

Overview

Cost Functions

Cost Minimization

- Accounting cost vs. economic cost
 - ▶ Manager shall make decision based on accounting cost or economic cost?
 - ▶ Economic cost is widely ignored because it is generally unobserved.
 - ▶ e.g. buying a property
- Simplifying assumption: $C = wl + vk$
 - ▶ labor costs
 - ▶ capital costs (rental rate)

Cost Minimization

- Cost-minimization problem

- ▶ Firm chooses the cheapest combination of inputs to produce output $q = f(k, l)$, given prices of inputs w and v

$$\min_{k, l} wl + vk \quad \text{s.t.} \quad f(k, l) = q$$

- ▶ Solution: contingent (conditional) demand for inputs

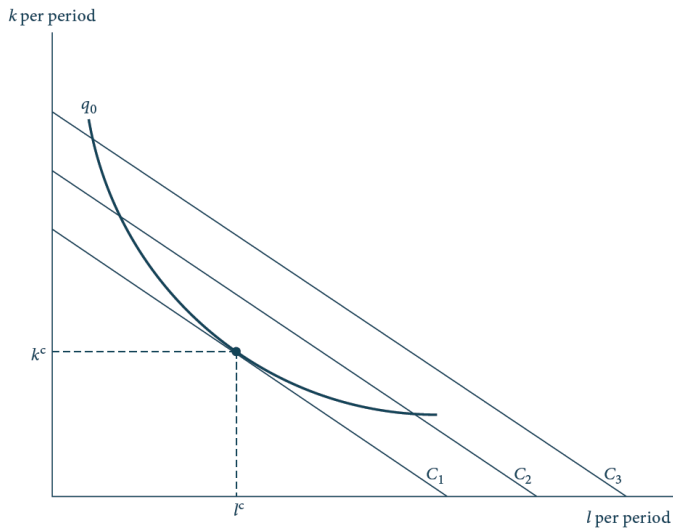
$$l^c(w, v, q), \quad k^c(w, v, q)$$

- Separate optimization at input market and output market

- ▶ For each output level q , find the cheapest way to produce q .
- ▶ Get the cost function $C(q)$.
- ▶ Then turn to the output market and decide how many to produce.

Cost Minimization

- Cost minimization



Cost Minimization

- Solving cost minimization problem

$$\min_{k,l} wl + vk \text{ s.t. } f(k,l) = q$$

- ▶ Graphical approach, condition for interior solution

$$\frac{w}{v} = \frac{f_l}{f_k} = RTS$$

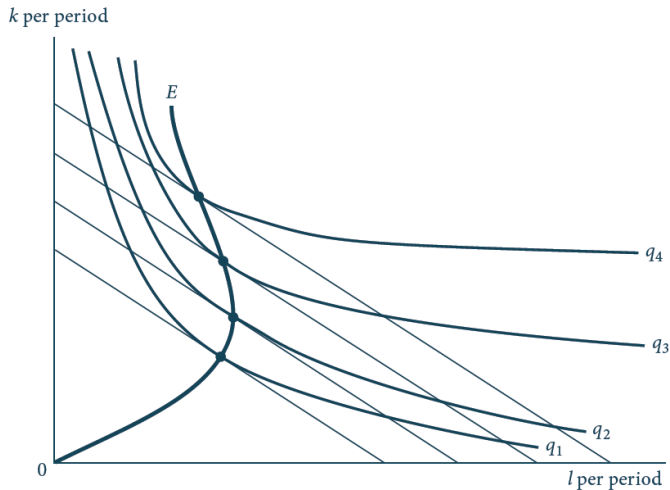
Elasticity of substitution by input price

$$\sigma = \frac{d \ln(k/l)}{d \ln(RTS)} = \frac{d \ln(k/l)}{d \ln(w/v)}$$

- ▶ Lagrangian approach

Cost Minimization

- With the expansion of output, capital-labor ratio may change



Cost Minimization

- Example 10.1, 10.2

$$q = f(k, l) = \min(\alpha k, \beta l)$$

$$q = f(k, l) = k^\alpha l^\beta$$

$$q = f(k, l) = (k^\rho + l^\rho)^{\gamma/\rho}$$

- ▶ Set up cost minimization problem and find the tangent condition.
- ▶ Find contingent input demands and cost function.

Cost Functions

- (Total) cost function

$$C(q) \equiv C(v, w, q) = wl(w, v, q) + vk(w, v, q)$$

- Results from envelop theorem (Shephard's lemma)

$$\mathcal{L} = vk + wl + \lambda(q - f(k, l))$$

$$\frac{\partial C(v, w, q)}{\partial v} = \frac{\partial \mathcal{L}(v, w, q, \lambda)}{\partial v} = k(v, w, q)$$

$$\frac{\partial C(v, w, q)}{\partial w} = \frac{\partial \mathcal{L}(v, w, q, \lambda)}{\partial w} = l(v, w, q)$$

Cost Functions

- Example 10.4, verify Shepard's lemma

- ▶ Fixed proportion

$$C(v, w, q) = q \left(\frac{v}{\alpha} + \frac{w}{\beta} \right)$$

- ▶ Cobb-Douglas

$$C(v, w, q) = q^{\frac{1}{\alpha+\beta}} B v^{\frac{\alpha}{\alpha+\beta}} w^{\frac{\beta}{\alpha+\beta}}$$

- ▶ CES

$$C(v, w, q) = q^{\frac{1}{\gamma}} \left(v^{1-\sigma} + w^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Cost Functions

- Short run vs. long run
 - ▶ Fixed and variable costs

$$SC = vk_1 + wl$$

- ▶ Short-run cost function

$$SC(q) = vk_1 + wl^{SR}(w, v, q, k_1)$$

- Example 10.5

$$q = k_1^\alpha l^\beta$$

Cost Functions

- Cost functions

- ▶ Total cost, summation of fixed cost and variable cost

$$C(q) = F + VC(q)$$

- ▶ Marginal cost

$$MC(q) = \frac{\partial C(q)}{\partial q}$$

- ▶ Average cost

$$AC(q) = \frac{C(q)}{q}$$

- ▶ Average variable cost

$$AVC(q) = \frac{VC(q)}{q}$$

- Example: Find AC, MC, and draw a diagram for

$$C(q) = q^3 - 4q^2 + 6q + 18$$

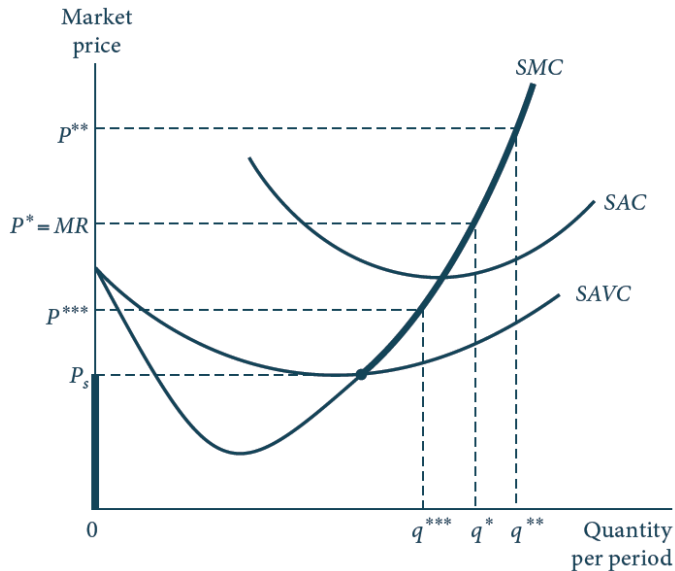
Cost Functions

- Properties of cost functions

- ▶ $C(q, v, w)$ are non-decreasing in q, v, w
- ▶ $C_{vv} < 0, C_{ww} < 0$ (due to change of input mix)
- ▶ TC, AC, and MC are homogeneous of degree 1 in input prices
- ▶ U-shape of AC
- ▶ MC pierces through AC at AC's minimum point, which is called *minimum efficient scale* (MES).
- ▶ Short-run cost is above long-run costs (except one point).

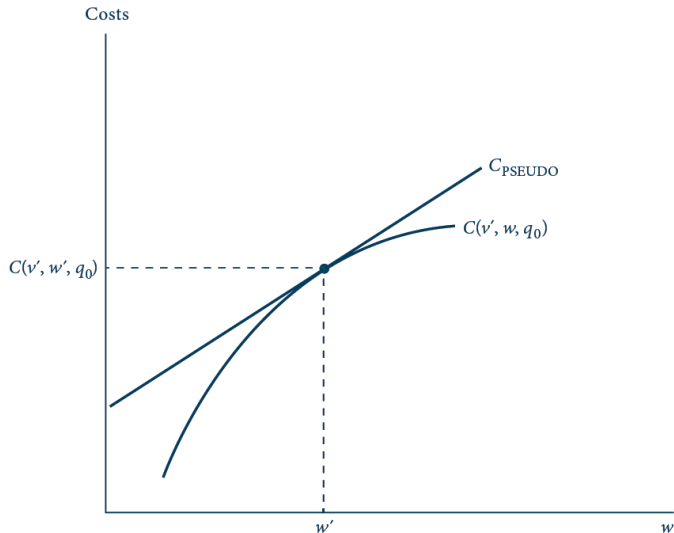
Cost Functions

- $MC(q)$, $AC(q)$, and $AVC(q)$



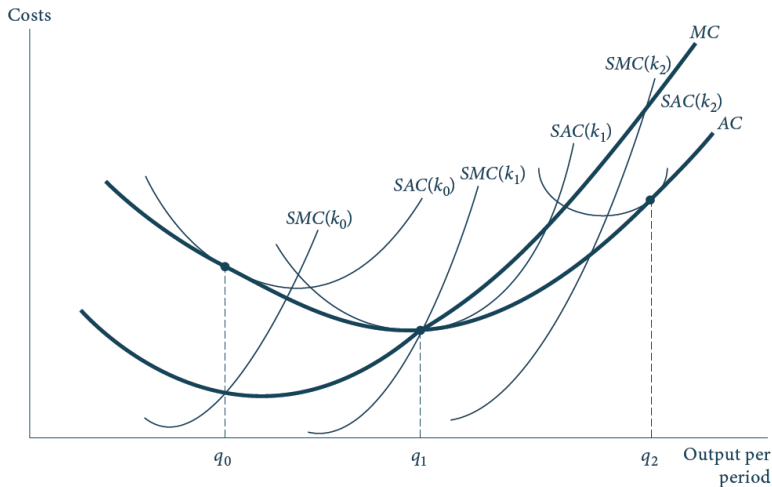
Cost Functions

- $C_{vv} < 0$, $C_{ww} < 0$



Cost Functions

- Short-run vs. long-run

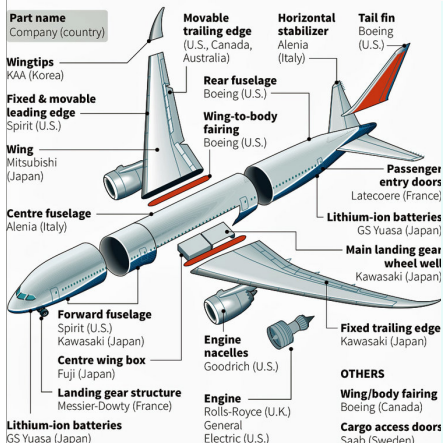


Discussion

● Global supply chain

787 Dreamliner structure suppliers

Selected component and system suppliers.



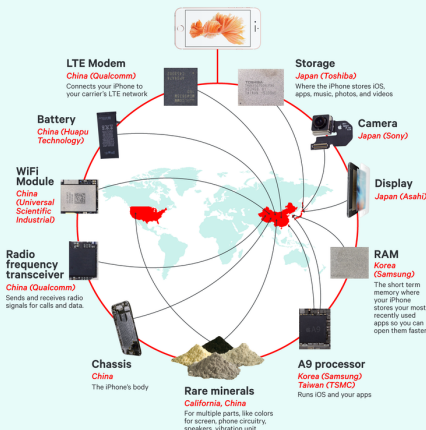
Sources: Boeing, Reuters

RWGS Staff, 09/10/2013

Note: Diagrams are not to scale

REUTERS

Where the parts of an iPhone 6s come from

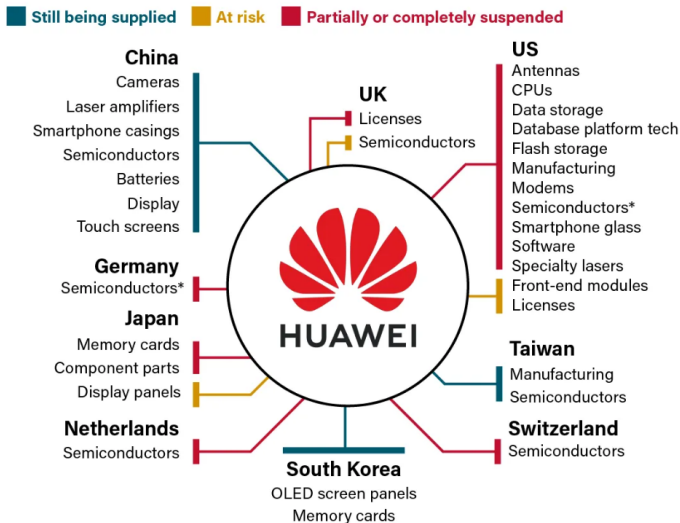


Sources: Foxconn

THE INSIDER

Discussion

Snapshot of Huawei's supply chain squeeze



*Includes critical design tools essential for making all semiconductors

Discussion

- Universities “supply” or outsource cleaning service

院校	外判商	平均月薪	時薪	每天工時	有薪用膳時間
中大	由中大直接聘用	\$12,333	\$53	7.5小時	有
理大	校園設施管理有限公司	\$5,010	\$40.4	4小時	有
		\$10,241	\$38.9	8.5小時	
教大	莊臣有限公司	\$10,600	\$38	9小時	有
嶺大	莊臣有限公司	\$10,526	\$37.7	9小時	有
城大	莊臣有限公司	\$9,736	\$36.9	8.5小時	有
浸大	惠康環境服務有限公司	\$7,193	\$38.7	6小時	有
		\$11,243	\$36.2	10小時	
港大	莊臣有限公司	\$10,005	\$35.8	9小時	有
科大	惠康環境服務有限公司	\$9,750	\$34.9	9小時	沒有

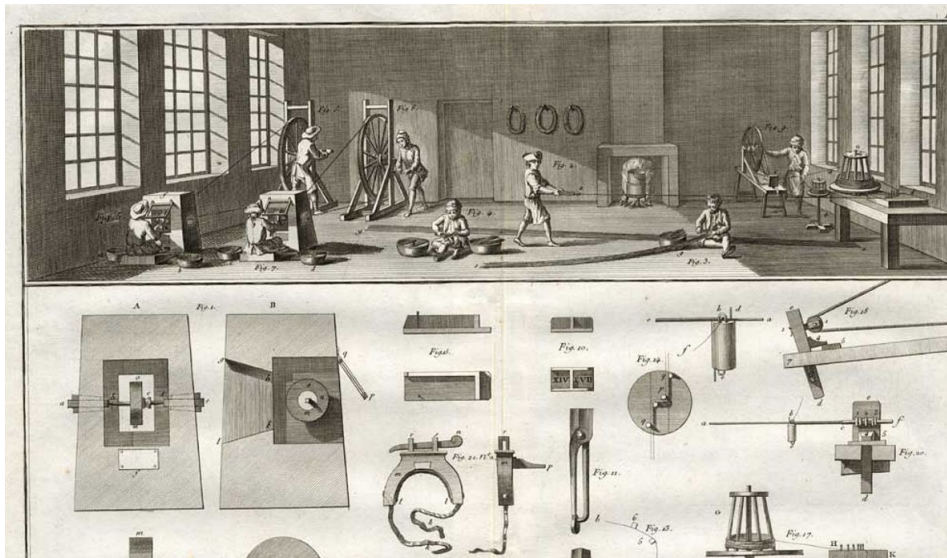
Discussion

- Why Apple do not produce iPhone by itself?



Discussion

- A pin factory in late 19th century.



Discussion

- Adam Smith's illustrate the productivity gain from division of labor by a pin factory in Chapter 1 of *The Wealth of Nations*.
 - ▶ Collaboration does not need to take place inside a firm.
 - ▶ Collaboration can happen across firms on a (vertically-related) market.
e.g. Milton Friedman's pencil (www.youtube.com/watch?v=67tHtpac5ws)
- Transaction cost and boundary of firm
 - ▶ Make or buy? What to outsource? Transaction within a firm or through a market?
 - ▶ If there is a competitive market, firm tend to buy instead of to make.
 - ▶ Need to consider transaction cost in various forms.
- Supply chain in modern economy
 - ▶ Outsourcing everything except for the core business that the firm has special advantage.
 - ▶ Global supply chain and within firm trade are very common.