MODERN OPTICS

P47 – Optics: Unit 9

Course Outline

~~Unit 1: Electromagnetic Waves~~
~~Unit 2: Interaction with Matter~~
~~Unit 3: Geometric Optics~~

~~Unit 4: Superposition of Waves~~
~~Unit 5: Polarization~~
~~Unit 6: Interference~~

~~Unit 7: Diffraction~~
~~Unit 8: Fourier Optics~~
Unit 9: Modern Optics

Two Criteria for Laser Operation

LASER: **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation

1. Coherent amplification of an optical field
   - Stimulated emission of radiation
   - Population inversion in a 3 (or more) level system

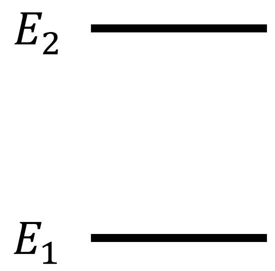
2. Feedback from an optical cavity
   - Fabry-Perot with partial reflector: Longitudinal modes
   - Mirror curvature: Transverse modes and Gaussian beams
   - Laser stability criteria, geometric optics, ABCD matrix.

Quantized Absorption & Emission

The quantum nature of light was first recognized in the context of absorption and emission by matter

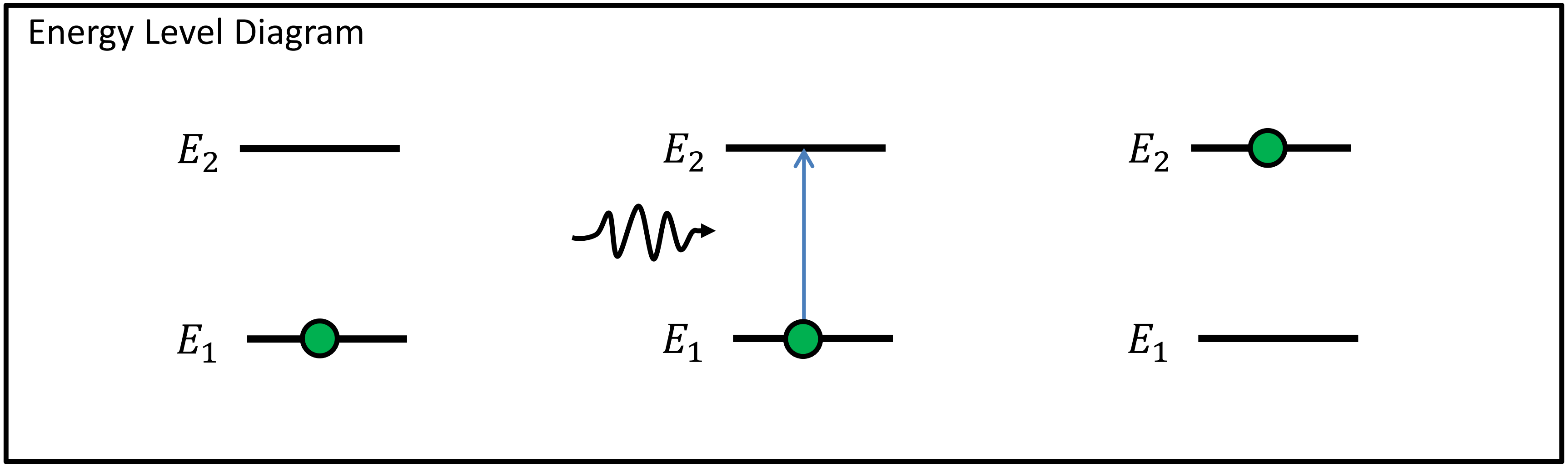
- Photoelectric effect
  - ejection of photoelectron by EM field
- Blackbody Radiation
  - Stefan-Boltzmann
  - Wien's Law
  - Planck Blackbody Spectrum

A Collection of Two-Level Atoms



- Have group of N two level atoms (two energy levels)
  - N_1 – the number of atoms in the ground state
  - N_2 – the number of atoms in the excited state
  - $N = N_1 + N_2$ – the total number of atoms
- Will consider a statistical mixture of populations (N_1 in E_1; N_2 in E_2)
- What happens when we introduce photons (EM fields)?

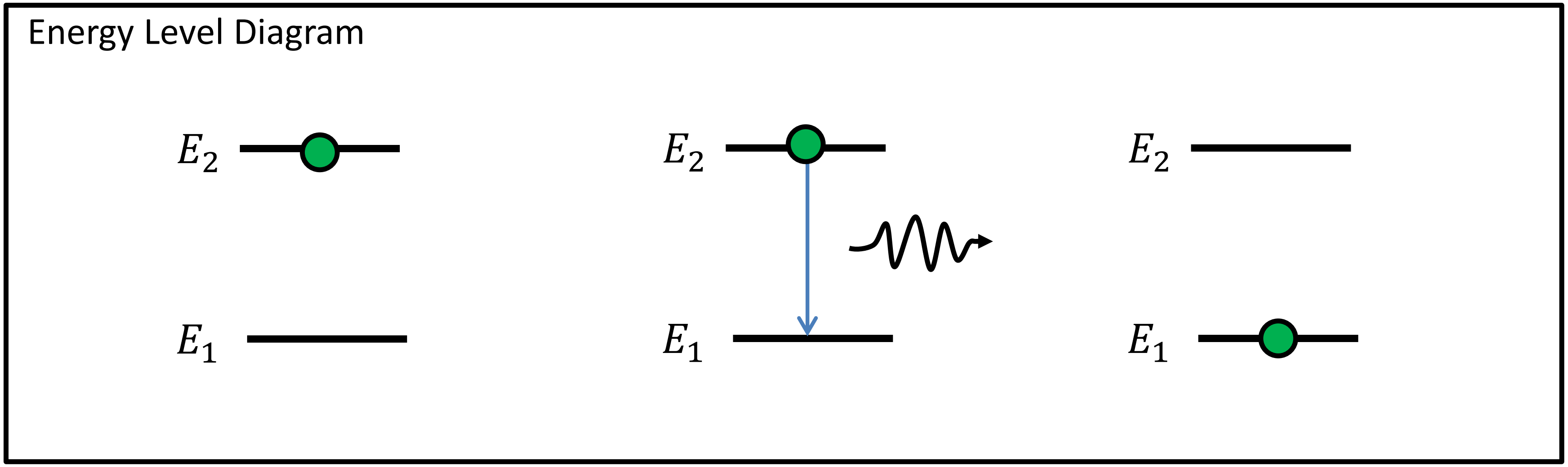
Optical Absorption



- Atom is initially in the ground state
- An incoming *resonant* photon is absorbed ($h\nu = \hbar\omega_{21} = E_{21}$)
- Atom gets promoted to a higher energy state
  (e.g. raising an electron to a higher energy orbital)

If this is the *only* allowed process, what will happen to a collection of atoms illuminated continuously?

Spontaneous Emission

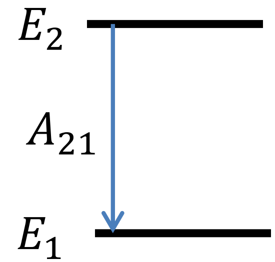


- Atom is initially in the <u>excited</u> state
- After some time, it will spontaneously decay to ground state
- Upon decay, a *resonant* photon is emitted ($h\nu = \hbar\omega_{21} = E_{21}$)
- This is called *fluorescence*

If this is the *only* allowed process, what will happen to a collection of atoms initially in the excited state?

Einstein Rate Equations

(No Driving Field)



Excited State: $$\frac{dN_2}{dt} = -A_{21}N_2$$

Ground State: $$\frac{dN_1}{dt} = A_{21}N_2$$

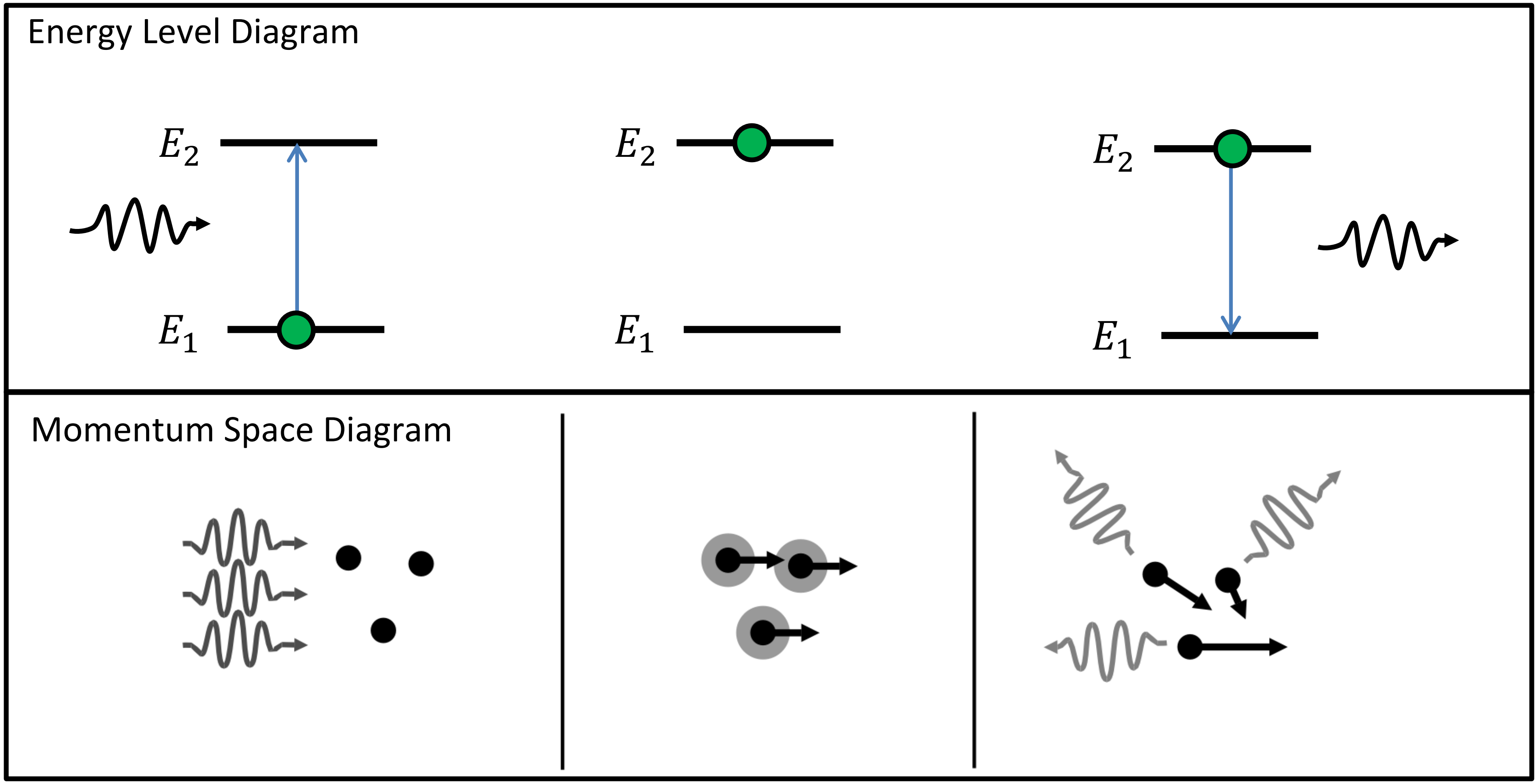
Spontaneous emission (A_{21}) takes atoms from the excited state to the ground state at a rate $\tau_{12} = 1/A_{21}$

(τ_{12} ~ nanoseconds)

Any population in the excited state decays exponentially

$$N_2(t) = N_2(0)e^{-A_{21}t} = N_2(0)e^{-t/\tau_{12}}$$

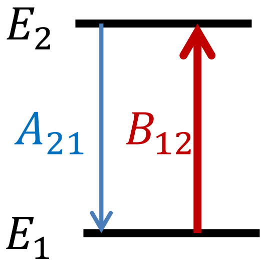
Absorption Followed by *Spontaneous* Emission



Assumption: Atoms in the excited state will *spontaneously* decay into the ground state at some rate τ_{12}

Why? Vacuum fluctuations – a perturbation is necessary.

Einstein Rate Equations
(with Driving Field)



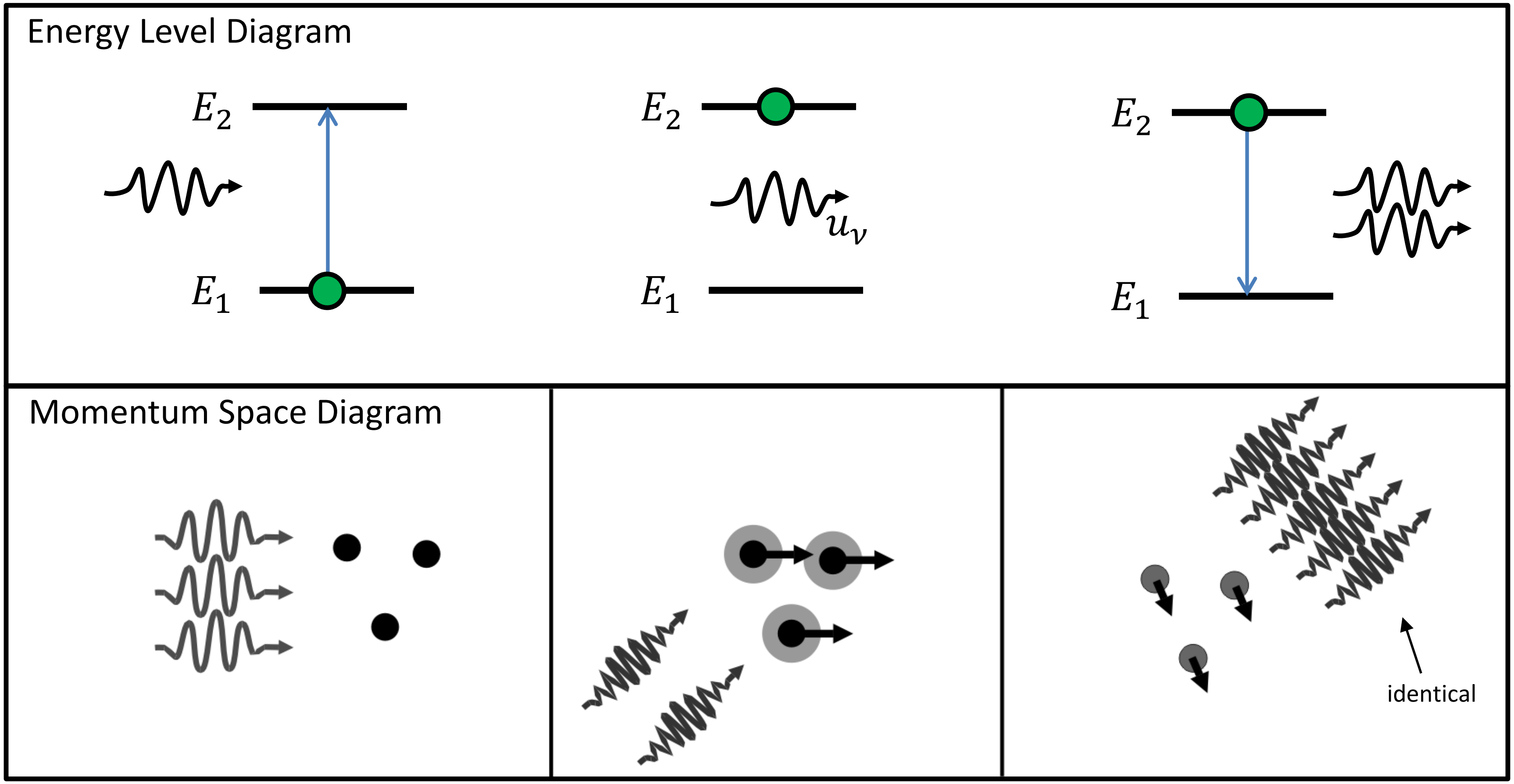
Excited State: $$\frac{dN_2}{dt} = -A_{21}N_2 + B_{12}u_\nu N_1$$

Ground State: $$\frac{dN_1}{dt} = A_{21}N_2 - B_{12}u_\nu N_1$$

u_ν is the energy density of the electromagnetic field

(This picture is missing something very important)

Absorption Followed by *Stimulated* Emission

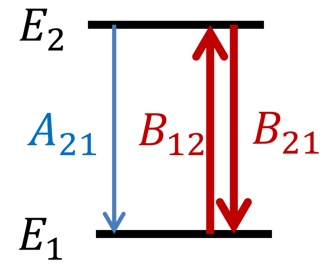


This time, the rate that photons are emitted depends on the spectral energy density of the external field, u_ν.

Einstein realized that this *must* happen for the theory to be self-consistent (ultimately what makes lasers possible…)

Einstein Rate Equations

(with *actual* Driving Field)



Excited State: $$\frac{dN_2}{dt} = -A_{21}N_2 - B_{21}u_\nu N_2 + B_{12}u_\nu N_1$$

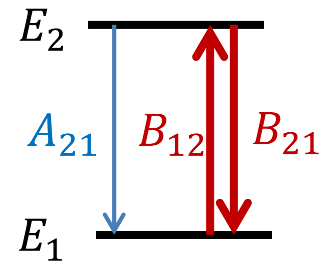
Ground State: $$\frac{dN_1}{dt} = A_{21}N_2 + B_{21}u_\nu N_2 - B_{12}u_\nu N_1$$

u_ν is the energy density of the electromagnetic field ($u_\nu = \frac{\vec{S}}{c^2} = \frac{\epsilon_0}{2c}E_0^2$)

have now incorporated both *stimulated emission* and *stimulated absorption* terms

Einstein Rate Equations

(Steady-State Case)



Take rate equation for the exited state:

$$\frac{dN_2}{dt} = -A_{21}N_2 - B_{21}u_\nu N_2 + B_{12}u_\nu N_1$$

In steady-state, we have $\frac{dN_2}{dt} = 0$

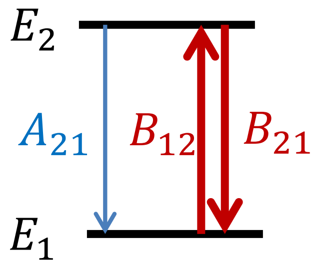
$$0 = -A_{21}N_2 - B_{21}u_\nu N_2 + B_{12}u_\nu N_1$$

$$\mathrm{N}_2(A_{21} + B_{21}u_\nu) = N_1\, B_{12}u_\nu$$

$$\boxed{\frac{\mathrm{N}_2}{N_1} = \frac{B_{12}u_\nu}{A_{21} + B_{21}u_\nu}}$$

- *without* stimulated emission, $N_2 \rightarrow N$ as $u_\nu \uparrow$
- *with* stimulated emission, as $u_\nu \rightarrow \infty$, then $\frac{\mathrm{N}_2}{N_1} \rightarrow 1$
- at most, half of atoms are excited
- no gain/amplification… no laser.

Einstein Rate Equations
(Coefficient Relations)



$$\frac{N_2}{N_1} = \frac{B_{12} u_\nu}{A_{21} + B_{21} u_\nu}$$

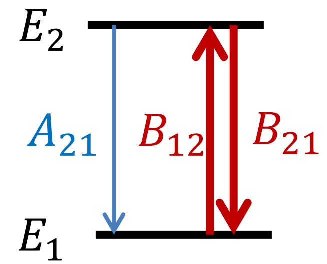
To get values for A and B requires knowledge of the specific atomic system and a quantum mechanical calculation…

BUT…

These are called the Einstein coefficients because he showed that there is an important connection between these three different coefficients, *independent of the fine details*.

Einstein Rate Equations

(Coefficient Relations)



$$\frac{N_2}{N_1} = \frac{B_{12} u_\nu}{A_{21} + B_{21} u_\nu}$$

We'll use specific information about the behavior of the two parts of this system **in thermal equilibrium**

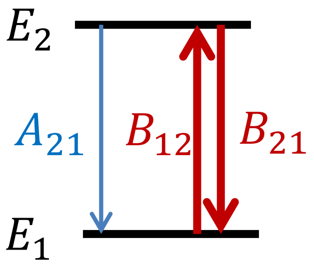
1) Boltzmann: How much population must be in state 2 at temperature *T* ?

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/k_B T} = e^{-h\nu_{21}/k_B T} = \frac{B_{12} u_\nu}{A_{21} + B_{21} u_\nu}$$

$$u_\nu = \frac{A_{21}/B_{21}}{(B_{12}/B_{21}) e^{h\nu_{21}/k_B T} - 1}$$

(population dependence eliminated!)

Einstein Rate Equations
(Coefficient Relations)



$$u_\nu = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})e^{h\nu_{21}/k_B T} - 1}$$

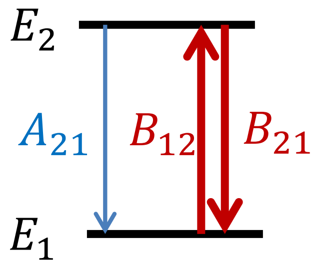
We've eliminated dependence on population, now eliminate dependence on spectral power…

2) Planck Blackbody Spectral Energy Density:

$$u_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{k_b T}} - 1}\right] = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})e^{h\nu_{21}/k_B T} - 1}$$

$$\boxed{\begin{gathered} B = B_{12} = B_{21} \\ A = \frac{8\pi h\nu^3}{c^3} B \end{gathered}}$$

Einstein Rate Equations
(Strong Driving Limit)



$$\frac{N_2}{N_1} = \frac{Bu_\nu}{A + Bu_\nu} < 1$$

$N_2 < N_1$ no matter how big the driving field is!

Take-Home Point: It's impossible to create a *steady-state population inversion* in a 2-level system!

If we can't make a laser with this system….
what do we do now?

Add another level!

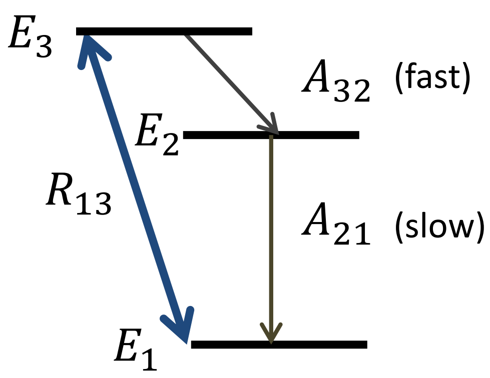
Laser Pumping Schemes
(Three-Level Laser)

Assume we have a third level and some process that excites the atoms to that level at a rate R_{13}

$$\frac{dN_3}{dt} = R_{13}(N_1 - N_3) - A_{32}N_3$$

$$\frac{dN_2}{dt} = A_{32}N_3 - A_{21}N_2$$

$$\frac{dN_1}{dt} = A_{21}N_2 + R_{13}(N_3 - N_1)$$



If the rate of spontaneous decay rate from 3 → 2 is much faster than the decay rate from 2 → 1 and the rate of pumping from 1 → 2, something very interesting becomes possible!

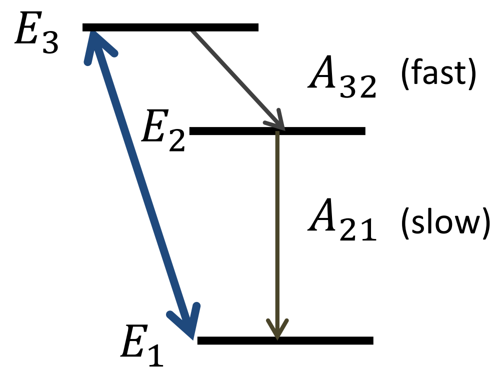
Laser Pumping Schemes

(Population Inversion)

Adiabatic approximation for level 3!

$$\frac{dN_3}{dt} = 0 = R_{13}(N_1 - N_3) - A_{32}N_3$$

$$N_3 = \frac{R_{13}}{A_{32} + R_{13}} N_1 \approx \frac{R_{13}}{A_{32}} N_1$$

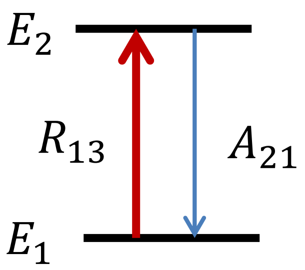


There's almost no population in state 3 because it decays so fast $(N_3 \ll N_1)$

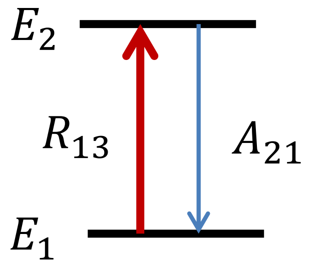
This gives us an asymmetric way to get atoms into state 2 without having the pump interaction drive them right back down again at the same rate!

$$\frac{dN_2}{dt} = R_{13}N_1 - A_{21}N_2$$

$$\frac{dN_1}{dt} = A_{21}N_2 - R_{13}N_1$$



Laser Pumping Schemes
(Population Inversion)



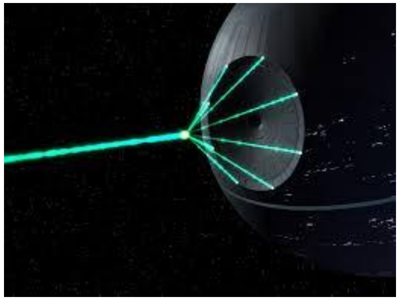
$$\frac{dN_2}{dt} = R_{13}N_1 - A_{21}N_2$$

$$\frac{dN_1}{dt} = A_{21}N_2 - R_{13}N_1$$

Again, look at the steady-state:

$$\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$$

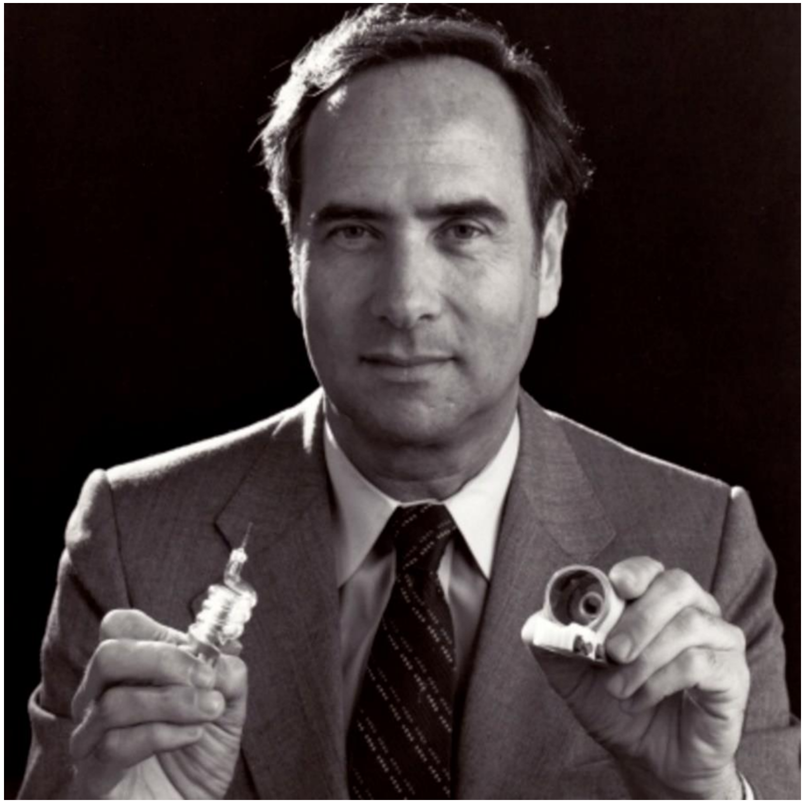
If we can make R_{13} bigger than A_{21}, we can now create a *population inversion* and make a laser!



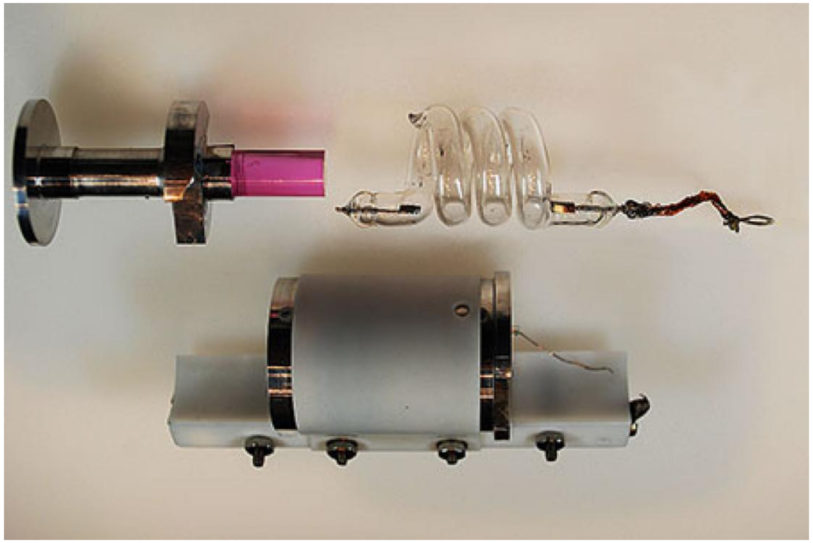
$$\frac{N_2}{N_1} = \frac{R_{13}}{A_{21}}$$

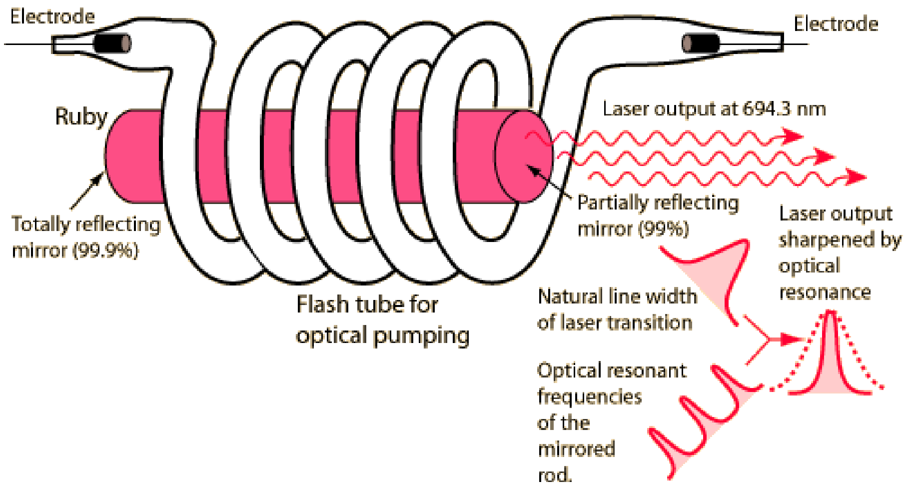
Maiman's Original Ruby Laser

(3-Level Pulsed Laser)

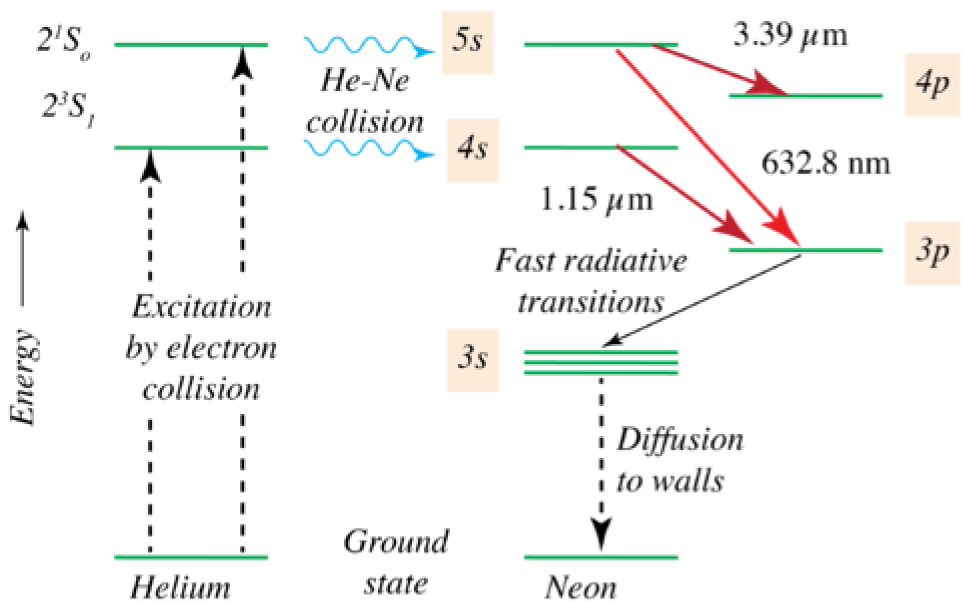


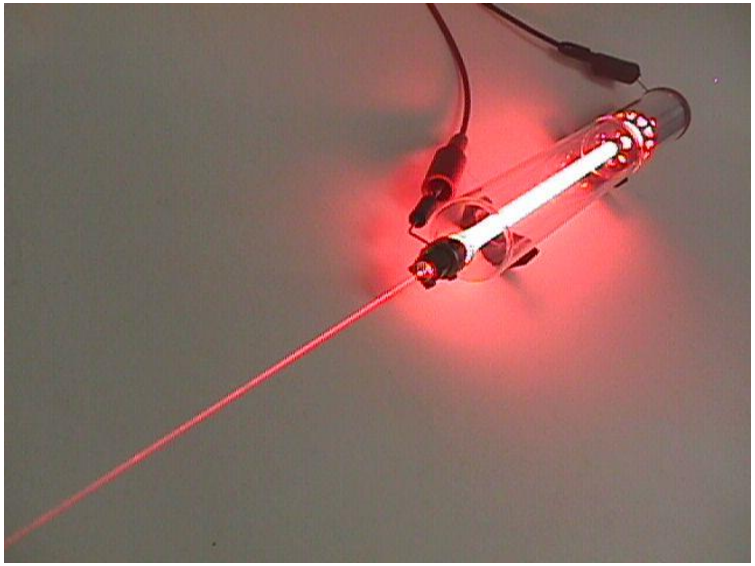
Theodore Harold "Ted" Maiman (July 11, 1927 – May 5, 2007)





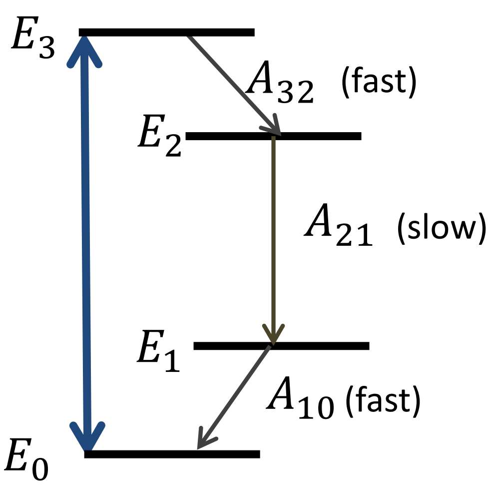
Helium-Neon (HeNe) Laser
(4-Level Continuous Wave Laser)





1. High voltage on electrodes creates a plasma
2. Electron collisions pump helium into several excited states (2s)
3. Helium collides with neon and transfers its energy
4. With sufficient pumping, a population inversion of the metastable 3s state can occur

Why would there be an advantage to this kind of setup?



Even if you screw-up engineering details of the laser, at least you've fabricated a neon lamp

Feedback and Laser Oscillation

Reflect some of the photons emitted from the gain medium back into it, where they can be amplified!

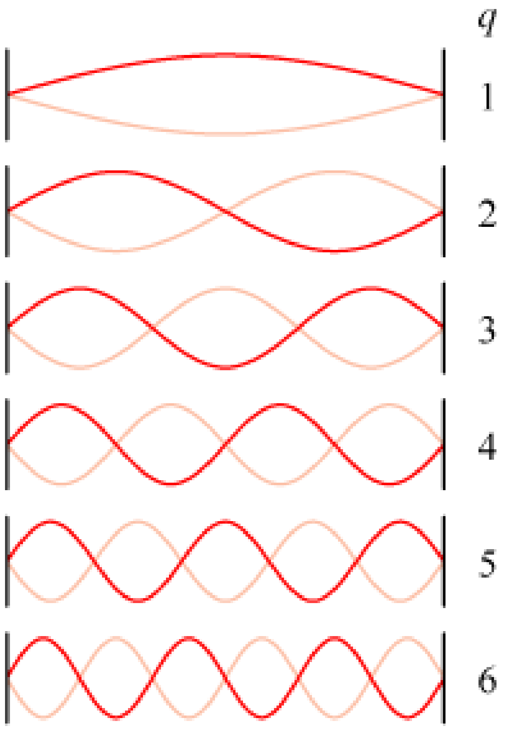
But not just any mirror configuration will do…

Cavity Longitudinal Modes

(Fabry-Perot Laser Cavity)

The cavity length must be an integer number of half-wavelengths of the light

For L $\approx$ 20.0 cm and $\lambda = 632.8$ nm:
$$q \approx 632{,}100$$



The laser can only operate at select frequencies separated by the cavity Free Spectral Range (FSR)

For L=20 cm:
$$\nu = q \cdot FSR = q\frac{c}{2L} = q \cdot 750 \text{ MHz}$$

Cavity Longitudinal Modes

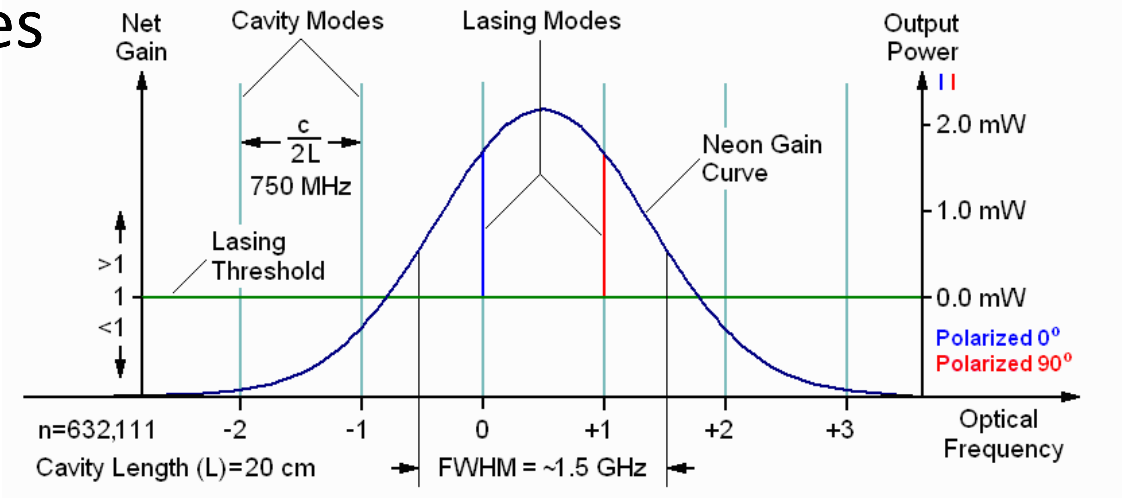
Lasing depends on overlap of the *gain profile* with the cavity modes

<u>Low-power (3 mW) cavity</u>

L = 20 cm

FSR = 750 MHz

Lasing occurs on only 2 modes



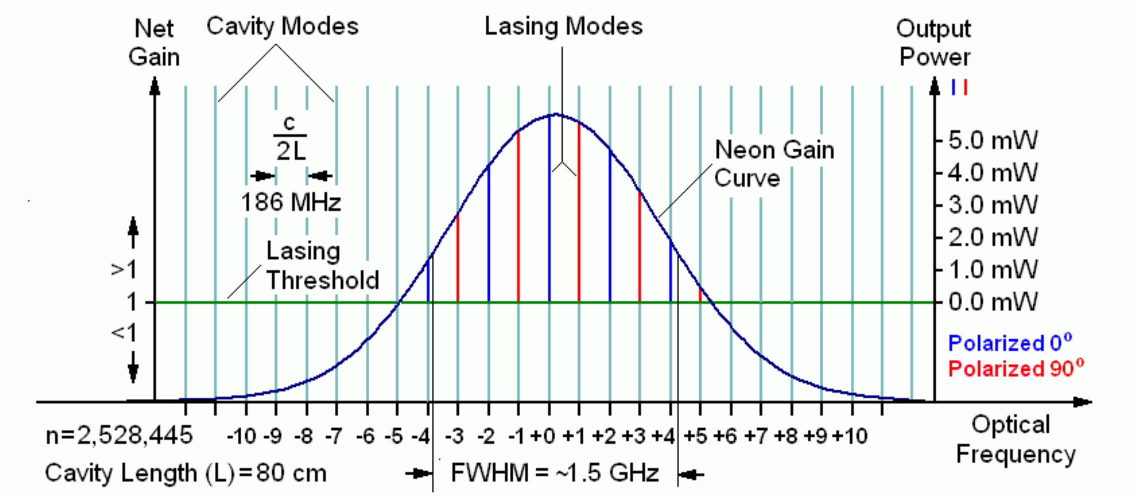
Longitudinal Modes of Typical Random Polarized 3 mW HeNe Laser

<u>Higher-power (30 mW) cavity</u>

L = 80 cm

FSR = 186 MHz

Lasing occurs on 10 modes



Longitudinal Modes of Typical Random Polarized 30 mW HeNe Laser

Cavity Transverse Modes

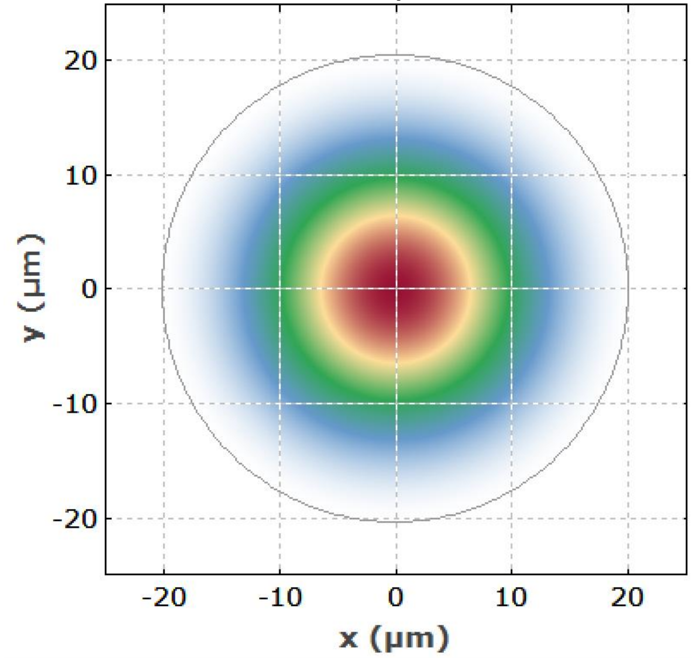
The mirrors are finite-sized…

Need solutions of the **paraxial Helmholtz equation** that vanish as r becomes large

The simplest solution we might consider is a **Gaussian Beam Mode**:

$$I(\rho, z) = I_0(z)e^{2\rho^2/w(z)^2}$$

Radial profile

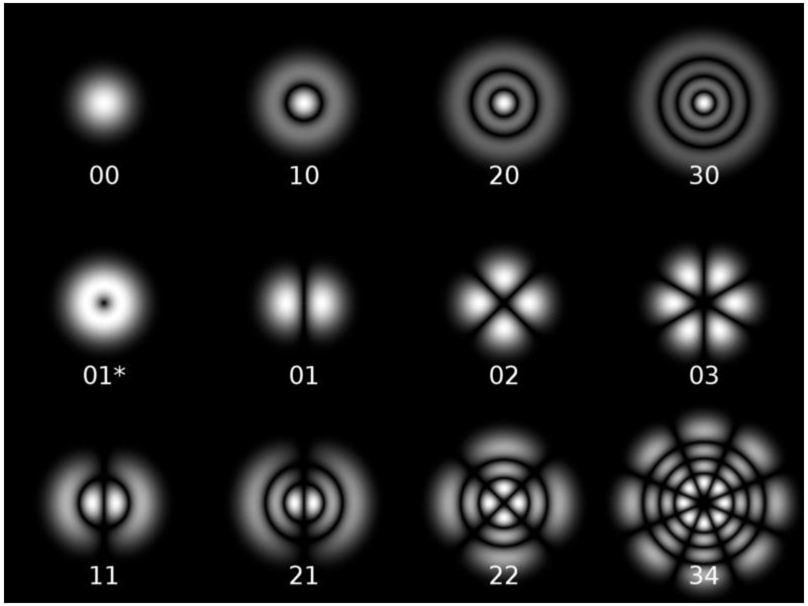


The simplest solution that we might use is the Gaussian or TEM_{00} electromagnetic field mode

Higher-Order Transverse Modes

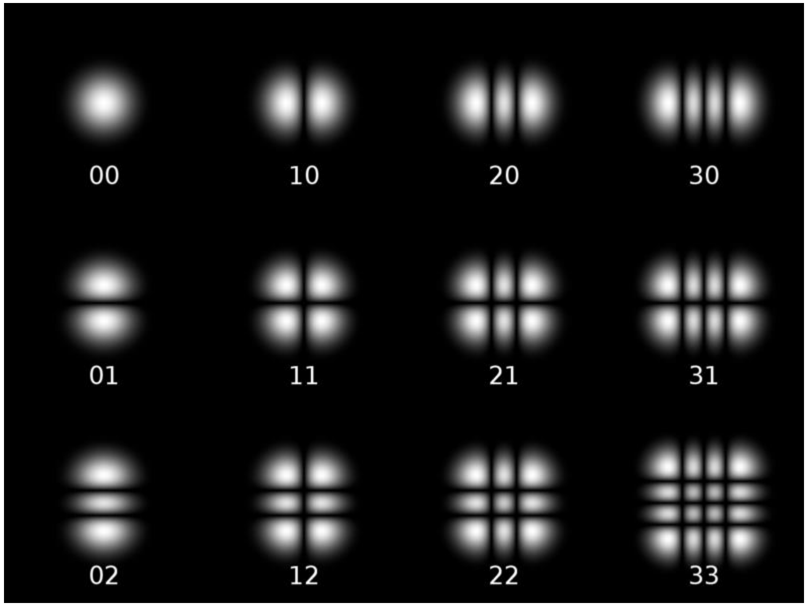
Laguerre-Gaussian Modes

(Cartesian – Laguerre polynomial basis)



Hermite-Gaussian Modes

(Cylindrical – Hermite Polynomial Basis)



Each one of these is its own Fourier transform!

Transverse Cavity Stability

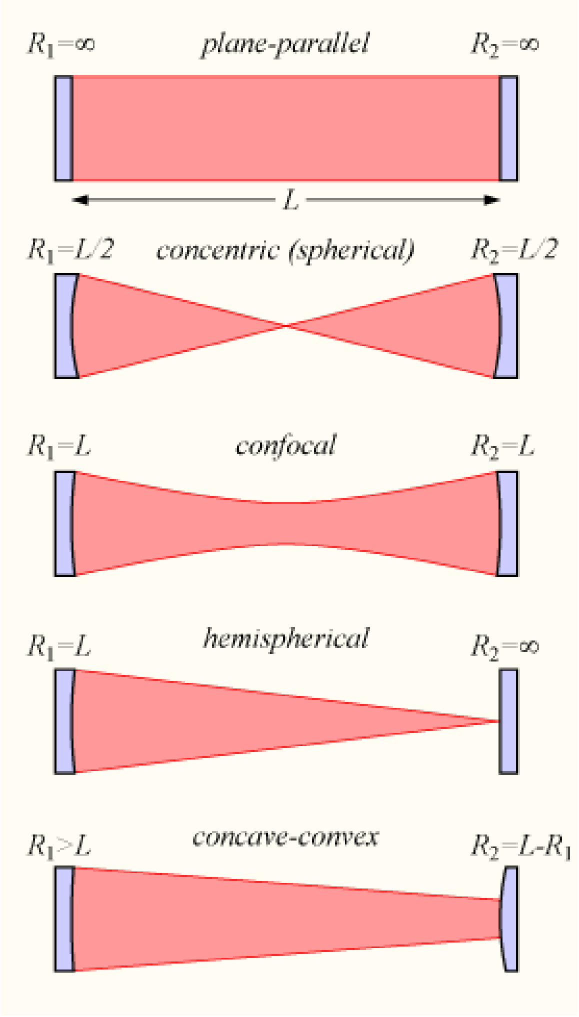
(will a photon stay inside?)

A flat mirror cavity is unstable
(Why?)

What affects whether a photon returns to the gain medium?

- mirror separation L
- mirror curvatures R_1 and R_2
- mirror diameter

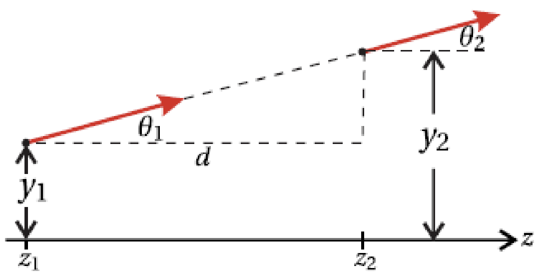
How can we quantify this?
Geometric optics…
Cavity stability analysis



Cavity Stability: The ABCD Matrix

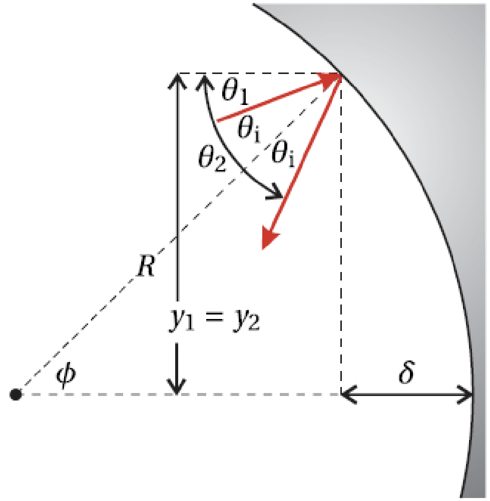
Propagation a distance d (index of refraction = 1)

$$\mathcal{M}_d = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$



Curved spherical optic (mirror or lens)

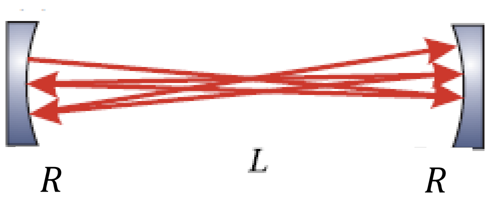
$$\mathcal{M}_f = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$



Can model the optical cavity by chaining these together

ABCD Matrix of Optical Cavity

$$\mathcal{M}_{cav} = \mathcal{M}_L \mathcal{M}_{R_2} \mathcal{M}_L \mathcal{M}_{R_1}$$



$$\mathcal{M}_{cav} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$$

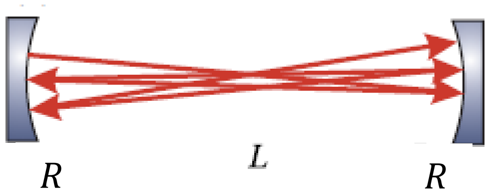
After some algebra.....

$$\mathcal{M}_{cav} = \begin{bmatrix} 4\left(\frac{L}{R}\right)^2 - 6\left(\frac{L}{R}\right) + 1 & 2L\left(1 - \frac{L}{R}\right) \\ 4/R\left(\frac{L}{R} - 1\right) & 1 - 2\left(\frac{L}{R}\right) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

This tells us exactly how the ray will be transformed after **one** round trip in the cavity!

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^N \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

Cavity Stability Criterion



The cavity will be stable **if** after many round trips the ray does not run off to infinity!

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^N \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

There is a straightforward formula for raising a 2x2 matrix to an arbitrary power:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = \frac{1}{\sin\theta} \begin{bmatrix} A\sin N\theta - \sin(N-1)\theta & B\sin N\theta \\ C\sin N\theta & D\sin N\theta - \sin(N-1)\theta \end{bmatrix}$$

Where $\cos\theta = \frac{1}{2}(A+D)$.

We need this expression to remain **finite**, given our cavity ABCD matrix

This implies that the ray will stay a finite distance from the cavity axis

Cavity Stability Criterion

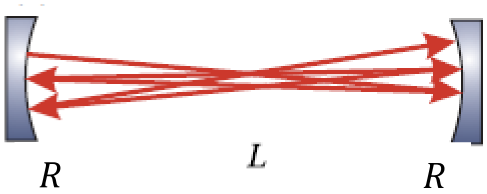
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = \frac{1}{\sin\theta}\begin{bmatrix} A\sin N\theta - \sin(N-1)\theta & B\sin N\theta \\ C\sin N\theta & D\sin N\theta - \sin(N-1)\theta \end{bmatrix}$$

Where $\cos\theta = \frac{1}{2}(A+D)$.

Point #1: N only appears within arguments of the sine function, which is bounded by [-1,1]

Point #2: The only problem we will run into is if θ becomes imaginary

As long as $-1 < \frac{1}{2}(A+D) < 1$, θ will be real-valued, and the cavity is stable!

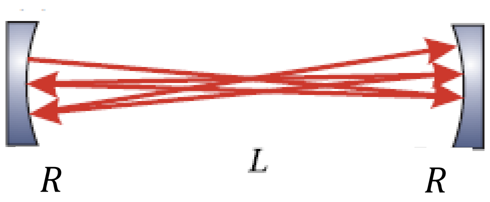


Cavity Stability Criterion

For our cavity earlier:

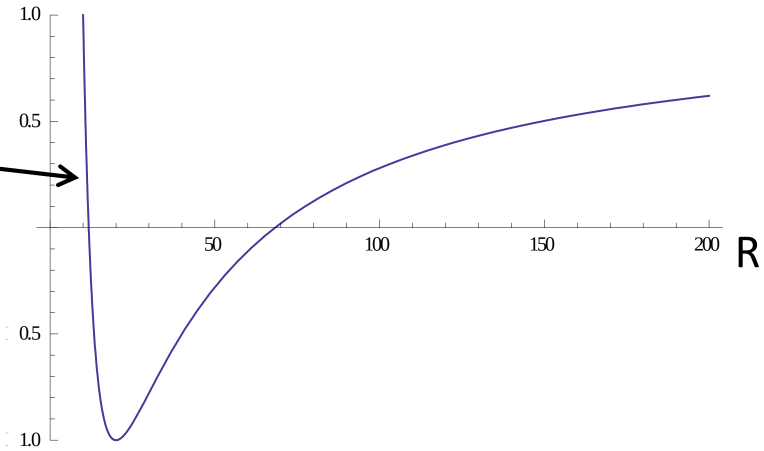
$$A = 4\left(\frac{L}{R}\right)^2 - 6\left(\frac{L}{R}\right) + 1$$

$$D = 1 - 2\left(\frac{L}{R}\right)$$



This gives us the criterion that

$$-1 < 2\left(\frac{L}{R}\right)^2 - 4\left(\frac{L}{R}\right) + 1 < 1$$



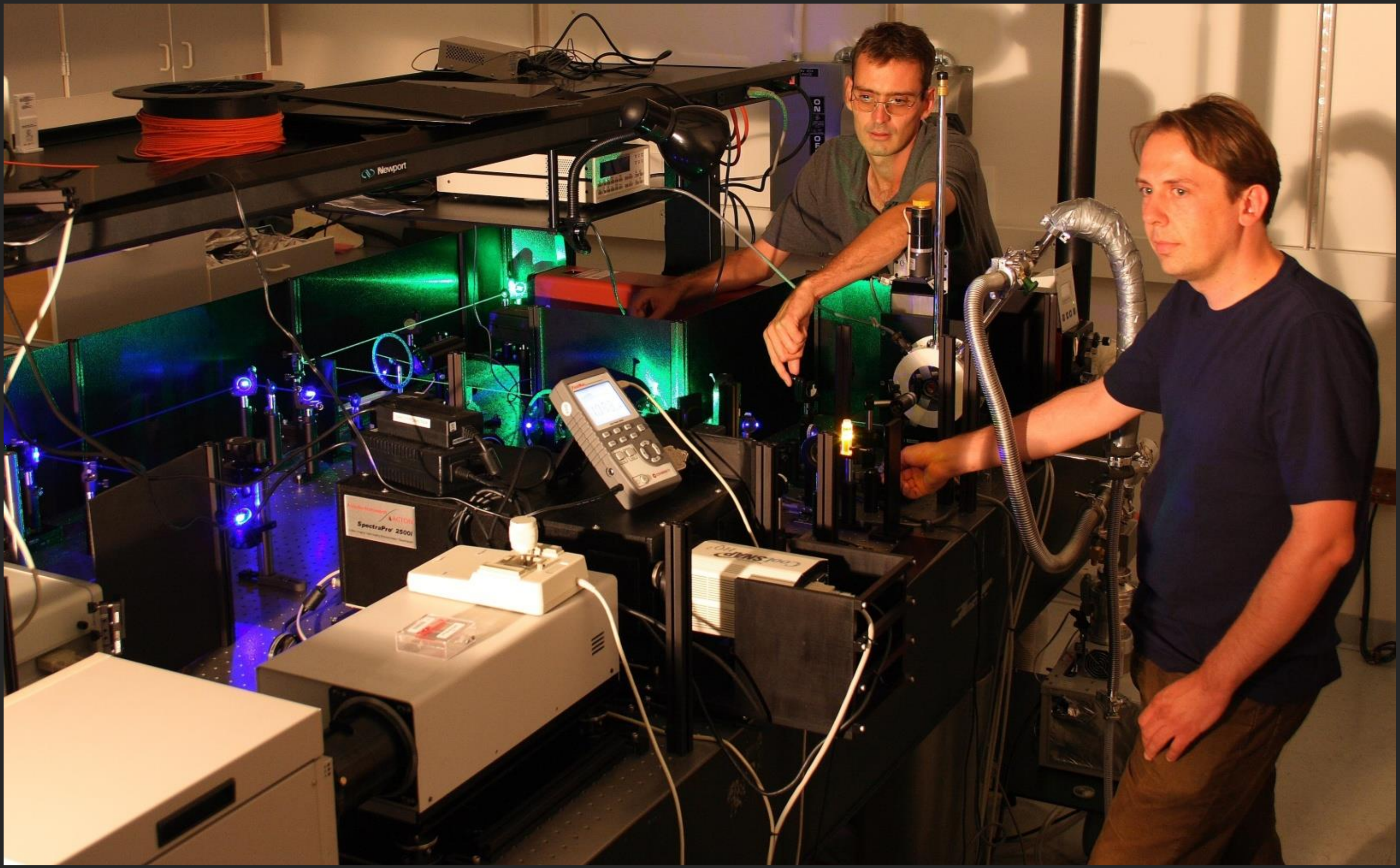
So, our L = 20 cm HeNe is stable for:

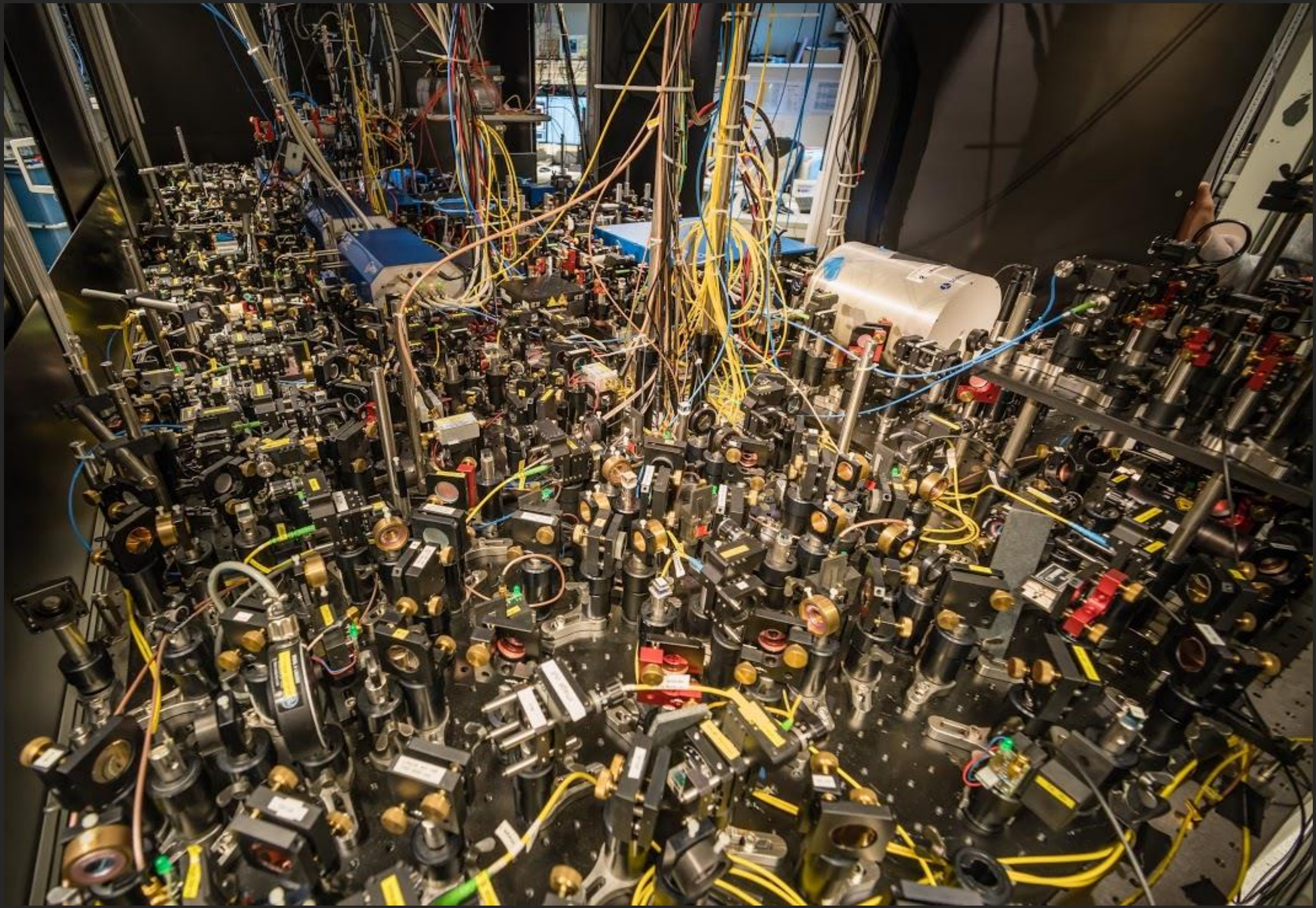
$$10 < R < \infty$$

Laser Summary

- Einstein Coefficients
  - A: spontaneous emission
  - B: stimulated absorption/emission
- Lasers require population inversion to work
- Can’t invert a 2-level system (in steady-state)
- 3 and 4 level systems make population inversion possible
- Need a stable cavity to enable feedback and laser oscillation

http://www.physik.uni-r.de/forschung/lupton/lupton/

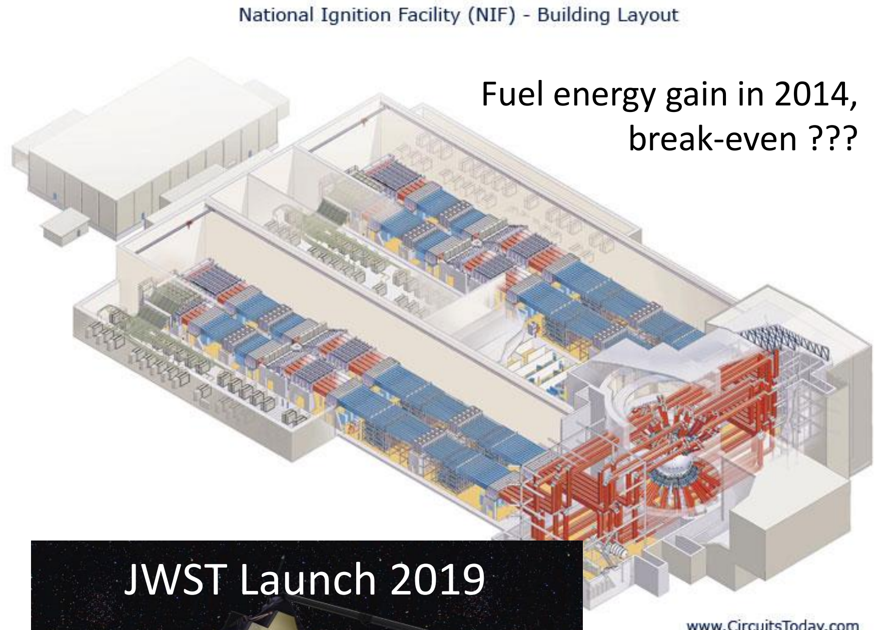




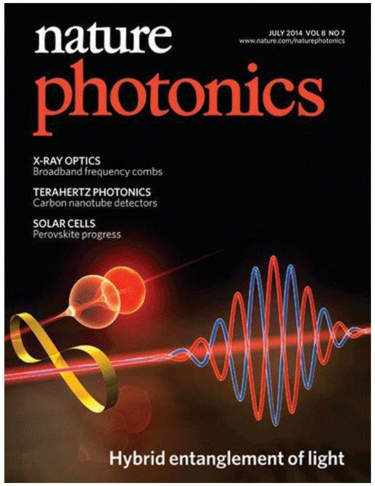
Immanual Bloch Lab @ Max Planck Institute of Quantum Optics

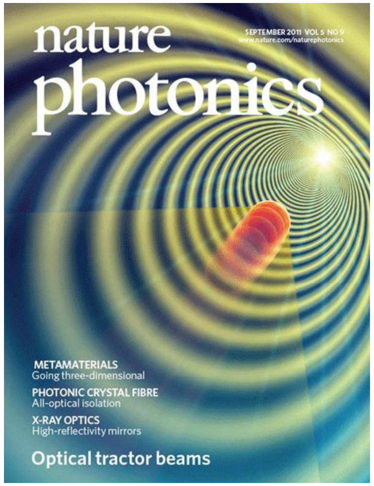
Current Optics Challenges Sampler Pack

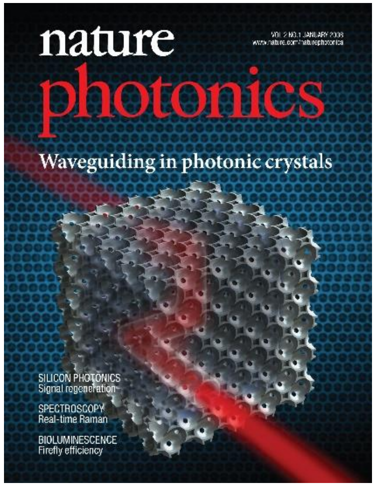
10 nm EUV lithography!

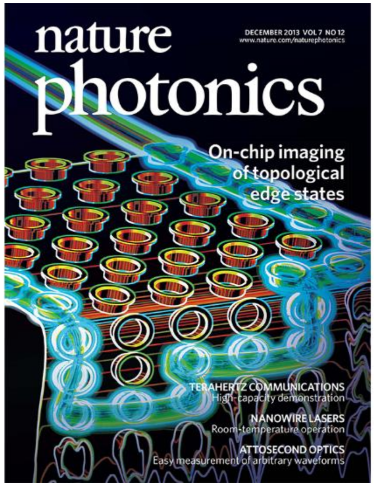


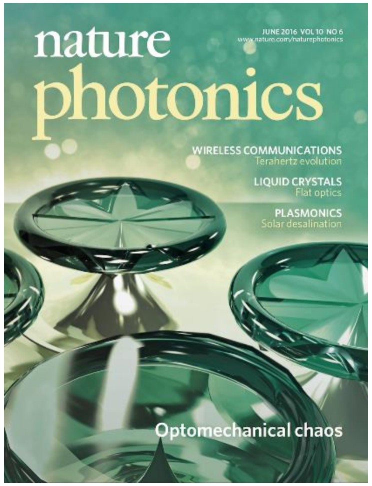
Current Optics Challenges Sampler Pack

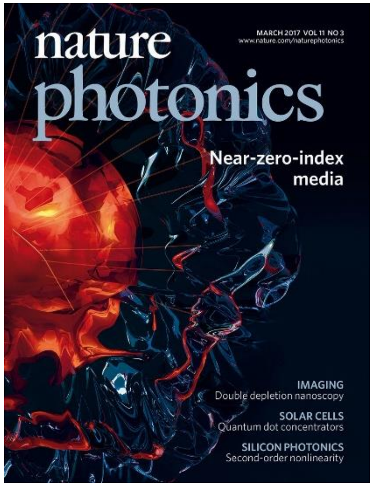


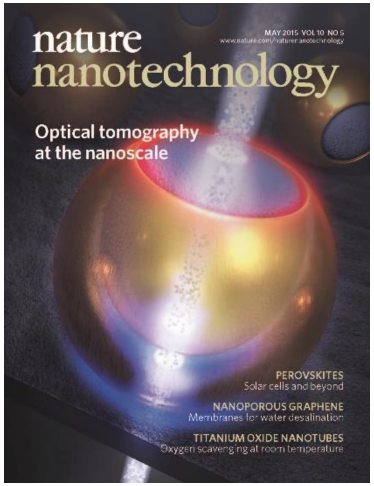


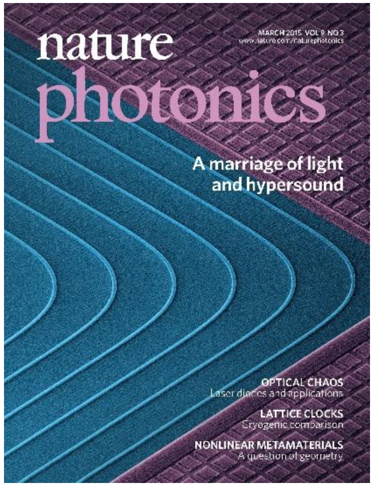












There are still plenty of epic optics challenges out there