# Physics 47 — Optics

Exam III

Due by 5pm on Friday, Nov 17, 2017

(give to me directly – Wilder 002)

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- Allowed items: any resources necessary for solving the problems, **except** other students. *All solutions must be an individual effort!*
- Show all work in detail for full credit. For partial credit, explain in detail your best guess towards solution.

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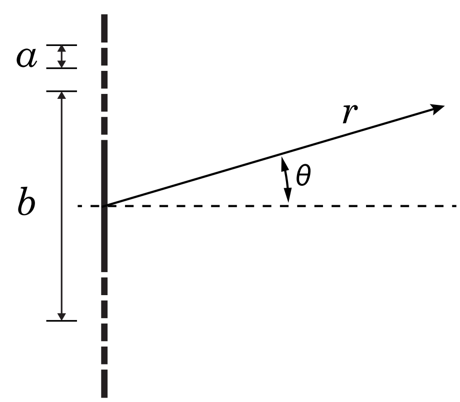
**Problem 1: Diffraction**

(a) A plane wave electromagnetic field of wavelength $\lambda$ and intensity $I_0$ is incident on an arrangement of identical slits, grouped into two identical arrays of $N$ sources each, as shown below.




Under the assumption that $r \gg a, b, \lambda$, find the far-field intensity distribution produced as a function of $I_0, \lambda, \theta, N, a,$ and $b$.

(b) As we've seen in class, the angular resolution limit for an imaging system is constrained by diffractive effects from the imaging system itself, resulting in a minimum observable angular difference of $\theta_{min} \simeq 1.22\lambda/D$, where $D$ is the aperture diameter of the imaging system. Normally, when considering the capture of some image, we tend to think in terms of collecting a distribution of photons which have been scattered from some object of interest. Alternatively, one may do just as well (or better) to exchange the photons for another type of particle as the probe to be scattered off some object.

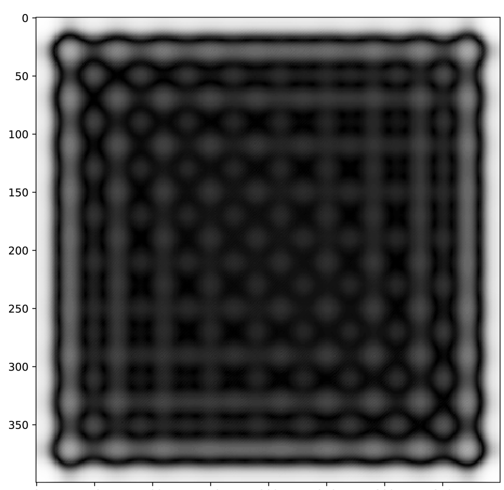
For this problem, explain what the de Broglie wavelength is and provide a simple (non-relativistic) derivation of this wavelength for an electron in terms of it's rest mass, $m_0$, and potential energy. Further, consider the case where you wish to image a certain 10 nm protein structure: compare the angular resolution limits you can expect from an optical microscope using (a typically biologically non-destructive) 532 nm laser versus a scanning electron microscope (SEM) operating with an accelerating electron potential of 10 keV. Assume equal aperture diameters. Your answer should make clear why the 2017 Nobel prize in Chemistry was awarded for cryo-electron microscopy.

**Problem 2: Fourier Optics**

For this problem, you are given a file containing a $400 \times 400$ matrix of complex field values, representing a particular image. However, upon checking the image by plotting the intensity/irradiance distribution, as shown below, the only discernible features are a set of dark geometric "blobs." Something is obviously amiss – somehow, the captured field distribution has been contaminated by a noise source *much larger* than the desired data source.

(a) Find and submit the uncontaminated *intensity distribution* as your solution to this problem, including the code used in your analysis. Both the noise and actual image will be abundantly obvious if you proceed correctly.

(b) Explain how your solution relates to the '4-f' setup you constructed in Lab III.

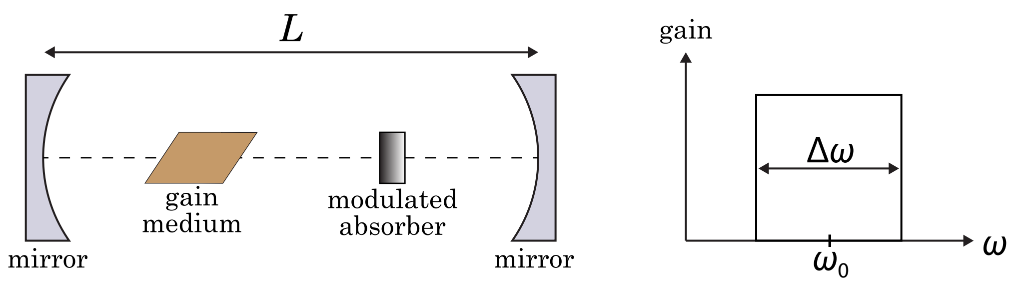



The file containing this complex field distribution can be found on the P47 Canvas website, at the bottom of the *Homework & All Solutions* page. There are two versions: *P47_ExamIII_2_noise.csv* and *P47_ExamIII_2_noise.mat*. These contain identical information, where the first is a generic comma-separated value file, while the second is a MATLAB data file.

Obviously, you will need to use some sort of computational resource. If you are unfamiliar with such tools, make use of the *Software* page on the Canvas homepage. If you clearly have in mind the operation you would like to perform, but are unable to figure out how to implement it (after much searching!), I am available for Python and MATLAB hints. However, I will not be giving hints about what operations you should be performing to the data.

**Problem 3: Lasers**

A particularly interesting type of laser is the mode-locked, or phase-locked, laser. The ultrashort light pulses it produces are especially useful in studying material and electrochemical changes occurring on sub-picosecond timescales. Like many lasers, it consists of an optical cavity of length $L$, bounded by two mirrors in a Fabry-Perot configuration. The amplifying medium (e.g. some solid, liquid, or gas) can be characterized by a gain spectrum; only light within this band of frequencies will be amplified. However, not all longitudinal modes within the gain spectrum will be in phase, which is the purpose of the modulated absorber, as described below.




Here, consider the amplifying medium to be Nd glass, which has $\omega_0 = 1.8 \times 10^{15} \frac{rad}{sec}$, $\Delta\omega_0 = 6 \times 10^{12} \frac{rad}{sec}$, and take the length of the laser to be $L = 150$ cm. Neglect any dispersive effects (i.e. index of refraction of Nd gain medium and throughout the cavity is one). To further simplify calculation, assume the gain spectrum is rectangular, as shown in the figure.

(a) Find the separation $\delta\omega_L$ between adjacent longitudinal modes of the cavity. For simplicity, neglect the modulator for this part.

(b) The rectangular gain spectrum will result in each mode within the gain band as having equal amplitude. If the intensity of an individual mode is $I_0$, what is the total intensity when the relative phases $\phi_n$ of the different modes are randomly distributed?

(c) Now, by inserting the modulated absorber into the cavity, one can lock the phases of all the modes to the same value, so that traveling waves interfere constructively, producing optical pulses. Find the intensity as a function of time for this mode-locked (or, rather, phase-locked) case. What is the pulse duration, pulse separation, and peak intensity (in terms of $I_0$)? Consider only one polarization.

(It's irrelevant to this problem, but for your information, a modulated absorber has an effective absorption that is a function of time – e.g. $\alpha = \alpha_1 + \alpha_2 cos(\omega t)$. Any cavity mode with the wrong phase relative to this modulation experiences excess attenuation, so only phase-locked modes are amplified.)