Final Exam Fall 2019

Feb. 20, 2020

1. *[30 marks, 10 marks each]* Unrelated questions on **concavity/quasi-concavity**

   (a) Let $f(x) = e^{-\left(x_1+x_2^2\right)}$, is $f(x)$ concave or convex? justify

   (b) Let $f(x) = e^{-\left(x_1+x_2^2\right)}$, is $f(x)$ quasi-concave or quasi-convex? justify

   (c) Give an example of a function on R^3 which is both concave and convex

2. *[30 marks]* **Utility maximization problem**: In an economy with 3 commodities, let x_1, x_2 and x_3 denote the amount of the three commodities, the utility function of consumer is $U(x) = x_1^2 x_2 x_3$. The price of a unit of commodity is $p_1 = 1$ and $p_2 = 1$ and $p_3 = 2$, and the income which the consumer has available to spend on these 3 commodities is $I = 8$ million. The consumer's goal is the following:

$$\max\left\{x_1^2 x_2 x_3\right\} \quad \text{s.t. } x_1 + x_2 + 2x_3 \leq 8$$

   (a) *[10 marks]* Solve the problem

   (b) *[10 marks]* Do you think $U(x)$ is concave or quasi-concave? justify

   (c) *[5 marks]* Claim that the solution in (a) is a global maximum and find the maximized utility M

   (d) *[5 marks]* Now instead of 8 million, suppose the income which the consumer has available to spend on the 3 commodities is $I = 8.1$ million, give an estimate of the maximized utility

3. *[30 marks, 10 marks each]* Consider the following problem:

$$\max\left\{x_1^2 x_2^2\right\}, \quad s.t. \;\; x_1^2 + x_2^2 = 2$$

   (a) Find all the solutions satisfying the first order conditions

   (b) Find all the solutions in (a) which satisfy sufficient conditions of local maximum by checking the properties of bordered Hessian matrix.

   (c) Do you think the solutions in (b) are global maximum? justify

4. *[10 marks]* Determine the value(s) of a for which the following matrix is positive definite, positive semi-definite, negative definite, negative semi-definite, or indefinite (There may be no values of a for which the matrix satisfies some of these conditions.)

$$A = \begin{pmatrix} a & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$