

Week 6 Assignment

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Week 6 Homework

A stock index is currently at 810 and has volatility of 20%.

The risk-free rate is 5% per year.

Assume that dividend yield equals 1%.

```
library(RQuantLib)
Option.Type<-"call"
Underlying.Price<-810
Strike<-800
Volatility<-.2
RFR<-.05
DivYield<-.01
Expiry<-.5
```

1. Price European 6-month call option with strike 800 using `EuropeanOption()` from `RQuantLib`.

```
call_price<-EuropeanOption(type=Option.Type, underlying = Underlying.Price, strike = Strike, dividendYield=DivYield, riskFreeRate=RFR, maturity=Expiry)
call_price$value
```

```
## [1] 58.73271
```

2. Calculate the same premium manually using the formulas on the last slide of the lecture notes. Think how dividend yield should affect option price.

```
#stock price at time=0
S_0<-810
K<-800 #strike price
R<-.05 #risk free return
Time<-.5 #expiry in years
Sigma<-.2 #volatility
d_1<-(log(S_0/K)+(R + Sigma^2/2)*Time)/(Sigma*sqrt(Time))
#N(d_1) or output of cumulative distribution function of standard normal distribution when input=d_1
N_d_1<-pnorm(d_1)
d_2<-d_1-Sigma*sqrt(Time)
#N(d_2) or output of cumulative distribution function of standard normal distribution when input=d_2
N_d_2<-pnorm(d_2)
#Price of call option
C_0<-S_0*N_d_1 - K*exp(-R*Time)*N_d_2
C_0
```

```
## [1] 61.25612
```

Experiment with the function `EuropeanOption()` with zero or non-zero dividend yield and find how the Black-Scholes formula on slide 17 should be modified for dividend yield.

The premium calculated using the Black-Scholes-Merton formula (61.25612) is the same as that calculated by the function `EuropeanOption()` with the `dividendYield` parameter = 0.

```
call_price_no_div<-EuropeanOption(type=Option.Type, underlying = Underlying.Price, strike = Strike, dividendYield=0, riskFreeRate=RFR, maturity=Expiry)
call_price_no_div$value
```

```
## [1] 61.25612
```

So as dividend yield of the underlying stock increases the value of the call option premium goes down, and as the dividend yield goes down the value of the call option goes up.

To modify the Black-Scholes formula for the dividend yield, discount the dividend yield:

```
#see slide 10 Week 7
#modify this
new_d_1<-(log(S_0/K)+(R -DivYield + Sigma^2/2)*Time)/(Sigma*sqrt(Time))
new_d_2<-new_d_1-Sigma*sqrt(Time)
NewN_d_1<-pnorm(new_d_1)
NewN_d_2<-pnorm(new_d_2)
#modify this
C_0_D<-S_0*exp(-DivYield*Time)*NewN_d_1 - K*exp(-R*Time)*NewN_d_2
C_0_D
```

```
## [1] 58.73271
```

This is the same as the value calculated by the function.

3. Calculate the premium of put option with the same expiry and same strike using put-call parity.

The equation for the premium of a put option is:

put premium = call premium + $e^{(-TR)}K$ - Stock price at time 0, with call premium being what we calculated in Questions 1 and 2

```
#see page 8 of Week 6 slides
#divide risk free rate by 100
call_premium<-call_price$value
call_premium
```

```
## [1] 58.73271
```

```
discounted_strike_price<-exp(-Time*RFR)*K
discounted_strike_price
```

```
## [1] 780.2479
```

```
stock_at_time_0<-Underlying.Price
stock_at_time_0
```

```
## [1] 810
```

```
put<-call_premium+discounted_strike_price-stock_at_time_0
put
```

```
## [1] 28.98064
```

So the premium of the put option for our given call premium as determined by using put-call parity is \$28.90.