Exercise 3 (No need to hand in)

1. Use elementary row operations to solve the following system of equations.

   (a) $\begin{cases} 2x_1 + x_2 = 4 \\ 6x_1 + 2x_2 + 6x_3 = 20 \\ -4x_1 - 3x_2 + 9x_3 = 3 \end{cases}$

   (b) $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 3x_1 + 2x_2 + x_3 = 1 \\ 5x_1 + 2x_2 - x_3 = 1 \end{cases}$

   (c) $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 3x_1 + 2x_2 + x_3 = 1 \\ 5x_1 + 2x_2 - x_3 = 2 \end{cases}$

2. Use elementary row operations to find the inverse of the matrix $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$

3. Consider the function $f(x, y) = x^2 - y$

   (a) Draw the three level curves $f(x, y) = -1, 0, 1$ in the same graph

   (b) Verify that $(1, 2), (-1, 1), (2, 3)$ are the three points on the three level curves respectively. Find the gradient vector of the function at each point and draw them in the same graph in part (a)

   (c) Find the Hessian matrix of $f(x, y)$

4. Find the gradient vectors and Hessian matrices of the following functons

   (a) $f(x, y, z) = 3xyz + x^2 y - xz^3$

   (b) $f(x, y, z) = 9 - x^2 - y^2 - z^2$

   (c) $f(x, y, z) = x^2 + 3y^2 - z^2 + xy - 2xz + 4yz$

5. Textbook questions:

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