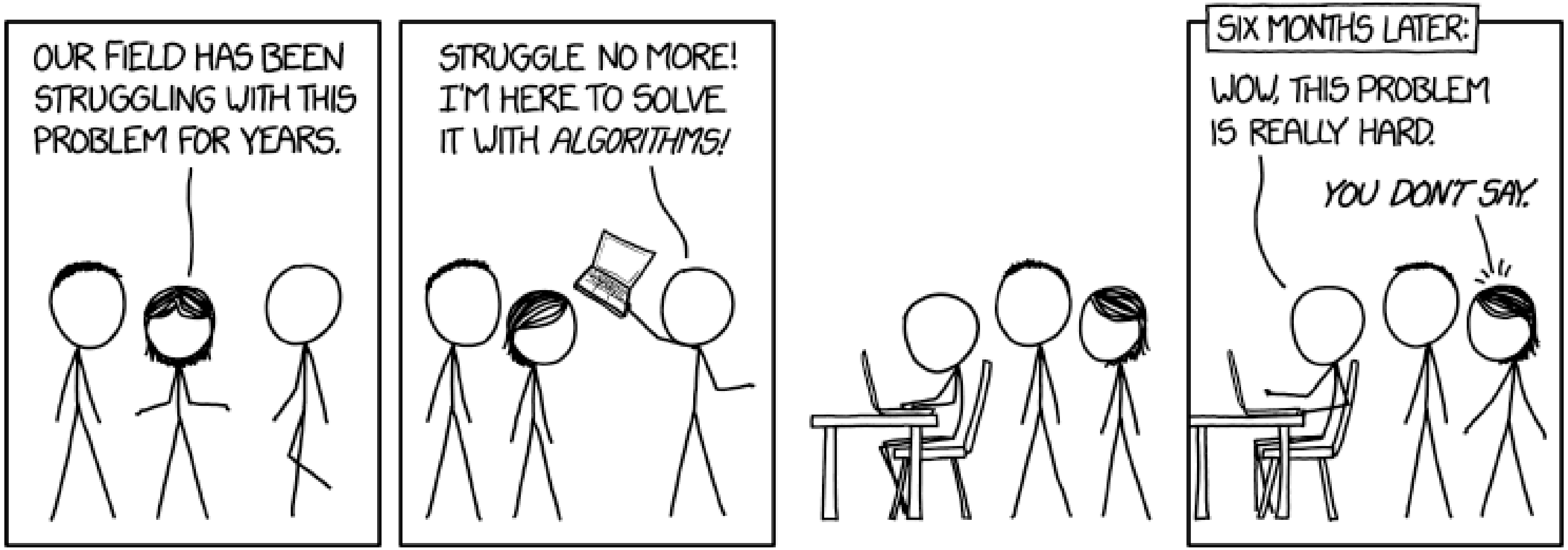
Multiple Linear Regression

ENTMLGY 6707 Entomological Techniques and Data Analysis



Learning objectives

1) Compare and contrast simple vs multiple linear regression

2) Become familiar with additional assumptions when using multiple linear regression and how to check for them / deal with them.

3) Interpret the outcome of a multiple linear regression

Simple linear regression

Table 4.1 Examples of the Generalized Linear Model as a Function of Independent Variable and Dependant Variable Type

<table>
<tr><th></th><th></th><th colspan="5">Responses</th></tr>
<tr><th></th><th></th><th>Continuous DV</th><th>Binary DV</th><th>Unordered Multicategory DV</th><th>Ordered Categorical DV</th><th>Count DV</th></tr>
<tr><td rowspan="3">Predictors</td><td>Continuous IV</td><td rowspan="2">OLS regression</td><td rowspan="2">Binary logistic regression</td><td rowspan="2">Multinomial logistic regression</td><td rowspan="3">Ordinal logistic regression</td><td rowspan="2">OLS, Poisson regression</td></tr>
<tr><td>Mixed continuous and categorical IV</td></tr>
<tr><td>Binary/ categorical IV only</td><td>ANOVA and t-test</td><td>Log-linear models</td><td>Log-linear models</td><td>Log-linear models</td></tr>
</table>

ANOVA, analysis of variance; DV, dependent variable; IV, independent variable; OLS, ordinary least squares.

Chapter 4: Simple Linear Models With Continuous Dependent Variables: Simple ANOVA Analyses
In: [Regression & Linear Modeling: Best Practices and Modern Methods]

Multiple linear regression

Table 4.1 Examples of the Generalized Linear Model as a Function of Independent Variable and Dependant Variable Type

Responses

Predictors

	Continuous DV	*Binary DV*	*Unordered Multicategory DV*	*Ordered Categorical DV*	*Count DV*
Continuous IV	OLS regression	Binary logistic regression	Multinomial logistic regression	Ordinal logistic regression	OLS, Poisson regression
Mixed continuous and categorical IV					
Binary/ categorical IV only	ANOVA and *t*-test	Log-linear models	Log-linear models		Log-linear models

ANOVA, analysis of variance; DV, dependent variable; IV, independent variable; OLS, ordinary least squares.

Chapter 4: Simple Linear Models With Continuous Dependent Variables: Simple ANOVA Analyses
In: Regression & Linear Modeling: Best Practices and Modern Methods

Linear regression

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

Response variable

***y*-intercept** (or the expected mean of *y* when *x*=0)

Slope coefficient (rise over run, or the expected change in *y* with a 1 unit change in *x*)

Predictor variable

Residuals

Comparing simple vs. polynomial linear regression

Simple linear regression

$$Y_i = \beta_0 + \beta_1 X_1 + \varepsilon_i$$

$$Height \sim DBH$$

Polynomial regression

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \varepsilon_i$$

$$Height \sim DBH + DBH^2$$

Comparing polynomial vs. multiple linear regression

Polynomial regression

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \varepsilon_i$$

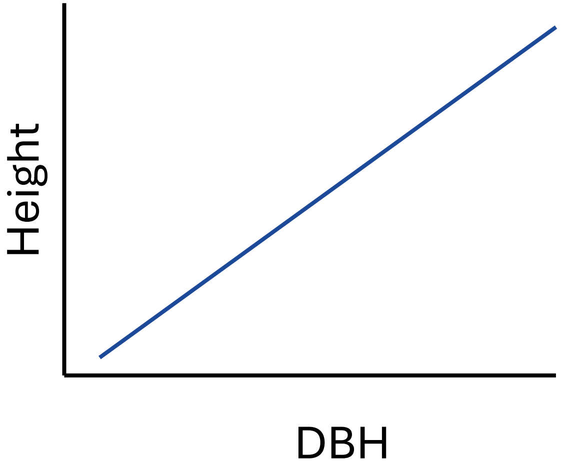
$$Height \sim DBH + DBH^2$$

Multiple linear regression

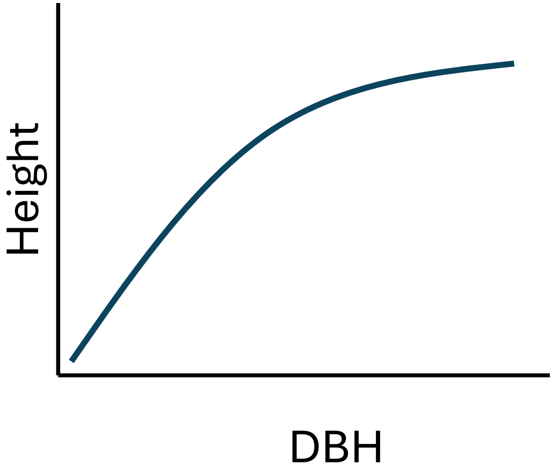
$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_i$$

$$Height \sim DBH + Nitrogen$$

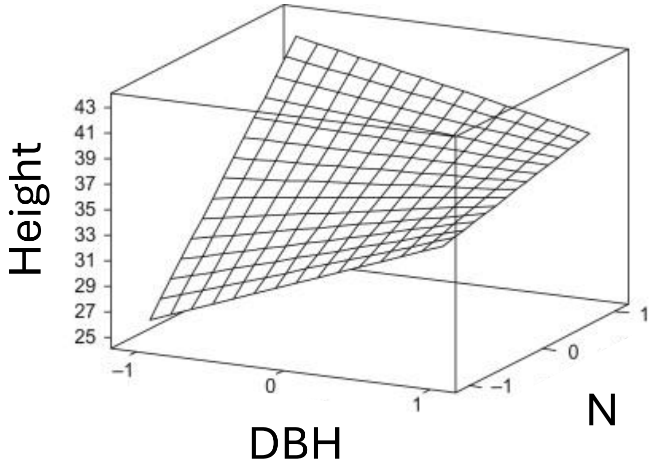
Single vs. polynomial vs. multiple regression models



$Height \sim DBH$



$Height \sim DBH + DBH^2$

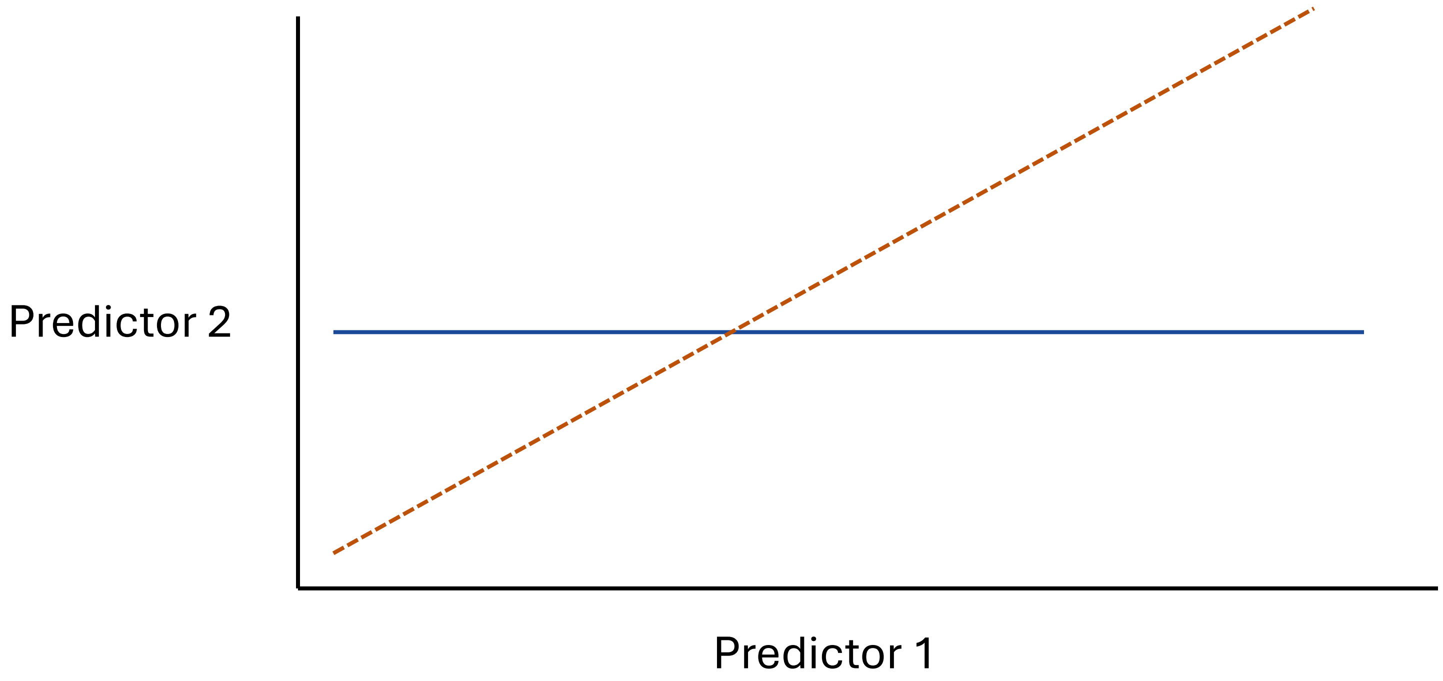


$Height \sim DBH + Nitrogen$

Multiple regression models: additional assumption

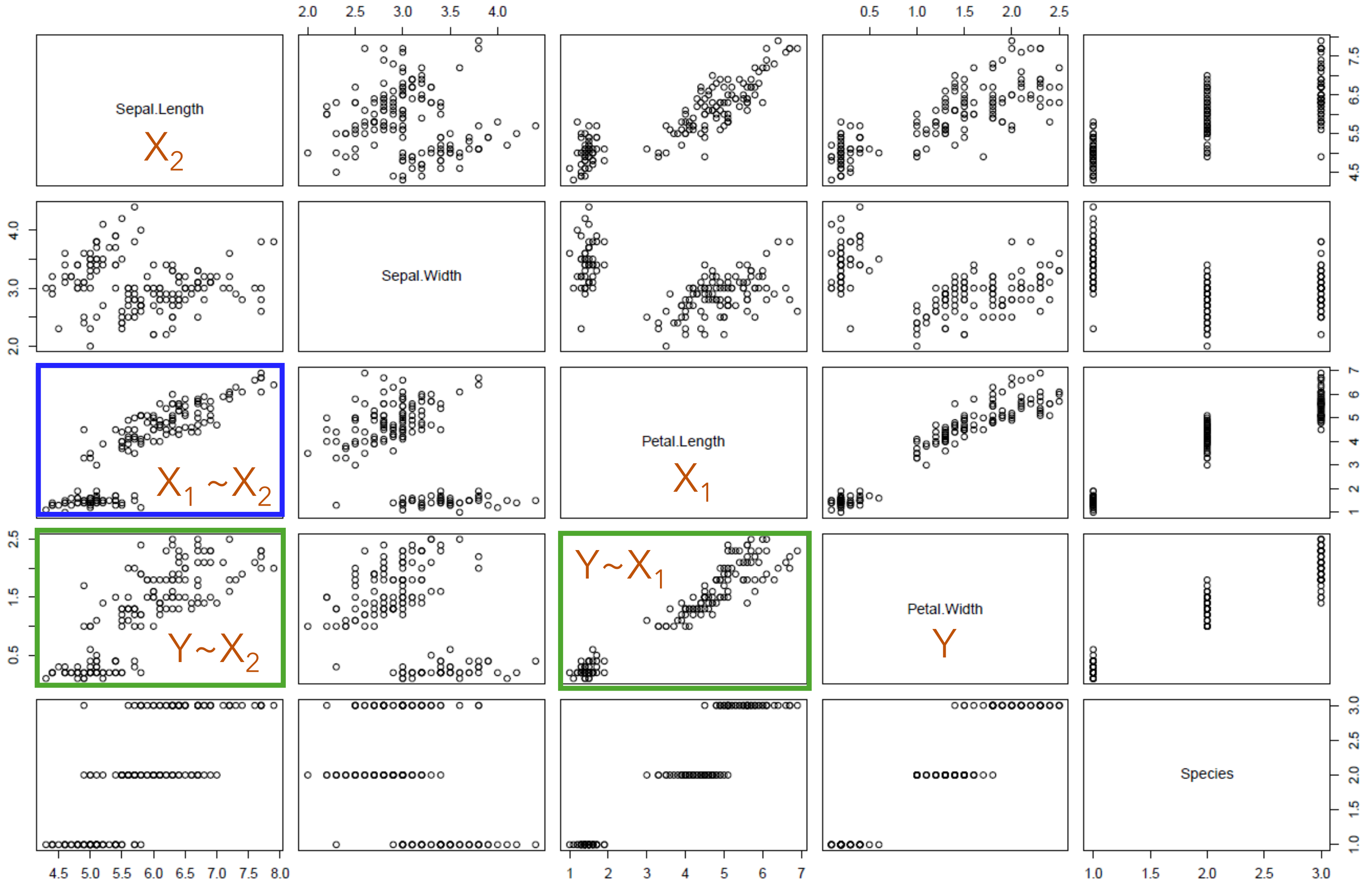
A correlation – negative or positive – between predictors is called **collinearity** which can cause problems in model fitting.

A common sign of collinearity is “large” changes in slope coefficients, including sign flipping (e.g., slope coefficient goes from negative to positive), depending on which predictors are fit in a model.



$$Petal.Width \sim Petal.Length + Sepal.Length$$

```
plot(iris)
```



```
fitA <- lm(Petal.Width ~ Petal.Length, data = iris)
summary(fitA)

Call:
lm(formula = Petal.Width ~ Petal.Length, data = iris)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.56515 -0.12358 -0.01898  0.13288  0.64272 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -0.363076   0.039762  -9.131  4.7e-16 ***
Petal.Length  0.415755   0.009582  43.387  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2065 on 148 degrees of freedom
Multiple R-squared:  0.9271,	Adjusted R-squared:  0.9266 
F-statistic:  1882 on 1 and 148 DF,  p-value: < 2.2e-16
```

```r
fitB <- lm(Petal.Width ~ Sepal.Length, data = iris)
summary(fitB)
```

```
Call:
lm(formula = Petal.Width ~ Sepal.Length, data = iris)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.96671 -0.35936 -0.01787  0.28388  1.23329 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -3.20022    0.25689  -12.46   <2e-16 ***
Sepal.Length  0.75292    0.04353   17.30   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.44 on 148 degrees of freedom
Multiple R-squared:  0.669,	Adjusted R-squared:  0.6668 
F-statistic: 299.2 on 1 and 148 DF,  p-value: < 2.2e-16
```

```
fitC <- lm(Petal.Width ~ Petal.Length + Sepal.Length, data = iris)
summary(fitC)

Call:
lm(formula = Petal.Width ~ Petal.Length + Sepal.Length, data = iris)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.60598 -0.12560 -0.02049  0.11616  0.59404 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -0.008996   0.182097  -0.049   0.9607    
Petal.Length  0.449376   0.019365  23.205   <2e-16 ***
Sepal.Length -0.082218   0.041283  -1.992   0.0483 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2044 on 147 degrees of freedom
Multiple R-squared:  0.929,	Adjusted R-squared:  0.9281 
F-statistic: 962.1 on 2 and 147 DF,  p-value: < 2.2e-16
```

```
fitA <- lm(Petal.Width ~ Petal.Length, data = iris)
```

Coefficients: fitA

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.363076   0.039762  -9.131  4.7e-16 ***
Petal.Length  0.415755   0.009582  43.387  < 2e-16 ***
```

```
fitB <- lm(Petal.Width ~ Sepal.Length, data = iris)
```

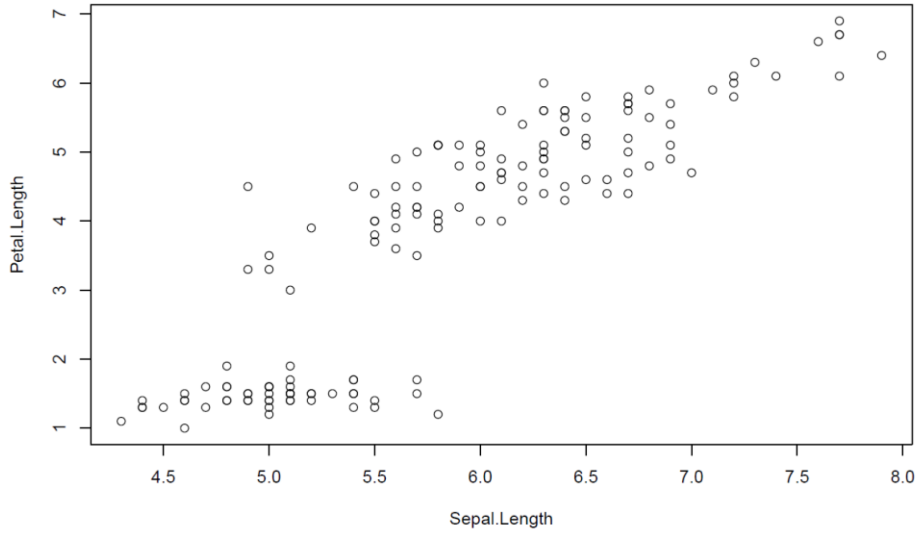
Coefficients: fitB

```
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -3.20022    0.25689  -12.46   <2e-16 ***
Sepal.Length  0.75292    0.04353   17.30   <2e-16 ***
```

```
fitC <- lm(Petal.Width ~ Petal.Length + Sepal.Length, data = iris)
```

Coefficients: fitC

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.008996   0.182097  -0.049   0.9607
Petal.Length  0.449376   0.019365  23.205   <2e-16 ***
Sepal.Length -0.082218   0.041283  -1.992   0.0483 *
```



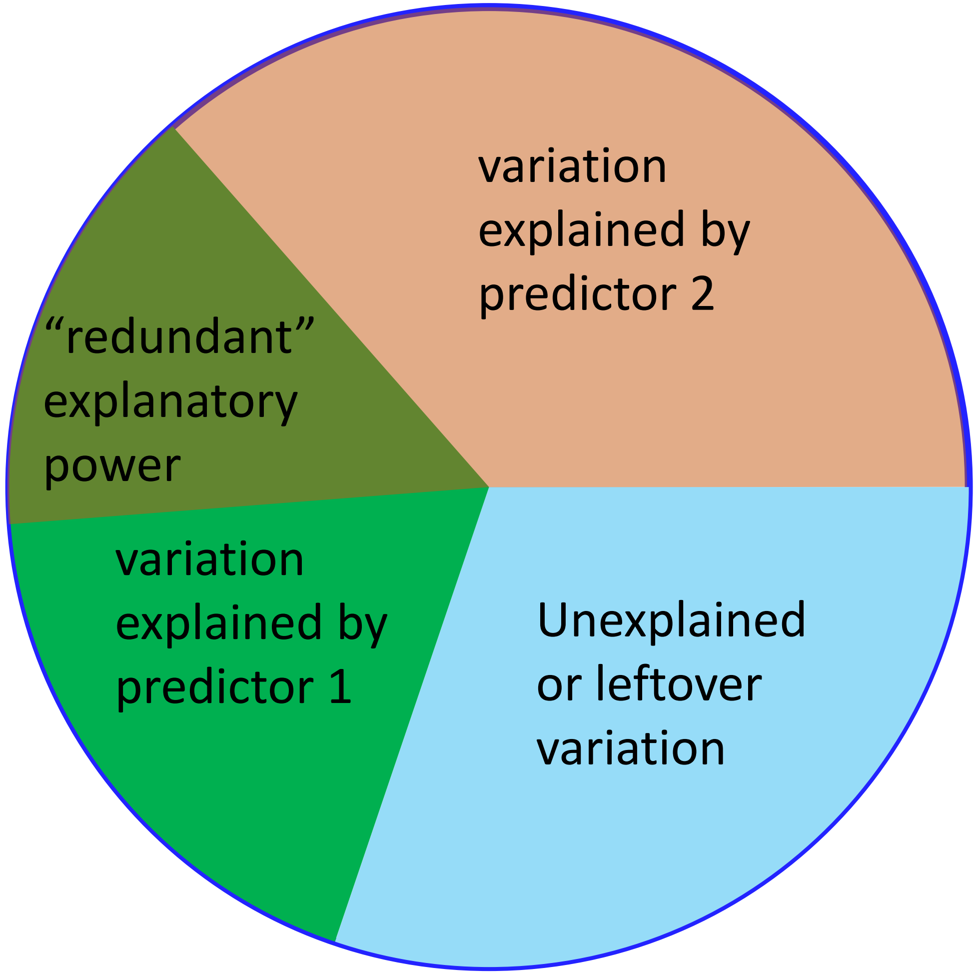
Interpreting simple vs multiple regression models

In **SLR**, we interpret the slope coefficient as follows: "a one unit increase in X_1 was associated with a β_1 unit increase in Y ."

In **MLR**, we interpret coefficients as follows: "<u>holding all else equal</u>, a one unit increase in X_1 was associated with a β_1 unit increase in Y ."

- That is part of the reason collinearity causes problems. If predictors X_1 and X_2 are highly correlated, it is difficult to "hold X_2 equal or constant" while estimating the effect of X_1.

Sequential vs. marginal fits in ANOVA



Sequential fitting

```r
grouseticks$f_YEAR <- as.factor(grouseticks$YEAR)
fit_ex1 <- lm(TICKS ~ f_YEAR + HEIGHT, data=grouseticks)
anova(fit_ex1)
```

```
## Analysis of Variance Table
##
## Response: TICKS
##            Df Sum Sq Mean Sq F value    Pr(>F)
## f_YEAR      2   7050  3524.9  24.995 5.928e-11 ***
## HEIGHT      1   6092  6092.0  43.199 1.550e-10 ***
## Residuals 399  56268   141.0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
fit_ex2 <- lm(TICKS ~ HEIGHT + f_YEAR, data=grouseticks)
anova(fit_ex2)
```

```
## Analysis of Variance Table
##
## Response: TICKS
##            Df Sum Sq Mean Sq F value    Pr(>F)
## HEIGHT      1   7692  7692.2  54.546 8.948e-13 ***
## f_YEAR      2   5450  2724.8  19.321 9.788e-09 ***
## Residuals 399  56268   141.0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Marginal fitting

```
library(car)
```

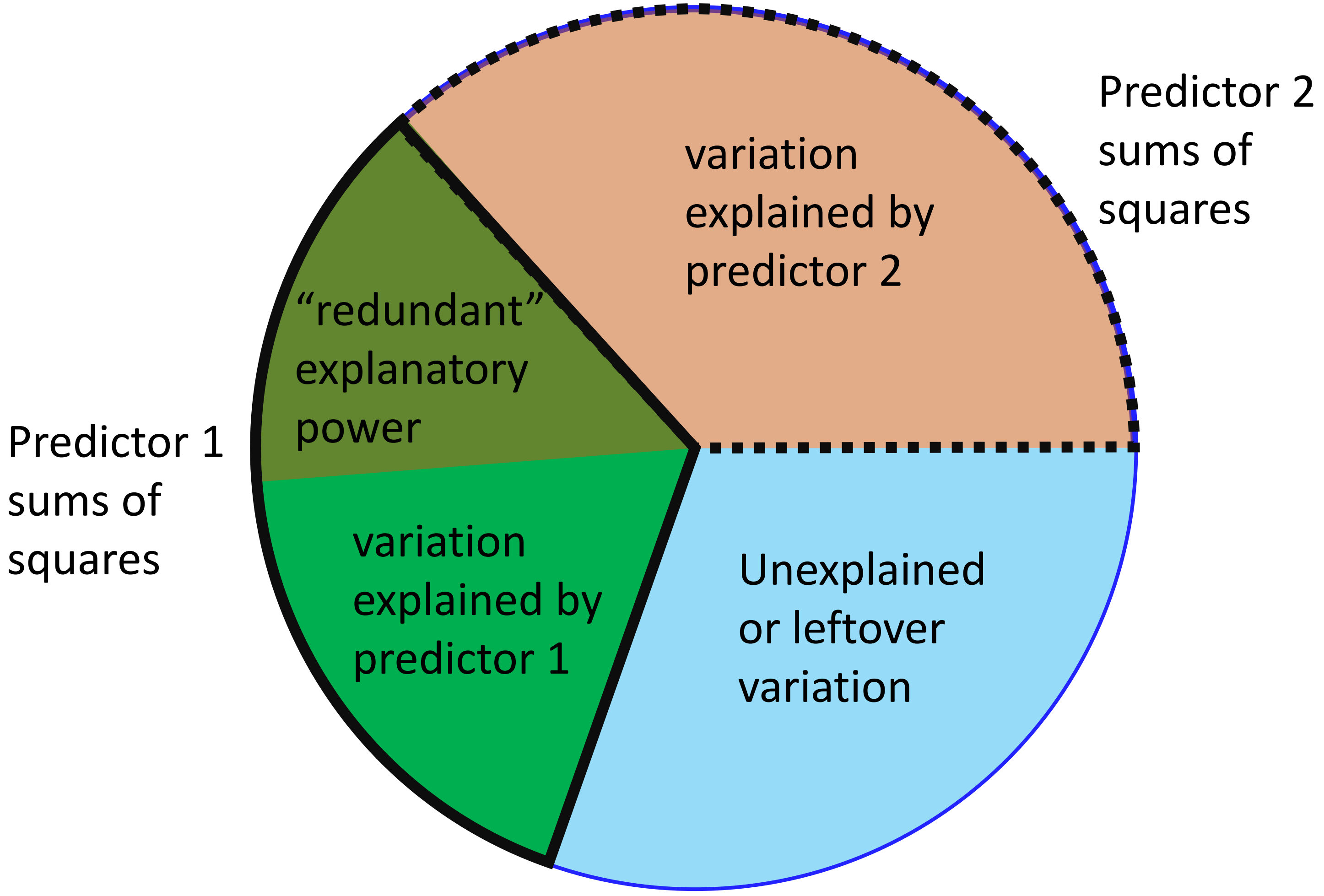
```
Anova(fit_ex1, type="III")
```

```
## Anova Table (Type III tests)
##
## Response: TICKS
##             Sum Sq  Df F value    Pr(>F)
## (Intercept)   7444   1  52.786 1.970e-12 ***
## f_YEAR        5450   2  19.321 9.788e-09 ***
## HEIGHT        6092   1  43.199 1.550e-10 ***
## Residuals    56268 399
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Anova(fit_ex2, type="III")
```

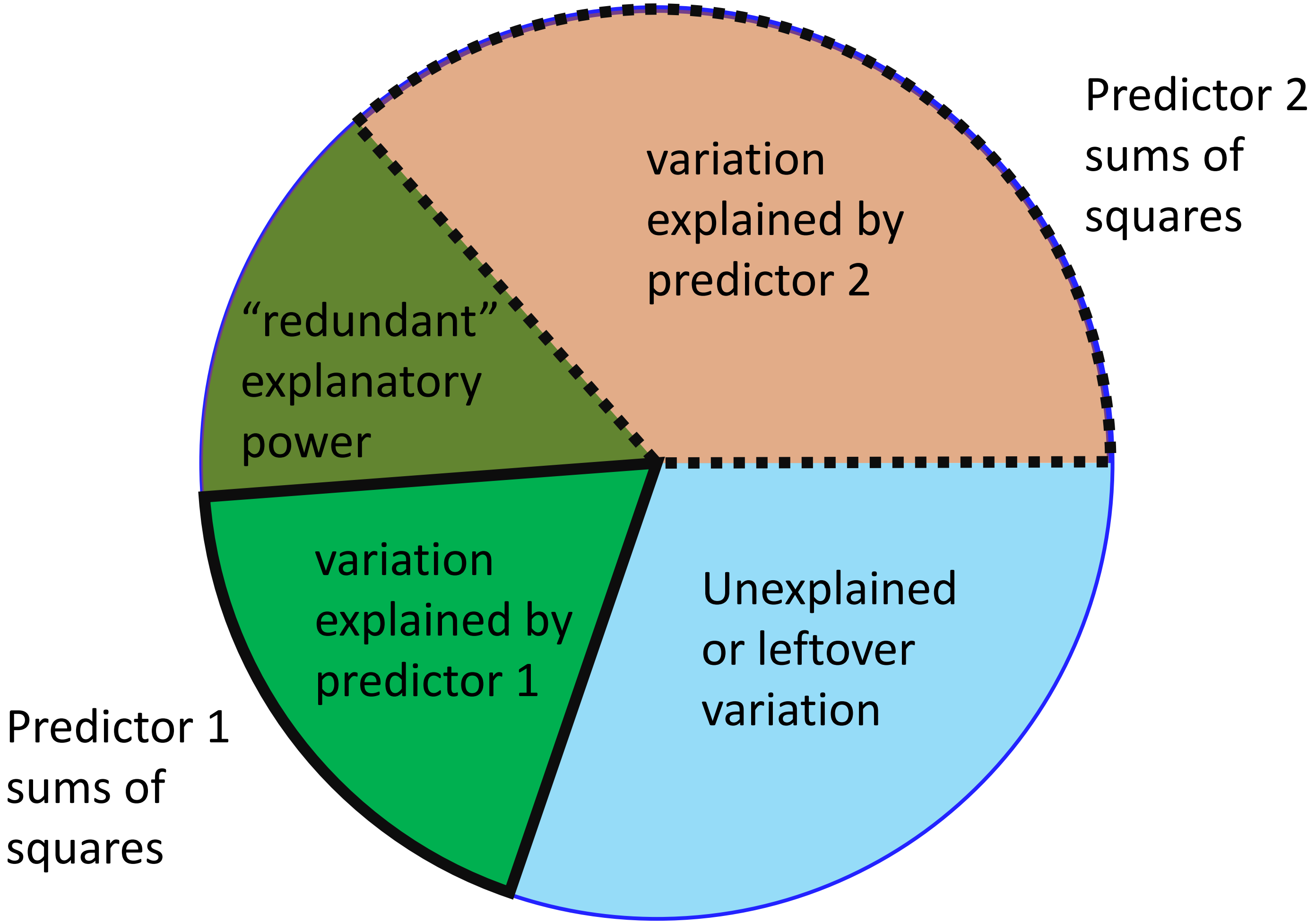
```
## Anova Table (Type III tests)
##
## Response: TICKS
##             Sum Sq  Df F value    Pr(>F)
## (Intercept)   7444   1  52.786 1.970e-12 ***
## HEIGHT        6092   1  43.199 1.550e-10 ***
## f_YEAR        5450   2  19.321 9.788e-09 ***
## Residuals    56268 399
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Sequential fit



Sequential fits change their answer depending on which variable/predictor is fit first.

Marginal fit



Sequential vs. marginal fitting

Table 1: ANOVA Table for fit_ex1_df (sequential fit)

Predictor	DF	SS	MS	F	P
f_YEAR	2	7049.72	3524.86	24.99	<0.0001
HEIGHT|f_YEAR	1	6091.98	6091.98	43.2	<0.01
Residuals	399	56268.21	141.02		

Table 2: ANOVA Table for fit_ex2_df (sequential fit)

Predictor	DF	SS	MS	F	P
HEIGHT	1	7692.19	7692.19	54.55	<0.0001
f_YEAR|HEIGHT	2	5449.52	2724.76	19.32	<0.0001
Residuals	399	56268.21	141.02		

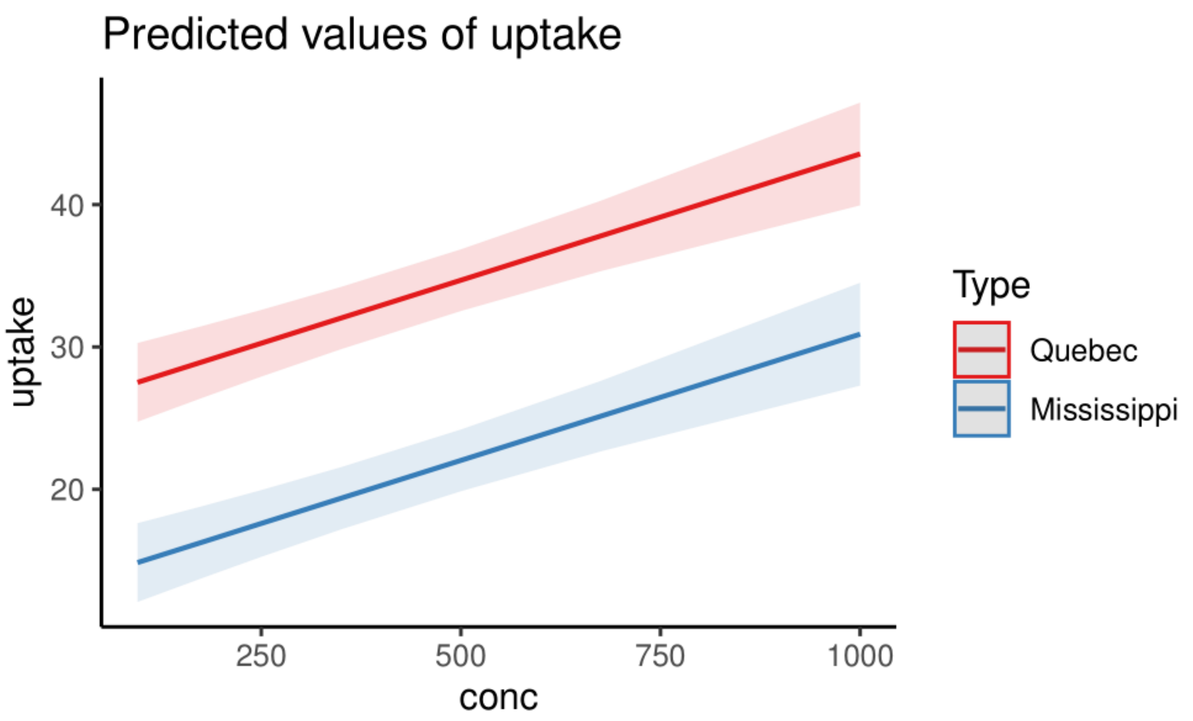
Table 3: ANOVA Table for fit_ex1_df (marginal fit)

Predictor	DF	SS	MS	F	P
f_YEAR|HEIGHT	2	5449.52	2724.7600	19.32	<0.0001
HEIGHT|f_YEAR	1	6091.98	6091.9800	43.2	<0.01
Residual	399	56268.21	141.0231		

Main effects

```
fit_plants_1_nointeraction <- lm(uptake~conc+Type,data=CO2)
summary(fit_plants_1_nointeraction)
```

```
##
## Call:
## lm(formula = uptake ~ conc + Type, data = CO2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.2145  -4.2549   0.5479   5.3048  12.9968
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      25.830052   1.579918  16.349  < 2e-16 ***
## conc              0.017731   0.002625   6.755 2.00e-09 ***
## TypeMississippi -12.659524   1.544261  -8.198 3.06e-12 ***
```



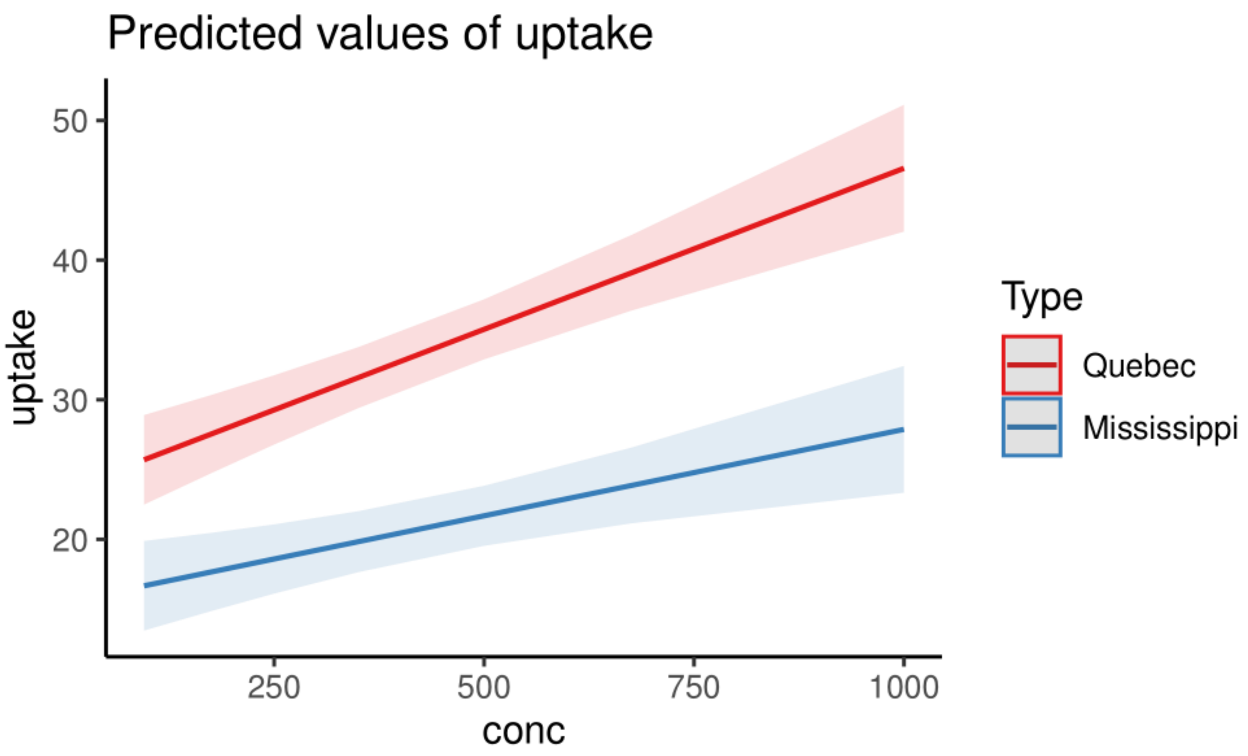
Interactions

```r
fit_plants_1_interaction <- lm(uptake~conc*Type,data=CO2)
```

```r
summary(fit_plants_1_interaction)
```

```
##
## Call:
## lm(formula = uptake ~ conc * Type, data = CO2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.3956  -5.5250  -0.1604   5.5724  12.0072
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           23.503038   1.910531  12.302  < 2e-16 ***
## conc                   0.023080   0.003638   6.344 1.25e-08 ***
## TypeMississippi       -8.005495   2.701899  -2.963  0.00401 **
## conc:TypeMississippi  -0.010699   0.005145  -2.079  0.04079 *
```

lm(uptake ~ conc + Type + conc*Type, data = C02)



Interactions

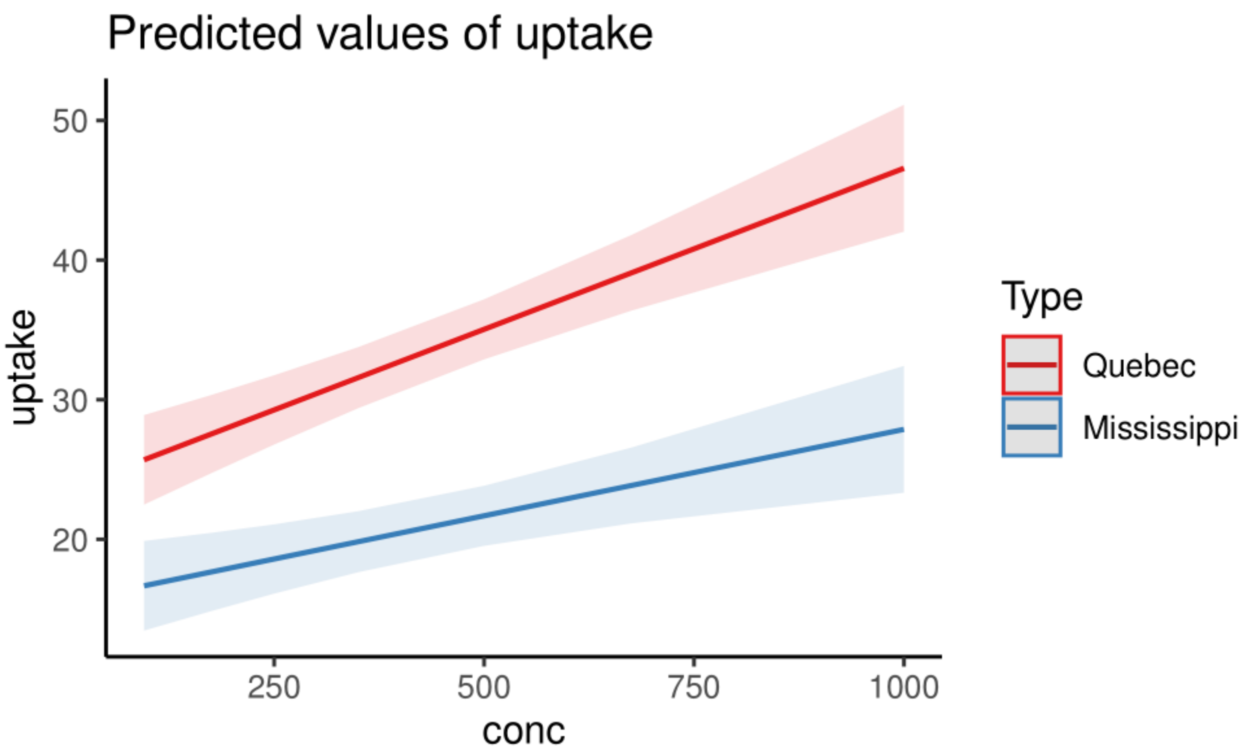
```
fit_plants_1_interaction <- lm(uptake~conc*Type,data=CO2)
summary(fit_plants_1_interaction)
```

```
##
## Call:
## lm(formula = uptake ~ conc * Type, data = CO2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.3956  -5.5250  -0.1604   5.5724  12.0072
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          23.503038   1.910531  12.302  < 2e-16 ***
## conc                  0.023080   0.003638   6.344 1.25e-08 ***
## TypeMississippi      -8.005495   2.701899  -2.963  0.00401 **
## conc:TypeMississippi -0.010699   0.005145  -2.079  0.04079 *
```

```
> Anova(fit_plants_1_interaction, type = "III")
Anova Table (Type III tests)

Response: uptake
            Sum Sq Df  F value    Pr(>F)
(Intercept) 7280.1  1 151.3352 < 2.2e-16 ***
conc        1935.9  1  40.2426 1.253e-08 ***
Type         422.3  1   8.7789  0.004012 **
conc:Type    208.0  1   4.3238  0.040787 *
Residuals   3848.4 80
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

lm(uptake ~ conc + Type + conc*Type, data = C02)



Example multiple linear regression

RESEARCH ARTICLE

Ecology and Evolution Open Access WILEY

Evolutionary and plastic variation in larval growth and digestion reveal the complex underpinnings of size and age at maturation in dung beetles

Patrick T. Rohner | Armin P. Moczek

Evolutionary and plastic variation in larval growth and digestion reveal the complex underpinnings of size and age at maturation in dung beetles

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Published Feb 16, 2023 on Dryad. https://doi.org/10.5061/dryad.j9kd51cdc

Cite this dataset

Rohner, Patrick T.; Moczek, Armin (2023). Evolutionary and plastic variation in larval growth and digestion reveal the complex underpinnings of size and age at maturation in dung beetles [Dataset]. Dryad. https://doi.org/10.5061/dryad.j9kd51cdc

Abstract

Age and size at maturity are key life history components, yet the proximate underpinnings that mediate intra- and interspecific variation in life history remain poorly understood. We studied the proximate underpinnings of species differences and nutritionally plastic variation in adult size and development time in four species of

Liatongus militaris