CMP

Composition of Movement Primitives

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Contents

1 ProMPs

1.1 Recap

- q_t: joint angle over time
- $\dot{q}_t$: joint velocity over time
- $\boldsymbol{\tau} = \{q_t\}_{t=0\ldots T}$: trajectory
- $\boldsymbol{w}$: weight vector of a single trajectory
- ϕ_t: basis function
- $\boldsymbol{\Phi}_t = [\phi_t, \dot{\phi}_t]$: $n \times 2$ dimensional time-dependent basis matrix
- $z(t)$: monotonically increasing phase variable
- $\boldsymbol{\epsilon}_y \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_y)$: zero-mean i.i.d. Gaussian noise

$$\boldsymbol{y}_t = \begin{bmatrix} q_t \\ \dot{q}_t \end{bmatrix} = \boldsymbol{\Phi}_t^\top \boldsymbol{w} + \boldsymbol{\epsilon}_y \tag{1}$$

$$p(\boldsymbol{\tau}|\boldsymbol{w}) = \prod_t \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Phi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \tag{2}$$

$$p(\boldsymbol{\tau};\boldsymbol{\theta}) = \int p(\boldsymbol{\tau}|\boldsymbol{w}) \cdot p(\boldsymbol{w};\boldsymbol{\theta}) d\boldsymbol{w} \tag{3}$$

Equation 3 is illustrated in Figure 1.

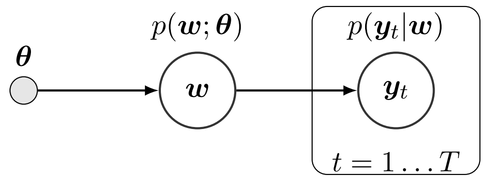

Figure 1: Hierarchical Bayesian model used in ProMPs.

1.2 Coupling between joints

$$p(\boldsymbol{y}_t|\boldsymbol{w}) = \mathcal{N}\left(\begin{bmatrix} \boldsymbol{y}_{1,t} \\ \vdots \\ \boldsymbol{y}_{d,t} \end{bmatrix} \middle| \begin{bmatrix} \boldsymbol{\Phi}_t^\top & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{\Phi}_t^\top \end{bmatrix} \boldsymbol{w}, \boldsymbol{\Sigma}_y \right) = \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \tag{4}$$

with:

- $\boldsymbol{w} = [\boldsymbol{w}_1^\top, \dots, \boldsymbol{w}_n^\top]^\top$: combined weight vector
- $\boldsymbol{\Phi}_t$: block-diagonal basis matrix containing the basis functions and their derivatives for each dimension
- $\boldsymbol{y}_{i,t} = [q_{i,t}, \dot{q}_{i,t}]^\top$: joint angle and velocity for the i^{th} joint

$$p(\boldsymbol{y}_t;\boldsymbol{\theta}) = \int \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \cdot p(\boldsymbol{w};\boldsymbol{\theta}) \tag{5}$$

$$= \int \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \cdot \mathcal{N}\Big(\boldsymbol{w}|\boldsymbol{\mu}_{\boldsymbol{w}}, \boldsymbol{\Sigma}_{\boldsymbol{w}}\Big) d\boldsymbol{w} \tag{6}$$

$$\dots \text{ToDo expand} \dots \tag{7}$$

$$= \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t^\top \boldsymbol{\mu}_{\boldsymbol{w}}, \boldsymbol{\Psi}_t^\top \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t + \boldsymbol{\Sigma}_y\Big) \tag{8}$$

1.3 Via-Points Modulation

- $\boldsymbol{x}_t^\star = [\boldsymbol{y}_t^\star, \boldsymbol{\Sigma}_t^\star]$: desired observation
- $\boldsymbol{y}_t^\star$: desired position and velocity vector at time t
- $\boldsymbol{\Sigma}_t^\star$: accuracy of the desired observation

$$p(\boldsymbol{w}|\boldsymbol{x}_t^\star) \propto \mathcal{N}\Big(\boldsymbol{y}_t^\star|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_t^\star\Big) \cdot p(\boldsymbol{w}) \tag{9}$$

$$\text{...ToDo expand...} \tag{10}$$

$$\boldsymbol{\mu}_{\boldsymbol{w}}^{[new]} = \boldsymbol{\mu}_{\boldsymbol{w}} + \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t \Big(\boldsymbol{\Sigma}_y^\star \boldsymbol{\Psi}_t^\top \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t\Big)^{-1} (\boldsymbol{y}_t^\star - \boldsymbol{\Psi}_t^\top \boldsymbol{\mu}_{\boldsymbol{w}}) \tag{11}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{w}}^{[new]} = \boldsymbol{\Sigma}_{\boldsymbol{w}} - \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t \Big(\boldsymbol{\Sigma}_y^\star \boldsymbol{\Psi}_t^\top \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t\Big)^{-1} \boldsymbol{\Psi}_t^\top \boldsymbol{\Sigma}_{\boldsymbol{w}} \tag{12}$$

2 Gaussian Mixture Models recap

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \sum_{k=1}^{K} \pi_k p_k(\boldsymbol{x}) \tag{13}$$

- p_k: kth mixture component
- π_k: mixture weights with $0 \leq \pi_k \leq 1$ and $\sum_{k=1}^{K} \pi_k = 1$

$$p(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \tag{14}$$

3 Composition of MPs

From Figure 2, we can compose a set of MPs by "stitching" them using the following mechanisms:

1. choosing appropriate via-points between each MP. One strategy is to define the via-point as the average between the ending MP and the starting one.
2. generating the MP using Equations (9), (11) and (12), from $z(t_{ini}) = \frac{i}{k}$ to $z(t_{end}) = \frac{i+1}{k}$.

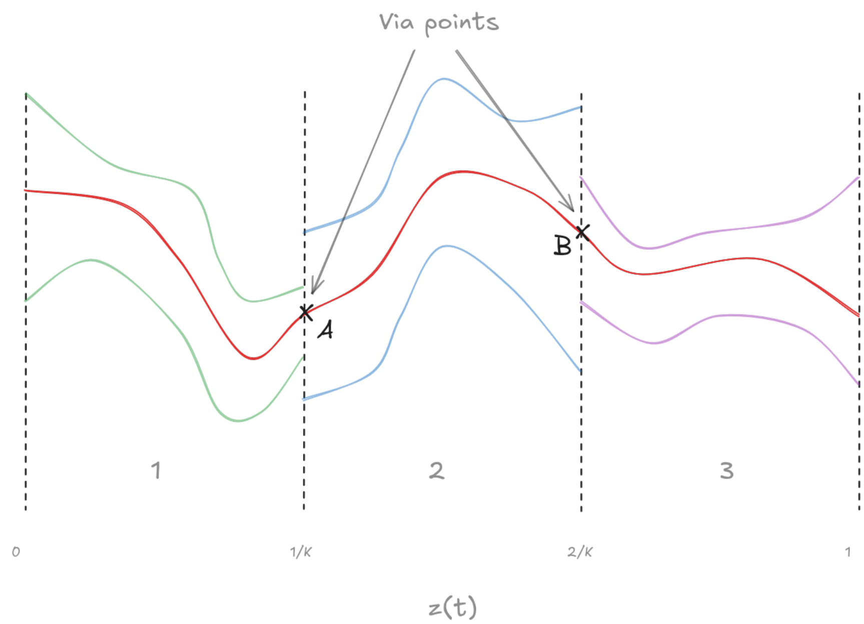

Figure 2: How to compose several ProMPs. Here $K = 3$, and $y_A = \frac{\mu_{1,t_A}+\mu_{2,t_A}}{2}$; $\quad y_B = \frac{\mu_{2,t_B}+\mu_{3,t_B}}{2}$.

Proposal on how to choose the via-points:

$$y_k = \frac{\mu_{k,t_{end}} + \mu_{k+1,t_{ini}}}{2}$$

$$\boldsymbol{y}_k = [\mu_{x_k}, \mu_{y_k}, \mu_{z_k}, \dots]^\top$$

$$\boldsymbol{Y}_k = \begin{bmatrix} \mu_{x_1} & \mu_{y_1} & \mu_{z_1} & \cdots \\ \vdots & \ddots & & \vdots \\ \mu_{x_k} & \mu_{y_k} & \mu_{z_k} & \cdots \end{bmatrix}$$

A comparison of features available in MPs frameworks is available in Table 1.

	DMP	ProMP	GMM	KMP	**CMP**
Probabilistic	-	✓	✓	✓	✓
Via-point	-	✓	-	✓	✓
End-point	✓	✓	-	✓	✓
Extrapolation	✓	-	-	✓	Check KMP
High-dimensional inputs	-	-	✓	✓	Check KMP
Composition	-	-	-	-	✓

Table 1: Comparison of features available for each Movements Primitives frameworks.