ECON3113
Microeconomic Theory I

Tutorial #2: The foundations of consumer choice

Today's tutorial

- Two components:
  - More on the foundations of consumer choice
  - A Practice Quiz (the last 15 minutes)

Where we got to last time and where we are going

- Towards a modern approach to the consumer choice problem
- We showed that a well-defined preference relation that obeys Completeness, Reflexivity and Transitivity generates a choice function:

  ie preference relation => choice function

- Next step:
  - To introduce the utility function
  - To show the equivalence of the utility function, the preference relation and the choice function as long as completeness, reflexivity and transitivity are met

Introducing utility

- Given a set of alternatives $X = \{x_1, x_2, \dots, x_n\}$
- Suppose that a consumer attaches a numerical score to each x_i according to its desirability
  - Call this score the consumer's <u>utility</u>
- Then the consumer's utility function, $\mathrm{u}(\mathrm{X})$, is the mapping from X to $\mathrm{u}(\mathrm{X})$:
  - $\{x_1, x_2, \dots, x_n\} \rightarrow \{\mathrm{u}(x_1), \mathrm{u}(x_2), \dots, \mathrm{u}(x_n)\}$
- Example:
  - {tea, espresso, latte} $\rightarrow \{30, 10, 30\}$

Notice that the utility function only gives us a <u>ranking</u> of desirability, <u>not a measure</u> of absolute or relative desirability

Utility and the preference relation

- The pairwise comparison of utilities defines a preference relation:
  - $x \geq y$ if and only if $\mathrm{u}(x) \geq \mathrm{u}(y)$
  - ie $\quad x \geq y \;\Leftrightarrow\; \mathrm{u}(x) \geq \mathrm{u}(y)$
- Example:

Preference relation

tea $>$ espresso
tea $\sim$ latte
latte $>$ espresso

$\Longleftrightarrow$

Utility

$\mathrm{u}(\mathrm{tea}) > \mathrm{u}(\mathrm{espresso})$
$\mathrm{u}(\mathrm{tea}) = \mathrm{u}(\mathrm{latte})$
$\mathrm{u}(\mathrm{latte}) > \mathrm{u}(\mathrm{espresso})$

From utility to the choice function

- We assume that given a selection of alternatives, A, the consumer chooses only those alternatives that have the highest utility score:

$$c_u(A) = \{x \in A : u(x) \geq u(y) \text{ for all } y \in A\}$$
$$= \left\{x \in A : u(x) = \max_{y \in A} u(y)\right\}.$$

- Example:
  - Given $A = \{\text{tea, espresso, latte}\}$ and $\mathrm{u}(A) = \{30, 10, 30\}$
  - $c_u(A) = \{\text{tea, latte}\}$
- So our utility function implies a choice function

The main theorem:

Theorem

Suppose X is finite.
(i) Given a utility function u, the implied choice function c_u is coherent. Moreover, the preference relation produced by u is complete and transitive.
(ii) Given a complete transitive preference relation $\succsim$, the implied choice function $c_{\succsim}$ is coherent. Moreover, there exists a utility function u that implies $\succsim$.
(iii) Given a coherent choice function c, there exist a complete transitive preference relation $\succsim$ and a utility function u that produces choices c.

- We illustrate each part (i)-(iii)

The main theorem: Part (i)

Theorem

Suppose X is finite.

(i) Given a utility function u, the implied choice function c_u is coherent. Moreover, the preference relation produced by u is complete and transitive.

- Given:
- $X =$ {tea, espresso, latte, hot chocolate}, $A =$ {tea, espresso, latte}, $B =$ {tea, espresso, hot chocolate}, and $A \cap B = \{\text{tea}, \text{espresso}\}$
- $\mathrm{u}(X) = \{30,10,30,20\}$
- Then:
  - $c_u(A) =$ {tea, latte}, $c_u(B) =$ {tea}
  - Since espresso is available in A and B and not chosen in either, the choice function implied by $\mathrm{u}(X)$ is coherent

The main theorem: Part (i)

Theorem

Suppose X is finite.

(i) Given a utility function u, the implied choice function c_u is coherent. Moreover, the preference relation produced by u is complete and transitive.

- We have $X = \{\text{tea, espresso, latte, hot chocolate}\}$ and $\mathrm{u}(X) = \{30,10,30,20\}$
- This implies:

	tea	espresso	latte	hot chocolate
tea	~	>	~	>
espresso	<	~	<	<
latte	~	>	~	>
hot chocolate	<	>	<	~

- Complete:
  - We have a comparison for each pair
- Transitive:
  - Can check
  - eg:
    - tea > hot chocolate
    - hot chocolate > espresso
    - and tea > espresso

The main theorem: Part (ii)

Theorem

Suppose X is finite.

(ii) Given a complete transitive preference relation $\succsim$, the implied choice function $c_{\succsim}$ is coherent. Moreover, there exists a utility function u that implies $\succsim$.

- Given:
- $X = \{$tea, espresso, latte, hot chocolate$\}$, $A = \{$tea, espresso, latte$\}$, $B = \{$tea, espresso, hot chocolate$\}$, and $A \cap B = \{\text{tea}, \text{espresso}\}$, and complete, transitive preference relation given by:

	tea	espresso	latte	hot chocolate
tea	$\sim$	>	$\sim$	>
espresso	<	$\sim$	<	<
latte	$\sim$	>	$\sim$	>
hot chocolate	<	>	<	$\sim$

The main theorem: Part (ii)

- Is the choice function $c_{\geq}$ coherent?
- For A, we have:

	tea	espresso	latte
tea	$\sim$	$>$	$\sim$
espresso	$<$	$\sim$	$<$
latte	$\sim$	$>$	$\sim$

- For B, we have:

	tea	espresso	hot chocolate
tea	$\sim$	$>$	$>$
espresso	$<$	$\sim$	$<$
hot chocolate	$<$	$>$	$\sim$

- $c_{\geq}(A) = \{\text{tea, latte}\}$
- $c_{\geq}(B) = \{\text{tea}\}$
- $A \cap B = \{\text{tea, espresso}\}$, and espresso not in $c_{\geq}(A)$ or $c_{\geq}(B)$
- Therefore $c_{\geq}$ is coherent

The main theorem: Part (ii)

- Is there a utility function that represents the preference relation $\succeq$?

	tea	espresso	latte	hot chocolate
tea	$\sim$	$>$	$\sim$	$>$
espresso	$<$	$\sim$	$<$	$<$
latte	$\sim$	$>$	$\sim$	$>$
hot chocolate	$<$	$>$	$<$	$\sim$

- Notice that any scoring scheme will do that represents the <u>ranking</u> implied by the preference relation
  - Examples:
    - $\mathrm{u}(X) = \{30,10,30,20\}$
    - $\mathrm{u}(X) = \{\pi, -4, \pi, e\}$
- So there is a utility function that represents $\succeq$

The main theorem: Part (iii)

Theorem

Suppose X is finite.
(iii) Given a coherent choice function c, there exist a complete transitive preference relation $\succsim$ and a utility function u that produces choices c.

- Given:
- X = {tea, espresso, latte, hot chocolate}, A = {tea, espresso, latte}, B = {tea, espresso, hot chocolate}, and $A \cap B = \{\text{tea}, \text{espresso}\}$, and coherent choice function $c_{\geq}$ such that $c_{\geq}(A) = \{\text{tea}\}$
- The following preference relation and utility function produce $c_{\geq}$:

	tea	espresso	latte	hot chocolate
tea	$\sim$	>	$\sim$	>
espresso	<	$\sim$	<	<
latte	$\sim$	>	$\sim$	>
hot chocolate	<	>	<	$\sim$

$$\mathrm{u}(X) = \{30,10,30,20\}$$

The equivalence of the utility function, choice function and preference relation

- We have shown that if completeness, reflexivity and transitivity of a preference relation are met, then:

Utility function $\Leftrightarrow$ Choice function $\Leftrightarrow$ Preference relation

- This is where we wanted to get to

Equivalent utility representations

- Can we distinguish between the following utility functions on the set $X = \{\text{tea, espresso, latte, hot chocolate}\}$?:

	u_1	u_2	u_3
tea	30	70	3.4012
espresso	10	30	2.3026
latte	30	70	3.4012
hot chocolate	20	50	2.9957

- No – only the ranking matters, and they give the same ranking
- In general, given a utility function $u_i(X)$, if f is a well-defined and strictly increasing function on the set of utilities, then utility functions $u_i(X)$ and $f[u_i(X)]$ are equivalent
- We have $u_2 = 10 + 2u_1$, $u_3 = \ln(u_1)$
  - Notice that these functions are well-defined and are strictly increasing for $u_1(x_i) > 0$

In this theory, utility is an ordinal concept

- From the lecture notes:

- In modern economic analysis, utility is an **ordinal** concept.
- It means that two utility functions are regarded as equivalent if the implied rankings over alternatives are identical.
- The following statements are *inconsistent* with the ordinal concept of utility.
  - John derives a utility of 10^{50} from a dish of char siu so he must be extremely satisfied after eating it.
  - Mary's utility of eating a dish of char siu is -100 so she must hate it.
  - John is happier than Mary because the char siu dish gives John more utility than Mary.
  - As $u\,(\text{char siu}) = 40$ and $u\,(\text{roasted duck}) = 20$, char siu is twice better than roasted duck.

- The absolute size of the utility number bears no relation to the magnitude of satisfaction
- Comparisons of utility between consumers cannot be made
- The relative magnitude of utility of different goods does not reflect the relative magnitude of the satisfaction between them
  - All completely different to the traditional theory