ECON3133 Microeconomic Theory II

Tutorial #8: Introduction to Game Theory

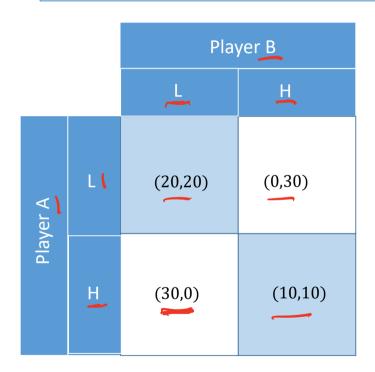
Today's tutorial:

- Introduction to Game Theory
- Pure strategies in static games
 - The Normal Form of a game
 - Dominant and secure strategies
 - Nash equilibrium
 - The socially optimal choice
 - Co-ordination and equilibrium choice
 - The power of a dominant strategy
 - Sequentially eliminating strictly dominated strategies
 - Games with three players
 - Voting games

Game theory: the setting

- The structure of a game
 - Players
 - · Strategies and choice of strategy
 - Pay-offs
 - Timing of making choices
 - Simultaneous or sequential
 - Frequency
 - How often is the game played?

. Once only.



- 2 players, A and B
- 2 strategies for each player, S_1 and S_2
- One pay-off per player per strategy (P_i^A, P_i^B)
 - Note: Player B is always in the second position in $(, \mathbb{C})$
- Eg if Player A chooses L and player B chooses H
 then the pay-off is (0,30), 0 to player A and 30
 to player B

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- Strategy types:
 - · A dominant strategy
 - A strategy that a player should choose regardless of the other player's choice
 - A secure strategy

at least

• A strategy that guarantees a sure pay-off of a given amount

layers max pay-off; mixed stortegy
game.

3. minimizes risk of loss

gnarantees a <u>certoin</u> minimum.

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		Player B	
		L	Н
Player A	L	(20,20)	(0,30)
Play	Н	(30,0)	(10,10)

- Does either Player have a dominant strategy in this example?
- Player *A*:

Player B chooses	Player A's Best Response	Pay-off to A
L	H	30
Н	H	10

We may write:

•
$$u_A(H, L) = 30 > u_A(L, L) = 20$$

•
$$s_A^* = H = BR_A(s_B = L)$$

•
$$u_A(H, H) = 10 > u_A(L, H) = 0$$

•
$$s_A^* = H = BR_A(s_B = H)$$

• $u_A(H, L) = 30 > u_A(L, L) = 20$ • $s_A^* = H = BR_A(s_B = L)$ • $u_A(H, H) = 10 > u_A(L, H) = 0$ • $s_A^* = H = BR_A(s_B = H)$ • $\Rightarrow s_A^* = H$ is a dominant strategy for player A

		Player B	
		L	Н
Player A	L	(20,20)	(0,30)
Play	Н	(30,0)	(10,10)

- Does either Player have a dominant strategy in this example?
- Player *B*:

Player A chooses	Player B's Best Response	Pay-off to B
L	H	30
Н	H	10

- We may write:
 - $u_R(L, H) = 30 > u_R(L, L) = 20$
 - $s_B^* = H = BR_B(s_A = L)$
 - $u_B(H, H) = 10 > u_B(H, L) = 0$
 - $s_B^* = H = BR_B(s_A = H)$
 - $\Rightarrow s_B^* = H$ is a dominant strategy for player B

		Player B	
		L	Н
er A	L	(20,20)	(0,30)
Player A	н	(30,0)	(10,10)

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- What secure strategies are available for either player?
- Player *A*:

• Player *B*:

· Alweys playing H earns at least 10.

Key idea: A Nash equilibrium

- A Nash equilibrium:
 - A NE is an equilibrium in which each player's strategy is the Best Response to the other players' Best Responses:
 - An equilibrium in which no player can unilaterally improve their outcome by changing their own strategy given all the other players' strategies
 - That is, an equilibrium in which one strategies are decided, no player has an incentive to cheat .
 - Note: any equilibrium is a combination of strategies and not pay-offs!

· (H, 4)

not

eg

(30, 10

		Player B	
		T	Н
Player A	L	(20,20)	(0, <u>30</u>)
Play	н	(30,0)	(10,10)

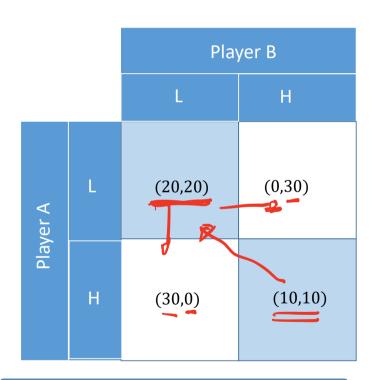
Is there a Nash equilibrium in this example?

Player B chooses	Player A's Best Response	Pay-off
L	H	30
Н	H	10

Player A chooses	Player B's Best Response	Pay-off
L	Н	30
Н	H	10

•
$$s_A^* = H = BR_A(s_B^* = H)$$

• $s_B^* = H = BR_B(s_A^* = H)$ Mutual Best Response of A and B



- What is the efficient outcome in this game?
- Pareto efficient:

Maximise total pay-off

- The efficient outcome is different to the Nash equilibrium
 - The socially optimal outcome is to co-operate

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theory II

Oue player tren not Paebo Efficient

Both player can chage, tren can reto efficient;

improve by going 120,20)

Example: a Normal Form game with multiple NE

		Player B	
		L	Н
Player A	L	(10,10)	(20,30)
Play	н	(30,20)	(10,10)

• Is there a Nash equilibrium in this example?

Player B chooses	Player A's Best Response	Pay-off to Player A
L	H	30
Н	7	20

Player A chooses	Player B's Best Response	Pay-off to Player B
L	H	30
Н	L	20-

- Therefore in this case we have two NE:
 - (L, H) and (H, L)

The Nash Equilibrium: some issues

- The Nash Equilibrium (NE) is such that once it has emerged, no player has an incentive to deviate from it
 - · But nothing in the game tells us how we get to a Nash Equilibrium
- A Nash Equilibrium is not unique
 - A game may have more than one NE (eg any symmetric two-player game will have an even number of NE), or none at all (eg Rock, Paper, Scissors)
 - A game may have too many NE to be useful
- Over time, information, expectations and pay-offs may change, so the NE in a game may change

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		Player B	
		L	Н
Player A	L	(10,10)	(20,20)
Play	Н	(30,30)	(10,10)

• What are the Nash equilibria in this example?

Player B chooses	Player A's Best Response	Pay-off to Player A
L	H	30
Н	2	20

Player A chooses	Player B's Best Response	Pay-off to Player B
L	Н	20.
Н	H	30.

• Therefore in this case we have two NE:

•
$$(L,H)$$
 and (H,L)

• A rational player will realise that (L,H) is better than (H,L) and so (L,H) is considered superior

B

		L	Н
A	L	(10,10)	(20,20)
	Н	(30,30)	(10,10)

B

		L	Н
H (30,20) (10,10)	L	(10,10)	(20,30)
	Н 1	(30,20)	(10,10)

- Two NE: (L,H) and (H,L)
- (HIL)

- (L,H)
- A rational player will realise that (L,H) is better than (H,L) and so (L,H) is considered superior
- Two NE: (*L*, *H*) and (*H*, *L*)
- No clear reason to expect one NE over the other

B

	L	Н
L	(30,20)	(10,10)
Н	(10,30)	(20,30)

- The NE are: (L,L), (H,H), (30,20), (20,30).
- Can we say anything about which NE is more likely?
 - Consider Player B's secure strategy:

Then consider Player A's response to Player B's secure strategy:

• Therefore (L) is the more likely NE

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Bared on risk preferences of player B.

	Ĺ	Н
L	(20,20)	(30,0)
Н	(0,30)	(50,50)

• The NE are: (し,し) (H,H)

- Is NE (50,50) more likely than NE (20,20)?

· What if Player B plays randomly?

A would like to be at (H,H) through

Co-operation, but if B plays randomly tren

Playing H might earn o for A. Playing L is more searce

· Player A's best strategy may be the secure strategy L to

earn a minimum of 20

- It's not enough to know that you are rational
- You also need to know that the other person is rational
- And the other person needs to know that you are rational

prefes a seure

Example: The power of a dominant strategy

	L	Н
L	(20,20)	(0,30)
Н	(30,0)	(10,10)

- However, if a Player has a dominant strategy, then it doesn't matter whether the other Player is rational or not
- The dominant strategy is more likely to be played
- Suppose that Player B is playing irrationally and Player A is playing rationally
 - Player A's best response is always the dominant strategy, which is to play H

Example: Two players and four strategies

			В		
		C1	C2	СЗ	C4
	R1	(6,2)	(8,0)	(4,4)	(8,2)
Α	R2	(6,0)	(2,12)	(0,4)	(16,2)
	R3	(8,8)	(6,2)	(2,8)	(8,4)
	R4	(4,2)	(4,4)	(0,2)	(12,0)

• The NE are:

Example: Two players and four strategies

			В		
		C1	C2	C3	C4
	R1	(4,0)	(1,4)	(2,1)	(6,2)
Α	R2	(8,2)	(2,2)	(6,3)	(8, <u>4</u>)
	R3	(3, <mark>1</mark>)	(6 , 1)	(8,2)	(1,2)
	R4	(6, <mark>3</mark>)	(8,4)	(4,6)	(6,6)
		F	-		-

- We can sometimes reduce the possible equilibria by eliminating those strategies that are strictly dominated by other strategies
 - Sequentially eliminating dominated strategies

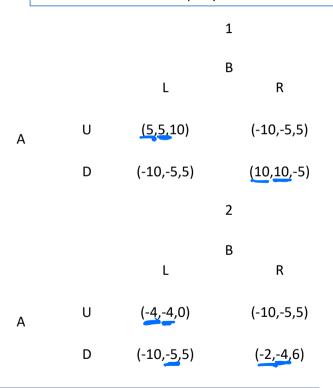
- 1. For Player B:
- 2. For Player A:
- 3. For Player B:
- 4. For Player A:

(R3, C3) and (R2, C4)

Games with three players

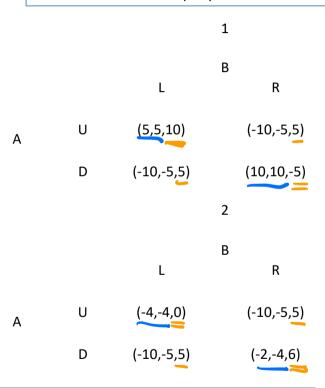
- We now have 3 players, A, B and C
- How to show a 3 dimensional game?
 - Players A and B choose U or D, L or R respectively
 - Player C chooses version 1 or 2 of the game

Games with three players



- Approach:
- 1. Fix Player C plays #1
 - 1.1 Player B plays $L \Rightarrow$ Player A plays U
 - 1.2 Player B plays $R \Rightarrow$ Player A plays D
 - 1.3 Player A plays $U \Rightarrow$ Player B plays L
 - 1.4 Player A plays $D \Rightarrow$ Player B plays R
- 2. Fix Player C plays #2
 - 1.1 Player B plays $L \Rightarrow$ Player A plays U
 - 1.2 Player B plays $R \Rightarrow$ Player A plays D
 - 1.3 Player A plays $U \Rightarrow$ Player B plays L
 - 1.4 Player A plays $D \Rightarrow$ Player B plays R

Games with three players



- 3. Player C's choices
- 3.1 Player A plays U, Player B plays L=> Player C chooses #1
- 3.2 Player A plays U, Player B plays $R \Rightarrow$ Player C indifferent between #1 and #2
- 3.3 Player A plays D, Player B plays $L \Rightarrow$ Player C indifferent between #1 and #2
- 3.4 Player A plays D, Player B plays $R \Rightarrow$ Player C chooses #2

To give 2 NE at:

- (*U*, *L*, 1)
- (D, R, 2)

Voting games: The intransitivity of collective preferences

• Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$
 $\pi_2(B) > \pi_2(C) > \pi_2(A)$
 $\pi_3(C) > \pi_3(A) > \pi_3(B)$

- What is the outcome if candidate C is disqualified?
 - Player 1: A
 - Player 2:
 - Player 3: *A*
 - The winner:

- Assume 3 players (1,2,3) and 3 strategies
 (A, B, C eg candidates, preferred election choices)
- A majority voting rule (ie all votes count equal, a simple majority wins)
- Pay-offs to the players π_1, π_2, π_3

Voting games: The intransitivity of collective preferences

Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$

$$\pi_2(B) > \pi_2(C) > \pi_2(A)$$

$$\pi_3(C) > \pi_3(A) > \pi_3(B)$$

- What is the outcome if B and C are the candidates?
 - Player 1:
 - Player 2:
 - Player 3:
 - The winner:



- What is the outcome if A and C are the candidates?
 - Player 1:
 - Player 2:
 - Player 3:
 - The winner: (

Voting games: The intransitivity of collective preferences

Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$

$$\pi_2(B) > \pi_2(C) > \pi_2(A)$$

$$\pi_3(C) > \pi_3(A) > \pi_3(B)$$

- Summary:
- Between A and B, A is preferred
- Between B and C, B is preferred
- Between A and C, C is preferred
- In terms of preference relations we have:
- A > B
- B > C
- Transitivity requires that therefore A > C, but in this case we have C > A
- Therefore the collective preference under simple majority voting is not transitive
- Note that individuals may have transitive preferences, but the collective preference need not be transitive

Voting games: the status quo

Suppose that preferences are as follows:

$$\pi_1(A) > \pi_1(B) > \pi_1(C)$$
 $\pi_2(B) > \pi_2(C) > \pi_2(A)$
 $\pi_3(C) > \pi_3(A) > \pi_3(B)$

- Suppose that if the voting result is 1-1-1 then nothing changes
 - The "status quo" is maintained
- Suppose that A is the incumbent/status quo/nothing changes candidate or policy
- What might Player #2 do in this situation?
 - In the simple majority, the result is 1-1-1
 - Then Player #2's least preferred candidate/policy wins
 - Knowing this beforehand, Player #2 might vote for their second preferred candidate/policy, C, and then C wins
 - But then Player #1 might know this beforehand, and so vote for B
 - And so on...