Derivative Securities (FINA 3203)
Solutions to Problem Set 4

Question 1: Basic Concepts on Options (2/10)

SOLUTION:

(1) American option is the same as European option, except that American option can be exercised anytime by the maturity date.

(2) It can be seen clearly by plotting the payoff function of European call and put options. In addition, the relationship between call and put options is the Put-Call Parity:

$$P(S, K, T) - C(S, K, T) = Ke^{-rT} - S.$$

Question 2 (Optional): Basic Concepts on Option Trading Strategies

SOLUTION:

(1) A butterfly spread is a neutral option strategy combining bull and bear spreads. Butterfly spreads use four option contracts with the same expiration but three different strike prices to create a range of prices the strategy can profit from. The trader sells two option contracts at the middle strike price and buys one option contract at a lower strike price and one option contract at a higher strike price. Both puts and calls can be used for a butterfly spread. No, if you expect volatility to increase, you should short the butterfly spread.

(2) A straddle is an options strategy in which the investor holds a position in both a call and put with the same strike price and expiration date, paying both premiums. This strategy allows the investor to make a profit regardless of whether the price of the security goes up or down, assuming the stock price changes somewhat significantly. Yes, if you expect volatility to increase, you should long the straddle.

Question 3 (Optional): More on Butterfly Spreads

SOLUTION:

i) The net premium that you receive in March from (i) writing a strike \$30 call, (ii) writing a strike \$30 put, iii) buying a strike \$27.5 put, and (iv) buying a strike \$32.5 call is

$$\$1.55 + \$0.85 - \$0.30 - \$0.45 = \$1.65.$$

ii) Let S_{April} denote the price of GE stock in April.

If $S_{\text{April}} \leq \$27.50$, then the put you bought and the put you sold are exercised and your net cash flow in April is

$$\$27.5 - S_{\text{April}} - (\$30 - S_{\text{April}}) = -\$2.50.$$

If $\$27.50 < S_{\text{April}} \leq \$30$, then only the put you sold is exercised and your net cash flow in April is

$$S_{\text{April}} - \$30.$$

If $\$30 < S_{\text{April}} \leq \$32.50$, then only the call you sold is exercised and your net cash flow in April is

$$\$30 - S_{\text{April}}$$

If $S_{\text{April}} > \$32.50$, then the call you bought and the call you sold are exercised and your net cash flow in April is

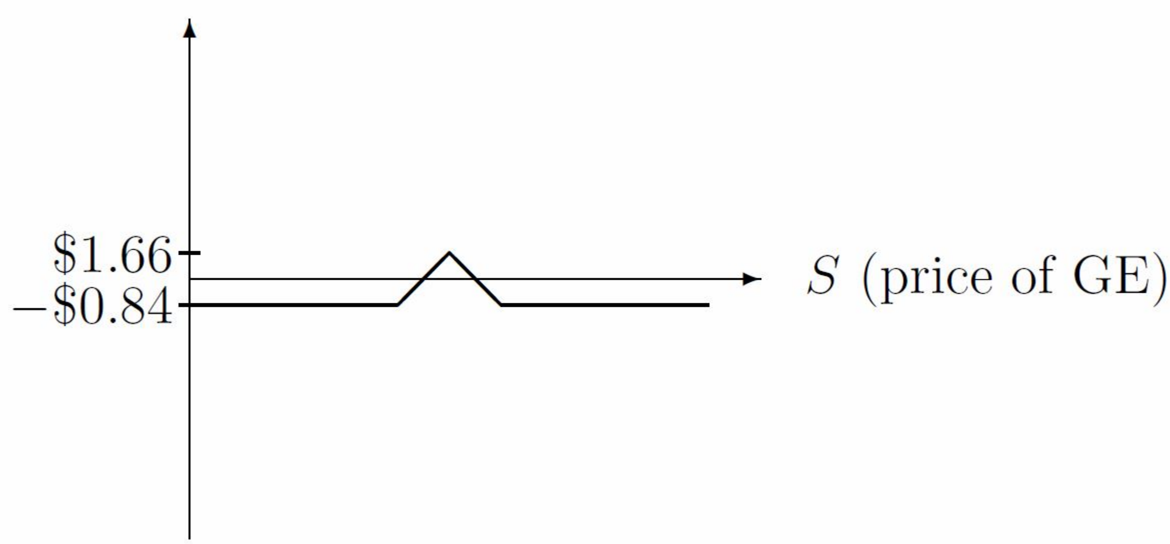
$$S_{\text{April}} - \$32.5 - (S_{\text{April}} - \$30) = -\$2.50$$

iii) The future value of the premium collected in March is $\$1.65e^{0.10\times 1/12} = \$1.66$

Hence, your profits are:

	Profits
$S_{\text{April}} \leq \$27.50$	$\$1.66 - \$2.50 = -\$0.84$
$\$27.50 < S_{\text{April}} \leq \$30$	$\$1.66 + S_{\text{April}} - \$30 = S_{\text{April}} - \$28.34$
$\$30 < S_{\text{April}} \leq \$32.50$	$\$1.66 + \$30 - S_{\text{April}} = \$31.66 - S_{\text{April}}$
$S_{\text{April}} > \$32.50$	$\$1.66 - \$2.50 = -\$0.84$

and



Question 4: One-Step Binomial Trees for Option Pricing (8/10)

SOLUTION:

An equivalent method is the backward-induction method using the risk neutral pricing approach. Here, we provide the risk-neutral pricing method. The risk-neutral probability for an up-move in the tree described is

$$q = \frac{e^{0.03\times 1/2} - 0.6}{1.3 - 0.6} = 0.593$$

The prices in the binomial tree are either $20 \times 1.3 = \$26$ or $20 \times 0.6 = \$12$ after 6 months, and $20 \times 1.3^2 = \$33.8$, or $20 \times 1.3 \times 0.6 = \$15.6$, or $20 \times 0.6^2 = \$7.2$ after a year.

(1)

$$C_E\,(T = 6 \text{ months}) = e^{-0.03\times 1/2} \times q \times (26 - 19) = \$4.09$$

(2)

$$P_E\,(T = 6 \text{ months}) = e^{-0.03\times 1/2} \times (1 - q) \times (19 - 12) = \$2.81$$

Put-call parity holds since

$$P_E - C_E = 2.81 - 4.09 = -1.28 = e^{-0.03\times 1/2} \times 19 - 20 = PV\,(K) - S$$

(3) You should buy the put (it is underpriced), and hedge by selling a synthetic put. In standard notation, the Δ of the put is

$$\Delta_{put} = \frac{0 - (19 - 12)}{26 - 12} = -\frac{1}{2}.$$

That is, you should sell $-\frac{1}{2}$ a share, i.e., buy 1/2 a share. So your cash flow today is $-1 - \frac{1}{2} \times 20 = -11$, while your cash flow in six months is either $0 + \frac{1}{2} \times 26 = 13$ (after an up movement) or $19 - 12 + \frac{1}{2} \times 12 = 13$ (after a down movement). So you have a certain six-month return of \$2 on an \$11 initial investment, which clearly beats the riskfree rate of 3%.

(4)

$$q = \frac{e^{0.03\times 1/2} - 0.6}{1.3 - 0.6} = 0.593$$

(5) See above equations.

Question 5 (Optional): Exotic Options

SOLUTION: Consider holding 1/40 shares of the stock. It is worth \$139/40. If you sell the position once the stock price reaches \$160, you will receive cash \$4 = \$160/40. Thus, this strategy pays you the same cash flow as the option. By non-arbitrage condition, the option must have the same price as the stock position \$139/40.

You might feel a bit puzzled after seeing the answer because we can replicate the payoff from this exotic option if and only if we sell the stock at price \$160. But now the question is why we want to sell the stock at price \$160? Why not wait a bit longer, for example, and sell the stock at a price potentially higher than \$160?

In other words, it seems that holding 1/40 shares of the stock gives you more flexibility than holding the exotic option, since you can always replicate the payoff of the exotic option by selling the share at \$160, and you can also choose to sell the stock at a different price and realize a different payoff. Because of the latter, it seems that the value of 1/40 shares of the stock should be higher than the value of the exotic option. How come we have the same value?

This is a non-trivial question that requires some further thinking. Basically, we want to know what is the value of being allowed to sell the stock share freely at any price? The answer is “zero”. Intuitively, this is because if selling the stock share itself has any value, we can always buy, sell, buy, sell, ..., and repeat infinite times to realize infinite profits. Because the value of this extra flexibility from holding 1/40 shares has zero value, it must be the case that the value of 1/40 shares is equal to the value of the exotic option.

Now let us try to get a deeper understanding by evaluating the flexibility embedded in American options. Intuitively, the value of an American option should be higher than a European option of the same characteristic, because holding an American option gives more flexibility by allowing buyers to exercise the option before the expiration date. We will learn next week, the value of this flexibility is zero for an American call with non-dividend paying stock. As a result, an American call’s price is equal to a European call’s price, although American call allows buyers to exercise early. But the value of this flexibility is positive for a put option, leading to a higher price for an American put.

Stay tuned!