PROBLEM SET 6 (NO NEED TO SUBMIT, NOT COUNTED FOR YOUR GRADES)

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In the following three questions, the assumptions of Merton's model hold. Let's assume a firm has assets of \$100 with $\sigma = 40\%$, the continuously-compounded expected return of assets is $\mu = 15\%$, the dividend yield is $q = 0$, and the continuous-compounded risk-free rate is $r = 8\%$.

Question 1: Credit Spread (3/10) The firm has a single outstanding debt issue with a promised maturity payment of \$120 in 5 years. What is the probability of default (bankruptcy)? What is the credit spread?

Question 2: Expected Loss Given Default (4/10) Suppose the firm issues a single zero-coupon bond with maturity value \$100.

1. Compute the credit spread and probability of default for times to maturity of $T = 1$, 3, and 10 years.

2. The expected loss given default has the following formula:

$$\mathbb{E}_0\left[F - V_T | V_T < F\right] = F - V_0 e^{\mu T}\frac{\mathcal{N}(-d_{1,\mu})}{\mathcal{N}(-d_{2,\mu})}, \tag{1}$$

   where

$$d_{1,\mu} = \frac{\ln\left(\frac{V_0}{F}\right) + \left(\mu + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}},$$

$$d_{2,\mu} = d_{1,\mu} - \sigma\sqrt{T}.$$

   Compute the expected loss given default for times to maturity of $T = 1$, 3, and 10 years.

3. For each time to maturity compute the approximation for the credit spread:

$$y - r = \frac{1}{T} \times \text{Prob Default} \times \text{Expected loss rate given default}$$

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The expected loss rate given default is defined as

$$\frac{\mathbb{E}_0\left[F - V_T | V_T < F\right]}{F}.\tag{2}$$

How accurate is the approximation?

Question 3: Expected Recovery (3/10)   Suppose the firm issues a single zero-coupon bond with time to maturity 3 years and maturity value \$110.

1. Compute the price of the bond, yield to maturity, and default probability.

2. (Optional) What is the expected recovery ($\mathbb{E}_0\left[D_T | \text{default}\right]$ with D_T being the final payment of the bond on maturity date). Hint: it is easy to derive the formula using equation (1).