Chapter 7: Integer Arithmetic

Chapter Overview

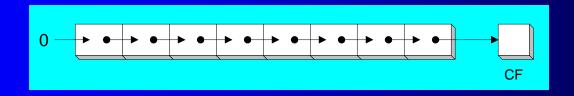
- Shift and Rotate Instructions
- Shift and Rotate Applications
- Multiplication and Division Instructions
- Extended Addition and Subtraction
- ASCII and Unpacked Decimal Arithmetic
- Packed Decimal Arithmetic

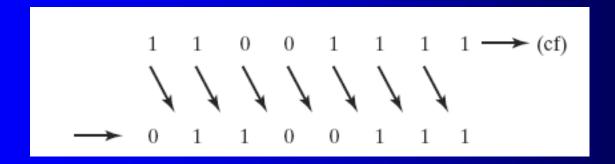
Shift and Rotate Instructions

- Logical vs Arithmetic Shifts
- SHL Instruction
- SHR Instruction
- SAL and SAR Instructions
- ROL Instruction
- ROR Instruction
- RCL and RCR Instructions
- SHLD/SHRD Instructions

Logical Shift

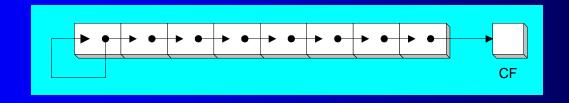
 A logical shift fills the newly created bit position with zero:

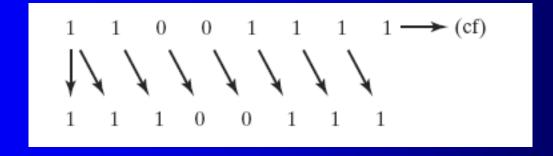




Arithmetic Shift

 An arithmetic shift fills the newly created bit position with a copy of the number's sign bit:





SHL Instruction

 The SHL (shift left) instruction performs a logical left shift on the destination operand, filling the lowest bit with 0.

 $(cf) \longleftarrow 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1$ $1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1$ $1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \leftarrow$

Operand types for SHL:

SHL reg,imm8

SHL mem,imm8

SHL reg,CL

SHL mem,CL

(Same for all shift and rotate instructions)

Fast Multiplication

Shifting left 1 bit multiplies a number by 2

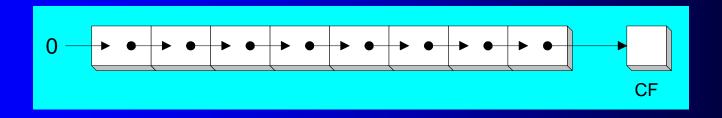
Shifting left *n* bits multiplies the operand by 2ⁿ

For example,
$$5 * 2^2 = 20$$

;
$$DL = 20$$

SHR Instruction

 The SHR (shift right) instruction performs a logical right shift on the destination operand. The highest bit position is filled with a zero.



Shifting right n bits divides the operand by 2^n

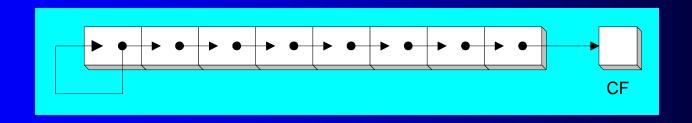
```
mov dl,80

shr dl,1 ; DL = 40

shr dl,2 ; DL = 10
```

SAL and SAR Instructions

- SAL (shift arithmetic left) is identical to SHL.
- SAR (shift arithmetic right) performs a right arithmetic shift on the destination operand.



An arithmetic shift preserves the number's sign.

```
mov dl,-80

sar dl,1 ; DL = -40

sar dl,2 ; DL = -10
```

Your turn . . .

Indicate the hexadecimal value of AL after each shift:

```
      mov al,6Bh

      shr al,1
      a. 35h

      shl al,3
      b. A8h

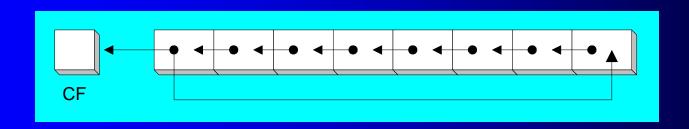
      mov al,8Ch
      c. C6h

      sar al,1
      c. C6h

      sar al,3
      d. F8h
```

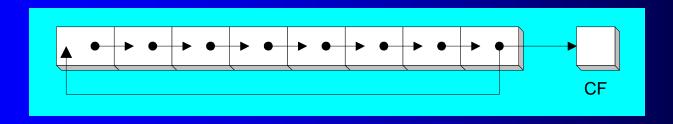
ROL Instruction

- ROL (rotate) shifts each bit to the left
- The highest bit is copied into both the Carry flag and into the lowest bit
- No bits are lost



ROR Instruction

- ROR (rotate right) shifts each bit to the right
- The lowest bit is copied into both the Carry flag and into the highest bit
- No bits are lost



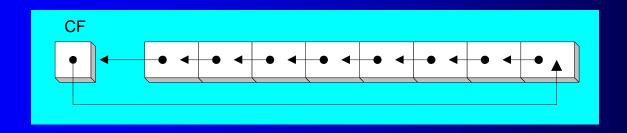
Your turn . . .

Indicate the hexadecimal value of AL after each rotation:

```
mov al,6Bh
ror al,1 a. B5h
rol al,3 b. ADh
```

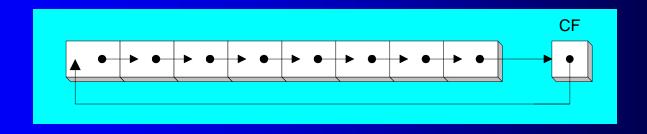
RCL Instruction

- RCL (rotate carry left) shifts each bit to the left
- Copies the Carry flag to the least significant bit
- Copies the most significant bit to the Carry flag



RCR Instruction

- RCR (rotate carry right) shifts each bit to the right
- Copies the Carry flag to the most significant bit
- Copies the least significant bit to the Carry flag



Your turn . . .

Indicate the hexadecimal value of AL after each rotation:

```
stc
mov al,6Bh
rcr al,1 a. B5h
rcl al,3 b. AEh
```

SHLD Instruction

- Shifts a destination operand a given number of bits to the left
- The bit positions opened up by the shift are filled by the most significant bits of the source operand
- The source operand is not affected
- Syntax:

SHLD destination, source, count

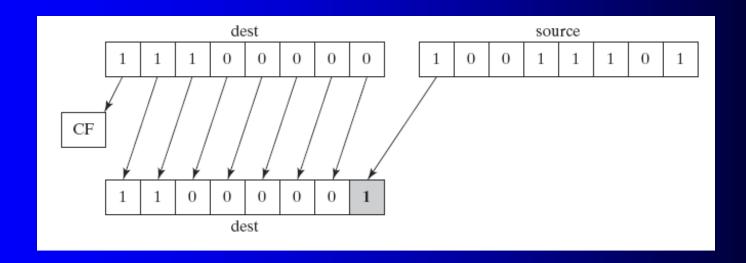
Operand types:

```
SHLD reg16/32, reg16/32, imm8/CL
SHLD mem16/32, reg16/32, imm8/CL
```

SHLD (Shift Left Double) Example

Shift count of 1:

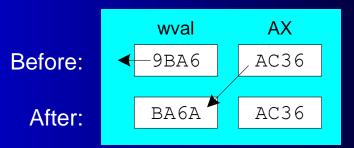
```
mov al,11100000b
mov bl,10011101b
shld al,bl,1
```



Another SHLD Example

Shift wval 4 bits to the left and replace its lowest 4 bits with the high 4 bits of AX:

.data
wval WORD 9BA6h
.code
mov ax,0AC36h
shld wval,ax,4



SHRD (Shift Right Double) Instruction

- Shifts a destination operand a given number of bits to the right
- The bit positions opened up by the shift are filled by the least significant bits of the source operand
- The source operand is not affected
- Syntax:

SHRD destination, source, count

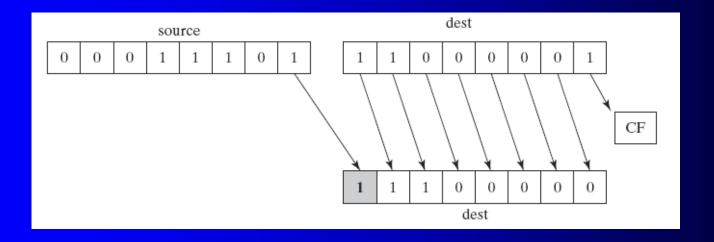
Operand types:

```
SHRD reg16/32, reg16/32, imm8/CL
SHRD mem16/32, reg16/32, imm8/CL
```

SHRD Example

Shift count of 1:

mov al,11000001b mov bl,00011101b shrd al,bl,1



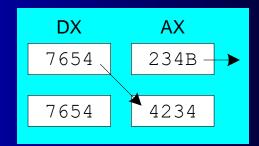
Another SHRD Example

Shift AX 4 bits to the right and replace its highest 4 bits with the low 4 bits of DX:

mov ax,234Bh mov dx,7654h shrd ax,dx,4

Before:

After:



Your turn . . .

Indicate the hexadecimal values of each destination operand:

```
mov ax,7C36h

mov dx,9FA6h

shld dx,ax,4 ; DX = FA67h

shrd dx,ax,8 ; DX = 36FAh
```

What's Next

- Shift and Rotate Instructions
- Shift and Rotate Applications
- Multiplication and Division Instructions
- Extended Addition and Subtraction
- ASCII and Unpacked Decimal Arithmetic
- Packed Decimal Arithmetic

Shift and Rotate Applications

- Shifting Multiple Doublewords
- Binary Multiplication
- Displaying Binary Bits
- Isolating a Bit String

Shifting Multiple Doublewords

- Programs sometimes need to shift all bits within an array, as one might when moving a bitmapped graphic image from one screen location to another.
- The following shifts an array of 3 doublewords 1 bit to the right (view complete source code MultiShf.asm):

Binary Multiplication

mutiply 123 * 36

```
01111011 123

× 00100100 36

01111011 123 SHL 2

+ 01111011 123 SHL 5

0001000101001100 4428
```

Binary Multiplication

- We already know that SHL performs unsigned multiplication efficiently when the multiplier is a power of 2.
- You can factor any binary number into powers of 2.
 - For example, to multiply EAX * 36, factor 36 into 32 + 4 and use the distributive property of multiplication to carry out the operation:

```
EAX * 36
= EAX * (32 + 4)
= (EAX * 32)+(EAX * 4)
```

Your turn . . .

Multiply AX by 26, using shifting and addition instructions. Hint: 26 = 16 + 8 + 2.

```
; test value
mov ax, 2
mov dx, ax
shl dx,4
                             ; AX * 16
push edx
                             ; save for later
mov dx, ax
shl dx,3
                             ; AX * 8
shl ax,1
                             ; AX * 2
                             ; AX * 10
add ax, dx
                             ; recall AX * 16
pop edx
                             ; AX * 26
add ax, dx
```

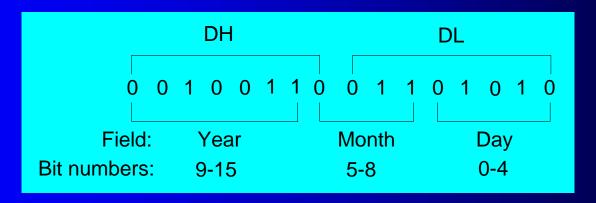
Displaying Binary Bits

Algorithm: Shift MSB into the Carry flag; If CF = 1, append a "1" character to a string; otherwise, append a "0" character. Repeat in a loop, 32 times (see WriteBin.asm).

```
.data
buffer BYTE 32 DUP(0),0
.code
    mov ecx,32
    mov esi,OFFSET buffer
L1: shl eax,1
    mov BYTE PTR [esi],'0'
    jnc L2
    mov BYTE PTR [esi],'1'
L2: inc esi
    loop L1
```

Isolating a Bit String

 The MS-DOS file date field packs the year, month, and day into 16 bits:



Isolate the Month field:

```
mov ax,dx ; make a copy of DX shr ax,5 ; shift right 5 bits and al,00001111b ; clear bits 4-7 mov month,al ; save in month variable
```

What's Next

- Shift and Rotate Instructions
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Multiplication and Division Instructions

- MUL Instruction
- IMUL Instruction
- DIV Instruction
- Signed Integer Division
- CBW, CWD, CDQ Instructions
- IDIV Instruction
- Implementing Arithmetic Expressions

MUL Instruction

- The MUL (unsigned multiply) instruction multiplies an 8, 16, or 32-bit operand by either AL, AX, or EAX.
- The instruction formats are:

MUL r/m8

MUL r/m16

MUL r/m32

Table 7-2 MUL Operands.

Multiplicand	Multiplier	Product
AL	reg/mem8	AX
AX	reg/mem16	DX:AX
EAX	reg/mem32	EDX:EAX

MUL Examples

100h * 2000h, using 16-bit operands:

```
.data
val1 WORD 2000h
val2 WORD 100h
.code
mov ax,val1
mul val2 ; DX:AX = 00200000h, CF=1
The Carry flag indicates whether or not the upper half of the product contains significant digits.
```

12345h * 1000h, using 32-bit operands:

IMUL Instruction

- IMUL (signed integer multiply) multiplies an 8-, 16-, or 32-bit signed operand by either AL, AX, or EAX
- Preserves the sign of the product by sign-extending it into the upper half of the destination register

Example: multiply 48 * 4, using 8-bit operands:

```
mov al,48
mov bl,4
imul bl ; AX = 00C0h, OF=1
```

OF=1 because AH is not a sign extension of AL.

IMUL Examples

Multiply 4,823,424 * -423:

OF=0 because EDX is a sign extension of EAX.

What will be the hexadecimal values of DX, AX, and the Carry flag after the following instructions execute?

```
mov ax,8760h
mov bx,100h
imul bx
```

DX = FF87h, AX = 6000h, OF = 1

DIV Instruction

- The DIV (unsigned divide) instruction performs 8-bit,
 16-bit, and 32-bit division on unsigned integers
- A single operand is supplied (register or memory operand), which is assumed to be the divisor
- Instruction formats:

DIV reg/mem8
DIV reg/mem16
DIV reg/mem32

Default Operands:

Dividend	Divisor	Quotient	Remainder
AX	r/m8	AL	АН
DX:AX	r/m16	AX	DX
EDX:EAX	r/m32	EAX	EDX

DIV Examples

Divide 8003h by 100h, using 16-bit operands:

```
mov dx,0 ; clear dividend, high
mov ax,8003h ; dividend, low
mov cx,100h ; divisor
div cx ; AX = 0080h, DX = 3
```

Same division, using 32-bit operands:

What will be the hexadecimal values of DX and AX after the following instructions execute? Or, if divide overflow occurs, you can indicate that as your answer:

```
mov dx,0087h
mov ax,6000h
mov bx,100h
div bx
```

DX = 0000h, AX = 8760h

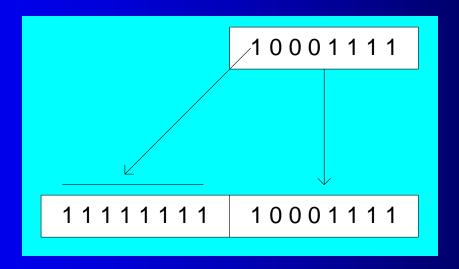
What will be the hexadecimal values of DX and AX after the following instructions execute? Or, if divide overflow occurs, you can indicate that as your answer:

```
mov dx,0087h
mov ax,6002h
mov bx,10h
div bx
```

Divide Overflow

Signed Integer Division (IDIV)

- Signed integers must be sign-extended before division takes place
 - fill high byte/word/doubleword with a copy of the low byte/word/doubleword's sign bit
- For example, the high byte contains a copy of the sign bit from the low byte:



CBW, CWD, CDQ Instructions

- The CBW, CWD, and CDQ instructions provide important sign-extension operations:
 - CBW (convert byte to word) extends AL into AH
 - CWD (convert word to doubleword) extends AX into DX
 - CDQ (convert doubleword to quadword) extends EAX into EDX

Example:

```
.data
dwordVal SDWORD -101  ; FFFFFF9Bh
.code
mov eax,dwordVal
cdq  ; EDX:EAX = FFFFFFFFFFFFF9Bh
```

IDIV Instruction

- IDIV (signed divide) performs signed integer division
- Same syntax and operands as DIV instruction

Example: 8-bit division of -48 by 5

```
mov al,-48

cbw ; extend AL into AH

mov bl,5

idiv bl ; AL = -9, AH = -3
```

IDIV Examples

Example: 16-bit division of –48 by 5

```
mov ax,-48

cwd ; extend AX into DX

mov bx,5

idiv bx ; AX = -9, DX = -3
```

Example: 32-bit division of -48 by 5

```
mov eax,-48

cdq ; extend EAX into EDX

mov ebx,5

idiv ebx ; EAX = -9, EDX = -3
```

What will be the hexadecimal values of DX and AX after the following instructions execute? Or, if divide overflow occurs, you can indicate that as your answer:

```
mov ax,0FDFFh ; -513
cwd
mov bx,100h
idiv bx
```

DX = FFFFh(-1), AX = FFFEh(-2)

Unsigned Arithmetic Expressions

- Some good reasons to learn how to implement integer expressions:
 - Learn how do compilers do it
 - Test your understanding of MUL, IMUL, DIV, IDIV
 - Check for overflow (Carry and Overflow flags)

```
; Assume unsigned operands
mov eax,var1
add eax,var2 ; EAX = var1 + var2
mul var3 ; EAX = EAX * var3
jc TooBig ; check for carry
mov var4,eax ; save product
```

Signed Arithmetic Expressions (1 of 2)

```
Example: eax = (-var1 * var2) + var3
      mov eax, var1
          eax
      neg
      imul var2
      jo TooBig
                         ; check for overflow
      add eax, var3
      jo TooBig
                         ; check for overflow
Example: var4 = (var1 * 5) / (var2 - 3)
                            ; left side
      mov eax, var1
      mov ebx,5
      imul ebx
                            ; EDX:EAX = product
                             ; right side
      mov ebx, var2
      sub ebx,3
      idiv ebx
                             ; EAX = quotient
      mov var4, eax
```

Signed Arithmetic Expressions (2 of 2)

```
Example: var4 = (var1 * -5) / (-var2 % var3);
                            ; begin right side
        eax, var2
   mov
   neg
        eax
                            ; sign-extend dividend
   cda
   idiv var3
                            ; EDX = remainder
                            ; EBX = right side
   mov ebx, edx
                            ; begin left side
   mov eax, -5
   imul var1
                            ; EDX:EAX = left side
   idiv ebx
                            ; final division
   mov var4,eax
                            ; quotient
```

Sometimes it's easiest to calculate the right-hand term of an expression first.

Implement the following expression using signed 32-bit integers:

```
eax = (ebx * 20) / ecx
```

mov eax,20
imul ebx
idiv ecx

Implement the following expression using signed 32-bit integers. Save and restore ECX and EDX:

```
eax = (ecx * edx) / eax
```

Implement the following expression using signed 32-bit integers. Do not modify any variables other than var3:

```
var3 = (var1 * -var2) / (var3 - ebx)
```

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Extended Addition and Subtraction

- ADC Instruction
- Extended Precision Addition
- SBB Instruction
- Extended Precision Subtraction

Extended Precision Addition

- Adding two operands that are longer than the computer's word size (32 bits).
 - Virtually no limit to the size of the operands
- The arithmetic must be performed in steps
 - The Carry value from each step is passed on to the next step.

ADC Instruction

- ADC (add with carry) instruction adds both a source operand and the contents of the Carry flag to a destination operand.
- Operands are binary values
 - Same syntax as ADD, SUB, etc.
- Example
 - Add two 32-bit integers (FFFFFFFF + FFFFFFFF), producing a 64-bit sum in EDX:EAX:

Extended Addition Example

- Task: Add 1 to EDX:EAX
 - Starting value of EDX:EAX: 00000000FFFFFFFFh
 - Add the lower 32 bits first, setting the Carry flag.
 - Add the upper 32 bits, and include the Carry flag.

EDX:EAX = 00000001 00000000

SBB Instruction

- The SBB (subtract with borrow) instruction subtracts both a source operand and the value of the Carry flag from a destination operand.
- Operand syntax:
 - Same as for the ADC instruction

Extended Subtraction Example

- Task: Subtract 1 from EDX:EAX
 - Starting value of EDX:EAX: 0000000100000000h
 - Subtract the lower 32 bits first, setting the Carry flag.
 - Subtract the upper 32 bits, and include the Carry flag.

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ASCII and Packed Decimal Arithmetic

- Binary Coded Decimal
- ASCII Decimal
- AAA Instruction
- AAS Instruction
- AAM Instruction
- AAD Instruction
- Packed Decimal Integers
- DAA Instruction
- DAS Instruction

Binary-Coded Decimal

- Binary-coded decimal (BCD) integers use 4 binary bits to represent each decimal digit
- A number using unpacked BCD representation stores a decimal digit in the lower four bits of each byte
 - For example, 5,678 is stored as the following sequence of hexadecimal bytes:

ASCII Decimal

- A number using ASCII Decimal representation stores a single ASCII digit in each byte
 - For example, 5,678 is stored as the following sequence of hexadecimal bytes:

35 36 37 38

AAA Instruction

- The AAA (ASCII adjust after addition) instruction adjusts the binary result of an ADD or ADC instruction. It makes the result in AL consistent with ASCII decimal representation.
 - The Carry value, if any ends up in AH
- Example: Add '8' and '2'

AAS Instruction

- The AAS (ASCII adjust after subtraction) instruction adjusts the binary result of an SUB or SBB instruction. It makes the result in AL consistent with ASCII decimal representation.
 - It places the Carry value, if any, in AH
- Example: Subtract '9' from '8'

AAM Instruction

 The AAM (ASCII adjust after multiplication) instruction adjusts the binary result of a MUL instruction. The multiplication must have been performed on unpacked BCD numbers.

AAD Instruction

- The AAD (ASCII adjust before division) instruction adjusts the unpacked BCD dividend in AX before a division operation
- 37/5

What's Next

- Shift and Rotate Instructions
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Packed Decimal Arithmetic

- Packed decimal integers store two decimal digits per byte
 - For example, 12,345,678 can be stored as the following sequence of hexadecimal bytes:

Packed decimal is also known as packed BCD.

Good for financial values – extended precision possible, without rounding errors.

DAA Instruction

- The DAA (decimal adjust after addition) instruction converts the binary result of an ADD or ADC operation to packed decimal format.
 - The value to be adjusted must be in AL
 - If the lower digit is adjusted, the Auxiliary Carry flag is set.
 - If the upper digit is adjusted, the Carry flag is set.

DAA Logic

```
If (AL(1o) > 9) or (AuxCarry = 1)
  AL = AL + 6
  AuxCarry = 1
Else
  AuxCarry = 0
Endif
If (AL(hi) > 9) or Carry = 1
  AL = AL + 60h
  Carry = 1
Else
  Carry = 0
Endif
```

If AL = AL + 6 sets the Carry flag, its value is used when evaluating AL(hi).

DAA Examples

Example: calculate BCD 35 + 48

```
mov al,35h
add al,48h ; AL = 7Dh
daa ; AL = 83h, CF = 0
```

Example: calculate BCD 35 + 65

```
mov al,35h
add al,65h
; AL = 9Ah
daa
; AL = 00h, CF = 1
```

Example: calculate BCD 69 + 29

```
mov al,69h
add al,29h ; AL = 92h
daa ; AL = 98h, CF = 0
```

- A temporary malfunction in your computer's processor has disabled the DAA instruction. Write a procedure in assembly language that performs the same actions as DAA.
- Test your procedure using the values from the previous slide.

DAS Instruction

- The DAS (decimal adjust after subtraction) instruction converts the binary result of a SUB or SBB operation to packed decimal format.
- The value must be in AL
- Example: subtract BCD 48 from 85

```
mov al,48h
sub al,35h ; AL = 13h
das ; AL = 13h CF = 0
```

DAS Logic

```
If (AL(lo) > 9) OR (AuxCarry = 1)
   AL = AL - 6;
   AuxCarry = 1;
Else
   AuxCarry = 0;
Endif
If (AL > 9FH) or (Carry = 1)
   AL = AL - 60h;
   Carry = 1;
Else
   Carry = 0;
Endif
```

If AL = AL – 6 sets the Carry flag, its value is used when evaluating AL in the second IF statement.

DAS Examples (1 of 2)

Example: subtract BCD 48 – 35

```
mov al,48h
sub al,35h ; AL = 13h
das ; AL = 13h CF = 0
```

Example: subtract BCD 62 – 35

```
mov al,62h
sub al,35h
; AL = 2Dh, CF = 0
das
; AL = 27h, CF = 0
```

Example: subtract BCD 32 – 29

```
mov al,32h
add al,29h ; AL = 09h, CF = 0
daa ; AL = 03h, CF = 0
```

DAS Examples (2 of 2)

Example: subtract BCD 32 – 39

```
mov al,32h
sub al,39h
; AL = F9h, CF = 1
das
; AL = 93h, CF = 1
```

```
Steps:
AL = F9h
CF = 1, so subtract 6 from F9h
AL = F3h
F3h > 9Fh, so subtract 60h from F3h
AL = 93h, CF = 1
```

- A temporary malfunction in your computer's processor has disabled the DAS instruction. Write a procedure in assembly language that performs the same actions as DAS.
- Test your procedure using the values from the previous two slides.

Summary

- Shift and rotate instructions are some of the best tools of assembly language
 - finer control than in high-level languages
 - SHL, SHR, SAR, ROL, ROR, RCL, RCR
- MUL and DIV integer operations
 - close relatives of SHL and SHR
 - CBW, CDQ, CWD: preparation for division
- Extended precision arithmetic: ADC, SBB
- ASCII decimal operations (AAA, AAS, AAM, AAD)
- Packed decimal operations (DAA, DAS)



55 74 67 61 6E 67 65 6E