Assignment 2

```matlab
clear;

one = [ 1 0 ];
two = [ 0 1 ];
v = [2;2];
Da = @(v,p) exp( one*( v - p ) )/( 1 + exp( one*( v - p ) ) +
 exp( two*( v - p ) ) );
Db = @(v,p) exp( two*( v - p ) )/( 1 + exp( one*( v - p ) ) +
 exp( two*( v - p ) ) );

% Q1

% When v_A = v_B = 2, the demand for each option if p_A = p_B = 1 is,
% D_A = e/(1+2e), D_B = e/(1+2e), D_0 = 1/(1+2e)
p1 = [ 1 ; 1 ];
Da(v,p1)
Db(v,p1)
(1 - Da(v,p1) - Db(v,p1))

% Q2 : Broyden's method

p = [ 0.2 ; 1.2 ];
fVal = foc(v,p);
focc = @(p) foc(v,p);
iJac = inv( myJac(focc, p ) );

maxit = 100;
tol = 1e-6;

for iter = 1:maxit
    fnorm = norm(fVal);
    fprintf('iter %d: p(1) = %f, p(2) = %f, norm(f(x)) = %.8f\n',
 iter, p(1), p(2), norm(fVal));
    if fnorm < tol
        break
    end
    d = - (iJac * fVal);
    p = p + d;
    fOld = fVal;
    fVal = focc(p);
    u = iJac*(fVal - fOld);
    iJac = iJac + ( (d - u)*(d'*iJac) )/(d'*u);
end

fVal
p

% Q3 : Gauss - Sidel method

p1 = 0.2;
```

```matlab
p2 = 1.2;

for iter = 1:maxit

    p1Old = 0;
    p2Old = 0;

    DaG = @(p) exp( v(1,1) - p )/( 1 + exp( v(1,1) - p ) + exp( v(2,1)
 - p2 ) );

    FOCa = @(p) ( DaG(p) - p * DaG(p) * ( 1 - DaG(p) ) );


    p10 = p1;
    p20 = p2;
    fOlda = FOCa(p1Old);



    for i = 1:maxit
        fValGa = FOCa(p10);
        if norm(fValGa) < tol
            break
        else
        p1New = p10 - ( ( p10 - p1Old )/( fValGa - fOlda ) ) * fValGa;
        p1Old = p10;
        p10 = p1New;
        fOlda = fValGa;
        end
    end

    DbG = @(p) exp( v(2,1) - p )/( 1 + exp( v(1,1) - p10 ) +
exp( v(2,1) - p ) );
    FOCb = @(p) ( DbG(p) - p * DbG(p) * ( 1 - DbG(p) ) );
    fOldb = FOCb(p2Old);

    for i = 1:maxit

        fValGb = FOCb(p20);
        if norm(fValGb) < tol
            break
        else
            p2New = p20 - ( ( p20 - p2Old )/( fValGb - fOldb ) ) *
fValGb;
            p2Old = p20;
            p20 = p2New;
            fOldb = fValGb;
        end
    end
    fprintf('iter %d: p(1) = %f, p(2) = %f, norm([ p1-p10 ; p2-p20 ])
= %.8f\n', iter, p10, p20, norm([ p1-p10 ; p2-p20 ]));
    if norm( [ p1-p10 ; p2-p20 ] ) < tol
        break
    else
```

```matlab
        p1 = p10;
        p2 = p20;
    end
end

[p1 ; p2]

% As you can see in the result from iteration, this method makes price
% converge faster than Broyden's method does. This is because Gauss-
seidel
% method uptades prices more quickly. You can check this in above code
 in
% which updated p1 is used in second sub-iteration for finding
 solution for
% FOC_b = 0 given p1.

% Q4

p = [0.2,1.2];

for iter = 1:maxit
    newp1 = 1/( 1 - Da(v,p) );
    newp2 = 1/( 1 - Db(v,p) );
    newp = [ newp1 ; newp2 ];
    fprintf('iter %d: p(1) = %f, p(2) = %f, norm(f(x)) = %.8f\n',
 iter, p(1), p(2), norm(newp - p));
    if norm( newp - p ) < tol
        break
    end
    p = newp;
end
p

% It converges to equilibrium price level p which is the same with
 previous
% results. But even though convergence criteria is different, you can
 check
% it converges slower than others. This is because previous two
 methods use
% first derivatives, which means they approach to maximum value more
% accurately in each iteration, compared to this method which exploits
 only
% demand function, thus more fluctuate since its step size is big.

% Q5

va = 2;
ppp = [ 0 0 ];
for vb = 0:0.2:3
    v = [ va ; vb ];
    p = [ 0.5 ; 0.5 ];
    fVal = foc(v,p);
    focc = @(p) foc(v,p);
    iJac = inv( myJac(focc, p ) );
```

```matlab
    for iter = 1:maxit
        fnorm = norm(fVal);
        fprintf('iter %d: p(1) = %f, p(2) = %f, norm(f(x)) = %.8f\n',
 iter, p(1), p(2), norm(fVal));
        if fnorm < tol
            break
        end
        d = - (iJac * fVal);
        p = p + d;
        fOld = fVal;
        fVal = focc(p);
        u = iJac*(fVal - fOld);
        iJac = iJac + ( (d - u)*(d'*iJac) )/(d'*u);
    end
    ppp = [ ppp ; p' ];
end

plot(ppp(2:17,1))
hold on
plot(ppp(2:17,2))
hold off
```

ans =

    0.4223

ans =

    0.4223

ans =

    0.1554

iter 1: p(1) = 0.200000, p(2) = 1.200000, norm(f(x)) = 0.60724192
iter 2: p(1) = 1.648743, p(2) = 1.623725, norm(f(x)) = 0.01713856
iter 3: p(1) = 1.604214, p(2) = 1.593479, norm(f(x)) = 0.00348205
iter 4: p(1) = 1.602510, p(2) = 1.602366, norm(f(x)) = 0.00143695
iter 5: p(1) = 1.599500, p(2) = 1.599517, norm(f(x)) = 0.00023284
iter 6: p(1) = 1.598939, p(2) = 1.598939, norm(f(x)) = 0.00000124
iter 7: p(1) = 1.598942, p(2) = 1.598942, norm(f(x)) = 0.00000000

fVal =

   1.0e-08 *

    0.1137
    0.1182

p =

```
        1.5989
        1.5989

iter 1: p(1) = 1.507385, p(2) = 1.578254, norm([ p1-p10 ; p2-p20 ]) =
  1.36100381
iter 2: p(1) = 1.594293, p(2) = 1.597898, norm([ p1-p10 ; p2-p20 ]) =
  0.08910021
iter 3: p(1) = 1.598706, p(2) = 1.598889, norm([ p1-p10 ; p2-p20 ]) =
  0.00452323
iter 4: p(1) = 1.598930, p(2) = 1.598939, norm([ p1-p10 ; p2-p20 ]) =
  0.00022952
iter 5: p(1) = 1.598939, p(2) = 1.598939, norm([ p1-p10 ; p2-p20 ]) =
  0.00000905
iter 6: p(1) = 1.598939, p(2) = 1.598939, norm([ p1-p10 ; p2-p20 ]) =
  0.00000000

ans =

        1.5989
        1.5989

iter 1: p(1) = 0.200000, p(2) = 1.200000, norm(f(x)) = 2.47364214
iter 2: p(1) = 1.831250, p(2) = 1.831250, norm(f(x)) = 0.40893712
iter 3: p(1) = 1.542088, p(2) = 1.542088, norm(f(x)) = 0.09960463
iter 4: p(1) = 1.612519, p(2) = 1.612519, norm(f(x)) = 0.02381903
iter 5: p(1) = 1.595676, p(2) = 1.595676, norm(f(x)) = 0.00572736
iter 6: p(1) = 1.599726, p(2) = 1.599726, norm(f(x)) = 0.00137543
iter 7: p(1) = 1.598753, p(2) = 1.598753, norm(f(x)) = 0.00033041
iter 8: p(1) = 1.598987, p(2) = 1.598987, norm(f(x)) = 0.00007937
iter 9: p(1) = 1.598931, p(2) = 1.598931, norm(f(x)) = 0.00001906
iter 10: p(1) = 1.598944, p(2) = 1.598944, norm(f(x)) = 0.00000458
iter 11: p(1) = 1.598941, p(2) = 1.598941, norm(f(x)) = 0.00000110
iter 12: p(1) = 1.598942, p(2) = 1.598942, norm(f(x)) = 0.00000026

p =

        1.5989
        1.5989

iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.64134545
iter 2: p(1) = 2.156789, p(2) = 1.389501, norm(f(x)) = 0.11703523
iter 3: p(1) = 1.893584, p(2) = 1.156927, norm(f(x)) = 0.01179938
iter 4: p(1) = 1.865908, p(2) = 1.144573, norm(f(x)) = 0.00057357
iter 5: p(1) = 1.867183, p(2) = 1.147618, norm(f(x)) = 0.00009476
iter 6: p(1) = 1.867088, p(2) = 1.148054, norm(f(x)) = 0.00000715
iter 7: p(1) = 1.867082, p(2) = 1.148094, norm(f(x)) = 0.00000006
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.62306220
iter 2: p(1) = 2.092968, p(2) = 1.390262, norm(f(x)) = 0.09931036
iter 3: p(1) = 1.866335, p(2) = 1.184548, norm(f(x)) = 0.00857906
iter 4: p(1) = 1.845803, p(2) = 1.172064, norm(f(x)) = 0.00036316
iter 5: p(1) = 1.846533, p(2) = 1.173960, norm(f(x)) = 0.00007200
iter 6: p(1) = 1.846461, p(2) = 1.174238, norm(f(x)) = 0.00000739
iter 7: p(1) = 1.846454, p(2) = 1.174272, norm(f(x)) = 0.00000004
```
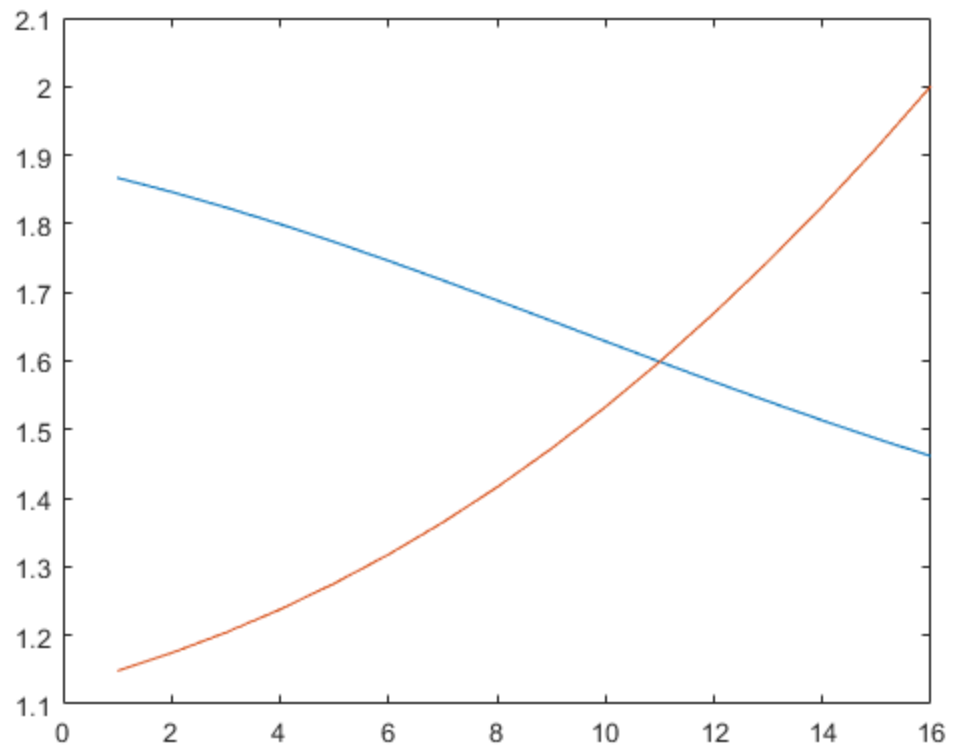
```
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.60254345
iter 2: p(1) = 2.027275, p(2) = 1.393174, norm(f(x)) = 0.08172780
iter 3: p(1) = 1.837950, p(2) = 1.215326, norm(f(x)) = 0.00587435
iter 4: p(1) = 1.823442, p(2) = 1.203105, norm(f(x)) = 0.00021270
iter 5: p(1) = 1.823921, p(2) = 1.203989, norm(f(x)) = 0.00002859
iter 6: p(1) = 1.823893, p(2) = 1.204077, norm(f(x)) = 0.00000477
iter 7: p(1) = 1.823888, p(2) = 1.204095, norm(f(x)) = 0.00000003
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.58000390
iter 2: p(1) = 1.961203, p(2) = 1.399022, norm(f(x)) = 0.06499437
iter 3: p(1) = 1.808649, p(2) = 1.249394, norm(f(x)) = 0.00384794
iter 4: p(1) = 1.799033, p(2) = 1.237865, norm(f(x)) = 0.00021496
iter 5: p(1) = 1.799473, p(2) = 1.237922, norm(f(x)) = 0.00003004
iter 6: p(1) = 1.799502, p(2) = 1.237842, norm(f(x)) = 0.00000601
iter 7: p(1) = 1.799509, p(2) = 1.237822, norm(f(x)) = 0.00000000
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.55593709
iter 2: p(1) = 1.896402, p(2) = 1.408736, norm(f(x)) = 0.04980597
iter 3: p(1) = 1.778666, p(2) = 1.286908, norm(f(x)) = 0.00269347
iter 4: p(1) = 1.772869, p(2) = 1.276498, norm(f(x)) = 0.00038605
iter 5: p(1) = 1.773392, p(2) = 1.275940, norm(f(x)) = 0.00008768
iter 6: p(1) = 1.773506, p(2) = 1.275678, norm(f(x)) = 0.00000360
iter 7: p(1) = 1.773502, p(2) = 1.275689, norm(f(x)) = 0.00000001
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.53121826
iter 2: p(1) = 1.834571, p(2) = 1.423402, norm(f(x)) = 0.03674202
iter 3: p(1) = 1.748260, p(2) = 1.328068, norm(f(x)) = 0.00237354
iter 4: p(1) = 1.745295, p(2) = 1.319159, norm(f(x)) = 0.00055380
iter 5: p(1) = 1.745943, p(2) = 1.318214, norm(f(x)) = 0.00012503
iter 6: p(1) = 1.746112, p(2) = 1.317887, norm(f(x)) = 0.00000378
iter 7: p(1) = 1.746107, p(2) = 1.317897, norm(f(x)) = 0.00000004
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.50720457
iter 2: p(1) = 1.777368, p(2) = 1.444277, norm(f(x)) = 0.02616252
iter 3: p(1) = 1.717714, p(2) = 1.373136, norm(f(x)) = 0.00234816
iter 4: p(1) = 1.716689, p(2) = 1.366010, norm(f(x)) = 0.00064251
iter 5: p(1) = 1.717430, p(2) = 1.364915, norm(f(x)) = 0.00013093
iter 6: p(1) = 1.717612, p(2) = 1.364616, norm(f(x)) = 0.00000192
iter 7: p(1) = 1.717609, p(2) = 1.364621, norm(f(x)) = 0.00000005
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.48579094
iter 2: p(1) = 1.726325, p(2) = 1.472819, norm(f(x)) = 0.01814610
iter 3: p(1) = 1.687331, p(2) = 1.422442, norm(f(x)) = 0.00214962
iter 4: p(1) = 1.687426, p(2) = 1.417233, norm(f(x)) = 0.00061931
iter 5: p(1) = 1.688166, p(2) = 1.416216, norm(f(x)) = 0.00010859
iter 6: p(1) = 1.688321, p(2) = 1.415995, norm(f(x)) = 0.00000060
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.46934272
iter 2: p(1) = 1.682809, p(2) = 1.510725, norm(f(x)) = 0.01256003
iter 3: p(1) = 1.657393, p(2) = 1.476363, norm(f(x)) = 0.00164412
iter 4: p(1) = 1.657854, p(2) = 1.473018, norm(f(x)) = 0.00048449
iter 5: p(1) = 1.658458, p(2) = 1.472267, norm(f(x)) = 0.00007382
iter 6: p(1) = 1.658566, p(2) = 1.472131, norm(f(x)) = 0.00000010
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.46040389
iter 2: p(1) = 1.648014, p(2) = 1.559991, norm(f(x)) = 0.00947708
iter 3: p(1) = 1.628097, p(2) = 1.535276, norm(f(x)) = 0.00089437
iter 4: p(1) = 1.628274, p(2) = 1.533542, norm(f(x)) = 0.00026401
iter 5: p(1) = 1.628604, p(2) = 1.533175, norm(f(x)) = 0.00004486
iter 6: p(1) = 1.628671, p(2) = 1.533099, norm(f(x)) = 0.00000006
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.46114029
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iter 2: p(1) = 1.622993, p(2) = 1.622993, norm(f(x)) = 0.00986255
iter 3: p(1) = 1.599478, p(2) = 1.599478, norm(f(x)) = 0.00022023
iter 4: p(1) = 1.598941, p(2) = 1.598941, norm(f(x)) = 0.00000044
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.47267854
iter 2: p(1) = 1.608721, p(2) = 1.702577, norm(f(x)) = 0.01475937
iter 3: p(1) = 1.571317, p(2) = 1.669073, norm(f(x)) = 0.00076582
iter 4: p(1) = 1.570012, p(2) = 1.669366, norm(f(x)) = 0.00022062
iter 5: p(1) = 1.569696, p(2) = 1.669641, norm(f(x)) = 0.00002359
iter 6: p(1) = 1.569658, p(2) = 1.669674, norm(f(x)) = 0.00000001
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.49470302
iter 2: p(1) = 1.606185, p(2) = 1.802191, norm(f(x)) = 0.02485558
iter 3: p(1) = 1.543058, p(2) = 1.743844, norm(f(x)) = 0.00116915
iter 4: p(1) = 1.541601, p(2) = 1.744893, norm(f(x)) = 0.00031518
iter 5: p(1) = 1.541061, p(2) = 1.745279, norm(f(x)) = 0.00000043
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.52557666
iter 2: p(1) = 1.616482, p(2) = 1.926026, norm(f(x)) = 0.04197529
iter 3: p(1) = 1.513740, p(2) = 1.823132, norm(f(x)) = 0.00126406
iter 4: p(1) = 1.513802, p(2) = 1.825481, norm(f(x)) = 0.00023567
iter 5: p(1) = 1.513429, p(2) = 1.825676, norm(f(x)) = 0.00003952
iter 6: p(1) = 1.513357, p(2) = 1.825718, norm(f(x)) = 0.00000017
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.56287302
iter 2: p(1) = 1.640927, p(2) = 2.079197, norm(f(x)) = 0.06906467
iter 3: p(1) = 1.481950, p(2) = 1.905794, norm(f(x)) = 0.00211900
iter 4: p(1) = 1.486725, p(2) = 1.910951, norm(f(x)) = 0.00001151
iter 5: p(1) = 1.486702, p(2) = 1.910923, norm(f(x)) = 0.00000020
iter 1: p(1) = 0.500000, p(2) = 0.500000, norm(f(x)) = 0.60397640
iter 2: p(1) = 1.681165, p(2) = 2.267960, norm(f(x)) = 0.10979617
iter 3: p(1) = 1.445826, p(2) = 1.990350, norm(f(x)) = 0.00496740
iter 4: p(1) = 1.460595, p(2) = 2.000910, norm(f(x)) = 0.00025395
iter 5: p(1) = 1.461153, p(2) = 2.000801, norm(f(x)) = 0.00002146
iter 6: p(1) = 1.461217, p(2) = 2.000803, norm(f(x)) = 0.00000046
```

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