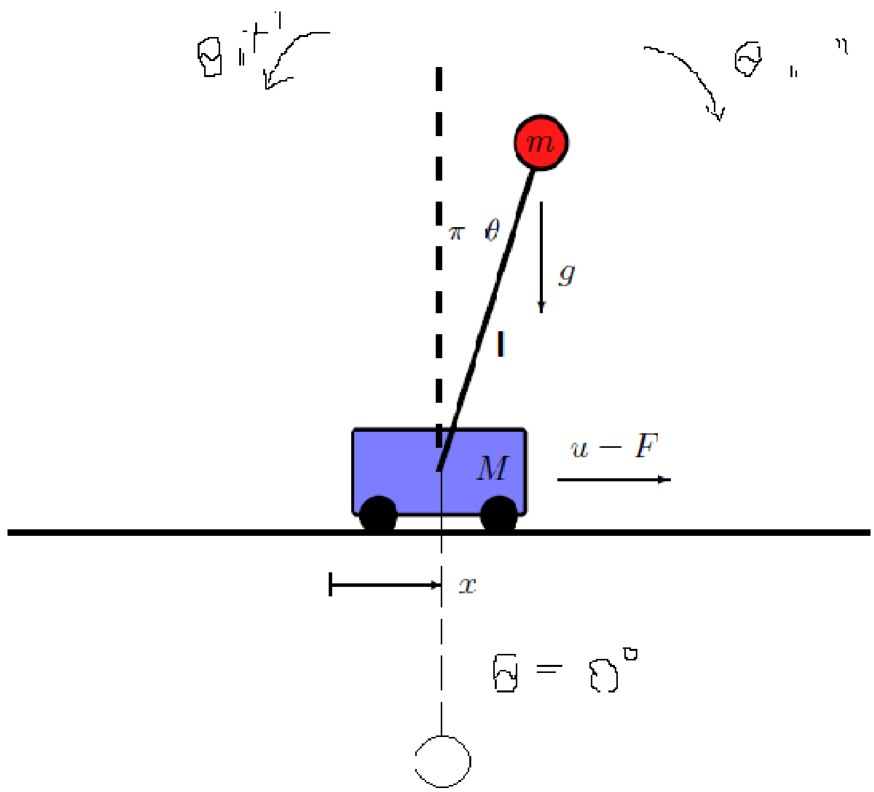
```
clc
clear all
close all

% System definition
```




```
m = 1;      % mass of pendulum
M = 5;      % mass of cart
L = 2;      % length of pend
g = -9.81;
d = 1;      % coeefficient of friction
b = 1;      % cos(theta) for theta = pi
```

Equations:

$$x = \begin{pmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\ddot{x} = \frac{-m^2.l^2.g.cos(\theta).sin(\theta) + m.l^2(m.l.\dot{\theta}^2.sin(\theta) - b.\dot{x}) + m.l^2.F}{m.l^2(M + m(1 - cos^2(\theta)))}$$

$$\ddot{\theta} = \frac{(m + M)m.g.l.sin(\theta) - m.l.cos(\theta)(m.l.\dot{\theta}^2.sin(\theta) - b.\dot{x}) - m.l.cos(\theta).F}{m.l^2(M + m(1 - cos^2(\theta)))}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{-m^2.l^2.g.cos(x_3).sin(x_3) + m.l^2(m.l.x_4^2.sin(x_3) - b.x_2 + m.l^2.F}{m.l^2(M + m(1 - cos^2(x_3)))}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{(m + M)m.g.l.sin(x_3) - m.l.cos(x_3)(m.l.x_4^2.sin(x_3) - b.x_2) - m.l.cos(x_3).F}{m.l^2(M + m(1 - cos^2(x_3)))}$$

Linearization:

```
% Linearizace kolem bodu theta = pi

A = [0 1 0 0;
    0 -d/M b*m*g/M 0;
    0 0 0 1;
    0 -b*d/(M*L) -b*(m+M)*g/(M*L) 0];
B = [0; 1/M; 0; b*1/(M*L)];
C = [[1 0 0 0];[0 0 1 0]];
D = [0; 0];
inv_pend_linsys = ss(A, B, C, D)
```

```
inv_pend_linsys =
 
  A = 
             x1       x2       x3       x4
   x1         0        1        0        0
   x2         0     -0.2   -1.962        0
   x3         0        0        0        1
   x4         0     -0.1    5.886        0
 
  B = 
        u1
   x1    0
   x2  0.2
   x3    0
   x4  0.1
 
  C = 
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   0   1   0
 
  D = 
```

```
       u1
   y1   0
   y2   0

Continuous-time state-space model.
```

```
% step(inv_pend_linsys)
[p, z] = pzmap(inv_pend_linsys)
```

```
p = 4×1
         0
   -2.4077
   -0.2336
    2.4414
z =

  0×1 empty double column vector
```

```
% controllability:
rank(ctrb(inv_pend_linsys))
```

```
ans = 4
```

```
% observability
rank(obsv(inv_pend_linsys))
```

```
ans = 4
```

LQR controller:

```
% state cost matrix
global Q R S
Q = [[1 0 0 0]; [0 1 0 0]; [0 0 10 0]; [0 0 0 1]];                        % state cost matrix
R = 0.01;                                 % input cost matrix
S = [[0]; [0]; [0]; [0]]                  % state input weight matrix
```

```
S = 4×1
     0
     0
     0
     0
```

```
global K
[K, W, ~] = lqr(inv_pend_linsys, Q, R, S)
```

```
K = 1×4
  -10.0000  -22.7468  314.2601  123.4261
W = 4×4
    2.1747    1.8596  -12.3426   -4.7192
    1.8596    3.1025  -22.1220   -8.4797
  -12.3426  -22.1220  198.7696   75.6700
   -4.7192   -8.4797   75.6700   29.3021
```

```
inv_pend_sys_lqr = ss(A-B*K, B, C, D)
```

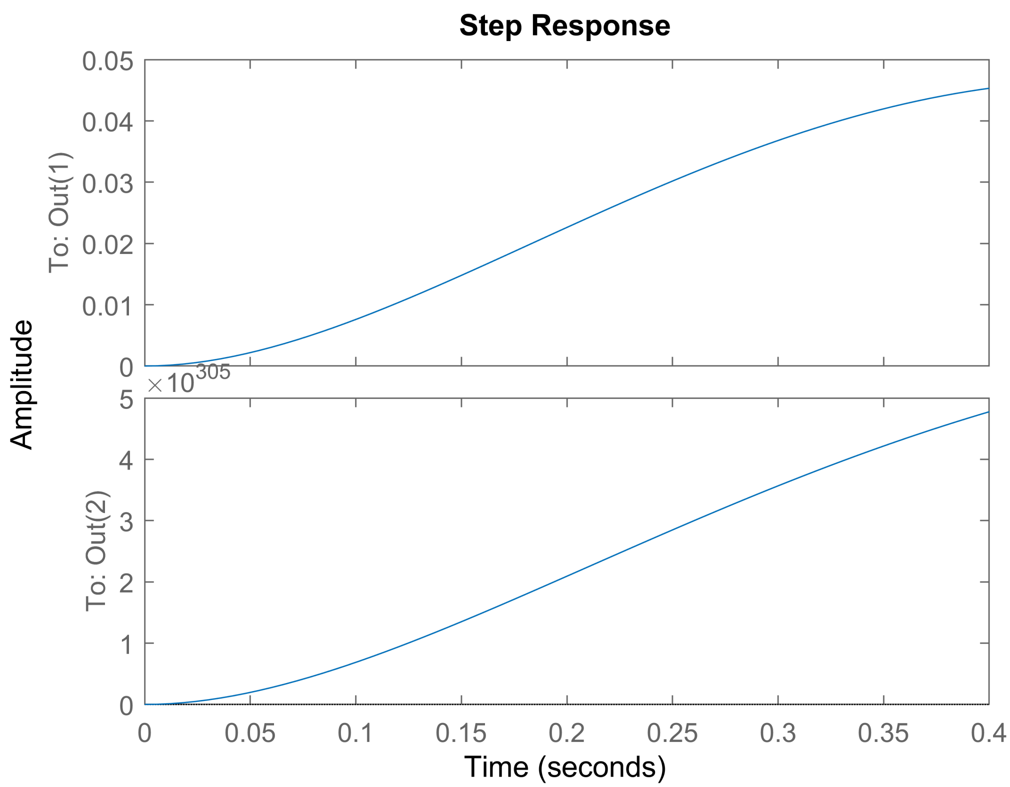
```
inv_pend_sys_lqr =
```

```
  A =
            x1       x2       x3       x4
   x1        0        1        0        0
   x2        2    4.349   -64.81   -24.69
   x3        0        0        0        1
   x4        1    2.175   -25.54   -12.34

  B =
         u1
   x1     0
   x2   0.2
   x3     0
   x4   0.1

  C =
        x1  x2  x3  x4
   y1    1   0   0   0
   y2    0   0   1   0

  D =
        u1
   y1    0
   y2    0

Continuous-time state-space model.
```

```
% stability of new system:
eig(A-B*K)
```

```
ans = 4×1 complex
  -2.5891 + 0.5460i
  -2.5891 - 0.5460i
  -1.5473 + 0.0000i
  -1.2677 + 0.0000i
```

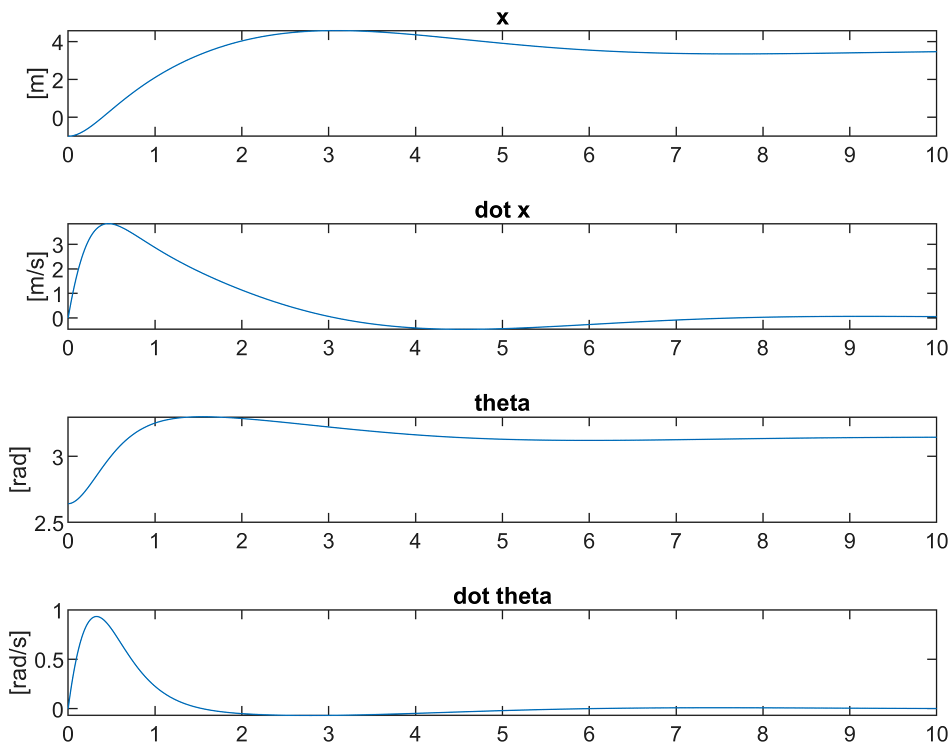
```
% !!! rescaling:

kr = -1./dcgain(inv_pend_sys_lqr);
kr(isinf(kr)) = realmax;      % subs inf for biggest double in matlab
inv_pend_sys_lqr.C = inv_pend_sys_lqr.C.*kr;
step(inv_pend_sys_lqr)
```



simulation with LQR controller:

```
tspan = 0:.001:10;
global x0
x0 = [-1; 0; pi-0.5; 0]; % initial condition
sp = [3.5; 0; pi; 0];     % set point
[t,x_lqr] = ode45(@(t,x)inv_pend(x, LQR(x, sp)),tspan,x0);

tiledlayout(4,1)

ax1 = nexttile;
plot(ax1,t,x_lqr(:, 1))
title(ax1,'x')
ylabel(ax1,'[m]')

ax2 = nexttile;
plot(ax2,t,x_lqr(:, 2))
title(ax2,'dot x')
ylabel(ax2,'[m/s]')

ax3 = nexttile;
plot(ax3,t,x_lqr(:, 3))
title(ax3,'theta')
ylabel(ax3,'[rad]')

ax4 = nexttile;
plot(ax4,t,x_lqr(:, 4))
```

```
title(ax4,'dot theta')
ylabel(ax4,'[rad/s]')
```



MPC:

discrete time model for MPC:

```
Ts = .001; % [s]

inv_pend_linsys_d = c2d(inv_pend_linsys, Ts, 'zoh')
```

```
inv_pend_linsys_d =

  A = 
                x1          x2          x3          x4
   x1            1   0.0009999  -9.809e-07   -3.27e-10
   x2            0      0.9998   -0.001962  -9.809e-07
   x3            0      -5e-08           1       0.001
   x4            0  -9.999e-05    0.005886           1

  B = 
               u1
   x1   9.999e-08
   x2      0.0002
   x3       5e-08
   x4   9.999e-05

  C = 
       x1  x2  x3  x4
   y1   1   0   0   0
```

```
   y2   0   0   1   0

  D =
         u1
   y1   0
   y2   0

Sample time: 0.001 seconds
Discrete-time state-space model.
```

```
% step(inv_pend_linsys_d)
```

Calculation of optimal input for first moment:

```
global A_d B_d W_d
A_d = inv_pend_linsys_d.A;
B_d = inv_pend_linsys_d.B;
W_d = idare(A_d,B_d,Q,R,S)
```

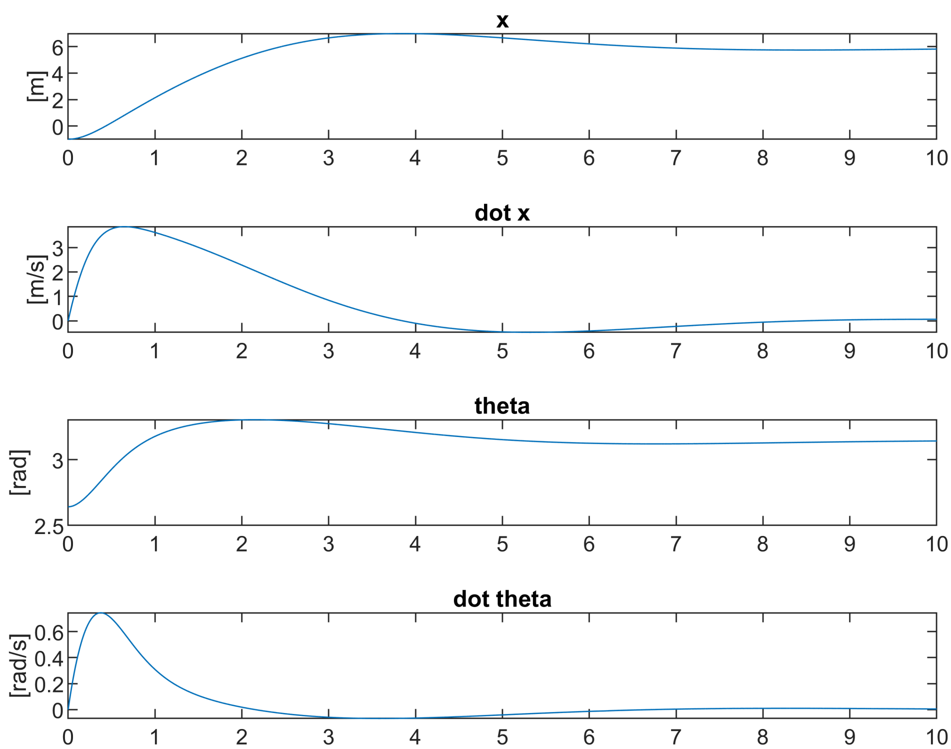
```
W_d = 4×4
10^5 ×
    0.0218    0.0186   -0.1234   -0.0472
    0.0186    0.0310   -0.2212   -0.0848
   -0.1234   -0.2212    1.9878    0.7567
   -0.0472   -0.0848    0.7567    0.2930
```

```
N = 100;                % horizont of prediction is 100 steps

up_0 = zeros([N,1]);    % initial input (optimised will be calculated)

% solving the optimisation problem, where JMPC is returning J (criterial
% func) and we have initial guess for optimal control (it's all zeros) and
% limits for "u": [-2, 2] [N]

% up_opt = fmincon(@(up)JMPC(x0,up,sp,A_d,B_d,Q,R,S,W_d),up_0,[],[],[],[],-2*ones(N,1),2*ones(N
```

Simulation with non-linear model:

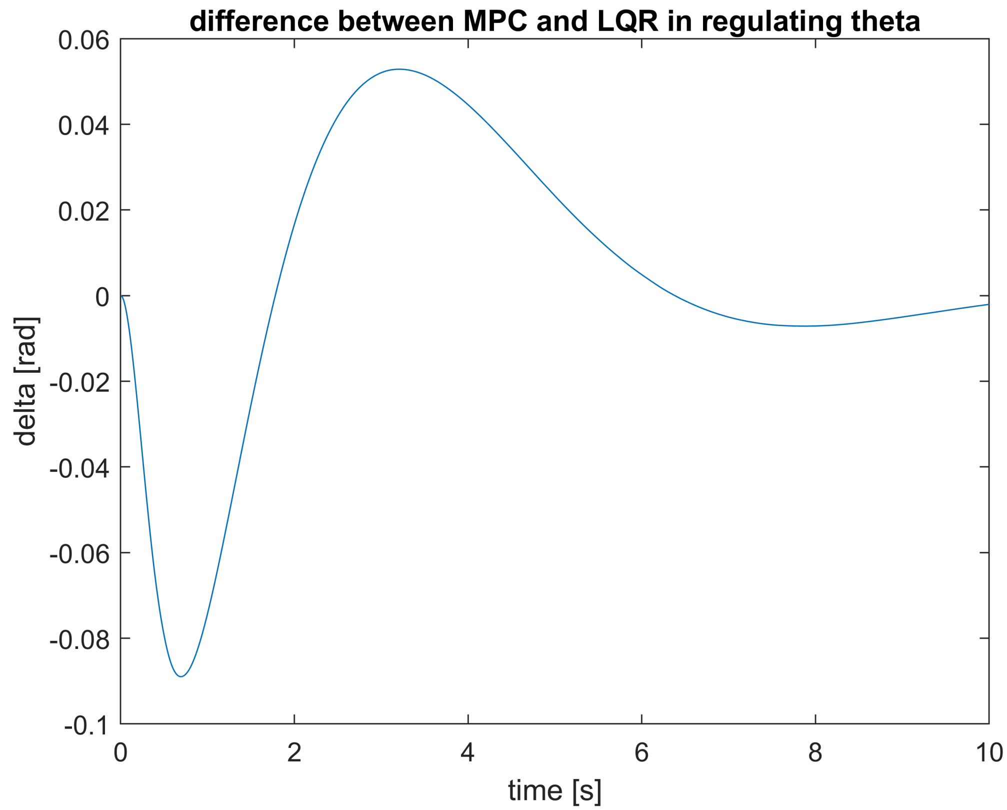
```
tspan = 0:Ts:10;

[t,x_mpc] = ode45(@(t,x)inv_pend(x, MPC(x, sp)),tspan,x0);
tiledlayout(4,1)

ax1 = nexttile;
plot(ax1,t,x_mpc(:, 1))
title(ax1,'x')
ylabel(ax1,'[m]')

ax2 = nexttile;
plot(ax2,t,x_mpc(:, 2))
title(ax2,'dot x')
ylabel(ax2,'[m/s]')

ax3 = nexttile;
plot(ax3,t,x_mpc(:, 3))
```

```
title(ax3,'theta')
ylabel(ax3,'[rad]')

ax4 = nexttile;
plot(ax4,t,x_mpc(:, 4))
title(ax4,'dot theta')
ylabel(ax4,'[rad/s]')
```



Difference between LQR and MPC:

```
tiledlayout(1,1)

plot(t, x_mpc(:,3) - x_lqr(:,3))
title('difference between MPC and LQR in regulating theta')
xlabel('time [s]')
ylabel('delta [rad]')
```



Inverted pendulum function definition (for simulating non-linear model):

```
function dx = inv_pend(x, F)

    M = 5;      % mass of the cart
    m = 1;      % mass of the pendulum
    b = 1;      % coefficient of the friction rotation
    g = -9.81;     % acceleration of gravity
    l = 2;      % length of shoulder of pendulum

    Sx = sin(x(3));
    Cx = cos(x(3));
    denom = m*(l^2)*(M+m*(1-Cx^2));

    dx1 = x(2);
    dx2 = (-(m^2)*(l^2)*g*Cx*Sx + m*(l^2)*(m*l*(x(4)^2)*Sx) - b*x(2) + m*(l^2)*F)/denom;
    dx3 = x(4);
    dx4 = ((m+M)*m*g*l*Sx - m*l*Cx*(m*l*(x(4)^2)*Sx-b*x(2)) - m*l*Cx*F)/denom;

    dx = [dx1 dx2 dx3 dx4]';

end
```

LQR regulator function definition:

```
function u = LQR(x, sp)
    global K
    u = -K*(x-sp);
end
```

MPC regulator function definition:

```
function u = MPC(x, sp)

    x0 = x;                         % shifting the window

    N = 10;                         % horizont of prediction is 10 steps

    up_0 = 0*ones([N,1]);           % initial input (optimised will be calculated)

    % solving the optimisation problem, where JMPC is returning J (criterial
    % func) and we have initial guess for optimal control (it's all zeros) and
    % limits for "u": [-100 [N], 100[N] ]

    global A_d B_d Q R S W_d
    up_opt = fmincon(@(up)JMPC(x0,up,sp,A_d,B_d,Q,R,S,W_d),up_0,[],[],[],[],-100*ones(N,1),100*

    u = up_opt(1);

end
```