

ECON 3113 Microeconomic Theory I

Lecture 7: Revealed Preference

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Introduction

- Recall we have seen that there is a tight connection between preference, utility, and choice behaviors.
- The past few weeks have been devoted to studying the implications of utility maximization (equivalently, preference maximization) on choice behaviors.
- Today, we go in the opposite direction: given some observed choice behaviors, what can we say about the consumer's utility function/preferences?
- How can we tell, just by observing the consumers' choice behaviors, whether she is a utility-maximizer?

Assumptions

- To proceed, we must make some assumption about the consumer's decision-making procedure.
 - Without any assumption, we can say nothing about her preferences.
- Key assumptions in this undertaking
 - 1 the consumer has a stable preference guiding all of her choices;
 - 2 she chooses her most preferred bundle given any budget set.
- We make another assumption, for simplification purpose, that for every budget set, the consumer is willing to pick one and only one bundle.

Direct Revealed Preference

- Suppose when presented with some budget set, the consumer chooses bundle (x_1, x_2) , but not (y_1, y_2) even though (y_1, y_2) is affordable. Then we say the consumer's choice *directly reveals* that she prefers (x_1, x_2) over (y_1, y_2) .

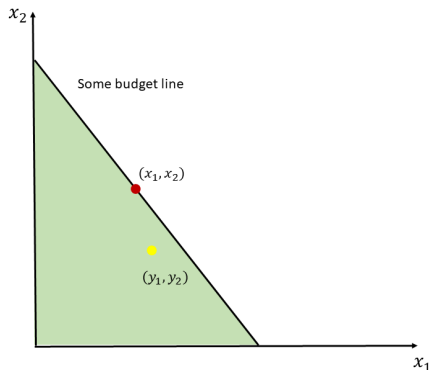
Definition

Bundle (x_1, x_2) is **directly revealed preferred** to bundle (y_1, y_2) if there is some budget set with which (x_1, x_2) is **chosen** and (y_1, y_2) is also affordable.

- The concept of revealed preference is defined over **observed choice behaviors**.

Direct Revealed Preference

- To say (x_1, x_2) is directly revealed preferred to (y_1, y_2) means that there is at least one **observed case** in which both bundles are affordable and (x_1, x_2) is **chosen over** (y_1, y_2) .
 - If (x_1, x_2) is chosen when (y_1, y_2) is affordable, one thing we know is $p_1 y_1 + p_2 y_2 \leq p_1 x_1 + p_2 x_2$.



The Principle of Revealed Preference

- Under the assumption that the consumer always chooses her most preferred bundle she can afford (which is always unique), the observation that bundle (x_1, x_2) is chosen when (y_1, y_2) is affordable implies that she strictly prefers (x_1, x_2) to (y_1, y_2) .

Theorem

Suppose the consumer always chooses her most preferred bundle she can afford. If (x_1, x_2) is directly revealed preferred to (y_1, y_2) , then she ranks $(x_1, x_2) \succ (y_1, y_2)$.

- This is a theorem, not a definition.
 - Direct revealed preference is about choice behavior, not preference (or utility).
 - Perhaps a better terminology is (x_1, x_2) is **chosen over** (y_1, y_2) .

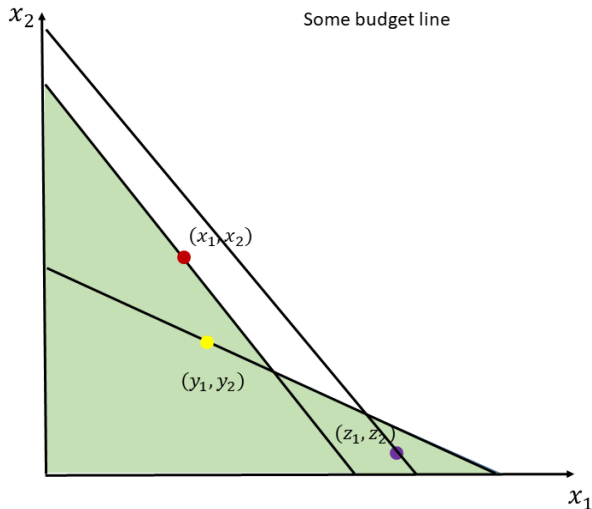
Indirect Revealed Preference

- Suppose we observe that (i) (x_1, x_2) is chosen when (y_1, y_2) is affordable and (ii) (y_1, y_2) is chosen when (z_1, z_2) is affordable.
- In the terminology introduced, (x_1, x_2) is directly revealed preferred to (y_1, y_2) , which is directly revealed preferred to (z_1, z_2) .
- We say (x_1, x_2) is **indirectly revealed preferred** to (z_1, z_2) .
 - In general, we say (x_1, x_2) is indirectly revealed preferred to (z_1, z_2) as long as we can find a *chain* of direct revealed preference connecting them.

Theorem (The Principle of Revealed Preference)

Suppose the consumer always chooses her most preferred bundle she can afford, based on a transitive preference. If (x_1, x_2) is directly or indirectly revealed preferred to (z_1, z_2) , then she ranks $(x_1, x_2) \succ (z_1, z_2)$.

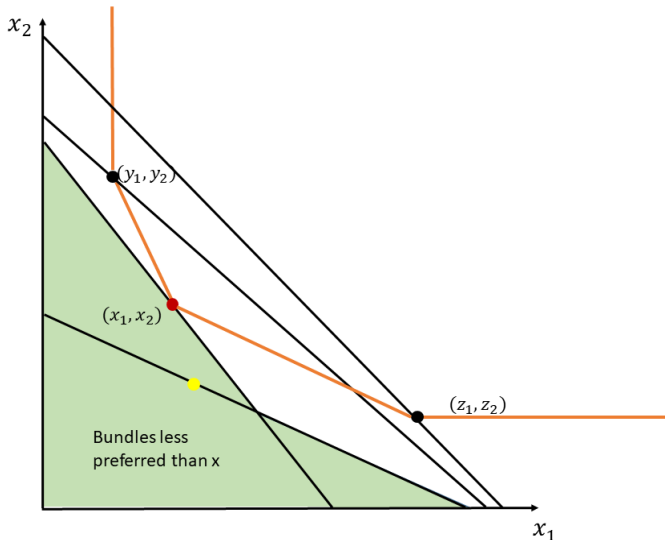
Indirect Revealed Preference



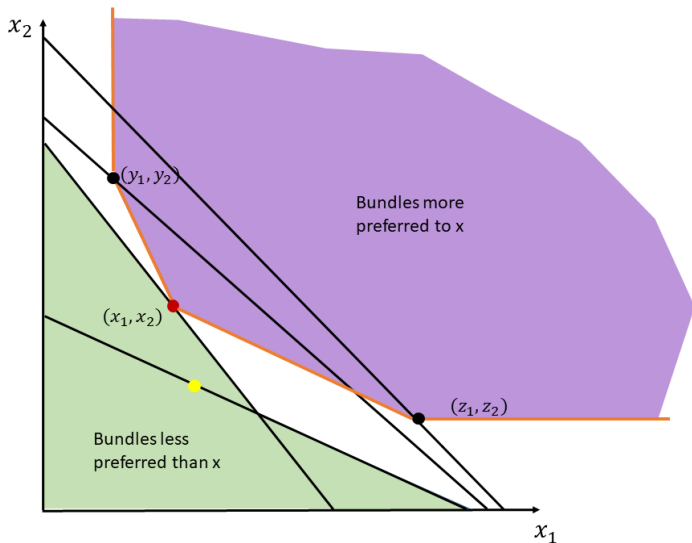
Recovering Preference

- We can try to reverse-engineer the consumer's preference by observing the choices she made.
- To illustrate, suppose we observe that bundles (y_1, y_2) and (z_1, z_2) are revealed preferred to (x_1, x_2) , either directly or indirectly.
- Assuming further that her preference is monotone and strictly convex, we can identify the regions of bundles better and worse than (x_1, x_2) .
- Consequently, we can "trap" the consumer's indifference curve to some tight region.

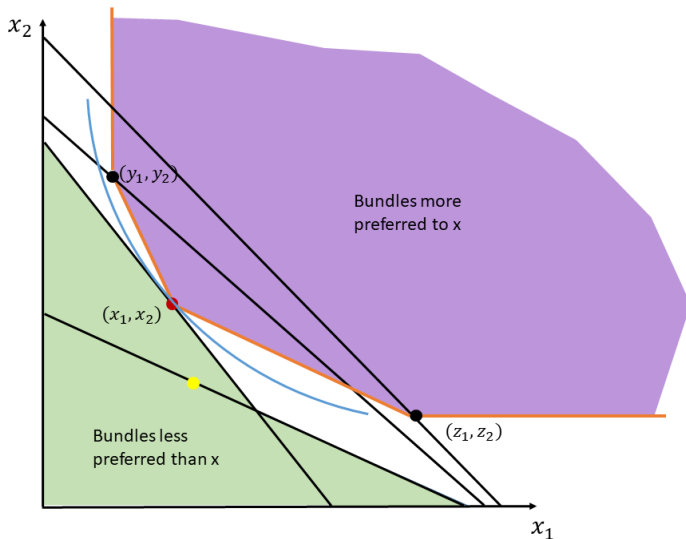
"Trapping" the indifference curve



"Trapping" the indifference curve



"Trapping" the indifference curve



Principle of Revealed Preference implies Negative Substitution Effect

- Suppose the following demand behaviour is observed:

Prices	(p_1, p_2)	(p'_1, p'_2)
Choice	(x_1, x_2)	(y_1, y_2)

- Suppose (x_1, x_2) and (y_1, y_2) are two distinct bundles that the consumer finds indifferent to.
- By the principle of revealed preference, neither one bundle is revealed preferred to the other, directly or indirectly.
- As (x_1, x_2) is chosen at prices (p_1, p_2) and (x_1, x_2) is **not** revealed preferred to (y_1, y_2) , we know that

$$p_1 x_1 + p_2 x_2 < p_1 y_1 + p_2 y_2.$$

- As (y_1, y_2) is chosen at prices (p'_1, p'_2) and (y_1, y_2) is **not** revealed preferred to (x_1, x_2) , we know that

$$p'_1 y_1 + p'_2 y_2 < p'_1 x_1 + p'_2 x_2.$$

Principle of Revealed Preference implies Negative Substitution Effect

- Adding these inequalities gives

$$\begin{aligned} (p_1 x_1 + p_2 x_2) + (p'_1 y_1 + p'_2 y_2) &< (p_1 y_1 + p_2 y_2) + (p'_1 x_1 + p'_2 x_2) \\ \Leftrightarrow (p_1 - p'_1)(x_1 - y_1) + (p_2 - p'_2)(x_2 - y_2) &< 0. \end{aligned}$$

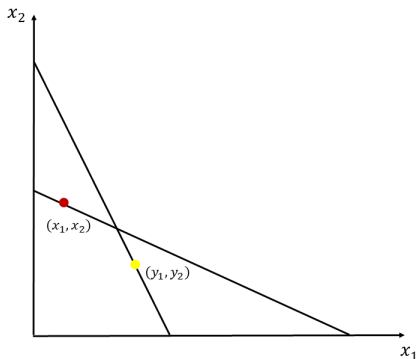
- Now suppose $p_1 > p'_1$ and $p_2 = p'_2$. Then we have

$$(p_1 - p'_1)(x_1 - y_1) < 0.$$

- Recall the consumer is indifferent between (x_1, x_2) and (y_1, y_2) .
- We have thus shown that holding utility constant, quantity of consumption of good 1 moves in the opposite direction to its price change.
 - Exactly the statement that substitution effect is negative.
 - We prove this without any assumption on preference convexity!

How to tell whether the consumer is utility-maximizing or not?

- What could happen if the consumer does not maximize utility in making her choices?
- Suppose (x_1, x_2) is chosen when (y_1, y_2) is affordable, and (y_1, y_2) is chosen when (x_1, x_2) is affordable.



How to tell whether the consumer is utility-maximizing or not?

- By the principle of revealed preference, the consumer has both $(x_1, x_2) \succ (y_1, y_2)$ and $(y_1, y_2) \succ (x_1, x_2)$, which is a contradiction.
- Either the consumer does not maximize utility in making her choices, or her preference has changed between the two choice problems.

Weak Axiom of Revealed Preference

Definition

The consumer's choices are said to satisfy the **Weak Axiom of Revealed Preference (WARP)** if the following property hold. Whenever (x_1, x_2) and (y_1, y_2) are distinct bundles and (x_1, x_2) is directly revealed preferred to (y_1, y_2) , then it cannot happen that (y_1, y_2) is directly revealed preferred to (x_1, x_2) .

- WARP is a property of the choice behaviors, not utility/preference.

Weak Axiom of Revealed Preference

- Suppose we observe the following choice behaviors.

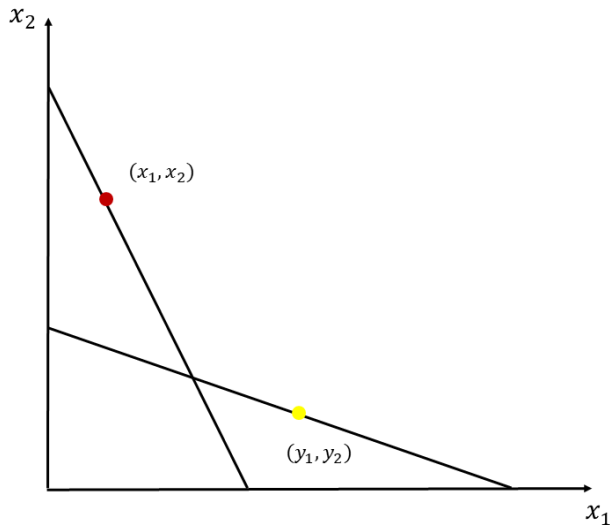
Prices	(p_1, p_2)	(p'_1, p'_2)
Choice	(x_1, x_2)	(y_1, y_2)

- The requirement of WARP can be stated as follows:

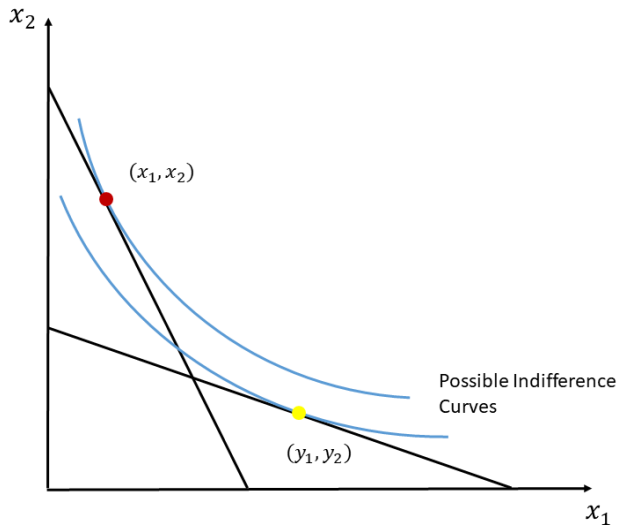
$$p_1 y_1 + p_2 y_2 \leq p_1 x_1 + p_2 x_2 \Rightarrow p'_1 y_1 + p'_2 y_2 < p'_1 x_1 + p'_2 x_2.$$

- If (x_1, x_2) is chosen when (y_1, y_2) is affordable, then (y_1, y_2) will be chosen **only** when (x_1, x_2) is not affordable.

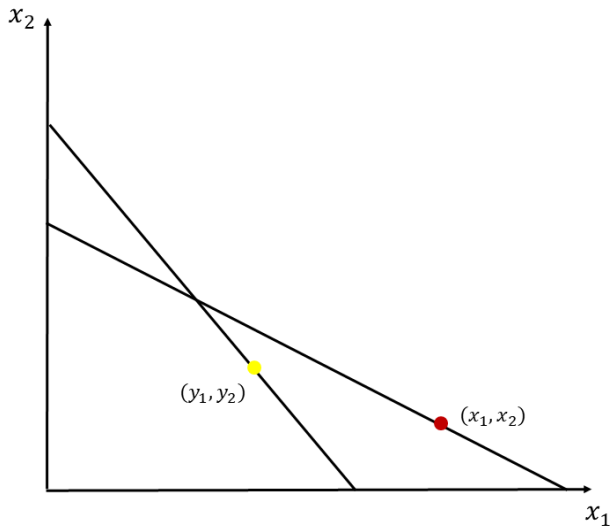
Checking WARP



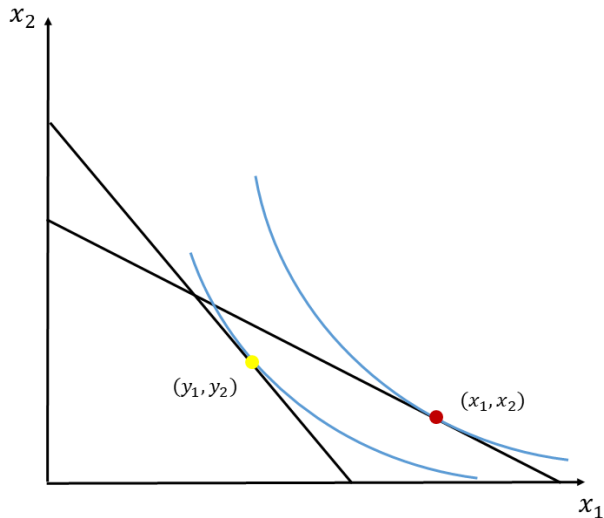
Checking WARP



Checking WARP



Checking WARP



Strong Axiom of Revealed Preference

- The weak axiom concerns direct revealed preference. If the consumer maximizes utility, it is natural to expect that similar condition holds for indirect revealed preference as well.

Definition

The consumer's choices are said to satisfy the **Strong Axiom of Revealed Preference (SARP)** if the following property hold. Whenever (x_1, x_2) and (y_1, y_2) are distinct bundles and (x_1, x_2) is directly or indirectly revealed preferred to (y_1, y_2) , then it cannot happen that (y_1, y_2) is directly or indirectly revealed preferred to (x_1, x_2) .

- SARP is a property of the choice behaviors, not utility/preference.

Strong Axiom of Revealed Preference

- Like WARP, SARP is also an immediate implication of the principle of revealed preference.
- Suppose a consumer is utility-maximizing. Suppose further we observe that (x_1, x_2) is revealed preferred to (y_1, y_2) , directly or indirectly, then we can infer that $(x_1, x_2) \succ (y_1, y_2)$.
- If we further observe that (y_1, y_2) is revealed preferred to (x_1, x_2) , directly or indirectly, then we can infer that $(y_1, y_2) \succ (x_1, x_2)$, which is a contradiction.
- A violation of SARP therefore implies that the consumer is not utility-maximizing or her preference has changed.

Relation between WARP and SARP

- Direct revealed preference can be thought of an indirect revealed preference in which the length of the chain is 1.
- SARP is a stronger than WARP:
 - there are more bundle pairs to check (indirect revealed preference covers more bundle pairs)
 - the requirement ("then...") is more restrictive (if bundle (y_1, y_2) is not indirectly revealed preferred to (x_1, x_2) , then bundle (y_1, y_2) is not directly revealed preferred to (x_1, x_2) , but not vice versa)

An example that violates SARP but not WARP

- Consider the following choice data with three goods:

Prices	$p_X = (2, 3, 3)$	$p_Y = (3, 2, 3)$	$p_Z = (3, 3, 2)$
Choice	$X = (3, 1, 7)$	$Y = (7, 3, 1)$	$Z = (1, 7, 3)$

- X is directly revealed preferred to Y and Y is directly revealed preferred to Z but Z is directly revealed preferred to X .
 - When X is chosen (at prices p_X), Y is affordable but Z is not affordable.
 - When Y is chosen (at prices p_Y), X is not affordable but Z is affordable.
 - When Z is chosen (at prices p_Z), X is affordable but Y is not affordable.
- As this is all we can say about the direct revealed preference relation here, there is no violation of WARP.
- However, it violates SARP because X is indirectly revealed preferred to Z (through Y), and at the same time Z is directly revealed preferred to X .

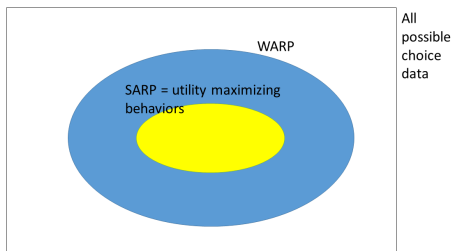
Strong Axiom of Revealed Preference

- SARP is a necessary implication of utility-maximizing behaviors.
- Somewhat surprisingly, SARP is effectively the only implication of utility-maximizing behaviors.
- If the observed choices satisfy SARP, we can always find nice, well-behaved preferences that could generate these choices.

Theorem

The consumer's choices satisfy SARP if and only if she maximizes a strictly increasing, continuous and quasi-concave utility function.

WARP, SARP, and Utility-Maximization



- The relations suggest a conceptual recipe to analyze choice behaviors:
 - First check WARP (preliminary test). If violated, the data cannot be explained by utility maximization.
 - If WARP is not violated, check SARP (in-depth test). If SARP is violated, again the data cannot be explained by utility maximization.
 - If SARP is not violated, the data can be explained by utility max. Preference can be partially recovered by identifying bounds on where the indifference curves lie.

- Given some observed choice behaviors, what can we say about the consumer's utility function/preferences?
 - We can "trap" the indifference curves using observed choice data.
 - The more data we have, the tighter we can trap.
- How can we tell, just by observing the consumers' choice behaviors, whether she is a utility-maximizer?
 - Check SARP.