

Part 3

5. Two pharmaceutical companies, A and B, form a joint venture to collaborate on developing a vaccine for the latest pandemics. The outcome is uncertain, with the probability of success increasing in the research effort exerted by the companies. Specifically, if Company A exerts effort x_A and Company B exerts effort x_B , then the probability of successfully developing an effective vaccine is $p(x_A, x_B) = x_A + x_B + \frac{1}{5}x_Ax_B$. Exerting effort level x requires a cost of $12x^2$. A success brings a profit of 4 **to each company**, whereas a failure brings zero. Suppose the companies choose their research effort simultaneously.

- (a) (5 points) What is the Nash equilibrium effort profile?
- (b) (1 point) Are the choices of research effort strategic complements or substitutes?
- (c) (3 points) Is effort over or under provided in equilibrium compared to the profit-maximizing level of the joint venture? What is the economic force behind this? *Explain without using any calculation.*
- (d) (6 points) Instead of choosing research effort simultaneously, suppose Company A takes the lead to commit to its x_A and makes it known to Company B before the latter chooses its x_B . Compared with the case of simultaneous move in part (b), would you expect the likelihood of project success to go up or down with sequential move? *Explain without using any calculation.*

(Total: 15 points)

6. Two sellers, A and B, can both supply a unit of **a homogeneous good at zero production cost**. There is a potential buyer for their good, who demands **at most one unit** and values it at $r > 0$. The buyer is initially informed about the existence of the sellers. To inform the buyer, each seller can independently send her a price quote. Receiving a price quote is free for the buyer, but sending a price quote requires **a quoting cost** $c \in (0, r)$ **to the firm**. If the buyer receives some price quotes, she can then decide whether and from which seller to buy. If she receives no price quote, then she does not know how and hence cannot buy from any seller.

Events unfold as follows. First, each seller simultaneously decides whether to send a price quote, and what price to ask for if she sends it. Next, depending on the price quotes received (if any), the buyer decides whether and from which seller to buy. Finally, all players collect their respective payoffs.

Clearly, the buyer will take the better deal as long as it is cheaper than her value r (suppose for simplicity that she randomizes equally if there is a tie in the price quotes). Therefore, we can focus attention on the first stage of price-quote competition between the sellers.

The payoffs facing a seller is as follows. If a seller does not send a price quote, her payoff is 0. If a seller sends a price quote, say p , that is accepted by the buyer, the seller's payoff is $p - c$. If a seller sends a price quote but is rejected, the seller's payoff is $-c$.

- (a) (3 points) Can it be an equilibrium to have both sellers sending a price quote of c (Bertrand paradox outcome)? Explain.
- (b) (3 points) Can it be an equilibrium to have both sellers sending a price quote of r (monopoly pricing outcome)? Explain.
- (c) For the rest of this question, we focus on **symmetric mixed strategies**. Here, a mixed strategy consists of two components: a probability $\beta \in [0, 1]$ of sending the price quote, and a price distribution $F(p)$ in case of sending a price quote. Furthermore, we will focus on price distribution $F(p)$ that has no mass point and no "gap", so it has a density function $f(p)$ such that $f(p) > 0$ if and only if $p \in (\underline{p}, \bar{p})$.
 - (i) (4 points) Explain why we **cannot** have $\beta = 0$ in any symmetric mixed-strategy equilibrium.
 - (ii) (4 points) Explain why $\bar{p} = r$ in any symmetric mixed-strategy equilibrium.
 - (iii) (4 points) Explain why we **cannot** have $\beta = 1$ in any symmetric mixed-strategy equilibrium. Note that together with part (c.i), this implies that the seller's equilibrium payoff is zero (so that she is indifferent between sending and not sending a price quote).
 - (iv) (4 points) Explain why $\underline{p} = c$ in any symmetric mixed-strategy equilibrium.
 - (v) (5 points) Use parts (c.i) to (c.iii) above to pin down the equilibrium value of β . (Note: your answer should depend on c and r .)
 - (vi) (8 points) Compute the equilibrium price distribution $F(p)$. (Note: your answer should depend on c and r .)

(Total: 35 points)

End of Part 3