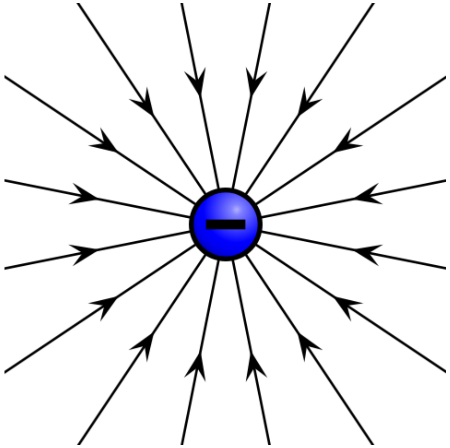
Interaction with Matter

P47 – Optics: Unit 2

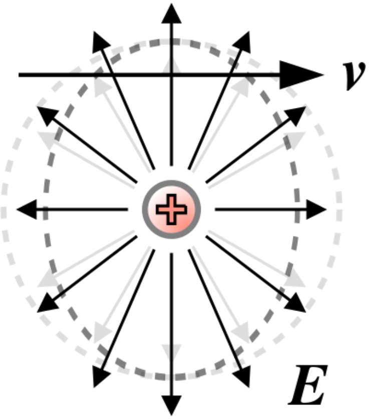
Electric Field of Point Charge

stationary

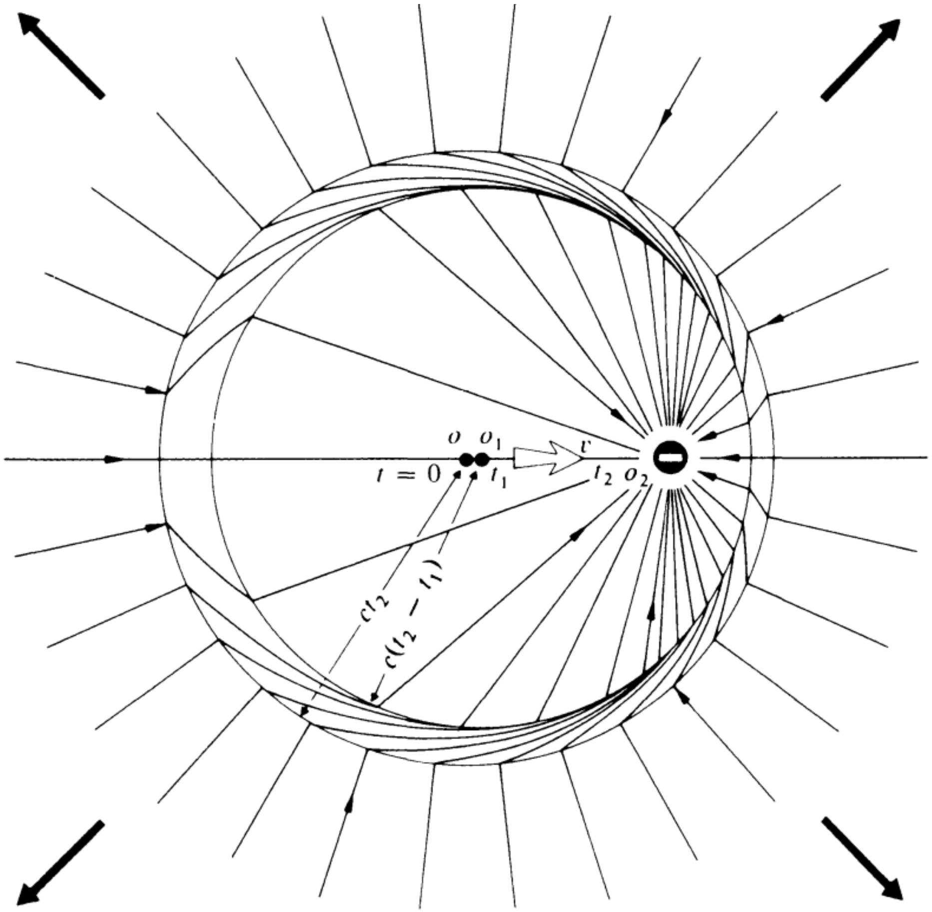


$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{|\vec{r}|^2}\hat{r}$$

constant velocity



Accelerating charges radiate!

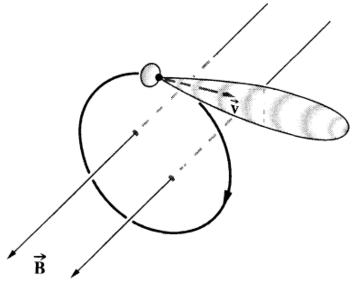


- t = 0: stationary charge ($E_C \sim 1/r^2$)
- t = 0→t_1: constant acceleration
- t = t_1→t_2: constant velocity

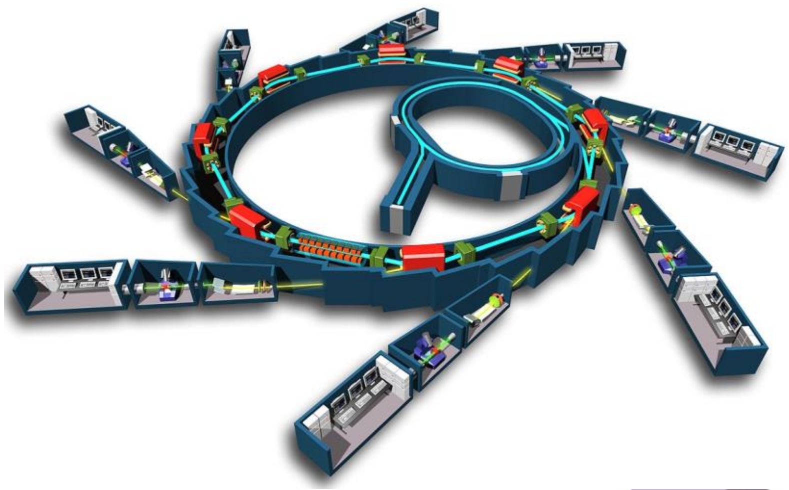
- all field lines must be connected (Gauss' Law)
- produces *transverse* E-field components during acceleration
- transverse components propagate outward ($E_T \sim 1/r$)

$$E_T(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{a}{c^2}\sin(\theta)\frac{q}{r}$$

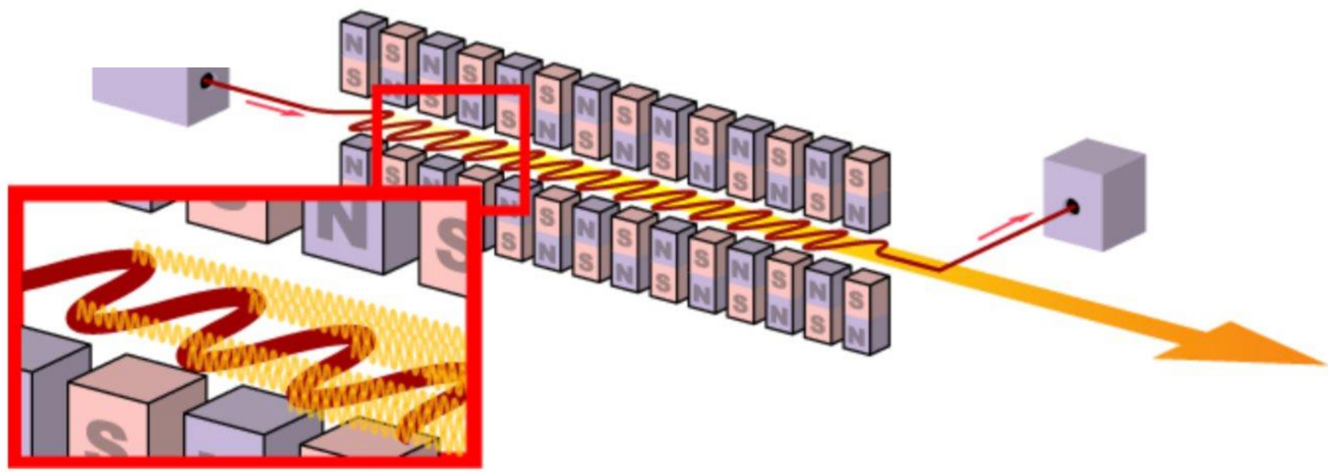
Why aren't E-field lines at O_2 radially symmetric?



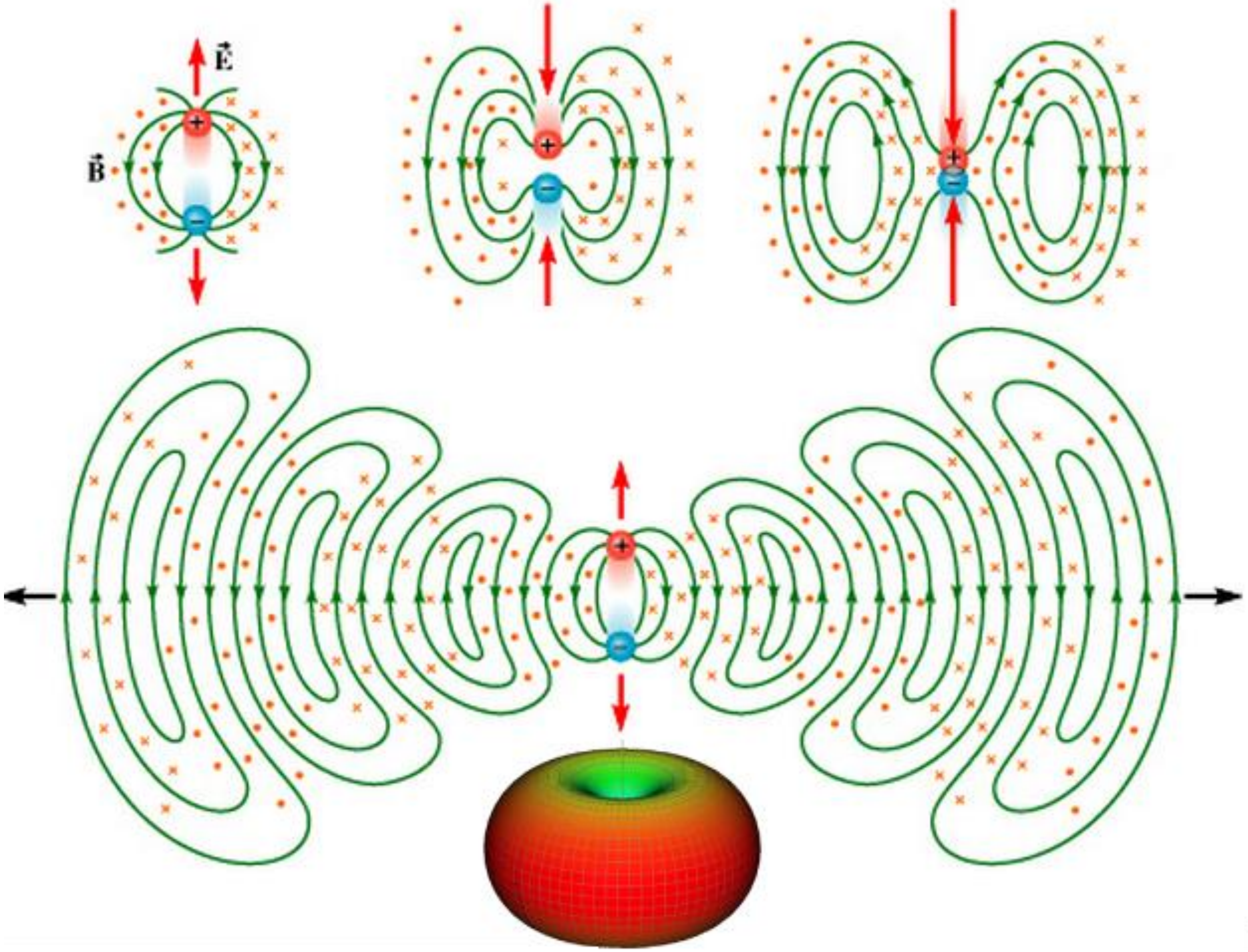
$$\omega_{cyc} = \frac{qB_0}{m}$$



Spring-8 synchotron & SACLA free-electron laser - Japan



…but in optics, normally only have dipole radiation



Electromagnetic Radiation from Oscillating Dipole

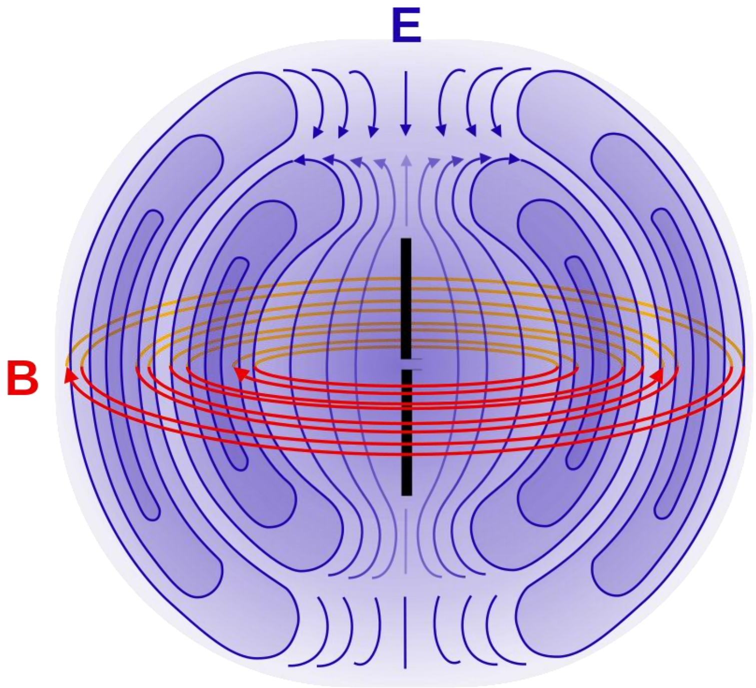
[credit: universe-review.ca]

…but in optics, normally only have dipole radiation

$$\boldsymbol{p}(t) = \hat{\boldsymbol{z}}\, p_0 \cos(\omega t) = \hat{\boldsymbol{z}}\, qd \cos(\omega t)$$

In the “far field” (r>>d) the radiated field is:

$$\boldsymbol{E}(r,\theta,t) = \frac{p_0 k^2 \sin\theta}{4\pi\epsilon_0} \frac{\cos(kr-\omega t)}{r} \hat{\boldsymbol{\theta}}$$



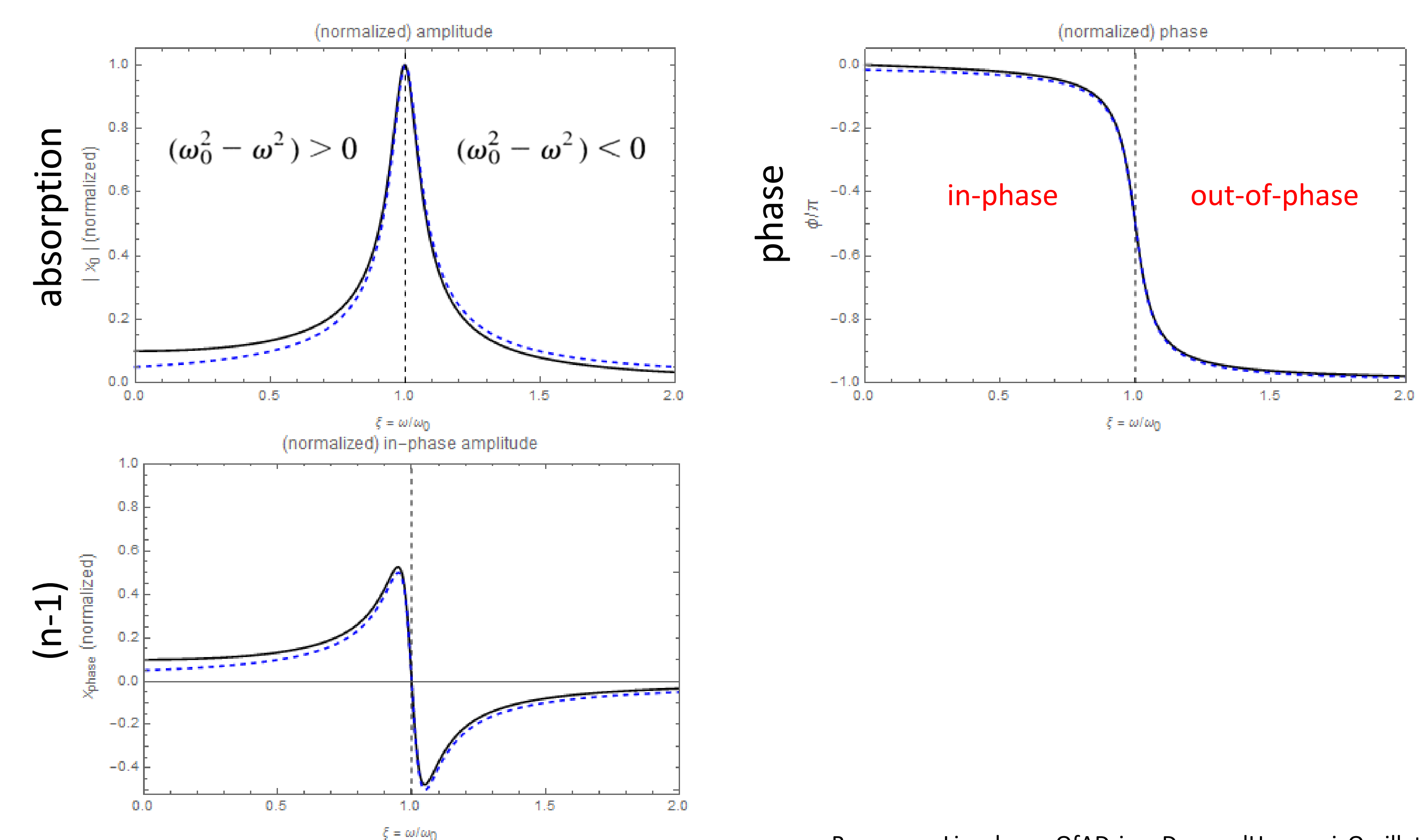
Radiation pattern of a dipole antenna

Note: No radiation at ϑ=0. This will matter later!

Lorentz Oscillator Resonance

$$n^2(\omega) = 1 + \frac{Nq_e^2}{\epsilon_0 m_e} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma_j\omega}$$

quality factor Q = $\omega_0/2\gamma$ 10.



ResonanceLineshapesOfADrivenDampedHarmonicOscillator.cdf

A material usually has multiple resonant frequencies and/or oscillator species

$$\frac{n^2-1}{n^2+2} = \frac{Nq_e^2}{3\epsilon_0 m_e} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma_j\omega} \qquad (3.73)$$

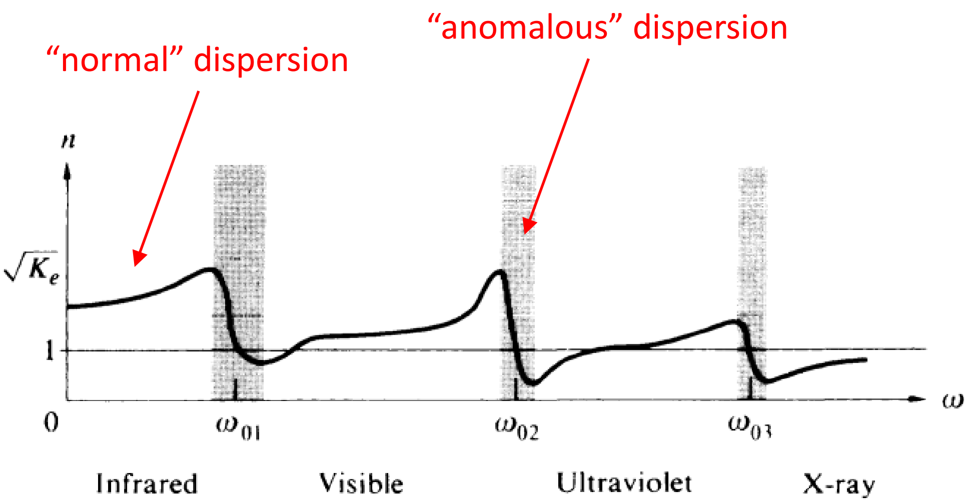
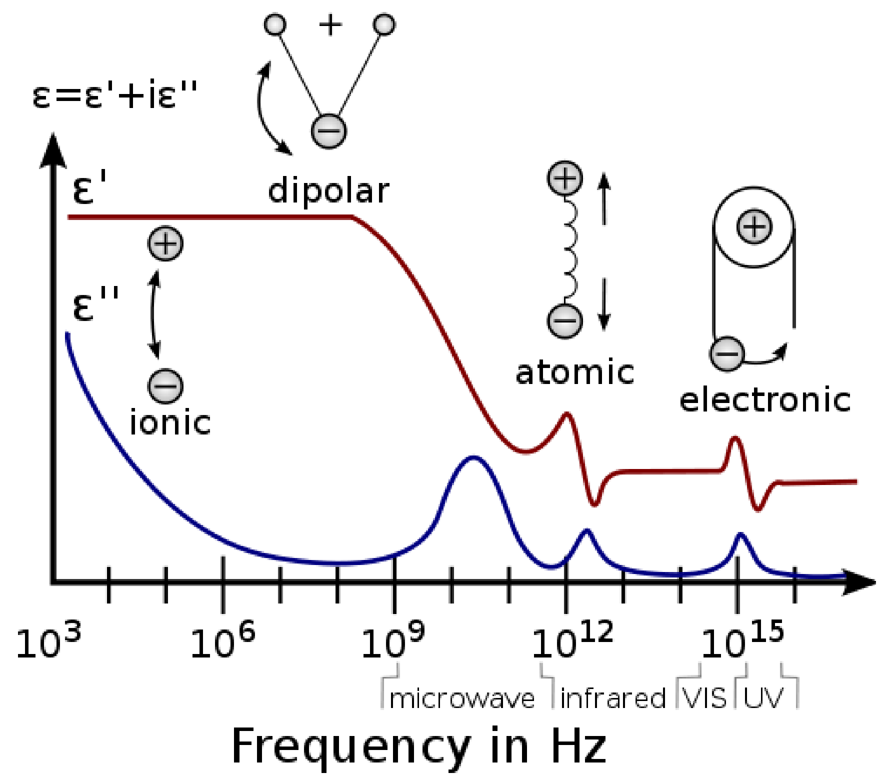
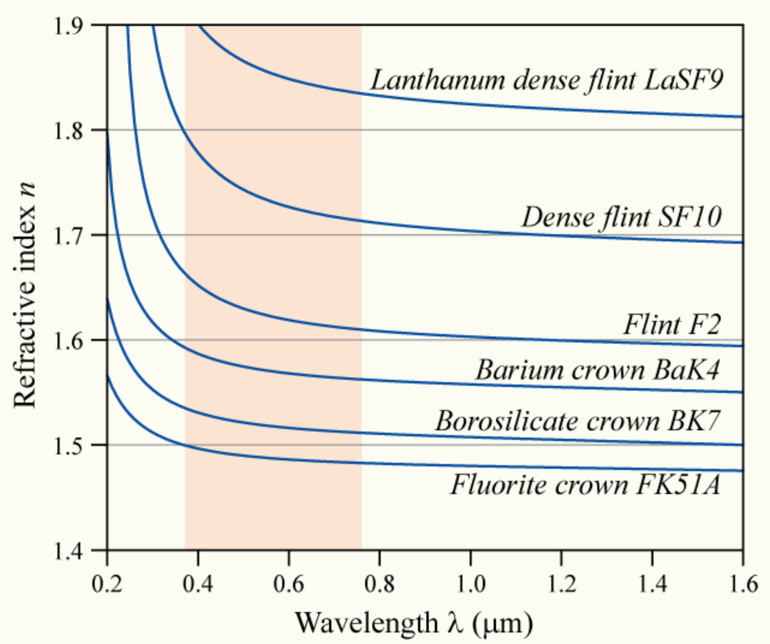
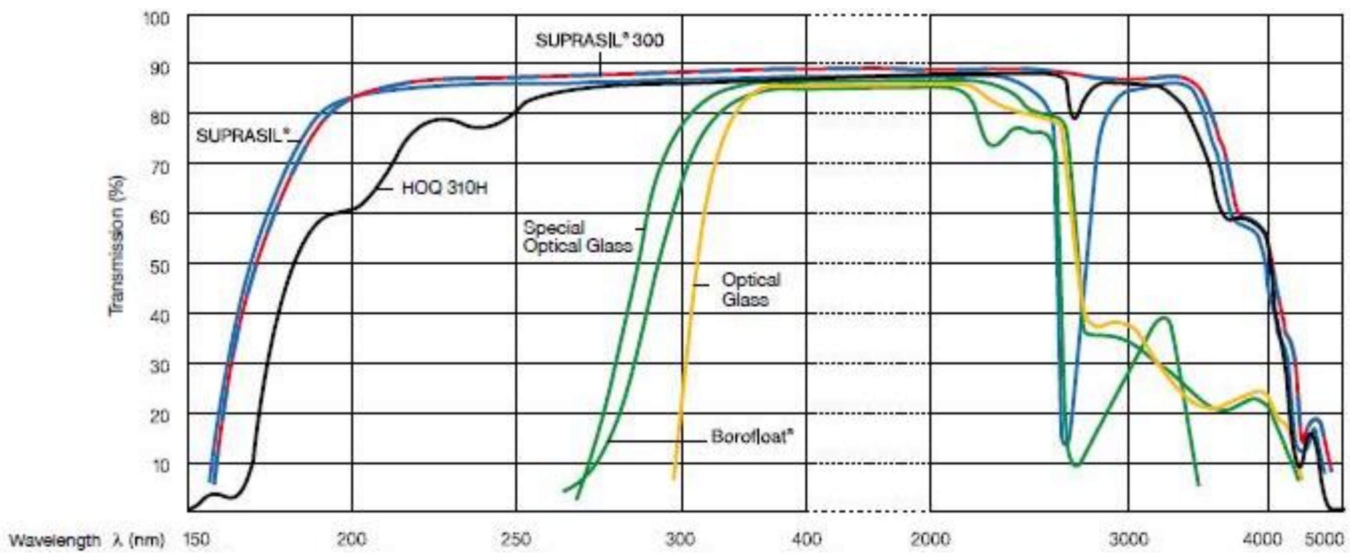


Figure 3.41 Refractive index versus frequency.



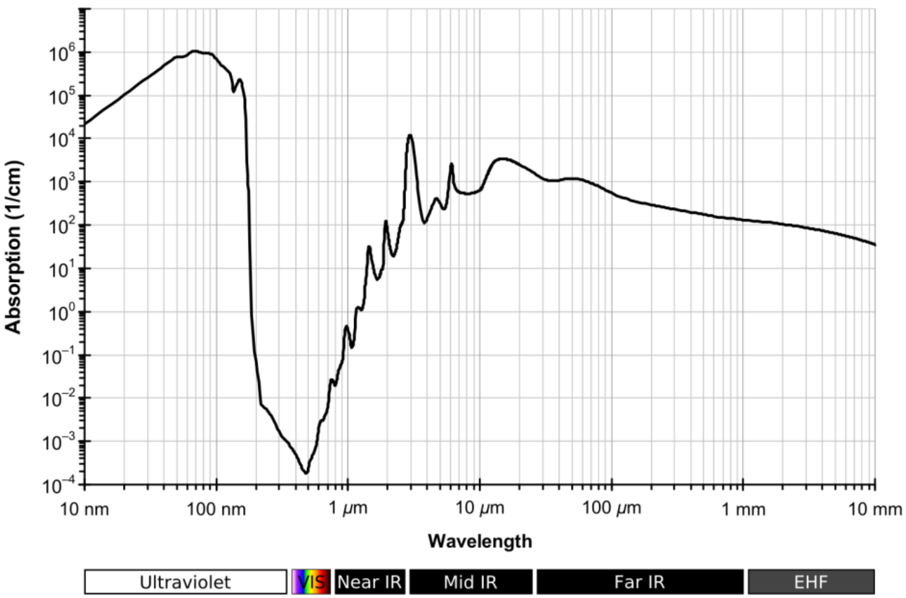
"Normal" Dispersion of Different Glasses

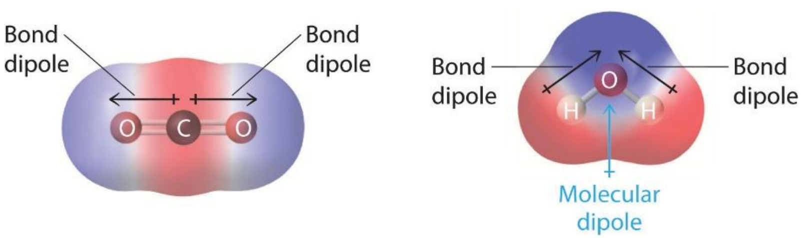




Absorption characteristics depend a lot on the energy level structure of the material

Water

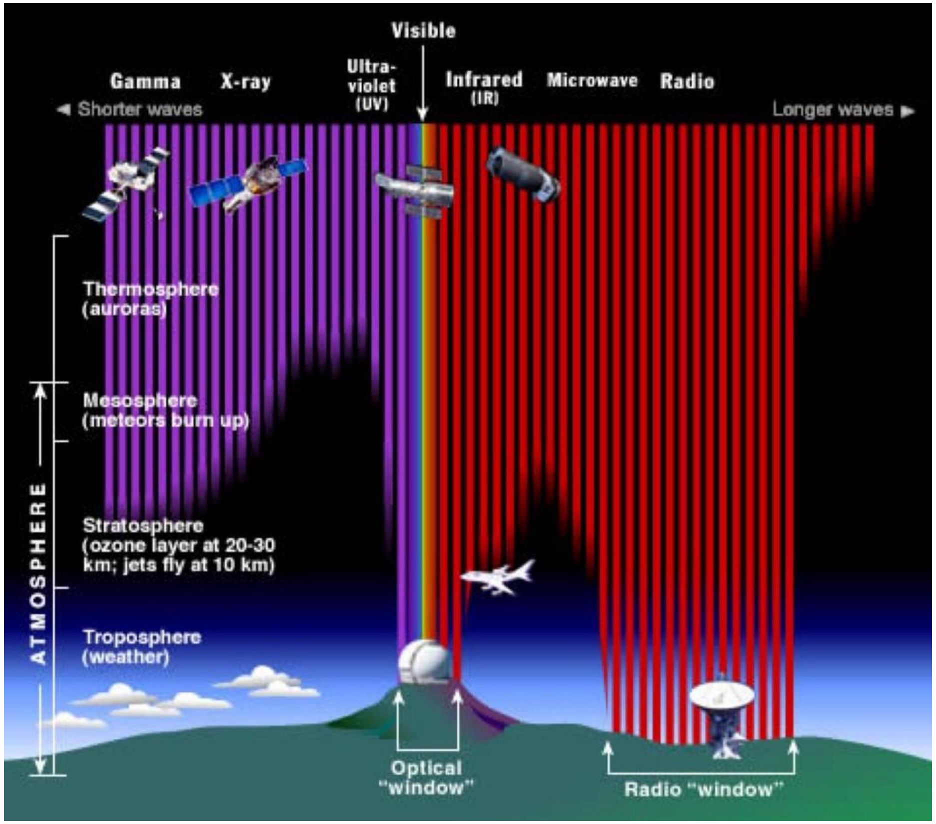




(a) No net dipole moment

(b) Net dipole moment

Atmosphere



Interaction of light with matter depends on:

- wavelength/frequency of light
- polarization
- atomic/molecular structure of the matter
- amount of incident EM energy

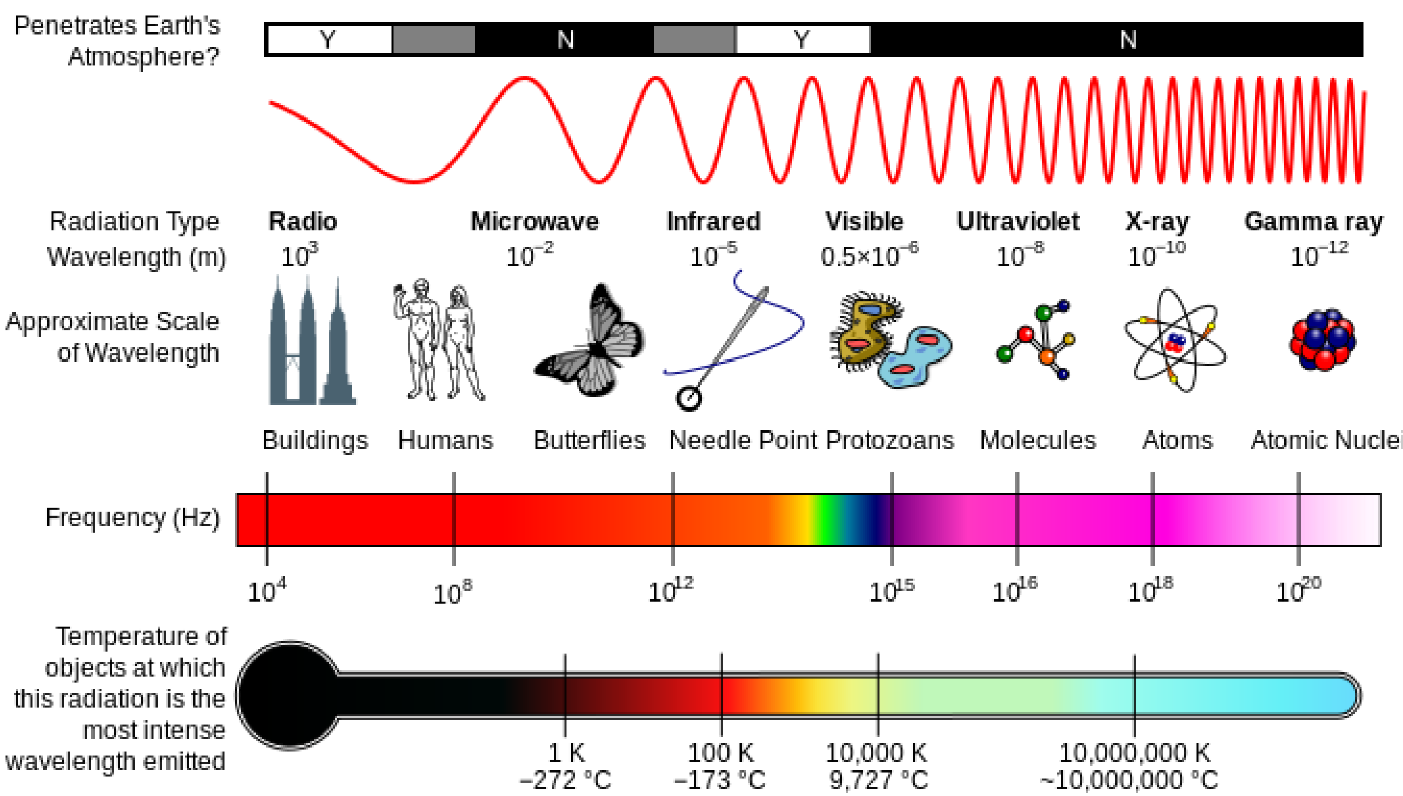


Image: http://en.wikipedia.org/wiki/File:EM_Spectrum_Properties_edit.svg

Optical Scattering

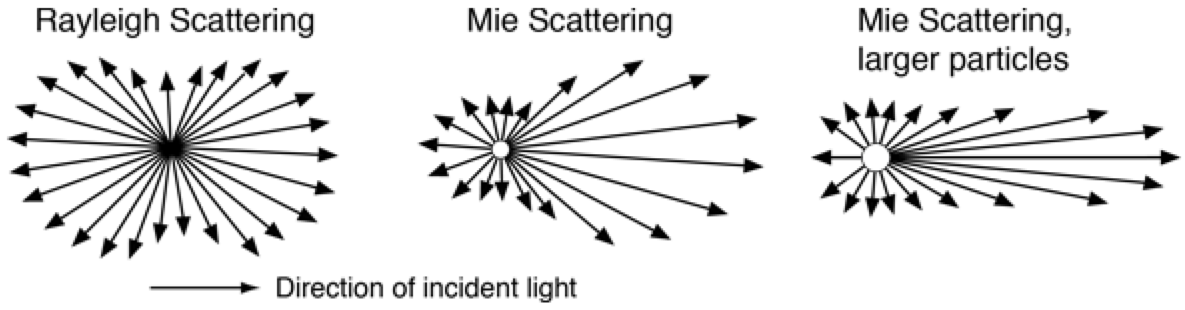
- Scattering by a polarizable particle

$$I(\theta) = \frac{d^2\omega^4}{32\pi^2 c^3} \frac{(\sin\theta)^2}{r^2}$$

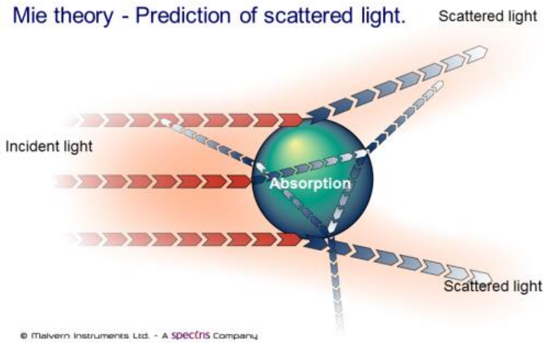
For $\omega<\omega_0$, phase of scattered wave is near 0

For $\omega>\omega_0$, phase of scattered wave is near π

- Interference of scattered waves from many particles depends on their density compared to the wavelength

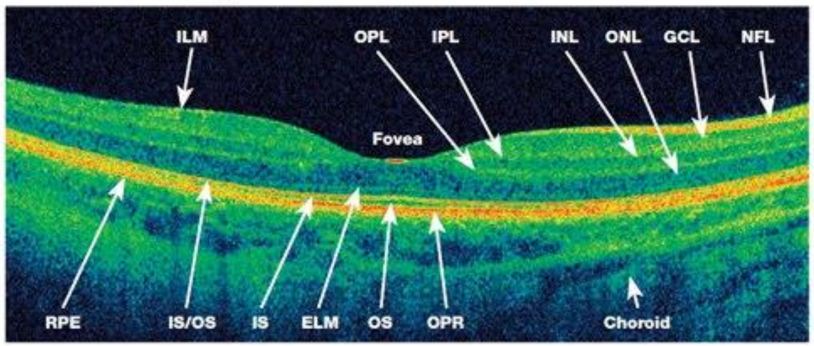


hyperphysics.phy-astr.gsu.edu

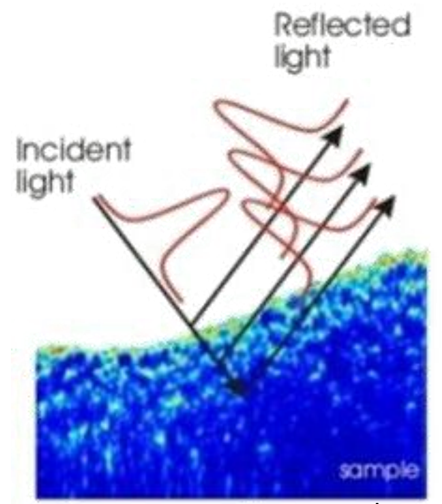


Optical Coherence Tomography

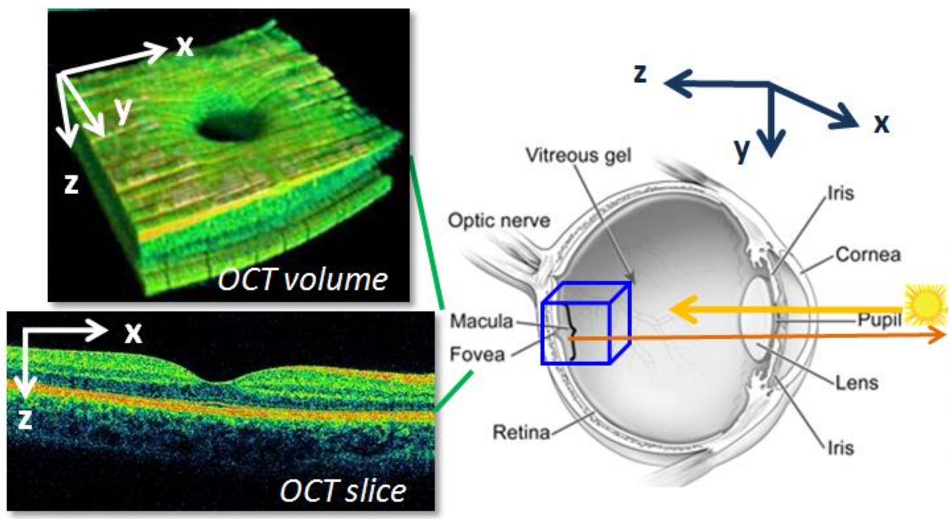
An HD-OCT scan of a healthy eye



NFL: Nerve fiber layer
ILM: Inner limiting membrane
GCL: Ganglion cell layer
IPL: Inner plexiform layer
INL: Inner nuclear layer
OPL: Outer plexiform layer
ONL: Outer nuclear layer
ELM: External limiting membrane
IS: Photoreceptor inner segment
OS: Photoreceptor outer segment
IS/OS: Interface between IS and OS
RPE: Retinal pigment epithelium
OPR: Outer photoreceptor/ RPE complex



www.recendt.at

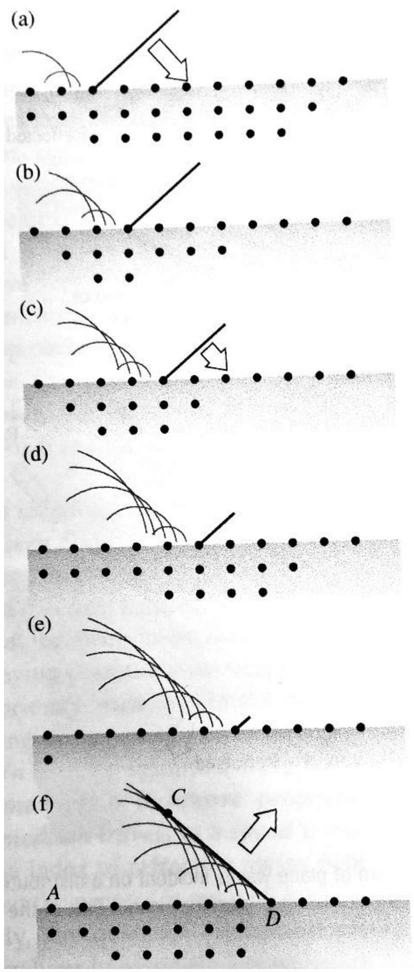


OCT machine

www.cc.gatech.edu

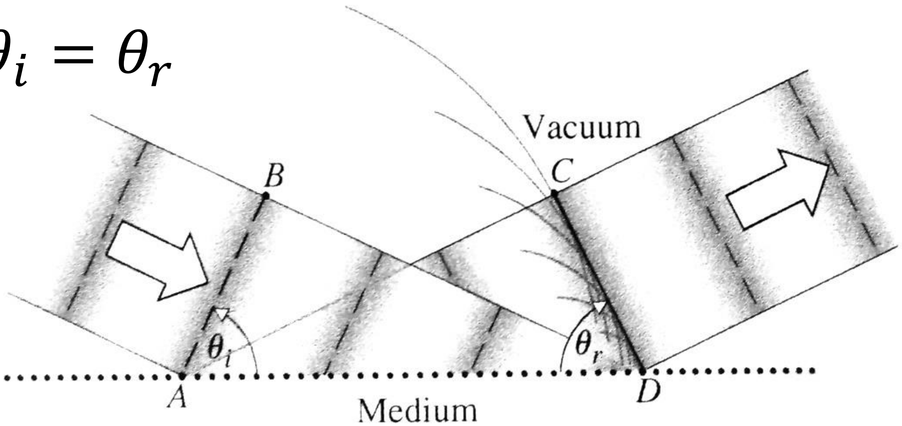
[see demo video: http://www.youtube.com/watch?v=pl-d8zE_Mhw]

Reflection

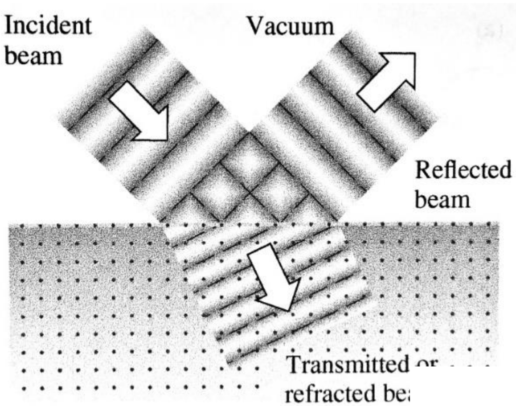


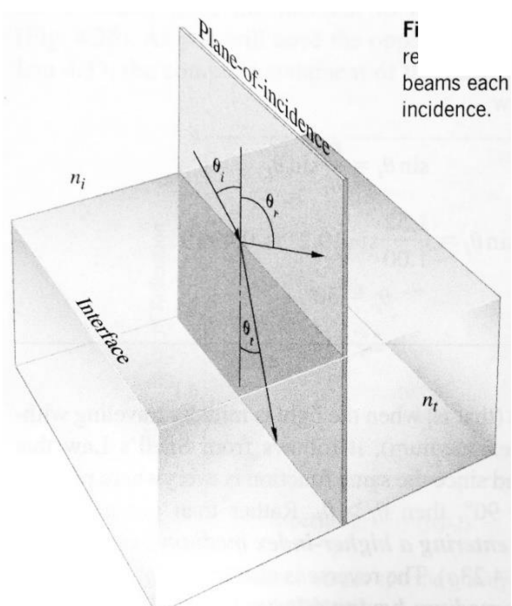
Law of Reflection

$$\theta_i = \theta_r$$



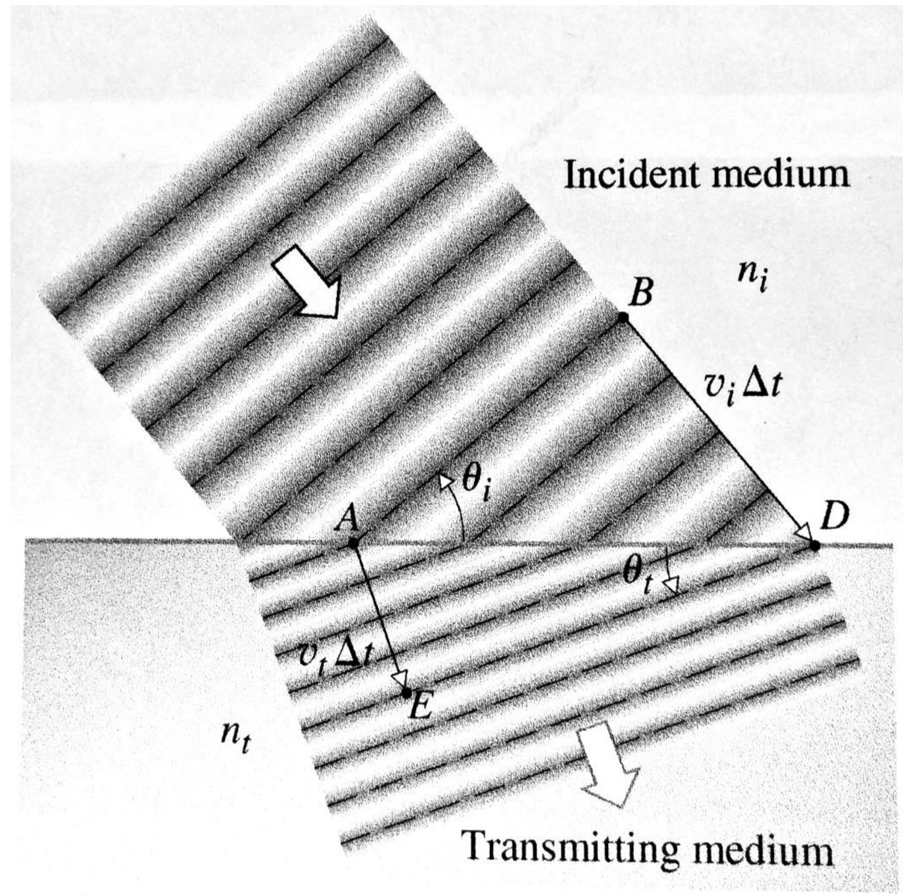
Refraction/Transmission





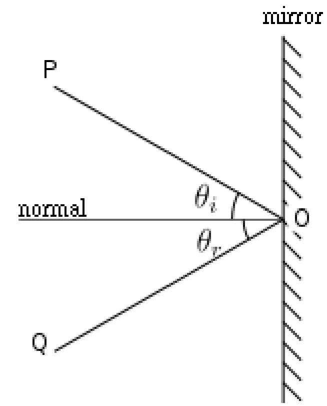
Law of Refraction (Snell's Law)

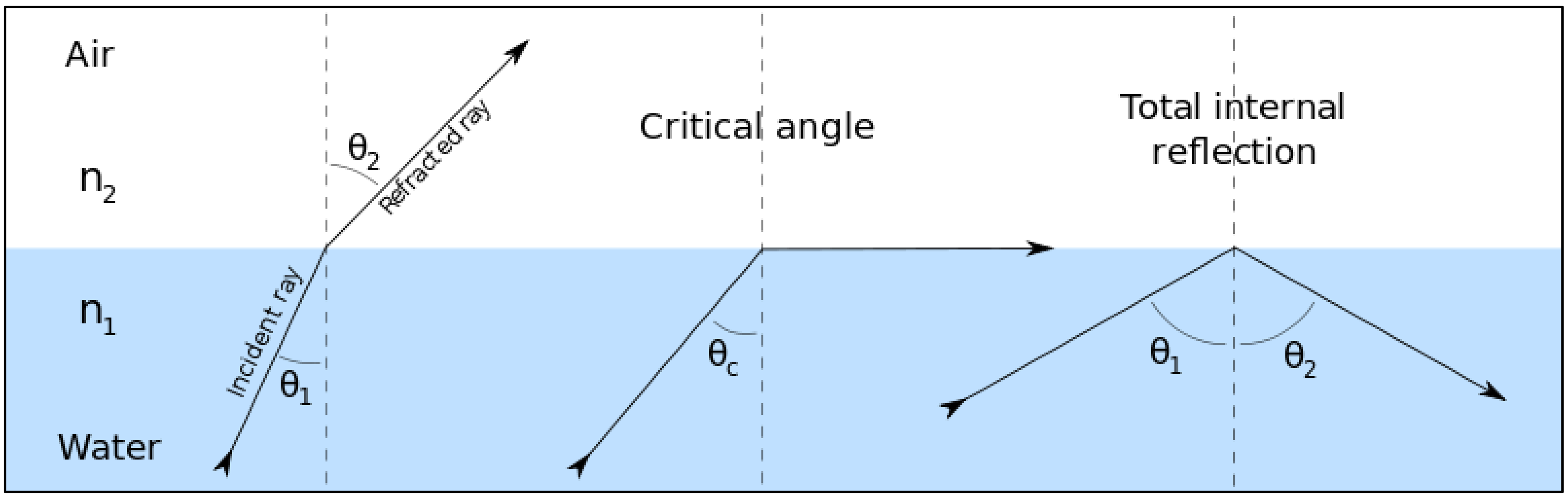
$$n_i \sin\theta_i = n_t \sin\theta_t$$



Reflection and Refraction

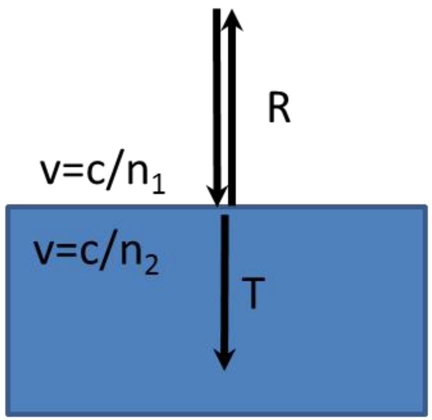
- Law of Reflection $\theta_i = \theta_r$
- Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$





Wikipedia

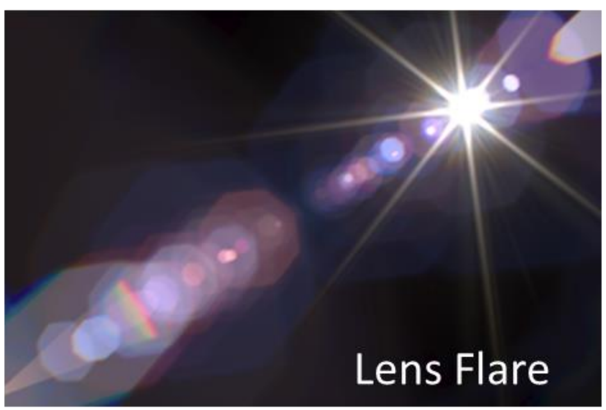
Reflection and Transmission at Normal Incidence



$$r_\perp = r_\parallel = \frac{n_t - n_i}{n_t + n_i}$$

$$t_\perp = t_\parallel = \frac{2n_i}{n_t + n_i}$$

Reflection off a single air-glass (n≈1.5) interface is about 4%



Wikimedia

$$R_\perp = R_\parallel = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

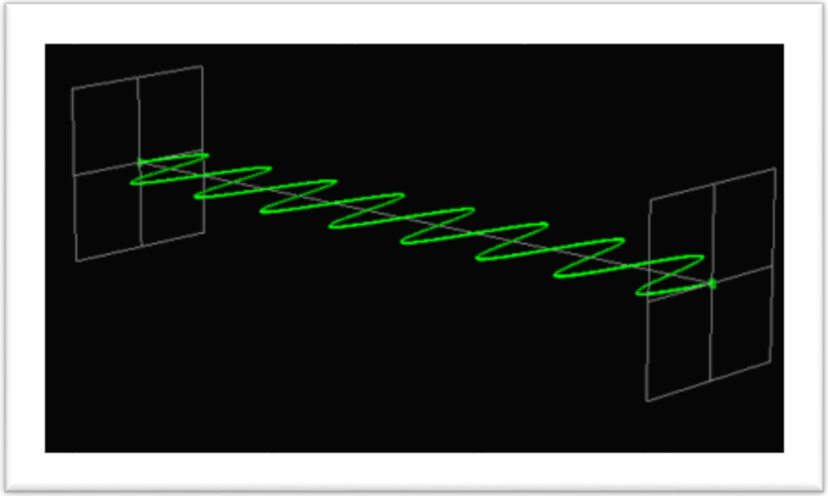
$$T_\perp = T_\parallel = \frac{4n_t n_i}{(n_t + n_i)^2}$$

EM waves (light) as a *vector* field, not a *scalar*

So far, we've mostly been considering waves like: $\boldsymbol{E}(z,t) = E\cos(kz - \omega t)$

We need to think about the more general case where the fields are vectors:

$$\boldsymbol{E}(z,t) = \hat{\boldsymbol{\imath}} E_{0x}(r,t)\cos(kz - \omega t + \varphi_x)$$



$$\boldsymbol{E}(z,t) = \hat{\boldsymbol{\jmath}}\, E_{0y}(r,t)\cos\left(kz - \omega t + \varphi_y\right)$$

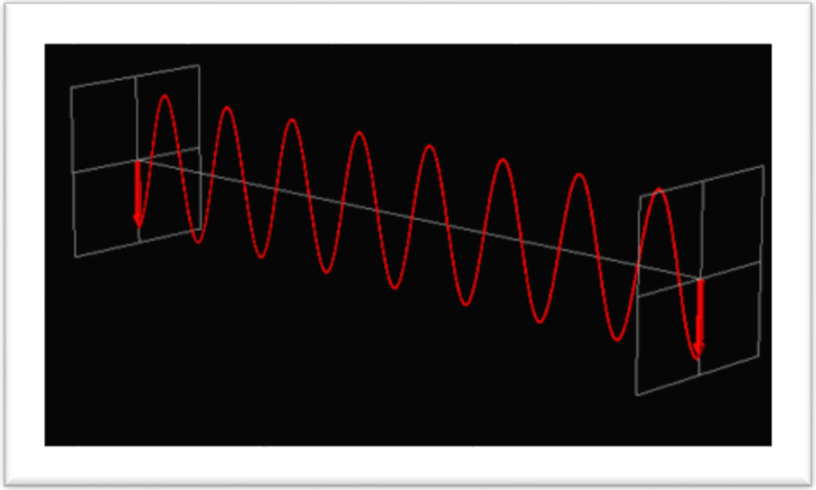
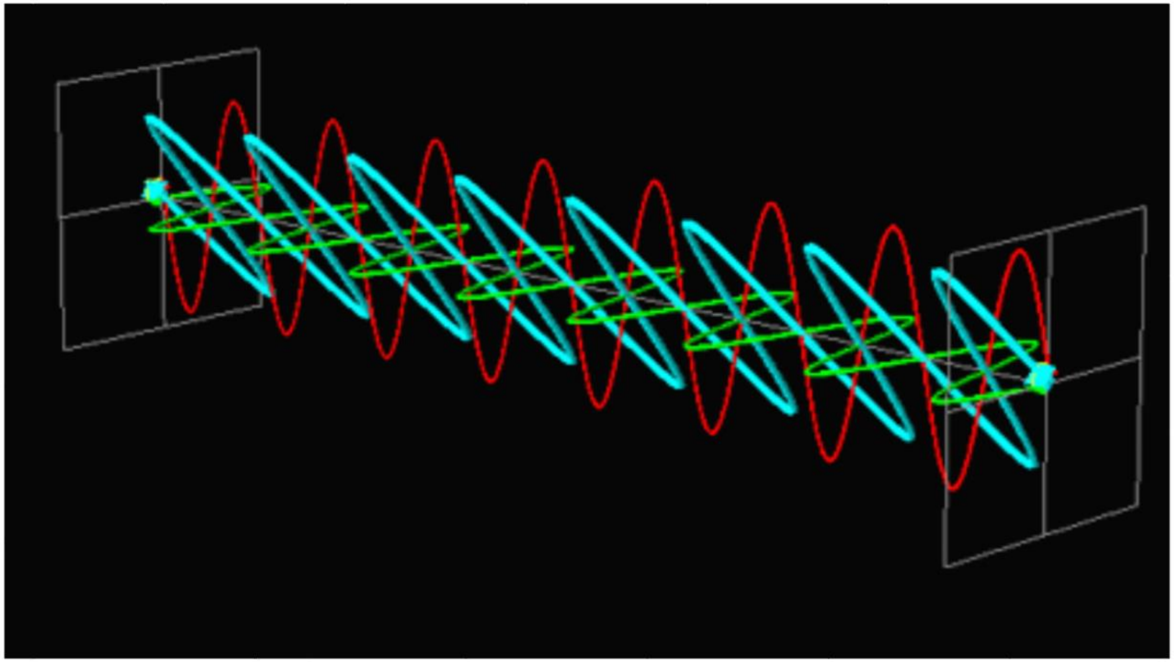


image credit: www.photophysics.com

Arbitrary Polarization with Superposition

$$\boldsymbol{E}(z,t) = \hat{\boldsymbol{\imath}} E_{0x}(r,t)\cos(kz - \omega t + \varphi_x) + \hat{\boldsymbol{\jmath}}\, E_{0y}(r,t)\cos\left(kz - \omega t + \varphi_y\right)$$



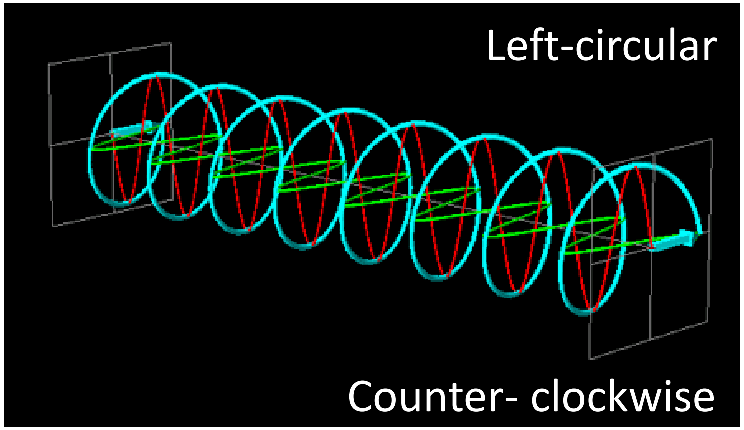
Circular Polarization

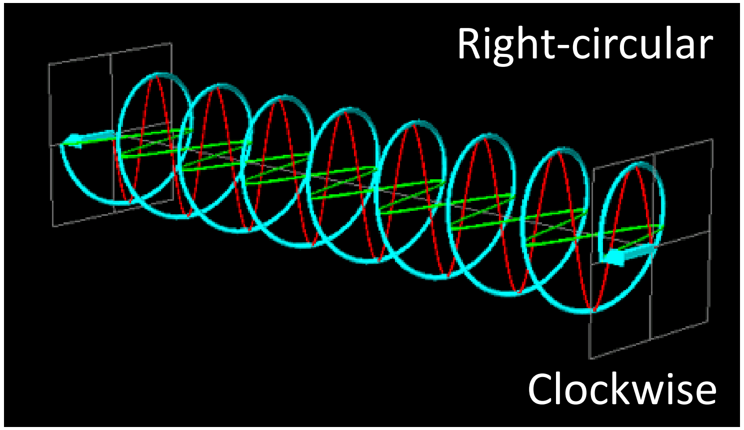
$$\boldsymbol{E}(z,t) = \hat{\boldsymbol{\imath}} E_{0x}(r,t)\cos(kz - \omega t + \varphi_x) + \hat{\boldsymbol{\jmath}}\, E_{0y}(r,t)\cos\left(kz - \omega t + \varphi_y\right)$$

if $E_{0x} = E_{0y} = E_0$ and $\varphi_x = \varphi_y + 2m\pi$ the two vector components are 90 degrees out of phase, and we can write the resultant wave as:

$$\boldsymbol{E}(z,t) = E_0\ (\ \hat{\boldsymbol{\imath}}\cos(kz - \omega t) \pm \hat{\boldsymbol{\jmath}}\ \sin(kz - \omega t)\ )$$

The electric field vector has a constant magnitude, but is rotating!



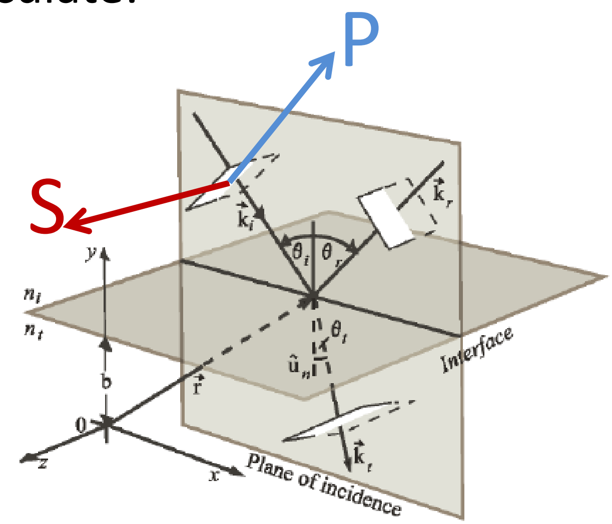


[more about this in Unit 5]

Dielectric Interfaces

Given incident wave amplitude, direction, polarization,
for the reflected and transmitted beams we can calculate:

- direction of propagation
- field amplitudes (or power, if desired)



There are some surprises hiding in the details…

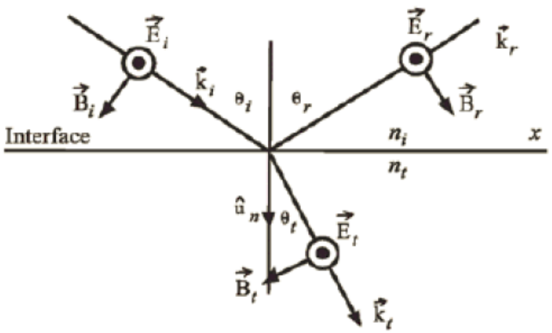
- What about the phase of the light after reflection/transmission?
- What happens to the transmitted light when the incident angle exceeds the critical angle?

Evanescent Waves

Clarification: Waveguide nomenclature for field orientation

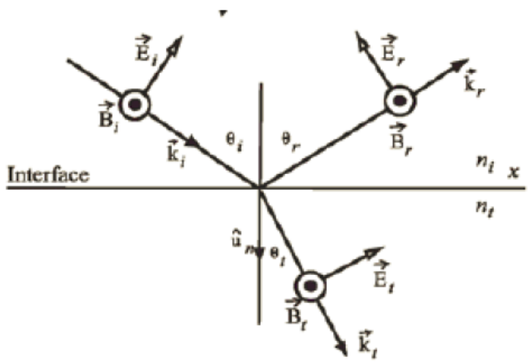
Transverse Magnetic: TM	Transverse Electric: TE
P-polarized	S-polarized
$E^{(p)} = E_{//}$	$E^{(s)} = E_{\perp}$

The Fresnel Equations



$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

These are electric field amplitude coefficients

The Fresnel Equations

These expressions can be written in a variety of ways, each one useful under different circumstances

[Note: The angles are *not* independent of each other and are constrained by Snell's law]

$$r_s \equiv \frac{E_\mathrm{r}^{(s)}}{E_\mathrm{i}^{(s)}} = \frac{\sin\theta_\mathrm{t}\cos\theta_\mathrm{i} - \sin\theta_\mathrm{i}\cos\theta_\mathrm{t}}{\sin\theta_\mathrm{t}\cos\theta_\mathrm{i} + \sin\theta_\mathrm{i}\cos\theta_\mathrm{t}} = -\frac{\sin(\theta_\mathrm{i}-\theta_\mathrm{t})}{\sin(\theta_\mathrm{i}+\theta_\mathrm{t})} = \frac{n_\mathrm{i}\cos\theta_\mathrm{i} - n_\mathrm{t}\cos\theta_\mathrm{t}}{n_\mathrm{i}\cos\theta_\mathrm{i} + n_\mathrm{t}\cos\theta_\mathrm{t}} \tag{3.18}$$

$$t_s \equiv \frac{E_\mathrm{t}^{(s)}}{E_\mathrm{i}^{(s)}} = \frac{2\sin\theta_\mathrm{t}\cos\theta_\mathrm{i}}{\sin\theta_\mathrm{t}\cos\theta_\mathrm{i} + \sin\theta_\mathrm{i}\cos\theta_\mathrm{t}} = \frac{2\sin\theta_\mathrm{t}\cos\theta_\mathrm{i}}{\sin(\theta_\mathrm{i}+\theta_\mathrm{t})} = \frac{2n_\mathrm{i}\cos\theta_\mathrm{i}}{n_\mathrm{i}\cos\theta_\mathrm{i} + n_\mathrm{t}\cos\theta_\mathrm{t}} \tag{3.19}$$

$$r_p \equiv \frac{E_\mathrm{r}^{(p)}}{E_\mathrm{i}^{(p)}} = \frac{\cos\theta_\mathrm{t}\sin\theta_\mathrm{t} - \cos\theta_\mathrm{i}\sin\theta_\mathrm{i}}{\cos\theta_\mathrm{t}\sin\theta_\mathrm{t} + \cos\theta_\mathrm{i}\sin\theta_\mathrm{i}} = -\frac{\tan(\theta_\mathrm{i}-\theta_\mathrm{t})}{\tan(\theta_\mathrm{i}+\theta_\mathrm{t})} = \frac{n_\mathrm{i}\cos\theta_\mathrm{t} - n_\mathrm{t}\cos\theta_\mathrm{i}}{n_\mathrm{i}\cos\theta_\mathrm{t} + n_\mathrm{t}\cos\theta_\mathrm{i}} \tag{3.20}$$

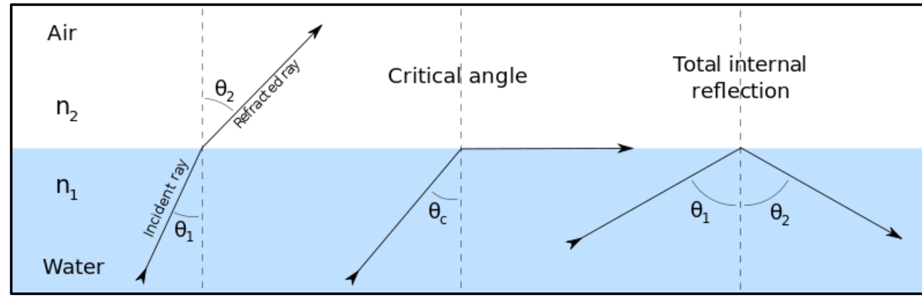
$$t_p \equiv \frac{E_\mathrm{t}^{(p)}}{E_\mathrm{i}^{(p)}} = \frac{2\cos\theta_\mathrm{i}\sin\theta_\mathrm{t}}{\cos\theta_\mathrm{t}\sin\theta_\mathrm{t} + \cos\theta_\mathrm{i}\sin\theta_\mathrm{i}} = \frac{2\cos\theta_\mathrm{i}\sin\theta_\mathrm{t}}{\sin(\theta_\mathrm{i}+\theta_\mathrm{t})\cos(\theta_\mathrm{i}-\theta_\mathrm{t})} = \frac{2n_\mathrm{i}\cos\theta_\mathrm{i}}{n_\mathrm{i}\cos\theta_\mathrm{t} + n_\mathrm{t}\cos\theta_\mathrm{i}} \tag{3.21}$$

The Fresnel Equations

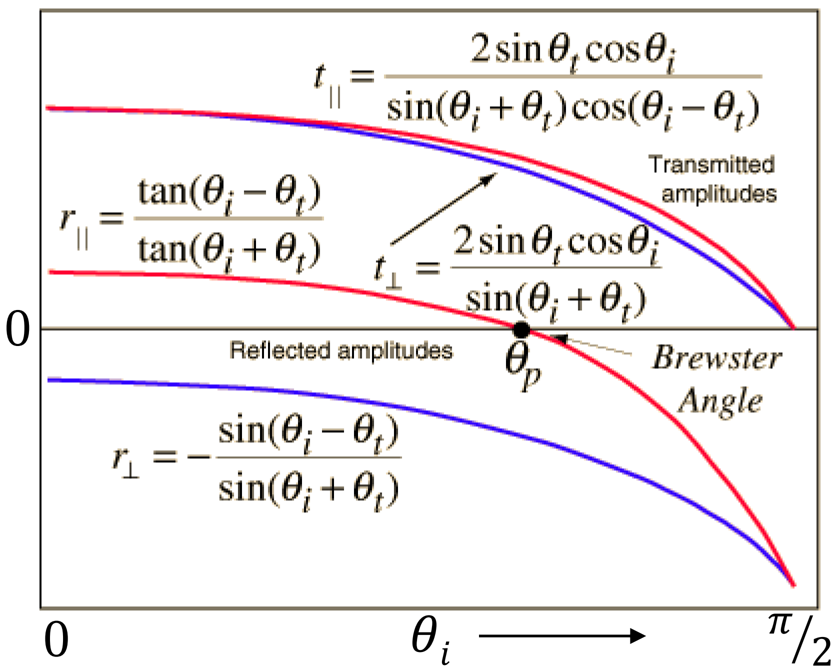
(amplitude coefficients)

$$r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

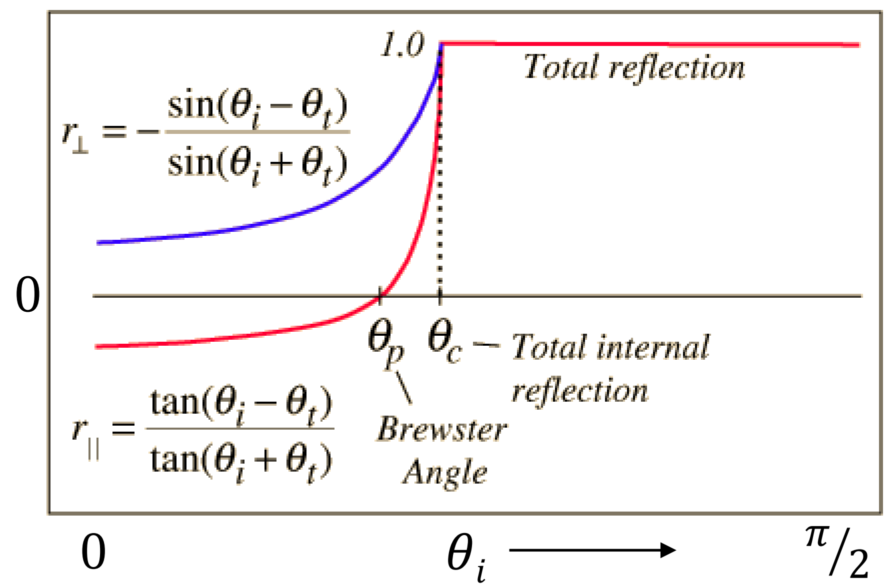
$$t_s = \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad t_p = \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$



$n_i < n_t$

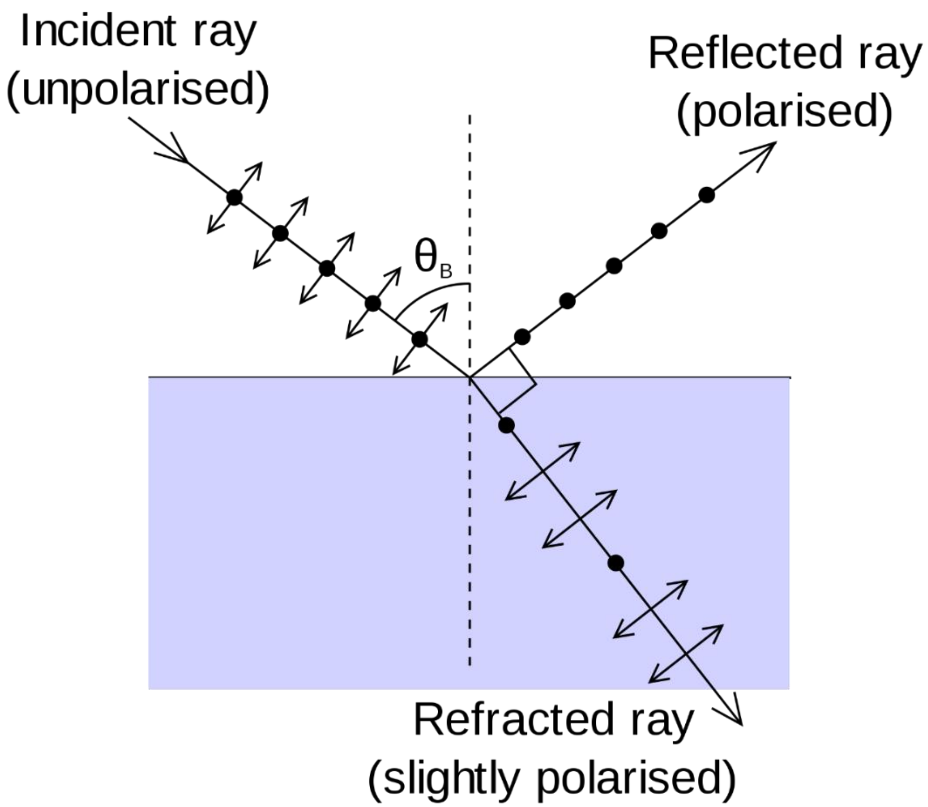


$n_i > n_t$

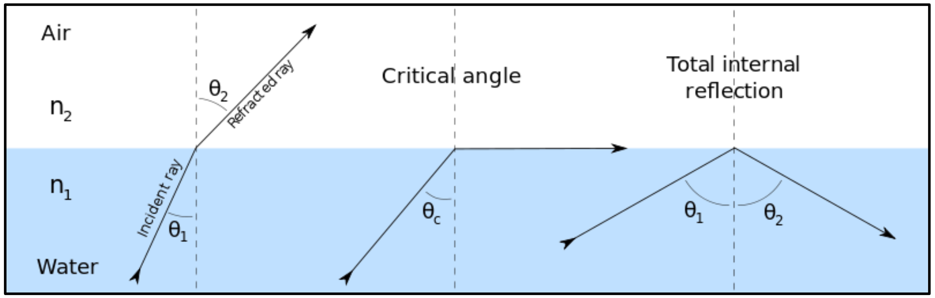


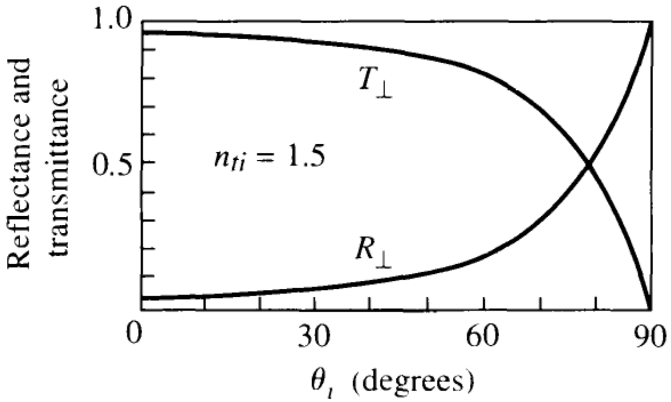
Brewster's Angle

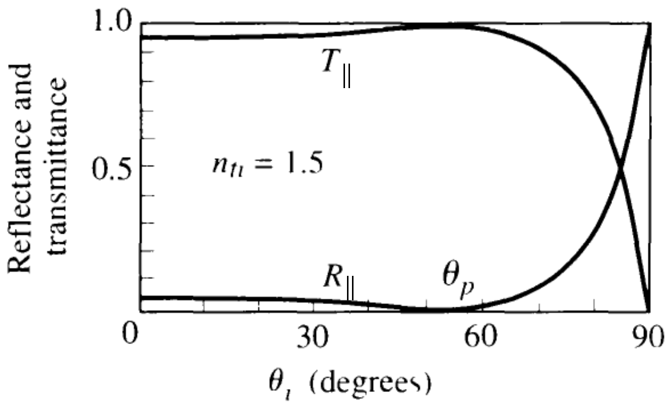
Dipoles cannot emit into the direction of their oscillation



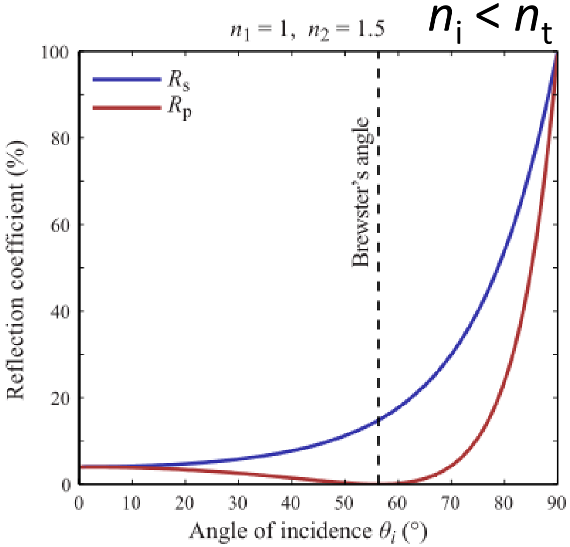
The Fresnel Equations
(power coefficients)



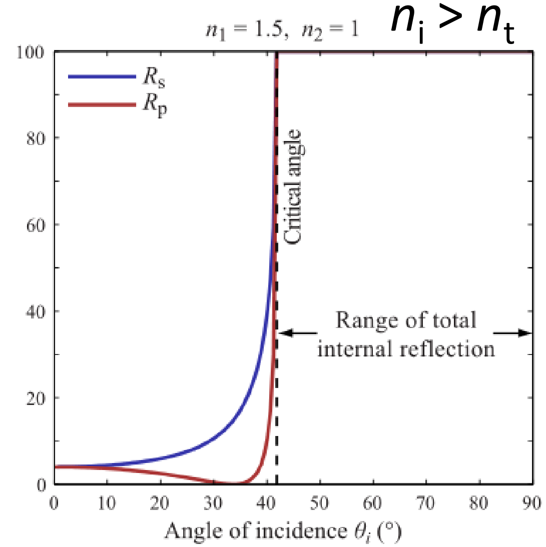




$n_i < n_t$

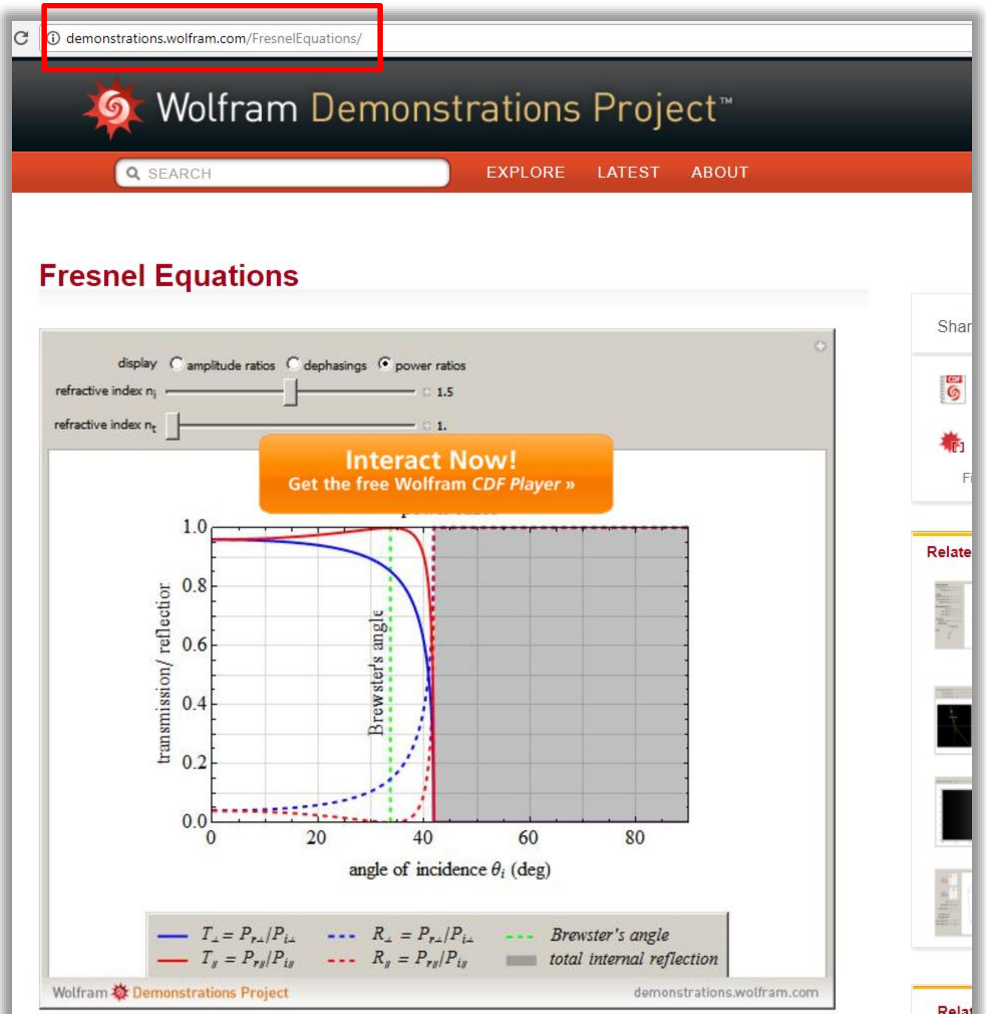


$n_i > n_t$



The Fresnel Equations

(can play around with these in a Mathematica widget)



Phase Shifts at Interfaces (normal incidence)

Transmitted light:

$\Delta\varphi = 0^0$

Reflected light:

If $n_1 < n_2$ then $\Delta\varphi = 180^0$

If $n_1 > n_2$ then $\Delta\varphi = 0^0$

Phase Shifts at Interfaces (arbitrary incidence)

Transmitted light:

$\Delta\varphi = 0^0$ unless $n_1 > n_2$ and $\theta_1 > \theta_{crit}$ (glass to air, TIR)

In which case $0 < \Delta\varphi < 90^0$, and depends on polarization and angle

Reflected light:

If $n_1 < n_2$ then $\Delta\varphi = 180^0$ (s-polarized flips to 0^0 for $\theta_1 > \theta_{brewster}$)

If $n_1 > n_2$ then $\Delta\varphi = 0^0$ (s-polarized flips to 180^0 for $\theta_1 > \theta_{brewster}$)

If $n_1 > n_2$ and $\theta_1 > \theta_{crit}$ (glass to air, TIR) then $0 < \Delta\varphi < 180^0$, and depends on both polarization and angle

Total Internal Reflection

Exponentially decaying
"evanescent" field:

$$E \propto e^{ik_R x} e^{-k_I y}$$

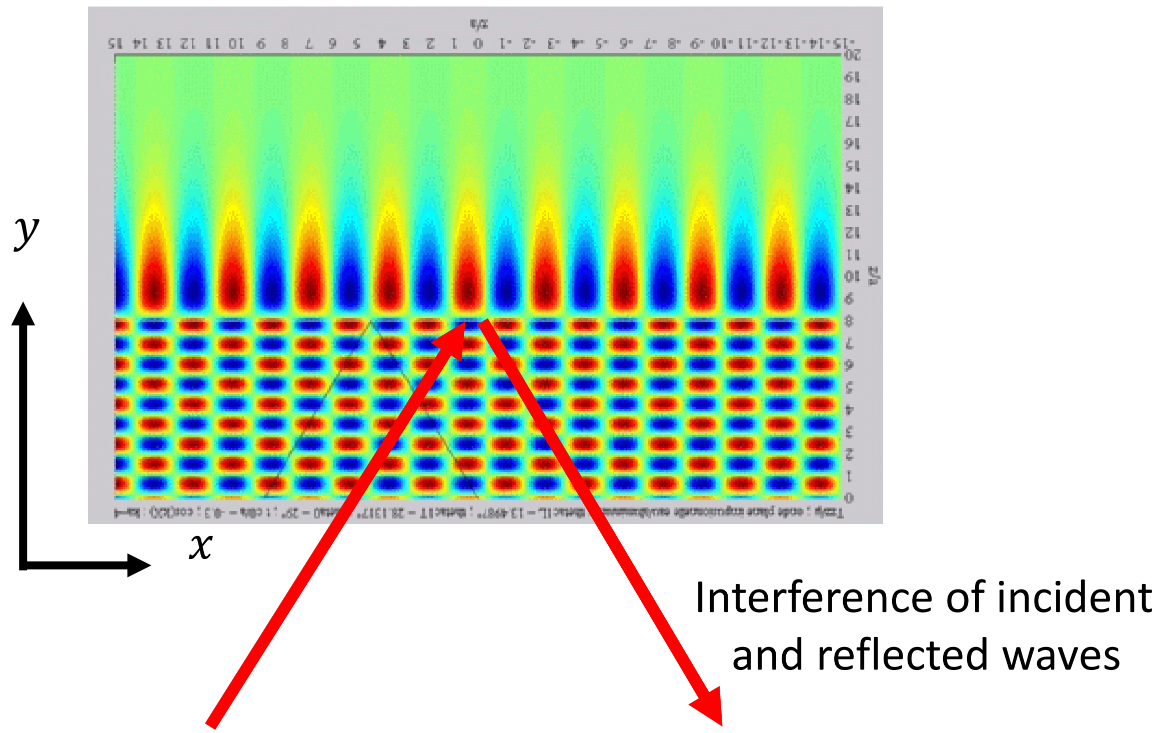
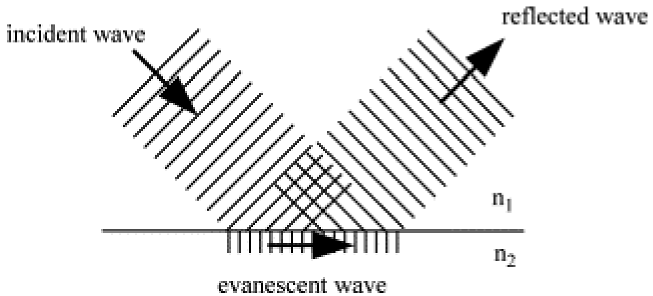


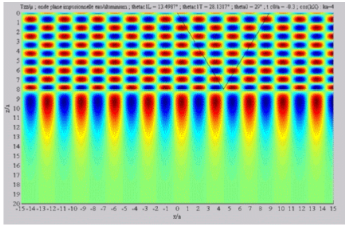
Image Credit: http://perso.univ-lemans.fr/~cpotel

Evanescent Waves

Beyond the critical angle, k_t is imaginary

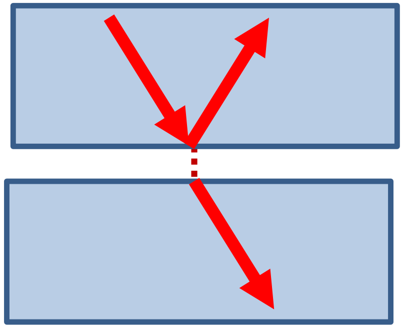
- Exponentially decaying field: $E \sim e^{-kx}$
- Still time-dependent: moves along interface





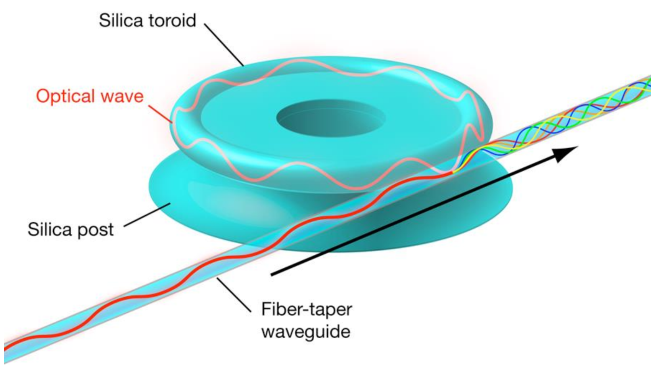
- What if we bring in another glass surface? Photon tunneling!

- Just like a quantum particle tunneling through a classically forbidden barrier.



Evanescent Wave Applications

optical ring resonators

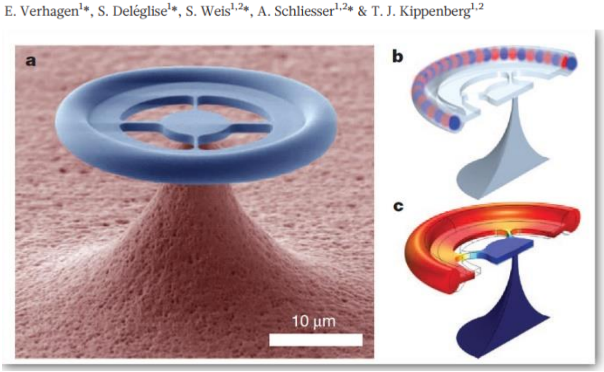


LETTER

doi:10.1038/nature10787

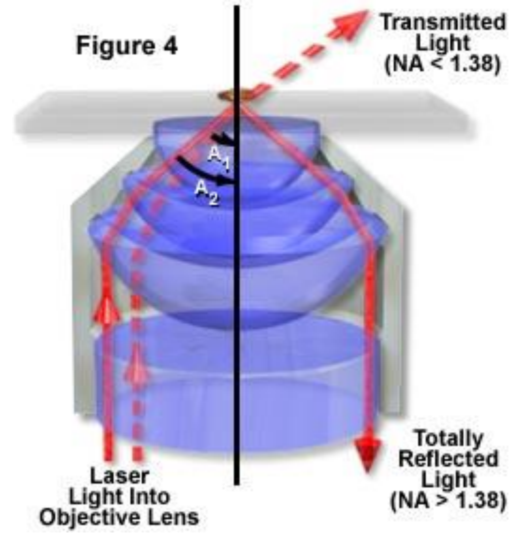
Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode

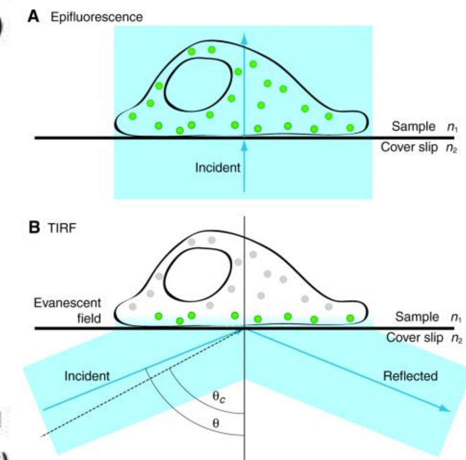
E. Verhagen[1]*, S. Deléglise[1]*, S. Weis[1,2]*, A. Schliesser[1,2]* & T. J. Kippenberg[1,2]

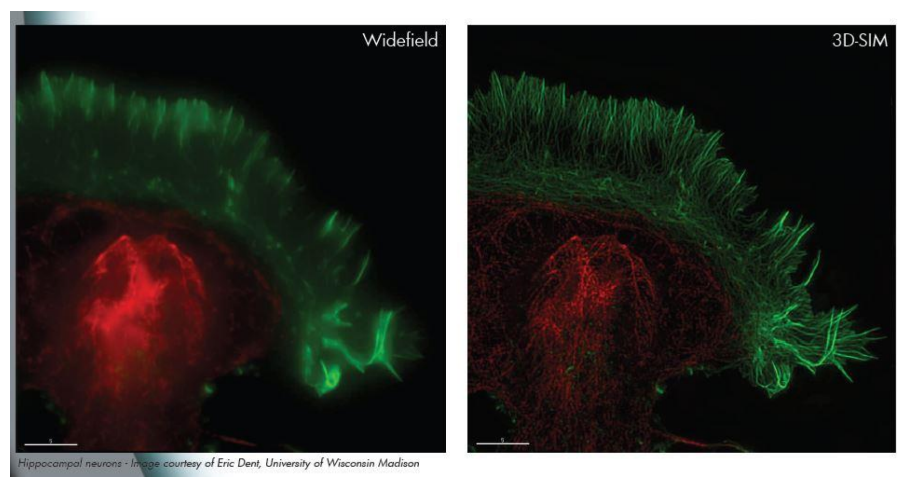


T. J. Kippenberg, *Nature 482*, 63 (2012)

total internal reflection microscope



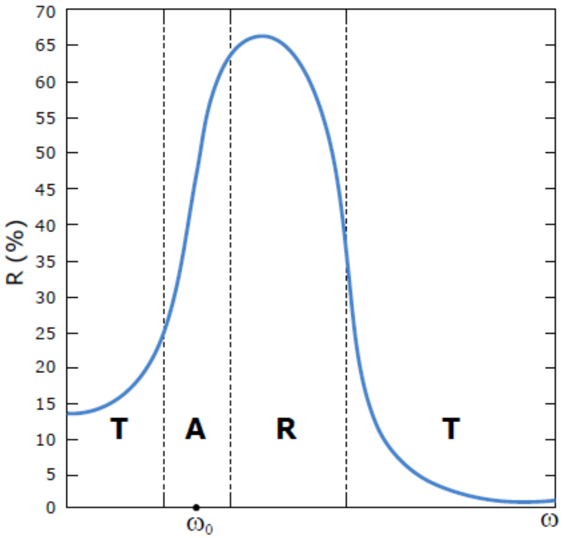




Plasma Frequency

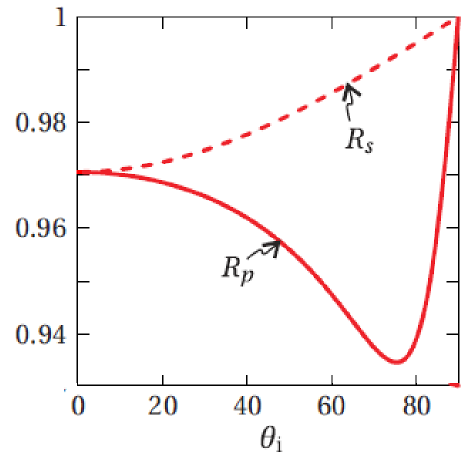
$$\omega_p^2 = \frac{Nq^2}{m\epsilon_0}$$

- "Collective" oscillation frequency for free electrons
  - *not* the same as the dipole resonant frequency ω_0
- Example: in the ionosphere, $\omega_p/2\pi \approx 9$ MHz,
  - important for long-range radio communications!
- The polarization of a conductor depends on the plasma frequency and the dipole oscillation frequency



$$r_s = \frac{\cos\theta_i - \sqrt{\mathcal{N}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\mathcal{N}^2 - \sin^2\theta_i}}$$

$$r_p = \frac{\sqrt{\mathcal{N}^2 - \sin^2\theta_i} - \mathcal{N}^2\cos\theta_i}{\sqrt{\mathcal{N}^2 - \sin^2\theta_i} + \mathcal{N}^2\cos\theta_i}$$



$\mathcal{N} = n_t$ is complex (here, $n_i = 1$)

*You might not want to compute these by hand…

Reflection by a good conductor

Air-silver interface at normal incidence, 780 nm light

$n_{Ag} = 0.27 + 4.47i$

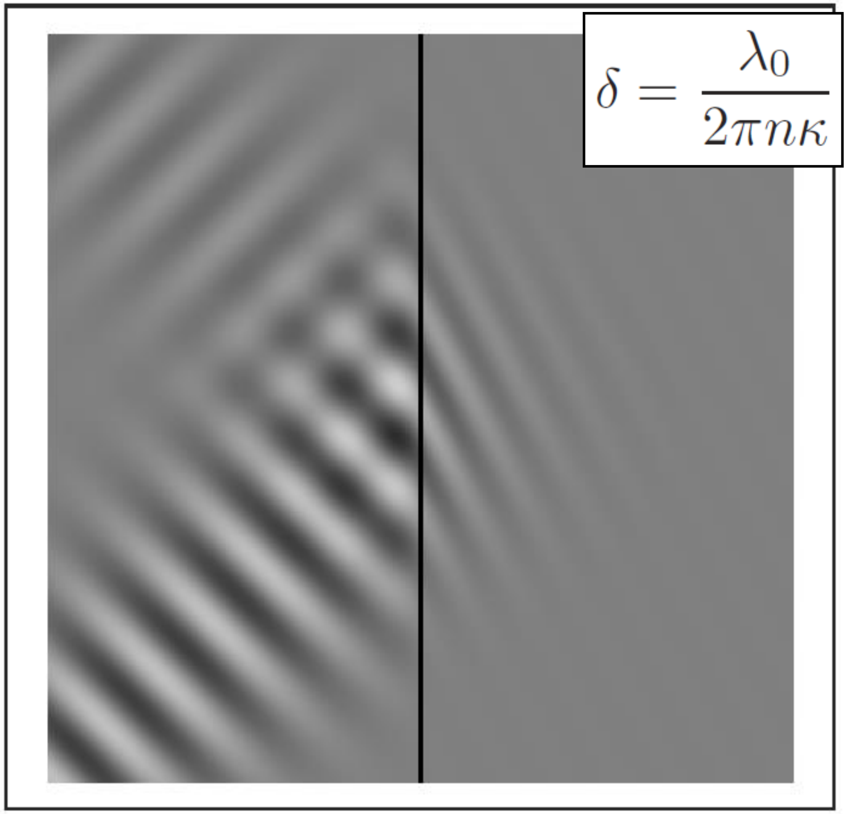
$$R = \left| \frac{n_1 - \tilde{n}_2}{n_1 + \tilde{n}_2} \right|^2$$

Gives a reflectivity of about 95%, which is pretty good.

How far does the wave penetrate?

Calculate the skin depth:

$$\delta = \frac{\lambda_0}{2\pi n\kappa} = \frac{\lambda_0}{28.1} = 27.8 \text{ nm}$$



$$\delta = \frac{\lambda_0}{2\pi n\kappa}$$

Credit: D. Steck