POLARIZATION

P47 – Optics: Unit 5

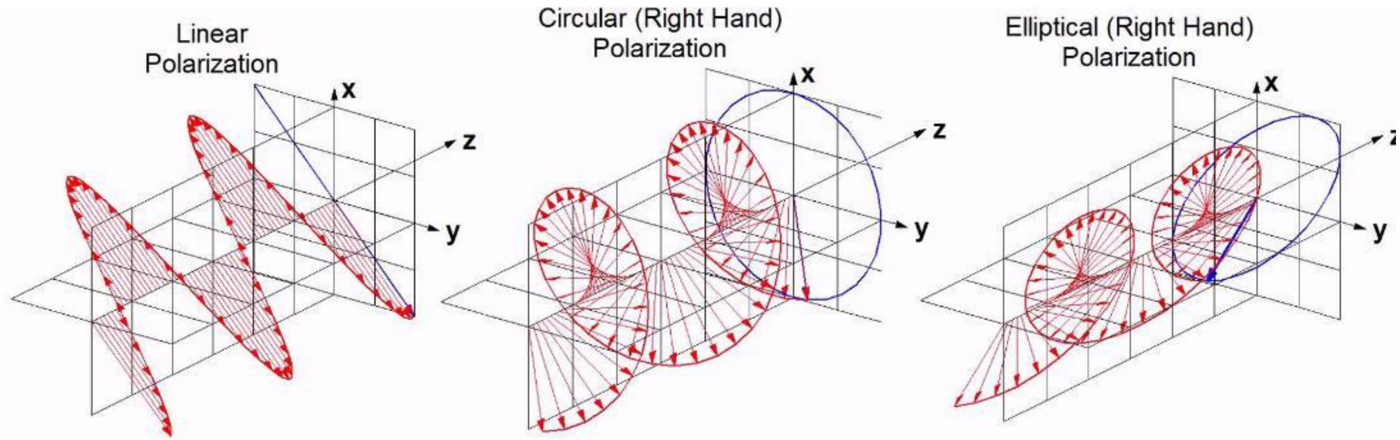
Course Outline

~~Unit 1: Electromagnetic Waves~~
~~Unit 2: Interaction with Matter~~
~~Unit 3: Geometric Optics~~

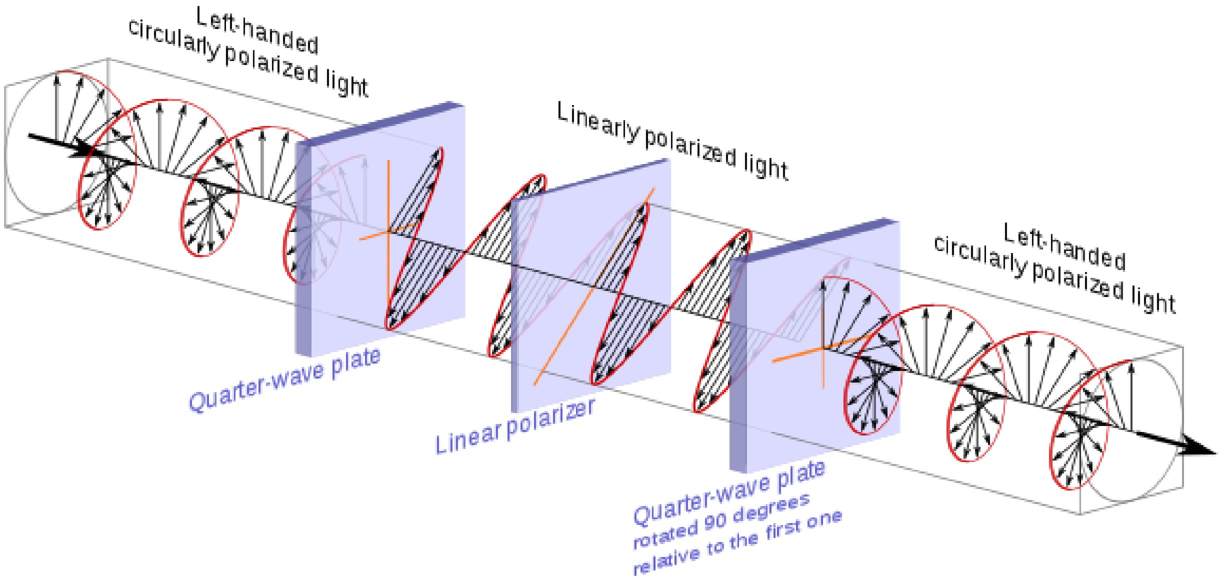
~~Unit 4: Superposition of Waves~~
Unit 5: Polarization
Unit 6: Interference

Unit 7: Diffraction
Unit 8: Fourier Optics
Unit 9: Modern Optics

I. Polarization States



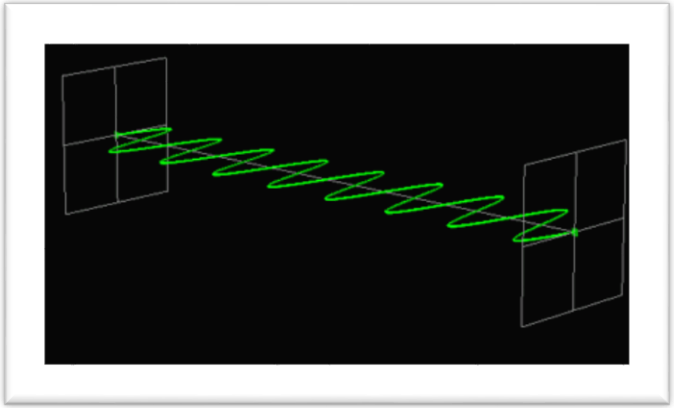
II. Controlling Polarization

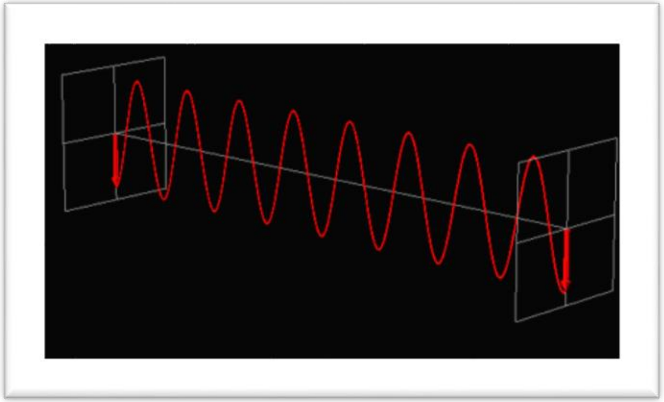


EM waves (light) are a *vector* field, not a *scalar.*

So far, we've mostly considered waves like:

$$\boldsymbol{E}(z,t) = E\cos(kz - \omega t)\hat{\boldsymbol{x}} \qquad \boldsymbol{E}(z,t) = E\cos(kz - \omega t)\hat{\boldsymbol{y}}$$





To represent polarization more generally, need a case where fields are superpositions:

$$\boldsymbol{E}(z,t) = \hat{\boldsymbol{x}}E_{0x}(r,t)\cos(kz - \omega t + \varphi_x) + \hat{\boldsymbol{y}}\,E_{0y}(r,t)\cos\left(kz - \omega t + \varphi_y\right)$$

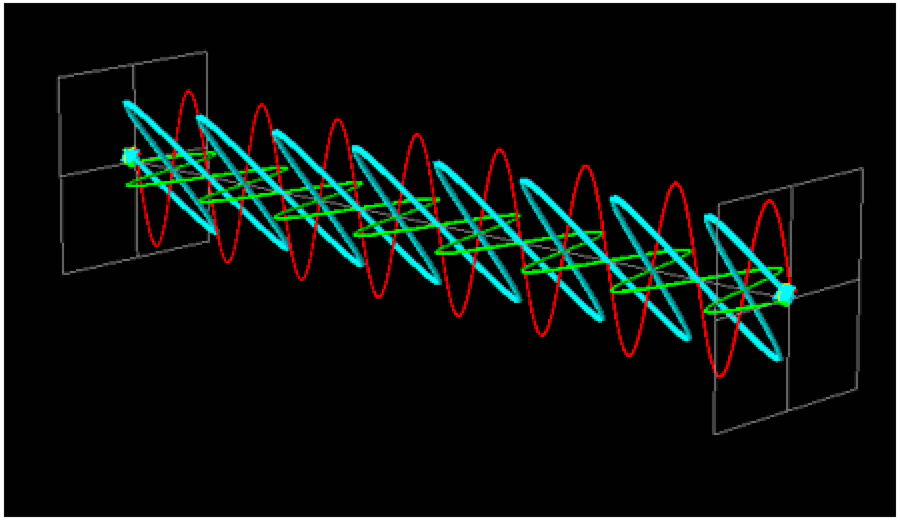
The polarization of the wave is related to the direction of ***E***,
but this direction can now be a *time–dependent* quantity.

Arbitrary Linear Polarization

$$\boldsymbol{E}(z,t) = \widehat{\boldsymbol{x}}E_{0x}(r,t)\cos(kz-\omega t+\varphi_x) + \widehat{\boldsymbol{y}}\,E_{0y}(r,t)\cos\left(kz-\omega t+\varphi_y\right)$$

if $\varphi_x = \varphi_y$, the two vector components are in phase and the resultant wave has a magnitude

$$\boldsymbol{E} = \widehat{\boldsymbol{x}}E_{0x} + \widehat{\boldsymbol{y}}E_{0y}$$



Depending on the magnitudes of E_{0x} and E_{0y} the vector E can be oriented at any angle in the x-y plane.

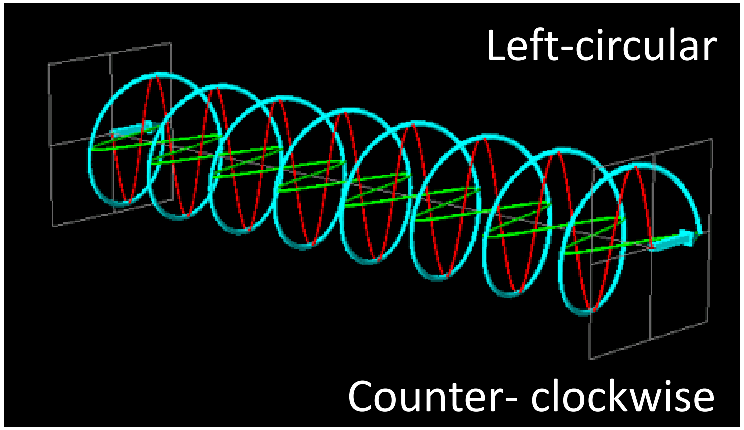
Circular Polarization

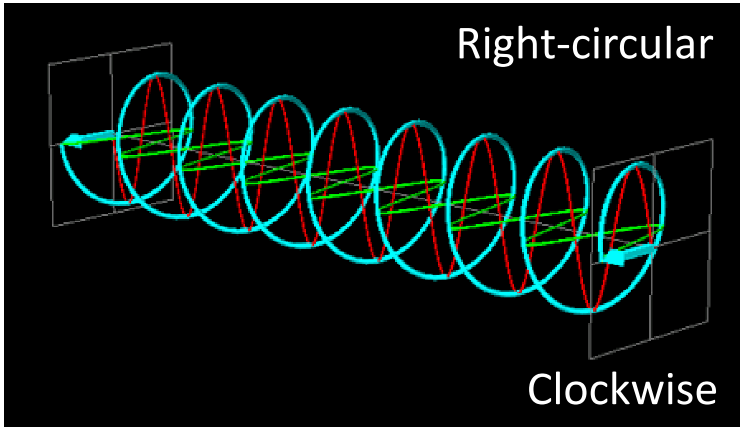
$$\boldsymbol{E}(z,t) = \hat{\boldsymbol{x}} E_{0x}(r,t)\cos(kz - \omega t + \varphi_x) + \hat{\boldsymbol{y}}\, E_{0y}(r,t)\cos\left(kz - \omega t + \varphi_y\right)$$

if $E_{0x} = E_{0y} = E_0$ and $\varphi_x = \varphi_y \pm \pi/2 \pm 2m\pi$ the two vector components are 90° out of phase, and we can write the resultant wave as:

$$\boldsymbol{E}(z,t) = E_0\,(\,\hat{\boldsymbol{x}}\cos(kz - \omega t) \pm \hat{\boldsymbol{y}}\,\sin(kz - \omega t)\,)$$

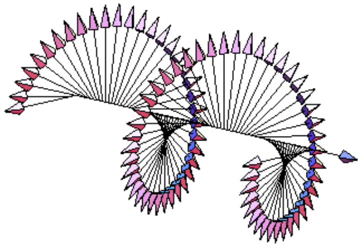
The electric field vector has a constant magnitude, but is rotating!



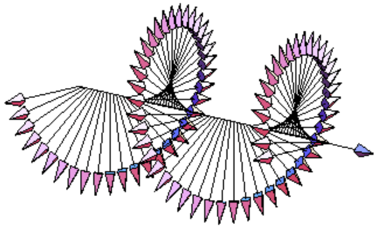


Angular Momentum of Photons

- We saw that photons carry linear momentum,
  but they also have angular momentum!
- The rotating electric field is associated with a non-zero spin angular momentum ($\pm\hbar$) of the photon.
- In particle physics, a photon is: relativistic, massless, spin-1 particle…



LC: $+\hbar$



RC: $-\hbar$

- If there's no such thing as a spin-0 photon, how do we get linearly polarized light?

Linearly polarized light is a coherent superposition of equal parts left and right!

Angular Momentum of Photons

- How can we observe this angular momentum affecting matter?
  - shine circularly polarized light on a very small object

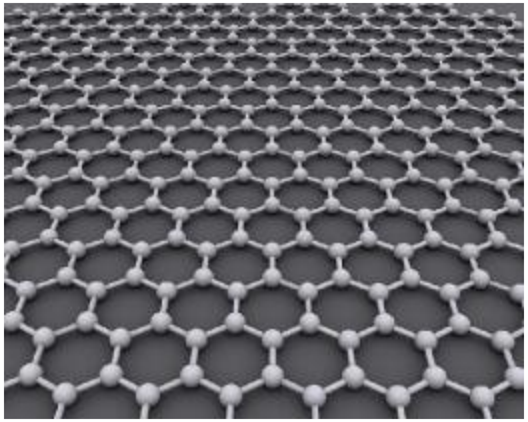
Video: Optical torque on a calcite crystal

Amazing stuff: Spinning up the world's fastest rotor with circularly polarized light!

Flake of graphene levitated in a trap in vacuum.
(has an extremely large Young's modulus)

Start it spinning with a circularly polarized laser

Spins up to 60 million RPM (!) before breaking apart

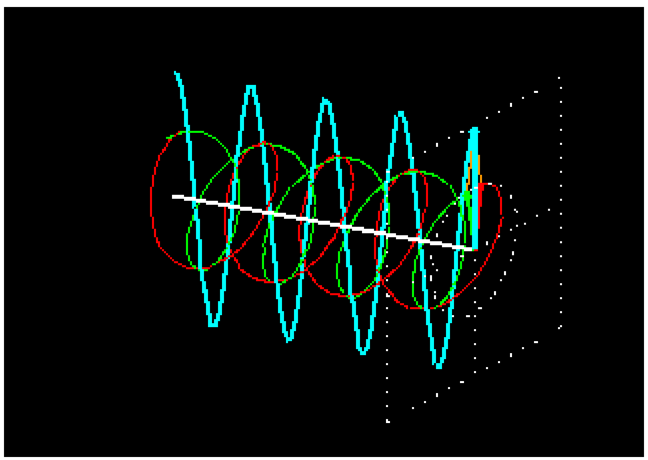


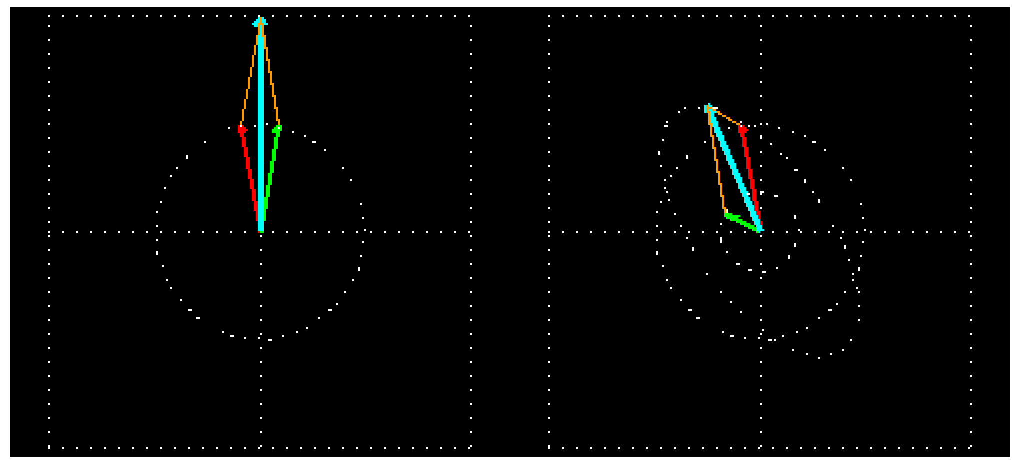
Bruce Kane. "Levitated spinning graphene flakes in an electric quadrupole ion trap." *Phys. Rev. B* 82, 115441 (2010). DOI:10.1103/PhysRevB.82.115441 . "Levitated Spinning Graphene." arXiv:1006.3774v1

Polarization Combinations

RH Circular + LH Circular Superposition
= Linear Polarization!

$E_{LHC} > E_{RHC}$
→ Elliptic Polarization?





Elliptical Polarization

The x and y components of the electric field have the same form as the parametric representation of an ellipse!

$$x = \mathrm{a}\cos(t + \varphi_x) \qquad E_x = E_{0x}\cos(kz - \omega t + \phi_x)$$
$$y = \mathrm{b}\cos(t + \varphi_y) \qquad E_y = E_{0y}\cos(kz - \omega t + \phi_y)$$

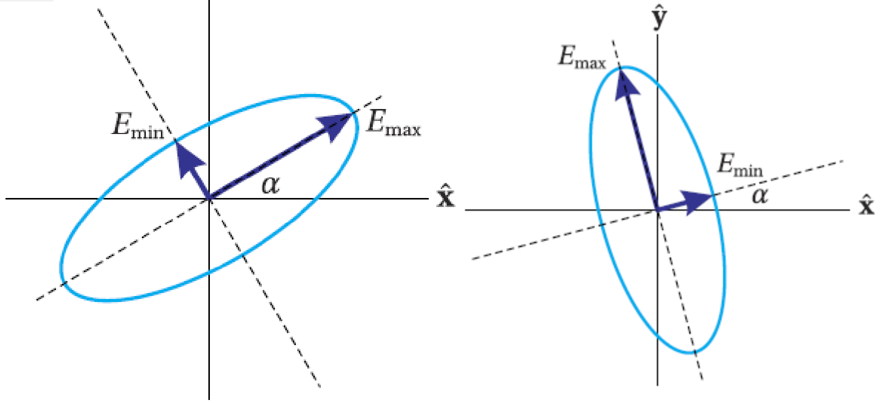
Combining equations and doing a little algebra to eliminate time dependence we get:

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_xE_y}{E_{0x}E_{0y}}\right)\cos\phi = \sin^2\phi,$$

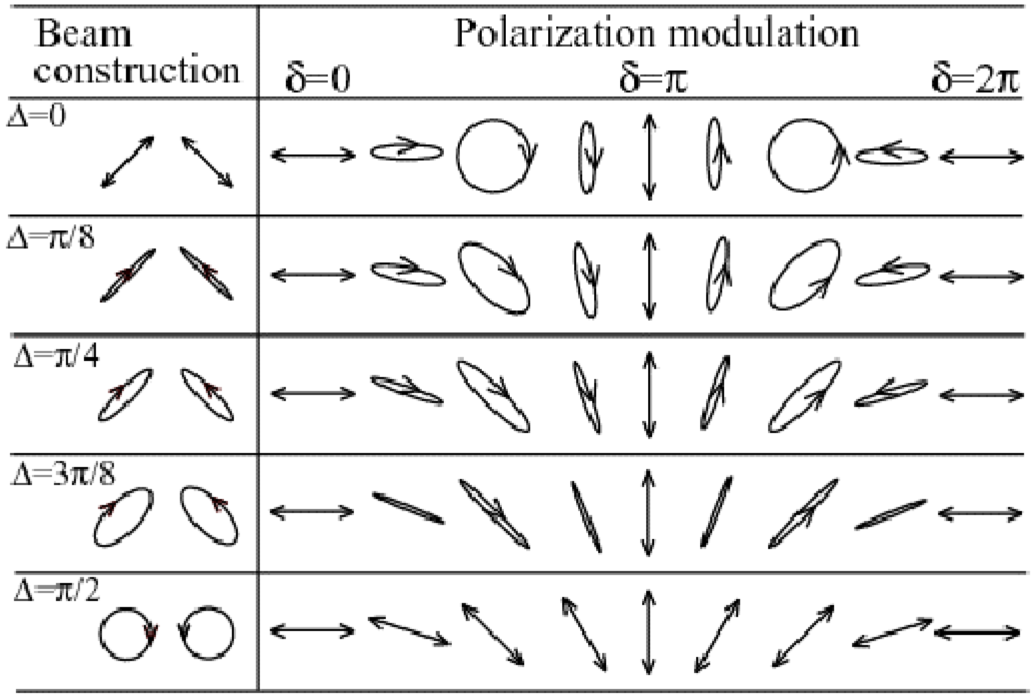
$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\phi}{E_{0x}^2 + E_{0y}^2}.$$

This is the equation for an ellipse *rotated* by an angle α with respect to the x-axis

Caution!



Optical Polarization States



Random & Partial Polarization

Most natural light does not have a well-defined polarization

The electric field has some instantaneous direction, but it changes extremely rapidly in an essentially random way.

In the fully general case, light is not *unpolarized*, but either *randomly* or *partially* polarized.

For example:
sunlight reflecting off the surface of a lake

Description of fully polarized light: Jones Matrices Description of partially polarized light: Mueller Matrices

Jones Matrix Representation

$$\mathbf{E}(z,t) = E_{\text{eff}}\left(A\hat{\mathbf{x}} + Be^{i\delta}\hat{\mathbf{y}}\right)e^{i(kz-\omega t)}$$

$$\mathbf{E}(z,t) = E_{\text{eff}}\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} e^{i(kz-\omega t)}$$

In many problems involving just polarization, we drop the oscillatory part.

$$\mathbf{E}(z,t) = E_{\text{eff}}\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$$

It's a *normalized* unit vector.

$$(A\hat{\mathbf{x}} + Be^{i\delta}\hat{\mathbf{y}}) \cdot (A\hat{\mathbf{x}} + Be^{i\delta}\hat{\mathbf{y}})^* = 1$$

$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2AB\cos\delta}{A^2 - B^2}\right)$$

Linearly polarized along x

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Linearly polarized along y

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Linearly polarized at angle α (measured from the x-axis)

$$\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

Right circularly polarized

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Left circularly polarized

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

Rotation Matrix Representation

To represent an arbitrary rotation θ in the x-y plane we use the rotation matrix R:

In a *passive* transformation, the coordinate system is moved.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

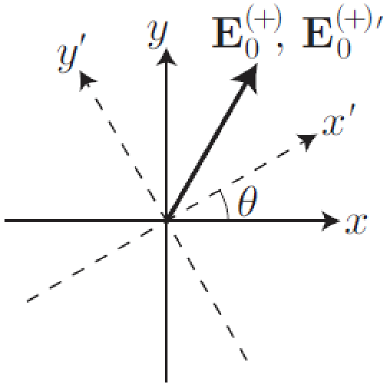
In an *active* transformation, we move the object (vector) instead.

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Special cases (passive rotation):

$$R(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad R(\pi/4) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$R(\pi) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad R(\pi/2) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



Jones Matrix Transformations

The real utility of the Jones formalism is that we can represent the effect of any linear optical element on the polarization of an input beam as follows:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A' \\ B' \end{bmatrix}$$

If light goes through a series of elements, we can chain the matrices:

$$\begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \cdots \begin{bmatrix} a_0 & b_0 \\ c_0 & d_0 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A' \\ B' \end{bmatrix}$$

The matrices are generally non-commutative and order is important!

Linear polarizer

$$\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Half wave plate

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

Quarter wave plate

$$\begin{bmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{bmatrix}$$

Right circular polarizer

$$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

Left circular polarizer

$$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$$

Reflection from an interface

$$\begin{bmatrix} -r_p & 0 \\ 0 & r_s \end{bmatrix}$$

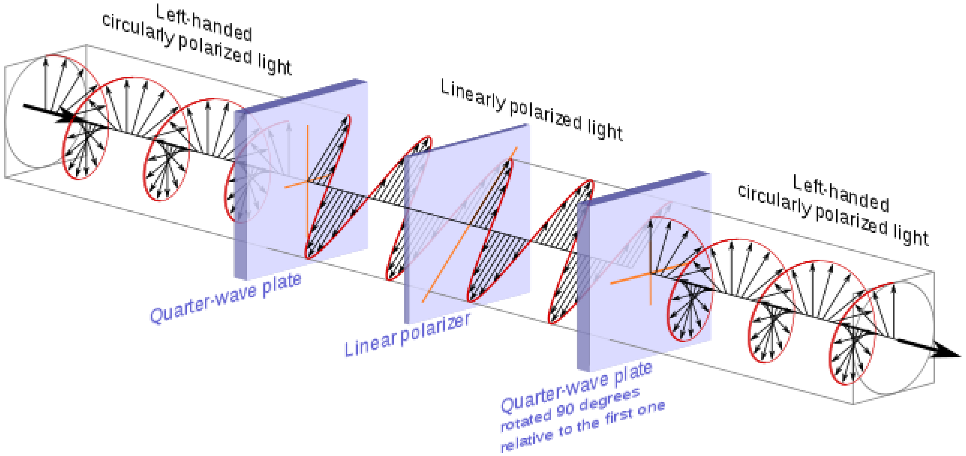
Transmission through an interface

$$\begin{bmatrix} t_p & 0 \\ 0 & t_s \end{bmatrix}$$

Polarization Optics

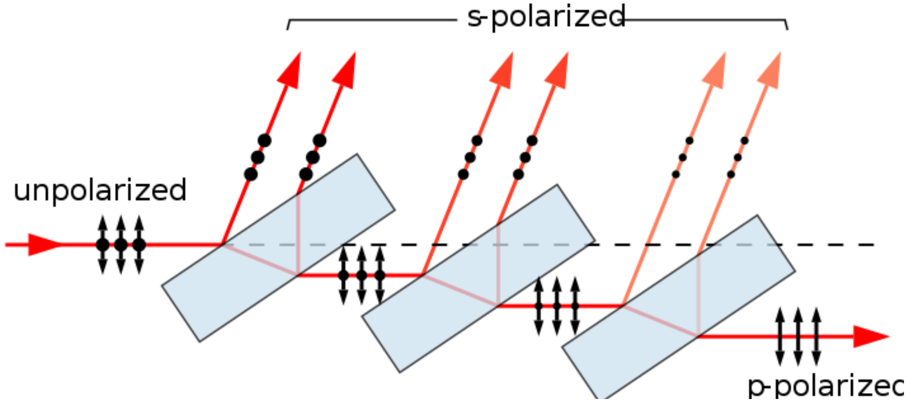
Anything that exerts an affect on polarization states

- Reflections from surfaces (Fresnel)
- Scattering from very small particles
- Non-isotropic physical materials
  - birefringent crystals like calcite
  - chiral materials like corn syrup
  - almost any clear material when stressed!

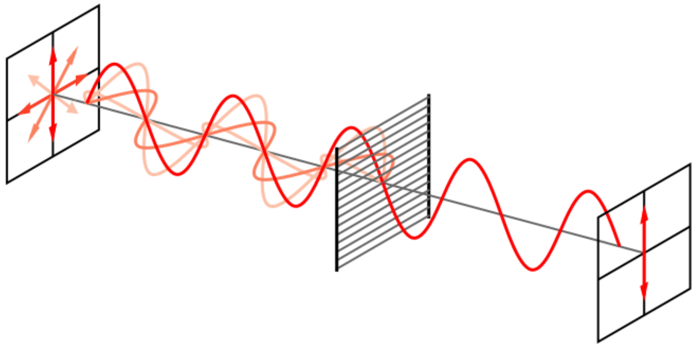


Polarization Optics

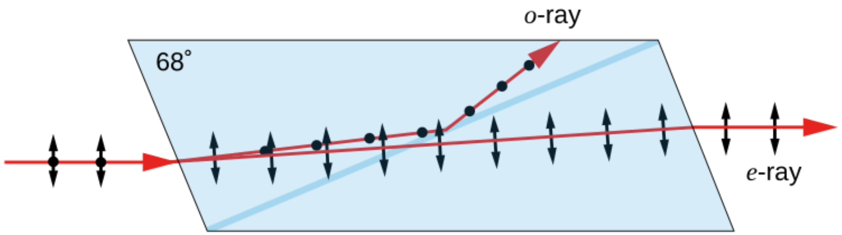
pile-of-plates
(Fresnel reflection)



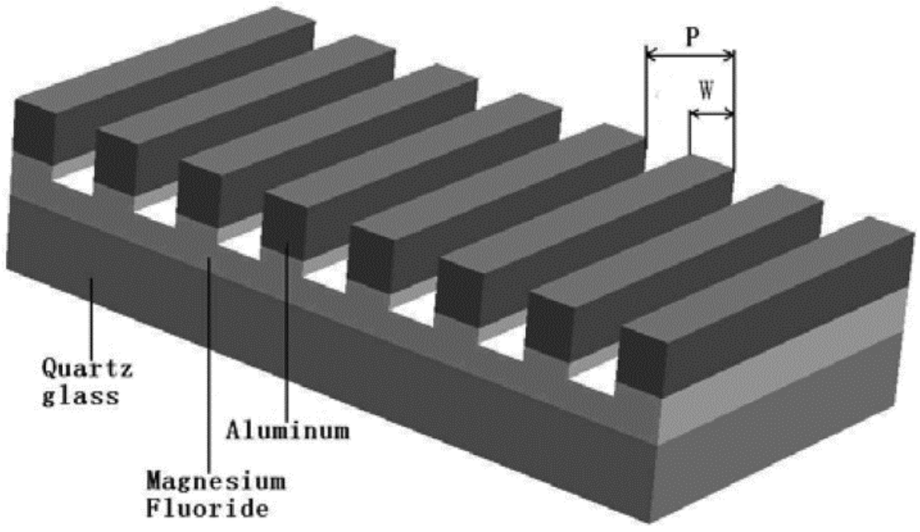
wire grid array
(reflection)

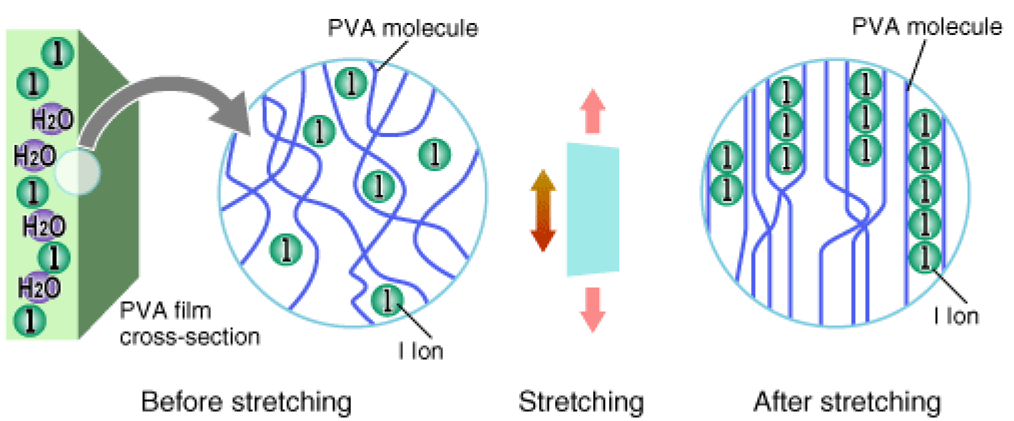


birefringent crystal
(Snell's Law!)

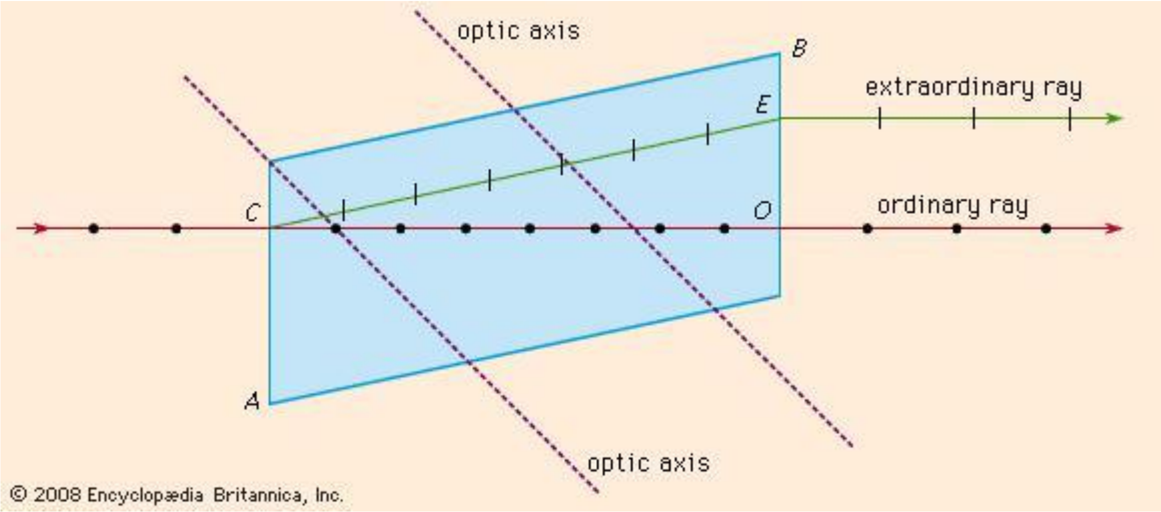


Polarization Filters

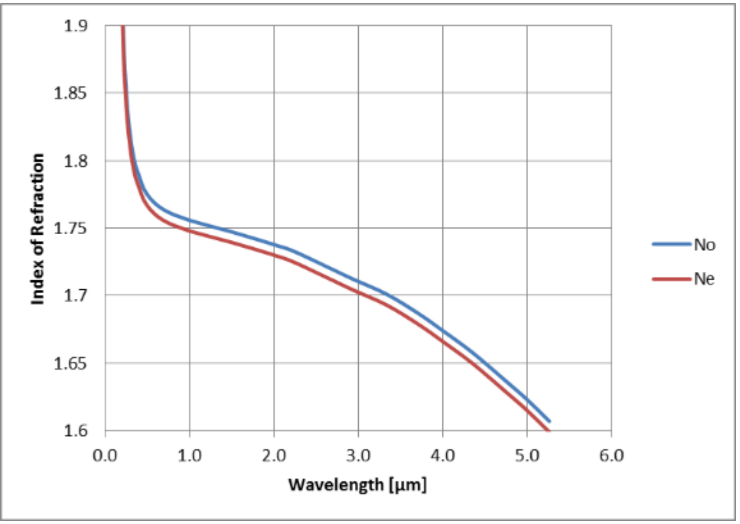




Birefringence (bi-*refractance*)



sapphire



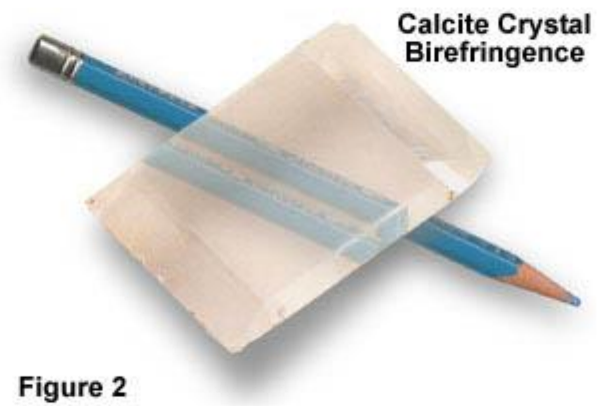


Figure 2

Isotropic Media

Assume our bulk matter is a linear dielectric [2-4]

$$\vec{P} = \epsilon_0 \chi_E \vec{E} \qquad \chi_E : \text{electric susceptibility}$$

$$= \epsilon_0 (K_E - 1)\vec{E} \qquad K_E : \text{relative permittivity (or dielectric constant)}$$

$$= \epsilon_0 \left(\frac{\epsilon}{\epsilon_0} - 1\right)\vec{E}$$

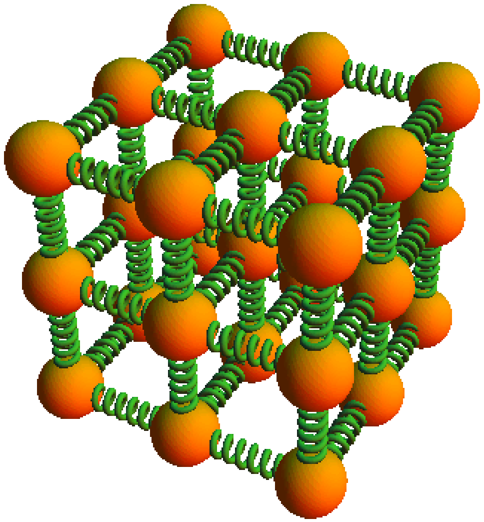
$$= (\epsilon - \epsilon_0)\vec{E}$$

then $\epsilon = \epsilon_0 + \frac{\vec{P}}{\vec{E}} = \frac{q^2 N/m}{(\omega_0^2 - \omega^2 + i\gamma\omega)} + \epsilon_0$

now remembering the connection between ϵ and the speed of the wave

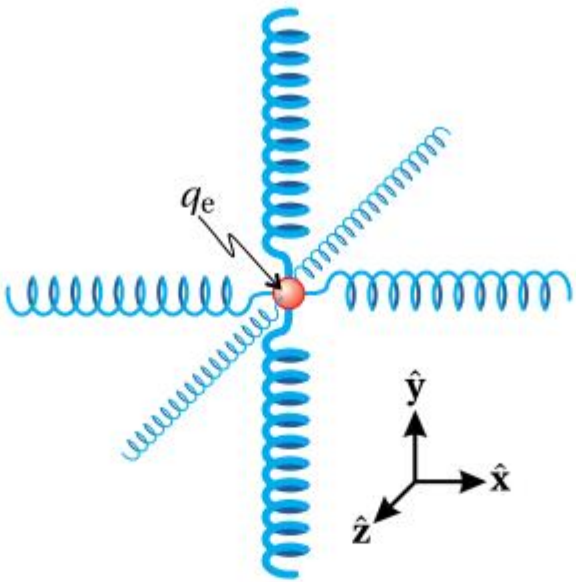
$n = \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} = \sqrt{K_E K_m}$, but $K_m \approx 1$ for <u>almost</u> all transparent materials

$$n^2 = K_E = \epsilon/\epsilon_0 = 1 + \frac{Nq^2}{\epsilon_0 m}\left(\frac{1}{\omega_0^2 - \omega^2 + i\gamma\omega}\right)$$



Anisotropic Media

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

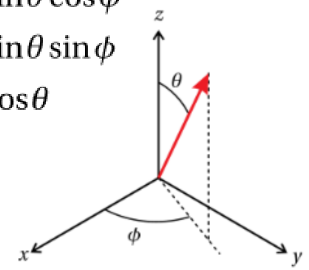


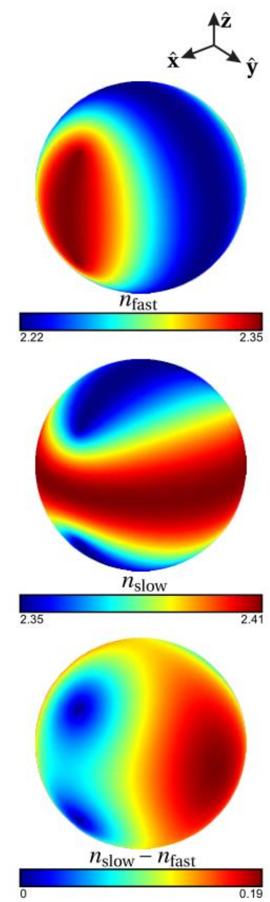
$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \chi_x & 0 & 0 \\ 0 & \chi_y & 0 \\ 0 & 0 & \chi_z \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\mathbf{P} = \hat{\mathbf{x}}\epsilon_0\chi_x E_x + \hat{\mathbf{y}}\epsilon_0\chi_y E_y + \hat{\mathbf{z}}\epsilon_0\chi_z E_z$$

$$n^2 = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

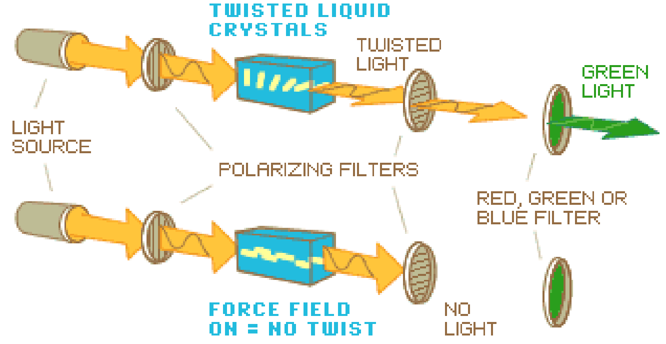
$$u_x = \sin\theta\cos\phi$$
$$u_y = \sin\theta\sin\phi$$
$$u_z = \cos\theta$$

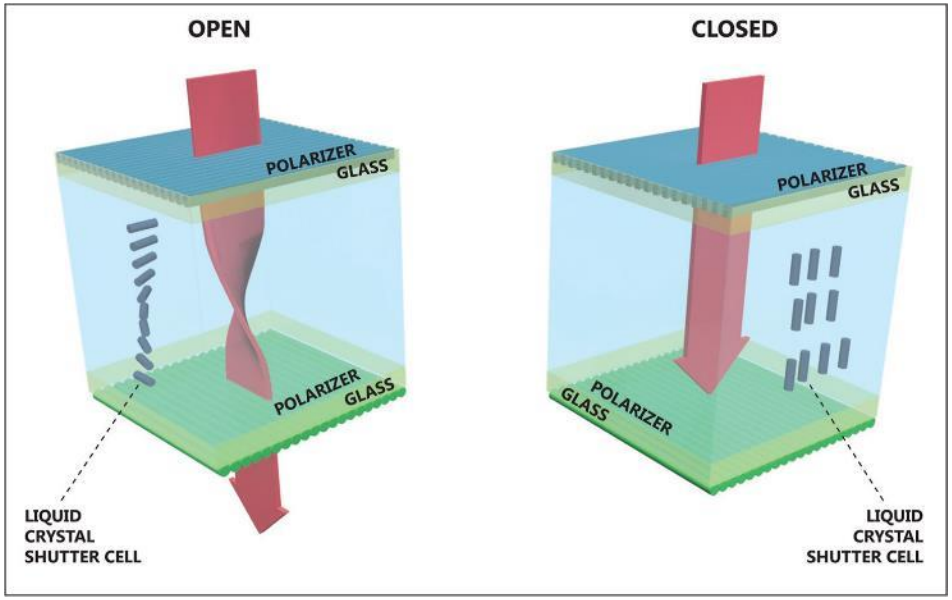


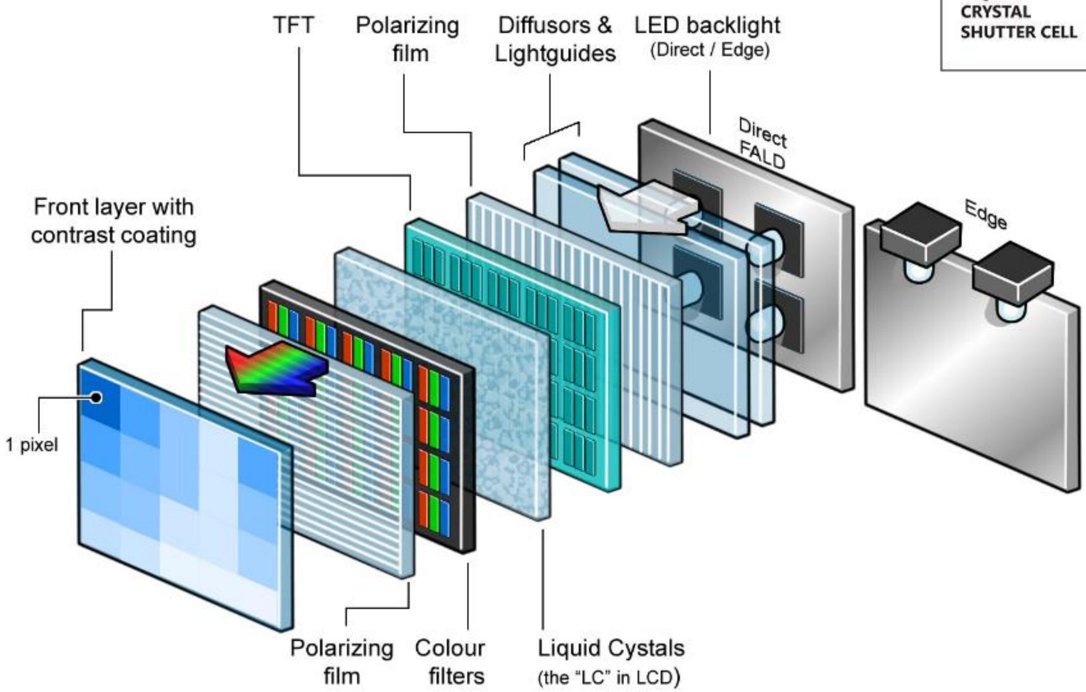


$$A \equiv u_x^2 n_x^2 + u_y^2 n_y^2 + u_z^2 n_z^2$$
$$B \equiv u_x^2 n_x^2 \left(n_y^2 + n_z^2\right) + u_y^2 n_y^2 \left(n_x^2 + n_z^2\right) + u_z^2 n_z^2 \left(n_x^2 + n_y^2\right)$$
$$C \equiv n_x^2 n_y^2 n_z^2$$

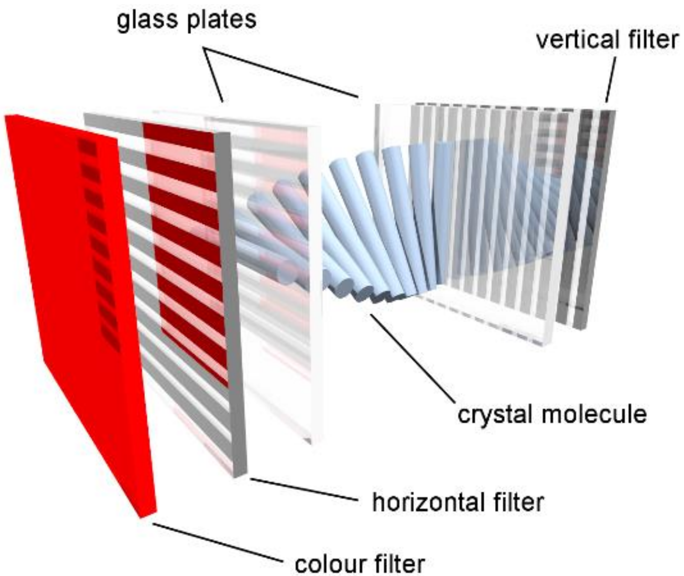
Applications: LCD

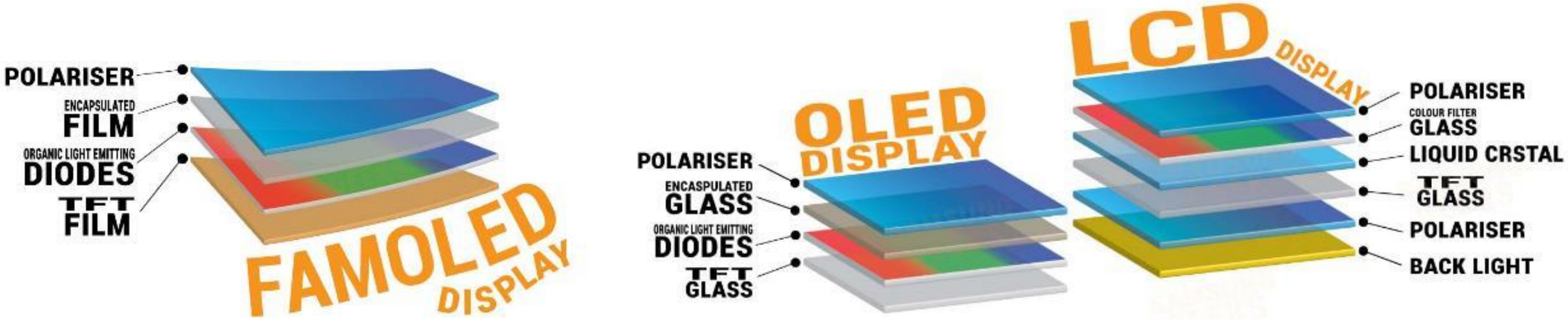






Applications: LCD



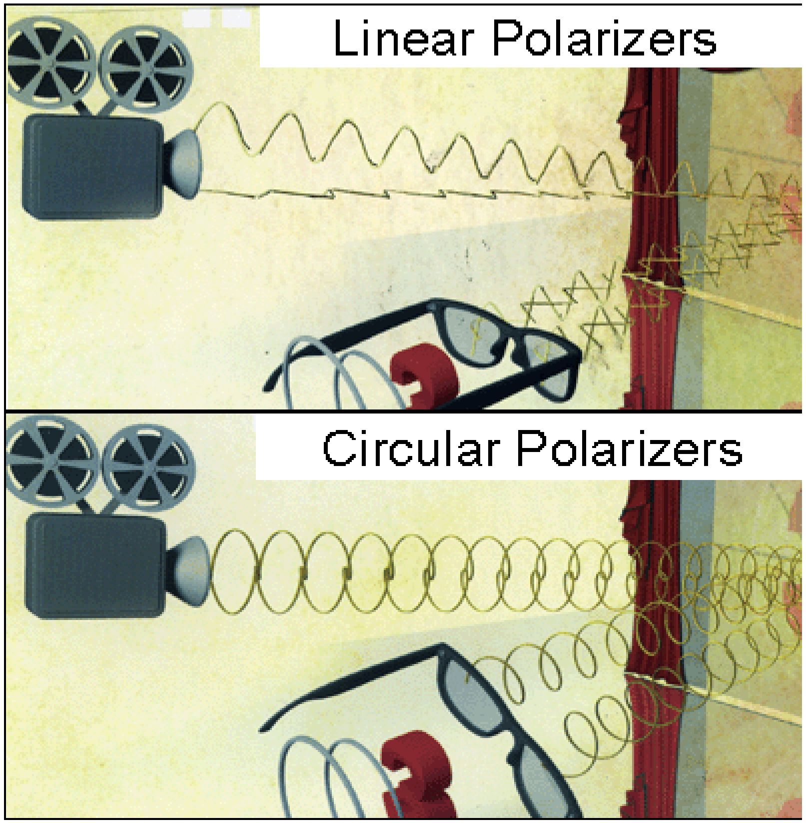


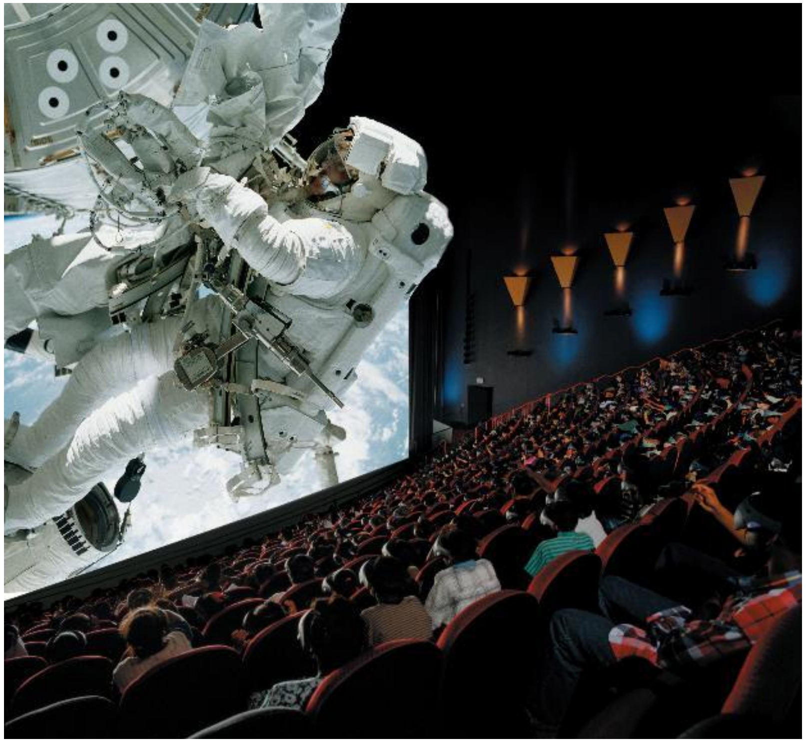
Applications: Ultra-Privacy Monitor

https://www.youtube.com/watch?v=zL_HAmWQTgA

Why replace the polarizer with another polarizer, though?...

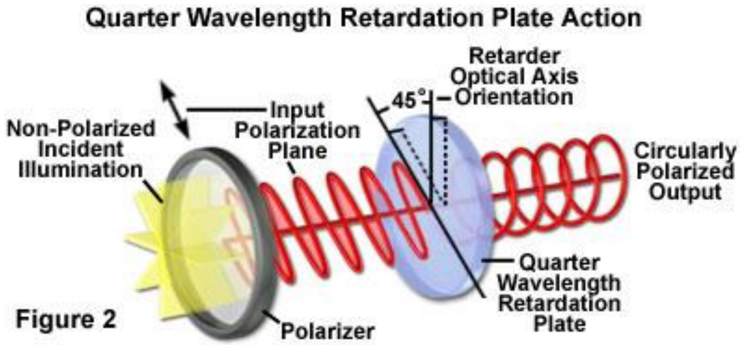
Applications: IMAX 3D

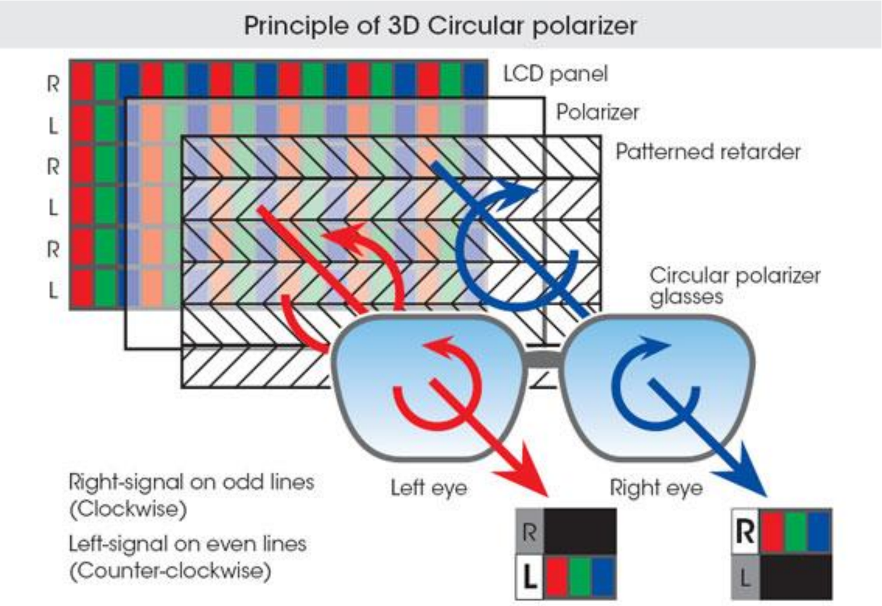




Projector(s)

Applications: 3D Television





OR