Composition of Movement Primitives

May 15, 2025

Contents

1 ProMPs

1.1 Recap

From (Paraschos et al., 2013, 2018):

- q_t: joint angle over time
- $\dot{q}_t$: joint velocity over time
- $\boldsymbol{\tau} = \{q_t\}_{t=0\dots T}$: trajectory
- $\boldsymbol{w}$: weight vector of a single trajectory
- ϕ_t: basis function
- $\boldsymbol{\Phi}_t = [\phi_t, \dot{\phi}_t]$: $n \times 2$ dimensional time-dependent basis matrix
- $z(t)$: monotonically increasing phase variable
- $\boldsymbol{\epsilon}_y \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_y)$: zero-mean i.i.d. Gaussian noise

$$\boldsymbol{y}_t = \begin{bmatrix} q_t \\ \dot{q}_t \end{bmatrix} = \boldsymbol{\Phi}_t^\top \boldsymbol{w} + \boldsymbol{\epsilon}_y \tag{1}$$

$$p(\boldsymbol{\tau}|\boldsymbol{w}) = \prod_t \mathcal{N}\Big(\boldsymbol{y}_t | \boldsymbol{\Phi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \tag{2}$$

$$p(\boldsymbol{\tau}; \boldsymbol{\theta}) = \int p(\boldsymbol{\tau}|\boldsymbol{w}) \cdot p(\boldsymbol{w}; \boldsymbol{\theta}) d\boldsymbol{w} \tag{3}$$

1.2 Coupling between joints

$$p(\boldsymbol{y}_t|\boldsymbol{w}) = \mathcal{N}\left(\begin{bmatrix} \boldsymbol{y}_{1,t} \\ \vdots \\ \boldsymbol{y}_{d,t} \end{bmatrix} \middle| \begin{bmatrix} \boldsymbol{\Phi}_t^\top & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \boldsymbol{\Phi}_t^\top \end{bmatrix} \boldsymbol{w}, \boldsymbol{\Sigma}_y \right) = \mathcal{N}\Big(\boldsymbol{y}_t | \boldsymbol{\Psi}_t \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \tag{4}$$

with:

- $\boldsymbol{w} = [\boldsymbol{w}_1^\top, \dots, \boldsymbol{w}_n^\top]^\top$: combined weight vector
- $\boldsymbol{\Phi}_t$: block-diagonal basis matrix containing the basis functions and their derivatives for each dimension
- $\boldsymbol{y}_{i,t} = [q_{i,t}, \dot{q}_{i,t}]^\top$: joint angle and velocity for the i^{th} joint

1.3 Hierarchical Bayesian Model

The Hierarchical Bayesian Model used in ProMPs is illustrated in Fig. 1.

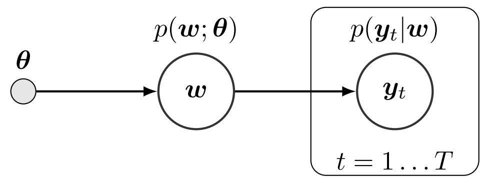

Figure 1: Hierarchical Bayesian Model used in ProMPs.

- $\boldsymbol{\theta} = \{\boldsymbol{\mu}_w, \boldsymbol{\Sigma}_w\}$
- $p(\boldsymbol{w};\boldsymbol{\theta}) = \mathcal{N}(\boldsymbol{w}|\boldsymbol{\mu}_w, \boldsymbol{\Sigma}_w)$: prior over the weight vector $\boldsymbol{w}$, with parameters $\boldsymbol{\theta}$, assumed to be Gaussian

$$
\begin{aligned}
p(\boldsymbol{y}_t;\boldsymbol{\theta}) &= \int \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \cdot p(\boldsymbol{w};\boldsymbol{\theta})\; d\boldsymbol{w} && (5)\\
&= \int \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \cdot \mathcal{N}\Big(\boldsymbol{w}|\boldsymbol{\mu}_{\boldsymbol{w}}, \boldsymbol{\Sigma}_{\boldsymbol{w}}\Big)\; d\boldsymbol{w} && (6)\\
&= \int \mathcal{N}\Big(\boldsymbol{w}|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_{\boldsymbol{w}} + \boldsymbol{\Sigma}_y\Big)\; d\boldsymbol{w} \quad \text{(product of Gaussian densities)} && (7)\\
&\dots \text{Missing development} \dots && (8)\\
&= \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t^\top \boldsymbol{\mu}_{\boldsymbol{w}}, \boldsymbol{\Psi}_t^\top \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t + \boldsymbol{\Sigma}_y\Big) && (9)
\end{aligned}
$$

1.4 Via-Points Modulation

- $\boldsymbol{x}_t^\star = [\boldsymbol{y}_t^\star, \boldsymbol{\Sigma}_t^\star]$: desired observation
- $\boldsymbol{y}_t^\star$: desired position and velocity vector at time t
- $\boldsymbol{\Sigma}_t^\star$: accuracy of the desired observation

Using Bayes rule:

$$
\begin{aligned}
p(\boldsymbol{w}|\boldsymbol{x}_t^\star) &= \frac{p(\boldsymbol{x}_t^\star|\boldsymbol{w}) \cdot p(\boldsymbol{w})}{p(\boldsymbol{x}_t^\star)} && (10)\\
p(\boldsymbol{w}|\boldsymbol{x}_t^\star) &\propto \mathcal{N}\Big(\boldsymbol{y}_t^\star|\boldsymbol{\Psi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_t^\star\Big) \cdot p(\boldsymbol{w}) && (11)
\end{aligned}
$$

From (Deisenroth et al., 2020; Soch et al., 2025; Bishop and Bishop, 2023), the parameters of a conditional multivariate Gaussian $p(\boldsymbol{x}|\boldsymbol{y}) = \mathcal{N}\big(\boldsymbol{\mu}_{x|y}, \boldsymbol{\Sigma}_{x|y}\big)$ are the following:

$$
\begin{aligned}
\boldsymbol{\mu}_{x|y} &= \boldsymbol{\mu}_x + \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}(\boldsymbol{y} - \boldsymbol{\mu}_y) && (12)\\
\boldsymbol{\Sigma}_{x|y} &= \boldsymbol{\Sigma}_{xx} - \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx} && (13)
\end{aligned}
$$

$$\text{... ToDo expand ...} \tag{14}$$

$$\boldsymbol{\mu}_{\boldsymbol{w}}^{[new]} = \boldsymbol{\mu}_{\boldsymbol{w}} + \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t \left(\boldsymbol{\Sigma}_y^{\star} \boldsymbol{\Psi}_t^{\top} \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t \right)^{-1} (\boldsymbol{y}_t^{\star} - \boldsymbol{\Psi}_t^{\top} \boldsymbol{\mu}_{\boldsymbol{w}}) \tag{15}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{w}}^{[new]} = \boldsymbol{\Sigma}_{\boldsymbol{w}} - \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t \left(\boldsymbol{\Sigma}_y^{\star} \boldsymbol{\Psi}_t^{\top} \boldsymbol{\Sigma}_{\boldsymbol{w}} \boldsymbol{\Psi}_t \right)^{-1} \boldsymbol{\Psi}_t^{\top} \boldsymbol{\Sigma}_{\boldsymbol{w}} \tag{16}$$

2 Composition of MPs

ToDo

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