

Investigation of Data-driven Modelling and Feedforward Control for a Two Speed Magnetorheological Fluid Dual Clutch Transmission of Electric Vehicles

Yiyi Liang¹, Jin Zhao¹, Lei Deng¹, Jing Zhao², Pak Kin Wong², Haiping Du^{1*}

¹ School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Wollongong, Australia

² Department of Electromechanical Engineering, University of Macau, Macau, China

*haiping_du@uow.edu.au

Abstract: This paper presents data-driven modelling and control of a two-speed magnetorheological fluid dual-clutch transmission (MRFDCT) developed for electric vehicles (EVs). Because of the intricate nonlinear dynamics and rate-dependent hysteresis, an accurate mathematical model for this transmission system remains a challenge. The study investigates a purely data-driven modelling technique called Dynamic Mode Decomposition with Control (DMDc) to explore the correlation between the input currents and output torques of the MRFDCT system. DMDc is a mathematical model-free modelling strategy that generates linear state space models for intricate nonlinear dynamical systems based on input and output data. In this study, DMDc was used to develop linear state space models for the input currents and output torques under several scenarios: sinusoidal and step responses of the upshifting process. Under different scenarios, the associated linear state-space models generated by DMDc are able to accurately predict experimental results, validating their effectiveness. The inverse models of MRFDCT were constructed by utilising the same DMDc method and reversing the input current and output torque. Based on the obtained state space models of the inverse models for gear 1 and gear 2, a feedforward proportional-integral-derivative (PID) control strategy is proposed for controlling torque. This strategy is found to be effective when the outputs of the feedforward PID controller are matched to reference torques.

Keywords: Dynamic mode decomposition with control, Data-driven modelling, Electric vehicle, Magnetorheological fluid dual clutch transmission.

1. Introduction

The energy crisis and environmental degradation boost the development of electric vehicles (EVs), which are optimal replacements for traditional vehicles because of their unique eco-friendliness, such as the absence of carbon emissions [1]. For an EV, the transmission system is crucial as it is directly in charge of the gear-shifting processes and determines the performance of the EV. Typical EV transmission systems include automatic manual transmission (AMT) [2], continuously variable transmission (CVT) [3], and dual clutch transmission (DCT) [4,5]. Among these transmissions, DCT demonstrates its own advantages over the others because it's more fuel-efficient, and the clutch-to-clutch shifting strategy enables DCT to minimise torque interruptions and achieve smoother shifting with minimal shift shock [4,5]. Hence, DCT has fantabulous driving comfort because the magnitude of the shifting shock can be handled by varying the forces applied to the clutch's friction material [6]. Those studies have proven that DCT is an ideal type of transmission for EVs by exhibiting the advantages and making a great contribution to the enhancement of driving performance.

On the basis of the conventional DCT, researchers developed a magnetorheological fluid dual clutch transmission (MRFDCT) by integrating magnetorheological technology [7,8]. Magnetorheological fluid (MRF) is an intelligent material that can change viscosity in response to an applied magnetic field [9]. The application of MRF is broad, including dampers [10], brakes [11], robotic legs [12], etc. As the MRF can make transformations between the liquid

and half-solid states within milliseconds, MRF clutches exhibit superior performance than conventional clutches made of hydraulic components because of the quick response ability [7, 8]. Based on those foundations, MRFDCT was developed.

One of the main challenges of MRFDCT is providing a precise model that describes its behaviours in operation. Due to the nonlinear and hysteresis properties of MRF clutches, modelling and control of MRFDCT remain a challenge. The representative mathematical modelling methods for MR devices include the Herschel-Bulkey model [13] and the Bingham model [14]. However, modelling a transmission with MR devices is different from other devices because the gears and tractions change over time [16]. For the MRFDCT, Zhao et al. [15] trained the neural BP network describing the relationship between the input currents and the rate-dependent hysteresis. The BP network was able to predict the output torque based on the input current. In addition, Zhao et al. [16] developed a modified Bouc-Wen model with an uncertainty observer that identified the relationship between the input currents and the output torques, taking the rate-dependent hysteresis into account and proposed a torque control strategy compiling the real-time hysteresis observer and super-twist algorithm.

In this study, a data-driven approach was used to identify the relationship between the input currents and the output torques. This approach is purely mathematical model-free. It does not require a complicated analysis of the construction of the mathematical model. Instead, it can be modelled directly based on the collected data. This study explored the dynamics of MRFDCT using data-driven

modelling using dynamic mode decomposition with control (DMDc). DMDc originated from DMD, which was introduced as a data-driven approach to project intricate fluid flow [17]. By avoiding large amounts of data that may not be interpretable or may not easily contain symmetries, DMDc has advantages over typical machine learning methods [18]. Based on DMD, DMDc contains the effect of actuation and control, and it includes the impact of actuation and can generate linear state-space models from the nonlinear complicated dynamical systems [19].

The structure of the study is outlined below. In section 2, the MRFDCT is introduced in terms of its structure and working principle. Section 3 discussed the background of DMD and the DMDc algorithm. Section 4 presents that the experimental results obtained from the MRFDCT test bench are applied to construct related DMDc models. The results of the linear state space models constructed by DMDc and the experimental results are discussed to validate the effectiveness of the DMDc models. In section 5, a feedforward control strategy is developed on the inverse MRFDCT model which aims at tracking the real-time torques to follow the reference trajectories.

2. Magnetorheological Fluid Dual Clutch Transmission (MRFDCT)

2.1. Structure of MRFDCT

The structure of the test bench of MRFDCT is illustrated in Fig.1. The bench contains a motor, battery pack, speed reducer, MRFDCT clutch, a planetary gearset and a brake. The control board (NI My RIO1900) is connected to the current sensor (Arduino 5A Current Sensor) and the motor controller (Roboteq MBL-1660). It allows the whole system to be controlled and signals to be sent to the computer so that the test bench can be operated, and data can be recorded by the computer. During operation, the system is driven by the motor and power is generated from the battery packs. The reducer enhances the motor torque and transmits the torque to the MRFDCT. The current sensor measures the current delivered to the MRFDCT, and the torque sensor connected to the brake that immobilizes the driveline, measures the torque.

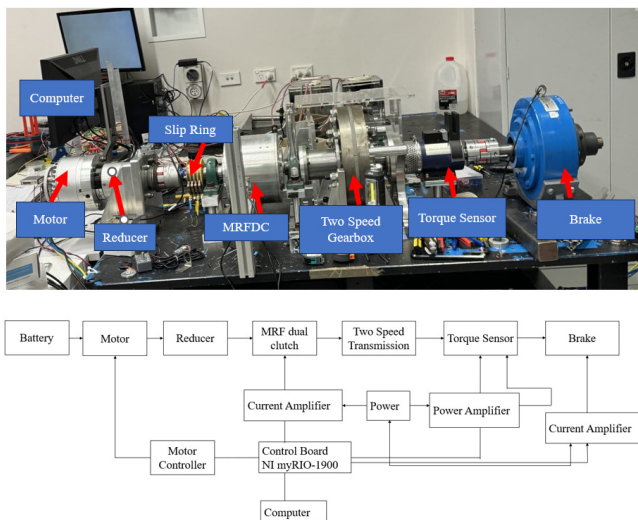


Fig. 1. Structure of the MRFDCT powertrain platform

2.2. Working Principle of MRFDCT

The MRFDCT system consists of two clutches: an external clutch (clutch 2) and an internal clutch (clutch 1), as depicted in Fig. 2. There are four planetary gears located between the ring gear and sun gear within the system. The internal clutch and the input shaft of the sun gear are connected by the output shaft 1, while the external clutch and the input shaft of the carrier are connected by the output shaft 2. When in operation, the input current will excite the coils through the slip rings; then magnetic field will be generated by the coils to energize the coils. Unlike traditional DCTs that perform gear shifting using hydraulic pressure, MRFDCTs use current to shift gears. During the operation on the first gear, the input current energises the internal clutch coil, leading to its engagement and consequently, the MRF transforming from liquid to semi-solid state. At this stage, the input current goes to the internal clutch and no current will go through the external clutch. Transmission torque is facilitated by solidified and shear stress operating MRF between the input and output shafts [7]. Upon activation of the external clutch coil, the external clutch will operate similarly, enabling the transfer of torque from the electric motor onto the input shaft [16].

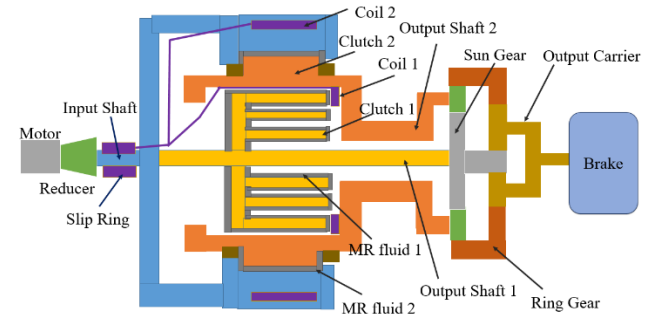
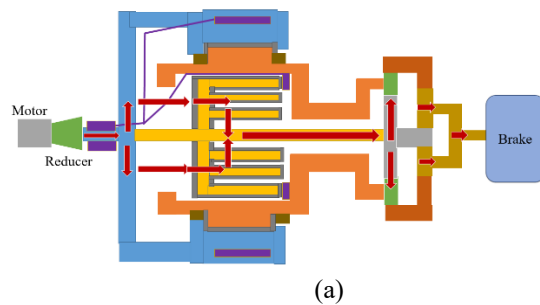


Fig. 2. Outline of MRFDCT

The paths for the transmissible torque are independent of each other and they are chosen by controlling the input currents. In the first gear, the excited current induced the magnetic circuit going through the internal clutch, and the torque travelled through the internal clutch to the gearbox as illustrated in Fig.3. a. Similarly, in the second gear, clutch 2 is energised and the flow for the torque is indicated in Fig. 3.b. When upshifting the gear, the current in coil 1 will decrease to zero and the current of coil 2 will increase to a certain value. The upshifting process is achieved by changing the torque transfer from the internal clutch to the external clutch by controlling the current [7,16].



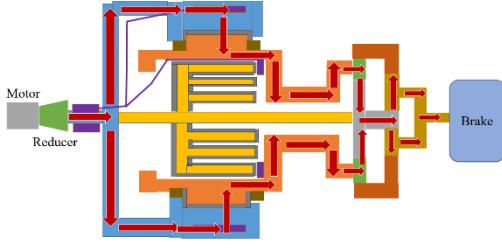


Fig. 3. Path of Torque Transmission. (a) for clutch 1, (b) for clutch 2

3. Algorithm of Dynamic Mode Decomposition with Control

This section first introduces DMD algorithm in section 3.1. Then, based on DMD, the DMDs algorithm is developed in section 3.2.

3.1. Dynamic Mode Decomposition (DMD)

DMD is based on proper orthogonal decomposition (POD), which is achieved by singular value decomposition (SVD) on the snapshot of the data matrix [19,20]. The principle of DMD is outlined briefly in this section.

A snapshot of data in the k elements is assumed to be approximately by the linear dynamical system, in which k implies the temporary iteration, and the linear dynamical system generated by DMD is written as:

$$X_{k+1} \approx AX_k \quad (1)$$

where $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^n$. Matrix A connects the snapshot of X_k and the subsequent snapshot X_{k+1} . The snapshots of data of the DMD have the following matrices if there are m elements:

$$X_1 = [x_1 x_2 \dots x_{m-1}], X_2 = [x_2 x_3 \dots x_m] \quad (2)$$

As a result, the linear system can be written as:

$$X' = AX \quad (3)$$

where X' is the time-shifted snapshot matrices. The target is to identify the best-fit operator A for the measurement data, and also find its leading spectral decomposition, such as eigenvectors and eigenvalues. The steps are:

First, perform single vector decomposition of X :

$$X = U\Sigma V^* \approx \tilde{U}\tilde{\Sigma}\tilde{V}^* \quad (4)$$

$$A = U^* \tilde{A} U \quad (5)$$

where Σ denotes as a diagonal singular values matrix, U is the POD mode and V is the right singular vector.

Second, matrix A can also be calculated from minimizing the distance between X' and AX :

$$\|X' - AX\|_F \quad (6)$$

where F is the Frobenius norm. The reduced matrix \tilde{A} is calculated to replace the matrix A of the high-dimensional system, and from the observation, it shares the same eigenvalues (not zero) as matrix A . Thus, the model defined by \tilde{A} can be written as:

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k \quad (7)$$

where \tilde{A} can be represented by:

$$\tilde{A} = U^* X' \tilde{V} \tilde{\Sigma}^{-1} \quad (8)$$

The DMD modes Φ can be computed by:

$$\Phi = X' \tilde{V} \tilde{\Sigma}^{-1} W \quad (9)$$

where W are the eigenvectors of the reduced system.

3.2 Dynamic Mode Decomposition with Control (DMDc)

As DMD itself is not able to produce an input and output model that can be applied to the control of dynamical systems and the dynamics may be corrupted by the external forcing [19], DMDc was developed based on the standard DMD. It can generate the input-output model and is applied for analysing the relationship between the future measurement based on the known current measurement and current control input. It is a method that can be utilised to identify the linear operators that satisfy the dynamics of the measurement [19,20]. A linear dynamical system is expressed as follows:

$$x_{k+1} = Ax_k + Bu_k \quad (10)$$

where A and B represent the state space matrices. x_{k+1} and x_k are the future and current state of the system, respectively. u_k is the control input. The snapshots of the measurements X, X' and the control input changing over time are used for the construction of data matrices. A new sequence of control input snapshots is denoted as:

$$Y_0 = [u_1 u_2 \dots u_{m-1}] \quad (11)$$

Thus, the new dynamic equation of the system can be written as Eq.12 to include the new input sequence:

$$X' \approx AX + BY_0 \quad (12)$$

DMDc aims to discover the best fit for A and B matrices. In this scenario of the study, B is unknown. Hence, A and B are figured simultaneously together. The dynamics in can be formulated as:

$$X' = [A \ B] \begin{bmatrix} X \\ Y_0 \end{bmatrix} = G\Omega \quad (13)$$

where $G = [A \ B]$ and it is obtained from the least square regression:

$$G = X' \Omega^\dagger = X' \begin{bmatrix} X \\ Y_0 \end{bmatrix}^\dagger \quad (14)$$

In Eq. 14, \dagger is the Moore-Penrose pseudo inverse, Ω encompasses the measurement and the snapshot information which is a high dimensional augmented data matrix that can be obtained by SVD. Ω is expressed as:

$$\Omega = \tilde{U}\tilde{\Sigma}\tilde{V}^* \quad (15)$$

where $\tilde{U} = \begin{bmatrix} \tilde{U}_1 \\ \tilde{U}_2 \end{bmatrix}$ that has been separated into two matrices that provide basis for the input space and it is originated from $X' = \tilde{U}\tilde{\Sigma}\tilde{V}^*$. Hence, the approximation of A and B are:

$$\tilde{A} = \tilde{U}^* X' \tilde{V} \tilde{\Sigma}^{-1} \tilde{U}_1^* \tilde{U} \quad (16)$$

$$\tilde{B} = \tilde{U}^* X' \tilde{V} \tilde{\Sigma}^{-1} \tilde{U}_2^* \quad (17)$$

The eigenvector Φ is figured through the eigen-decomposition of matrix \tilde{A} , which is expressed as $\tilde{A}W = W\lambda$. Hence, the eigenvector is recovered from:

$$\Phi = X_2 \tilde{V} \tilde{\Sigma}^{-1} \tilde{U}_1^* \tilde{U} W \quad (18)$$

Based on those steps, the identification of the linear operators can be completed. The system can be built from the linear state-space equation from the identification.

4. Data-driven Modelling of MRFDCT

In this study, the DMDc method is applied to discover the relationship between the input current and the output torque. The experimental data is recorded via LABVIEW on the computer and the computer is in connection to the control board to conduct the acquisition of data and transferring the input and output signals. In order to verify the effectiveness of DMDc on modelling the dynamics of MRFDCT, the experiments are carried out under these scenarios: sine waves as the reference trajectories and the upshifting process from gear 1 to gear 2 to realize a reference target torque.

4.1. Case 1: Sine Wave for Gear 1

In this case, sine waves are set as reference trajectories for gear 1 that operates on the internal clutch. The input current waves are sine waves, and the corresponding output torques are also in sinusoidal form. The associated state space model of gear 1 via sinusoidal waves includes three input variables, which are the input current I , the first derivative of the input current I' , and the second derivative of the input current I'' . The state variable is the output torque T . The data used to identify the state space model of gear 1 is shown in Fig.4. It is found that the majority of the results from the DMDc model are consistent with the true dynamic values, indicating that the DMDc model is able to fit the real system. Additionally, the linearized model is applied for testing as illustrated in Fig.5. The testing values cover the frequency of 0.5 Hz, 1 Hz and 2 Hz. We can see that the DMDc model perfectly predicts the dynamics and the errors between the real dynamics and the predicted state values are minimal, with maximum errors merely ranging from -2 to 6.

The linearized state space model identified for gear 1 under sine wave scenario can be written as:

$$x_{k+1} = A_{\text{gear1}} x_k + B_{\text{gear1}} u_k \quad (19)$$

where x is the state variable (output torque) and u represents the input variables (current, the first derivative of the current and the second derivative).

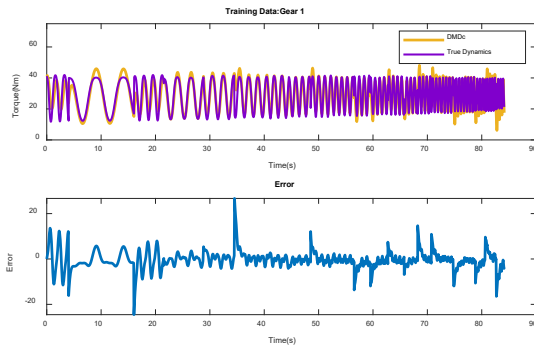


Fig. 4. Training Data of Gear 1 on Model Identification

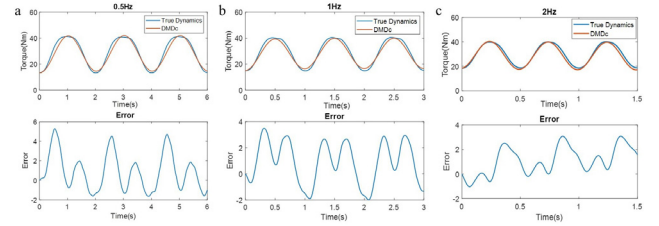


Fig. 5. Testing Data of Gear 1: (a)0.5Hz, (b)1Hz, (c)2Hz

4.2. Case 2: Sine Wave for Gear 2

Similarly, the model identification of gear 2 is conducted using the same approach as for gear 1. The data applied for model identification is demonstrated in Fig.6, which the linearized state-space model for gear 2 is generated. Also, the testing of the model is conducted through applying the sine waves of 1.5 Hz, 2 Hz and 2.5 Hz, as shown in Fig.7. The three sets of testing data attest to the validity of the linearised model, the predicted values of which are also consistent with the true dynamics.

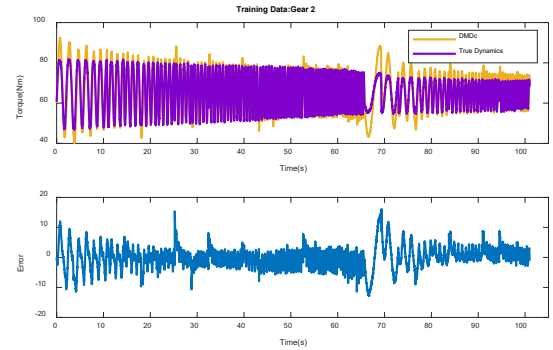


Fig. 6. Training Data of Gear 2

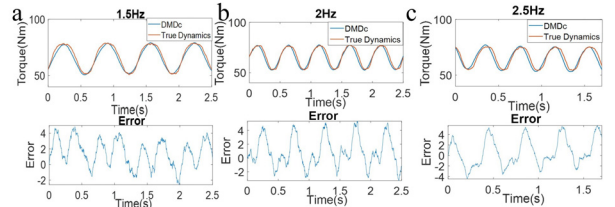


Fig. 7. Testing Data of Gear 2: (a)1 Hz, (b)1.5Hz, (c)2Hz

Likewise, the linearized state space model generated for gear 2 under sine wave scenario by DMDc can be written as:

$$x_{k+1} = A_{\text{gear2}} x_k + B_{\text{gear2}} u_k \quad (20)$$

where x is the state variable (output torque for gear 2) and u represents the input variables, including the current for gear 2, the first derivative of the current and the second derivative.

4.3. Case 3: Upshifting from Gear 1 to Gear 2

Upshifting experiments were conducted in order for the real-time torque to reach the reference torque during the shifting process under the PID controller. Under this scenario, local DMDc models are generated corresponding to each group of data. DMDc models are compared with the experimental data in Fig.8. In all cases, local DMDc models generated by the group data match the experimental data. By using the DMDc algorithm, linear state space models can accurately represent torque values.

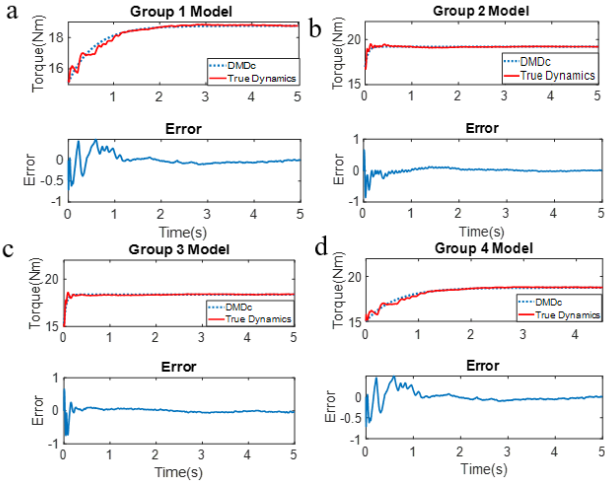


Fig. 8. Experimental data versus DMDc and the error (a) group 1, (b) group 2, (c) group 3, (d) group 4

5. PID Feedforward Control on the Inverse Model of MRFDCT

As previously discussed, DMDc is based solely on data and does not have any connection to the underlying mechanisms of the models. Thus, the data obtained from the MRFDCT platform can be directly used to generate linear state space models which are further useful in designing a control strategy for the system. Through utilizing DMDc, we generated the inverse models of MRFDCT in gear 1 and gear 2 separately by reversing the input data and the output data collected from the experiments. The inputs in the inverse model consist of torque, the first and second derivatives of torque, while the output is the current. A feedforward PID controller was developed based on inverse state-space models to track real-time torques and ensure they followed the reference trajectories accurately. Similarly, it is divided into two cases: case 1 is the feedforward controller on the inverse model of gear 1 and case 2 is for gear 2 respectively and experiments were performed on the MRFDCT test bench to test the performance of the controller. The diagram of the control strategy with inverse model is illustrated below.

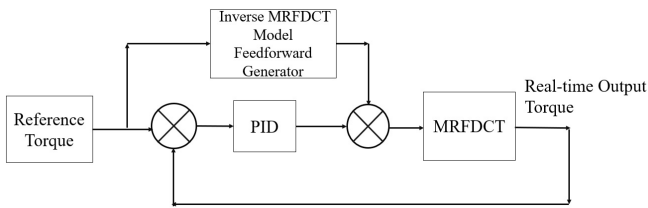


Fig. 9. Diagram of the control strategy with inverse model

5.1. Feedforward Control on the Inverse Model of Gear 1

The control performances of PID controller with feedforward compensator are evaluated through the adjustment of the reference trajectories for the torque. The reference trajectories set for the torque are various sinusoidal waves, as depicted in Fig.10-11. The sinusoidal waves applied for testing comprise of different frequencies, biases, and amplitudes. The amplitudes of the sine waves range from 30 to 50, and the values of the frequencies range from 1 Hz to 3 Hz. It is observed that real-time torques are able to follow

the references accurately, and the tracking errors between the reference and the real-time signals are minimal. The root mean square error (RMSE) of the reference output torque and the real-time output torque of gear 1 for group a to group f are: 3.855, 4.54, 2.6818, 4.9542, 3.0481, 3.7036 respectively. These RMSE values are within a small range, which validates the effectiveness of the proposed controller.

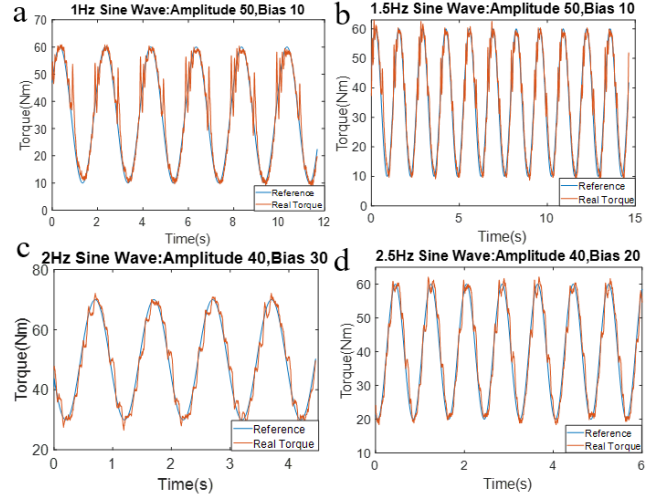


Fig. 10. Control performance of the real-time torque versus the reference value in Gear 1: (a) 1 Hz, Amplitude 50, Bias 10, (b) 1.5 Hz, Amplitude 50, Bias 10, (c) 2 Hz, Amplitude 40, Bias 30, (d) 2.5 Hz, Amplitude 40, Bias 20

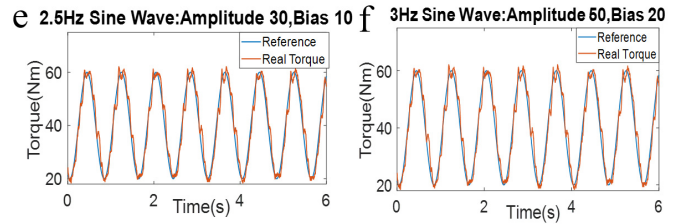


Fig. 11. Control performance of the real-time torque versus the reference value in Gear 1: (e) 2.5 Hz, Amplitude 30, Bias 10, (f) 3 Hz, Amplitude 50, Bias 20

5.2. Feedforward Control on the Inverse Model of Gear 2

The method of the feedforward control on the inverse model of gear 2 is the same as for gear 1, except that the MRFDCT works on the external clutch in this situation. The reference trajectories selected for testing this control strategy in gear 2 are also sinusoidal waves with varying amplitudes, biases, and frequencies. Real-time output torque and reference torques are displayed in Fig.12. Corresponding RMSE values between the reference and output torques for the four sets are presented as 2.0491, 1.7599, 2.6974, and 2.3820. The tiny magnitudes of RMSE values also verify the control strategy that can be applied in tracking the system output torque to match the desired references.

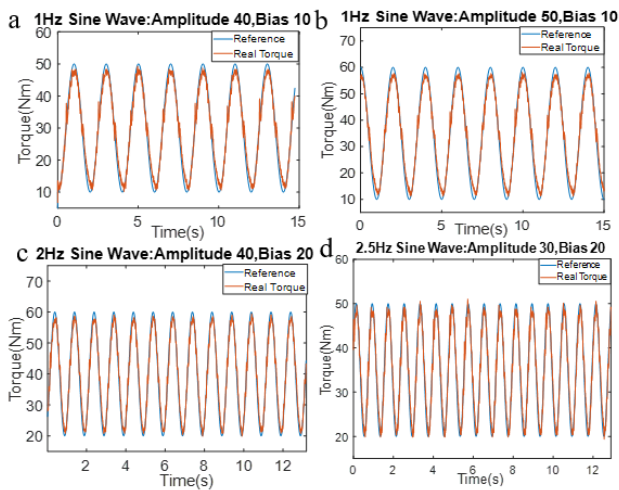


Fig. 12. Control performance of the real-time torque versus the reference value in Gear 2: (a) 1 Hz, Amplitude 40, Bias 10, (b) 1Hz, Amplitude 50, Bias 10, (c) 2Hz, Amplitude 40, Bias 20, (d) 2.5 Hz, Amplitude 30, Bias 20

6. Conclusion

In this study, a data-driven modelling technique called DMDc was employed to identify the relationship between input currents and output torques of the MRFDCT transmission system. Based on the data collected from the powertrain platform under different scenarios, the corresponding linear state space models were obtained. The results indicate that the linear state space models estimated using the DMDc algorithm are consistent with the true dynamics. Furthermore, a PID controller with a feedforward compensator has been implemented as the control strategy on the inverse models for gear 1 and gear 2 separately. The inverse models were also developed by DMDc and have been applied to the feedforward compensation. The proposed control strategy was conducted in experiments on the platform and thus demonstrated the efficacy of the PID feedforward controller in enabling the real-time torque to track with the desired trajectories. To summarize, this paper presents a data-driven modelling technique that generates linear state space models regardless of the high nonlinearity and complexity of the MRFDCT dynamics, and implemented a control strategy that enables the capability for the real-time torque to track with the desired references.

7. Acknowledgments

This research is supported by the ARC Grant (No. LP190100603).

8. Reference

- [1] Gelmanova, Z.S., Zhabalova, G.G., Sivyakova, G.A., et al.: 'Electric cars. Advantages and disadvantages', Journal of Physics: Conference Series, 2018, 1015, (5), p. 052029.
- [2] Ahssan, M.R., Ektesabi, M.M., Gorji, S.A.: 'Electric Vehicle with Multi-Speed Transmission: A Review on Performances and Complexities', SAE International Journal of Alternative Powertrains, 2018, 7, (2), pp. 169–181.
- [3] Ruan, J., Walker, P., Zhang, N.: 'Comparison of Power Consumption Efficiency of CVT and Multi-Speed Transmissions for Electric Vehicle', International Journal of Automotive Engineering, 2018, 9, (4), pp. 268–275.
- [4] Mishra, Kirti D., Srinivasan, K.: 'Robust control and estimation of clutch-to-clutch shifts', Control Engineering Practice, 2017, 65, pp. 100–114.
- [5] Lu, X., Chen, H., Gao, B., Zhang, Z., Jin, W.: 'Data-Driven Predictive Gearshift Control for Dual-Clutch Transmissions and FPGA Implementation', IEEE Transactions on Industrial Electronics, 2015, 62, (1), pp. 599–610.
- [6] Ogawa, K., Aihara, T.: 'Development of two-speed dual-clutch transmission for seamless gear shifting in EVs', Transportation Engineering, 2021, 6, p. 100097.
- [7] Zhang, H., Du, H., Sun, S., et al.: 'A novel magneto-rheological fluid dual-clutch design for two-speed transmission of electric vehicles', Smart Materials and Structures, 2021, 30, (7), p. 075035.
- [8] Zhang, H., Du, H., Sun, S., Li, W., Wang, Y.: 'Design and Analysis of a Novel Magnetorheological Fluid Dual Clutch for Electric Vehicle Transmission', SAE technical paper series, 2019.
- [9] De Vicente, J., Klingenberg, D.J., Hidalgo-Alvarez, R.: 'Magnetorheological fluids: a review', Soft Matter, 2011, 7, (8), pp. 3701–3710.
- [10] Wang, D. H., and Liao, W. H., "Magnetorheological fluid dampers: a review of parametric modelling." Smart materials and structures 20.2 (2011): 023001.
- [11] Olabi, A.G., Grunwald, A.: 'Design and application of magneto-rheological fluid', Mater. Des., 2007, 28, (10), pp. 2658–2664.
- [12] Jiang, N., Sun, S., Ouyang, Y., Xu, M., Li, W., Zhang, S.: 'A highly adaptive magnetorheological fluid robotic leg for efficient terrestrial locomotion', Smart Mater. Struct., 2016, 25, (9), pp. 095019.
- [13] Farjoud, A., Vahdati, N., Yap, F.F.: 'Mathematical Model of Drum-type MR Brakes using Herschel-Bulkley Shear Model', J. Intell. Mater. Syst. Struct., 2007, 19, (5), pp. 565–572.
- [14] Fernandez, M.F., Chang, J.-Y.: 'Development of magnetorheological fluid clutch for robotic arm applications', 2016 IEEE 14th International Workshop on Advanced Motion Control (AMC), 2016.
- [15] Zhao, J., Du, H., et. al.: 'Modelling of a Magnetorheological Fluid Dual Clutch with BP Neural Network', International Journal of Powertrains, in press.
- [16] Zhao, J., Deng, L., et.al.: 'Modelling and torque control against rate-dependent hysteresis of a magnetorheological fluid dual clutch in an electric vehicle transmission system', unpublished.

- [17] Schmid, P.J.: ‘Dynamic mode decomposition of numerical and experimental data’, *Journal of Fluid Mechanics*, 2010, 656, pp. 5–28.
- [18] Kaiser, E., Kutz, J.N., Brunton, S.L.: ‘Sparse identification of nonlinear dynamics for model predictive control in the low-data limit’, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 2018, 474, (2219), p. 20180335.
- [19] Proctor, J.L., Brunton, S.L., Kutz, J.N.: ‘Dynamic Mode Decomposition with Control’, *SIAM Journal on Applied Dynamical Systems*, 2016, 15, (1), pp. 142–161.
- [20] Brunton, S.L., J. Nathan Kutz: ‘Data-Driven Science and Engineering’ (Cambridge: Cambridge University Press., 2022)