Obtaining the α Parameter from FMR Linewidth

October 1, 2014

The Gilbert damping parameter, α, can be obtained from the FMR linewidth if the saturation magnetization, M_{eff}, and magnetization angle, ϑ_M, are known.

See articles:
[1] Maksymowicz, A. Z. & Leaver, K. D. Angular dependence of line width in nickel platelets. *J. Phys. F Met. Phys.* **3**, 1031–1038 (1973)
[2] Mizukami, S., Ando, Y. & Miyazaki, T. Effect of spin diffusion on Gilbert damping for a very thin permalloy layer in Cu/permalloy/Cu/Pt films. *Phys. Rev. B* **66**, 104413 (2002).

In [1]:
```python
from __future__ import division
from sympy import *
init_printing(use_latex=True)
```

In [2]:
```python
# define some symbolic factors
factor_symbols = 'Delta, omega, gamma, theta_H, theta_M, 4piM_eff, alpha'
field_symbols = 'H_1, H_2, H_res, H_in, H_ex, H_pp'
delta_symbols = 'd4piMeff, dThetaH'

Delta, omega, gamma, thetaH, thetaM, Meff, alpha = symbols(factor_symbols, real=True)
H_1, H_2, H_res, H_in, H_ex, H_pp = symbols(field_symbols, real=True)
Delta4piMeff, DeltaThetaH = symbols(delta_symbols, real=True)

DeltaH_in = Delta*H_in
DeltaH_ex = Delta*H_ex
DeltaH_pp = Delta*H_pp
```

Once the microwave field and precessing magnetization are substituted into the Landau-Lifshitz-Gilbert equation, the FMR resonance condition can be obtained after considering the following relations (consequences of linear approximation):

In [3]:
```python
H1 = H_res*cos(thetaH - thetaM) - Meff*cos(2*thetaM)
Eq(H_1, H1)
```

Out[3]:

$$H_1 = -4piM_{eff}\cos(2\theta_M) + H_{res}\cos(\theta_H - \theta_M)$$

In [4]:
```
H2 = H_res*cos(thetaH - thetaM) - Meff*cos(thetaM)**2
Eq(H_2, H2)
```

Out[4]:

$$H_2 = -4piM_{eff}\cos^2(\theta_M) + H_{res}\cos(\theta_H - \theta_M)$$

In [5]:
```
Larmor = sqrt(H1*H2)
Eq(omega/gamma, sqrt(H_1*H_2))
```

Out[5]:

$$\frac{\omega}{\gamma} = \sqrt{H_1 H_2}$$

In [6]:
```
Eq(omega/gamma, Larmor)
```

Out[6]:

$$\frac{\omega}{\gamma} = \sqrt{\left(-4piM_{eff}\cos^2(\theta_M) + H_{res}\cos(\theta_H - \theta_M)\right)\left(-4piM_{eff}\cos(2\theta_M) + H_{res}\cos(\theta_H - \theta_M)\right)}$$

In [7]:
```
# expand the terms under the radical
Eq(omega/gamma, expand(Larmor))
```

Out[7]:

$$\frac{\omega}{\gamma} = \sqrt{4piM_{eff}^2\cos^2(\theta_M)\cos(2\theta_M) - 4piM_{eff}H_{res}\cos^2(\theta_M)\cos(\theta_H - \theta_M) - 4piM_{eff}H_{res}\cos(2\theta_M)\cos(\theta_H - \theta_M) + H_{res}^2\cos^2(\theta_H - \theta_M)}$$

Angular dependence of magnetization can be obtained from:

In [8]:
```
Hres_vs_thetaM = Eq(2*H_res*sin(thetaM - thetaH), Meff*sin(2*thetaM)); Hres_vs_thetaM
```

Out[8]:

$$-2H_{res}\sin(\theta_H - \theta_M) = 4piM_{eff}\sin(2\theta_M)$$

Where the FMR resonance center behaves as

```
In [9]: Hres =  solve(Hres_vs_thetaM, H_res)[0]
        Eq(H_res, Hres)
```

Out[9]:

$$H_{res} = -\frac{4piM_{eff}\sin\left(2\theta_M\right)}{2\sin\left(\theta_H - \theta_M\right)}$$

For cases where the magnetization has low perpendicular anisotropy, the FMR FWHM linewidth is determined by two principle components: an intrinsic and extrinsic broadening.

```
In [10]: Eq(DeltaH_pp, (DeltaH_in + DeltaH_ex)/sqrt(3))
```

Out[10]:

$$\Delta H_{pp} = \frac{\sqrt{3}}{3}\left(\Delta H_{ex} + \Delta H_{in}\right)$$

The intrinsic broadening is governed by the Gilber damping factor

```
In [11]: Eq(DeltaH_in, alpha*(H_1 + H_2)/abs(Derivative((omega/gamma), H_res)))
```

Out[11]:

$$\Delta H_{in} = \frac{\alpha\left(H_1 + H_2\right)}{\left|\frac{\partial}{\partial H_{res}}\left(\frac{\omega}{\gamma}\right)\right|}$$

```
In [12]: deltaH_in = alpha*(H1 + H2)/abs(Derivative(Larmor, H_res))
         Eq(DeltaH_in, deltaH_in)
```

Out[12]:

$$\Delta H_{in} = \frac{\alpha\left(-4piM_{eff}\cos^2\left(\theta_M\right) - 4piM_{eff}\cos\left(2\theta_M\right) + 2H_{res}\cos\left(\theta_H - \theta_M\right)\right)}{\left|\frac{\partial}{\partial H_{res}}\sqrt{\left(-4piM_{eff}\cos^2\left(\theta_M\right) + H_{res}\cos\left(\theta_H - \theta_M\right)\right)\left(-4piM_{eff}\cos\left(2\theta_M\right) + H_{res}\cos\left(\theta_H - \theta_M\right)\right)}\right|}$$

```
In [13]: deltaH_in = deltaH_in.doit()
         Eq(DeltaH_in, simplify(deltaH_in))
```

Out[13]:

$$\Delta H_{in} = \frac{2\alpha\left(-3\cdot 4piM_{eff}\cos^2\left(\theta_M\right) + 4piM_{eff} + 2H_{res}\cos\left(\theta_H - \theta_M\right)\right)}{\left|\frac{\left(-3\cdot 4piM_{eff}\cos^2\left(\theta_M\right) + 4piM_{eff} + 2H_{res}\cos\left(\theta_H - \theta_M\right)\right)\cos\left(\theta_H - \theta_M\right)}{\sqrt{2\cdot 4piM_{eff}^2\cos^4\left(\theta_M\right) - 4piM_{eff}^2\cos^2\left(\theta_M\right) - 3\cdot 4piM_{eff}H_{res}\cos^2\left(\theta_M\right)\cos\left(\theta_H - \theta_M\right) + 4piM_{eff}H_{res}\cos\left(\theta_H - \theta_M\right) + H_{res}^2\cos^2\left(\theta_H - \theta_M\right)}}\right|}$$

The extrinsic broadening follows the dispersion in magnetization magnitude and angle

In [14]:
```
deltaH_ex = abs(Derivative(H_res, Meff))*Delta4piMeff + abs(Derivative(H_res, thetaH))*DeltaThetaH
Eq(DeltaH_ex, deltaH_ex)
```

Out[14]:

$$\Delta H_{ex} = d4piMeff \left| \frac{d}{d4piM_{eff}} H_{res} \right| + dThetaH \left| \frac{d}{d\theta_H} H_{res} \right|$$

In [15]:
```
deltaH_ex = abs(Derivative(Hres, Meff))*Delta4piMeff + abs(Derivative(Hres, thetaH))*DeltaThetaH
deltaH_ex = deltaH_ex.doit()
Eq(DeltaH_ex, simplify(deltaH_ex))
```

Out[15]:

$$\Delta H_{ex} = \frac{d4piMeff}{2} \left| \frac{\sin(2\theta_M)}{\sin(\theta_H - \theta_M)} \right| + \frac{dThetaH \left|4piM_{eff} \sin(2\theta_M) \cos(\theta_H - \theta_M)\right|}{2 \sin^2(\theta_H - \theta_M)}$$

Then the total FMR linewidth is

In [16]:
```
deltaH_pp = (deltaH_in + deltaH_ex)/sqrt(3)
Eq(DeltaH_pp, deltaH_pp)
```

Out[16]:

$$\Delta H_{pp} = \frac{\sqrt{3}}{3} \left(\frac{\alpha \left(-4piM_{eff} \cos^2(\theta_M) - 4piM_{eff} \cos(2\theta_M) + 2H_{res} \cos(\theta_H - \theta_M)\right)}{\left| \frac{\sqrt{(4piM_{eff} \cos^2(\theta_M) - H_{res} \cos(\theta_H - \theta_M))(4piM_{eff} \cos(2\theta_M) - H_{res} \cos(\theta_H - \theta_M))}}{(4piM_{eff} \cos^2(\theta_M) - H_{res} \cos(\theta_H - \theta_M))(4piM_{eff} \cos(2\theta_M) - H_{res} \cos(\theta_H - \theta_M))} \left(\frac{1}{2} \left(4piM_{eff} \cos^2(\theta_M) - H_{res} \cos(\theta_H - \theta_M)\right) \cos(\theta_H - \theta_M) + \frac{1}{2} \left(4piM_{eff} \cos(2\theta_M) - H_{res} \cos(\theta_H - \theta_M)\right) \cos(\theta_H - \theta_M) \right) \right|} + \frac{d4piMeff}{2} \left| \frac{\sin(2\theta_M)}{\sin(\theta_H - \theta_M)} \right| + \frac{dThetaH \left|4piM_{eff} \sin(2\theta_M) \cos(\theta_H - \theta_M)\right|}{2 \sin^2(\theta_H - \theta_M)} \right)$$

Now can solve for the Gilbert damping parameter, alpha

In [17]:
```
D_in_expr = alpha*(H_1 + H_2)/abs(Derivative((omega/gamma), H_res))
D_ex_expr = abs(Derivative(H_res, Meff))*Delta4piMeff + abs(Derivative(H_res, thetaH))*DeltaThetaH
D_pp_expr = (D_in_expr + D_ex_expr)/sqrt(3)
Eq(DeltaH_pp, D_pp_expr)
```

Out[17]:

$$\Delta H_{pp} = \frac{\sqrt{3}}{3} \left(\frac{\alpha (H_1 + H_2)}{\left| \frac{\partial}{\partial H_{res}} \left(\frac{\omega}{\gamma}\right) \right|} + d4piMeff \left| \frac{d}{d4piM_{eff}} H_{res} \right| + dThetaH \left| \frac{d}{d\theta_H} H_{res} \right| \right)$$

In [18]:
```
alpha_expr = solve(Eq(DeltaH_pp, D_pp_expr), alpha)[0]
Eq(alpha, alpha_expr)
```

Out[18]:

$$\alpha = \frac{\left|\frac{\partial}{\partial H_{res}} \left(\frac{\omega}{\gamma}\right)\right|}{H_1 + H_2} \left(\sqrt{3}\Delta H_{pp} - d4piMeff \left|\frac{d}{d4piM_{eff}} H_{res}\right| - dThetaH \left|\frac{d}{d\theta_H} H_{res}\right|\right)$$

In [19]:
```
alpha_eqn = solve(Eq(DeltaH_pp, deltaH_pp), alpha)[0]
Eq(alpha, alpha_eqn)
```

Out[19]:

$$\alpha = \frac{1}{4\left(3 \cdot 4piM_{eff} \cos^2(\theta_M) - 4piM_{eff} - 2H_{res} \cos(\theta_H - \theta_M)\right) \sin^2(\theta_H - \theta_M)} \left(-2\sqrt{3}\Delta H_{pp} \sin^2(\theta_H - \theta_M) + d4piMeff \sin^2(\theta_H - \theta_M) \left|\frac{\sin(2\theta_M)}{\sin(\theta_H - \theta_M)}\right| + dThetaH \left|4piM_{eff} \sin(2\theta_M) \cos(\theta_H - \theta_M)\right|\right) \left|\frac{\left(-3 \cdot 4piM_{eff} \cos^2(\theta_M) + 4piM_{eff} + 2H_{res} \cos(\theta_H - \theta_M)\right) \cos(\theta_H - \theta_M)}{\sqrt{2 \cdot 4piM_{eff}^2 \cos^4(\theta_M) - 4piM_{eff}^2 \cos^2(\theta_M) - 3 \cdot 4piM_{eff} H_{res} \cos^2(\theta_M) \cos(\theta_H - \theta_M) + 4piM_{eff} H_{res} \cos(\theta_H - \theta_M) + H_{res}^2 \cos^2(\theta_H - \theta_M)}}\right|$$

In [19]: