

# ML ASSIGNMENT # 01

23K-0842

Q2)  $x = [x_1, x_2]^T$        $l = \begin{cases} 1 & \text{if } x \text{ is positive family oriented apartment} \\ 0 & \text{if } x \text{ is negative non family oriented apartment} \end{cases}$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a)  $D = \{ (x^{(i)}, l^{(i)}) \mid i = 1, 2, 3, \dots, M \}$

where

$$x^{(i)} = \begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \end{bmatrix}, \quad l^{(i)} \in \{0, 1\}$$

b)  $H(x) = \begin{cases} 1 & \text{if } \theta_0 + \theta_1 x_1 + \theta_2 x_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$

where

$\theta_0$  = bias or intercept

$\theta_1$  = weight for  $x_1$

$\theta_2$  = weight for  $x_2$

Decision boundary is the line  $\theta_1 x_1 + \theta_2 x_2 + \theta_0 = 0$ .

c) Hypothesis error:-

o/s Loss function :-

$$L_{0/1} = \frac{1}{M} \sum_{i=1}^M (H(x^{(i)}) \neq l^{(i)})$$

$$L_{0/1}(H(x^{(i)}), l^{(i)}) = \begin{cases} 0 & \text{if } H(x^{(i)}) = l^{(i)} \\ 1 & \text{if } H(x^{(i)}) \neq l^{(i)} \end{cases}$$

Mean Squared Error :-

$$L_{MSE} = \frac{1}{M} \sum_{i=1}^M (H(x^{(i)}) - l^{(i)})^2$$

Root Mean Squared Error :-

$$L_{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^M (H(x^{(i)}) - l^{(i)})^2}$$

Q1) Most specific hypothesis :-

It classifies only the exact positive training example as positive, (no generalization beyond them). It is too restrictive.

Most general hypothesis :-

It classifies the largest possible set as positive. It accepts everything, no constraints. It will have zero false negatives but many false positives.

Q2)

id	Price $x_1$ (PKR)	size $x_2$ (sq.ft)	Label( $h$ )
A	55	450	Standard (0)
B	75	900	luxury (1)
C	65	500	standard (0)
D	85	850	luxury (1)
E	45	400	standard (0)

Labels =  $\begin{cases} \text{standard} & , h_i = 0 \\ \text{luxury} & , h_i = 1 \end{cases}$

$x_{\text{new}} = (70, 700)$

Chebyshev Distance ( $L_\infty$  Norm) :-

$$d_\infty(x, y) = \max(|x_1 - y_1|, |x_2 - y_2|)$$

$$d_\infty(x_{\text{new}}, A) = \max(|70 - 55|, |700 - 450|) = \max(15, 250) = 250$$

$$d_\infty(x_{\text{new}}, B) = \max(|70 - 75|, |700 - 900|) = \max(5, 200) = 200$$

$$d_\infty(x_{\text{new}}, C) = \max(|70 - 65|, |700 - 500|) = \max(5, 200) = 200$$

$$d_\infty(x_{\text{new}}, D) = \max(|70 - 85|, |700 - 850|) = \max(15, 150) = 150$$

$$d_\infty(x_{\text{new}}, E) = \max(|70 - 45|, |700 - 400|) = \max(25, 300) = 300$$

With  $k=3$ , B, C, D are nearest neighbours.

luxury ← standard luxury → majority within  $x_{\text{new}}$  is luxury (1)



City Block / Manhattan (L1 Norm) :-

$$d(x, y) = \sum_{i=1}^n |x_i - y_i|$$

$$d(x_{\text{new}}, A) = |70 - 55| + |700 - 450| = 15 + 250 = 265$$

$$d(x_{\text{new}}, B) = |70 - 75| + |700 - 900| = 5 + 200 = 205$$

$$d(x_{\text{new}}, C) = |70 - 65| + |700 - 500| = 5 + 200 = 205$$

$$d(x_{\text{new}}, D) = |70 - 85| + |700 - 850| = 15 + 150 = 165$$

$$d(x_{\text{new}}, E) = |70 - 45| + |700 - 800| = 25 + 300 = 325$$

With  $k=3$ , B, C, D are nearest.  
luxury   standard   luxury  $\xrightarrow{\text{majority voting}}$  new is luxury (1).

Computing weights :-

$$w_i = \frac{1}{d_i}$$

Chebyshev Weights :-

$$w_A = \frac{1}{250} = 0.00400 \rightarrow \text{standard (0)}$$

$$w_B = \frac{1}{200} = 0.00500 \rightarrow \text{luxury (1)} \quad \checkmark$$

$$w_C = \frac{1}{200} = 0.00500 \rightarrow \text{standard (0)} \quad \checkmark$$

$$w_D = \frac{1}{150} \approx 0.00667 \rightarrow \text{luxury (1)} \quad \checkmark$$

} Nearest

$$w_E = \frac{1}{300} \approx 0.00333 \rightarrow \text{standard (0)}$$

$k=3$ , so nearest were B, C, D

sum weights per class :-

- luxury (1) :  $0.00667 + 0.005 = 0.01167$

- standard (0) :  $0.005$

luxury (0.01167) > standard (0.005), so  $x_{\text{new}}$  can be classified as luxury (1).

## Manhattan weights 2-

$$w_A = \frac{1}{265} = 0.00377 \rightarrow \text{Standard (0)}$$

$$w_B = \frac{1}{205} = 0.00488 \rightarrow \text{Luxury (1)}$$

$$w_C = \frac{1}{205} = 0.00488 \rightarrow \text{Standard (0)}$$

$$w_D = \frac{1}{165} = 0.00606 \rightarrow \text{Luxury (1)}$$

$$w_E = \frac{1}{325} = 0.00308 \rightarrow \text{Standard (0)}$$

D, C, B  $\rightarrow$  nearest neighbours

Sum :-

$$\bullet - \text{Luxury (1)} :- 0.00488 + 0.00606 = 0.01094$$

$$\bullet - \text{Standard (0)} :- 0.00488$$

$\text{Luxury (1)} = 0.01094 > \text{Standard (0)} = 0.00488$  so new is  $\text{Luxury (1)}$ .

Final Answers:-

$\bullet$  - using Manhattan (city - block), weighted KNN with  $k=3$  predicts  ~~$k=3$  predicts~~  $\text{Luxury (1)}$ .

$\bullet$  - using Chebyshev ( $L_\infty$ ), weighted KNN with  $k=3$  predicts  $\text{Luxury (1)}$ .