1) $y = \sqrt{x\sqrt{x\sqrt{x}}}$

$\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$ $\left[\frac{d\sqrt{x}}{dx} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \right.$

$\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x\sqrt{x\sqrt{x}}}} \cdot \frac{d}{dx}\left(x+\sqrt{x\sqrt{x}}\right)$

$= \frac{1}{2\sqrt{x+\sqrt{x\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x\sqrt{x}}} \cdot \frac{d}{dx}\left(x+\sqrt{x}\right)\right]$

$= \frac{1}{2\sqrt{x+\sqrt{x}+\sqrt{x}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \frac{d}{dx}(x)\right]$

$= \frac{1}{2\sqrt{x+x+x}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)\right.$

$= \frac{1+2\sqrt{x}+1}{2\sqrt{x}} = \frac{1+2\sqrt{x}+}{4\sqrt{x+\sqrt{x}+\sqrt{x}}}$

Dhruti

1cont

$$\frac{4\sqrt{x}\sqrt{x+\sqrt{x}} + 2\sqrt{x}+1}{8\sqrt{x}\cdot\sqrt{x+\sqrt{x}}\cdot\sqrt{x+\sqrt{x+\sqrt{x}}}}$$

answer: $\boxed{\dfrac{4\sqrt{x}\sqrt{x+\sqrt{x}} + 2\sqrt{x}+1}{8\sqrt{x}\cdot\sqrt{x+\sqrt{x}}\cdot\sqrt{x+\sqrt{x+\sqrt{x}}}}}$

that was confusing, almost lost my method there

2) $f(x) = \int_{\sin x}^{1} \sqrt{1-T^2}\,dT$

$\cancel{f(x)}$

$f(x) \int_{p(x)}^{q(x)} h(t)\,dt$ Derivative

$f'(x) = h(q(x))q'(x) - h(p(x)p'(x)))$

$p(x) = \sin x$, $q(x) = 1$

$p'(x) = \cos x$, $q'(x) = 0$ $h(t) = \sqrt{1-t^2}$

$f'(x) = h(1)(0) - h(\sin x)\cos x$

$= -\cos x \, h(\sin x)$

$f'(x) = \cos x \sqrt{1-\sin^2 x}$

$f'(x) = -\cos x \sqrt{1-\sin^2 x}$

hope It right.

3)

$$y = \frac{1}{x^2-9}$$

$x^2 - 9 = 0$ $x = 3$, $x = -3$

Domain = $x < -3$ or $-3 < x < 3$ or $x > 3$

Range:

$$\frac{1}{x^2-9}(x^2-9) = (x^2-9)$$

$yx^2 - 9y - 1$

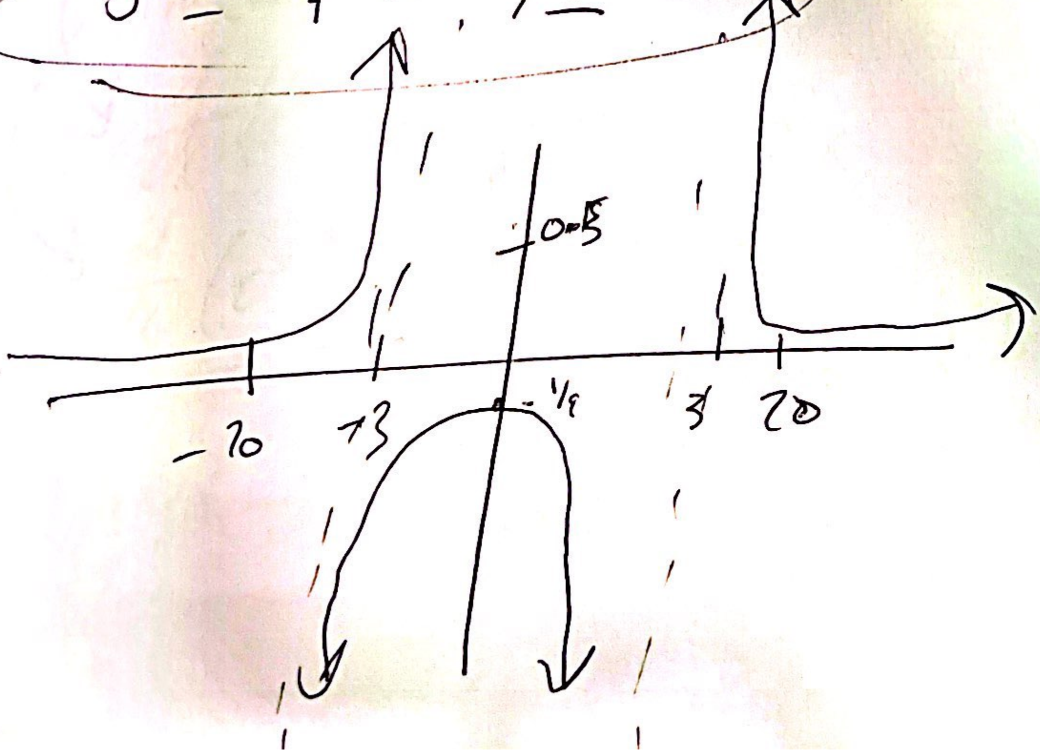
$1 = x^2 - 9$ $\quad 1 = x^2y - 9y$

quadratic

$0 \geq 4y(-9y-1) \to 36y^2 + 4y$

$36y^2 + 4y \geq 0 = 9y^2 + y \geq 0 = y(9y+1) \geq 0$

$y \leq -\frac{1}{9}$ or $y \geq 0$



4)

$$\lim_{x \to \infty} \frac{x^2}{e^x + 1}$$

$$\frac{x^2}{e^x+1} \to \frac{\frac{x^2}{e^x}}{\frac{e^x}{e^x} + \frac{1}{e^x}} \to \frac{\frac{x^2}{e^x}}{1 + \frac{1}{e^x}}$$

$$\lim_{x \to \infty} \left(\frac{x^2}{e^x}\right)$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{e^x}\right)$$

L'hosp

$$\lim_{x \to \infty} (x^2) = \infty$$

$$\lim_{x \to \infty} (e^x) = \infty$$

Power rule

$$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$= \lim_{x \to \infty} \left(\frac{2x}{e^x}\right) \to$$ another L'hosp

$2x = \infty$

$e^x = \infty$ → next

4 Cont.

$\frac{d}{dx}(2x)$

$2\frac{d}{dx}(x) = 2 \cdot 1 = 2$

$\frac{d}{dx}(e^x) = \cancel{\ } e^x = \lim_{x \to \infty} \frac{2}{e^x}$

$2 \cdot \lim_{x \to \infty} \left(\frac{1}{e^x}\right) =$

$\lim_{x \to \infty} (1) \to = 1$

common limit infinity prop

$\lim_{x \to \infty} (e^x) \to 2 \cdot \frac{1}{\infty} = \frac{1}{\infty} = 0$

$2 \cdot 0 = 0$ next limit: $\lim_{x \to \infty} \left(1 + \frac{1}{e^x}\right)$

$\lim_{x \to \infty} (1) + \lim_{x \to \infty} \left(\frac{1}{e^x}\right)$

$\lim_{x \to \infty} (1) = 1$

$\lim_{x \to \infty} (1) = 1$

$\lim_{x \to \infty} \left(\frac{1}{e^x}\right) \to \lim_{x \to \infty} (e^x) = \frac{1}{\infty} = 0$

$1 + 0 = 1$ $\frac{0}{1} = 0$

answer to all is

0

5)

eliminate absolutes

$\int_{-3}^{1} |1-x|\,dx + \int^{2} |-1+x|\,dx$

$= \int_{-3}^{1} (1-x)\,dx + \int_{1}^{2} (-1+x)\,dx$

$= \int_{-3}^{1} (1-x)\,dx = \int_{-3}^{1} 1-x\,dx = \int_{-3}^{1} 1\,dx - \int_{-3}^{1} x\,dx$

sum rule

$\int_{-3}^{1} 1\,dx = [1 \cdot x]_{-3}^{1} = [x]_{-3}^{1}$

boundries

$\lim x \to -3 + (x) = -3$

$\to 1-(x) = 1 \to 1-(-3) = 4$

$x \to -\int_{1}^{2} 1\,dx + \int_{1}^{2} x\,dx$

integral of constant

$x \to [1 \cdot x]_{1}^{2} \pm [x]_{1}^{2}$

5 COVET

$\lim x \to 1^+ (x) = 1$

$\lim x \to 2^- (x) = 2$

$= 1$

$\int_1^2 x\,dx \xrightarrow{\text{Power rule}} \left[\frac{x^{1+1}}{1+1}\right]_1^2 \to \left[\frac{x^2}{2}\right]_1^2$

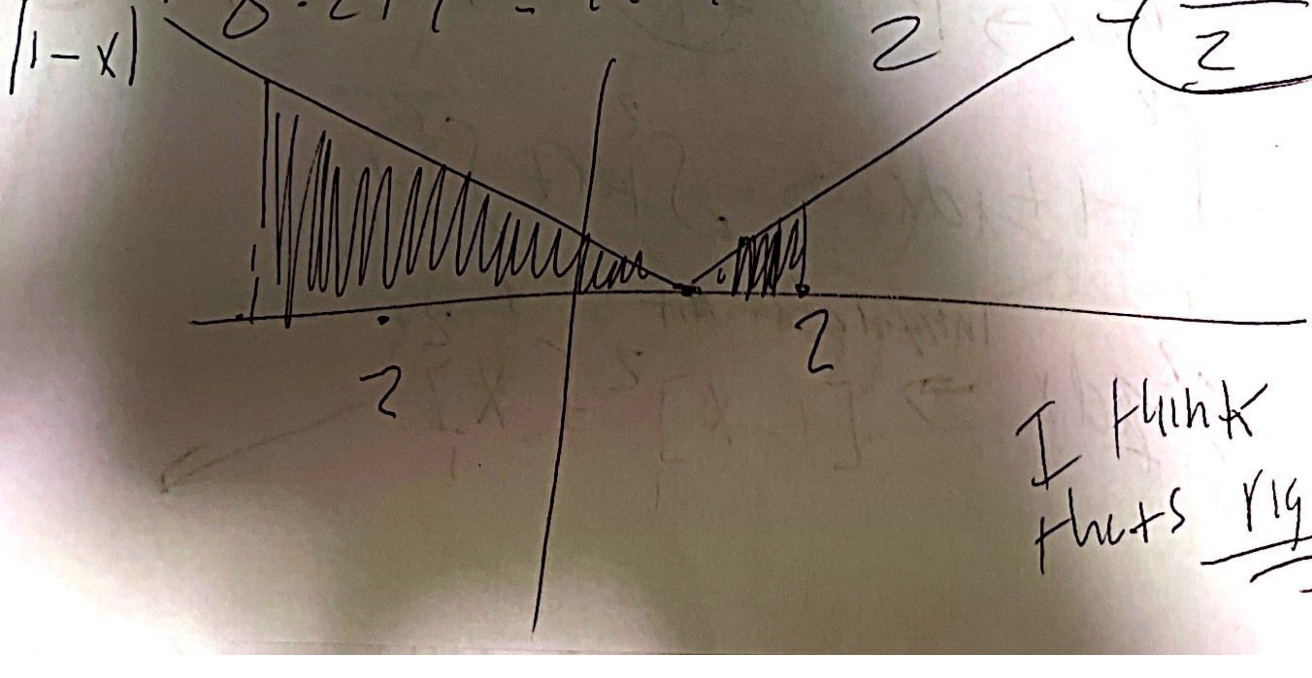
$\lim x \to 1^+ \left(\frac{x^2}{2}\right) = \frac{1^2}{2} = \frac{1}{2}$

$\lim x \to 2^- \left(\frac{x^2}{2}\right) = \frac{2^2}{2} = 2$

$2 - \frac{1}{2} = \frac{3}{2}$

$-1 + \frac{3}{2} = \frac{1}{2} \to 8 + \frac{1}{2} = \frac{8 \cdot 2}{2} + \frac{1}{2}$

$8 \cdot 2 + 1 = 16 + 1 = \frac{17}{2} = \frac{17}{2}$



I think thats right

(b) $x^2+4xy+y^2=13$

$$2x+4y+4x\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

Take 2 common

$$x+2y+2x\frac{dy}{dx}+y\frac{dy}{dx}=0$$

$$=\frac{2x\,dy}{dx}+y\frac{dy}{dx}=-(x+2y)$$

$$=\frac{dy}{dx}(2x+y)=-(x+2y)$$

$$=\frac{dy}{dx}=\frac{-(x+2y)}{(2x+y)}$$

$$=\frac{dy}{dx}=\frac{-(x+2y)}{(2x+y)}$$

this was not too bad!

y' or dx/dy

Scanned with CamScanner

(7) $y = \sqrt{x}$ Closet to $(2,0)$

1ST point on curve

$x = t^2, y = t$

Point is at (T^2, T) and $(2,0)$

$d = \sqrt{(t^2-2)^2 + (T-0)^2}$

$= d' = \dfrac{[2(T^2-2) \cdot 2t + 2T]}{2\sqrt{(T^2-2)^2 + T^2}}$

$= d' = 0$

$= 4t^3 - 8T + 2t = 0$

$= 2t(2T^2 - 3) = 0$

$T = 0$ $dt = \pm\sqrt{\dfrac{3}{2}}$

$d = \cancel{[4t^3 - 8T + 2T]}$

next

Scanned with CamScanner

$$d' = \frac{\left[4t^3 - 8t + 2t\right]}{2\sqrt{(t^2-2)^2 + t^2}}$$

$$d' = \frac{4t^3 - 6t}{2\sqrt{(t^2-2)^2 + t^2}}$$

$$d'' = \frac{1}{2}\left[\frac{\sqrt{(t^2-2)^2+t^2}\,(12t^2-1) - \frac{(4t^3-6t)}{2}\,\frac{(2(t^2-2))\cdot 2t + 2t}{}}{(t^2-2)^2+t^2}\right]$$

$$d'' = \frac{1}{2}\left[\frac{8(t^2-6)\left((t^2-2)^2+(t^2)\right) - (4t^3-6t)\left((t^2-2)2t+t\right)}{\left((t^2-2)^2+t^2\right)^{3/2}}\right]$$

$$d'' = \frac{1}{2}\left[\frac{(12t^2-6)\left((t^2-2)^2+t^2\right) - (4t^3-6t)(2t^3-3t)}{\left((t^2-2)^2+t^2\right)^{3/2}}\right]$$

next

~~Q~~. 7 cont..

now at $T=0$, $d''=0$ = inflection point

now at $T=\pm\sqrt{\frac{3}{2}}$ = $d''>0$

min is at $T=\pm\sqrt{\frac{3}{2}}$

So point $(x,y) = \left(\frac{3}{2}, \pm\sqrt{\frac{3}{2}}\right)$

that was some serious math, really great question, I think I got IT. I DID get lost a bit though?.

-thanks

...st point on curve closest to (?,0)

n...

7 cont...

now 8) $\int_{-\pi}^{\pi}\left(\frac{x}{1+18x^6}+\cos x\right)dx$

now

mi...

let $I_1=\int_{-\pi}^{\pi}\frac{x}{1+18x^6}dx$

$f(x)=\frac{x}{1+18x^6}$

$f(-x)=\frac{-x}{1+18x^6}=-f(x)$

if $\int_{-a}^{a}f(x)\,dx=0$ if f is odd $I_1=0$

$I_2=\int_{-\pi}^{\pi}\cos x\,dx$ $\cos(-x)=\cos x$

$=\int_{-a}^{a}f(x)dx=2\int_{0}^{a}f(x)\,dx$

$=2\int_{-\pi}^{\pi}\cos(x)dx=2(\sin x)_{0}^{\pi}$

$=2(0-0)=0$

I_2

$I_1+I_2=0$

$I=0$ $\int_{-\pi}^{\pi}\left(\frac{x}{1+18x^6}+\cos x\,dx\right)=0$

9. $\lim_{x \to \infty} \frac{\sin^2 x}{x^2+x+2}$

$$\frac{\frac{\sin^2(x)}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}} \Rightarrow \lim_{x\to\infty}\left(\frac{\frac{\sin^2(x)}{x^2}}{1+\frac{1}{x}+\frac{2}{x^2}}\right)$$

$$\frac{\lim_{x\to\infty}\left(\frac{\sin^2(x)}{x^2}\right)}{\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{2}{x^2}\right)}$$

Sqz theorem

$$\lim\left(\frac{\sin^2 x}{x^2}\right) = \lim_{x\to\infty}\left(\frac{0}{x^2}\right) = 0$$

$$\lim_{x\to\infty}\frac{1}{x^2} = 0$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{2}{x^2}\right) = 1$$

$$\lim_{x\to\infty} = (1) = 1$$

$$\lim_{x\to\infty}\frac{1}{x} = 0$$

$$\lim_{x\to\infty}\frac{2}{x^2} = 0$$

answer

$$\boxed{\frac{0}{1}}$$