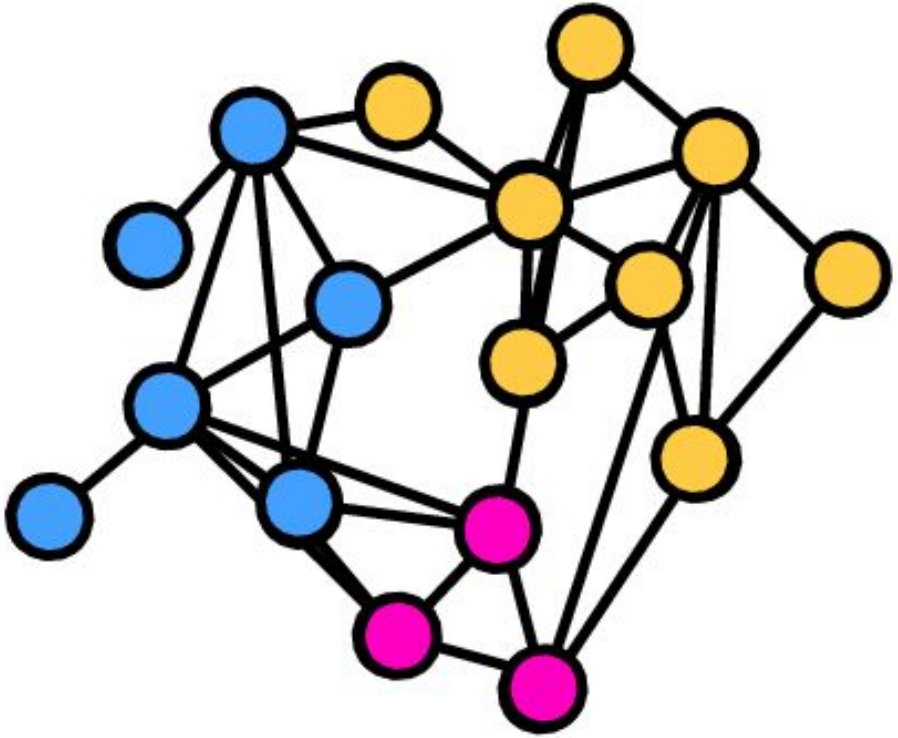
2025

Graphs



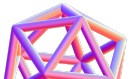
A2SV

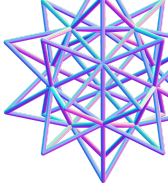
Lecture Flow

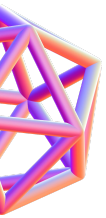
1. Pre-requisites
2. Definition of Graphs
3. Types of Graphs
4. Graph terminologies
5. Checkpoint
6) Graph representations
7) Graphs & Trees
8) Receiving Inputs on Graph Problems
9) Practice questions
10) Ending quote

Pre-requisites

- Arrays / Linked list
- Matrices
- Dictionaries
- Time and space complexity analysis







What do we mean by Graph?

Definition

- A way to represent **relationships** between objects.
- A collection of **nodes** that have data and are connected to other nodes through edges.

Graph = **Nodes** + **Edges**

Example: Friendship Graph

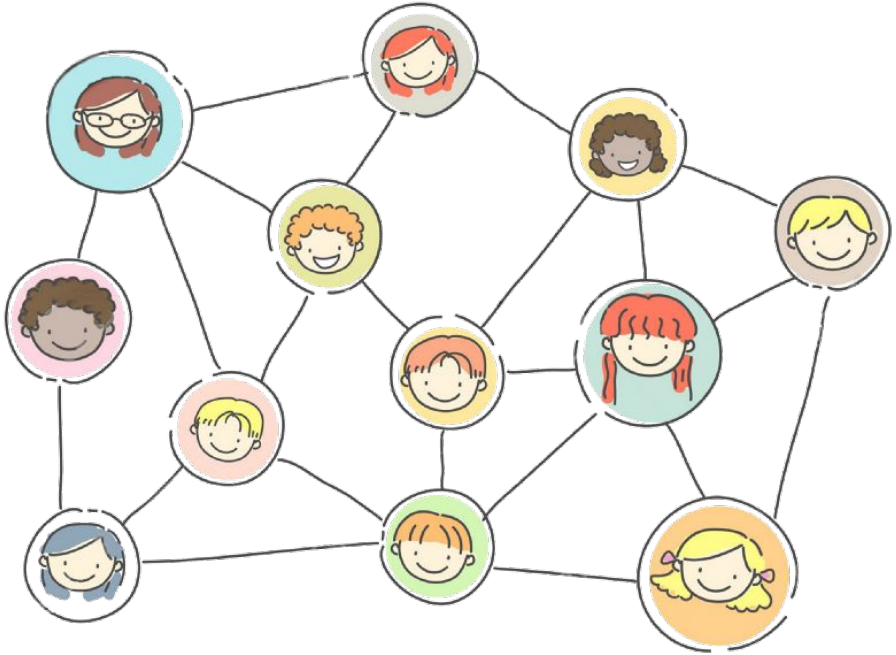
- Nodes or vertices: The objects in graph
  - These people our case
- Edges: the relation between nodes.
  - The friendship between them (the lines)

Types of graphs

Undirected Graph

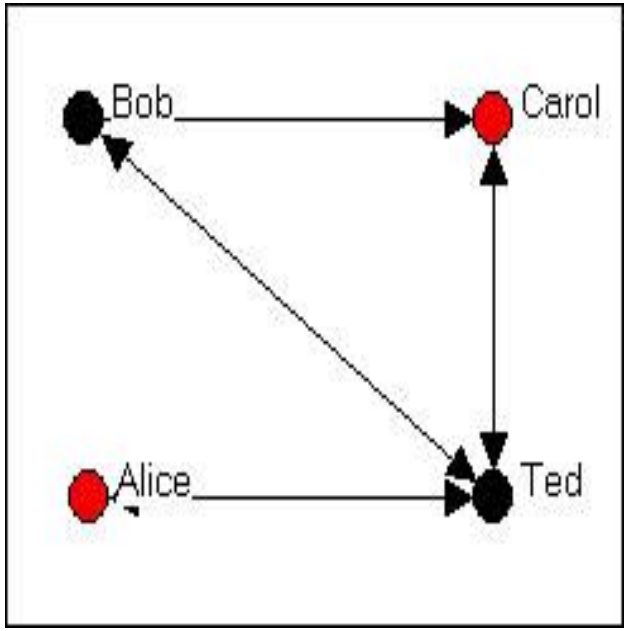
- Facebook friendship
- If Alice is friends with Bob,

  Bob is also friends with alice



Directed Graph

- Linkedin and instagram "following" relation.
- If Bob follows Carol, that does not mean that Carol follows Bob.

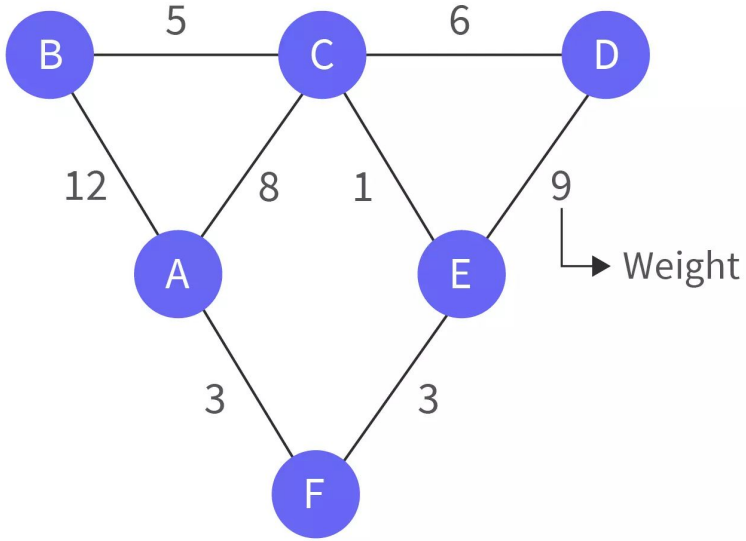


Let's say we want to add a weight parameter which represents the strength of the friendship.

How can we do that?

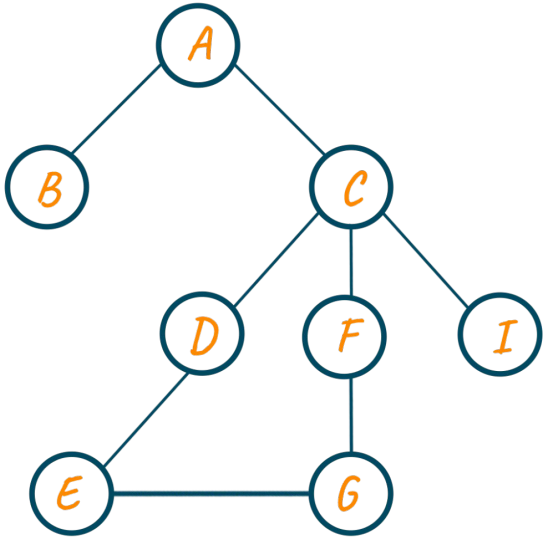
Weighted Graph

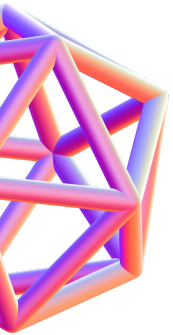
Weighted graphs assign **numerical values** to edges.



Unweighted Graph

All edges have the **same value.**

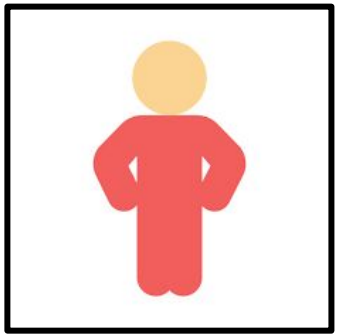


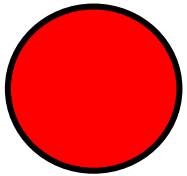


Graph Terminologies

Terminology: Node / Vertex

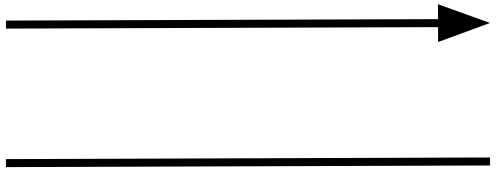
- Represent entities (e.g., people, devices).
- Connected to other nodes by edges.





Terminology: Edge

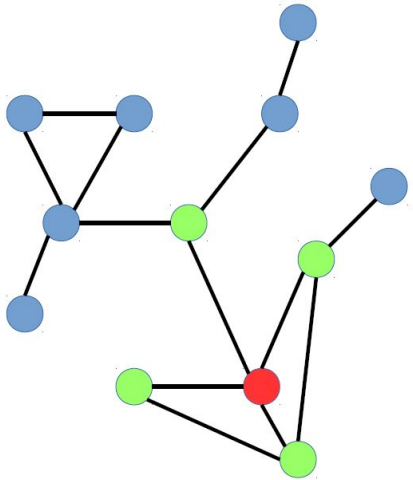
- Connection between two nodes.
- Represented by lines or arrows.
- model relationships in various systems.



Terminology: Neighbors

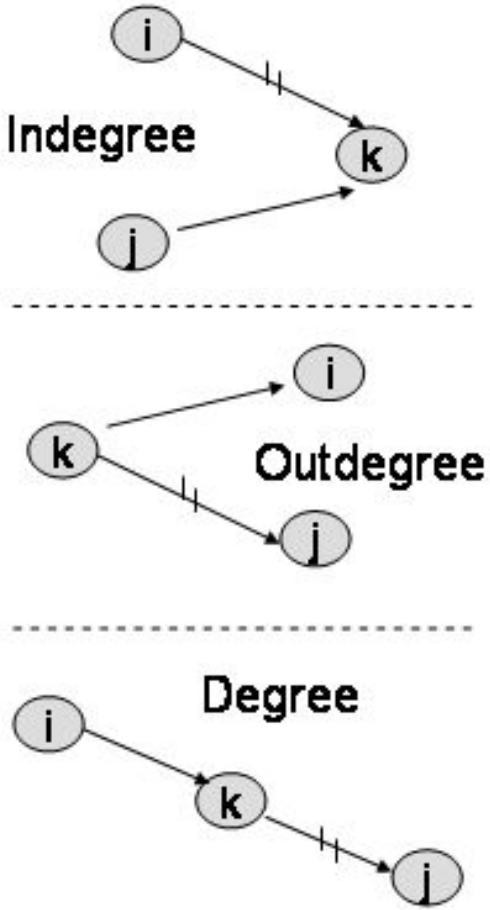
- Two nodes are **neighbors** or **adjacent** if there is an edge **between** them

Graph Neighborhood



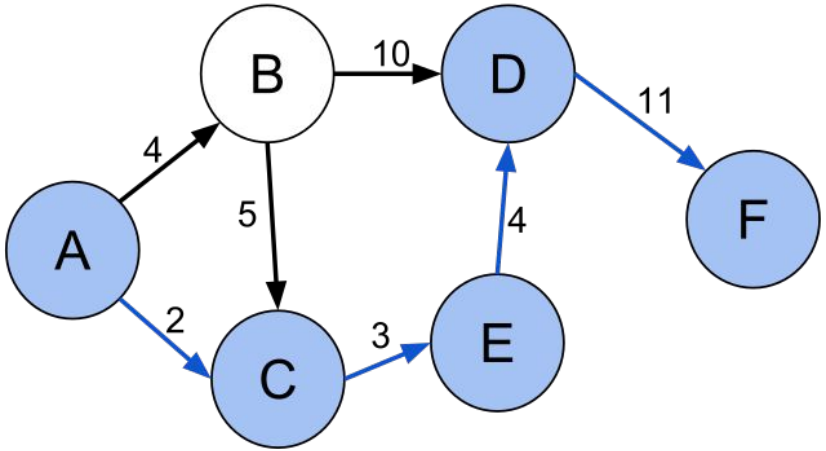
Terminology: Degree

- Node degree = number of neighbors.
- In directed graphs, consider node k:
  - Indegree of k = edges ending at node k.
  - Outdegree of k = edges starting at node k.



Terminology: Path

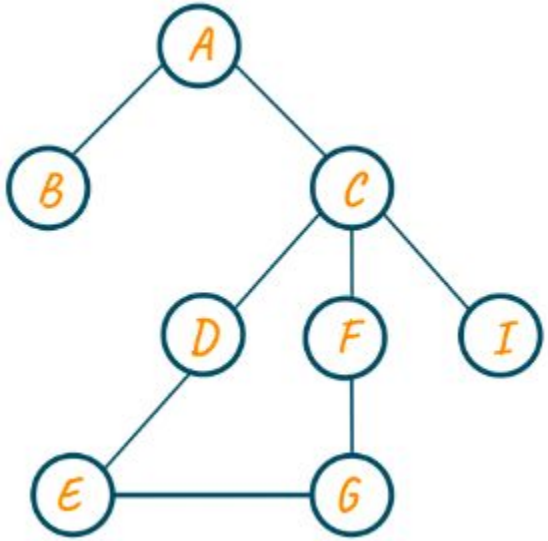
- A **path** leads from one node to another.
- The length of a path is the number of edges in it.
- Let's consider the path between node A and node F



Terminology: Cycle

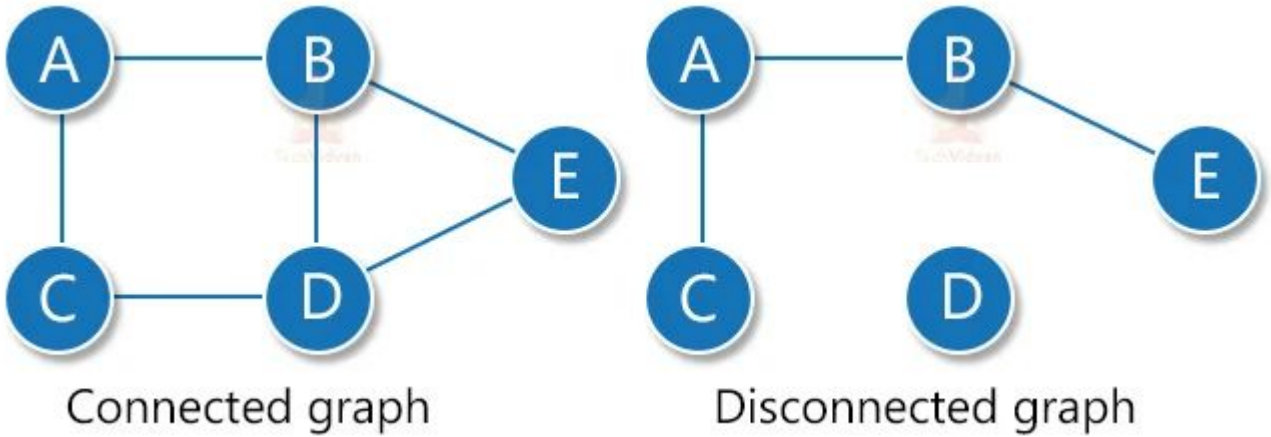
A path is a **cycle** if the first and the last node of the path is the same.

Cycle = C-D-E-G-F-C



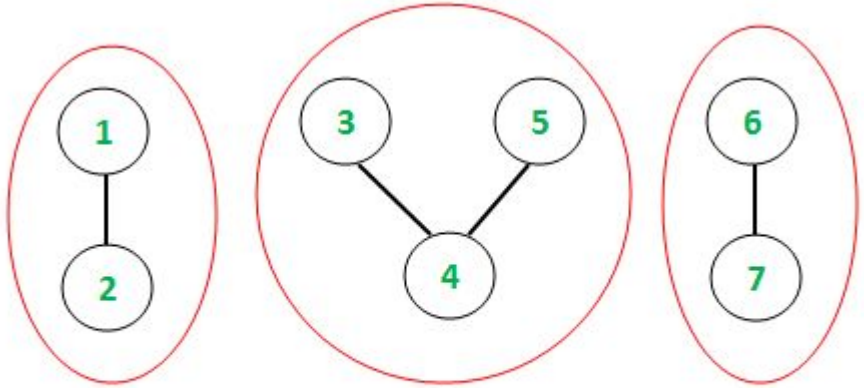
Terminology: Connectivity

A graph is **connected** if there is a path between any two nodes



Terminology: Components

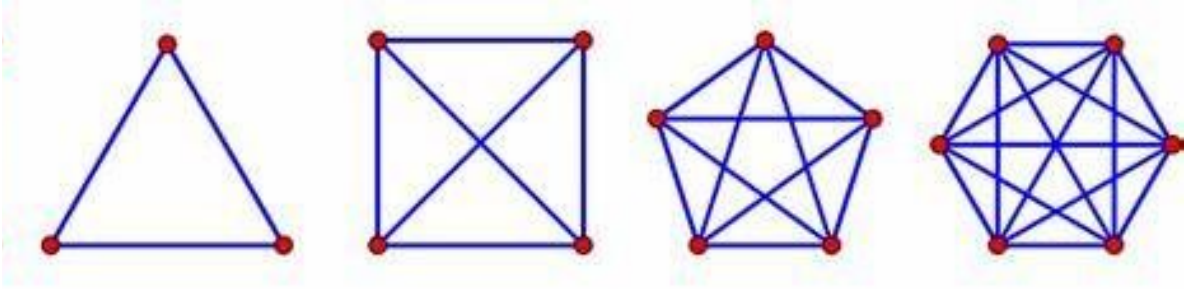
The connected parts of a graph are called its **components.**



The **counts** of connected components are - **2, 3 and 2**

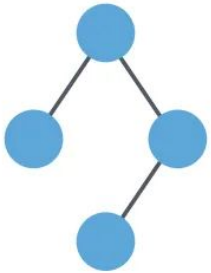
Terminology: Complete Graph

A complete graph is a graph in which **each pair of node** is connected by an edge.

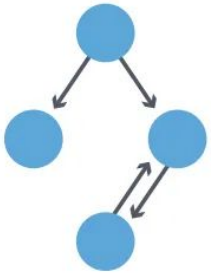


Summary

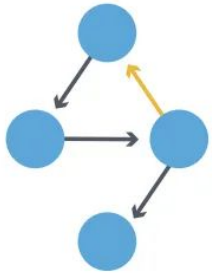
Undirected



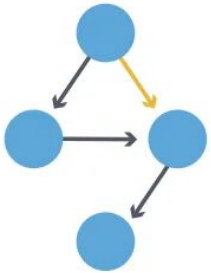
Directed



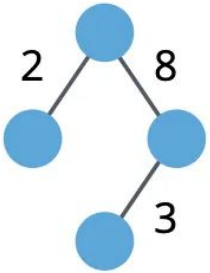
Cyclic



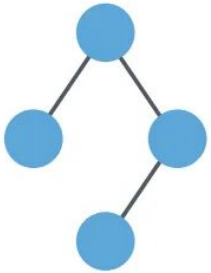
Acyclic



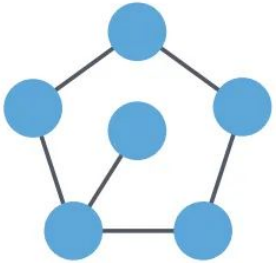
Weighted



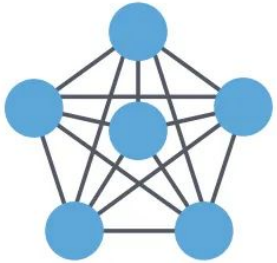
Unweighted



Sparse



Dense



Common Terminologies

Node

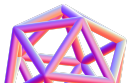
Edge

Path

Cycle

Connectivity

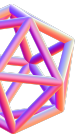
Components



Common Terminologies

Node	Element in a graph.
Edge	Connection between two nodes.
Path	Sequence of edges connecting nodes.
Cycle	Path that starts and ends at the same node.
Connectivity	The ability to reach any node from another.
Components	Independent connected subgraphs.

Common Terminologies



Neighbors

Degree

In-degree

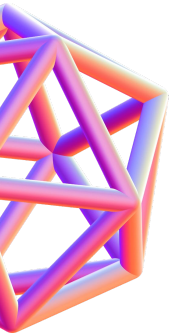
Out-degree

Complete

Traverse

Common Terminologies

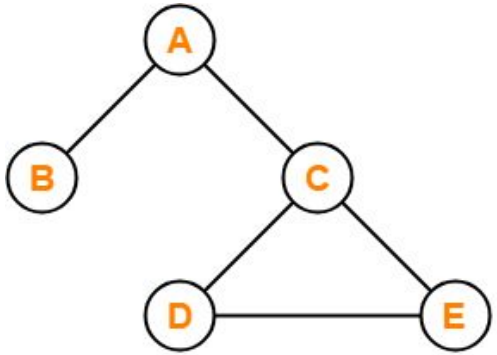
Neighbors	Nodes connected by an edge.
Degree	Number of neighbors a node has.
In-degree	Number of edges ending at a node.
Out-degree	Number of edges starting from a node.
Complete	Each node is connected to every other node.
Traverse	Visiting nodes using edges.



Graph and Tree

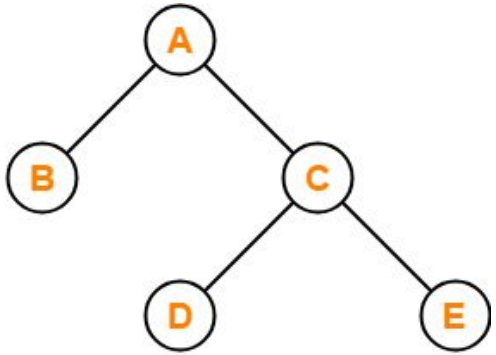
Tree

- Tree is special type of graph.
- A tree is a **connected** and **acyclic graph.**
- A tree has a **unique** path between any two vertices.
- How many edges does a tree have?



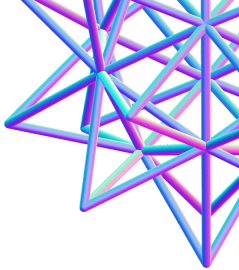
✗

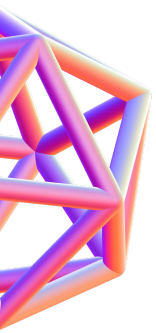
This graph is not a Tree



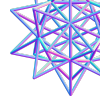
✓

This graph is a Tree



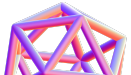


Graph Representations

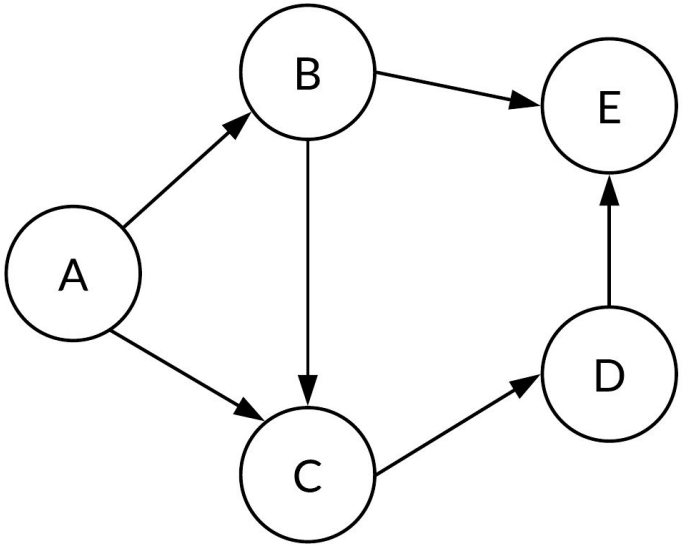


Graph Representations

- Edge List
- Adjacency Matrix
- Adjacency List
- Grids as Graphs



Graph Representation: Edge list




```
Edge list:

graph = [('A', 'B'),
('A', 'C'),
('B', 'C'),
('B', 'E'),
('C', 'D'),
('D', 'E')
]
```

Advantages and Disadvantages of Edge List

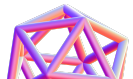
Graph Representation: Edge list

Advantages:

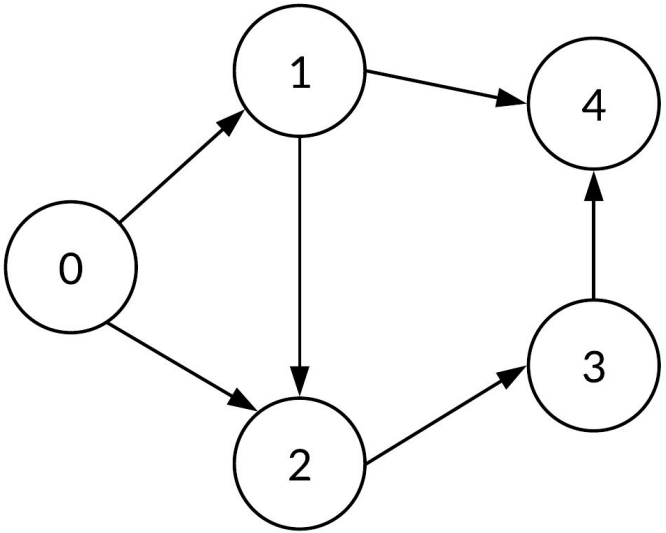
- It uses less memory.
- Easy to represent

Disadvantages:

- Edge look up is slow
- Hard to traverse



Graph Representation: Adjacency matrix



Adjacency Matrix

	0	1	2	3	4
0	0	1	1	0	0
1	0	0	1	0	1
2	0	0	0	1	0
3	0	0	0	0	1
4	0	0	0	0	0

Graph Representation: Adjacency matrix

[Leetcode Playground: Convert Edge List to Adjacency Matrix]

Implementation

```python
def buildAdjacencyMatrix(n, edge_list) -> list[list]:

    # Initialize n x n matrix with zeros
    matrix = [[0] * n for _ in range(n)]
    for u, v in edge_list:
        matrix[u][v] = 1
        matrix[v][u] = 1
        # Since the graph is undirected

    return matrix
```

Advantages and disadvantages of Adjacency Matrix

Graph Representation: Adjacency matrix

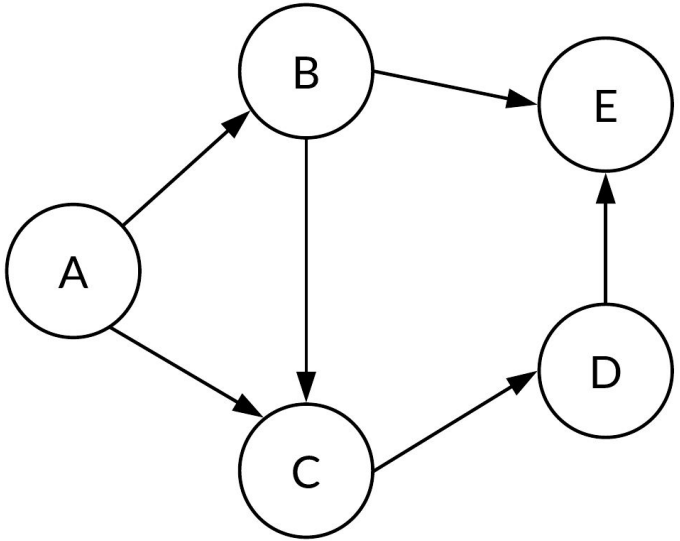
Advantages:

- To represent dense graphs.
- Edge lookup is fast

Disadvantages:

- It takes more time to build and consume more memory (**O(N**2)** for both cases)
- Finding neighbors of a node is costly

Graph Representation: Adjacency List using List



Adjacency list:

```
graph = {
'A': ['B', 'C'],
'B': ['C', 'E'],
'C': ['D'],
'D': ['E'],
}
```

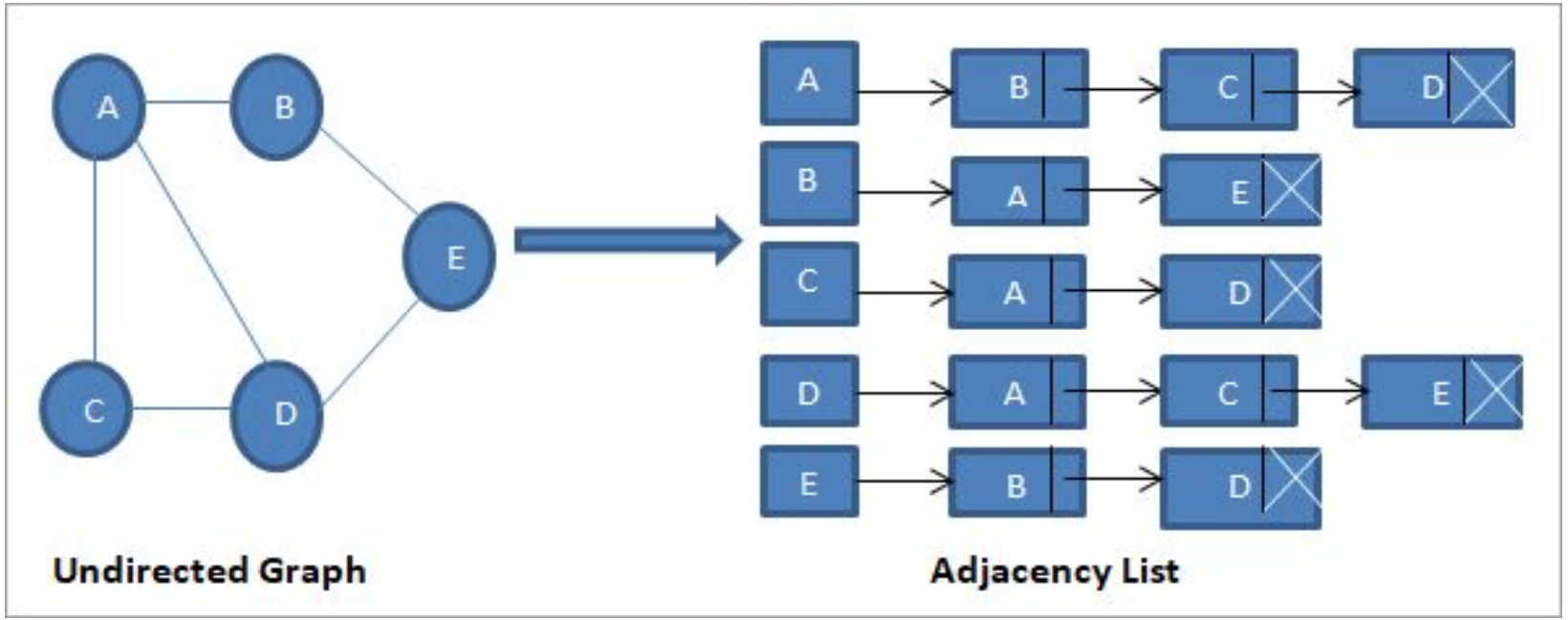
Graph Representation: Adjacency List

[Leetcode Playground: Convert Edge List to Adjacency List]

Implementation

```
def buildAdjacencyList(n, edge_list) -> list[list]:
    graph = [[] for i in range(n)]

    for u, v in edge_list:
        graph[u].append(v)
        graph[v].append(u)

    return graph
```

Graph Representation: Adjacency List using Linked List



Advantages and Disadvantages of Adjacency List

Graph Representation: Adjacency list

Advantages:

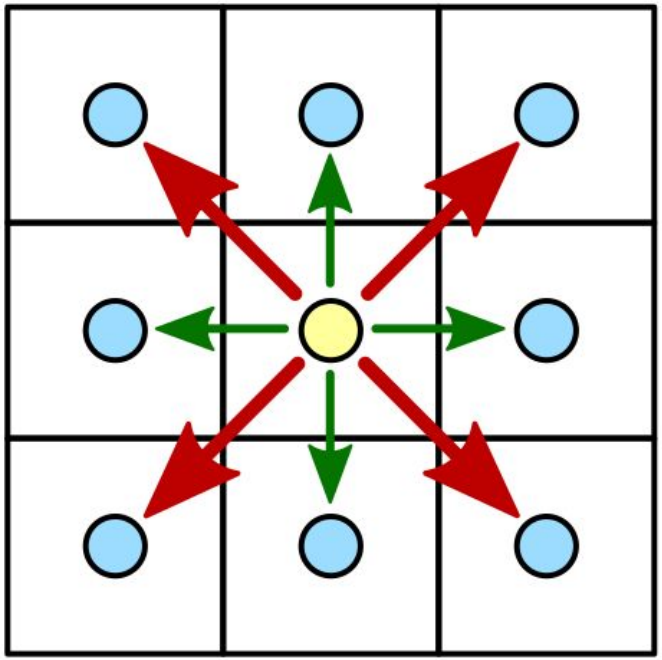
- It uses less memory
- Neighbours of a node can be found pretty fast
- Best for sparse graphs

Disadvantages:

- Edge look up is slow

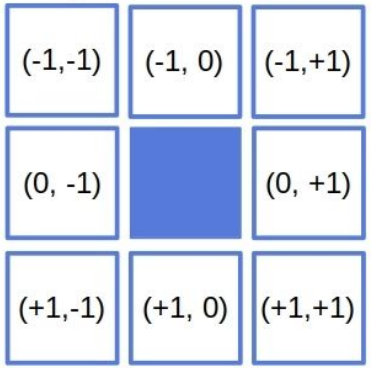
Graph Representation: Grids as Graph

- Matrix cells = **nodes**
- **Edges** between adjacent cells:
  - **4 perpendicular**
  - **2 diagonal**
  - **2 antidiagonal.**

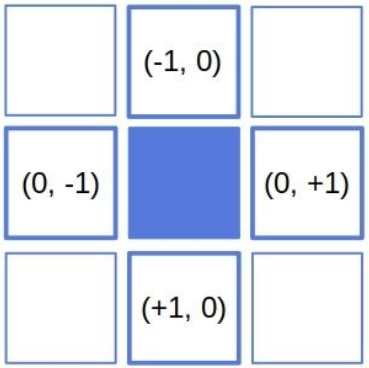


Grids – Direction vectors

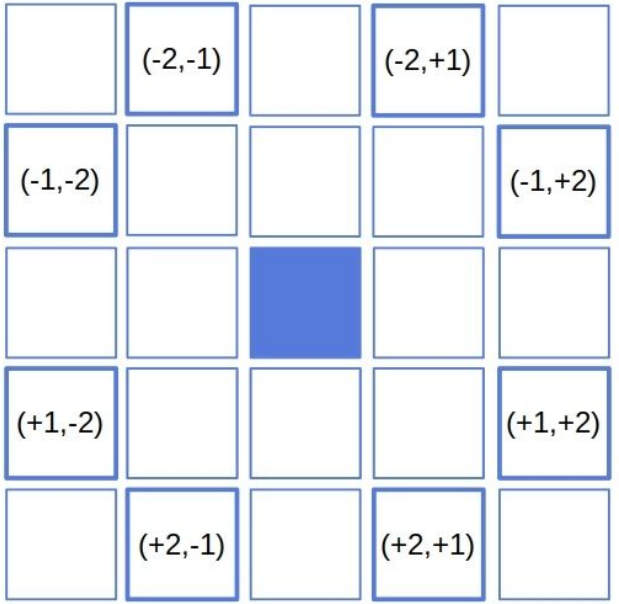
The standard square neighborhood



Only vertical and horizontal neighbors



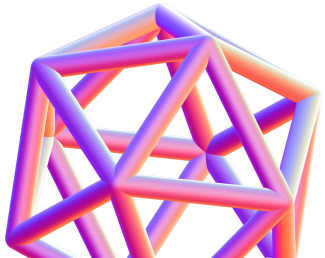
The chess knight neighborhood

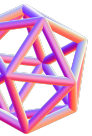


Generic template — 4 Directional Traversal

```python
# Direction vectors for moving up, down, left, and right
directions = [(-1, 0), (1, 0), (0, -1), (0, 1)]
def is_inbound(x, y, rows, cols):
    """Checks if the given (x, y) position is inside the grid"""
    return 0 <= x < rows and 0 <= y < cols

def traverse_grid(grid):
    rows, cols = len(grid), len(grid[0])
    for i in range(rows):
        for j in range(cols):
            # Explore all 4 directions
            for dx, dy in directions:
                new_x, new_y = i + dx, j + d
                if is_inbound(new_x, new_y, rows, cols):
                    # Do Something
                    pass
```

Receiving Inputs on Graph Problems



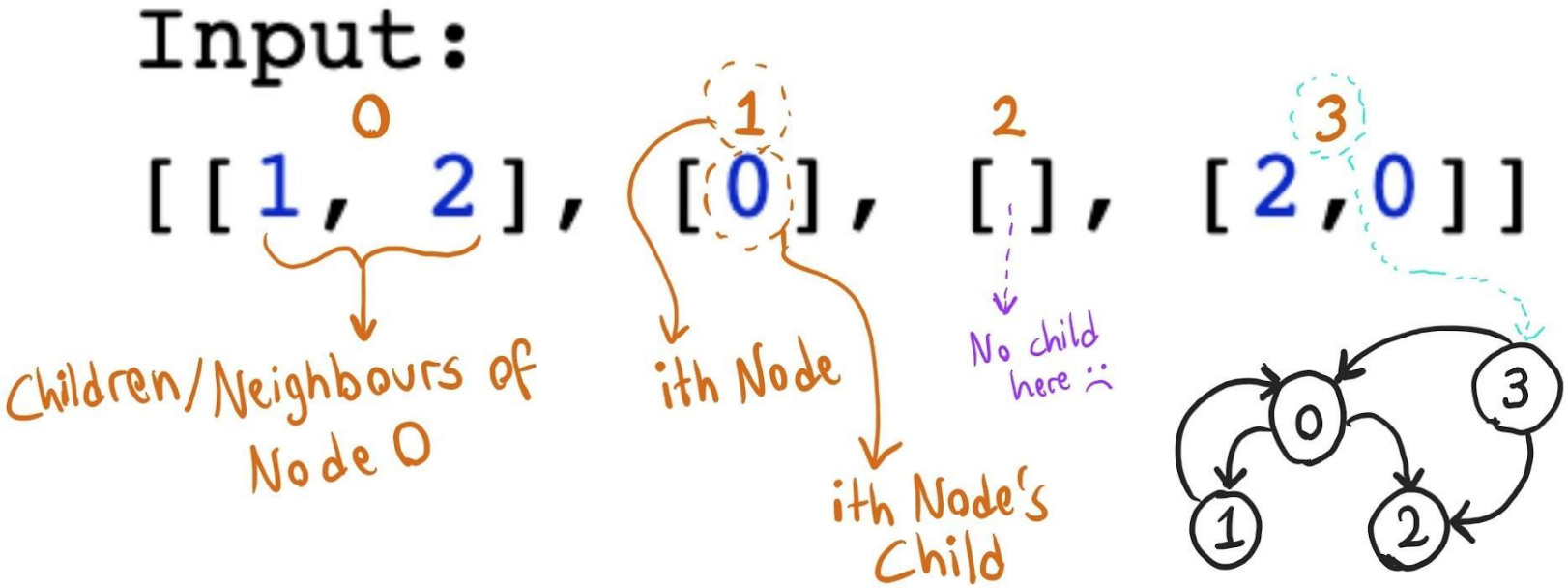


Adjacency List Inputs

Receiving Inputs: Directed Unweighted Graphs - AL

```
Input:
[[1, 2], [0], [], [2, 0]]
```

Receiving Inputs: Directed Unweighted Graphs - AL

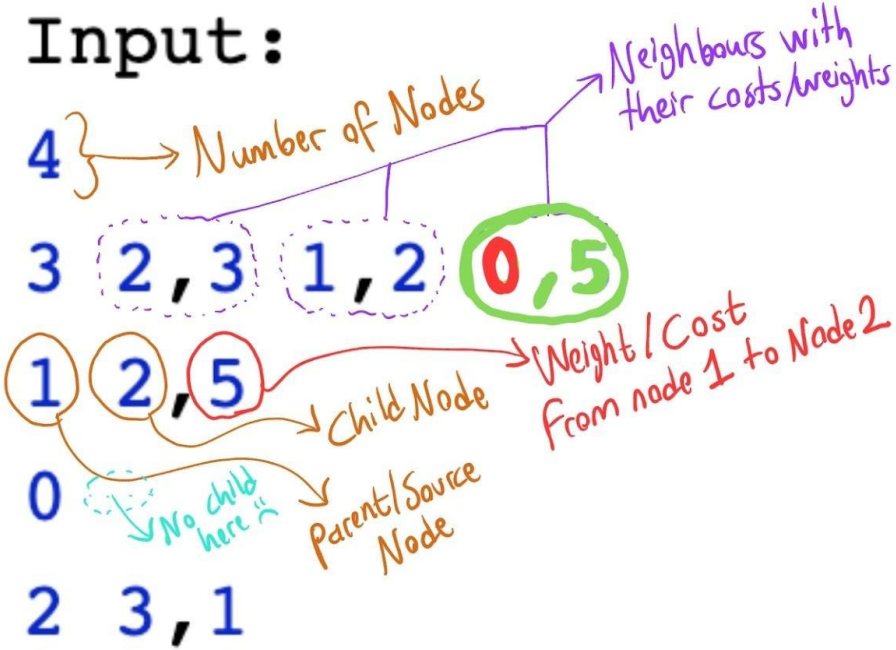


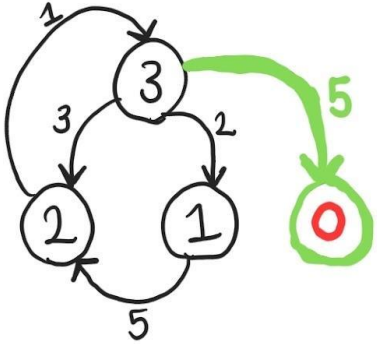
Think of ways to implement this

Receiving Inputs: Directed Weighted Graphs - AL

```
Input:
4
3 2,3 1,2 0,5
1 2,5
0
2 3,1
```

Receiving Inputs: Directed Weighted Graphs - AL





Receiving Inputs: DWG - AL - Implementation

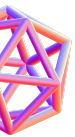
```
from collections import defaultdict

n = int(input())
graph = defaultdict(list)

for i in range(n):
    line = input().split()
    node = int(line[0])
    for neighbor in range(1, len(line)):
        adj_node, weight = map(int, line[neighbor].split(","))
        graph[node].append((adj_node, weight))
```

Receiving Inputs: DWG Complexity Analysis - AL

Time Complexity: O(n + E)

Space Complexity: O(E)

n = number of nodes
E = number of edges



Adjacency Matrix Inputs

Receiving Inputs: Directed Weighted Graphs - AM

```
Input:
3
0 1 2
3 4 3
2 6 4
```

Receiving Inputs: DWG Illustration - AM

Input:

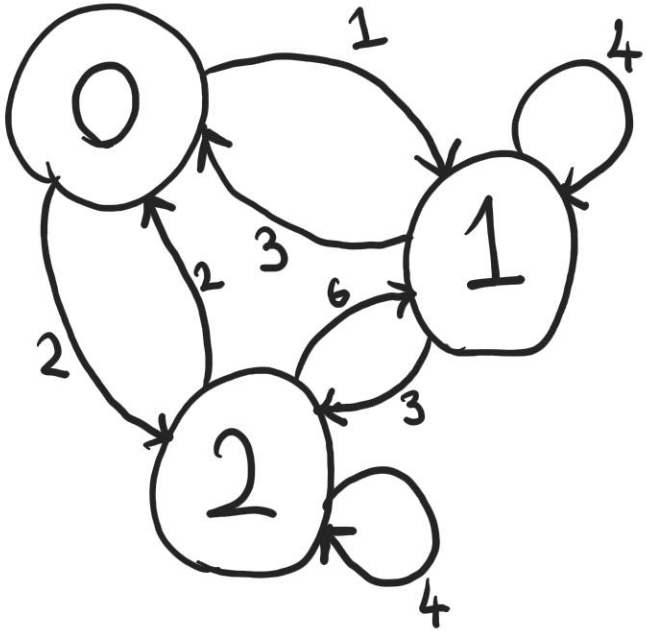
3 → Matrix Size (N x N)

0 1 2 → Indices (Our Nodes)

0 0 1 2 → First Row

1 3 4 3 → Weight between Node 1 and Node 2

2 2 6 4



Receiving Inputs: DWG - Implementation - AM

```
from collections import defaultdict

n = int(input())
graph = defaultdict(list)

for i in range(n):
    row = list(map(int, input().split()))

    for j in range(len(row)):
        graph[i].append((j, row[j]))
```

Receiving Inputs: DWG Complexity Analysis - AM

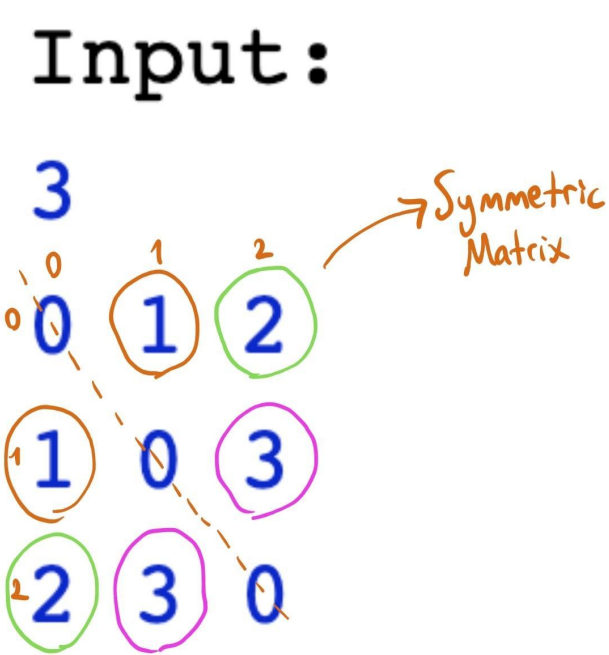
- Time Complexity: **O(n^2)**
- Space Complexity: **O(n^2)**

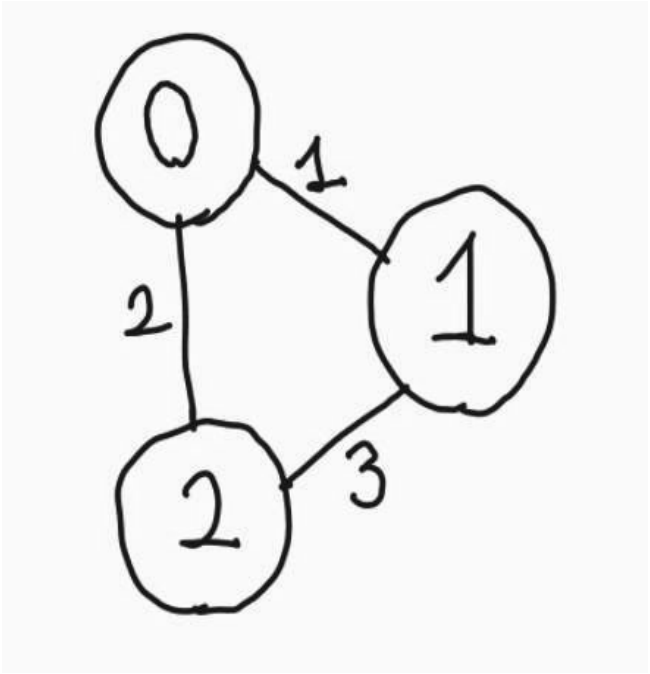
n = number of nodes(matrix length)

Receiving Inputs: Undirected Weighted Graphs - AM

```
Input:
3
0 1 2
1 0 3
2 3 0
```

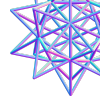
Receiving Inputs: UDWG Illustration - AM





Receiving Inputs: UDWG - Implementation - AM

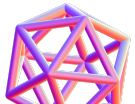
```
from collections import defaultdict


n = int(input())
graph = defaultdict(list)

for i in range(n):
    row = list(map(int, input().split()))

    for j in range(i + 1, len(row)):
        graph[i].append((j, row[j]))
        graph[j].append((i, row[i]))
```

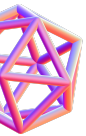


Receiving Inputs: UDWG Complexity Analysis - AM

- Time Complexity: **O(n^2)**
- Space Complexity: **O(n^2)**

n = number of nodes (matrix length)



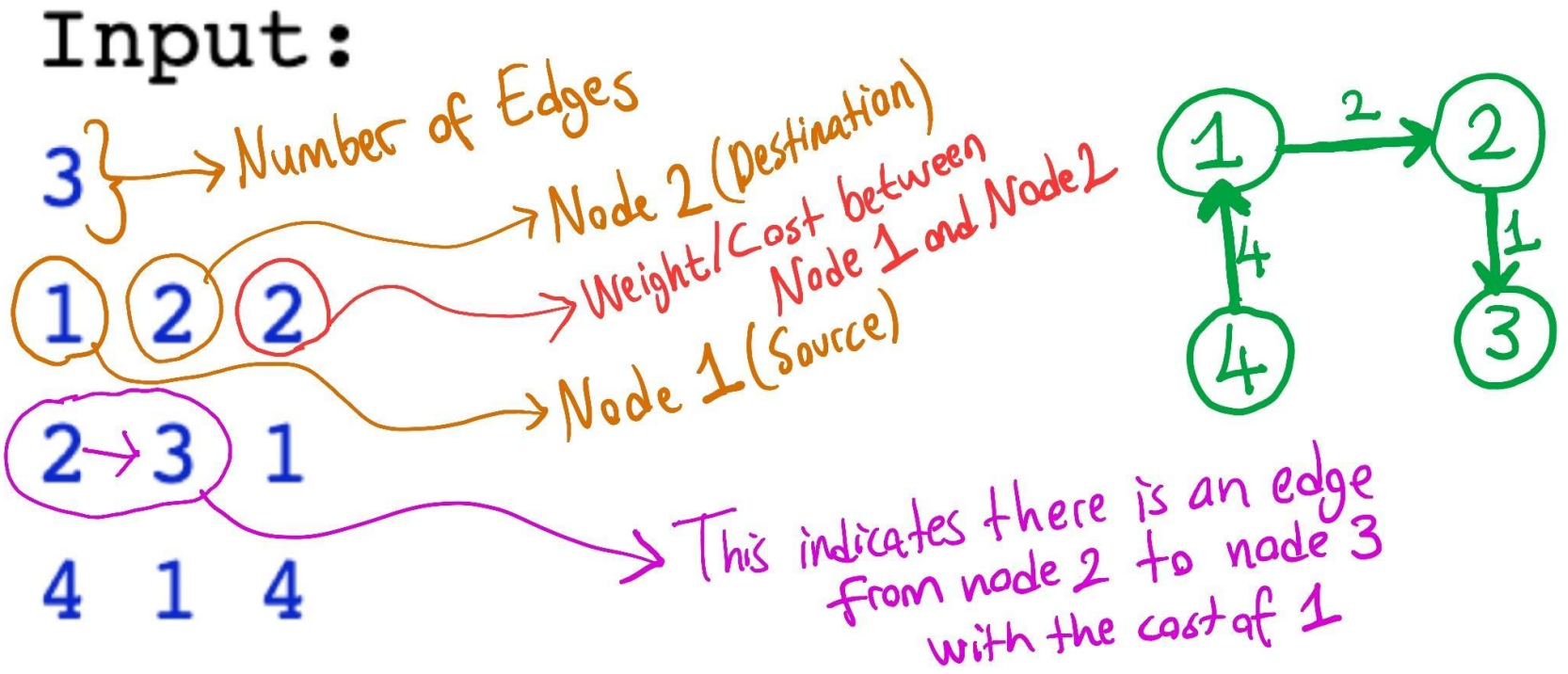


Edge List Inputs

Receiving Inputs: Directed Weighted Graphs - EL

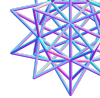
```
Input:
3
1 2 2
2 3 1
4 1 4
```

Receiving Inputs: DWG Illustration - EL



Receiving Inputs: DWG - Implementation - EL

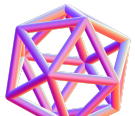
```
from collections import defaultdict

n = int(input())
graph = defaultdict(list)

for i in range(n):
    src, dest, w = list(map(int, input().split()))

    graph[src].append((dest, w))
```

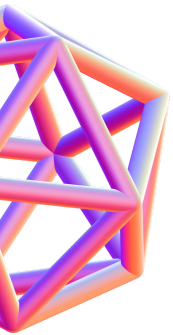


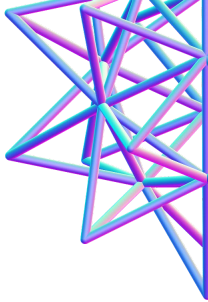
Receiving Inputs: DWG Complexity Analysis - EL

- Time Complexity: **O(E)**
- Space Complexity: **O(E)**

E = number of edges





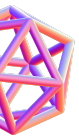


For Multiple Graph Problems (Generic Templates)

Generic template - I

```
t = int(input()) # number of test cases

for _ in range(t):
    # number of nodes and edges
    n, m = map(int, input().split())
    graph = [[] for _ in range(n)]
    for j in range(m):
        u, v = map(int, input().split())
        graph[u - 1].append(v - 1)
        graph[v - 1].append(u - 1)

    # do something with the graph
```

Which kind of input is the graph?



OR

Generic template - II

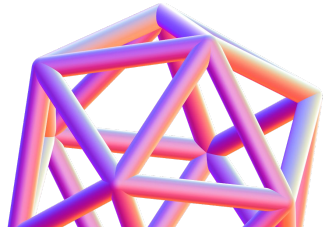
```
from collections import defaultdict

t = int(input()) # number of test cases
for _ in range(t):
    # number of nodes and edges
    n, m = map(int, input().split())
    graph = defaultdict(list)
    for j in range(m):
        u, v = map(int, input().split())
        graph[u].append(v)
        graph[v].append(u)
```

Tip - To make your inputs faster...

```
import sys
input = sys.stdin.readline.strip()
```

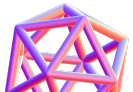
- This replaces the input() function with sys.stdin.readline() which is faster.

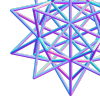
Common Pitfalls



Common Pitfalls

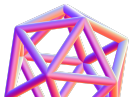
- Not considering cycles in the graph.
- Not checking whether the graph is directed or undirected
- Not understanding input format well
- Falling in to infinite loop (because of cycles)





Types of Graph Questions

- Graph questions can be classified into different categories based on the problem requirements.
- Some common types of graph questions include:
  - Shortest path: find the shortest path between two vertices.
  - Connectivity: determine if there is a path between two vertices.
  - Cycle detection: detect cycles in the graph.
  - Topological sorting: order the vertices in a directed acyclic graph.



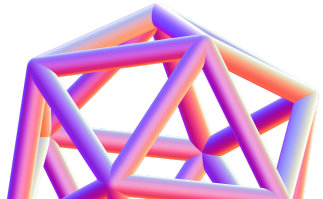
Approaches to Solving Graph Problems

There are several approaches to solving graph problems, including:

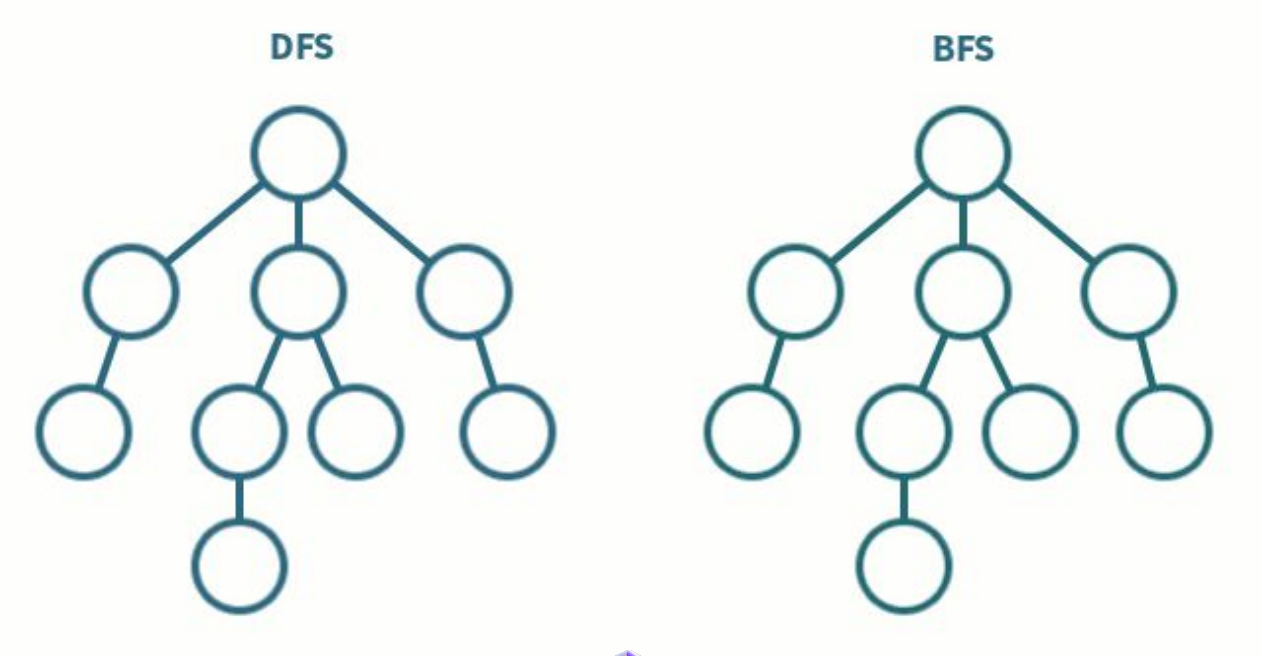
- Breadth-first search (BFS)
- Depth-first search (DFS)
- Dijkstra's algorithm
- Bellman-Ford algorithm
- Kruskal's algorithm
- Floyd-Warshall algorithm

The choice of algorithm depends on the **problem's requirements.**

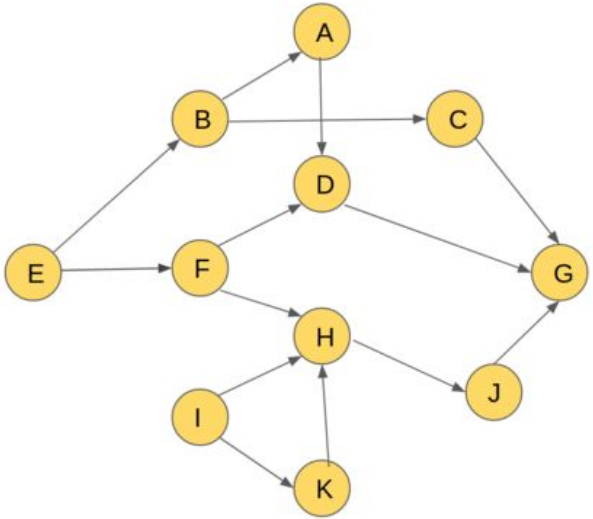
What is next?



Graph Traversal: DFS and BFS

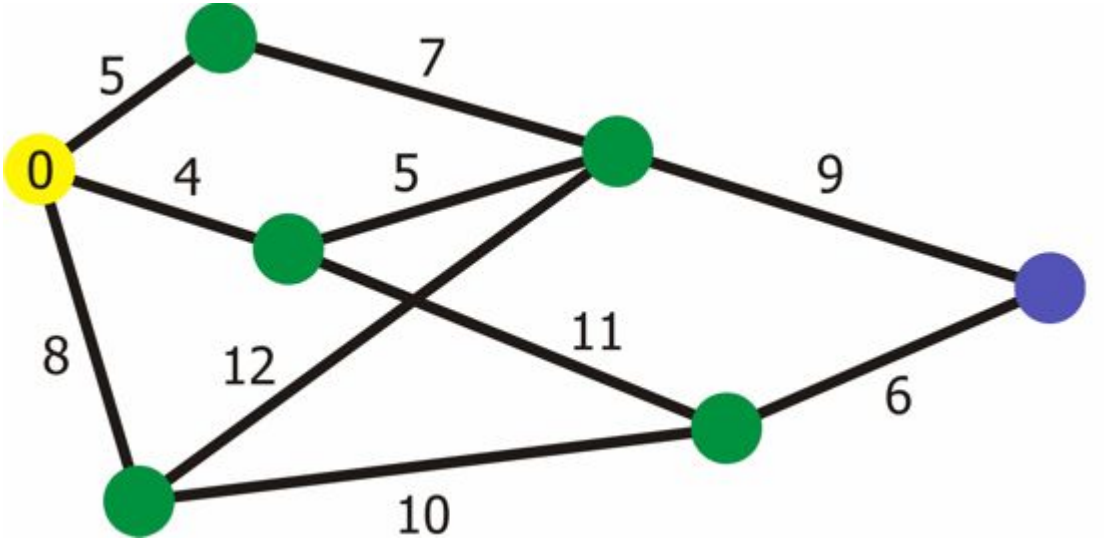


Topological Sort

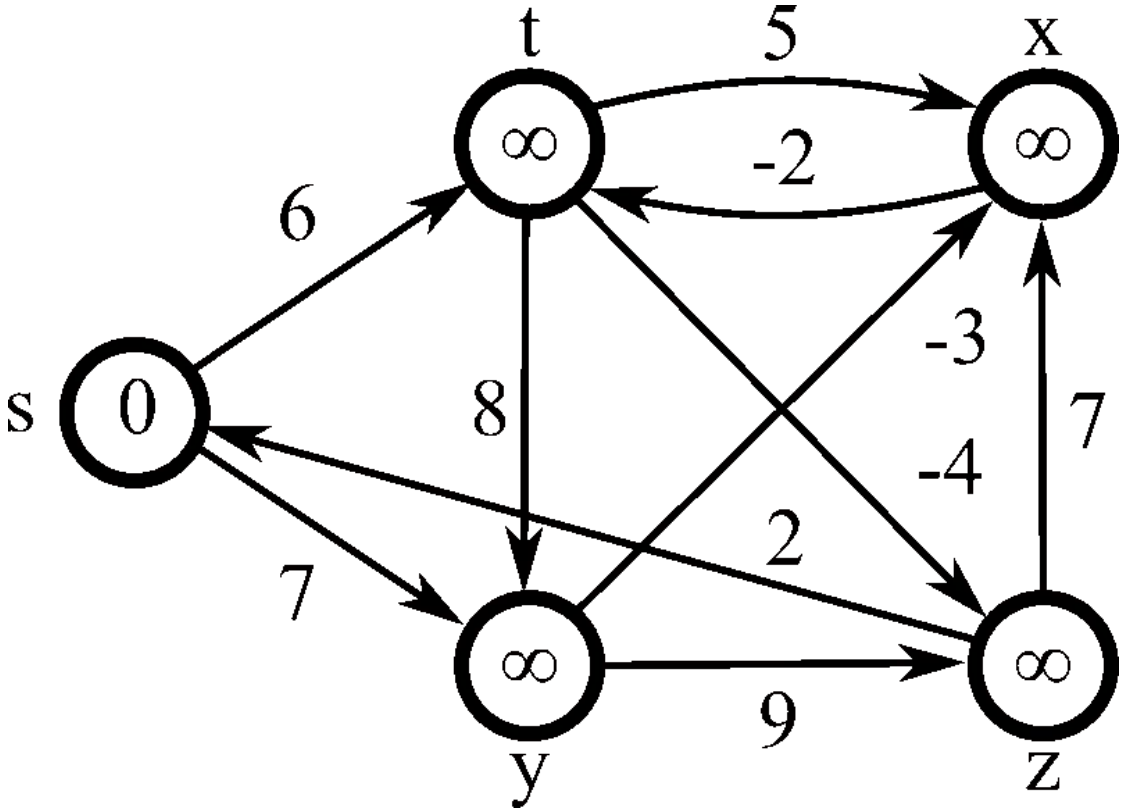


RESULT :

Shortest path: Dijkstra



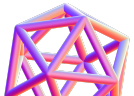
Shortest path: Bellman Ford



Exercise problems...

1. Operations on graph
2. Cities and roads
3. From adjacency matrix to adjacency list
4. From adjacency list to adjacency matrix
5. Regular graph
6. Sources and sinks

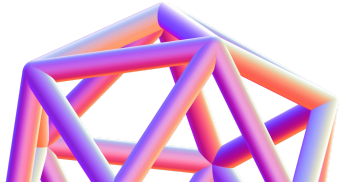
For more graph representation Problems: Link



Ending Quote

"*You can always recognize truth by its beauty and simplicity.*"

– **Richard P. Feynman**



Slide Assets

Colors:

- Color 1
- Color 2
- Color 3

Fonts:

- Headings Font (Lora)
- Body Font (Inter)

Graphic Elements