Composition of Movement Primitives

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Contents

1 ProMPs

1.1 Recap

From [1, 2]:

- q_t: joint angle over time
- $\dot{q}_t$: joint velocity over time
- $\boldsymbol{\tau} = \{q_t\}_{t=0\ldots T}$: trajectory
- $\boldsymbol{w}$: weight vector of a single trajectory
- ϕ_t: basis function
- $\boldsymbol{\Phi}_t = [\phi_t, \dot{\phi_t}]$: $n \times 2$ dimensional time-dependent basis matrix
- $z(t)$: monotonically increasing phase variable
- $\boldsymbol{\epsilon}_y \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_y)$: zero-mean i.i.d. Gaussian noise
- $p(\boldsymbol{w}; \boldsymbol{\theta})$: prior over the weight vector $\boldsymbol{w}$, with parameters $\boldsymbol{\theta}$, assumed to be Gaussian

$$\boldsymbol{y}_t = \begin{bmatrix} q_t \\ \dot{q}_t \end{bmatrix} = \boldsymbol{\Phi}_t^\top \boldsymbol{w} + \boldsymbol{\epsilon}_y \tag{1}$$

$$p(\boldsymbol{\tau}|\boldsymbol{w}) = \prod_t \mathcal{N}\Big(\boldsymbol{y}_t | \boldsymbol{\Phi}_t^\top \boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \tag{2}$$

$$p(\boldsymbol{\tau}; \boldsymbol{\theta}) = \int p(\boldsymbol{\tau}|\boldsymbol{w}) \cdot p(\boldsymbol{w}; \boldsymbol{\theta}) d\boldsymbol{w} \tag{3}$$

Eq. (3) is illustrated in Fig. 1.

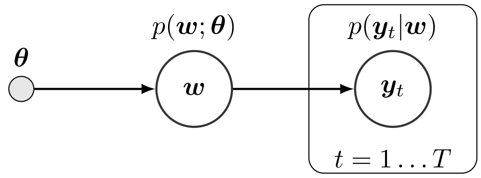

Figure 1: Hierarchical Bayesian model used in ProMPs.

1.2 Coupling between joints

$$p(\boldsymbol{y}_t|\boldsymbol{w}) = \mathcal{N}\left(\begin{bmatrix} \boldsymbol{y}_{1,t} \\ \vdots \\ \boldsymbol{y}_{d,t} \end{bmatrix} \middle| \begin{bmatrix} \boldsymbol{\Phi}_t^\top & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \boldsymbol{\Phi}_t^\top \end{bmatrix} \boldsymbol{w}, \boldsymbol{\Sigma}_y \right) = \mathcal{N}\Big(\boldsymbol{y}_t|\boldsymbol{\Psi}_t\boldsymbol{w}, \boldsymbol{\Sigma}_y\Big) \tag{4}$$

with:

- $\boldsymbol{w} = [\boldsymbol{w}_1^\top, \dots, \boldsymbol{w}_n^\top]^\top$: combined weight vector
- $\boldsymbol{\Phi}_t$: block-diagonal basis matrix containing the basis functions and their derivatives for each dimension
- $\boldsymbol{y}_{i,t} = [q_{i,t}, \dot{q}_{i,t}]^\top$: joint angle and velocity for the i^{th} joint

1.3 Via-Points Modulation

- $\boldsymbol{x}_t^\star = [\boldsymbol{y}_t^\star, \boldsymbol{\Sigma}_t^\star]$: desired observation
- $\boldsymbol{y}_t^\star$: desired position and velocity vector at time t
- $\boldsymbol{\Sigma}_t^\star$: accuracy of the desired observation

Using Bayes rule: $P(A|B) = \frac{p(B|A) \cdot P(A)}{P(B)}$

$$p(\boldsymbol{w}|\boldsymbol{x}_t^\star) = \frac{p(\boldsymbol{x}_t^\star|\boldsymbol{w}) \cdot p(\boldsymbol{w})}{p(\boldsymbol{x}_t^\star)} \tag{5}$$

$$p(\boldsymbol{w}|\boldsymbol{x}_t^\star) \propto \mathcal{N}\Big(\boldsymbol{y}_t^\star|\boldsymbol{\Psi}_t^\top\boldsymbol{w}, \boldsymbol{\Sigma}_t^\star\Big) \cdot p(\boldsymbol{w}) \tag{6}$$

$$\dots \text{ToDo expand} \dots \tag{7}$$

$$\boldsymbol{\mu}_{\boldsymbol{w}}^{[new]} = \boldsymbol{\mu}_{\boldsymbol{w}} + \boldsymbol{\Sigma}_{\boldsymbol{w}}\boldsymbol{\Psi}_t\Big(\boldsymbol{\Sigma}_y^\star\boldsymbol{\Psi}_t^\top\boldsymbol{\Sigma}_{\boldsymbol{w}}\boldsymbol{\Psi}_t\Big)^{-1}(\boldsymbol{y}_t^\star - \boldsymbol{\Psi}_t^\top\boldsymbol{\mu}_{\boldsymbol{w}}) \tag{8}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{w}}^{[new]} = \boldsymbol{\Sigma}_{\boldsymbol{w}} - \boldsymbol{\Sigma}_{\boldsymbol{w}}\boldsymbol{\Psi}_t\Big(\boldsymbol{\Sigma}_y^\star\boldsymbol{\Psi}_t^\top\boldsymbol{\Sigma}_{\boldsymbol{w}}\boldsymbol{\Psi}_t\Big)^{-1}\boldsymbol{\Psi}_t^\top\boldsymbol{\Sigma}_{\boldsymbol{w}} \tag{9}$$

2 Composition of MPs

ToDo

References

[1] A. Paraschos, C. Daniel, J. R. Peters, and G. Neumann, "Probabilistic Movement Primitives," in *Advances in Neural Information Processing Systems*, vol. 26, Curran Associates, Inc., 2013.

[2] A. Paraschos, C. Daniel, J. Peters, and G. Neumann, "Using probabilistic movement primitives in robotics," *Autonomous Robots*, vol. 42, pp. 529–551, Mar. 2018.