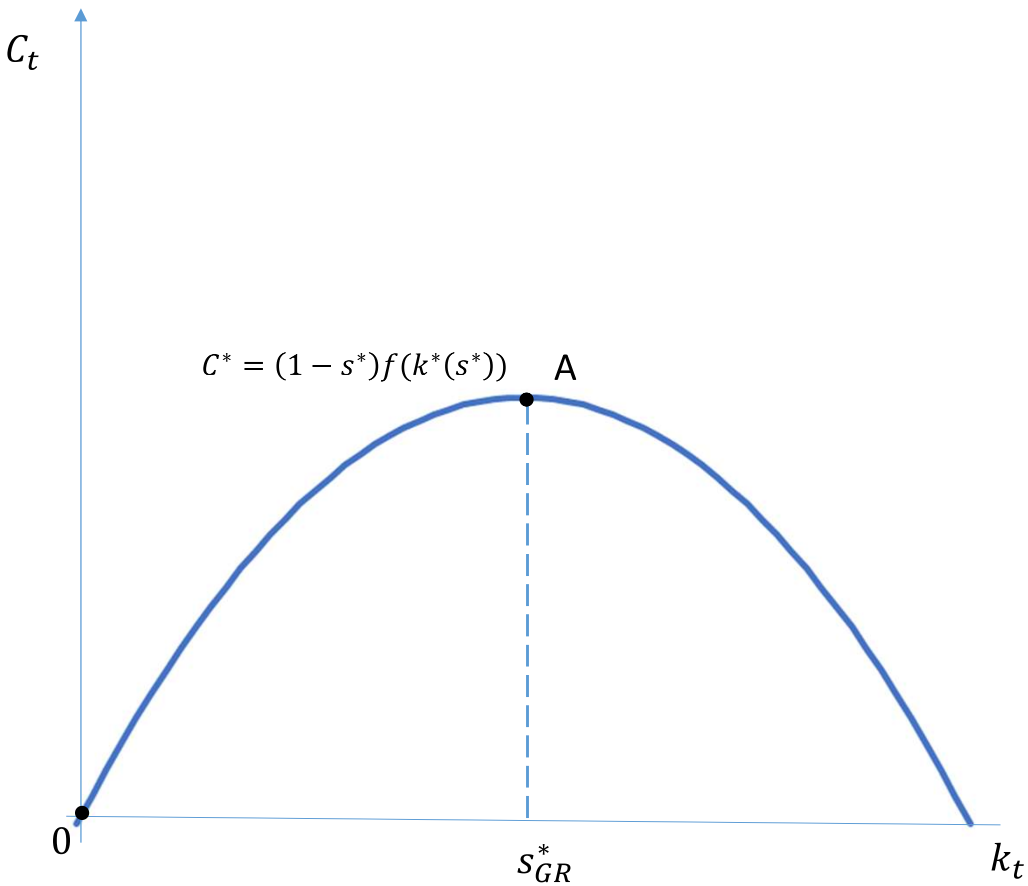
- In the steady state equilibrium, investment per worker (and therefore savings per worker) is equal to depreciation:
  - $\delta k_t^* = s\, y_t^*$
- Therefore in the steady state:
  - $c^* = y^* - \delta k^*$
  - This is the difference between the **black** line and the **green** line

Consumption in the model and in the steady state



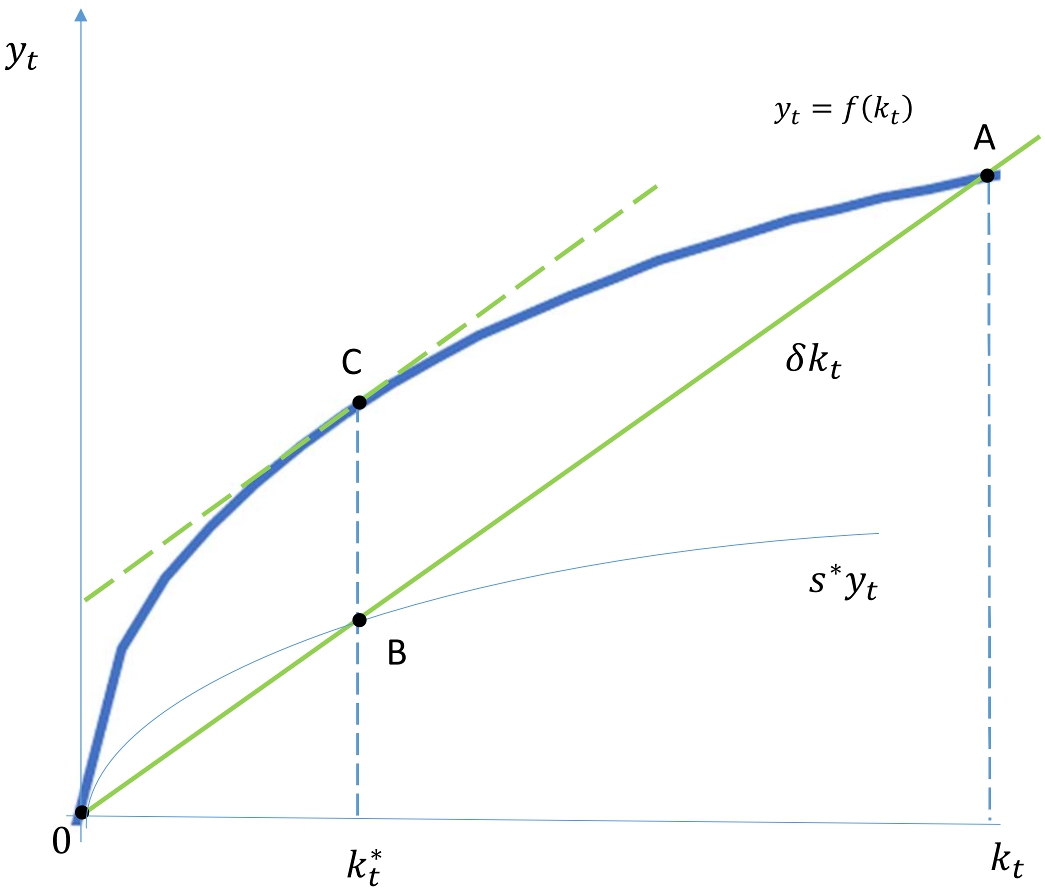
- What can we say about consumption in the steady state?
  - At the origin:
  - At the point B
- Steady state consumption reaches a maximum somewhere between the origin and the point B
- There is a value of s that determines the k^* that maximises the distance between y^* and δk^*
  - ie that maximises c^*, steady state consumption

The model: Savings, consumption and the golden rule



- So steady-state consumption reaches a maximum at the golden rule savings rate s^*_{GR}
- If the savings rate is below s^*_{GR}, then increasing savings increases consumption in the long run
- If the saving rate is below s^*_{GR}, then reducing savings increases consumption in the long run

The model: Savings, consumption and the golden rule



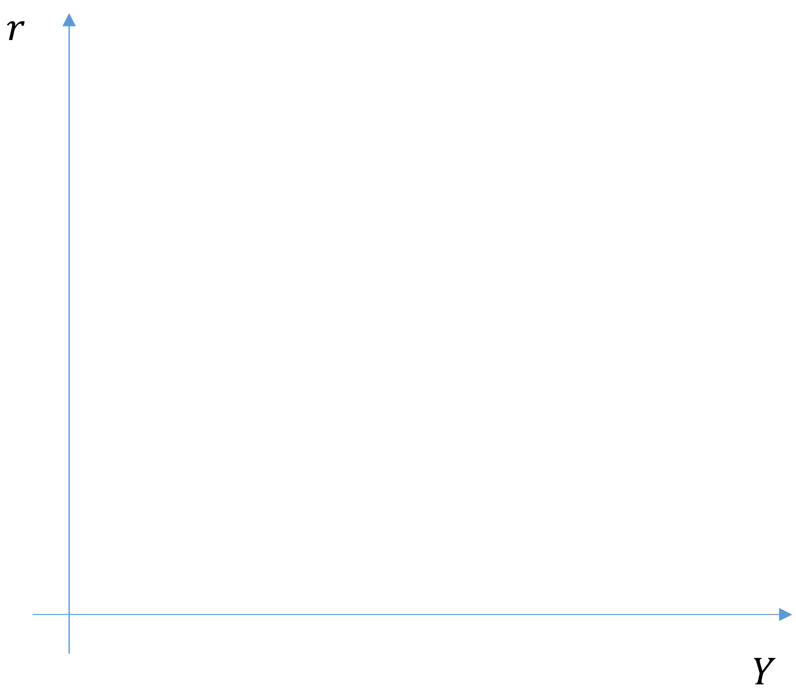
- How do we find the 'Golden Rule savings rate?
- And the associated level of capital?
- Consider the following:
    - At $s = 0$, $y_t = 0$ and so consumption per worker $= 0$
    - At $s = 1$, $y_t =$ investment per worker, and so consumption per worker $=$
    - Using some calculus we can show that golden rule consumption reaches a maximum between the two boundary values of zero where the slope of $y_t = f(k_t) = \delta$
- $C^* = (1 - s^*) y_t^* = (1 - s^*) f(k_t^*)$

Exercise 1: Blanchard ch.11 q.2

- Suppose that the head of the Finance Ministry in your country were to go on the record advocating an effort to restrain current consumption, arguing that ‘lower consumption now means higher savings; and higher savings now means a permanent higher level of consumption in the future.’ What would you make of such a statement?

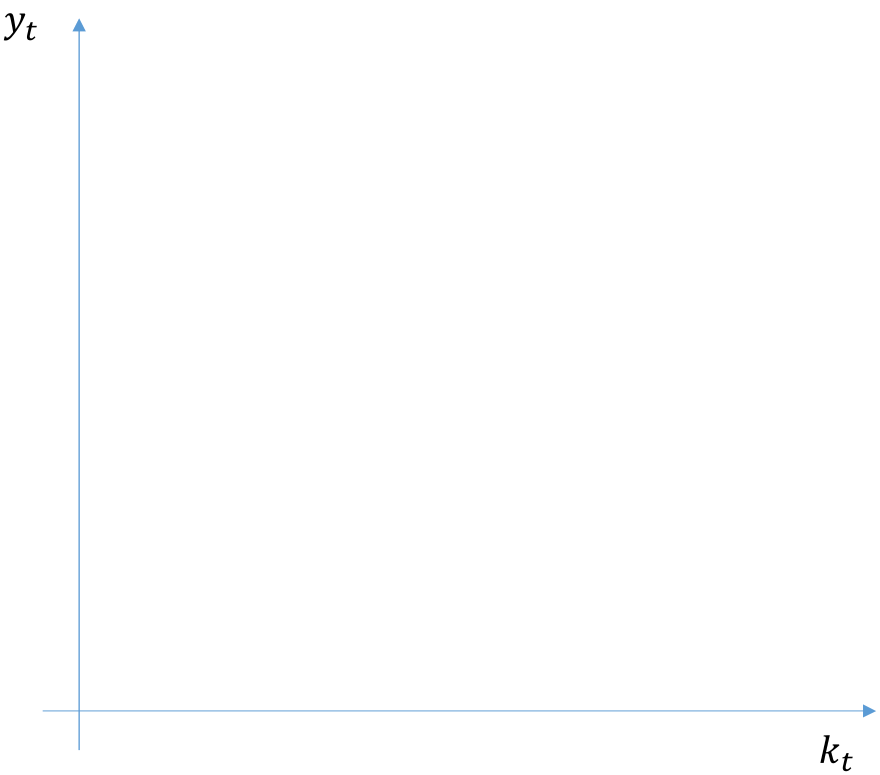
Exercise 1: Blanchard ch.11 q.2

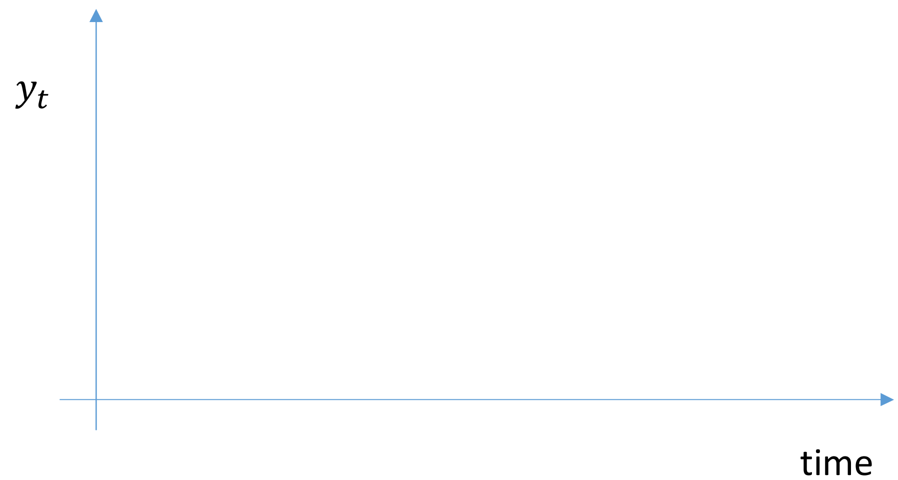
- What happens if savings increase in the short and medium term?
- How do we model increased savings in this case?



Exercise 1: Blanchard ch.11 q.2

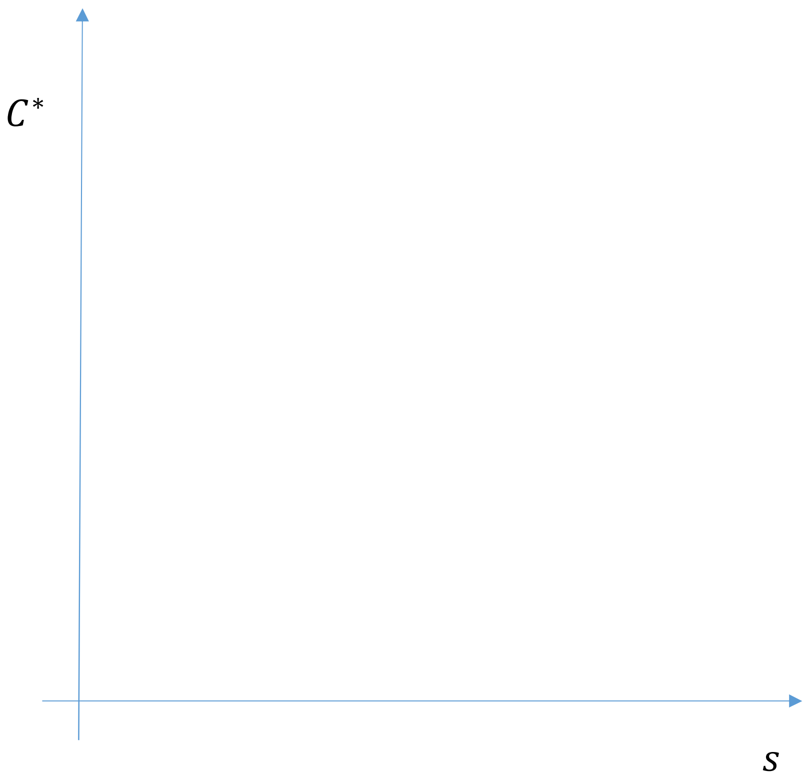
- What happens in the long run?





Exercise 1: Blanchard ch.11 q.2

- But does this mean that the a higher savings rate will increase well-being?



Exercise 2: Blanchard ch.11 q.10

A) Consider the aggregate production function $Y_t = K_t^{0.5} N_t^{0.5}$. Express steady state capital and output per worker in terms of the savings rate s and depreciation rate δ.

Exercise 2: Blanchard ch.11 q.10

B) Using a value of 10% for the depreciation rate δ and the data below for gross private savings as a percentage of GDP, calculate the steady state capital and output per worker for the countries shown (data for 2018)

	Gross Private Savings (% GDP), s	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$
France	22.9		
Germany	29.3		
Italy	20.8		
Spain	22.3		
UK	13.4		

Exercise 2: Blanchard ch.11 q.10

B) Using a value of 10% for the depreciation rate $\delta$and the data below for gross private savings as a percentage of GDP, calculate the steady state capital and output per worker for the countries shown (data for 2018)

	Gross Private Savings (% GDP), s	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$
France	22.9	5.2	2.3
Germany	29.3	8.6	2.9
Italy	20.8	4.3	2.1
Spain	22.3	5.0	2.2
UK	13.4	1.8	1.3

Exercise 2: Blanchard ch.11 q.10

C) So far we have assumed that a country's budget deficit is zero and therefore that $S_t = I_t$. Now assume that $S_t = I_t + G - T$. Use the data below for budget deficit as a percentage of GDP to adjust the steady state capital and output per worker estimates. Does a budget deficit increase or decrease steady state capital and output per worker?

	Gross Private Savings (% GDP), s	Budget deficit as % GDP	Total savings as % GDP	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$
France	22.9	-2.5	20.4	4.1	2.0	5.2	2.3
Germany	29.3	1.9	31.2	9.7	3.1	8.6	2.9
Italy	20.8	-2.2	18.6	3.5	1.9	4.3	2.1
Spain	22.3	-2.5	19.8	3.9	2.0	5.0	2.2
UK	13.4	-2.2	11.2	1.3	1.1	1.8	1.3

Exercise 2: Blanchard ch.11 q.10

D) Now assume that the four countries that do not have the lowest budget deficit (or highest budget surplus) have the same budget position as the lowest budget deficit country. What happens to their steady state capital and output per worker? Do you think it would be easy for these countries to achieve these results in practice?

	Gross Private Savings (% GDP), s	Budget deficit as % GDP	Total savings as % GDP	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{Y_t^*}{N_t} = \frac{s}{\delta}$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$	$\frac{K_t^*}{N_t} = \left(\frac{s}{\delta}\right)^2$
France	22.9	1.9	24.8	6.1	2.5	4.1	2.0
Germany	29.3	1.9	31.2	9.7	3.1	9.7	3.1
Italy	20.8	1.9	22.7	5.1	2.3	3.5	1.9
Spain	22.3	1.9	24.2	5.8	2.4	3.9	2.0
UK	13.4	1.9	15.3	2.3	1.5	1.3	1.1