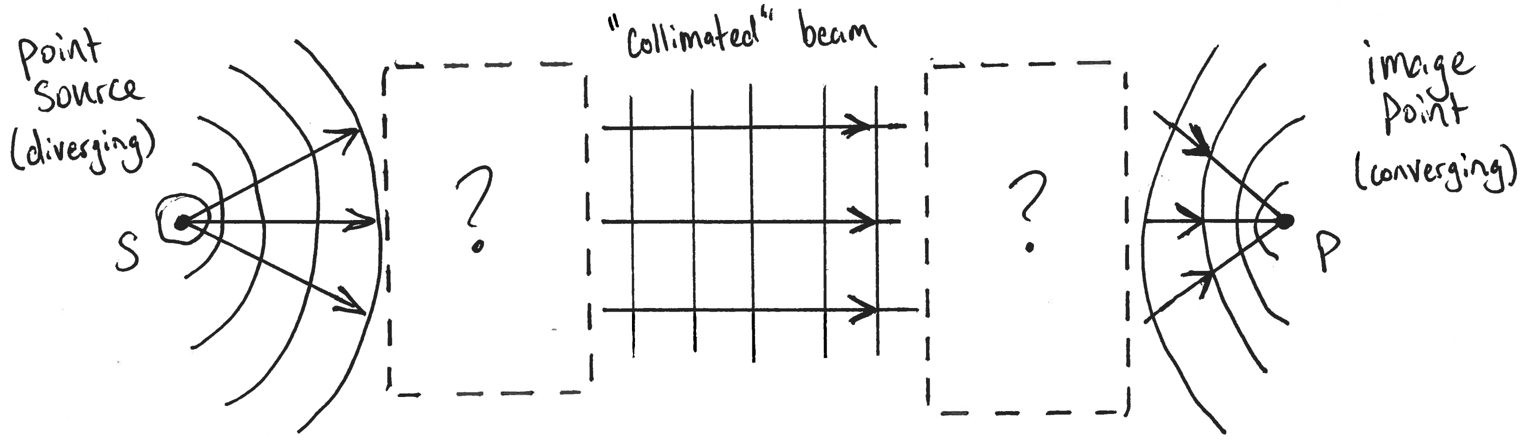
Basics of Ray Optics

What is ray optics?

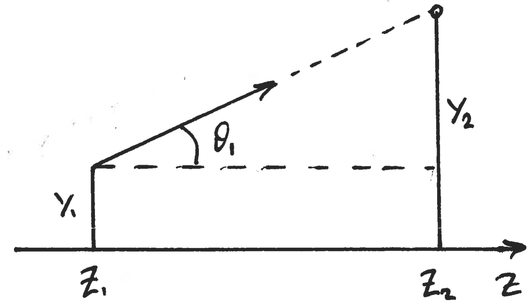
- neglect the wave nature of light entirely ($\lambda_0 \to 0$), except to imagine objects to be composed of collection of radiating pt. sources
- treat light as "rays" moving collinear to $\vec{S}$ ($\perp$ to wavefront of $\vec{S}$)
- these rays can be manipulated and redirected through interactions with different materials (i.e. reflection, refraction)
- limit ourselves to objects (and images) much larger than λ
- assume optical elements/systems are homogeneous and isotropic (have constant index, n) and are bounded by sharp ($\ll \lambda$) interfaces

[Note: there is a way to handle situations where the index of refraction varies continuously (mirages, human eye, etc.) called the eikonal equation (just the wave equation in the limit $\lambda \to 0$), but we won't be able to cover it this term.]



In general, treating light as rays allows us to use Law of Reflection, Snell's Law, and simple geometry to mathematically describe effects of optical elements on light propagation [3-2]

Description of Rays



- generally only going to consider cylindrically symmetric systems oriented along some <u>optical axis</u> (here, z)
- a ray can be described by 2 parameters $\begin{cases} y \text{ - distance from optical axis} \\ \theta \text{ - angle w.r.t. optical axis} \end{cases}$
- given these, any point further down the ray is:

$$y_2 = y_1 + (z_2 - z_1)\tan\theta_1$$

$$\theta_2 = \theta_1$$

- BUT, this $\tan\theta$ (and $\sin\theta$ from Snell's Law) makes for tedious calculations

Paraxial Approximation

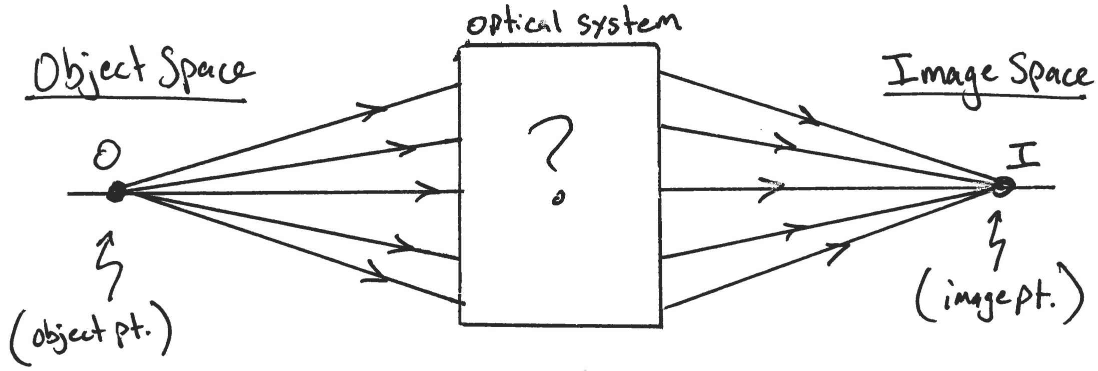
- θ is usually small, therefor can make <u>small-angle approximation</u>
- trig functions get Taylor expanded to first order $\left(\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \cdots\right)$

$$\sin\theta \simeq \tan\theta \simeq \theta \quad , \quad \cos\theta \simeq 1$$

- "First-order" or "Gaussian" Optics (3rd order isn't too bad, but we won't cover it)
- allows lens makers to fabricate simpler "spherical" lenses

[<u>Note</u>: aspherical lenses are ideal for maintaining wavefront, but traditionally difficult to manufacture – see Hecht's section on this if interested]

General Concept of Paraxial Imaging

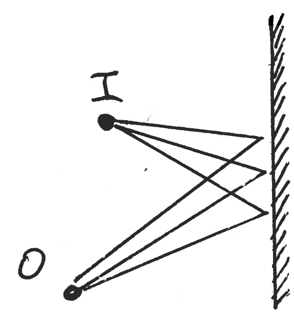


For a "perfect" (stigmatic) system, every point in the object space can be mapped to a point in the image space (conjugates or stigmatic pair)

In fact, Gauss developed method for reducing a paraxial optical system to a characteristic series of cardinal points for ray calculations (...later)

Physically, perfect stigmatic imaging doesn't exist due to λ of light (diffraction limit)

Note that sometimes the physical object and image spaces can overlap (mirror)



Can already sense need for nomenclature to describe relationship between objects and images

- ~~same as opposite side of system~~
- real or virtual
- erect or inverted
- magnified or diminished

Before describing these, need to know a little of optical elements making them necessary

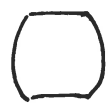
Basic Optical Elements

positive/"converging"		negative/"diverging"	
Lens	Mirror	Lens	Mirror

(convex)	(concave)	(concave)	(convex)

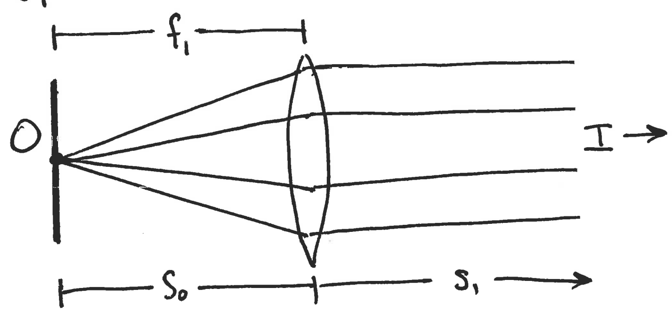
But lenses have 2 surfaces ⟶ can come in different shapes

biconvex   plano-convex   convex meniscus   biconcave   plano-concave   concave meniscus

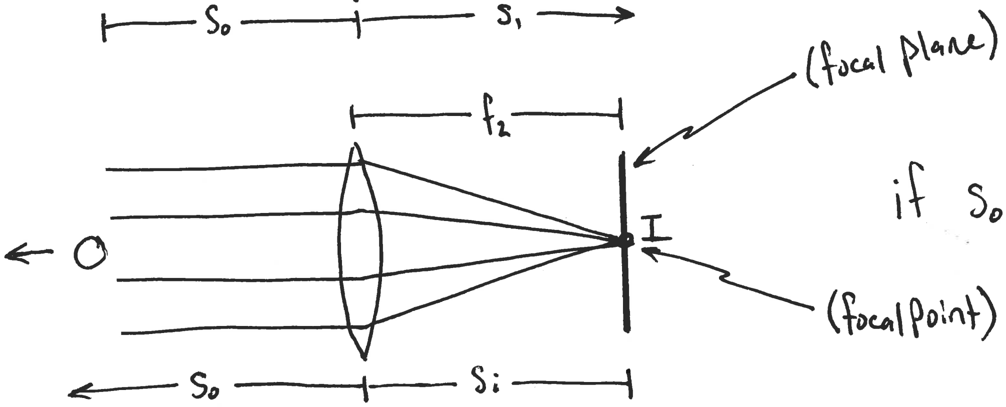
And can also vary in thickness :  (but we'll neglect this for now)

Focal Points & Planes

Optical elements are mainly characterized by their focal length (at least these types)



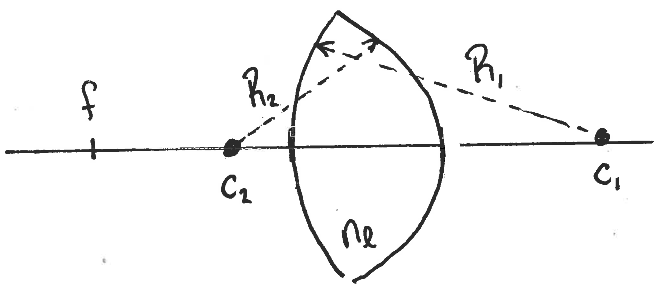
if $s_o = f$, then $s_i = \infty$



if $s_o = \infty$, then $s_i = f$

In general, can have $f_1 \neq f_2$, but for now we'll only consider <u>thin lenses</u> where $f_1 = f_2$

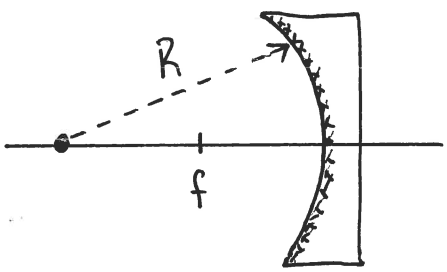
focal length of lens



(thin) Lens Maker's Equation

$$\frac{1}{f} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

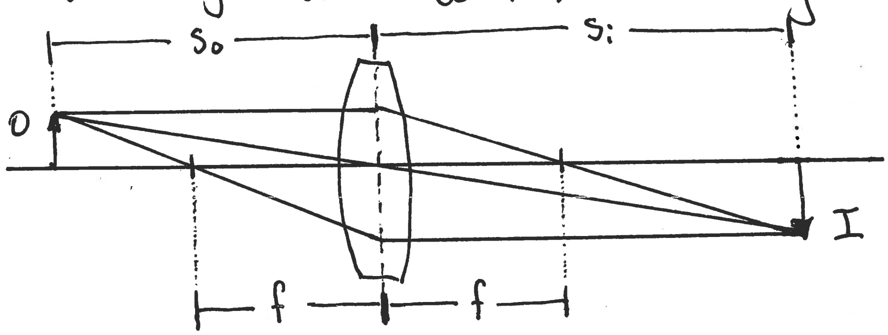
focal length of mirror



Mirror Maker's Equation

$$f = -R/2$$

Given the focal length, there is a simple relation between the object distance (s_o) and image distance (s_i)

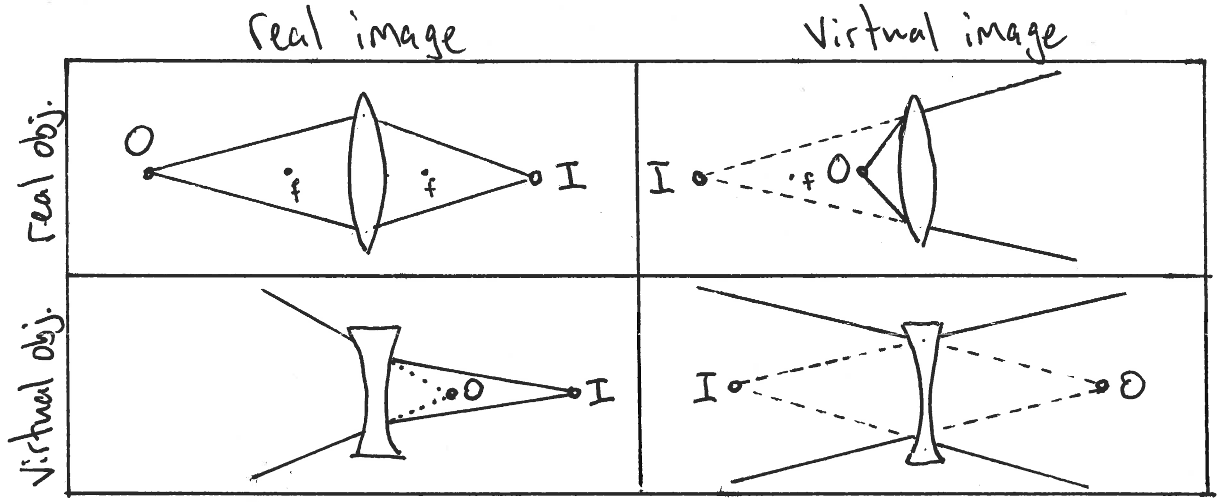


Gaussian Lens Formula

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

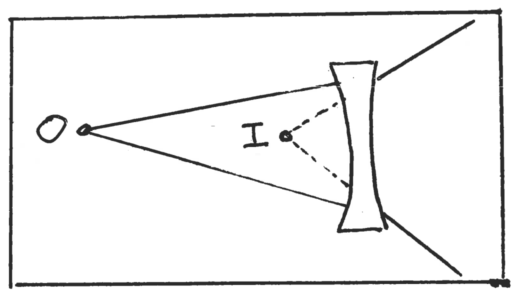
☆ [See Slide on Sign Conventions]

Real vs. Virtual Image/Object



Remember that the image and object pts. actually exist on a plane (object plane, image plane)

This is certainly not an exhaustive set of possibilities



Thin Lens Image Magnification

Size of image and orientation depends on ratio of object and image distance

$\updownarrow$ Transverse Magnification: $M_T = \frac{y_i}{y_o} = \frac{-S_i}{S_o} = \frac{-x_i}{f} = \frac{-f}{x_o}$ (many ways of writing it)

$\longleftrightarrow$ Longitudinal Magnification: $M_L = \frac{dx_i}{dx_o} = \frac{-f^2}{x_o^2} = -M_T^2$

Angular Magnification: $M_\alpha = \frac{\theta_i}{\theta_o} = -f_1/f_2$ $\leftarrow$ (for telescopes and microscopes)

These expressions are identical when formulated for mirrors!

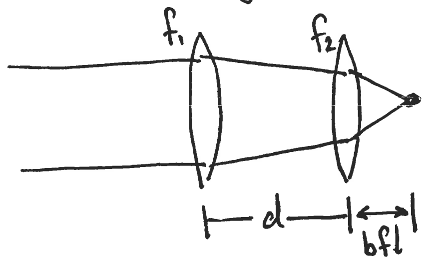
Note: Sign conventions vary!

Just be careful and consistent. Explain what you've calculated in words. ~~Ex~~ $S_i = -15\text{ cm} \Rightarrow$ inverted? left of lens? virtual? best to just write it out.

Combining "Thin" Optics (with small separations)

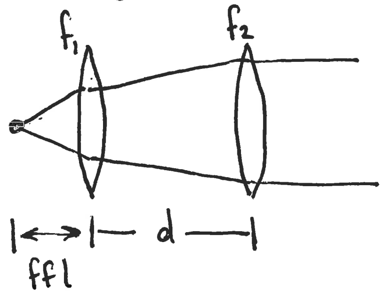
- We'll get to this in much more detail later
- basically just want to introduce some terms: back, front, effective focal lengths

back focal length (b.f.l.)

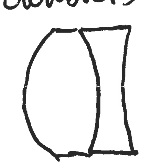


	finite d	$d \to 0$
$\text{b.f.l.} =$	$\dfrac{f_2(d-f_1)}{d-(f_1+f_2)}$	$\dfrac{f_1 f_2}{f_1+f_2}$
$\text{f.f.l.} =$	$\dfrac{f_1(d-f_2)}{d-(f_1+f_2)}$	$\dfrac{f_1 f_2}{f_1+f_2}$

front focal length (f.f.l.)



Ex: achromatic doublets



effective focal length (e.f.l.)

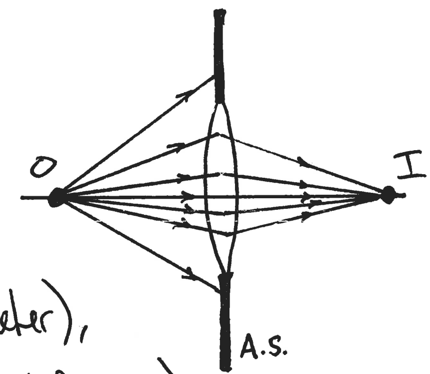
- Simple way to characterize a system of thin lenses if they are in contact ($d \ll f_1, f_2$)

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \left(\text{in fact, } \frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots + \frac{1}{f_N}\right)$$

- Otherwise $\dfrac{1}{f_{eff}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}$

Apertures

So far, haven't said any thing about lens diameter

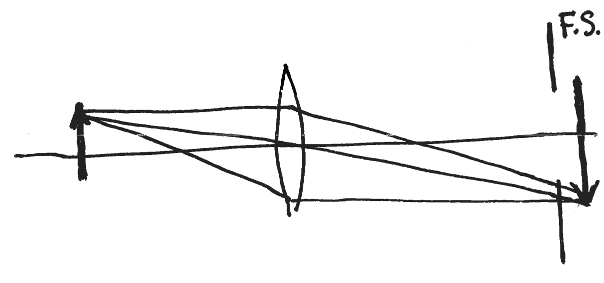


Aperture Stop:

- Some limiting diameter imposed on the collection optics
- either the physical limit of optics (ie. lens diameter), or an additional mask (ex: diaphram, pinhole, pupil of eye...)
- affects brightness (limits cone of rays collected), aberrations (ie: spherical aberration from lens edges), and resolution (f/#)

Pupil:

- basically just the image of the aperture stop, as viewed from either the front of a lens system ("entrance" pupil) or behind it ("exit" pupil)
- when the aperture stop is co-located with objective $\Rightarrow$ A.S. = pupil



Field Stop:

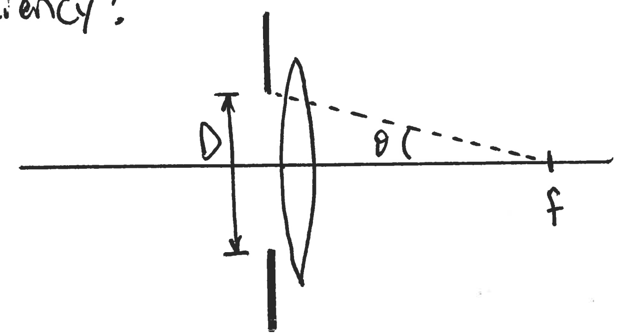
- trivial – some limit to the image plane
- either some diaphram or size of CCD chip

Chief & Marginal Rays:

- a construction used to determine location and diameter of entrance/exit pupils w.r.t. lens and aperture stop
- we won't concern ourselves with this (or pupils beyond the A.S. for that matter)

Characterizing Apertures:

- how does aperture impact collection efficiency?
- have 2 related metrics to do so
  - f-number ($f/\#$) (not a fraction!)
  - Numerical Aperture (NA)



- $f/\#$ (or "speed") of a lens comes from considering:

$$\frac{I_{image}}{I_{A.S.}} = \frac{A_{AS}}{A_i} \propto \frac{D^2}{y_i^2} \propto \left(\frac{D}{f}\right)^2, \quad \text{where } M_T = \frac{y_i}{y_o} = -\frac{f}{x_o}$$

$$\Longrightarrow \boxed{f/\# \equiv f/D} \qquad (f = D \Rightarrow f/1 \,,\; f = 2D \Rightarrow f/2)$$

$\llcorner$ $\frac{1}{4}$ brightness of $f/1$ requires 4x exposure time

- NA is analagous (and related) to $f/\#$

$$\boxed{NA = n_i \sin\theta_{max}}$$ where θ_{max} is max angle entering lens due to A.S.

but also see that $\tan\theta_{max} = \frac{(D/2)}{f}$

$$\Longrightarrow \quad NA = n\sin\theta = n\sin\left[\tan^{-1}\left(D/2f\right)\right] \simeq n\frac{D}{2f}$$

so $\boxed{f/\# = 1/2NA}_{n \simeq 1 \text{ (air)}}$

Dioptric Power:

- defined as $\mathcal{D} \equiv \frac{1}{f}$ Why? Optometrists can't handle fractions!

Combining lenses: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \longrightarrow \mathcal{D} = \mathcal{D}_1 + \mathcal{D}_2$

- also have notion of "vergence": $\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i} \longrightarrow \mathcal{D} = V_o + V_i$

(convergence / divergence)

What limits our resolution? $\longrightarrow$ Diffraction!

We'll come back to the wave nature of light later on to see this.

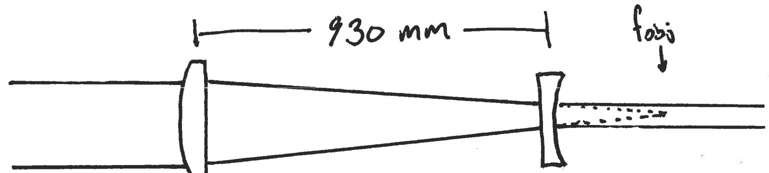
For now, we'll just say the resolution limits are:

<u>Spatial Resolution</u>: $\Delta l \approx 1.22 \frac{f\lambda}{D}$ (reminder: $f/\# = \frac{f}{D} = \frac{1}{2NA}$)

<u>Angular Resolution</u>: $\Delta\theta \approx 1.22 \frac{\lambda}{D}$

Example Optical Systems

Galileo's Original Telescope



$D_{obj} = 37\text{ mm}$ $D_{ep} = 20\text{ mm}$

$f_{obj} = 980\text{ mm}$ $f_{ep} = -50\text{ mm}$

- had an angular magnification of

$$M_\alpha = \frac{-f_{obj}}{f_{ep}} = \frac{-(980\text{ mm})}{(-50\text{ mm})} = \underline{\underline{19.6}}$$

- had an f/# of: $f/\# = \frac{f_{obj}}{D_{obj}} = \frac{(980\text{ mm})}{(37\text{ mm})} = \underline{\underline{26.5}}$

that's pretty "slow" in camera-speak $\longrightarrow$ slower means dimmer

- okay, then how did it's light-gathering area compare to a human eye?

$$\frac{D_{obj}^2}{D_{eye}^2} = \frac{(37\text{ mm})^2}{(5\text{ mm})^2} = \underline{\underline{54.8}} \quad \text{(not bad..)}$$

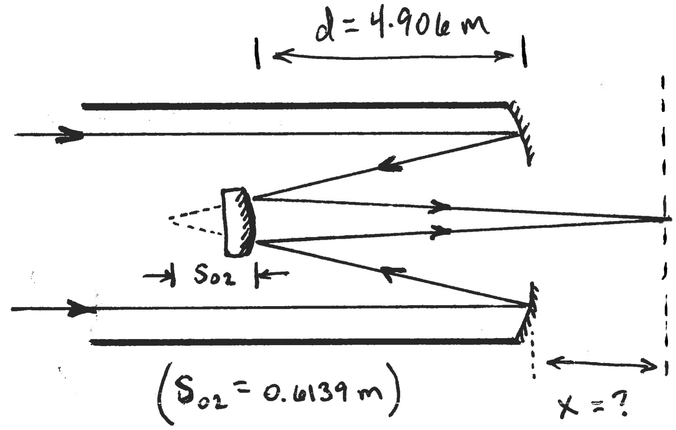
- when viewing Jupiter, what was his resolution limit? (Jupiter is kind of orange, so $\lambda = 600\text{ nm}$)

$$\Delta\theta \approx 1.22\frac{\lambda}{D} = 1.22\frac{(600\text{ nm})}{(37\text{ mm})} = \underline{1.98 \times 10^{-5}\text{ radians}}$$

(or $0.001°$, or 4 arcsec)

Hubble Space Telescope

Okay, let's look at the Hubble and then compare it with what we found for Galileo's telescope



"primary" mirror

$R_1 = -11.04\,m$

$D_1 = 2.4\,m$

$f_1 = \frac{-R_1}{2} = 5.52\,m$

$f/\# = f_1/D_1 = \frac{(5.52m)}{(2.4m)} = 2.3$

$NA = n\sin\theta \simeq n\frac{D_1}{2f_1} = \frac{1}{2\cdot(2.3)} = 0.22$

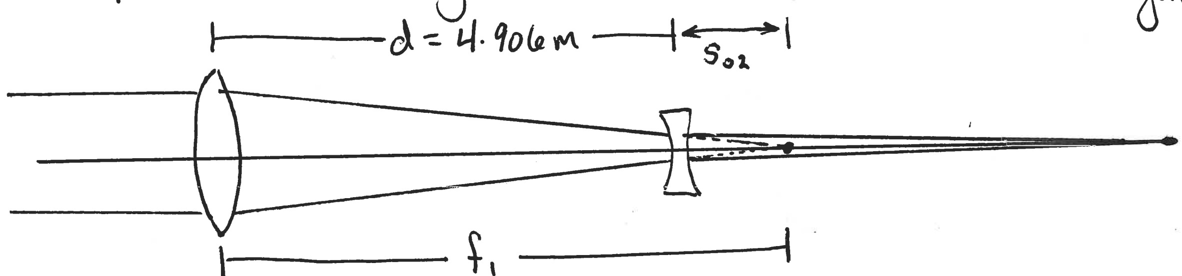
"secondary" mirror

$R_2 = 1.358\,m$

$D_2 = 0.3\,m$

$f_2 = \frac{-R_2}{2} = -0.679\,m$

Can simplify our reasoning a bit with a (crude) trick $\longrightarrow$ turn it into a galilean refractor!



$S_{o2} = d - f_1$
$= 0.6139\,m$

So, see that the second optic has a virtual object at

Then where is the image? How far behind the primary should the CCD be located?

$$\frac{1}{f_2} = \frac{1}{S_{o2}} + \frac{1}{S_{i2}} \implies S_{i2} = \left[\frac{1}{f_2} - \frac{1}{S_{o2}}\right]^{-1} = \left[\frac{1}{f_2} - \frac{1}{d-f_1}\right]^{-1} = \underline{\underline{6.414\,m}}$$

And see the image is located 6.414 m behind the secondary

Hubble (cont.)

Now, go back to the Cassegrain diagram.

How far behind the primary is this image?

$$x = S_{i2} - d = \left[\frac{1}{f_2} - \frac{1}{d-f_1}\right]^{-1} - d = (6.414\,m) - (4.906\,m)$$

$$\Rightarrow \underline{\underline{x = 1.508\ m}}$$

- the Hubble's focus is a diffraction-limited spot on a CCD camera
  → NOT setup for human observers!

  How would you modify the design for human compatibility?

  Can see the virtual object at S_{o2} is closer to the secondary mirror than f_2 ($S_{o2} < f_2$), so need mirror to move <u>towards the primary by $f_2 - S_{o2} = 65.1$ mm to get parallel rays</u>

- what is the angular magnification?

$$M_\alpha = \frac{\theta_i}{\theta_o} = \frac{-f_1}{f_2} = \frac{-(5.52\,m)}{(-0.679\,m)} = \underline{\underline{8.13}}$$

(doesn't seem like very much magnification, does it?)

- Okay, let's check its angular resolution then. To compare against the galilean telescope, assume $\lambda = 600$ nm (a hot red giant, like Arcturus)

$$\Delta\theta \approx 1.22\frac{\lambda}{D} = 1.22\frac{(600\,nm)}{(2.4\,m)} = \underline{\underline{3.05 \times 10^{-7}\ \text{radians}}}$$

$(1.75 \times 10^{-5}\ \text{deg},\ 0.063\ \text{arcsec})$

(So about 2 orders magnitude better!)

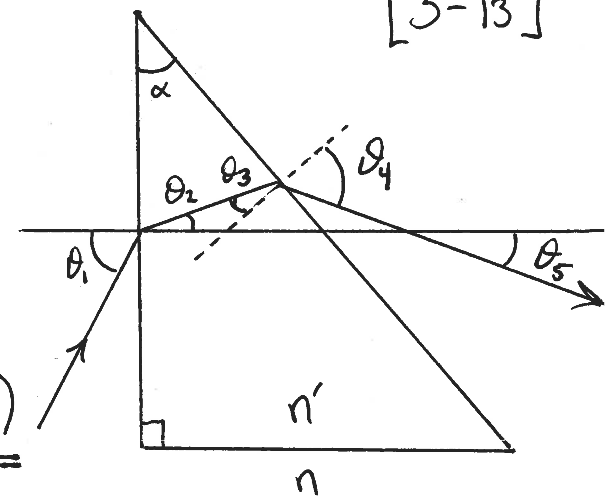
- what is the lateral resolution? (how far apart are any 2 spots on CCD?)

need $f_{eff} = \left[\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}\right]^{-1} = 57.66\,m \Rightarrow \Delta l \approx 1.22\frac{f_{eff}\lambda}{D} = \underline{\underline{17.6\ \mu m}}$

(just a bit larger than CCD pixel size!)

Prisms:

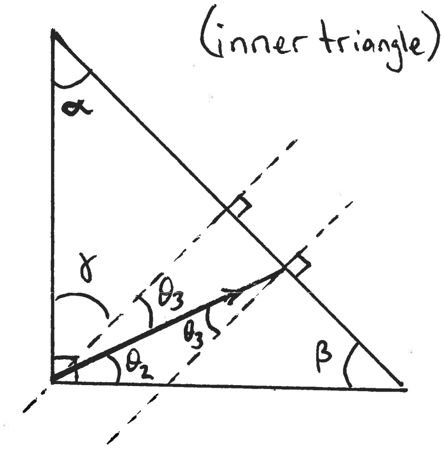
Let's solve for the total deviation, $\delta = \theta_1 + \theta_5$, in terms of θ_1 and wedge angle, α.



Have Snell's Law at first interface

$$n \sin\theta_1 = n' \sin\theta_2 \Rightarrow \underline{\underline{\theta_2 = \sin^{-1}\left(\frac{n}{n'}\sin\theta_1\right)}}$$

Now, with the beam inside the prism, have Snell's Law again at second interface



$$n' \sin\theta_3 = n \sin\theta_4 \Rightarrow \underline{\underline{\theta_4 = \sin^{-1}\left(\frac{n'}{n}\sin\theta_3\right)}}$$

Can write θ_3 in terms of α & θ_2 by considering the inner triangle

$$\alpha + \beta = \gamma + \theta_2 + \theta_3 = \frac{\pi}{2} \quad \underline{\text{AND}} \quad \alpha + \gamma = \pi/2$$

$$\Rightarrow \left(\frac{\pi}{2} - \alpha\right) + \theta_2 + \theta_3 = \frac{\pi}{2}$$

$$\Rightarrow \underline{\underline{\theta_3 = \alpha - \theta_2}}$$

in exactly the same way, another pair of triangles can be used to find

$$\Rightarrow \underline{\underline{\theta_5 = \theta_4 - \alpha}}$$

(this is already long enough, so not showing)

$$\text{Now } \delta = \theta_1 + \theta_5 = \theta_1 + \theta_4 - \alpha = \theta_1 - \alpha + \sin^{-1}\left[\frac{n'}{n}\sin\theta_3\right]$$

$$\delta = \theta_1 - \alpha + \sin^{-1}\left[\frac{n'}{n}\sin(\alpha - \theta_2)\right]$$

$$\underline{\underline{\delta = \theta_1 - \alpha + \sin^{-1}\left[\frac{n'}{n}\sin\left(\alpha - \sin^{-1}\left(\frac{n}{n'}\sin\theta_1\right)\right)\right]}}$$

Prisms (cont.)

But, what if both θ_1 and α are small? ($\sin x \simeq \sin^{-1} x \simeq x$)

$$\Rightarrow \delta \simeq \theta_1 - \alpha + \frac{n'}{n}\left(\alpha - \frac{n}{n'}\theta_1\right)$$

$$\delta \simeq \theta_1 - \alpha + \frac{n'}{n}\alpha - \theta_1$$

$$\delta \simeq \alpha\left(\frac{n'}{n} - 1\right)$$

Remember: with dispersion, refraction is freq. dependent

of prism in air

$$\Rightarrow \delta(\omega) \simeq \alpha\left(\frac{n'(\omega)}{n} - 1\right)$$