ENTMLGY 6707 Entomological Techniques and Data Analysis Supplemental activity (do not submit): Split Plots

1 Introduction

Stratified designs are used to increase precision of estimates by experimentally controlling or reducing variation. This tutorial covers how to analyze split-plot and split-split-plot designs. The examples include multiple ways of analyzing the same exact data.

You might recall that F-values in ANOVAs are calculated from F-ratios (e.g., mean squares for treatment divided by means squares for errors). Historically, for more complicated designs like split-plots, folks had to compute the F-ratios "by-hand": these designs have whole plot and sub plot factors and errors, and so F-ratios for whole plot factors use the whole plot error as the denominator whereas sub plot factors use the sub plot error. When examining the R output below for split plots and split-split plots, note which mean square values are used in the F-ratios.

Nowadays, some folks use mixed-effects models to analyze stratified designs. Be aware that a fixed-effect only approach might be preferred, given that we assume random effects are normally distributed and it is hard to test that assumption when there are only a few levels of a random effect (e.g., 3 blocks or 4 whole plots).

Either way, you still have to be careful about specifying the random effects correctly. And you will notice that if you do, the sums of squares and F-statistics for each treatment are typically equivalent. There are sometimes differences, but rarely do they influence the overall conclusions.

The following packages are necessary to complete this tutorial.

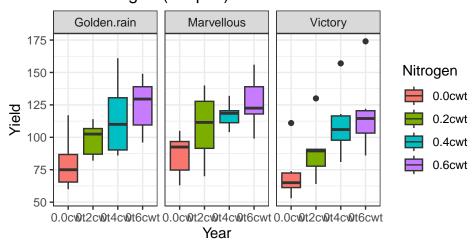
```
library(car)
library(lme4)
library(lmerTest)
library(tidyverse)
library(agricolae)
library(emmeans)
```

2 Split plot

These data report the yield of oats from a split-plot field trial. The treatment structure used in the experiment was a 3×4 full factorial, with three varieties of oats and four concentrations of nitrogen. The experimental units were arranged into six blocks, each with three whole-plots subdivided into four subplots. The varieties of oats were assigned randomly to the whole-plots and the concentrations of nitrogen to the subplots. All four concentrations of nitrogen were used on each whole-plot.

```
oats_df <- read.table("oats.txt", header=T, sep="\t",</pre>
                      colClasses = c("factor", "factor", "factor",
                                     "factor", "numeric"))
summary(oats df)
##
    Block
                    Variety
                                 Nitrogen
                                                Yield
##
    I :12
             Golden.rain:24
                               0.0cwt:18
                                           Min.
                                                   : 53.0
    II:12
             Marvellous :24
                               0.2cwt:18
                                            1st Qu.: 86.0
##
##
    III:12
             Victory
                         :24
                               0.4cwt:18
                                           Median :102.5
##
    IV:12
                               0.6cwt:18
                                                   :104.0
                                           Mean
##
    V
      :12
                                            3rd Qu.:121.2
    VI:12
                                           Max.
                                                   :174.0
ggplot(oats_df, aes(x=Nitrogen, y=Yield, fill=Nitrogen)) +
  geom_boxplot()+ theme_bw()+
  xlab("Year")+
  ylab("Yield")+
```

Split plot: Variety (whole plot) and Nitrogen (subplot)



ggtitle("Split plot: Variety (whole plot) \n and Nitrogen (subplot)")

2.1 Agricolae package

facet wrap(~Variety)+

```
library(agricolae)
sp.plot(oats_df$Block, oats_df$Variety, oats_df$Nitrogen, oats_df$Yield)
##
## ANALYSIS SPLIT PLOT: oats_df$Yield
## Class level information
```

```
##
## oats_df$Variety : Victory Golden.rain Marvellous
## oats df$Nitrogen
                      : 0.0cwt 0.2cwt 0.4cwt 0.6cwt
## oats_df$Block : I II III IV V VI
## Number of observations: 72
## Analysis of Variance Table
##
## Response: oats_df$Yield
                                   Df Sum Sq Mean Sq F value
                                                                 Pr(>F)
## oats_df$Block
                                    5 15875.3 3175.1 17.9297 9.525e-10 ***
                                                893.2 1.4853
## oats_df$Variety
                                    2 1786.4
                                                                 0.2724
## Ea
                                   10 6013.3
                                                601.3
## oats_df$Nitrogen
                                    3 20020.5 6673.5 37.6856 2.458e-12 ***
## oats_df$Variety:oats_df$Nitrogen 6
                                       321.8
                                                 53.6 0.3028
                                                                 0.9322
                                   45 7968.7
## Eb
                                                177.1
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## cv(a) = 23.6 \%, cv(b) = 12.8 \%, Mean = 103.9722
2.2
     aov()
fit_aov_SP <- aov(Yield ~ Variety*Nitrogen + Error(Block/Variety), data=oats_df)</pre>
summary(fit_aov_SP)
##
## Error: Block
            Df Sum Sq Mean Sq F value Pr(>F)
## Residuals 5 15875
                         3175
##
## Error: Block: Variety
            Df Sum Sq Mean Sq F value Pr(>F)
## Variety
                 1786
                        893.2
             2
                                1.485 0.272
## Residuals 10
                 6013
                        601.3
##
## Error: Within
##
                   Df Sum Sq Mean Sq F value
                                               Pr(>F)
## Nitrogen
                    3 20020
                                6673 37.686 2.46e-12 ***
## Variety:Nitrogen 6
                         322
                                  54
                                       0.303
                                                0.932
## Residuals
                   45
                        7969
                                 177
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
2.3 mixed-effects model
fit_lmer_SP <- lmer(Yield ~ Variety*Nitrogen + (1|Block) + (1|Block:Variety), data=oats_df)</pre>
anova(fit_lmer_SP, type=3)
## Type III Analysis of Variance Table with Satterthwaite's method
                    Sum Sq Mean Sq NumDF DenDF F value
##
                                                          Pr(>F)
## Variety
                     526.1
                             263.0
                                       2
                                            10 1.4853
                                                          0.2724
## Nitrogen
                   20020.5 6673.5
                                       3
                                            45 37.6857 2.458e-12 ***
```

```
6 45 0.3028
## Variety:Nitrogen
                   321.8
                          53.6
                                                   0.9322
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

2.4 pairwise comparisons

```
emmeans(fit_lmer_SP, pairwise~"Nitrogen")
## NOTE: Results may be misleading due to involvement in interactions
## $emmeans
## Nitrogen emmean
                     SE
                         df lower.CL upper.CL
## 0.0cwt
            79.4 7.17 6.79
                                62.3
                                         96.5
## 0.2cwt
              98.9 7.17 6.79
                                81.8
                                        116.0
             114.2 7.17 6.79
## 0.4cwt
                                97.2
                                        131.3
## 0.6cwt
             123.4 7.17 6.79
                               106.3
                                        140.5
##
## Results are averaged over the levels of: Variety
## Degrees-of-freedom method: kenward-roger
## Confidence level used: 0.95
##
## $contrasts
## contrast
                 estimate
                             SE df t.ratio p.value
## 0.0cwt - 0.2cwt -19.50 4.44 45 -4.396 0.0004
## 0.0cwt - 0.4cwt -34.83 4.44 45 -7.853 <.0001
## 0.0cwt - 0.6cwt -44.00 4.44 45 -9.919 <.0001
## 0.2cwt - 0.4cwt
                    -15.33 4.44 45 -3.457 0.0064
## 0.2cwt - 0.6cwt -24.50 4.44 45 -5.523 <.0001
## 0.4cwt - 0.6cwt -9.17 4.44 45 -2.067 0.1797
##
## Results are averaged over the levels of: Variety
## Degrees-of-freedom method: kenward-roger
## P value adjustment: tukey method for comparing a family of 4 estimates
```

3 Split-split splot

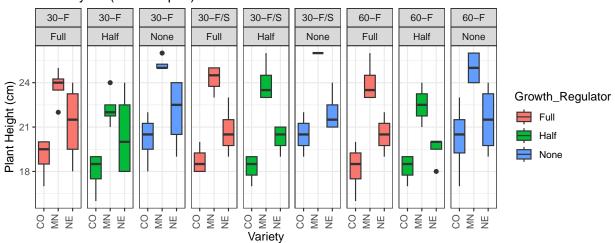
Prairie junegrass is a native species that has potential as a low input turf grass. Seed production strategies need to be investigated to determine best management practices. An experiment was conducted at the Sand Plain Research Center near Becker, MN to investigate varieties, fertility, and the use of growth regulators. Growth regulators in grasses tend to reduce plant height by influencing stem elongation. The experiment was a split split plot treatment design in a randomized complete block with 4 replications (blocks).

```
• Whole plots: Fertility
       -30-F = 30 lbs N applied in Fall
       -30-F/S = 30 lbs N applied in Fall + 30 lbs N applied in Spring
       -60-F = 60 lbs N applied in Fall
  • Subplots: Growth regulator applications
       - None = no Apogee growth regulator
       - Full = full rate (8 oz.)
       - Half = half rate (4 oz.)
  • Sub-Subplots: 3 prairie junegrass populations

    CO = Koeleria Colorada

       - NE = Koeleria Nebraska
       - MN = Koeleria Weaver Dunes, MN
june_df <- read.table('EPP_junegrass.txt', header=T, sep="\t",</pre>
                        colClasses = c("factor", "factor", "factor", "factor",
                                        "numeric", "numeric", "numeric"))
summary(june_df)
    Variety Growth_Regulator Fertility
                                                    Seed_Yield_samples
                                            Rep
##
    CO:36
             Full:36
                               30-F :36
                                             1:27
                                                    Min.
                                                            : 2.00
    MN:36
             Half:36
                                             2:27
##
                               30-F/S:36
                                                    1st Qu.:11.00
##
    NE:36
             None:36
                               60-F :36
                                             3:27
                                                    Median :15.00
                                                            :15.81
##
                                             4:27
                                                    Mean
##
                                                    3rd Qu.:20.25
##
                                                    Max.
                                                            :40.00
##
    Seed Yield lbs per acre Plant Height
##
    Min.
           : 18.0
                              Min.
                                      :16.00
    1st Qu.: 98.0
                              1st Qu.:19.00
##
    Median :134.0
                              Median :21.00
            :140.7
                                      :21.36
##
    Mean
                              Mean
##
    3rd Qu.:180.2
                              3rd Qu.:24.00
                                      :26.00
##
   Max.
            :356.0
                              Max.
```

Split-split plot Fertility (whole plot) Growth regulator (subplot) Variety (sub-subplot)



3.1 Agricolae package

```
##
## ANALYSIS SPLIT-SPLIT PLOT: june_df$Plant_Height
## Class level information
##
## june_df$Fertility
                      : 30-F 30-F/S 60-F
## june_df$Growth_Regulator
                                : None Full Half
## june_df$Variety : CO NE MN
## june_df$Rep : 1 2 3 4
##
## Number of observations: 108
##
## Analysis of Variance Table
##
## Response: june_df$Plant_Height
                                                               Df Sum Sq Mean Sq
##
## june df$Rep
                                                                3 109.88 36.627
                                                                           2.778
## june_df$Fertility
                                                                    5.56
## Ea
                                                                6
                                                                  13.26
                                                                           2.210
## june_df$Growth_Regulator
                                                                2 84.39 42.194
## june_df$Fertility:june_df$Growth_Regulator
                                                                    1.39
                                                                           0.347
## Eb
                                                               18 18.44
                                                                           1.025
## june_df$Variety
                                                                2 470.72 235.361
## june_df$Variety:june_df$Fertility
                                                                    5.72
                                                                           1.431
## june_df$Variety:june_df$Growth_Regulator
                                                                    2.72
                                                                           0.681
## june_df$Variety:june_df$Fertility:june_df$Growth_Regulator
                                                                    2.67
                                                                8
                                                                           0.333
## Ec
                                                               54
                                                                  64.17
                                                                           1.188
##
                                                                F value
                                                                           Pr(>F)
                                                                30.8234 9.508e-12
## june_df$Rep
```

```
## june_df$Fertility
                                                               1.2570
                                                                         0.3500
## Ea
## june df$Growth Regulator
                                                              41.1777 1.921e-07
## june_df$Fertility:june_df$Growth_Regulator
                                                               0.3389
                                                                         0.8482
## june df$Variety
                                                              198.0701 < 2.2e-16
## june_df$Variety:june_df$Fertility
                                                                         0.3198
                                                               1.2039
## june_df$Variety:june_df$Growth_Regulator
                                                               0.5727
                                                                         0.6836
## june_df$Variety:june_df$Fertility:june_df$Growth_Regulator
                                                               0.2805
                                                                         0.9696
## Ec
##
## june_df$Rep
                                                              ***
## june_df$Fertility
## Ea
## june_df$Growth_Regulator
                                                              ***
## june_df$Fertility:june_df$Growth_Regulator
## Eb
## june df$Variety
## june_df$Variety:june_df$Fertility
## june_df$Variety:june_df$Growth_Regulator
## june_df$Variety:june_df$Fertility:june_df$Growth_Regulator
## Ec
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## cv(a) = 7 \%, cv(b) = 4.7 \%, cv(c) = 5.1 \%, Mean = 21.36111
3.2 aov()
fit_aov_SSP <-aov(Plant_Height</pre>
                               ~ Rep + Fertility*Growth_Regulator*Variety +
                   Error(Rep/Fertility/Growth_Regulator),data=june_df)
summary(fit_aov_SSP)
##
## Error: Rep
      Df Sum Sq Mean Sq
## Rep 3 109.9
                 36.63
##
## Error: Rep:Fertility
           Df Sum Sq Mean Sq F value Pr(>F)
## Fertility 2 5.556 2.778
                                1.257 0.35
## Residuals 6 13.259
## Error: Rep:Fertility:Growth_Regulator
                             Df Sum Sq Mean Sq F value
                                                         Pr(>F)
## Growth_Regulator
                              2 84.39
                                         42.19 41.178 1.92e-07 ***
                                                 0.339
## Fertility:Growth_Regulator 4
                                 1.39
                                          0.35
                                                          0.848
                             18 18.44
                                          1.02
## Residuals
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Error: Within
##
                                     Df Sum Sq Mean Sq F value Pr(>F)
                                      2 470.7 235.36 198.070 <2e-16 ***
## Variety
```

```
## Fertility: Variety
                                           5.7
                                                  1.43
                                                         1.204 0.320
## Growth_Regulator:Variety
                                      4
                                                  0.68
                                                         0.573 0.684
                                           2.7
                                                  0.33
                                                         0.281 0.970
## Fertility:Growth_Regulator:Variety
                                     8
                                           2.7
## Residuals
                                     54
                                          64.2
                                                  1.19
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
    mixed-effects model
fit lmer SSP <- lmer(Plant Height ~ Fertility*Growth Regulator*Variety +
                   (1|Rep) + (1:Rep:Fertility) + (1:Rep:Fertility:Growth_Regulator),
                  data=june df)
anova(fit_lmer_SSP, type=3)
## Type III Analysis of Variance Table with Satterthwaite's method
                                     Sum Sq Mean Sq NumDF DenDF F value
##
## Fertility
                                       5.56
                                              2.778
                                                        2
                                                             78
                                                                  2,2600
                                                             78 34.3293
## Growth_Regulator
                                      84.39 42.194
                                                        2
## Variety
                                     470.72 235.361
                                                        2
                                                             78 191.4895
## Fertility:Growth_Regulator
                                                        4
                                                             78 0.2825
                                       1.39
                                              0.347
## Fertility: Variety
                                       5.72
                                              1.431
                                                             78 1.1639
                                                        4
                                                             78 0.5537
## Growth Regulator: Variety
                                       2.72
                                              0.681
## Fertility:Growth Regulator:Variety
                                              0.333
                                                             78 0.2712
                                       2.67
##
                                        Pr(>F)
## Fertility
                                        0.1111
## Growth_Regulator
                                     2.022e-11 ***
## Variety
                                     < 2.2e-16 ***
## Fertility:Growth Regulator
                                        0.8885
## Fertility:Variety
                                        0.3333
## Growth_Regulator:Variety
                                        0.6969
## Fertility:Growth_Regulator:Variety
                                        0.9735
## ---
```

3.4 pairwise comparisons

emmeans(fit_lmer_SSP, pairwise~Variety)

```
## NOTE: Results may be misleading due to involvement in interactions
## $emmeans
## Variety emmean
                          df lower.CL upper.CL
                     SE
             19.1 0.602 3.42
                                 17.3
                                          25.9
## MN
             24.1 0.602 3.42
                                 22 3
             20.9 0.602 3.42
                                 19.1
                                          22.7
##
## Results are averaged over the levels of: Fertility, Growth_Regulator, Rep
## Degrees-of-freedom method: kenward-roger
## Confidence level used: 0.95
##
## $contrasts
## contrast estimate
                        SE df t.ratio p.value
## CO - MN
            -5.06 0.261 78 -19.347 <.0001
## CO - NE
               -1.86 0.261 78 -7.122 <.0001
```

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

```
## MN - NE 3.19 0.261 78 12.225 <.0001
##
## Results are averaged over the levels of: Fertility, Growth_Regulator, Rep
## Degrees-of-freedom method: kenward-roger
## P value adjustment: tukey method for comparing a family of 3 estimates</pre>
```

4 R Activity

You will need the following packages to complete this problem set:

```
library(tidyverse)
library(car)
library(lme4)
library(lmerTest)
library(emmeans)
```

An experiment was designed to assess the effect of nitrogen fertilizer on yield (grams) from three varieties of wheat. The experimental design was a split plot with replicates (=blocks). Five rates of nitrogen fertilizer were applied to whole plots at rates of 0, 40, 80, 120 and 160 kg/ha, and the three varieties were planted in sub plots. The data are in the "EPP_yield.txt" data file. Even though split plots are often designed to evaluate interactions, please ignore interactions for this activity.

1. Load in the data. Note that column names starting with Rep represent a block.

```
##
   Nitrogen Variety
                           Rep1
                                           Rep2
                                                            Rep3
##
   0 :3
             1:5
                      Min.
                              :3.60
                                              :4.30
                                                              :4.200
                                      Min.
                                                      Min.
   120:3
##
             2:5
                      1st Qu.:5.35
                                      1st Qu.:5.30
                                                      1st Qu.:5.300
##
   160:3
             3:5
                      Median:6.40
                                      Median:6.50
                                                      Median :5.900
##
   40 :3
                              :6.58
                                              :6.58
                                                              :6.313
                      Mean
                                      Mean
                                                      Mean
##
    80:3
                      3rd Qu.:8.00
                                      3rd Qu.:7.75
                                                      3rd Qu.:7.300
##
                      Max.
                              :9.20
                                      Max.
                                              :9.70
                                                      Max.
                                                              :9.300
```

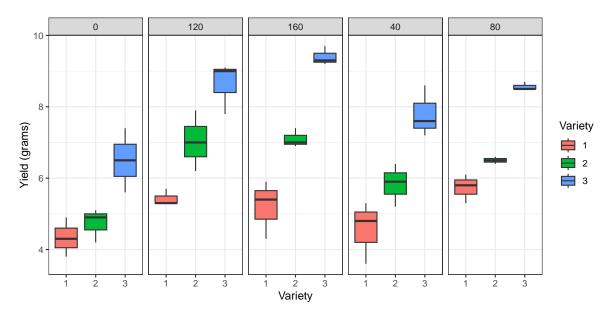
2. You might notice the data do not have one observation per row (which R expects when fitting linear models). Convert the data from "wide" to "long" format using R. Check out last week's R activity for some example code.

```
yield_df_long <- yield_df %>%
  pivot_longer(
  cols = starts_with("Rep"),
  names_to = "Rep",
  values_to = "Yield")
summary(yield_df_long)
```

```
Rep
   Nitrogen Variety
                                              Yield
##
   0 :9
             1:15
                      Length:45
                                                 :3.600
                                          Min.
   120:9
             2:15
                      Class : character
                                          1st Qu.:5.300
##
   160:9
                                          Median :6.400
             3:15
                      Mode :character
   40:9
##
                                          Mean
                                                 :6.491
##
    80:9
                                          3rd Qu.:7.600
##
                                          Max.
                                                 :9.700
```

3. Graph the data using a boxplot. In the plot, group the data by the whole plot factor (one panel for each level of Nitrogen) and display the sub plot factor on the x-axis.

```
ggplot(yield_df_long, aes(x=Variety, y=Yield, fill=Variety)) +
  geom_boxplot()+ theme_bw()+
  xlab("Variety")+
  ylab("Yield (grams)")+
  facet_wrap(~Nitrogen, ncol=5)
```



4. Conduct an analysis of variance (ANOVA) using the aov() command (i.e., assess if Nitrogen and Variety explain variation in yield). Treat Nitrogen as a factor for all analyses.

```
aov_yield_1 <- aov(Yield ~ Rep + Nitrogen + Variety + Error(Rep/Nitrogen),
data=yield_df_long)
summary(aov_yield_1)</pre>
```

```
##
## Error: Rep
      Df Sum Sq Mean Sq
## Rep 2 0.7111 0.3556
##
## Error: Rep:Nitrogen
##
            Df Sum Sq Mean Sq F value
                                       Pr(>F)
                        6.562
                                23.25 0.000184 ***
## Nitrogen
             4 26.248
## Residuals 8 2.258
                        0.282
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Error: Within
            Df Sum Sq Mean Sq F value
##
                                       Pr(>F)
                                90.66 5.87e-13 ***
## Variety
             2 74.74
                        37.37
## Residuals 28
               11.54
                         0.41
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

5. Conduct the same ANOVA as you did in the previous step but use the lmer() command from the lme4 package and the anova() command from the lmerTest package. Note: if you get an error that says boundary (singular) fit: see ?isSingular, ignore it.

```
lmer_yield_1 <- lmer(Yield ~ Nitrogen + Variety + (1|Rep) + (1|Rep:Nitrogen),
    data=yield_df_long)</pre>
```

```
## boundary (singular) fit: see help('isSingular')
anova(lmer_yield_1, type=3)
```

```
## Type III Analysis of Variance Table with Satterthwaite's method
              Sum Sq Mean Sq NumDF DenDF F value
                                     38 17.185 3.976e-08 ***
  ## Nitrogen 26.248
                      6.562
                                4
  ## Variety 74.739 37.370
                                     38 97.868 1.024e-15 ***
  ## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
6. Pick one of the variables to conduct pairwise comparisons of treatment levels. Explain the reasoning for
  your choice and conduct the comparisons using the lmer() model.
  emmeans(lmer_yield_1, pairwise~Nitrogen)
  ## $emmeans
  ## Nitrogen emmean
                        SE df lower.CL upper.CL
  ## 0
                5.19 0.206 10
                                  4.73
                                           5.65
  ## 120
                7.03 0.206 10
                                  6.57
                                           7.49
  ## 160
                7.23 0.206 10
                                  6.77
                                          7.69
  ## 40
                6.07 0.206 10
                                  5.61
                                         6.53
  ## 80
                 6.93 0.206 10
                                           7.39
                                  6.47
  ##
  ## Results are averaged over the levels of: Variety
  ## Degrees-of-freedom method: kenward-roger
  ## Confidence level used: 0.95
  ##
  ## $contrasts
  ## contrast
                               estimate
                                           SE df t.ratio p.value
  ## Nitrogen0 - Nitrogen120
                               -1.844 0.291 8 -6.332 0.0015
  ## Nitrogen0 - Nitrogen160
                                 -2.044 0.291 8 -7.018 0.0008
  ## Nitrogen0 - Nitrogen40
                                 -0.878 0.291 8 -3.013 0.0914
  ## Nitrogen0 - Nitrogen80
                                 -1.744 0.291 8 -5.989 0.0022
  ## Nitrogen120 - Nitrogen160
                                -0.200 0.291 8 -0.687 0.9539
  ## Nitrogen120 - Nitrogen40
                                 0.967 0.291 8
                                                   3.319 0.0602
  ## Nitrogen120 - Nitrogen80
                                  0.100 0.291 8
                                                   0.343 0.9964
  ## Nitrogen160 - Nitrogen40
                                 1.167 0.291 8
                                                   4.005 0.0239
  ## Nitrogen160 - Nitrogen80
                                 0.300 0.291 8
                                                   1.030 0.8352
  ## Nitrogen40 - Nitrogen80
                                 -0.867 0.291 8 -2.975 0.0963
  ##
  ## Results are averaged over the levels of: Variety
  ## Degrees-of-freedom method: kenward-roger
  ## P value adjustment: tukey method for comparing a family of 5 estimates
  emmeans(lmer_yield_1, pairwise~Variety)
  ## $emmeans
  ## Variety emmean SE df lower.CL upper.CL
  ## 1
               5.05 0.16 14
                                         5.40
                                4.71
  ## 2
               6.24 0.16 14
                                5.90
                                         6.58
  ## 3
               8.18 0.16 14
                                7.84
                                         8.52
  ## Results are averaged over the levels of: Nitrogen
  ## Degrees-of-freedom method: kenward-roger
  ## Confidence level used: 0.95
  ##
  ## $contrasts
  ## contrast
                          estimate
                                     SE df t.ratio p.value
```

Variety1 - Variety2 -1.19 0.226 28 -5.259 <.0001

```
## Variety1 - Variety3 -3.13 0.226 28 -13.857 <.0001
## Variety2 - Variety3 -1.94 0.226 28 -8.598 <.0001
##
## Results are averaged over the levels of: Nitrogen
## Degrees-of-freedom method: kenward-roger
## P value adjustment: tukey method for comparing a family of 3 estimates</pre>
```

7. Write 4-5 sentences comparing the conclusions from the two approaches (i.e., using aov() vs. lmer()) including conclusions drawn from any pairwise comparisons you conducted. At least 1-2 of your sentences should include a conclusion written in "biologically meaningful" terms.

Answer: The analyses using aov() and lmer() were equivalent in terms of sums of squares and mean squares, but the F-values differed (FYI: it's due to the differences in residual degrees of freedom). Yield varied across nitrogen levels ($F_{4,38} = 17.19, p < 0.0001$) and between varieties ($F_{2,38} = 97.87, p < 0.0001$). Pairwise comparisons indicated that variety 3 produced a higher yield than variety 1 (Tukey's range test (TRT): $t_{28} = 13.86, p < 0.0001$) or variety 2 (TRT: $t_{28} = 8.60, p < 0.0001$). Even though nitrogen applications of 160 kg/ha were associated with the highest yield, yield at that rate did not differ from yield at applications 80 kg/ha (TRT: $t_8 = 1.03, p = 0.84$); thus, to maximize yield, plant variety 3 should be planted and returns on yield from nitrogen applications beyond 80-100 kg/ha might be considered negligible.