Lecture 12. Saving, Capital Accumulation, and Output (The Solow Model)

Reading: Blanchard, Chapter 11.

The aggregate production function

- $Y = F(K, N)$
- Constant returns to scale $\Rightarrow y = \frac{Y}{N} = F\left(\frac{K}{N}, 1\right) = f(k)$
- An example: $Y = \mathcal{A}K^{\alpha}N^{1-\alpha}$, where $0 < \alpha < 1$.

Divide both hand sides by N.

$$\Rightarrow \quad y = \frac{Y}{N} = \mathcal{A}K^{\alpha}\frac{N^{1-\alpha}}{N} = \mathcal{A}K^{\alpha}N^{-\alpha} = \mathcal{A}\left(\frac{K}{N}\right)^{\alpha} = \mathcal{A}k^{\alpha}.$$

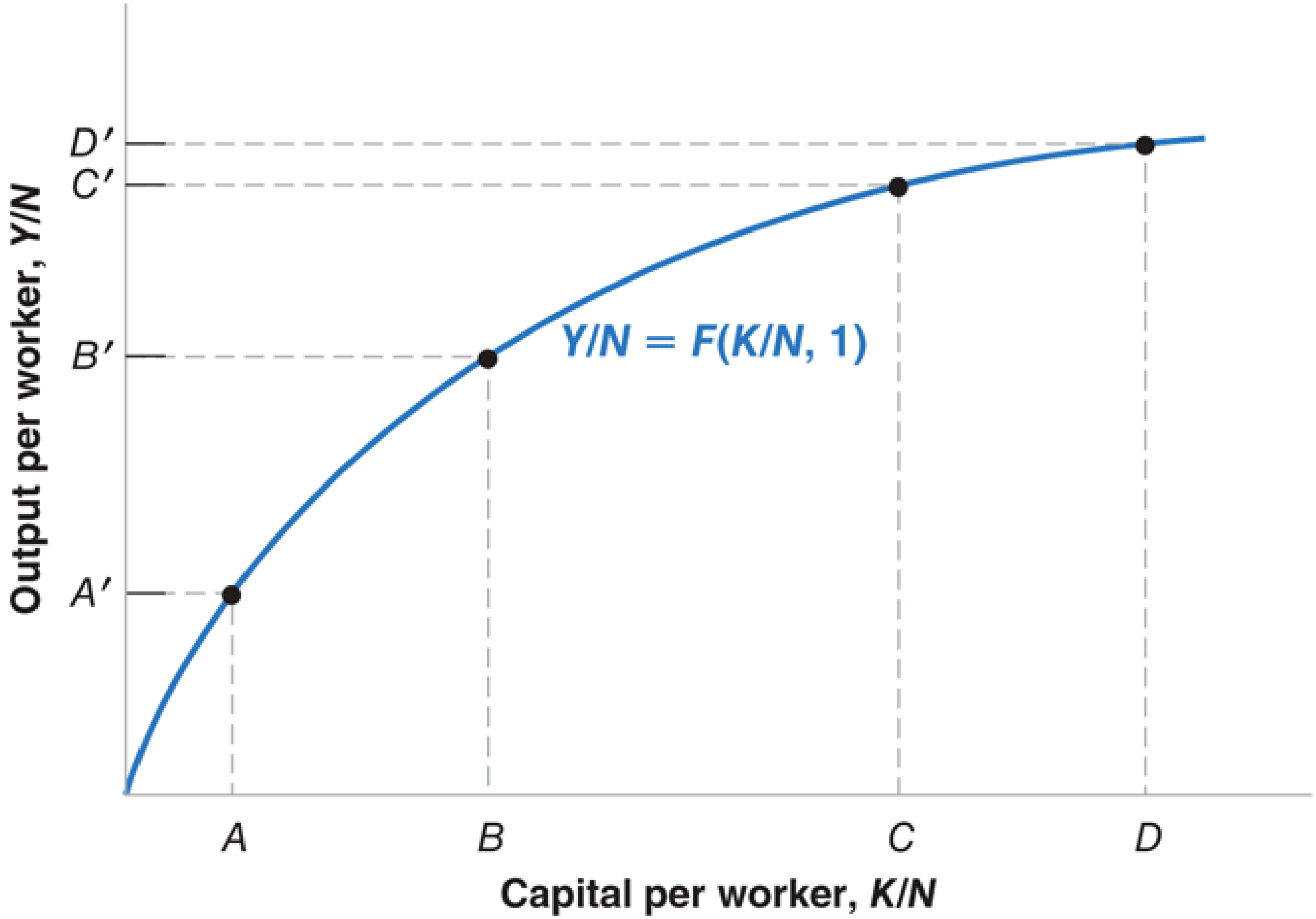
- To make y grow, we need to make either k or $\mathcal{A}$ grow.

output per worker capital per worker or productivity

Growth due to capital accumulation

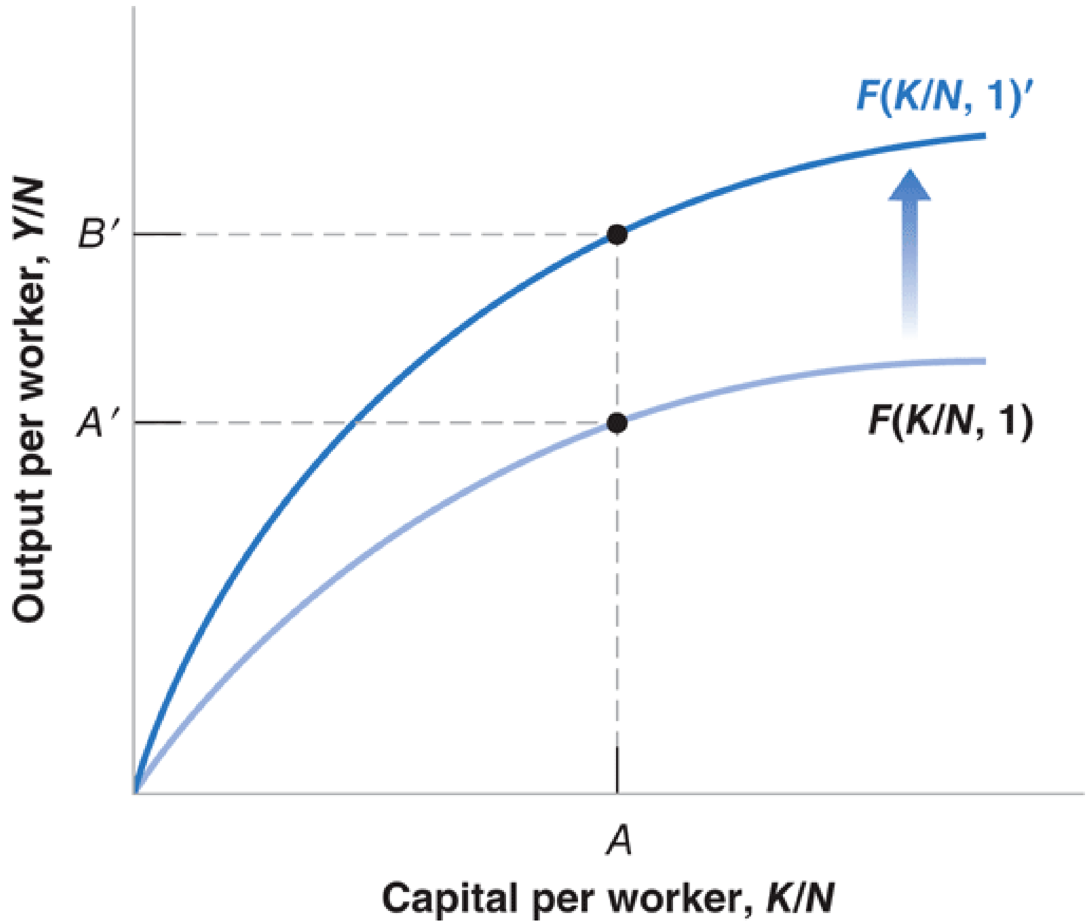
- Given $\mathcal{A}$,
  as k increases from A to D,
  y increases from A’ to D’.

- f is concave, i.e., the slope decreases as k increases (This property holds because we assume decreasing returns to capital).



Growth due to technological progress

- Given k at A, as $\mathcal{A}$ increases, y increases from A' to B'.



Outline

- The Solow Growth Model
- Examples

- The Saving Rate, Output, and Consumption
- An Analytical Example ($Y = \sqrt{\mathrm{K}}\sqrt{N}$)

- Population Growth, Technological Progress, and Human Capital Accumulation

Outline

- The Solow Growth Model
- Examples

- The Saving Rate, Output, and Consumption
- An Analytical Example ($Y = \sqrt{\mathrm{K}}\sqrt{N}$)

- Population Growth, Technological Progress, and Human Capital Accumulation

Conceptual framework

- The long-run dynamics
- Trend, not cyclical fluctuations

- Capital Stock (K_t)

→ Output/Income (Y_t) via the aggregate production function

→ Saving/Investment (S_t or I_t)

→ Change in the capital stock ($K_{t+1} - K_t$)

→ Capital Stock in the next period (K_{t+1})

Simplifying assumptions

- N is constant.
- No change in demographics such as population, participation rate, employment rate, etc.

- $\mathcal{A}$ is constant.

- Unrealistic. But these assumptions allow us to concentrate on how capital accumulation affects growth.

- We will return to these assumptions later.

The Solow model; Assumptions

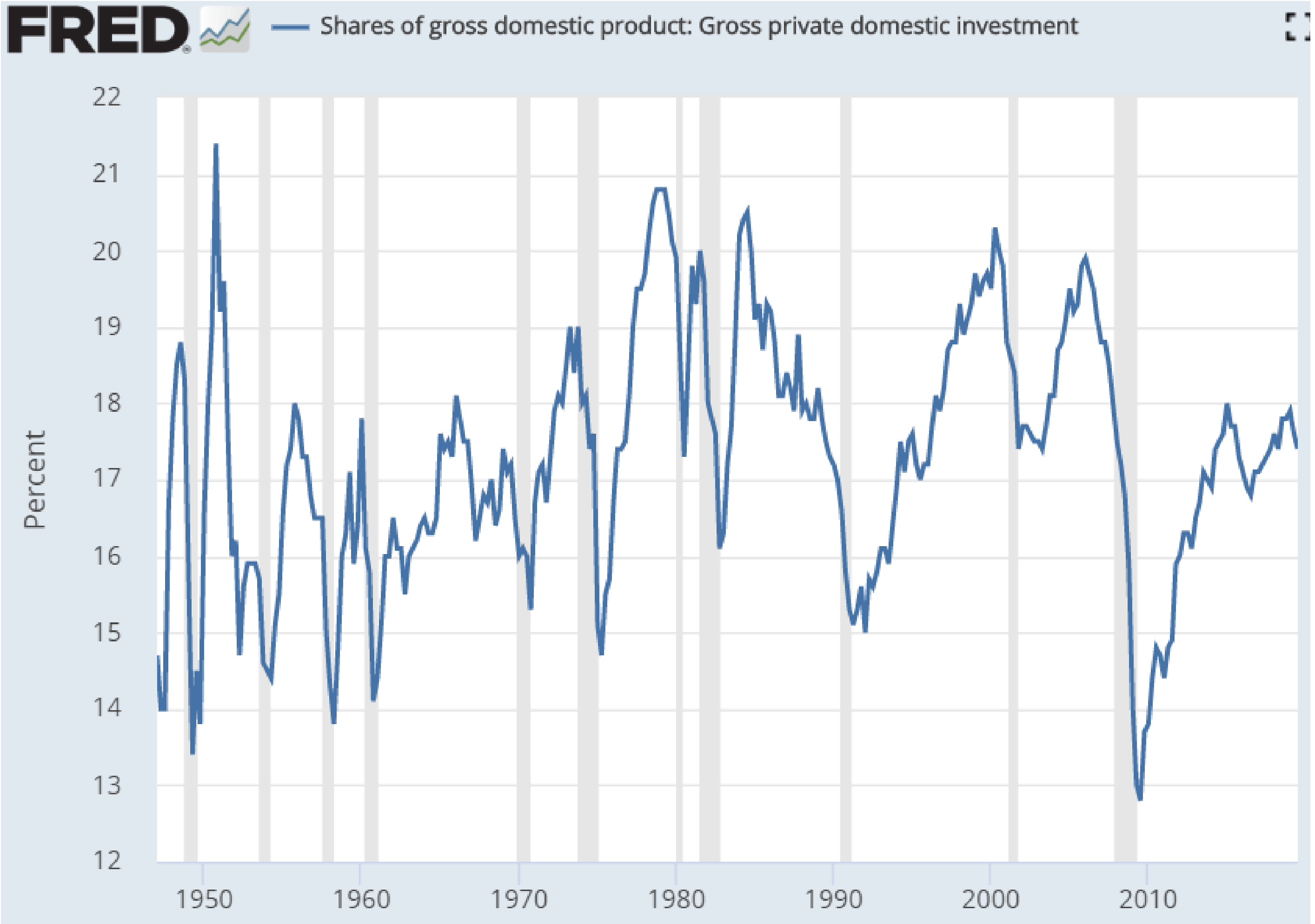
- Closed-economy
- Balanced budget: $T - G = 0$
- Saving is proportional to income: $S = sY$, where s is the saving rate.

$\Rightarrow$

- $C =$

- s in data?

I/Y in the US



- Stable around 16-18%

The evolution of the capital stock

- K_t: the capital stock at the beginning of year t,
  which is used for production in year t.
- The evolution of capital:
$$K_{t+1} = (1 - \delta)K_t + I_t,$$
  where capital depreciates at rate δ.
  The value of a car, computer, and others decrease over time as they are used. It is known that δ is about 5-10 percent per year.
- Given that N is constant and $I_t = sY_t$,

$$\frac{K_{t+1}}{N} = (1 - \delta)\frac{K_t}{N} + \frac{I_t}{N} = (1 - \delta)\frac{K_t}{N} + s\frac{Y_t}{N}$$

The evolution of the capital stock (cont'd)

- $\frac{K_{t+1}}{N} = (1-\delta)\frac{K_t}{N} + \frac{I_t}{N} = (1-\delta)\frac{K_t}{N} + s\frac{Y_t}{N}$

$$\Rightarrow \quad k_{t+1} = (1-\delta)k_t + sy_t$$
$$= (1-\delta)k_t + sf(k_t)$$

- This equation characterizes the dynamics of k_t.

- $\Delta k_{t+1} = k_{t+1} - k_t =$

change in capital = investment – depreciation

per worker per worker per worker

$$\Delta k_{t+1} = sf(k_t) - \delta k_t$$

- $k_t < k^* = K^*/N$

$\Rightarrow sf(k_t) > \delta k_t$

(investment p.w. > depreciation p.w.)

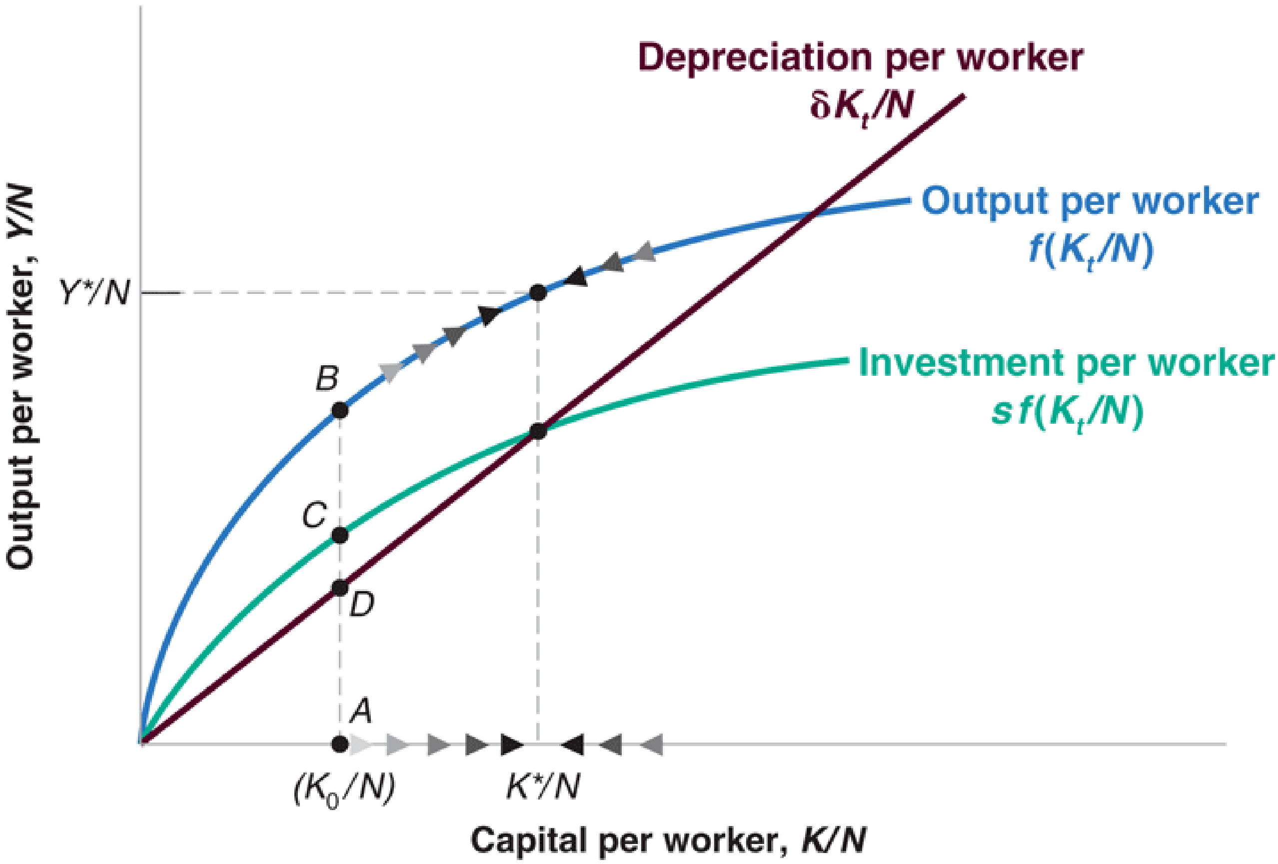
$\Rightarrow \Delta k_{t+1}$ 0, i.e., k and $y = f(k)$.

- $k_t > k^* = K^*/N$

$\Rightarrow sf(k_t) < \delta k_t$

(investment p.w. < depreciation p.w.)

$\Rightarrow \Delta k_{t+1}$ 0, i.e., k and $y = f(k)$.



$$\Delta k_{t+1} = sf(k_t) - \delta k_t$$

- $k_t < k^* \Rightarrow \Delta k_{t+1} > 0$, i.e., $k \uparrow$
- $k_t > k^* \Rightarrow \Delta k_{t+1} < 0$, i.e., $k \downarrow$

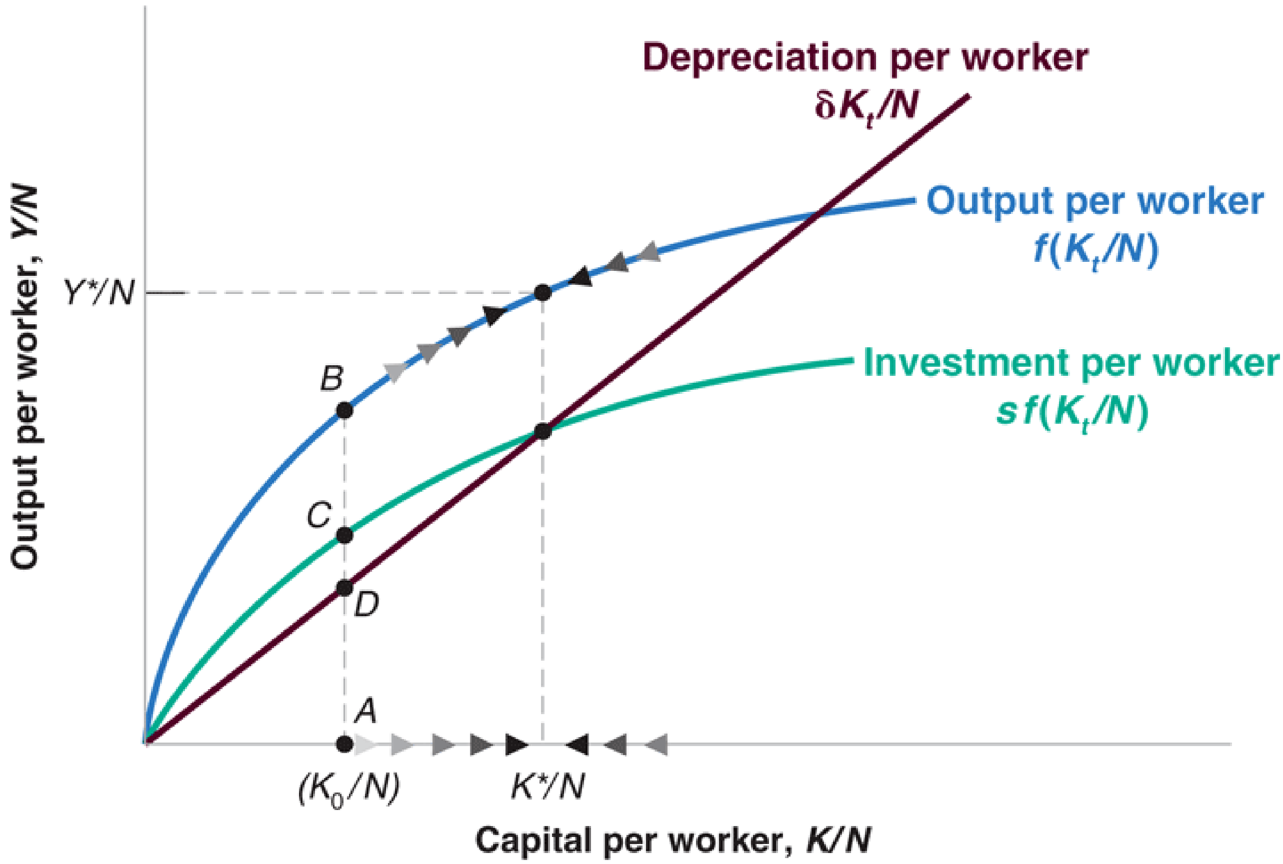
- The :

$$k_t = k^* \Rightarrow k_{t+1} = k_{t+2} = \cdots = k^*$$

$$\Rightarrow y_t = y_{t+1} = \cdots = f(k^*)$$

- k^* satisfies

$$sf(k^*) = \delta k^*.$$



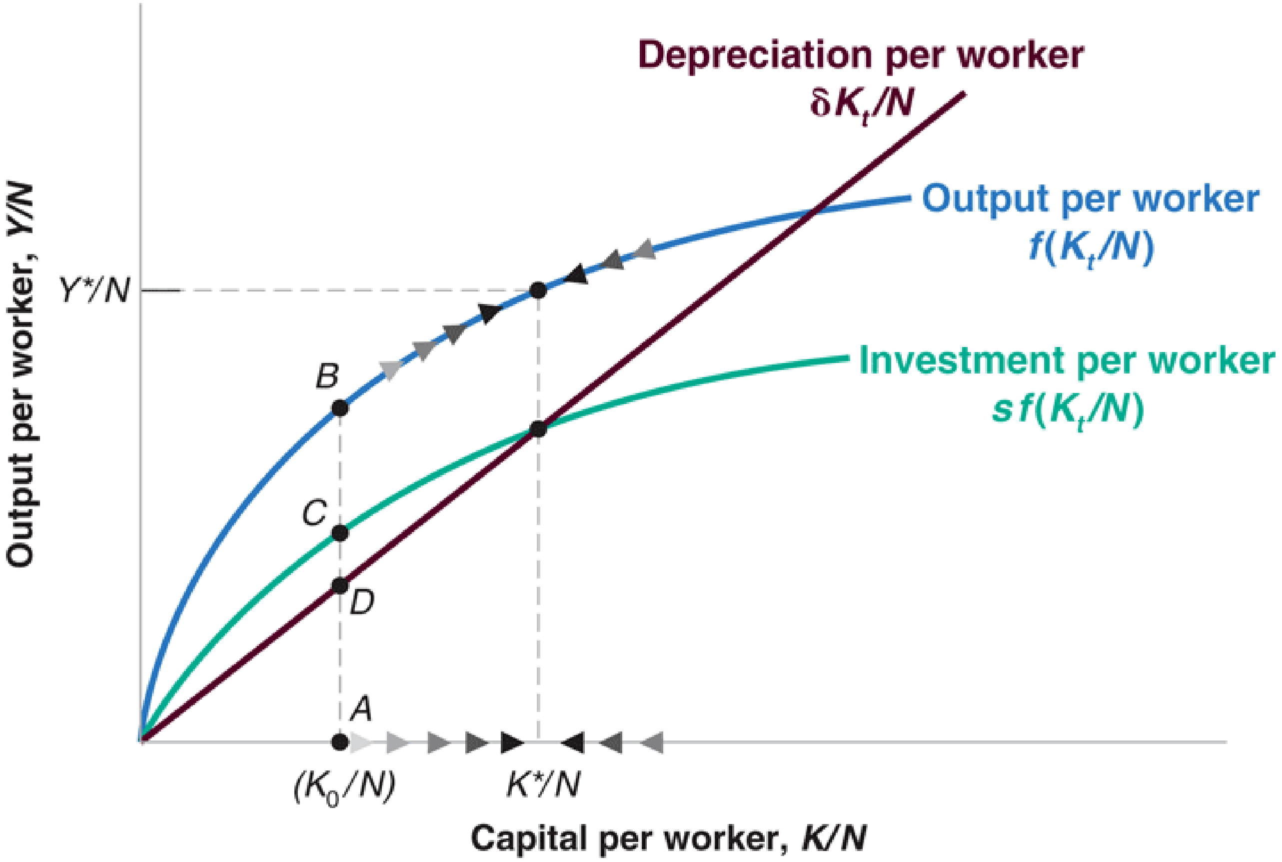
Outline

- The Solow Growth Model
- Examples

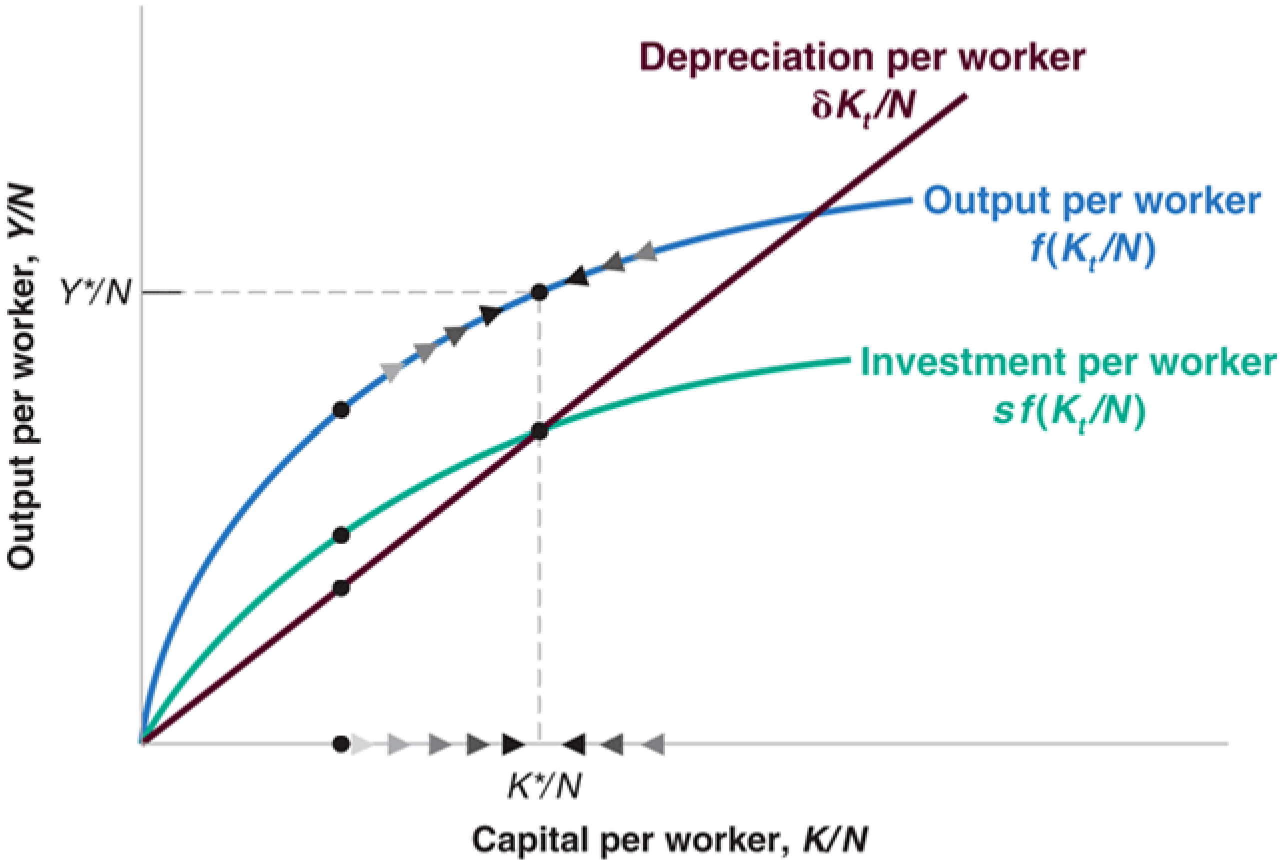
- The Saving Rate, Output, and Consumption
- An Analytical Example ($Y = \sqrt{\mathrm{K}}\sqrt{N}$)

- Population Growth, Technological Progress, and Human Capital Accumulation

Level and growth rate of Y/N when $K_0 < K^*$



Level and growth rate of Y/N when $K_0 > K^*$



- Suppose that $K_0 = K^*$. In period $t = 5$, a war breaks out. As a result of bombing, the country loses its capital stock more than its population. Illustrate dynamics of k, y, growth rate of y (g_y), and Y.

Capital Accumulation and Growth in France in the Aftermath of WWII (p.244)

- “There is plenty of anecdotal evidence that small increases in capital led to large increases in output. Minor repairs to a major bridge would lead to the reopening of the bridge. Reopening the bridge would significantly shorten the travel time between two cities, leading to much lower transport costs. The lower transport costs would then enable a plant to get much needed inputs, increase its production, and so on.”

Outline

- The Solow Growth Model
- Examples

- The Saving Rate, Output, and Consumption
- An Analytical Example ($Y = \sqrt{\mathrm{K}}\sqrt{N}$)

- Population Growth, Technological Progress, and Human Capital Accumulation

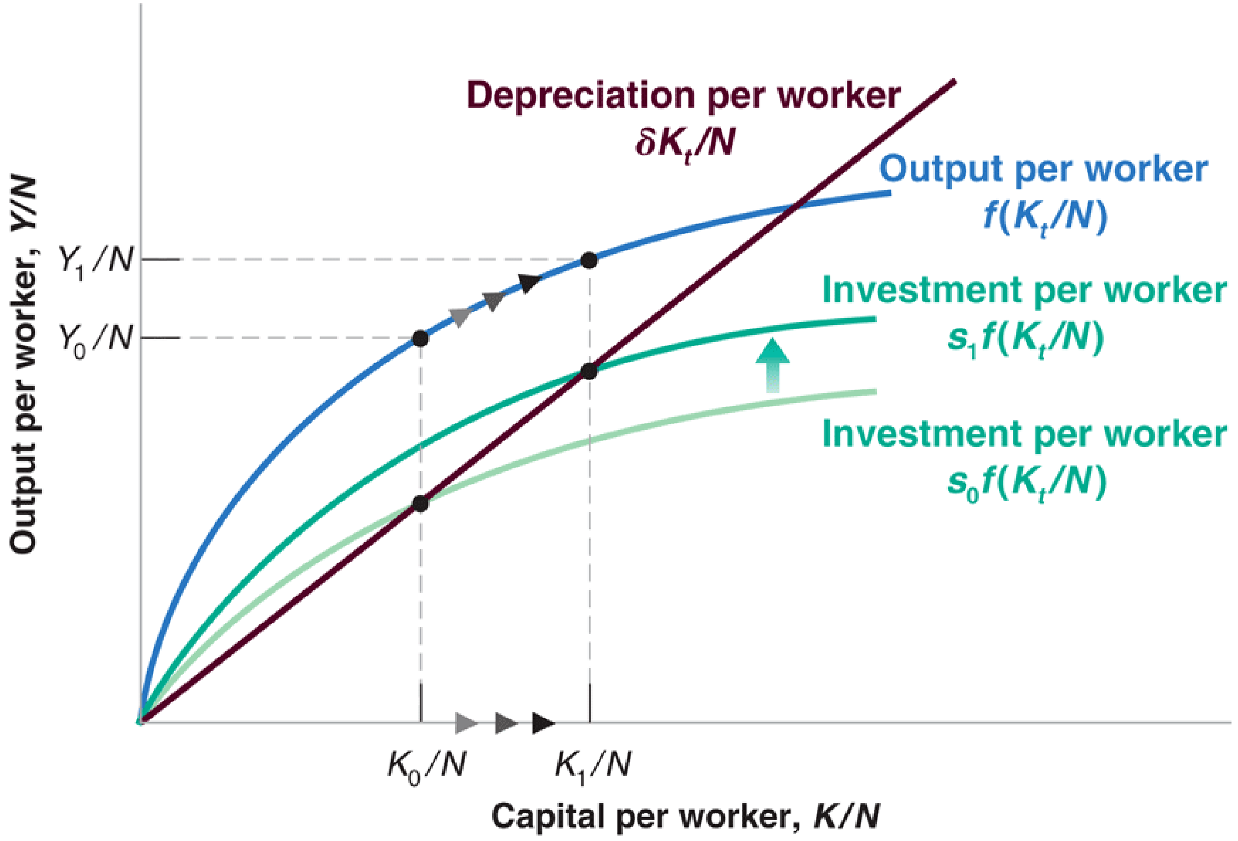
The saving rate and growth

- Suppose that an economy is in its steady state.
- In year t, the saving rate increases from s_0 to s_1 ($s_1 > s_0$).

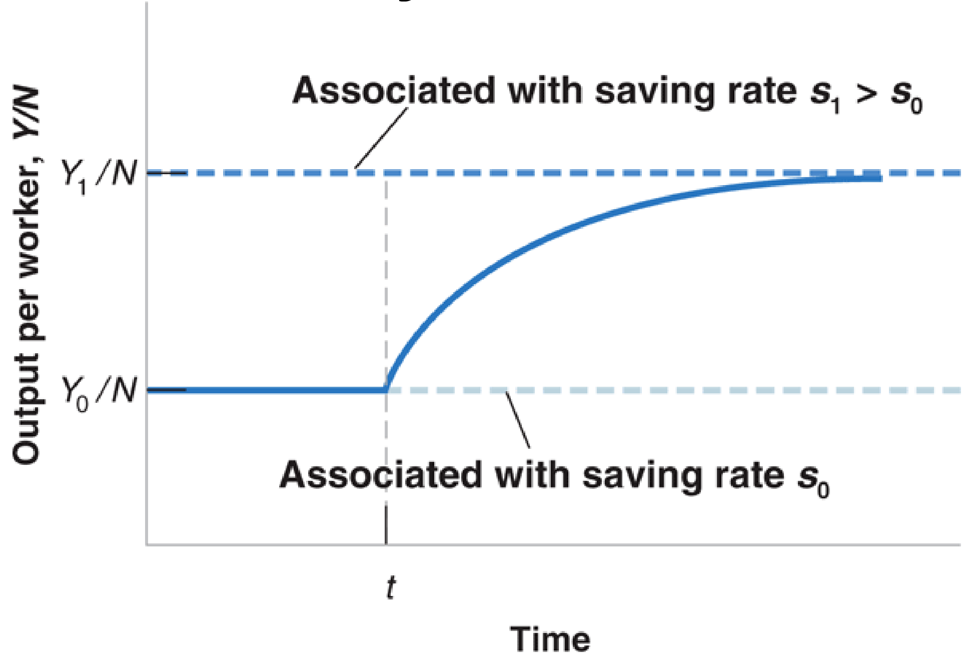
- What happens to k, y, and g_y?

The saving rate, s, increases from s_0 to s_1

- k starts to grow from $k_0(=$



y and g_y



- The saving rate has no effect on the long-run growth rate of output per worker, which is equal to zero.
- Nonetheless, the saving rate determines the level of output per worker in the long run.
- An increase in the saving rate will lead to higher growth of output per worker for some time, but not forever.

The paradox of thrift, revisited

- The effects of $s \uparrow$ in different time horizons
- The short run (Keynesian cross / IS-LM): $Y \downarrow$ (recession)
- The medium run (IS-LM-PC): Y- and $I \uparrow$
- The long run (Solow): $I \uparrow \rightarrow K \uparrow \rightarrow Y \uparrow$

The saving rate and consumption

- Althouth $s \uparrow$ cannot sustain higher growth of output per worker forever, it raises k^* and y^*.
- Then, will it be optimal to increase s as much as possible?

- To evaluate the welfare implication of such a change, we need to investigate consumption per worker, not output per worker.
- Consumption per worker in steady state, c^*, is given by income minus saving, $y^* - sy^* = (1 - s)y^* = (1 - s)f(k^*)$.

$$c^* = (1 - s)f(k^*)$$

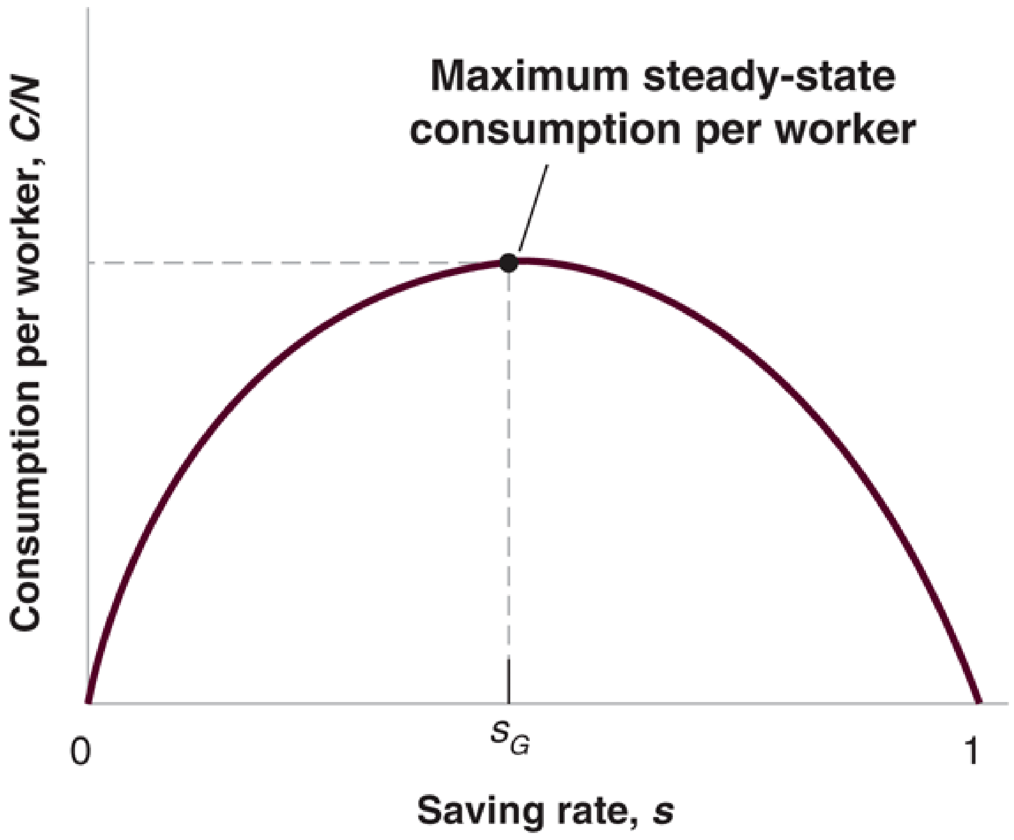
- Remarks) By focusing on the average consumption in steady state, c^*, we ignore transition dynamics between steady states under different s. We also ignore the distribution of consumption across people.

- We compare consumption in different steady states.
- When s is low (close to 0), investment, k^*, and $y^* = f(k^*)$ is small.
- When s is high (close to 1), investment, k^*, and y^* are large; but most of the product is used to replace depreciation in this case (remember that $\delta k^* = sf(k^*)$).

$$c^* = (1 - s)f(k^*)$$

- As $s\uparrow$, $(1-s)\downarrow$ and $f(k^*)\uparrow$.
- There exists the rate of saving, s_G, that maximizes c^*. The corresponding value of k^* is the golden-rule level of capital (per worker).

- $s < s_G \quad \Rightarrow \quad s\uparrow$ increases c^*
- $s > s_G \quad \Rightarrow \quad s\uparrow$ decreases c^*



s_G in the real world

- For example, when $Y = \mathcal{A}K^{\alpha}N^{1-\alpha}$, it is known that $s_G = \alpha$ (You do not need to know how to derive this result).
- In the US, α is known to be about 1/3, which is far greater than s. The situation is similar in other developed countries.
- Then, should governments try to raise s?

- $s \uparrow$ hurts current generation by initiating a recession in the short run. But in the long run, c^* increases. So, there is a trade-off between the welfare of current and future generations.
- A political issue: the future generations do not vote now.

Outline

- The Solow Growth Model
- Examples

- The Saving Rate, Output, and Consumption
- An Analytical Example ($Y = \sqrt{\mathrm{K}}\sqrt{N}$)

- Population Growth, Technological Progress, and Human Capital Accumulation

An example

- $Y = F(K, N) = \sqrt{\mathrm{K}}\sqrt{N}$

  (a Cobb-Douglas function with $\mathcal{A} = 1$ and $\alpha = .5$)

- Output per worker:
  $y =$

- The evolution of capital per worker over time:
  $k_{t+1} = (1 - \delta)k_t + sy_t$

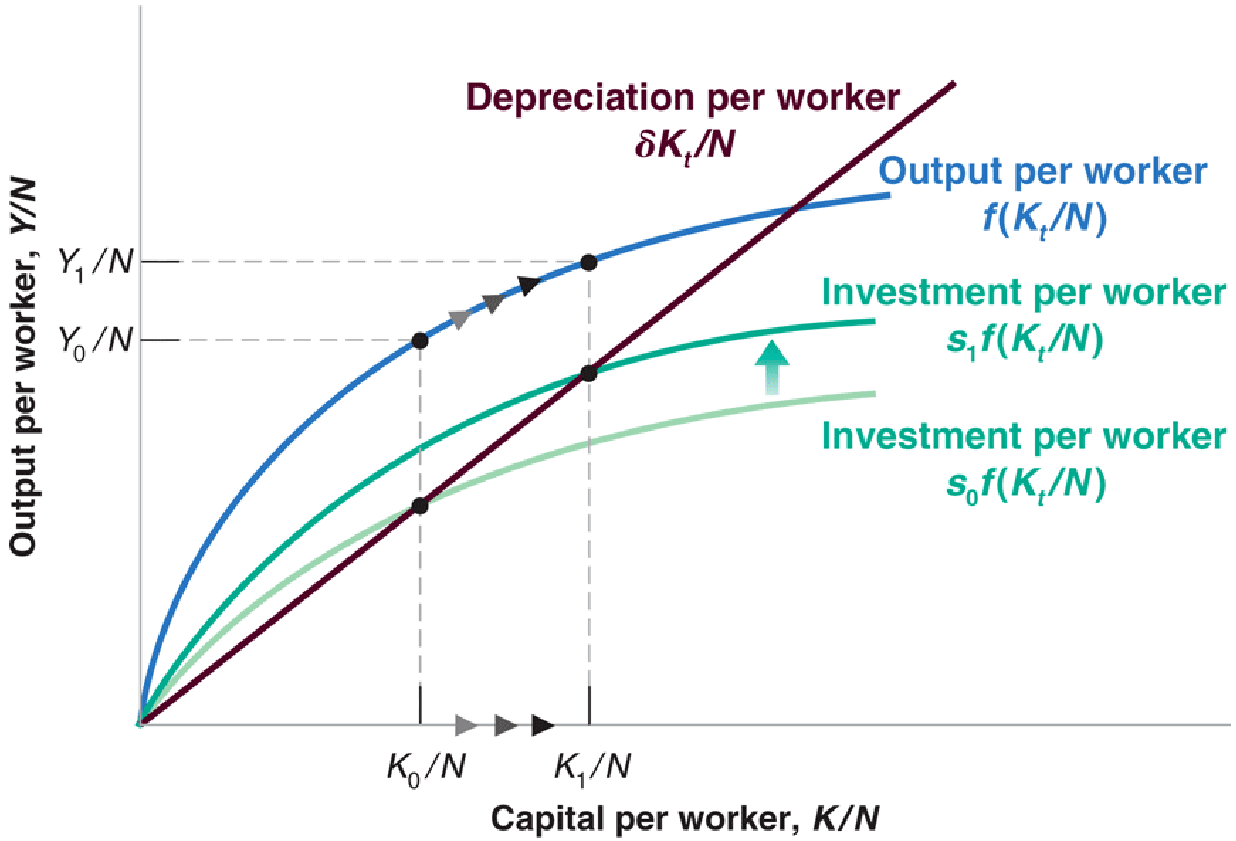
$\Rightarrow$

The steady state

- Saving/investment per worker = depreciation per worker
  $sf(k^*) = s\sqrt{k^*} = \delta k^* \quad \Rightarrow$

- The steady-state capital per worker: $k^* = \left(\frac{s}{\delta}\right)^2$
- The steady-state output per worker: $y^* = \sqrt{k^*} =$
- The steady-state saving/investment per worker: $sy^* =$
- The steady-state consumption per worker: $c^* = (1-s)y^* =$
- $s_G = argmax_s\, c^* = 0.5$

Why $k^* \uparrow$ when $s \uparrow$ and $\delta \downarrow$?



$s \uparrow$ from s_0 to s_1

$\delta \downarrow$ from δ_0 to δ_1

Outline

- The Solow Growth Model
- Examples

- The Saving Rate, Output, and Consumption
- An Analytical Example ($Y = \sqrt{\mathrm{K}}\sqrt{N}$)

- Human Capital as a Source of Economic Development

Human capital

- Physical capital: machines, plants, office buildings, etc.
- Human capital: the set of skills of the workers in the economy
- $Y = F(K, HN)$
- As workers become more skilled, $H \uparrow$.

- Human capital is a very broad concept. In practice, we usually measure it using average educational attainment of workers.
- As people get a higher education, they become more skilled and receive higher salaries.

- There was a significant increase in years of schooling during the 20th century in the US.
- $H \uparrow$ contributed to $\frac{Y}{N} \uparrow$.
- But this trend cannot be sustained forever as many people have already attained a college degree.

- Source: Goldin, Claudia and Lawrence F. Katz (2007), *The Race between Education and Technology.*

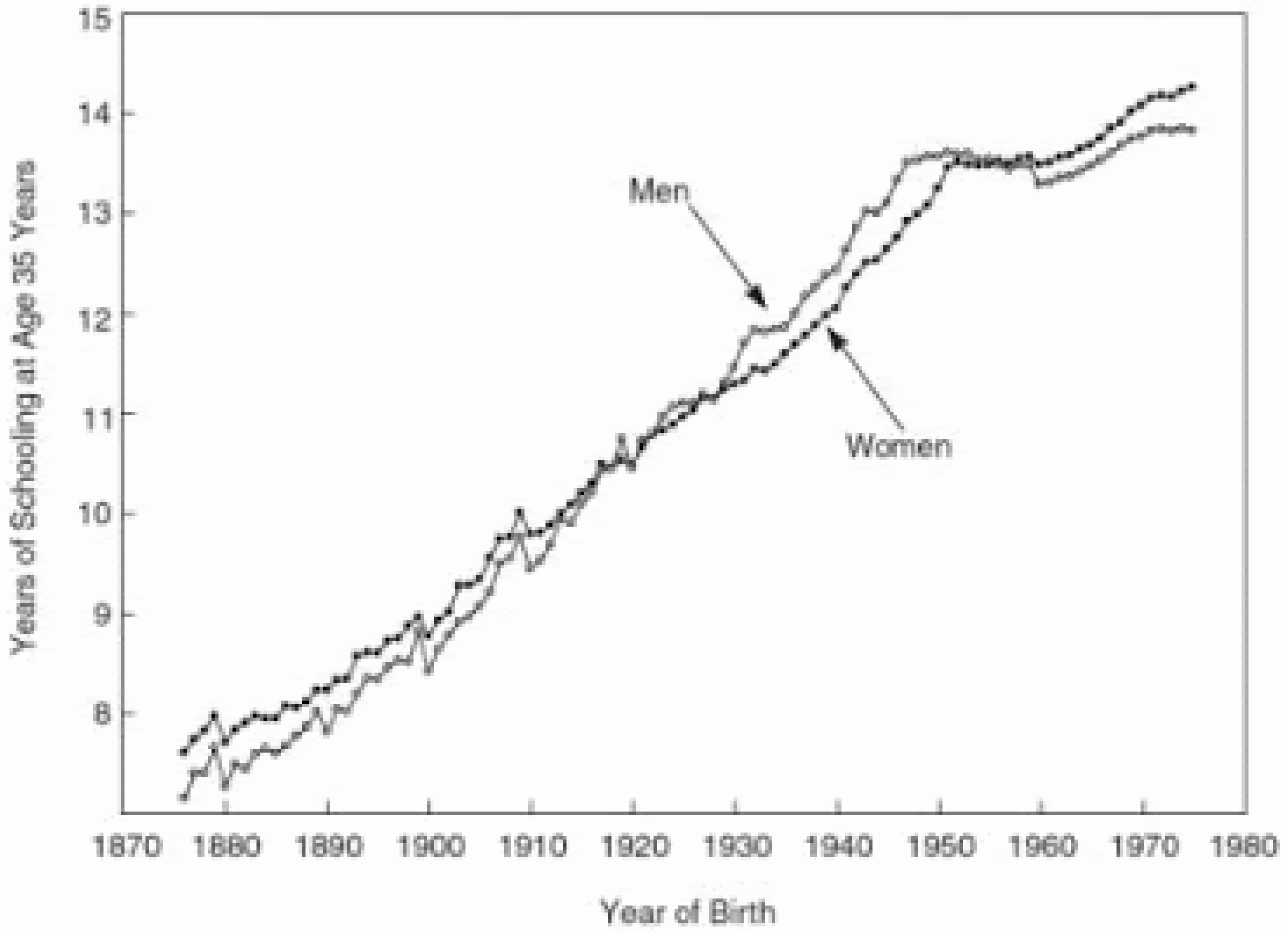


Figure 1.5. Years of Schooling by Birth Cohorts, U.S. Native-Born, by Sex: 1876 to 1975. This figure plots the mean years of completed schooling for U.S. native-born residents by birth cohort and sex, adjusted to age 35 using the approach described in the notes to Figure 1.4. Sources: 1940 to 2000 IPUMS.

- Education in Hong Kong
- Source: Snapshot of the Hong Kong Population, 2016 Population By-census

https://www.bycensus2016.gov.hk/data/snapshotPDF/Snapshot02.pdf

表 1 2006 年、2011 年及 2016 年按年齡組別及性別劃分的 3 歲及以上人口就學比率(1)

Table 1 School attendance rate(1) of population aged 3 and over by age group and sex, 2006, 2011 and 2016

年齡組別 Age group	2006 男性 Male	2006 女性 Female	2006 合計 Both sexes	2011 男性 Male	2011 女性 Female	2011 合計 Both sexes	2016 男性 Male	2016 女性 Female	2016 合計 Both sexes
	就學比率（百分比） School attendance rate (%)								
3 – 5(2)	89.9	88.3	89.1	91.0	91.6	91.3	92.7	92.3	92.5
6 – 11	99.9	99.9	99.9	100.0	100.0	100.0	100.0	100.0	100.0
12 – 17	96.4	97.4	96.9	96.6	97.7	97.1	97.6	98.0	97.8
18 – 24(3)	43.5 (43.5)	41.9 (45.9)	42.7 (44.7)	48.9 (49.0)	49.3 (51.7)	49.1 (50.3)	50.8 (50.8)	52.7 (54.4)	51.8 (52.6)
25+	0.5	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6

註釋：(1) 在各年齡組別中，就讀全日制院校的人數佔該年齡組別總人數的百分比。

(2) 有關教育特徵的數據是根據當年上半年的情況作訪問。故此，剛滿 3 歲的兒童可能因學期初（通常是早一年的 9 月份）還未達入學年齡的最低要求而在當年上半年仍未入讀學前教育。

(3) 括號內數字是把有關年齡及性別組別人口中的外籍家庭傭工扣除後，編製的就學比率。

Notes : (1) The percentage of population attending full-time educational institutions in the respective age groups.

(2) Data related to educational characteristics were enquired with reference to the first half of the year. Hence, children just reaching age 3 might not be attending pre-primary education in the first half of the year as they had not yet reached the minimum age for entrance at the beginning of the school term (usually in September of the previous year).

(3) Figures in brackets are school attendance rates compiled with foreign domestic helpers excluded from the population in the respective age-sex groups.

Human capital as a source of growth

- In the underdeveloped and the developing countries, encouraging people to attend schools and accumulate human capital can be an important driver of economic growth.
- Example) Primary education → Workers can read and understand written instructions and manuals for equipment.

- In the developed countries where most children get secondary education and significant fraction of people have a college degree, it is evident that further accumulation of human capital cannot be the major source of future economic growth.

In the next class…

- We discuss the importance of technological progress as a determinant of economic growth in the developed countries.
- We review the recent trends in productivity in the US.
- We study policy tools to promote innovations, which lead to higher productivity.