

11

1)

$$\frac{d}{dx} \left(\frac{\sqrt{x}-1}{\sqrt{x+1}} \right)$$

Quotient rule

$$\frac{\frac{d}{dx} (\sqrt{x}-1) (\sqrt{x+1}) - \frac{d}{dx} (\sqrt{x+1}) (\sqrt{x}-1)}{(\sqrt{x+1})^2}$$

Sum/Diff

$$\frac{d}{dx} (\sqrt{x}-1) \rightarrow (\sqrt{x}) - (1) \rightarrow \frac{d}{dx} (\sqrt{x})$$

Power rule

$$\frac{d}{dx} (x^{\frac{1}{2}}) \rightarrow \frac{1}{2} x^{\frac{1}{2}-1} \rightarrow \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} (1) = 0 = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} (\sqrt{x+1}) = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}} (\sqrt{x+1}) - \frac{1}{2\sqrt{x}} (\sqrt{x}-1)$$

$$= \text{simplify} \Rightarrow \frac{\sqrt{x+1}}{2\sqrt{x}} - \frac{\sqrt{x}-1}{2\sqrt{x}}$$

$$\frac{\sqrt{x+1} - (\sqrt{x}-1)}{2\sqrt{x}}$$

$$\frac{2}{\sqrt{x}(\sqrt{x+1})^2}$$

answer

$$1B) \frac{d}{d\theta} \left(\frac{1 - \sec(\theta)}{\tan(\theta)} \right)$$

Quotient

$$\frac{\frac{d}{d\theta} (1 - \sec(\theta)) \tan(\theta) - \frac{d}{d\theta} \tan(\theta) (1 - \sec(\theta))}{(\tan(\theta))^2}$$

(1 - sec θ) sum diff

$$\frac{d}{d\theta} (1) - \frac{d}{d\theta} (\sec(\theta))$$

$$(1) = 0$$

$$\frac{d}{d\theta} (\sec(\theta)) = \text{Common Derivative}$$

$$(\sec(\theta) \tan(\theta))$$

$$0 - \sec(\theta) \tan(\theta)$$

$$= -\sec(\theta) \tan(\theta)$$

$$\frac{d}{d\theta} (\tan(\theta)) = \sec^2(\theta) = \frac{(-\sec(\theta) \tan(\theta)) \tan(\theta) - \sec^2(\theta) (1 - \sec(\theta))}{\tan^2(\theta)}$$

answer :

$$\frac{-\tan^2(\theta) \sec(\theta) - \sec^2(\theta) (1 - \sec(\theta))}{\tan^2 \theta}$$

simplify

$$2) \quad a) (f+g)'(3)$$

~~Answer~~

$$(f+g)'(x) = f'(x) + g'(x)$$

$$\text{Sub } x=3,$$

$$(f+g)'(3) = f'(3) + g'(3)$$

$$-6 + 5 = -1$$

$$\text{So } f+g'(3) = -1$$

$$b) (f-g)'(x) = f'(x) - g(x) - g'(x) - f(x)$$

$$\text{Sub } x=3$$

$$(-6) - 2 + 5.4$$

$$= -12 + 20$$

$$= 8 \text{ therefore } (f-g)'(3) \text{ is } \boxed{8}$$

2C)

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$$

$$\left(\frac{f}{g}\right)'(3) = \frac{g(3) \cdot f'(3) - f(3) \cdot g'(3)}{g(3)^2}$$

$$= \frac{2 \cdot (-6) - 4 \cdot 5}{2^2}$$

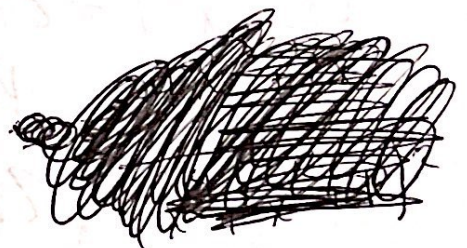
$$= \frac{-12 - 20}{4} = \frac{-32}{4} = -8$$

$$\left(\frac{f}{g}\right)'(3) = \boxed{-8}$$

3)

$$\frac{1}{x^4 + x^2 + 1}$$

$$\frac{d}{dx} \left(\frac{1}{x^4 + x^2 + 1} \right)$$



exp rule

$$\frac{d}{dx} \left((x^4 + x^2 + 1)^{-1} \right)$$

chain rule

$$\frac{d}{du} (u^{-1}) \frac{d}{dx} (x^4 + x^2 + 1) \quad \text{power rule}$$

$$\frac{d}{du} (u^{-1})$$

$$= -1 \cdot u^{-1-1} = -\frac{1}{u^2}$$

4)

$$= u^{-\frac{1}{2}} \frac{d}{dx} (x^4 + x^2 + 1)$$

Sub u

$$= \frac{1}{(x^4 + x^2 + 1)^{\frac{1}{2}}} \frac{d}{dx} (x^4 + x^2 + 1)$$

$$\frac{d}{dx} (x^4 + x^2 + 1) \quad \text{Sum/Rule}$$

$$\frac{d}{dx} (x^4) + \frac{d}{dx} (x^2) + \frac{d}{dx} (1)$$

$$4x^{4-1} \text{ simp} = 4x^3 \quad \text{Power Rule}$$

$$\frac{d}{dx} (x^2) = 2x$$

$$2x^{2-1} \text{ Power rule} = 2x$$

$$2x \frac{d}{dx} (1) = 0$$

$$= 4x^3 + 2x$$

$$= \frac{4x^3 + 2x}{(x^4 + x^2 + 1)^{\frac{1}{2}}}$$

multiply fac

$$= \frac{4x^3 + 2x}{(x^4 + x^2 + 1)^{\frac{1}{2}}}$$

Answer

4)

$$\frac{dy}{dx} = \frac{(x+1) \cdot 1 - (x-1) \cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Parallel to $x - 2y = 2$

$$\frac{2}{(x+1)^2} = \frac{1}{2} \Rightarrow x = -3, 1$$

$$y(-3) = \frac{-3-1}{-3+1} = \frac{-4}{-2} = 2$$

$$y(1) = \frac{1-1}{1+1} = 0$$

Equation of the Tangents

$$y-2 = \frac{1}{2}(x+3), \quad y-0 = \frac{1}{2}(x-1)$$

$$y-2 = \frac{1}{2}x + \frac{7}{2}, \quad y = \frac{1}{2}x - \frac{1}{2}$$

5)

$$H(\theta) = \theta$$

$$H'(\theta) = \frac{d}{d\theta} (\theta \cdot \sin \theta)$$

$$\theta \cdot \frac{d}{d\theta} (\sin \theta) + \sin \theta \cdot \frac{d}{d\theta} (\theta)$$

Prod Rule

$$H'(\theta) = \theta \cdot \cos \theta + \sin \theta$$

Then

$$H''(\theta) = \frac{d}{d\theta} (H'(\theta))$$

$$= \frac{d}{d\theta} (\theta \cdot (\cos \theta + \sin \theta))$$

$$= \frac{d}{d\theta} (\theta \cdot \cos \theta + \frac{d}{d\theta} (\sin \theta))$$

$$\downarrow \theta \frac{d}{d\theta} (\cos \theta) + \cos \theta \frac{d}{d\theta} (\theta) + (\cos \theta$$

$$H''(\theta) = -\theta \sin \theta + 2 \cos \theta$$

6) deriv of $f(x) = \sqrt{x}\sqrt{x}\sqrt{x}$

$$f'(x) = f'(x)$$

$$f'(x) = \sqrt{x}\sqrt{x}\sqrt{x} = f'(x)$$

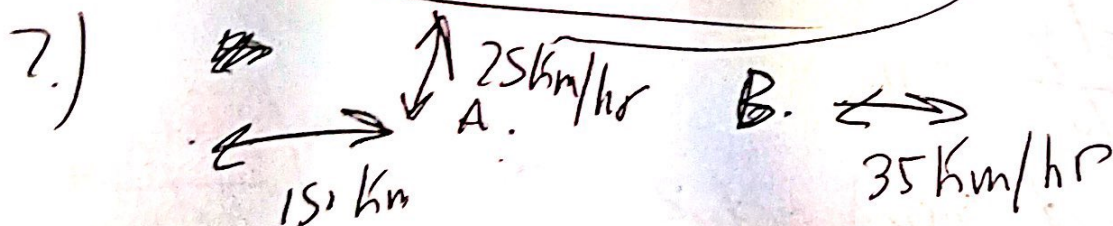
$$f'(x) = x \quad \text{so } f(x) = \int g(x) dx$$

$$f(x) = \int x dx$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} \quad a \neq -1$$

$$\frac{x^{1+1}}{1+1} = \frac{x^2}{2} + C$$

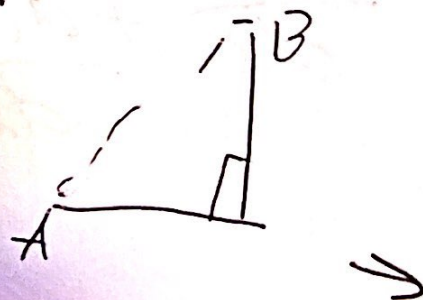
$$f(x) = \frac{x^2}{2} + C_1 \quad \text{I think}$$



4:00 pm

$$\text{dist A} = 35 \times 4 = 140 \text{ km}$$

$$\text{dist B} = 25 \times 4 = 100 \text{ km}$$



Let $AO = h$

Let $BO = y$

Distn = $D = \sqrt{x^2 + y^2}$

$$\frac{d(D)}{dt} = \frac{(x \frac{dx}{dt} + y \frac{dy}{dt})}{\sqrt{x^2 + y^2}}$$

So at 4 $x = 10$ $x = 100$ $\frac{dx}{dt} = 35$ $\frac{dy}{dt} = 25$

$$\frac{dD}{dt} = \frac{2 \times 10 \times 35 + 25 \times 25 \times 100}{2\sqrt{(10)^2 + (100)^2}} = \frac{700 + 5000}{200.99} = 28.3 \text{ km/h}$$

(w/c = $x^2 + 4xy + y^2 = 13$)

Slop = $\frac{dy}{dx} = m$

$$\frac{d(x^2)}{dx} + \frac{d(4xy)}{dx} + \frac{d(y^2)}{dx} = 0$$

$$\rightarrow 2x + 4\left(x \cdot \frac{dy}{dx} + y \frac{dx}{dx}\right) + \frac{dy^2}{dx} \cdot \frac{dy}{dx} = 0$$

8 (cont.)

$$2x + 4x \frac{dy}{dx} + 4y + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x+2y) = -(2x+4y)$$

$$\frac{-(2x+4y)}{2x+2y} \Rightarrow \frac{dy}{dx} = -\frac{(x+2y)}{x+y}$$

$$(2,1) \quad \frac{-2+2}{4+1} = -\frac{4}{5} \quad m = -\frac{4}{5}$$

Equation of tangent line at $(2,1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{4}{5}(x - 2)$$

$$y = \frac{-4x}{5} + \frac{8}{5} + 1$$

$$y = -\frac{4x}{5} + \frac{13}{5}$$

Answer